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Price Formation and Order Placement Strategies in a Dynamic Order Driven Market[*]

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Abstract

This paper provides an auction theoretic analysis of price formation and optimal trading strategies in a dynamic order-driven market. Investors can choose to post either limit orders or to submit market orders. Limit orders provide better prices but run execution risk. The execution probability of a limit order is endogenous and depends on the order placement strategies followed by the other traders. As a result the order placement strategies of all the traders are interdependent. Solving for the equilibrium of this game, the properties of the order placement strategies and the transaction prices are derived. No order submission strategy is strictly dominant. The traders with a high willingness to trade use market orders while the traders with a low willingness to trade use limit orders. Bids and asks prices are shown to be closely related to the composition of the order flow. Some testable implications are proposed.

1. Introduction.

Several securities markets (such as the Paris Bourse, the Tokyo Stock Exchange or the Toronto Stock Exchange) are organized as continuous order-driven markets. In these markets, public customers can continuously either post limit orders or submit market orders[1]. Limit orders are stored in a limit order book, waiting for future execution according to price priority. This execution is triggered by incoming market orders which are executed upon arrival against the best quote on the opposite side of the book. With a market order, a trader is executed with certainty while accepting the available quoted price. With a limit order, a trader can determine his execution price while running uncertainty on his execution. The goal of this paper is to analyze the double auction mechanism which is at work in a continuous order-driven market[2].

A limit order trader plays the same role as a market-maker in a dealership market[3]. He supplies immediacy to market order traders. However, in contrast with a dealership market, there is no obligation for a trader to quote bid and ask prices in an order-driven market. An investor can choose to trade with market orders (demand liquidity) or with limit orders (supply liquidity). The viability of a limit order market depends crucially on this choice since it depends on the existence of limit orders which provide liquidity to market orders. In this paper, a modeling of the equilibrium process between the demand and supply of liquidity in an order-driven market is proposed. In particular, the choice between limit and market order strategies is analyzed. In the model, the traders arrive sequentially in the market and make their decisions according to the state of the limit order book and their expectations on the trading strategies of the future traders. Consequently, as in real world

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[1] A limit order specifies a limit price and a quantity. For a buy limit order, the limit price is the maximum price that a buyer will pay and for a sell limit order, the limit price is the minimum price that a seller will obtain. Market orders are orders to buy or sell a given quantity at any price.

[2] Friedman (1993) defines the basic continuous double auction in the following way : "It is a continuous-time, two-sided auction in which messages (i.e., traders' actions) consist of bids and asks for single units of the goods, and of acceptances of the current best bid and ask." Order-driven markets are a variant of the basic continuous double auction.

[3] In a dealership market, market-makers quote continuously bid and ask prices at which they stand ready to execute public customers' orders. Public customers are not allowed to quote prices. Numerous papers have studied price formation in dealership markets (see for example Stoll (1978) or Glosten and Milgrom (1985)). In these papers, the choice between market and limit orders is not analyzed. The traders who are allowed to post limit orders (the market-makers) are determined exogenously. See Pagano and Roell (1990), (1992) for a discussion about the differences between order-driven and dealership markets. See Cohen, Kalman, Maier, Schwartz and Whitcomb (1986) for more institutional details about the structure of financial Exchanges.

markets, the equilibrium order placement strategies followed by the traders are the product of a dynamic interactive process.

Another important feature of order-driven markets is that the execution price of a limit order trader is in general equal to the quote he has offered [4]. Consequently, in this kind of market structure, there is a strong incentive for the buyers to underbid and for the sellers to overbid in order to gain a surplus from trade. In fact the buyers and the sellers, as in a bargaining situation, choose their quotes in order to get as much as possible of the gains from trade. In this study, the bidding strategies of the limit order traders are characterized and the determinants of the market power of each side of the market are identified.

In summary, this paper aims to provide a simple model of dynamic trading in an order-driven market in which both the optimal order submission strategy and price formation can be analyzed. Several questions are addressed. Is there a dominant strategy for all the traders? Who are the traders who post limit orders and those who submit market orders? What are the determinants of the quotes posted by the agents? Why is there a spread between bid and ask prices? Do the strategies of the traders depend on the state of the book?

The present paper is not the first attempt to model price formation and order placement decision in order-driven markets. Cohen, Maier, Schwartz and Withcomb (1981), Bronfman and Schwartz (1992), Angel (1992) and Friedman (1992) analyze the order placement strategies in order-driven markets. However, in these models, the distribution of prices and the execution probability of an order are exogenous. Consequently, in these models, the problem of the equilibrium prices which result from the order placement strategies of the agents is not adressed.

Rock (1990) and Glosten (1992) study price formation in a limit order market in presence of asymmetric information. They assume that limit orders traders are uninformed while market order traders are informed. Hence, limit order prices reflect the adverse selection risk run by limit order traders. The optimal choice between market orders and limit orders is not analyzed since informed traders are

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[4] It is not the case for limit orders with a price that meets or exceeds the counterpart quotes. These limit orders are excuted at the best quotes as market orders and are called marketable limit orders.

restricted to use market orders and uninformed traders are restricted to use limit orders.

Kumar and Seppi (1993) analyze the optimal choice between limit and market orders. In their model, the execution probability of a limit order is endogenous, depending upon the order placement strategies followed by all the traders. As a result, the order placement strategies of all the traders are dependent and the traders are in a game situation. Solving for the Nash equilibrium of this game, they derive the properties of the limit order book in equilibrium. However, in this model, the dynamics of the choice between market and limit orders is not studied[5].

The distinctive feature of the present model is that both the order placement strategies and the transaction prices are endogenous and derived in a dynamic framework, without adverse selection risk.

Price formation is modelled as a game between sellers and buyers. The traders are either sellers who have one unit to sell or buyers who seek to buy one unit. They differ by the values they assign to the asset. The agents arrive sequentially in the market. Upon arrival, a trader observes the state of the limit order book in which the previous limit orders are stored. Then he has to choose either to submit a market order and trade at the best quote or to post a limit order. With a market order, an agent trades with certainty but does not get a good execution price (for example, a buyer is executed at the ask). In order to obtain a better execution price, he can decide to post a limit order. However in that case he bears the risk of nonexecution. Finally a trader chooses to submit a market order if the gain he can obtain by hitting the quote is greater than the expected gain he can obtain by posting a limit order. Now, the expected gain of a limit order trader depends on his execution probability. This execution probability is endogenous in the model. For a buyer (a seller), it depends on the market order strategies followed by the sellers (buyers). Consequently, the bids and the asks quoted by all the traders and the prices for which they decide to submit a market order are interdependent and must be determined simultaneously. Solving for the equilibrium of this game, the properties of the transaction prices and the bidding strategies of the traders are derived.

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[5] Beyond this theoretical literature, there are some empirical analyzes of order-driven markets. See for example Biais, Hillion and Spatt (1992), Harris and Hasbrouck (1992), Handa and Schwartz (1992) and Hamon et al. (1993).

The description of the model shows that it is related to the literature on double auctions[6]. Following this literature, I describe a trading process in which both buyers and sellers can post prices. I assume that traders are risk-neutral with unit demands and private valuations as well. However, this literature has mainly focused on one shot double auctions, in which the traders have to quote bids and asks simultaneously. On the contrary, I analyse a dynamic trading process in which the traders arrive sequentially. As a consequence, the trading strategies are more complex, since the traders can either quote prices or trade at the quotes. Two main assumptions are necessary for tractability. First, it is assumed that a limit order is valid for only one period and that a trader leaves the market without gains if his limit order is not executed. Second, the private valuations of the buyers (sellers) are assumed to be drawn from a two-point distribution. The model is highly stylized but it provides some insights on the properties of the order placement strategies in an order-driven market. The main results of the paper are the following :

○ _Order Placement Strategies._ Three equilibria are derived. The trading strategies and the transactions prices are different for each equilibrium. However, in each case, both market and limit orders are used and there is no strategy (market or limit order) which is strictly dominant for all the traders. The traders who are relatively more willing to trade (the buyers with high private valuations and the sellers with low private valuations) are those who use market orders while the traders who are relatively less willing to trade are more likely to post limit orders. Because of strategic bidding, buyers (sellers) always quote lower bids (higher asks) than the maximum (minimum) prices at which they are willing to trade immediately by submitting market orders. Actually traders are ready to pay a premium when they demand immediacy and require a premium when they supply immediacy.

○ _Gains from Trade_ . The bids and the asks quoted by the buyers and the sellers determine the splitting of the gains from trade between the traders. The higher (lower) the ask (the bid) of a seller (buyer), the higher his gain in case of execution but the lower his probability of execution. In equilibrium, the bidding strategies of the buyers and the sellers depend on the composition of the order flow. The buyers and the sellers post higher (lower) quotes when the number of buy orders is high (low) relative to the number of sell orders. This result is explained by the competition over time which occurs in a dynamic market. Intuitively, when there are more buyers than sellers, the incentive for a seller to reject a low bid is increased because the probability of trading with another buyer at a better price is

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[6] See for example Satterthwaite and Williams (1991) or McAfee (1992).

greater, ceteris paribus. Consequently, the buyers must increase their bids in order to have a chance to be hit by a sell market order.

• *Transaction Prices.* By assumption, all the buyers have higher private valuations than the sellers. However, each kind of trader wishes to capture as much as possible of the gains from trade. As a result, the ask prices required by the sellers are higher than the bid prices required by the buyers. Hence, in equilibrium, the market exhibits a spread which is due to the incentive to overbid for the sellers and to underbid for the buyers. Due to the competition over time, the size and the location of this spread depend on the proportion of buyers relative to sellers.

• *Empirical Implications.* Testable implications of the model are the following. The spread must be lower when the imbalance between buy and sell orders is high, ceteris paribus. Moreover, a larger spread is associated with a lower frequency of transactions. On the other hand, it turns out that the value of a limit order strategy relative to the value of a market order strategy depends on the state of the book. For this reason, the order flow depends on the quotes. In particular, the probability of observing a buy (sell) limit order is higher when the ask (bid) price is high (low) than when the ask (bid) price is low (high). These results on the order flow are consistent with the empirical findings in Biais, Hillion and Spatt (1993), Harris and Hasbrouk (1992) and Handa and Schwartz (1992).

