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Was Prometheus Unbound by Chance? Risk, Diversification and Growth*

Daron Acemoglu†
and
Fabrizio Zilibotti‡

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† Massachusetts Institute of Technology.

‡ Universitat Pompeu Fabra.

Abstract

We construct a model where, at early stages of development, the presence of indivisible projects limits the degree of risk–spreading (diversification) that the economy can achieve. The desire to avoid highly risky investments slows down capital accumulation and the inability to diversify all the idiosyncratic risks introduces high uncertainty in the growth process. The typical development pattern of a society will consist of a lengthy period of "primitive accumulation" followed by take–off and financial deepening and finally, steady industrial growth. Lucky countries will spend relatively less in the primitive accumulation stage while with sufficiently risk–averse agents, it is possible for economies that receive a series of unlucky draws to get locked into underdevelopment. We also identify a source of inefficiency in the decentralized equilibrium; in the presence of indivisibilities, missing markets endogenously arise and as a result, the average speed of industrialization is suboptimal. The source of inefficiency, a pecuniary externality, is demonstrated to be robust across various market structures. It is also shown that our results generalize to an open economy and that capital flows may increase or reduce the rate of industrialization. In particular, at the early stages they tend to work on the side of convergence while later contribute to convergence.

1) Introduction

"The advance occurred very slowly over a long period and was broken by sharp recessions. The right road was reached and thereafter never abandoned only during the eighteenth century and then only by a few privileged countries. Thus, before 1750 or even 1800 the march of progress could still be affected by unexpected events, even disasters." F. Braudel (1962), p. xi.

This view of slow and uncertain progress between the tenth and early nineteenth century is shared by many economic historians, for instance North and Thomas (1973) who describe the fourteenth and fifteenth centuries as times of *"contractions, crisis and depression"* (p. 71) or De Vries (1990) who refers to this period as *"The Age of Crisis"*. The same phenomenon is observed today; developing countries exhibit considerably higher variation in their growth rates than their more advanced counterparts and for most, the process of development is painfully slow. Why are the early stages of development slow and also subject to considerable randomness? The models of economic development based on threshold effects, for instance Azariadis and Drazen (1990), may predict the slow progress at the early stages of development but without additional assumptions have no implications about randomness of growth. This paper in contrast argues that these patterns are predicted by the neoclassical growth model augmented with the natural assumptions of micro-level indivisibilities and micro level uncertainty. In particular, we will link the availability of diversification opportunities to the process of accumulation of an economy and show that such links both make the early stages highly random and, by endogenously reducing the rate of return on savings, slow down the process of growth.

We start from a number of observations that will be elaborated and empirically supported in the next section. First, most economies have access to a large number of imperfectly correlated projects, thus a significant part of the risks they face can be diversified. Secondly, a large proportion of these projects are subject to significant indivisibilities, especially in the form of *minimum size requirements* or *start-up costs*. Thirdly, agents normally dislike risk. Fourthly, there often exist investment opportunities that are less productive but relatively safe. And finally, societies at the early stages of development have less capital to invest than do developed countries. These features naturally lead to a number of important implications. First, due to the scarcity of capital, only a limited number of imperfectly correlated projects can be undertaken and an economy at the early stages of development will seek insurance by investing in safe but unproductive assets. As a result, poor countries will endogenously have lower productivity and this will contribute to their slower development. Second, since a large part of the savings are invested in safe investments, the more productive ventures that are undertaken will bear more of the diversifiable risks. This will make the earlier stages of development highly random. It will also slowdown economic progress even further, since many runs towards progress will be stopped by crises and failed take-offs. Third, chance will play a very important role; economies that are lucky enough to receive good draws at the early stages will have more capital, thus will achieve better risk-diversification and higher productivity. Thus although Prometheus will not be unbound accidentally, chance will always play a key role in his unchaining and may even condemn him to permanent imprisonment.

In our model, agents decide how much to save and how much of their money to invest in a safe asset with low return. The rest of the funds are used to invest in imperfectly correlated risky projects taking into account the minimum size restrictions of these projects. The more "sectors" (projects) are open, the higher is the proportion of the savings that agents are willing to put in risky investments. In turn the higher is the capital stock of the economy, the higher are the incomes and savings, and the more sectors can be opened. However, this process is full of perils, because with limited investment in imperfectly correlated projects, the economy is subject to considerable randomness and spends a long time fluctuating in the stage of low accumulated capital. At this stage economies that receive "lucky draws" will grow while those that are unfortunate enough to receive a series of "bad news" will stagnate. As the lucky economies grow, eventually the take-off stage will be reached and a larger stock of savings will be made available to be invested in productive investments, to use Braudel's terms, to *"open the sluice-gates"* of the economy. At this stage the number of projects expands quickly, and the induced financial development provides better opportunities for risk-spreading. Eventually, industrial growth is reached with idiosyncratic risks being completely diversified and *"the right road thereafter will never be abandoned"*.

This last point that industrial growth will be eventually reached is a feature of our baseline model. Formulated differently, our economy has a unique equilibrium path and the stochastic process associated with it has a unique ergodic set that corresponds to full diversification and "industrialization"; all economies will eventually reach this stage and they will never abandon it. However, we show that this feature of a unique ergodic set only arises when preferences induce a rate of relative risk aversion less or equal to 1. We demonstrate that with more risk-aversion, the equilibrium stochastic process no longer has a unique ergodic set. This implies that economies that receive a series of unlucky draws may fall to a region from where there is no escape, thus they will be trapped in underdevelopment. Yet it has to noted that the nature of this underdevelopment trap is quite different than others in the development literature that are driven by the form of the technology (Boldrin (1992), Matsuyama (1991)) where increasing returns to scale play the key role. In our case, although the sectoral indivisibilities are important, it is attitudes towards risk that condemn an economy to underdevelopment; when only a few sectors can be opened, risk-averse agents are willing to invest a limited amount in risky assets and the rate of productivity is endogenously low.