The paper is organized as follows. The model is spelled out in Section 2. In Section 3, the equilibrium of the trading game is defined and some general results on the properties of the bidding strategies are offered. In Section 4, the different equilibria of the game are investigated. For each equilibrium, the bidding strategies are characterized and the properties of the equilibrium are discussed. Section 5 concludes.

2. A model of trading in an order-driven market.

2.1 The trading institution.

A market for a financial asset is considered. The traders arrive sequentially to the market at times $t = 0, 1, ... + \infty$. There is only one arrival at each point in time. A trader can be either a buyer with

probability k, or a seller with probability $(1 - k)$. k is a measure of the imbalance between supply and demand. For instance, when the number of buyers is high relative to the number of seller, k is high. The sellers and the buyers differ with respect to their private valuation of the asset. There are two types of buyers and two types of sellers. With probability π, a buyer has a private valuation v_h for the asset and with probability $(1-\pi)$, his private valuation is v_l. In the same way, a seller assigns to the asset a value c_h with probability $(1 - \pi)$ or a value c_l with probability π. I assume that $c_l < c_h < v_l < v_h$. Thus the private valuations are chosen so that there will be always gains from trade between a seller and a buyer, regardless of their private valuations[7]. Moreover, it is assumed that $(v_h - v_l) = (c_h - c_l)$ [8].

No rationale is given here for the differences in private valuations. In a more elaborate framework, these differences could be explained either by differences in endowments or by differences of opinion on the expected values of the asset (see Harris and Raviv (1993)). As the parameters $\{k, \pi, v_h, v_l, c_h, c_l\}$ remain constant over time, the market is assumed to be in steady state. The question of how such a steady state can be obtained is not addressed here.

When an agent arrives in the market at time t, he observes the state of the limit order book. In particular he observes the best bid and the best ask, denoted $B^m(t)$ and $A^m(t)$. If an ask (bid) price is available when he arrives, a buyer (a seller) can either submit a market order for one unit of the asset or post a limit order for one unit. If there is no offer in the book when he arrives, a trader has to quote a price. A market order is immediately executed at the best counterpart price in the market. A limit order can be executed only against a market order. As it will become clear further, it runs a risk of non-execution which is endogenous in the model. When he has transacted, a trader must leave the market and if he decides to post a limit order, a trader is no longer allowed to use a market order.

Two kind of competition are likely to occur in this dynamic setting. For example, consider the buy side of the market. Two buyers at the same time in the market will be in a direct competition for the

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[7] Without changing the main insights of the model, I could have assumed $c_l < v_l < c_h < v_h$. However this configuration is less interesting because in this case there is a natural room for a spread since the high valuation sellers do not want to sell for less than c_h and the low valuation buyers do not want to buy for more than v_l.

[8] This assumption and the assumption that the proportion π of high valuation buyers is equal to the proportion of low valuation sellers are only for the sake of simplicity. They could be relaxed without altering the results qualitatively.

sell orders. In that case they are likely to act as Bertrand competitors. Now consider the following sequence of arrivals : a buyer who posts a limit order, a seller and a buyer. In that case the second seller can choose to trade with the first buyer by submitting a market order or with the second buyer by posting a limit order. The first buyer is therefore in an indirect competition with the second buyer. In order to simplify the analysis of the competitive interactions, it is assumed that a limit order is valid for only one period. This assumption implies that two buy (sell) limit orders cannot coexist at the same time in the limit order book. Consequently, there is no direct competition between the traders in the model and the attention is focused on the indirect competition. Without this assumption, the analysis of the competitive interactions between the traders becomes rapidly very complex and is left for future research.

[INSERT FIG.1 HERE]

The trading structure is depicted in Figure 1. At each date, Nature randomly selects whether the trader arriving on the market is a seller or a buyer and his private valuation according to the probabilistic structure described previously. Then the trader observes the state of the book and chooses his trading strategy accordingly. At the next date, the trader selection process is repeated in the same way[9].

2.2 Preferences.

All agents are assumed to be expected utility maximisers and risk neutral. The utility of purchasing the asset at price P for a buyer is given by :

$$U_j(v_j, P) = (v_j - P) \quad j = \{h, l\}.$$

Similarly for a seller, the utility of selling the asset at price P is :

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[9]This sequential trading process bears some similarities with the trading process which is analyzed in Glosten and Milgrom (1985). However in their model, the traders have to trade with market orders while here they can choose between market and limit orders. Another major difference stems from the absence of adverse selection risk in my model.

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$$U_j(c_j, P) = (P - c_j) \quad j = \{h, l\}.$$

The reservation utility of all the agents if they do not trade is normalized to zero. Moreover, all agents have a common discount factor δ which is assumed to be equal to 1. Therefore, the utility of a transaction for a trader is independent of the date of completion of this transaction. This assumption could be relaxed without changing qualitatively the results. However, with a discount factor less than one, the propensity to place a market order would be increased. For a given price, the traders with types v_h and c_l obtain higher utility than the traders with types v_l or c_h. Consequently they are more willing to trade and I will sometimes refer to these traders as the traders with a high willingness to trade.

3. The equilibrium.

The goal of this section is to give a formal definition of the equilibrium. First the strategies of the agents are described. Second the strategic interactions between the buyers and the sellers are discussed. Finally, the equilibrium definition is given.

3.1 Strategies.

[INSERT FIG.2 HERE]

When a buyer arrives in the market at time t, he can decide to trade immediately at the ask price $A^m(t)$ (if this quote is available) with a market order, or to post a limit order (See Figure 2). Consequently, his strategy must specify for which states of the book he decides to submit a market order and at which price he posts a limit order. The following proposition characterizes the decision to submit a market order for a buyer.

Proposition 1 : *The decision to submit a buy market order can be described by a cut-off rule : a buyer, with private valuation v, arriving at time t, decides to trade immediately at the best ask if the best ask is less than or equal to a given price determined by the buyer. This price is called the buyer's*

cut-off price and is denoted $B_0(v,t)$.

Proof : Assume that the buyer who arrives at time t, with private valuation v, quotes a bid $B(v,t)$ if he chooses to post a limit order. With that bidding strategy, he gets an expected utility denoted $E(U(v, B(v,t)))$. He will decide to submit a market order if the utility he can obtain by trading immediately at the best ask is greater than the expected utility he can get with a limit order:

$$v - A^m(t) \geq E(U(v, B(v,t))),$$

or

$$A^m(t) \leq (v - E(U(v, B(v,t)))).$$

Let $B_0(v,t) = (v - E(U(v, B(v,t))))$, the last inequality is equivalent to : $A^m(t) \leq B_0(v,t)$. Finally, the buyer submits a market order if $A^m(t) \leq B_0(v,t)$. Otherwise he posts a limit order with a price equal to $B(v,t)$.

Q.E.D

Proposition 1 shows that a buyer 's order placement strategy is characterized by his bid $B(v,t)$ and his corresponding cut-off price. Hence let $\Sigma(v,t) = (B_0(v,t), B(v,t))$ be the order placement strategy of a type-v buyer at time t. Analogously, $\Phi(v,t) = (A_0(c,t), A(c,t))$ describes the order placement strategy of a seller with a private valuation c at time t. The seller will submit a market order if $B^m(t) \geq A_0(c,t)$. Otherwise, he will quote an ask $A(c,t)$. The interpretation of these results is that a trader compares the value of a market order strategy and the expected value of a limit order strategy and decides to choose the strategy with the highest value.

I have allowed the strategies to depend on the time of arrival of a trader. However in the following, the analysis is restricted to stationary strategies.

Definition 1 :

- *A stationary strategy for a buyer with private valuation v is a function $\Sigma(.,.)$ such that $\Sigma(v,t) = \Sigma(v,t')$ $\forall t$ $\forall t' \in \{1, 2..., +\infty\}$.*

o *A stationary strategy for a seller with private valuation c is a function $\Phi(.,.)$ such that $\Phi(c,t) = \Phi(c,t')$ $\forall t$ $\forall t' \in \{1,2,...+\infty\}$.*

A stationary strategy does not depend on the time of arrival of an agent in the market. Consequently, the subscript t in the strategies of the agents is suppressed. I focus on stationary strategies because 1) the probabilities k, π and the private valuations of the traders do not change over time and 2) the horizon of the model is infinite. Consequently, two traders who have the same private valuation and who differ only by their arrival time face exactly the same problem. It is therefore natural to assume that they will use the same strategy.

3.2 Execution risk.

If a trader chooses to post a limit order, he runs an execution risk because of the uncertainty on the actual execution of his order. For example, if a buyer chooses to quote a price which is inferior to the cut-off prices of all the sellers, his probability of execution is zero. Let $\Gamma(B)$ be the execution probability of a buy limit order with price B and $\Psi(A)$ as the execution probability of a sell limit order with price A. These probabilities characterize the execution risk of a given limit order. The lower $\Gamma(B)$, the higher the execution risk of a buy limit order with price B. With these definitions :

$$E(U(v_j, B)) = \Gamma(B).(v_j - B) \quad for \quad j = \{h, l\}.$$

$$E(U(c_j, A)) = \Psi(A).(A - c_j) \quad for \quad j = \{h, l\}.$$

$E(U(v, B))$ is the expected utility of a buyer with a private valuation v if he quotes a bid equal to B. Similarly, $E(U(c, A))$ is the expected utility for seller if he decides to quote an ask equal to A.

The limit order price chosen by a trader will affect the probability of his trade and the price he will pay. He faces a trade-off between a good execution price and a low execution risk[10]. For example, if a buyer quotes a low bid, he gets a high utility if he trades but he has a low probability to trade. In

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[10]This is similar to the trade-off faced by the bidders in a one shot double auction.

order to determine his optimal limit order price, a trader must compute the execution probabilities $\Gamma(.)$ and $\Psi(.)$.