Theoretically our model corresponds to a general equilibrium economy with an endogenous commodity space because the set of traded financial assets (or open sectors) is determined endogenously. We use the competitive equilibrium concept suggested by Makowski (1980) for this type of economies. This equilibrium is Walrasian conditional upon the number of sectors that are open; therefore all agents are still kept as price-takers and there are no unexploited gains in any activity. However, although the indivisibilities in this economy are very simple and look innocuous, the competitive economy is inefficient and this inefficiency will take the form of too few projects being undertaken. The underlying problem is that each sector creates a positive "*pecuniary externality*" on the already existing sectors as consumers now bear less risks when they buy the shares of these sectors. Not only do we show that the competitive equilibrium is inefficient but we also establish the considerably stronger result that there is no decentralized market structure that can avoid this inefficiency.

It may be conjectured that since our mechanism is related to capital shortages, its validity will be limited in the presence of international capital flows. We turn to this problem in section 6 and identify two factors that need to be taken into account. First, foreign capital may flow in, and secondly, domestic capital may flow out in order to obtain outside insurance. To discuss these issues, we extend our model to a two-country world. In this case the problems that our paper emphasizes do not disappear. With a neoclassical production function, it is not profitable to invest all the savings of the world economy in the same place and agents will like to invest in both countries. However, unless capital is abundant, not all the sectors in both countries will be open and adverse shocks affect capital accumulation in both countries but not always to the same degree. The general pattern of development in this two-country world takes an interesting form that matches the historical facts of the development of Western Europe; at the early stages, capital flows more into one of the countries, thus capital flows create divergence; but as the world economy becomes richer, capital flows increase the rate of convergence. Yet, in many cases, domestic capital going abroad for outside insurance will be harmful to industrialization and to the welfare of future generations. This latter effect can also be important in the case of a small open economy with enforcement problems that limit the inflow of capital.

Our model is closely related to the large and growing literature on credit and growth. Greenwood and Jovanovic (1990) and Zilibotti (1994) investigate the role of credit markets in allocating funds to the most productive users. Bencivenga and Smith (1991) emphasize the importance of liquidity and show how financial intermediation may enable faster growth. Greenwood and Smith (1993) also deal with the financial development but mostly build on the importance of liquidity provision role of financial intermediation. These papers make valuable contributions but derive most of their dynamic effects from the presence of fixed costs. Since financial intermediation is costly, it only develops slowly. Differently from these contributions, our model concentrates on the diversification role of the credit markets (as emphasized by Gurley and Shaw (1955)) and relates this to the allocation of funds to most productive investments. We show that in the presence of micro-level non-convexities (and without fixed costs of financial intermediation), endogenous dynamics that link economic growth to financial deepening are introduced. Further, not only do we obtain a two-way causality between growth and financial development (as empirically found in Goldsmith (1969), Jung (1986), Atje and Jovanovic (1992), King and Levine (1993)) but we also predict the typical pattern of development as laid out in the next section; slow and uncertain progress followed by more steady industrial growth. The paper most closely related to ours is Saint-Paul (1992) who discusses the insurance (risk-diversification) role of the stock market. Saint-Paul's main argument is that when stock market insurance is possible there will be more productive specialization. Our paper differs from his in a number of important ways; the degree of diversification is endogenized, the dynamic interaction between financial deepening, and growth is investigated and the implications for the role and evolution of randomness in development are derived. Also none of the above papers investigate the links between credit markets and international capital flows.

The plan of the paper is as follows. The next section starts by providing historical and empirical support to the main observations that motivated this investigation. Section 3 lays out the basic model and determines the decentralized equilibrium. Section 4 shows why the decentralized equilibrium is not Pareto efficient and characterizes the Pareto optimal allocation. Section 5 shows that risk-aversion plays a very important role in our model and that in the presence of a rate of relative risk-aversion greater than one, an economy that receives a series of unlucky draws may get locked into an underdevelopment. Section 6 discusses how our results would be affected in the presence of international (or inter-economy) capital flows. Section 7 concludes while an appendix contains all the proofs.

2) Motivation and Historical Evidence

Many economic historians (e.g. Braudel (1962,1979), North and Thomas (1973), De Vries (1990)) report evidence that countries at the early stages of development experienced much more risk and uncertainty. McCloskey (1976) calculates the coefficient of variation of output net of seed in Medieval England at 0.347 and that "famines" were occurring on average every 13 years. Part of the variability was certainly due to the fact that agricultural productivity was largely dependent on weather, but this heavy reliance on agriculture is itself a symptom of an undiversified economy. Additionally, there is considerable evidence that non-agricultural activities were also subject to large uncertainties. Braudel describes the development of industry before 1750 as *"subject to halts and breakdowns"* (1979, p.312) and points out the presence of failed take-offs. More specifically, he documents *"three occasions in the West when there was an expansion of banking and credit so abnormal as to be visible to the naked eye* (Florence 1300s, Genoa late 1500s and Amsterdam 1700s)...*three substantial successes, which ended every time in failure or at any rate in some kind of withdrawal.*" (1979, p. 392). Also the pattern of these failures is informative; these cities seem to have grown gradually by expanding the scope of industrial and commercial activities and yet, the collapse took the form of an abrupt end ignited by a few bankruptcies - which is suggestive of large undiversified risks and that the resolution of these risks impacts significantly on the rest of the economy. For instance in the case of Genoa and that of Antwerp, another such failed take-off, the Spanish state default of 1557 (Braudel (1984, p. 153)) or in the case of Amsterdam, the Clifford bankruptcy of December 1772 (Braudel (1984, p. 271) or more recently in the case of Austria, the Bourse Crash of 1873 (Rudolph (1972, p.29)) appear as the sparks that opened the way of more widespread failures.

And also today, poor countries exhibit considerably higher variability of output (and of consumption) than more developed economies. Using data from the Summers and Heston (1991) for 115 countries from the period 1960-88, we first calculate the standard deviation of the growth rate of each country from a fitted linear time trend, then run a non-linear (second order polynomial) cross-country regression of this measure of variability on the logarithm of GDP per capita in 1960. The result is the following (t-statistics in brackets);

$$\underset{}{sd(\Delta \log GDP)} = \underset{(1.27)}{-0.141} + \underset{(2.13)}{0.065 \log GDP60} - \underset{(-2.5)}{0.0052 (\log GDP60)^2} \qquad \underset{F(2,112)=13.1}{R^2 = 0.20}$$

Figure 1 plots our measure of variability as a function of the (log of) initial period GDP, with the solid curve tracing the fitted values from the regression. From this plot, the relation appears non-monotonic (inverse U-shaped), though only the poorest 15 out of 115 countries lie in the increasing region of the curve. Thus, for a large majority of the countries, the variability is a decreasing function of the initial GDP and the variability of output seems especially high for low-middle income countries.

FIGURE 1 HERE

Recent empirical works by Quah (1993,1994), and Benhabib and Gali (1994) provide further evidence supportive of the general implications of our model. On the one hand it seems difficult for very poor countries to achieve growth and improve their condition. On the other hand there is a significant degree of "reshuffling" and variety of experiences when we look at the lower-middle income and middle income countries, with the probability of unusually poor performance decreasing with the level of income. Quah (1993) studies the cross country dynamics of growth by estimating a Markov chain transition matrix by classifying the countries in four groups according to their GDP per capita relative to the world average (Table 1, p.431). Table 1 below report a summary of his results for a 23-year transition matrix (1962-1985).

Table 1. Estimated 23-year transition matrix (Quah 1993, Table 1).

	Prob(↓)	Prob(=)	Prob(↑)
x<1/4	****	0.76	0.24
1/4<x<1/2	0.52	0.31	0.17
1/2<x<1	0.29	0.46	0.26
1<x<2	0.24	0.53	0.24
x>2	0.05	0.95	****

x= GDP per capita relative to the world average.

In the three different columns, we report the estimated probability that a country belonging to a certain group falls to a relatively poorer group, remains in the same group, or moves up to a richer group, respectively. Although there is some considerable persistence at both the highest and the lowest level, the middle income countries experience much higher rates of mobility. Additionally, the probability of moving to a worse position becomes lower as we consider higher income groups. The findings of Benhabib and Gali (1994, p. 11) are along the same lines. They classify countries according to their capital-output ratios. Their estimate of a 20-year (1965-85) transition frequency matrix (relatively to its mean value) is summarized in Table 2:

Table 2. Estimated 20-year transition matrix (Benhabib-Gali, 1994).

	Prob(↓)	Prob(=)	Prob(↑)
z<0.8	****	0.76	0.24
0.8<z<1.2	0.27	0.52	0.20
z<1.2	0.10	0.89	****

z = Capital-Output ratio relative to the world average.

Again, both high persistence at the lowest and highest levels and much more variability for middle income countries can be detected. It therefore appears that the uncertainties faced by countries at different levels of development vary considerably, in particular, low-middle income face much higher variability than the rich and developed economies.

A possible explanation for both these and the previous findings is technological. Countries at the early stages of development may only have access to technologies that are risky and lowly productive (e.g. agriculture). This however does not seem to be the whole story. Quite often savings rather than technology appear to be the main constraint. Even societies at early stages of development usually have access to a number of activities with high returns and although technological constraints no doubt played an important role, these were by no means always the binding constraints. Braudel (1984) documents the presence of very advanced technologies in Ancient Egypt as well as Europe in Medieval Times. North and Thomas (1973) and Rosenberg and Birdzell (1985) also show that many of the technologies that were later used in improving agricultural productivity were actually known in Medieval Europe and attribute the failure to adopt these technologies earlier to the lack of monetary incentives.. Hobsbawm (1968) goes even further and argues that there was nothing that was new in the technology of the British Industrial Revolution and most of the new productive methods could have been developed at least 150 years before. Reynolds (1983, p.950) writes for the pre-industrial stages, *"The economy is not uncommercial and unmonetized. A considerable percentage of output is exchanged among households. Local trade is always important and long-distance trade is often quite important".* However, as is the case today, most high return projects also had high risks; for instance despite sophisticated insurance systems as described by Braudel (1979), there were substantial risk-premia paid to the financiers of the very profitable long-distance trade ventures.

As an obstacle to expansion and growth, capital requirements and limited savings appear more important in many instances. Bagehot (1873, p.4) more than a century ago argued that indivisibilities and capital shortages were a significant hurdle to be passed, in his words, *"...in poor countries, there is no spare money for new and great undertakings.."*, even when they were very promising. And Gerschenkron (1962, p.14) also echoed the same view in writing; *"...in a relatively backward country capital is scarce and diffused, the distrust of industrial activities is considerable and finally there is greater pressure for bigness..."* The size of the required activity was certainly a relevant factor in the minds of innovators, Scherer (1984, p.13) quotes, Boulton, James Watt's partner, writing to Watt that the production of the engine was not profitable for just a few countries but would be so if the whole world were the market. In line with the view of capital constraints, certain economic historians, for instance Wrigley (1988), argue capital shortages that lead to energy shortages to be the main reason for heavy reliance on agriculture and a key constraint to industrialization, while a large part of the development literature, implicitly or explicitly, defines underdeveloped countries as those that have shortages of capital (see for instance Viner (1958), Singer (1958), Lewis (1958)).