These probabilities depend on the market order strategies followed by the future traders since the execution of a limit order is triggered by the submission of a market order. Hence they are endogenous and the traders deduce these probabilities of their knowledge of the other traders strategies[11]. Consider the problem of a buyer who has to choose a bid. Assume that the cut-off price of a low valuation seller is lower than the cut-off price of a high valuation seller (this will be the case in equilibrium, see Proposition 2). If the buyer quotes a bid which is greater or equal to the cut-off price of a high valuation seller, then his limit order will be executed if the next trader is a seller, regardless of the private valuation of this seller. Hence in this case, the execution probability of his limit order is equal to the probability that the next trader will be a seller : $\Gamma(B) = (1 - k)$. If he posts a bid higher than the cut-off price of a low valuation seller but lower than the cut-off price of a high valuation seller, then his limit order is executed only if the next trader is a low valuation seller and in this case his execution probability is $\Gamma(B) = (1 - k)\pi$. Finally, if the buyer posts a very low bid, inferior to the cut-off prices of all the sellers, his execution probability is zero.

Because the execution probability of a limit order is endogenous, the optimal order placement strategy of a trader depends on the strategies followed by the other traders. Therefore the traders are in a game situation. In the following, the equilibrium of this game is defined.

3.3 Equilibrium definition.

Definition 2 : *A stationary market equilibrium is a set of strategies* $(\Sigma^*(v_j))_{\{j=h,l\}}$ *for the buyers and* $(\Phi^*(c_j))_{\{j=h,l\}}$ *for the sellers such that for all* $j \in \{h, l\}$ *and for all* $t \in \{1, 2, ... + \infty\}$ *:*

a)

$$B^*(v_j) \in Arg \max_{B} E(U(v_j, B)) \qquad given \qquad (\Sigma^*(v_i), \Phi^*(c_i))_{\{i=h,l\}} \qquad (1),$$

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[11]In Cohen, Maier, Shwartz and Withcomb (1981), Bronfman and Schwartz (1991), Angel (1992) or Friedman (1992), these probabilities are exogenous. In this case, a trader faces a problem which is very similar to a search problem.

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and

$$B_0^*(v_j) = v_j - E(U(v_j, B^*(v_j)))$$ (2).

b)

$$A^*(c_j) \in Arg \max_A E(U(c_j, A)) \quad given \quad (\Sigma^*(v_i), \Phi^*(c_i))_{\{i=h,l\}}$$ (3),

and

$$A_0^*(c_j) = c_j + E(U(c_j, A^*(c_j)))$$ (4).

Conditions a) and b) require that the strategies of all the traders be optimal given the strategies of the other traders. The conditions a.2) and b.4) determine the optimal market order strategies of the agents, as explained in Proposition 1. They require that a trader submit a market order if the utility he can obtain by trading at the best quote in the market is greater or equal to the expected utility he can get by optimally posting a limit order. The quote chosen by a buyer (seller) depends on the market order strategies of the future sellers (buyers) (conditions a.1) and b.3)). Consequently, the market order and limit order placement strategies of the traders are linked and must be determined simultaneously. Finally, note that the equilibrium has the flavor of a Nash perfect equilibrium. Actually, the stationarity insures that a trader's strategy is optimal even when he observes prices ($B^m(t)$ or $A^m(t)$) which should not be observed if the players follow their equilibrium strategies.

The following proposition gives some general properties of the trading strategies.

Proposition 2 :

The equilibrium strategies satisfy the following relationships for all $j \in \{h, l\}$:

a) The price at which a buyer (a seller) is willing to buy (sell) immediately by submitting a market order is always smaller (greater) than or equal to his private valuation.

b) The bid (ask) posted by a buyer (a seller) is always smaller (greater) than or equal to the price at which he is willing to buy (sell) immediately by submitting a market order.

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c) The price at which a high valuation buyer (low valuation seller) is willing to buy (sell) immediately by submitting a market order is always greater (smaller) than or equal to the price at which a low valuation buyer (high valuation seller) is willing to buy immediately.

(See appendix.)

The first part of Proposition 2 implies that a buyer or a seller may choose to bypass the opportunity to trade at a favorable price and obtain a surplus from trade. This result is due to the option for an agent to try for a better execution by posting a limit order. The value that a trader gives to the asset when he arrives in the market ($A_0^*(.)$ or $B_0^*(.)$ in the model) takes into account the value of this option. For this reason, the price at which he is ready to trade immediately may differ from his private valuation and may be more related to his expectations concerning his future trading opportunities[12]. For example, by definition of the equilibrium, the value assigned by a seller to the asset when he arrives in the market is equal to his private valuation plus his expected gain from trade if he posts a limit order ($A_0^*(c) = c + E(U(c, A^*))$). This last term measures the value of the option to try for a better execution.

The second part of Proposition 1 states that there is a positive difference between the price at which a buyer is willing to trade immediately and his bid. This difference reflects the price markup required by a buyer to supply immediacy to the sell market orders. This price markup compensates the trader for the execution risk that he runs when he chooses to post a limit order. By the same token, there is a positive difference between the ask quoted by a seller and his cut-off price. It will be shown in the next section that this price markup and the value of the option to delay a trade depend on the composition of the order flow.

The buyers with high valuations are more willing to trade because they benefit more from a trade ceteris paribus. Consequently they are less willing to run an execution risk and they are ready to pay higher prices than the low valuation buyers in order to trade with certainty. For this reason, their cut-off price is higher than it is for the buyers with low valuation. A similar intuition explains that the cut-off price of the low valuation sellers is lower than the cut-off price of the high valuation sellers.

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[12]This result is common in models of dynamic trading. See for example the extensive literature on search behaviors or models of speculative behaviors such as Biais and Bossaerts (1993).

An implication of this result is that the market orders are more likely to be used by traders with a high willingness to trade.[13]

<u>Corollary 1</u> :

In equilibrium, the execution probability of a buy limit order, $\Gamma(.)$ is increasing with its bid while the excution probability of a sell limit order $\Psi(.)$ is decreasing with its ask.

Proof :

This result is a direct consequence of part c) in Proposition 2.

An empirical implication of Corollary 1 is that the order flow is dependent on the state of the book. In particular, the frequency of new sell limit orders when the best bid is low should be higher than or equal the frequency of new sell limit orders when the best bid is high. A symmetric observation should be obtained for the frequency of buy limit orders. These results are consistent with the empirical findings of Biais, Hillion and Spatt (1993) that investors tend to post limit orders when the spread is large instead of hitting the quotes.

In the market structure considered here, the buyers have an incentive to underbid and the sellers to overbid in order to obtain a good execution. The buyers (the sellers) try in fact to reap as much as possible of the gains from trade by quoting low (high) prices. This strategic misrepresentation of their real private valuations could lead to inefficiency because some trades which could occur, do not necessarily occur. The goal of the next section is to study how the gains from trade are shared in equilibrium and to analyze to which extent the strategic behavior of the traders results in inefficiency.

4. An analysis of the equilibria of the trading process.

In this section, the stationary equilibria of the trading process are derived and their properties are

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[13]See Proposition 3, Proposition 4 and Proposition 5.

investigated.

4.1 Construction of equilibria.

<u>Optimal Bids and Asks.</u>

Lemma 1 : *For a buyer (a seller), the optimal bidding strategy is to quote a price equal to the cut-off price of a seller (buyer). As a consequence, the bids and the asks in the market can only be :*

$$B^*(v) = A_0^*(c_l) \quad or \quad B^*(v) = A_0^*(c_h),$$

$$A^*(c) = B_0^*(v_l) \quad or \quad A^*(c) = B_0^*(v_h).$$

Proof : Consider the case of a buyer. The execution probability of a bid greater than or equal to the cut-off price of a high valuation seller is equal to $(1-k)$. The execution probability of a bid lower than the cut-off price of a high valuation seller but greater than or equal to the cut-off price of a low valuation seller is equal to $(1-k)\pi$. Now, note that for a given execution probability, it is optimal for the buyer to choose the lowest bid. Consequently, it is optimal for a buyer to quote a bid equal either to the cut-off price of a high valuation seller or to the cut-off price of a low valuation seller. Q.E.D

A similar argument shows that there are only two possible optimal asks for a seller. A seller will choose to quote an ask equal either to the cut-off price of a low valuation buyer or an ask equal to the cut-off price of a high valuation buyer. In the first case, his ask is lower than in the second case and consequently the seller obtains a better execution probability. In the following, I say that a trader runs a low (high) execution risk if he decides to post a limit order with a high (low) execution probability.

<u>Existence Conditions.</u>

Using Lemma 1, three equilibria can be investigated. In the first equilibrium, all the traders choose to run a low execution risk, whatever their private valuations. In this case, the expected gain of a

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buyer if he posts a limit order is : $E(U(v, A_0^*(c_h))) = (1 - k)(v - A_0^*(c_h))$. But the buyer can run the chance of a better execution by quoting a bid equal to the cut-off price of a low valuation seller. In this case, his expected gain is : $E(U(v, A_0^*(c_l))) = (1 - k)\pi(v - A_0^*(c_l))$. So in the first equilibrium, the following condition must be satisfied for all the buyers :

$$E(U(v, A_0^*(c_h))) > E(U(v, A_0^*(c_l))),$$

or

$$(v - A_0^*(c_h)) > \pi.(v - A_0^*(c_l)) \qquad I.C(1).$$

A similar argument shows that for all the sellers, the following inequality must be satisfied in equilibrium :

$$(B_0^*(v_l) - c) > \pi.(B_0^*(v_h) - c) \qquad I.C(2).$$

In contrast with the first equilibrium, in the second equilibrium, all the traders choose to run a high execution risk. The bids of the buyers are equal to the cut-off prices of the low valuation sellers and the asks of the sellers are equal to the cut-off prices of the high valuation buyers. The existence conditions are in this case :

$$(v - A_0^*(c_h)) \le \pi.(v - A_0^*(c_l)) \qquad I.C'(1),$$
$$(B_0^*(v_l) - c) \le \pi.(B_0^*(v_h) - c) \qquad I.C'(2).$$

In the last equilibrium, only the traders who are the less willing to trade decide to run a high execution risk, while the traders who are the more willing to trade decide to run a low execution risk. In this case, buyers (sellers) with different valuations will quote different bids (asks). Consequently, the existence conditions are different for each type of trader. A high valuation buyer quotes a bid equal to the cut-off price of a high valuation seller in this equilibrium. Hence the existence condition I.C(1) must be satisfied for a high valuation buyer. On the contrary, a low valuation buyer quotes a bid equal to the cut-off price of a low valuation seller. Hence the constraint that must be satisfied for a low valuation buyer is the condition I.C'(1). Similarly, the condition I.C(2) must be satisfied for a low valuation seller and the condition I.C'(2) must be satisfied for a high valuation seller.