The implication is that the indivisibilities introduced by the fact that it is impossible (or unprofitable) to build a half dam, or a half railway network or because of the considerable start-up costs of most industrial activities will pose problems in the process of capital accumulation. Also, as emphasized by the literature on the Big Push (Rosenstein Rodan (1947), Murphy, Shleifer and Vishny (1989)), pecuniary externalities will often make certain sectors unprofitable when they are in isolation of the rest of the economy, for instance, investments into a railway and a new industrial district can be profitable only if undertaken simultaneously. This kind of complementarities will also increase the extent of non-convexities in the economy and would imply that large outlays will be necessary to reach a viable rate of return. The important empirical question of course concerns at which level these indivisibilities (non-convexities) would apply. For instance, the start-up investment for an industrial activity will definitely be too large for a single village or even town. However, if all the resources of Medieval Europe or even those of a small developing country today were pulled together, it is unlikely that many sectors would have binding minimum size requirements. On the other hand, both actual transaction costs and those created by incentive and enforcement problems imply that the relevant unit of accumulation will often not be a whole continent nor even a whole country. What the relevant unit of accumulation will be is at the heart of the empirical relevance of this question and does not have an easy answer. It would be appealing to have a theoretical model in which the relevant size of the accumulation unit is endogenous and jointly determined with the set of traded assets to explain, for instance, why the unit of accumulation in the Feudal Europe is not the same as in the contemporary world. However, in this paper we leave this problem aside and investigate the implications of binding minimum size requirements or start-up costs, assuming that the unit of accumulation is given[1].

An important aspect of the mechanism offered in this paper is that agents choose the composition of investment in order to reduce the risks that they bear and this entails the use of low return but relatively safe investments. At earlier stages of development, when capital shortages are particularly severe, a large proportion of the portfolio consists of relatively safe but less productive investments. There is wide-spread agreement among economists and economic historians that economic agents are willing to pay a price to avoid large risks. Many complex institutions have developed to deal with the problems of risks (e.g. the organization of villages that enables households to pool risks, see Persson (1988), chapter 2, Townsend (1990)). The storage technology and the scattering of fields widely used Medieval Europe are clearly inferior technologies chosen because of their relative safety[2]. The pattern of change in the portfolios between the eighteenth and nineteenth century Britain

[1] If the relevant accumulation unit is taken to be determined by the political borders, our mechanism would imply "size-effects" in the take-off process, i.e. larger countries are likely to take-off before others. This is a feature of most growth models with non-convexities and there is not much evidence in support of this implication. However, as emphasized in the above paragraph, "size" is beneficial when by "size" we mean the availability of a large stock of savings conditional on the extent of the accumulation unit and the size of non-convexities. When comparing China to Belgium, it is obvious that neither the accumulation units nor the non-convexities facing the two countries are the same. This reasoning also underlies the use of per capita income in the motivating cross-sectional regression reported above.

[2] Storage is widely believed to be common in Medieval Europe and yet McCloskey and Nash (1976) argue that storage was scarcely used but interpret the open field system and scattering of cultivation as an inefficient technology adopted for insurance reasons.

also illustrates how the use of relatively safe assets has decreased as the array of available assets has expanded and the income level increased, thus giving agents better diversification opportunities through a wider variety of risky assets (see Kennedy (1987), table 5.1). Based on this and additional evidence, Kennedy (1987, p.113) argues; *"The larger is a portfolio the more opportunities that exist for laying risks off against each other; that is, the set of optimal feasible choices may be more favorable for larger portfolios than smaller ones... "*. Braudel, emphasizes in line with the thesis of this paper that the shortage of credit is a serious constraint to the adoption of certain projects; that the relaxation of this constraint requires a simultaneous advance on a number of fronts and notes that unproductive hoarding (storage) frequently occurs in poor economies. More specifically he writes (1979, p.386); *"Every society accumulates capital which is then at its disposal, either to be saved and hoarded unproductively, or to replenish the channels of the active economy... If the flow was not strong enough to open all the sluice-gates, capital was almost inevitably immobilized, its true nature as it were unrealized"*. To conclude on this point, we can note that the low savings rate that Rostow (1978) sees as the source of slow growth until the industrial revolution may also follow as a result of high risks ad low returns at early stages.

Finally, this paper stresses that at early stages lack of diversification gives an important role to "luck". In particular, we emphasize that if large and risky investments undertaken at early stages are successful, the economy grows, more risk-diversification becomes possible and new projects can be undertaken. Here the historical role of the railways can be viewed as an example of the importance of the success of large projects in opening the way to steady growth. For instance, according to Alfred Chandler (1977), railways constitute a turning point in the historical development of US capitalism because after the efforts of the Wall Street to raise the necessary funds required for the large railway investments, it was also possible to finance other large scale projects (although the main aspect emphasized by Chandler is that Wall Street "learned-by-doing" to cut big deals). In Spain railroads attracted more than 15 times the amount invested in all other manufacturing by the end of 1864 but, in contrast to the US experience, the returns in the second half of the nineteenth century were very disappointing and all the sectors of the economy suffered turmoil and capital scarcities as a result of these heavy losses in railways (Tortella (1972, pp. 118-121)). Regarding this episode and a similar one in Italy, Cameron (1972, p. 14) writes, *"Spain in the 1850's undertook a vigorous program of railway construction and financial promotion not unlike the later Russian program; Italy did the same in the 1860's. But in both cases the result was a fiasco which set back the progress of industrialization and economic development by at least a generation."*