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The following table summarizes the bidding strategies of the agents in each equilibrium. L denotes a bidding strategy with a low execution risk while H denotes a bidding strategy with a high execution risk.

	c_l	c_h	v_l	v_h
equilibrium 1	L	L	L	L
equilibrium 2	H	H	H	H
equilibrium 3	L	H	H	L

Table 1 : Execution risk in each equilibrium.

In these three equilibria, the traders with the same willingness to trade choose strategies with the same execution risk. For example, in the first equilibrium, the traders with a low willingness to trade choose to quote prices with a high execution probability while in the two other equilibria they quote prices with a low execution probability. In this sense, these equilibria are symmetric. The existence of nonsymmetric equilibria is discussed at the end of section 4.2.3.

4.2 Equilibria.

4.2.1 Equilibrium 1.

<u>Proposition 3</u> :

if $(v_l - c_h) > (1 - k.(1 - k)).\left(\frac{\pi}{(1-\pi)}\right).(c_h - c_l)$ then an equilibrium in which all the traders choose to run a low execution risk exists. In equilibrium, the traders always submit a market order if there is a quote available. Otherwise the buyers quote a bid equal to :

$$B^* = B^*(v_h) = B^*(v_l) = k\lambda(k).v_l + (1 - k\lambda(k)).c_h.$$

Similarly, the sellers quote an ask equal to

$$A^* = A^*(c_h) = A^*(c_l) = \lambda(k).v_l + (1 - \lambda(k)).c_h,$$

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with $\quad \lambda(k) = \frac{k}{1-k.(1-k)}$.

(I will refer to this equilibrium as Equilibrium 1.)

Using the definition of the equilibrium, it is easy to derive the cut-off prices which characterize the market order strategies of the traders. In particular, one can check that $B_0^*(v_l) = A^*$ and that $A_0^*(c_h) = B^*$. As stated in the proposition, it means that in equilibrium each trader chooses to run a low execution risk when he posts a limit order. Note that in this case, high willingness traders strictly prefer market order strategies while low willingness traders are <u>indifferent</u> between the market and limit orders strategies.

The existence condition states that the equilibrium described in Proposition 3 exists if 1) the difference between the private valuations of a high valuation seller (buyer) and a low valuation seller (buyer) is low enough and if 2) the proportion π of the traders with a high willingness to trade is low. The interpretation of these conditions is the following. Each buyer can choose to quote a high bid or a low bid. With a low bid, a buyer obtains more surplus in case of execution but runs a higher execution risk. Hence, a buyer has no incentive to quote a low bid if 1) the increase in surplus compared to the surplus he can get with a high bid is low (which is the case if $(c_h - c_l)$ is small) and if 2) a low bid has a very low execution probability (which is the case if π is low). A similar argument explains that π and $(v_h - v_l)$ must be small to induce the sellers to quote a low ask in equilibrium.

Gains from trade and competition over time.

The gains from trade obtained by the traders depend on the prices they quote. The lower (higher) the bid (ask) quoted by a buyer (a seller), the higher the surplus he can extract from the sellers (the buyers). The function $\lambda(.)$ drives the splitting of the gains from trade. $\lambda(.)$ is increasing in k. Consequently the bids B^* quoted by the buyers and their cut-off prices B_0^* are increasing in k. In the same way, the asks A^* and the cut-off prices A_0^* of the sellers are increasing in k. Moreover we have :

$$\lim_{k \to 1} B^* = \lim_{k \to 1} A^* = v_l,$$

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and

$$\lim_{k \to 0} B^* = \lim_{k \to 0} A^* = c_h.$$

Finally, the higher (lower) the proportion k of buyers in the market, the larger (smaller) are the gains from trade obtained by the sellers. The following intuition is offered for this result. The sellers benefit from an increase in the proportion of buyers for two complementary reasons. First, ceteris paribus, a seller runs a lower execution risk when he posts a limit order. Hence, his incentive to reject a bid and to quote an ask is increased. This behavior forces a buyer to increase his bid in order to give a sufficient incentive to a seller to hit his quote by submitting a sell market order. This result is due to the fact that the option to try for a better execution by posting limit orders creates competition over time between the buyers[14]. On the other hand, the execution risk of a given bid increases when the proportion of buyers increases. Therefore, a buyer is more willing to trade immediately and is ready to pay higher prices to avoid execution risk. In equilibrium, it allows the sellers to increase their ask prices.

Transaction prices.

All the buy market orders are executed at the ask price A^* and all the sell market orders are executed at the bid price B^*.

$$S_1 = A^* - B^* = \frac{k.(1-k).(v_l - c_h)}{1 - k.(1-k)} \geq 0.$$

Consequently all the buy market orders are executed at a price higher than the sell market orders. Hence $(A^* - B^*)$ can be interpreted as the spread paid by the traders who submit market orders and who benefit from the liquidity offered by the limit orders. In this model, there is no cost of trading (transaction costs, inventory costs or adverse selection costs) which could explain the spread. Actually if each trader chooses to quote prices equal to their private valuations, the difference between the ask and the bid price would be negative because $(v_l - c_h) > 0$ by assumption. The spread appears here for two complementary reasons. On the first hand, the limit orders traders must receive a compensation for the execution risk in order to be induced to post limit orders. This compensation takes the form of a premium for the sellers and a discount for the buyers relative to their private valuations. On

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[14]This kind of competition over time is common in models of pairwise matching (see for example Rubinstein and Wolinsky (1985)). However, in contrast with the present model, in these models the prices are determined by a bargaining process.

the other hand limit order traders try to get high gains from trade. Consequently limit order buyers choose low bids and limit order sellers choose high asks. Finally, these strategic behaviors result in a spread.

The size of the spread is maximum when the demand pressure is equal to the supply pressure ($k = 0.5$). When there is a high demand or supply pressure ($k \rightarrow 1$ or $k \rightarrow 0$), the spread becomes nul because all the buyers are forced to trade at the ask or all the sellers are forced to trade at the bid. An empirical implication of this result is that the size of the spread must be related to the composition of the order flow[15]. More specifically, other things equal, stocks for which the imbalance between buy and sell orders is high must exhibit larger spreads than stocks for which this imbalance is low.

It is clear that the bid and the ask in Equilibrium 1 belong to the interval $[c_h, v_l]$. The location of the spread in this interval can be characterized by the mid-quote which is : $P^* = \frac{A^* + B^*}{2} = c_h + \frac{(1+k)\lambda(k)(v_l - c_h)}{2}$. Hence the location of the spread changes with the proportion of buyers and sellers. If there is a high proportion of buyers (sellers) in the market, the midquote is high (low) and close to v_l (c_h). If the proportion of buyers is just equal to the proportion of sellers, the midquote is just equal to the midpoint of the interval $[c_h, v_l]$.

It turns out that for a different set of parameters, another equilibrium with different properties is obtained.

4.2.2 Equilibrium 2.

Proposition 4 :

If $max \left(\frac{1}{1-(1-k).\pi}, \frac{1}{1-k.\pi} \right).(1 - k(1 - k)\pi^2).(c_h - c_l) \leq (v_h - c_l) \leq \frac{(c_h - c_l)}{1-\pi}.(1 - k(1 - k)\pi^2)$ then an equilibrium in which all the traders run a high execution risk exists. The order placement strategies of the traders in this equilibrium are the following :

1) The buyers with low valuation and the sellers with high valuation never submit a market order and trade only by posting limit orders. The buyers with high valuation and the sellers with low valuation

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[15]This result is obtained by Angel (1992) in a different setting.

submit a market order if there is a quote in the book, otherwise they post a limit order.

2) Both types of buyers quote a bid equal to :

$$B^{*'} = k.\pi\lambda'(k).v_h + (1 - k\pi\lambda'(k)).c_l,$$

Similarly both types of sellers quote an ask equal to :

$$A^{*'} = \lambda'(k).v_h + (1 - \lambda'(k)).c_l.$$

with $\quad \lambda'(k) = \frac{1-(1-k)\pi}{1-k.(1-k)\pi^2}.$

(I will refer to this equilibrium as Equilibrium 2.)

In this equilibrium, one can check that the bid is equal to the cut-off price of a low valuation seller $(B^{*'} = A_0^{*}(c_l))$ and that the ask is equal to the cut-off price of a high valuation buyer $(A^{*'} = B_0^{*'}(v_h))$. For this reason, the value of a market order strategy is strictly lower than the value of a limit order strategy for the low valuation buyers and the high valuation sellers. Consequently, they never use market orders and all the traders run a high execution risk when they post a limit order.

The following corollary sheds some light on the differences between the existence conditions of Equilibria 1 and 2.

Corollary 2 :

a) There is no set of parameters $\{k, \pi, c_h, c_l, v_h, v_l\}$ for which the two equilibria can exist simultaneously.

b) For given values of $\{k, c_h, c_l, v_h, v_l\}$, there are at least, for parameter π, one value π_1 such that Equilibrium 1 exists and one value π_2 such that Equilibrium 2 exists. Moreover, it is always the case that : $\pi_1 < \pi_2$.

(The proof of this corollary is tedious and is omitted for the sake of brevity. Details can be obtained upon request.)

The first part of the corollary shows that the set of parameters which satisfy the existence condition

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of Equilibrium 1 is different from the set of parameters which satisfy the existence condition of Equilibrium 2. The second part of the corollary points out that the proportions of the different types of traders in the market are the main reason of this difference. As explained previously, the existence condition of Equilibrium 1 requires that the proportion of high valuation buyers and low valuation sellers be low to induce the sellers to quote a low ask and the buyers to quote a high bid. Now, in Equilibrium 2, the traders choose a high ask and a low bid. Consequently, the proportion π of the traders which are ready to submit market orders for low bids and high asks must be increased (compared to Equilibrium 1). This explains that, other things equal, Equilibrium 2 exists only for higher values of π.

Transaction prices and transaction frequency.

As in the first equilibrium and for the same reasons, the bid and the ask quoted by the traders depend on the proportion of buyers and sellers in the market. However, in the second equilibrium, the ask price and the bid price depend on the proportion π of the traders with a high willingness to trade, as well. This is due to the fact that the execution probability of a limit order depends on this proportion in this case. It is easy to show that, other things equal, the buyers increase their bids and the sellers decrease their ask if the proportion of the traders who are the more willing to trade increases ($\frac{\partial A^{*'}}{\partial \pi} < 0 \quad ; \quad \frac{\partial B^{*'}}{\partial \pi} > 0$).

This result stems from the link between the limit order strategies and the market order strategies. When the proportion of the low valuation sellers increases, the execution probability of the buyers is higher ceteris paribus. Consequently, they are more reluctant to submit a market order. For this reason, their cut-off price decreases and the sellers must decrease the ask they require when they post a limit order ($\frac{\partial A^{*'}}{\partial \pi} < 0$). A symmetric argument explains that $\frac{\partial B^{*'}}{\partial \pi} > 0$.

Now the spread is given by :

$$S_2(k, \pi) = \frac{1 - (1 - k)\pi}{1 - k.(1 - k)\pi}(1 - k\pi)(v_h - c_l).$$

As in Equilibrium 1, the size of the spread is dependent on the composition of the order flow (the larger the imbalance between buy and sell orders, the lower the spread). Equilibrium 1 and Equilibrium 2 do not exist for the same set of parameters. Hence the comparison between the spread in the first

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equilibrium and the spread in the second equilibrium is not straightforward. However, using Corollary 1, one can make the following comparison. Assume that there are two markets which are identical with respect to the values of the parameters (k, c_h, c_l, v_h, v_l) but which differ with respect to the proportion of the high valuation buyers and the low valuation sellers. In the first market, the proportion of these types of traders is low and Equilibrium 1 is obtained. In the second market, the proportion of these traders is high and the second equilibrium is obtained. How does the spread differ in the two markets? The following corollary answers this question.

Corollary 3 : *For given values of* (k, c_h, c_l, v_h, v_l), *the spread is always lower when the traders run a low execution risk than when they run a high execution risk :*

$$S_2 > S_1.$$

Hence the size of the spread is always higher in the second equilibrium. This result implies that the bids are always lower or/and the asks higher in Equilibrium 2 compared to their values in Equilibrium 1. It suggests that the size of the spread in an order-driven market is linked to the willingness to trade of the agents in the market.

In Equilibrium 2, there are less traders to submit market orders than in Equilibrium 1. This intuitively suggests that the volume of trading is lower in the the second equilibrium than it is in the first. A way to assess the volume of trading is to measure the transactions frequency for a given interval of time. Let $P(e)$ be the probability, conditional on the state of the book, to observe a transaction during the interval $[t, t+2)$, where e indicates the state of the book. If there is no order in the book, $e = 0$. $e = 1$ if there is a buy limit order and $e = 2$ if there is a sell limit order. The values of these probabilities are for each equilibrium :

$P(e)$	Equilibrium 1	Equilibrium 2
$P(0)$	$2k(1-k)$	$2(1-k)k\pi$
$P(1)$	$1 - k^2$	$(1-k^2).\pi + (1-k)k\pi(1-\pi)$
$P(2)$	$1 - (1-k)^2$	$P(2) = k(1-k).\pi(1-\pi) + \pi.(1-(1-k)^2)$

Table 2 : Probability to observe a trade.

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Consequently for all the states of the book, the probability to observe a transaction in [t,t+2) is higher in the first equilibrium than in the second. So the increase in the spread is combined here with a decrease in the transactions frequency[16].

This decrease in the transactions frequency is due to the fact that, in the second equilibrium, only the traders who are the more willing to trade submit market orders. Consequently, the low valuation buyers and the high valuation sellers never trade together, although there could be a possibility for these traders to reach an agreement in another market structure since $c_h < v_l$. Equilibrium 2 exists only for high values of π. Consequently the incentive is increased for the sellers to overbid and for the buyers to underbid in order to obtain the best execution for their orders and to reap a large share of the gains from trade. Finally some trades that could occur, do not occur in equilibrium. In this sense the market is inefficient due to the strategic behavior of the traders.

4.2.3 Equilibrium 3.

Proposition 5 :

If : $\left(\frac{1-\pi^2 k(1-k)}{1-\pi} \right) (c_h - c_l) \leq (v_h - c_l) \leq \left(1 + \frac{k^2\pi - k\pi + 1}{1-\pi} \right) (c_h - c_l)$

then an equilibrium with the following properties exist :

a) The traders with a high willingness to trade submit a market order if there is a quote available and post a limit order otherwise.

A buyer (a seller) with a low (high) private valuation submits a market order only if the ask (bid) is quoted by a low (high) valuation seller (buyer). Otherwise he posts a limit order.

b) The bids quoted by the buyers change with their private valuations. The bid of a high valuation buyer is equal to :

$$B^{*''}(v_h) = k\pi\gamma_1(k).v_h + (1 - k\pi\gamma_1(k)).c_h,$$

while the bid of a low valuation buyer is equal to :

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[16] An inverse relationship between the spread and the frequency of trading is documented in McInish and Wood (1992) for NYSE stocks.

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$$B^{*''}(v_l) = k\gamma_2(k).v_l + (1 - k\gamma_2(k)).c_l.$$

Similarly, the asks quoted by the sellers depend on their private valuations. They are equal to :

$$A^{*''}(c_l) = \gamma_2(k).v_l + (1 - \gamma_2(k)).c_l,$$

$$A^{*''}(c_h) = \gamma_1(k).v_h + (1 - \gamma_1(k)).c_h,$$

with $\gamma_1(k) = \frac{k}{1-(1-k)k\pi}$. *and* $\gamma_2(k) = \frac{1-\pi(1-k)}{1-(1-k)k\pi}$.

(I will refer to this equilibrium as Equilibrium 3.)

For this equilibrium, one can check that :

$$A^{*''}(c_h) = B_0^{*''}(v_h) \quad A^{*''}(c_l) = B_0^{*''}(v_l),$$

$$B^{*''}(v_h) = A_0^{*''}(c_h) \quad B^{*''}(v_l) = A_0^{*''}(c_l).$$

The ask quoted by a low valuation seller is equal to the cut-off price of a low valuation buyer while the ask quoted by a high valuation seller is equal to the cut-off price of a high valuation buyer. Hence, as stated in Proposition 5, a low valuation buyer submits a market order only if the ask quoted in the book has been posted by a low valuation seller. Similarly, a high valuation seller hits the bid, only if this bid is quoted by a high valuation buyer. Consequently, in equilibrium, the strategies of the traders with a low willingness to trade depend on the state of the book. For some states, they post limit orders while for other state they submit market orders [17]. The traders with a high willingness to trade run a low execution risk, while the traders with a low willingness to trade run a high execution risk in equilibrium.

In this case, the buyers (the sellers) with different valuations use different bidding strategies. The buyers with low valuations always quote lower bids than the buyers with high private valuations $(B^{*''}(v_l) < B^{*''}(v_h))$ and the sellers with high valuations quote higher ask than the sellers with low valuations $(A_0^{*''}(c_h) > A_0^{*''}(c_l))$. In the previous equilibria, only two transaction prices can be observed. Now, four transaction prices are obtained in equilibrium : $A^{*''}(c_h)$, $A^{*''}(c_l)$, $B^{*''}(v_h)$,

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[17]This result is consistent with the empirical findings of Harris and Hasbrouk (1992). They obtain that the relative values of market and limit orders strategies change according to the state of the book.

$B^{*''}(v_l)$. Moreover, the lowest bid $(B^{*''}(v_l))$ is always smaller than the asks. Similarly, the highest ask $(A^{*''}(c_h))$ is always greater than the bids. However, in some cases, the highest bid may be higher than the lowest ask. Consequently, now, the size and the sign of the spread can change and depend on the sequence of arrivals of the traders in the market.

There is no equilibrium in which one strategy (the market or the limit order strategy) strictly dominates the other for all the traders. In the first equilibrium, the market order strategy strictly dominates the limit order strategy for the traders with a high willingness to trade but the other traders are indifferent between the two strategies. Conversely, in the second equilibrium, the limit order strategy strictly dominates the market order strategy for the traders with a low willingness to trade while the other traders are indifferent between the two strategies. Finally in the last equilibrium, the strategy which is chosen by the agents depends on the state of the book. However, in all cases, the traders with a low willingness to trade prefer limit orders or are indifferent and the traders with a high willingness to trade prefer market orders or are indifferent[18]. These results suggest that market orders are used by the traders who are the more eager to trade while limit orders are posted by the traders who are the less eager to trade. In fact, the traders with a low willingness to trade have a competitive advantage in the production of liquidity because their opportunities costs of being not executed are lower than for the traders with a high willingness to trade.

The following corollary characterizes the third equilibrium in comparison to the equilibria 1 and 2.