Another case of a large project's destiny getting interwoven with that of the whole economy is that of the Dutch versus the English East India companies. At the beginning of the Eighteenth Century the two companies were attracting substantial shares of the savings of the period. During this century, the English company doubled its capital and paid a high level of dividends to shareholders. The performance of the Dutch company instead, affected among other things by a unfortunate crisis in the trade of tea and pepper with China, steadily worsened; the capital level of the company did not grow during the Eighteen Century, and the company first stopped paying dividends and finally collapsed and was taken over by the Dutch State. While the success of the British East India company contributed to the financial development of Britain, the failure of the Dutch Company appears to be an important cause of the decline of the Netherlands (Neal (1990), especially p. 121). More importantly, as would be suggested by our mechanism, the success of English Company appears to have reduced the overall risk-premium of long-distance trade (Neal (1991, p.129)). Therefore, all these events appear to support the third implication that once a number of large projects are successful, it is possible to expand into new ones because there are more funds to invest and better diversification opportunities to exploit; as a result investors are rationally less shy towards large and risky projects. Overall, both the observations that constitute our starting point and the implications that our analysis draws from those seem to receive support from historical evidence.

3) The Basic Model and the Decentralized Equilibrium

We consider a simple overlapping generations model with non-altruistic agents that live for two periods. There is a continuum of agents normalized to 1 in each living generation and agents of the same generation are all identical. The production side of the economy consists of a single final good sector and a continuum of intermediate sectors normalized to 1. The final good sector transforms the total capital of the economy into final output. The intermediate sectors transform savings of time t into capital to be used at time t+1. We can think of our final good as wheat which can be combined (as capital) with labor to produce consumption goods (bread) but part of this final output can also be used as seeds to obtain wheat next period. The main issue in our economy is how to transform the seeds to wheat for next period. We can plant it in the back garden which will be a safe investment with low productivity. Alternatively, we can plant it in the fields but some of these fields will be attacked by grasshoppers and bugs that will destroy the output. Also to complicate matters, investment in some of these fields will only be possible if a large amount of wheat is planted there.

In their youth, our agents work in a final sector firm, receive the competitive wage rate of this sector. At the end of this period they take their consumption and saving decision.

This decision entails how much to save and how much of their savings to put in a safe asset. The safe asset has a certain rate of return equal to r, therefore one unit of capital at time t is transformed in to r units of capital at time t+1. Alternatively, they can invest part of their savings in some of the risky projects of this economy. After the investment decisions, the uncertainty unravels and the amount of capital that is brought forward to the next period is determined. The capital that agents own in their retirement period is rented by final sector firms and old agents consume the rent they receive. Figure 2 summarizes the sequence of events in our model.

FIGURE 2 HERE

The crucial issue is the form of uncertainty in the intermediate sectors. We assume that there is a continuum of states again normalized to 1 and that project j pays R in state j and nothing in all other states[3], where R is greater than r. Hence investing in a sector is equivalent to buying a basic Arrow security that only pays in one state of nature. We also assume that each state of nature in this world is equally likely. This implies that if the economy invests \$1 in each of p projects (thus a total of \$p invested), there will be a probability p that the return will be \$R and 1-p that it will be zero. This formalization captures two features that will drive all our results; first, the risky investment is more profitable than the safe investment; second, different risky investment projects are imperfectly correlated so that there is safety in numbers.

Formulated in this way the allocation problem of the economy is straightforward. Invest all the savings in each of the risky sectors simultaneously. This result will also be true in more general models as long as all these sectors exhibit decreasing or constant returns to scale. However, in the presence of some projects with non-convexities as in our case, a trade-off is introduced between diversification and high productivity. We capture this feature in a very simple way by assuming that certain sectors have a minimum size requirements. In other words, sector j has the following production function

$$f_j(F_j)=\begin{cases} RF_j & \textit{if state } j \textit{ occurs and } F_j > M(j) \\ 0 & \textit{otherwise} \end{cases}$$

where F_j is the amount of resources invested in this sector. This functional form implies that all intermediate sectors exhibit constant returns to scale but some sectors require a certain

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[3] More formally, the rate of return of project j is $\lim_{dj \to 0} \frac{R}{dj}$ and occurs with probability dj. Thus in the absence of size restrictions, with 1 unit of capital, an investor can invest in all j $\in$ [0,1] and get a certain rate of return of R units of capital at t+1.

size (M(j)) before being productive. We also assume the following function which associate to each sector j a minimum size requirement M(j) (see Figure 3 for a diagrammatic representation):

$$M(j)=Max\ \{0,\frac{D}{1-\gamma}(j-\gamma)\} \tag{1}$$

This specification implies that all sectors $j \leq \gamma$ have no minimum size requirement and for the rest of the sectors, the minimum size requirement increases linearly. Our results are not however in any way dependent on this linear specification. Obviously the ranking of projects from lower to higher size is without loss of any generality and imposes no timing constraint; any project can be adopted first, but obviously in equilibrium, sectors with smaller minimum size requirements will be opened before the others. The minimum size requirement we are introducing may be thought as an internal increasing returns to scale; for instance, we cannot have half a dam to provide only a limited number of fields with water or build half a ship for long distance travel and trade. We will therefore think of the minimum size requirement to be important at the individual firm-project level[4].

Preferences over final goods

The preferences of consumers over final goods is defined as

$$E_tU(c_t,c_{t+1})=\log c_t+\beta\int_0^1 \log c^j_{t+1}dj \tag{2}$$

where j represents the states of nature which are assumed, as noted above, to be equally likely. Each agent discounts the future at the rate β and has logarithmic preferences which will give us a constant saving rate. Note that, although the state of nature does not influence the productivity of the final good sector, the final consumption of our representative agent depends on it because the state of nature determines how much capital he takes into the final good production stage.