Corollary 4:

a) There is no set of parameters $\{k, \pi, v_h, v_l, c_h, c_l\}$ for which equilibrium 1 and equilibrium 3 can exist simultaneously.

b) There is no set of parameters $\{k, \pi, v_h, v_l, c_h, c_l\}$ for which Equilibrium 2 and Equilibrium 3 can exist simultaneously.

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[18] Handa and Schwartz (1992) show empirically that limit order and market order strategies give the same expected gains. They conclude that no strategy is strictly dominant for all the traders. They suggest that this is in fact a requirement for the existence of an equilibrium between the supply and the demand of liquidity in order driven markets. Our results support this finding.

c) For given values of $\{k, v_h, v_l, c_h, c_l\}$, *there always exists a value* π_3, *for parameter* π, *such that Equilibrium 3 exists. Moreover if* π_1 *is a probability for which Equilibrium 1 exists and* π_2 *a probability for which Equilibrium 2 exists, the following relationship is satisfied :* $\pi_1 < \pi_3 < \pi_2$.

(As for Corollary 2, the proof of this corollary is long and is omitted for the sake of brevity.)

This corollary provides intuition for the existence condition of Equilibrium 3. For given values of k, the existence condition of this third equilibrium requires that π be neither too low, nor too high. The interpretation of this condition is the following. In this case, in equilibrium, the high valuation buyers choose to quote high bids while the low valuation buyers choose to quote low bids. As explained previously, the low valuation buyers have an incentive to follow this risky strategy only if π is high enough. Conversely, the high valuation buyers have an incentive to quote high bids only if π is low. Finally, for the equilibrium to exist, it is necessary that π is neither low, nor high. A symmetric argument explains that π must be medium for the sellers. Hence, it turns out that, other things equal, Equilibrium 3 exists only for values of π which are greater than the values of π for which equilibrium 1 exists but lower than the values of π for which Equilibrium 2 exists.

4.3 On the existence of other equilibria.

[INSERT FIG.3 HERE]

Figure 3 depicts for each value of k, the sets of values of π for which the different equilibria exist. It turns out that for some values of the parameters (k very low and π very high or k very high and π very high), there is no symmetric equilibrium. The explanation of this result is the following. If k is very low and π very high, the probability that the next trader is a seller with a high willingness to trade is very high. Thus, the execution probability of a buy limit order is very high, even if the bid quoted by the buyer is low. Consequently, the cut-off prices of the buyers are very low, whatever their private valuations. Now, for the values of (k, π) in the area I_1 of Figure 3, it turns out that the cut-off price of the high valuation buyer is so low that it is less than the private valuation of the high valuation sellers. Consequently, it is not possible in this case to obtain a symmetric equilibrium in which the high valuation sellers quote an ask equal to the cut-off price of the high valuation buyers. A similar intuition explains that there is no symmetric equilibrium if k and π are very high ($(k, \pi) \in I_2$).

In this paper, the attention has been focused on symmetric equilibria. However, if $(k, \pi) \in I_1$ there exists a nonsymmetric equilibrium. In this equilibrium, the traders have the same behavior as in Equilibrium 2, except the sellers of type c_h who do not trade. Similarly, if $(k, \pi) \in I_2$, there exists another nonsymmetric equilibrium in which the traders follow the same strategies as in equilibrium 2, except the buyers of type v_l who do not trade. These equilibria are nonsymmetric in the sense that some traders do not trade while all the others are trading. Due to the symmetry of the assumptions concerning the parameters of the model, no other nonsymmetric equilibria can be obtained[19].

Concluding remarks.

This paper presents a simple model which attempts to capture some of the features of the trading in order-driven markets. The analysis is focused on order placement decision and on price formation. The bidding strategies of the agents are characterized and it is shown that their strategic behaviors result in the existence of a bid-ask spread. The order placement strategies turn out to be dependent of the composition of the order flow, the state of the book and the beliefs of the traders on the other traders' strategies. Three empirical implications of the model are that 1) the spread is small if the imbalance between supply and demand is large, 2) the order flow (the probability to observe a market or a limit order) depends on the state of the book and 3) the relative values of market and limit order strategies depend on the state of the book.

One of the main determinant of the spread is the imbalance between supply and demand. This result is related to the competition over time occuring in a dynamic market with sequential arrival of the traders. However, this model does not capture the instantaneous competition between the traders which occurs in an order-driven market. Actually, if two buyers (sellers) are in the market simultaneously, they are likely to act as Bertrand competitors[20]. In the present framework, a way to capture this instantaneous competition would be to increase the time validity of a limit order to two periods. This is left for future research.

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[19] The proof of this result is long and does not add any intuition to the previous results. Therefore it is omitted for the sake of simplicity.

[20] Evidences of undercutting behaviors between limit order traders are documented in Biais, Hillion and Spatt (1992)

Appendix.

Proof of Proposition 2.

a-d : A buyer(a seller) will never post a limit order at a price higher (lower) than his private valuation. Consequently, $E(U(v, B^*) \geq 0$. From the definition of the equilibrium, one can deduce that : $B_0^*(v) \leq v$ and $A_0^*(c) \geq c$.

b-e : By definition of the equilibrium :

$$v_j - B_0^* = \Gamma(B^*).(v_j - B^*).$$

Since $\Gamma(.) \leq 1$, it implies that $B_0^* \geq B^*$. In the same way, one proves $A^* \geq A_0^*$.

c-f : It is possible to write $(v_h - B_0^*(v_l))$:

$$v_h - B_0^*(v_l) = (v_h - v_l) + v_l - B_0^*(v_l).$$

Using the definition of the equilibrium, this equality can be rewritten :

$$v_h - B_0^*(v_l) = (v_h - vl) + E(U(v_l, B^*(v_l))).$$

Since $B^*(v_l)$ is the optimal bid of the buyer v_l, it must be the case that :

$$E(U(v_l, B^*(v_l))) \geq E(U(v_l, B^*(v_h))),$$

So the last equality can be rewritten:

$$v_h - B_0^*(v_l) \geq (v_h - v_l) + \Gamma(B^*(v_h)).(v_l - B^*(v_h)),$$

This inequality is rewritten :

$$v_h - B_0^*(v_l) \geq (v_h - v_l) + \Gamma(B^*(v_h)).(v_l - v_h) + \Gamma(B^*(v_h)).(v_h - B^*(v_h)).$$

Consequently :

$$v_h - B_0^*(v_l) \geq (1 - \Gamma(B^*(v_h))).(v_h - v_l) + \Gamma(B^*(v_h)).(v_h - B^*(v_h)).$$

This last inequality implies :

$$v_h - B_0^*(v_l) \geq v_h - B_0^*(v_h).$$

Therefore :

$$B_0^*(v_h) \geq B_0^*(v_l).$$

Using the same kind of argument, one proves that :

$$A_0^*(c_h) \geq A_0^*(c_l).$$

Q.E.D

Proof of Proposition 3.

In equilibrium, the traders choose to run a low execution risk. As a consequence, the following inequalities must be satisfied :

$$(B_0^*(v_l) - c) > \pi.(B_0^*(v_h) - c) \qquad (B.1),$$

$$(v - A_0^*(c_h)) > \pi.(v - A_0^*(c_l)) \qquad (B.2),$$

The first condition insures that the sellers quote an ask equal to $B_0^*(v_l)$ and the second condition insures that the buyers quote a bid equal to $A_0^*(c_h)$. Therefore the bid and the ask are :

$$B^*(v_l) = B^*(v_h) = A_0^*(c_h) \qquad (B.3),$$

$$A^*(c_l) = A^*(c_h) = B_0^*(v_l) \qquad (B.4).$$

According to the definition of the equilibrium, $A_0^*(.)$ and $B_0^*(.)$ must satisfy the following equations :

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$$A_0^*(c_h) = (1-k).c_h + k.B_0^*(v_l) \qquad (B.5),$$

$$A_0^*(c_l) = (1-k).c_l + k.B_0^*(v_l) \qquad (B.6),$$

$$B_0^*(v_h) = k.v_h + (1-k).A_0^*(c_h) \qquad (B.7),$$

$$B_0^*(v_l) = k.v_l + (1-k).A_0^*(c_h) \qquad (B.8).$$

I obtain closed form solutions for $A_0^*(.)$ and $B_0^*(.)$ by solving this system of equations. These solutions are :

$$B_0^*(v_l) = \frac{k.v_l + (1-k)^2.c_h}{1 - k(1-k)},$$

$$B_0^*(v_h) = B_0^*(v_l) + k.(v_h - v_l),$$

$$A_0^*(c_h) = \frac{k^2.v_l + (1-k).c_h}{1 - k(1-k)},$$

$$A_0^*(c_l) = A_0^*(c_h) + (1-k).(c_h - c_l).$$

From these solutions and from B.3 and B.4, one deduces that :

$$B^* = B^*(v_h) = B^*(v_l) = \frac{k^2.v_l + (1-k).c_h}{1 - k(1-k)} = k.\lambda(k).v_l + (1 - k.\lambda(k)).c_h \qquad (B.9),$$

$$A^* = A^*(c_h) = A^*(c_l) = \frac{k.v_l + (1-k)^2.c_h}{1 - k(1-k)} = \lambda(k).v_l + (1 - \lambda(k)).c_h \qquad (B.10).$$

I now replace $A_0^*(c_h), A_0^*(c_l), B_0^*(v_h), B_0^*(v_l)$, by their expressions in B.1 and B.2 in order to derive the set of parameters for which the equilibrium given by B.9 and B.10 exist. Thus the condition B.1 and B.2 are rewritten :

$$v_l - c_h > \left(\frac{\pi}{1 - \pi}\right).(v_h - v_l).(1 - k(1-k)),$$

$$v_l - c_h > \left(\frac{\pi}{1 - \pi}\right).(c_h - c_l).(1 - k(1-k)).$$

As $(c_h - c_l) = (v_h - v_l)$ by assumption, these two conditions can be rewritten :

$$v_l - c_h > \left(\frac{\pi}{1 - \pi} \right) . (c_h - c_l) . (1 - k(1 - k)).$$

Q.E.D

Proof of Proposition 4.

The proof of Proposition 4 follow the lines of the previous proof. Now the traders choose to run a high execution risk. Consequently, the following inequalities must be satisfied in equilibrium :

$$(B_0^{*'}(v_l) - c) \leq \pi . (B_0^{*'}(v_h) - c) \tag{C.1},$$

$$(v - A_0^{*'}(c_h)) \leq \pi . (v - A_0^{*'}(c_l)) \tag{C.2}.$$

The limit orders strategies of the buyers and the sellers are :

$$B^{*'}(v_l) = B^{*'}(v_h) = A_0^{*'}(c_l) \tag{C.3},$$

$$A^{*'}(c_l) = A^{*'}(c_h) = B_0^{*'}(v_h) \tag{C.4}.$$

According to the definition of the equilibrium, the following relationships must be satisfied in this case :

$$A_0^{*'}(c_h) = (1 - k\pi) . c_h + k\pi . B_0^{*'}(v_h) \tag{C.5},$$

$$A_0^{*'}(c_l) = (1 - k\pi) . c_l + k\pi . B_0^{*'}(v_h) \tag{C.6},$$

$$B_0^{*'}(v_h) = (1 - (1 - k)\pi) . v_h + (1 - k)\pi . A_0^{*'}(c_l) \tag{C.7},$$

$$B_0^{*'}(v_l) = (1 - (1 - k)\pi) . v_l + (1 - k)\pi . A_0^{*'}(c_l) \tag{C.8}.$$

Solving this system for $A_0^{*'}(.)$ and $B_0^{*'}(.)$, I obtain :

$$B_0^{*'}(v_h) = \frac{(1 - (1 - k)\pi) . v_h + \pi(1 - k)(1 - k\pi) . c_l}{1 - k(1 - k)\pi^2},$$

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$$B_0^{*'}(v_l) = B_0^*(v_h) - (1 - (1-k)\pi).(v_h - v_l),$$

$$A_0^{*'}(c_l) = \frac{k\pi(1 - (1-k)\pi).v_h + (1 - k\pi).c_l}{1 - k(1-k)\pi^2},$$

$$A_0^{*'}(c_h) = A_0^*(c_l) + (1 - k\pi).(c_h - c_l).$$

It follows that :

$$A^{*'} = A^*(c_h) = A^*(c_l) = \frac{(1 - (1-k)\pi)v_h + \pi(1-k)(1-k\pi)c_l}{1 - k(1-k)\pi^2} = \lambda'(k)v_h + (1 - \lambda'(k)c_l,$$

$$B^{*'} = B^*(v_h) = B^*(v_l) = \frac{k\pi(1 - (1-k)\pi)v_h + (1 - k\pi)c_l}{1 - k(1-k)\pi^2} = k\pi\lambda'(k)v_h + (1 - k\pi\lambda'(k))c_l.$$

I now turn to the conditions of existence of this equilibrium. Using the expressions for $A_0^{*'}(.)$ and $B_0^{*'}(.)$, one can check that the conditions C.1 and C.2 are equivalent to :

$$(v_h - c_l) \leq \frac{(1 - k(1-k)\pi^2)}{1 - \pi}.(c_h - c_l) \qquad (C.9).$$

In this case, these conditions are not sufficient to insure the existence of the equilibrium. Actually, for $A^{*'}$ and $B^{*'}$ to be an equilibrium, it is necessary that $B^{*'}(v_l) \leq v_l$ and $A^{*'}(c_h) \geq c_h$ as well. Otherwise the traders of types v_l and c_h will not post these quotes. In equilibrium, these two conditions imply :

$$(v_h - c_l) \geq \frac{(1 - k(1-k)\pi^2)}{1 - k\pi}.(v_h - v_l),$$

$$(v_h - c_l) \geq \frac{(1 - k(1-k)\pi^2)}{1 - (1-k)\pi}.(c_h - c_l).$$

These four inequalities impose finally the following restriction on the parameters for the equilibrium defined by $A^{*'}$ and $B^{*'}$ to exist :

$$\text{Max}\left(\frac{c_h - c_l}{1 - (1-k).\pi}, \frac{v_h - v_l}{1 - k.\pi}\right).(1 - k(1-k)\pi^2) \leq (v_h - c_l) \leq \frac{c_h - c_l}{1 - \pi}(1 - k(1-k)\pi^2).$$

Q.E.D

Proof of Corollary 2.

$$S_2(k, \pi) - S_1 = \frac{(1 - (1-k)\pi).(1 - k\pi)}{1 - k(1-k)\pi^2}.(v_h - c_l) - \frac{k(1-k)}{1 - k(1-k)}.(v_l - c_h).$$

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Now $S_2(.,.)$ is decreasing in π. It implies that for all values of π and k :

$$S_2(k,\pi) - S_1 > S_2(k,1) - S_1,$$

$$S_2(k,\pi) - S_1 > \frac{k(1-k)}{1-k(1-k)}.((v_h - c_l) - (v_l - c_h)) \geq 0.$$

Q.E.D

Proof of Proposition 5.

The mechanisms at work in the proof of proposition 4 are very similar to those in the proof of Propositions 3 and 4. So for the sake of simplification, I sketch just the main lines of the proof of Proposition 5.

Now the high valuation buyers and the low valuation sellers choose to run a low execution risk while the low valuation buyers and the high valuation sellers choose to run a high execution risk. Therefore the following inequalities must be satisfied in equilibrium :

$$(B_0^{*''}(v_l) - c_l) \geq \pi.(B_0^{*''}(v_h) - c_l) \tag{E.1},$$

$$(v_h - A_0^{*''}(c_h)) \geq \pi.(v_h - A_0^{*''}(c_l)) \tag{E.2},$$

$$(B_0^{*''}(v_l) - c_h) \leq \pi.(B_0^{*''}(v_h) - c_h) \tag{E.3},$$

$$(v_l - A_0^{*''}(c_h)) \leq \pi.(v_l - A_0^{*''}(c_l)) \tag{E.4}.$$

The limit order strategies of the buyers are $B^{*''}(v_l) = A_0^{*''}(c_l)$ and $B^{*''}(v_h) = A_0^{*''}(c_h)$. The limit order strategies of the sellers are $A^{*''}(c_l) = B_0^{*''}(v_l)$ and $A^{*''}(c_h) = B_0^{*''}(v_l)$. According to these strategies and the definition of the equilibrium, the equilibrium cut-off prices must solve the following system of equations :

$$A_0^{*''}(c_h) = (1 - k\pi).c_h + k\pi.B_0^{*''}(v_h) \tag{E.5},$$

$$A_0^{*''}(c_l) = (1 - k).c_l + k.B_0^{*''}(v_l) \tag{E.6},$$

$$B_0^{*''}(v_h) = k.v_h + (1 - k).A_0^{*''}(c_h) \tag{E.7},$$

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$$B_0^{*"}(v_l) = (1 - (1 - k)\pi).v_l + (1 - k)\pi.A_0^{*"}(c_l) \qquad (E.8).$$

Solving this system for $A_0^{*"}(c_l)$, $A_0^{*"}(c_h)$, $B_0^{*"}(v_h)$, $B_0^{*"}(v_l)$ and using the relation between these cut-off prices and the limit order prices of the traders in equilibrium, one obtains the limit order prices given in proposition 4. Plugging the closed form of the cut-off prices in the inequalities E.1, E.2, E.3 and E.4, one obtains the existence condition given in the Proposition 5. In this case, one can show easily that this condition is sufficient.

Q.E.D

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Figure 1 : Events and decisions at time t.