[4] We think of our projects as individual projects but as our use of the term "sectors" suggests, these can be thought of as different sectors of the economy as well. In this case, this minimum size requirement would also be consistent with decreasing returns at the firm level and increasing returns at some sectoral level; for instance, large railway companies may run into managerial inefficiencies but also unless there is a critical mass of tracks, the railway sector will not be profitable. Also note that because these projects are never perfect substitutes to each other (due to imperfect correlation of their returns), the minimum size requirements give rise to non-convexities in the aggregate production set and this is the reason why introducing lotteries will not make any difference in this setting.

Final good production technology

Output of a typical final good producer is given by

$$y_t = A k_t^{\alpha} l_t^{1-\alpha} K_t^{\delta} \tag{3}$$

where lower case letters denote the levels hired by the firm, the upper case is the aggregate amount and where $0 \leq \delta \leq 1-\alpha$. The last term captures the possibility of Romer (1986) type externality that may lead to endogenous growth. The case with $\delta = 0$ corresponds to the basic neoclassical growth model upon which we focus on in most of the paper.

Bearing in mind that the labor and capital markets are competitive and that aggregate labor supply is equal to 1, the relative prices of these factors, w_t and ρ_t respectively, are obtained as

$$\begin{aligned} w_t &= (1-\alpha) A K_t^{\alpha+\delta} \\ \rho_t &= \alpha A K_t^{\alpha+\delta-1} \end{aligned} \tag{4}$$

Organization of the Stock Market

We will think of each intermediate sector as represented by a set of firms that sell the particular Arrow security and compete "a la Bertrand" by calling prices P^j for each unit of security (one unit of security entitles the holder to R units of t+1 capital in state j) and each agent decides how much of each traded security to buy. Naturally, for the sectors that have binding minimum size requirements only one firm will be active but the potential of competition will force it to make zero profits (see below for the precise equilibrium concept). For simplicity we assume that intermediate sector firms function without using any additional resources. The intermediate sector trading can be thought as a stock market, but for our present purposes this can also correspond to a situation in which a financial intermediary collects the funds from the consumers (again by selling similar securities) and then lends it directly to the intermediate sector firms.

In the analysis of this section, we will assume that inter-firm share trading is not allowed. This assumption will rule the situation where some firm or intermediary can buy-up the whole "security market". Although at this stage this looks quite *ad hoc*, it is in fact only a simplifying assumption that enables us to derive the main results using a simple concept of equilibrium (to be introduced in the next subsection). In section 4, it will be shown that a natural refinement of our equilibrium concept will make this assumption redundant.

An immediate implication of ruling our inter-firm share trading and of competition among financial intermediaries is that investors will face a constant (linear) price for each share equal to the marginal cost of supplying it; thus $P^j = 1$ for all securities that are traded. To see this note that Bertrand competition will lead to zero profits and in the absence of inter-firm share trading, this implies zero profit in each state of nature. If a firm demands a price higher than \$1 for any unit of its security, a competitor can enter and undercut this price. The same applies if a firm pays less than \$R when it is successful and since firm j is not allowed to hold shares of other firms, it has to pay zero in all states of nature other than j.

The Equilibrium Concept

A full Arrow-Debreu equilibrium is defined as a price mapping P^* that assigns a price to each commodity (project) in each time period such that for all $P_t^{j*}>0$, the excess demand for security j at time t, $ed_t^j(P^*)$, is equal to zero, and for all $P_t^{j*}=0$, $ed_t^j(P^*)\leq 0$. Note that this concept of equilibrium assigns a price level to all commodities, irrespective of whether they are being traded or not. It will be shown in the next section that such an equilibrium does not exist due to the sectoral non-convexities[5]. However, this full Arrow-Debreu equilibrium is too strong a concept for an economy with endogenously determined commodity space (proportion of open projects) and is not normally used in these circumstances (see for instance Hart (1979), Makowski (1980)). We will instead offer an alternative equilibrium concept that captures all the salient features of a competitive situation; in particular, it will keep all agents as price-takers and exploit all the gains from trade that can be exploited via a decentralized trading procedure.

An equilibrium is defined as a proportion of open sectors and a price function that assigns a price level to each open sector. More formally;

Definition 1: Consider the vector $(n_t^*, \{P_t^{*j}\}_{0\leq j\leq n})_{t\geq 0}$ where n_t^* denotes the proportion of open sectors at time t and P_t^* determines prices of securities at time t. This vector characterizes an equilibrium iff

(i) $ed_t^j(n_{t*},P^*)=0$ for all $j\leq n_t^*$ (supply is equal to demand in all the open (n_t^*) sectors);

(ii) no additional firm can enter at any time, offer a security conditional on a feasible production plan and make positive profit even if consumers re-optimize.

Note that we have defined the equilibrium as a dynamic concept. However, we will see that because of the limited intertemporal interaction, the equilibrium prices and active sectors can be determined separately for each period, given the earned income of the old

[5] The important point is not whether there is price for the non-traded securities but whether this price is the same as the price that an entrant contemplates to receive if it enters. In our context, there are well-defined prices conditional on entry and an entrant would enter in case it would make positive profits at those prices. There are also prices that would induce the consumers to choose zero demands. However, because of the non-convexity, these two set of prices are not equal to each other.

generation.

This equilibrium concept is related to at least two others that have been used in the literature. The first is the famous Rothschild-Stiglitz (1976) concept of equilibrium in the presence of informational asymmetries. Rothschild and Stiglitz assume that an allocation is an equilibrium if consumers take insurance contracts as given and there is no contract that can be offered and, after consumers re-optimize about which contracts to choose, would make positive profit. This is obviously very similar to our equilibrium concept. Here there is no informational considerations but, the re-optimization of consumers is more involved than the one in Rothschild and Stiglitz; the consumers do not only decide to drop one seller of a particular security in favor of a new entrant but they can change their complete portfolio when a new firm enters. The other equilibrium concept that is similar to ours is Makowski (1980)'s which is formulated, as is ours, for an economy with endogenous market formation. Makowski defines a Walrasian Equilibrium as a feasible competitive allocation sustained *by the set of traded commodities.* A Full Walrasian Equilibrium (FWE) is then a Walrasian Equilibrium with the added condition that *"no firm sees that it can increase its profits by altering its trade decisions assuming that the set of marketed commodities other than its own will remain the same"* (p.228). In other words, at the first stage of the trading procedure, the set of traded commodities is determined by the condition that a new commodity must not make positive profit. Thus overall, the equilibrium concept we offer is common in this type of economies and is close in spirit to the Arrow-Debreu equilibrium concept as it keeps all agents competitive (see next section on efficiency issues).

Individual Maximization Problem

We denote the saving of each individual at time t by s_t (by our representative agent assumption this will also the aggregate savings), the amount they devote to the safe asset by ϕ_t and their investment in sector j by F_t^j. Since we have a representative agent model, these also correspond to their respective aggregate amounts. Also we denote the number of sector open at time t by n_t. Each agent solves the following maximization problem

$$\max_{s_t,\phi_t,\{F_t^j\}_{0\leq j\leq 1}} \log c_t + \beta \int_0^1 \log c_{t+1}^j \, dj \tag{5}$$

subject to

$$\phi_t + \int_0^1 F_t^j dj = s_t \tag{6}$$

and obviously for the non-traded securities, $F_t^j=0$ for all $j > n_t$. The consumer also takes w_t,

$$c_{t+1}^{j}=\rho_t^j(r\phi_t+RF_t^j) \tag{7}$$

$$c_t+s_t\leq w_t \tag{8}$$

n_t and ρ_t^j as given and where ρ_t^j denotes the marginal product of capital in state j at time t. Note also, we have used the fact that $P_t^j=1$ for all j that is traded in writing the budget constraint of the individual consumer. It is particularly important to note that n_t is taken as given by these price taking individuals and it will be determined by the competition among intermediate sector firms and the aggregate resource constraint that is obtained by summing (6) across all individuals. Using (8), this aggregate constraint is expressed as

$$\int_0^{n_t} F_t^j dj=s_t-\phi_t \tag{9}$$

Finally, though each individual takes the marginal product of capital as given, in equilibrium this is determined as

$$\rho_t^j=\alpha A(r\phi_t+RF_t^j)^{\alpha+\delta-1} \tag{10}$$

Individual Decision Rules

We start our analysis with the basic case of no endogenous growth, $\delta=0$ and no fixed endowment (but our results carry over to all cases with $\delta<1-\alpha$). From logarithmic preferences we directly obtain the following saving rule

$$s_t=\frac{\beta}{1+\beta}w_t \tag{11}$$

In other words, the logarithmic specification implies that the exact risk-return trade-off that the agents face will not influence their saving rate. Nevertheless it will crucially influence the allocation of funds into safe and risky assets. In particular ϕ_t and F_t^j will be chosen so as to maximize the static expected return in the second period.

Next we can state a simple Lemma that will be important throughout our analysis[6];

[6] Since we have a maximization problem with respect to a continuum of choice variables, all our claims should be read as "almost everywhere". However, this technical detail does not affect any of our results.

Lemma 1: Let J(t) be the set of securities traded in equilibrium at t; then $F_t^j = F_t^{j'} \quad \forall\, j, j' \in J(t)$.

The proofs of all lemmas and propositions are provided in the Appendix. The main idea of this lemma is very simple; because of Bertrand competition in the stock market, each individual is facing a constant price for each of these symmetric Arrow securities and therefore he will want to purchase an equal amount of all the securities that are being traded. Therefore, when $j \leq n_t$ sectors are open, it follows that $F_t^j = F_t$. This reduces our problem to the following maximization program where as before n_t is treated as a parameter;

$$\max_{\phi_t, F_t} \; n_t \log(RF_t + r\phi_t) + (1-n_t)\log(r\phi_t) \tag{12}$$

subject to $s_t = \phi_t + n_t F_t$. This maximization yields the following decision rules;

$$\phi_t = \frac{(1-n_t)R}{R - rn_t} s_t \tag{13}$$

and

$$F_t^j = \begin{cases} \dfrac{R-r}{R-rn_t} s_t & \forall j \leq n_t \\ 0 & \forall j > n_t \end{cases} \tag{14}$$

Equation (14) is expressed diagrammatically in Figure 3 as a relationship between a pseudo-aggregate demand for each risky asset (solved for the equilibrium price) and the number of sectors that are already open. It is clear from both the diagram and equation (14) that the demand for each asset grows as the number of sectors that are open increases. This is due to the fact that the more sectors are open, the better are the risk diversification opportunities and the more willing are the consumers to further reduce their investment in the safe asset and increase their investment, F_t^j, in the sectors that are open.

At this stage, it is convenient to introduce a restriction on the region of admissible parameters, which, as we will show, guarantees the uniqueness of the equilibrium process. It will be shown in section 5 that with a refinement of our equilibrium concept, multiplicity of equilibria will not arise even when assumption (A) is not satisfied, but for now adopting this assumption simplifies our exposition.

Assumption (A): $R \geq (2-\gamma)r$

Equilibrium

Proposition 1:

The Equilibrium exists. If (A) is satisfied the equilibrium is unique, and the proportion of sectors open is given by

$$n_t^* = \min \left\{ \frac{(R+r\gamma) - \sqrt{(R+r\gamma)^2 - 4r\left[\frac{(R-r)(1-\gamma)}{D}s_t + \gamma R\right]}}{2r}, 1 \right\} \tag{15}$$

All security prices are identical and demand for each security is given by (14).