Fig 1 : Tree diagramm of the trading structure : events and decisions at time t
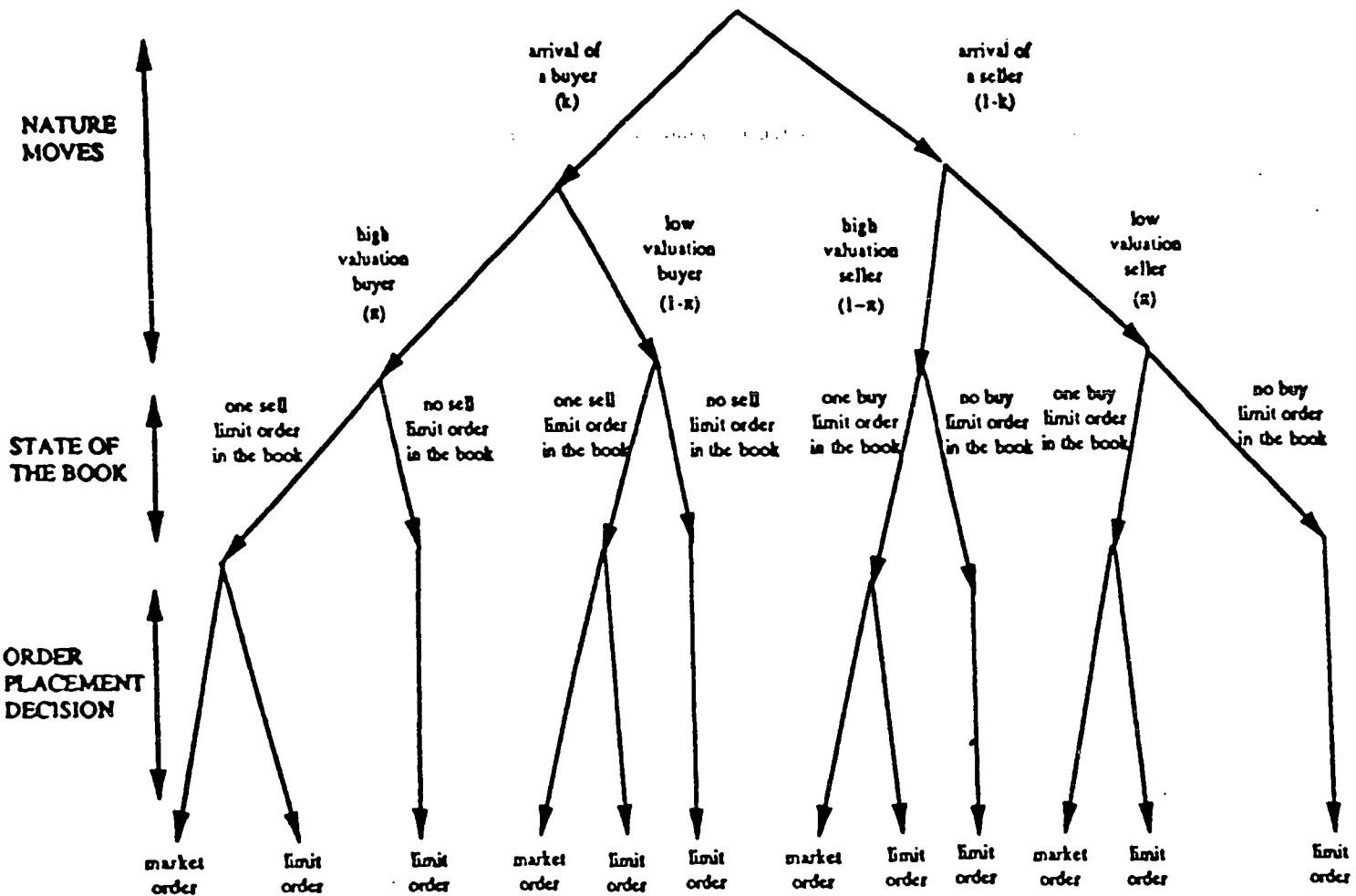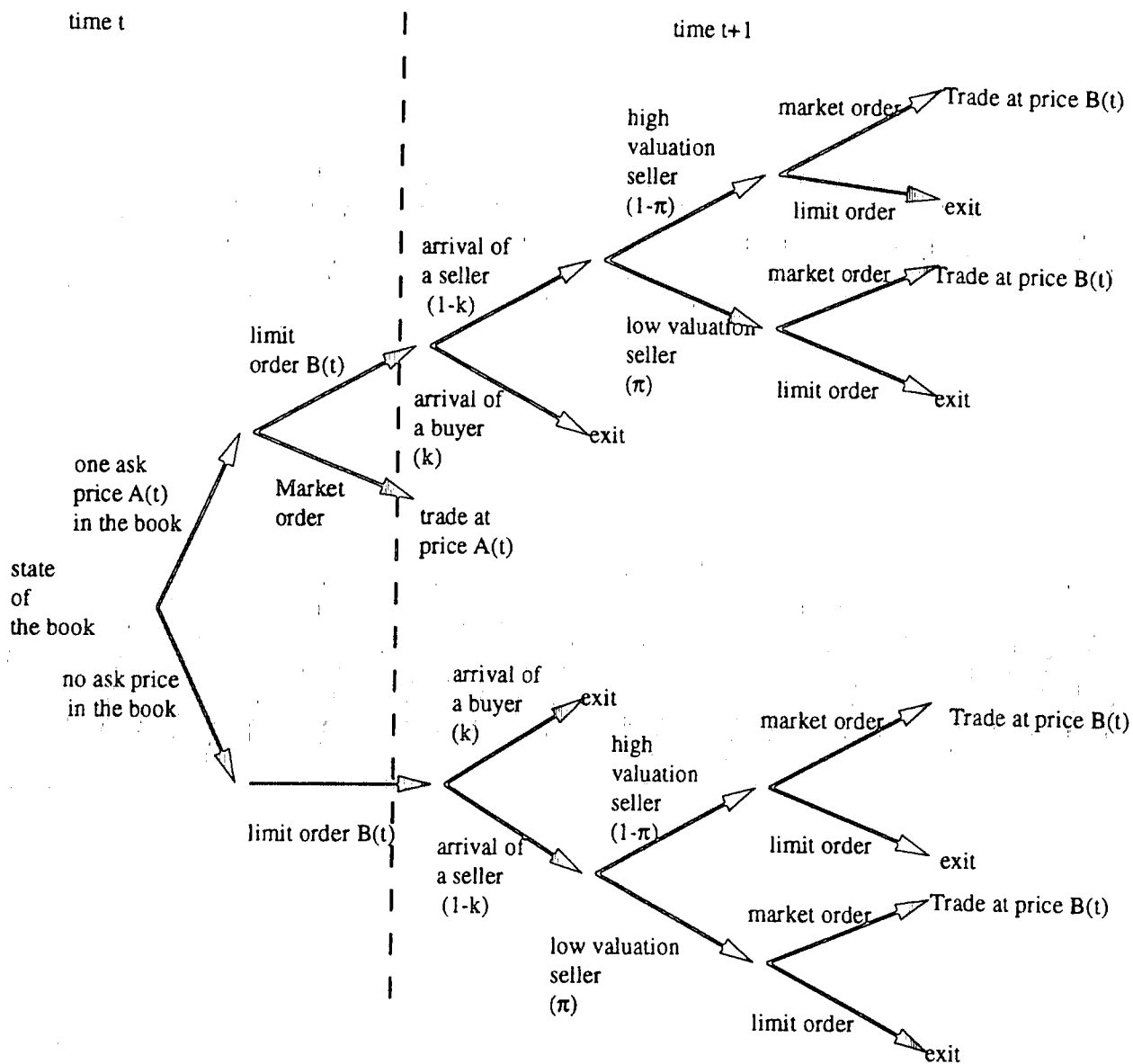

NATURE
MOVES

arrival of
a buyer
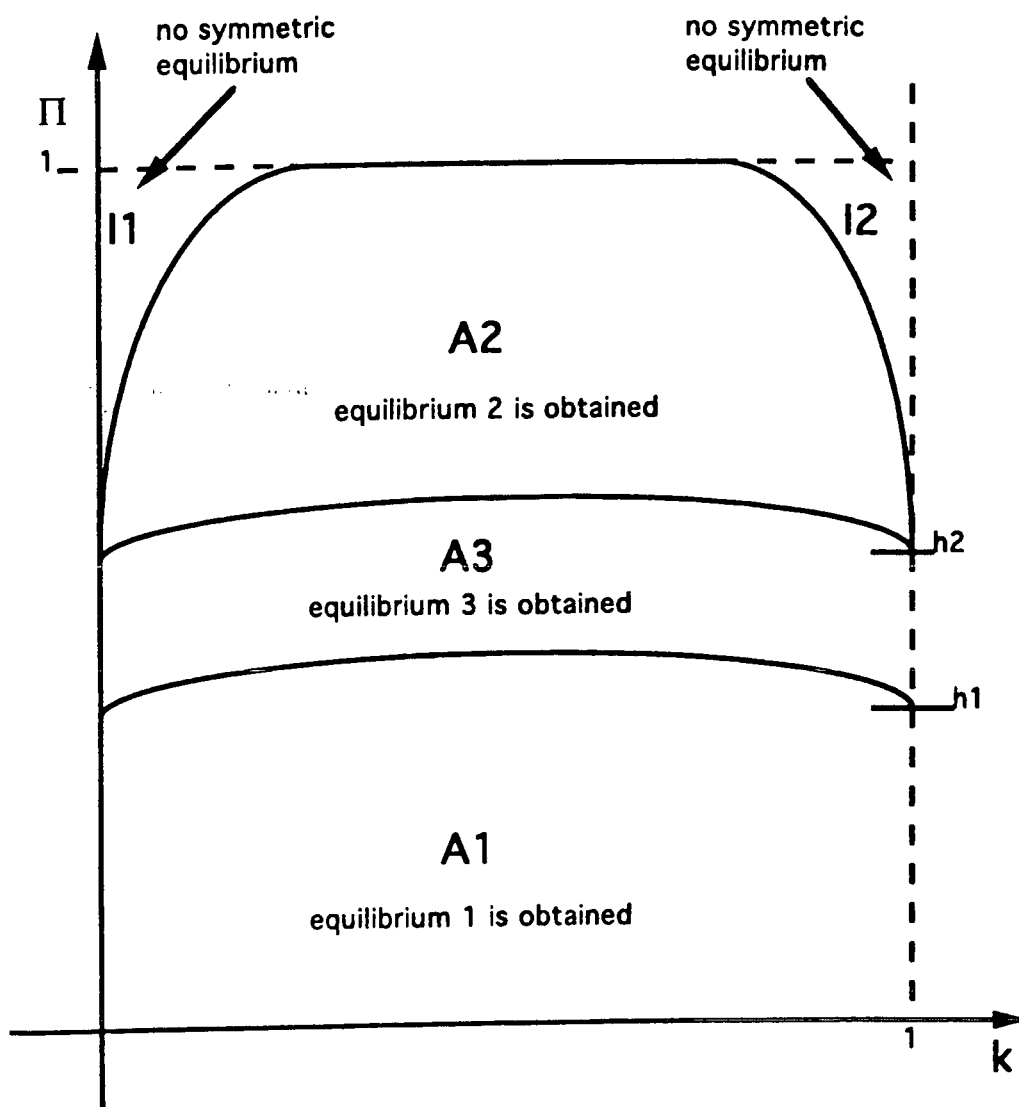(k)

arrival of
a seller
(1-k)

high
valuation
buyer
(π)

low
valuation
buyer
(1-π)

high
valuation
seller
(1-π)

low
valuation
seller
(π)

STATE OF
THE BOOK

one sell
limit order
in the book

no sell
limit order
in the book

one sell
limit order
in the book

no sell
limit order
in the book

one buy
limit order
in the book

no buy
limit order
in the book

one buy
limit order
in the book

no buy
limit order
in the book

ORDER
PLACEMENT
DECISION

market
order

limit
order

limit
order

market
order

limit
order

limit
order

market
order

limit
order

limit
order

market
order

limit
order

limit
order

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Figure 2 : Decision tree of a buyer.

Figure 3 : The different equilibria.

$$h_1 = \frac{(v_l - c_h)}{(v_l - c_l)} \qquad h_2 = \frac{v_l - c_l}{[(v_l - c_l + (c_h - c_l)]}$$

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