FIGURE 3 HERE

This equilibrium is expressed as the intersection of the pseudo-aggregate demand of each risky asset with the minimum size requirement of sectors in Figure 3. It can be checked that as it should be expected, when $s_t = D$ so that there is enough money to open all the intermediate sectors, n_t^* indeed takes the value 1 and thus (16) defines a continuous function as in Figure 3. Note that n_t^* is one of the roots of a second order polynomial; if assumption (A) were violated existence would be still guaranteed, but the two schedule F(n) and M(n) could cross more than once in the admissible range (0,1), so multiple equilibria would arise.

From the equilibrium of Proposition 1, the dynamic transition equations also readily follow;

$$K_{t+1} = \begin{cases} \frac{r(1-n_t^*)}{R-rn_t^*} R\Gamma K_t^\alpha \equiv \sigma_B(n^*(K_t),r,R)\Gamma K_t^\alpha & prob.\ 1-n_t^* \\ R\Gamma K_t^\alpha \equiv \sigma_G(n^*(K_t),r,R)\Gamma K_t^\alpha & prob.\ n_t^* \end{cases} \tag{16}$$

where $n_t^* = n^*(K_t)$ is given by equation (15) and Γ is a constant term defined as

$$\Gamma = \frac{\beta}{1+\beta} A(1-\alpha) \tag{17}$$

The important feature to note is that the level of capital stock next period will depend on whether the society is lucky in the current period or not. Moreover, the probability of this event also changes over time. As the economy develops, it can afford to open more sectors, i.e. higher n_t^*, and the probability of transferring a large capital stock to the next period, that is the probability of a lucky event, increases. Also note from (16) that the expected the productivity of an economy depends on its level of development and diversification; as n_t increases, expected productivity, which is equal to $\sigma(n^*,r,R) \equiv n^*\sigma_G(n^*,r,R) + (1-n^*)\sigma_B(n^*,r,R)$, increases as well. However interestingly, the productivity level conditional on good news,

$\sigma_G(n^*,r,R)$, is independent of the diversification level of the economy. This is a feature of the logarithmic presences whereby the effect of higher investment in risky assets as n increases is exactly offset by the fact that no money was invested in projects that did not pay-off. This issue will be discussed further when we consider more general preferences in section 5.

To formalize the dynamics of development, we define the following concepts;

(i) QSSB: "quasi steady state" of an economy which always has unlucky draws in the sense that the sectors invested by this economy never pay-off.

(ii) QSSG: "quasi steady state" corresponding to an economy which always receives good news.

The capital stocks corresponding to these two quasi steady states will respectively be

$$K^{QSSB}=\left\{\frac{r(1-n^*(K^{QSSB}))}{R-rn^*(K^{QSSB})}R\Gamma\right\}^{\frac{1}{1-\alpha}} \tag{18}$$

and

$$K^{QSSG}=\{R\Gamma\}^{\frac{1}{1-\alpha}} \tag{19}$$

In particular, if uncertainty can be completely removed (i.e. $n(K^{QSSG})=1$), there will exist a steady state; a point, if reached, from which the economy will never depart. From equation (15), the condition for this steady state to exist is that the saving level that corresponds to K^{QSSG} to be sufficient to ensure investment in all the intermediate sectors. Thus

$$D<\Gamma^{\frac{1}{1-\alpha}}R^{\frac{\alpha}{1-\alpha}} \tag{20}$$

We will refer to this particular case of (19) as K^{SS}.

When this steady state exists, the pattern of development will typically look as expressed in Figure 4. At very low capital levels (region I), the concavity of the production function guarantees positive growth. Then, there is a range (region II) in which growth only occurs conditional on good draws, when some of the risky investments pays off. Remember that (16) implies that the probability of a good realization is increasing with the capital stock, so growth is relatively unlikely close to K^{QSSB}, which is a sort of point of attraction for a poor economy. As the economy receives a lucky draw, the probability of a lucky draw next period also increases (but also bad news now becomes more damaging since more is invested in risky projects[7]). Even when growth occurs over a number of periods, however, the

[7] The current specification of the model has the unrealistic prediction that set-backs in the pre-take-off stage can only take the form of true economic disasters, taking the economy back to a very primitive stage of development. The reason is that the more sectors are open, the less will be the amount that agents are willing to put in the safe asset for insurance purposes. Therefore, a bad realization, though now much

economy is still exposed to large undiversified risks, and will typically experience serious set-backs. As the economy enters region III, finally, all idiosyncratic risk is removed (since all sectors are open and an equal amount is invested in all sectors) and we observe a deterministic convergence to K^{SS}.

FIGURE 4 HERE

We have established so far that an economy that receives a series of lucky draws will be unchained from its low productivity "primitive accumulation" process and will reach full diversification and full industrialization. As Figure 4 suggests, given the specification of this economy the equilibrium stochastic process has a unique ergodic set - which in this case is just a point. Therefore, no underdevelopment trap exists and take off will occur almost surely, although it will take longer and may be painfully slow for countries that are unfortunate.

Corollary 1: Suppose (20) is satisfied, then $plim_{t\to\infty}(K_t)=K^{SS}$.

We will return to this issue in section 5, where we show that this result no longer holds when agents have a relative risk aversion greater than one.

What will happen to the randomness of the growth process? To answer this question, the most natural measure to look at is the variance of the productivity of the capital stock defined above, $\sigma(n^*(K_t),r,R)$, which will be denoted by $Var(\sigma(n))$. This function corresponds to the variance of the growth rate of capital (and output) conditional on the capital level and after removing the "convergence effects" induced by the neoclassical technology. This variance will be subject to forces; first, as the economy develops more money is invested in risky assets and secondly, as the economy develops, more sectors are opened, thus idiosyncratic risks are better diversified. Corollary establishes that under some parameter configurations the first effect may dominate at the very early stages of development and the beginning of the development process may be associated with an increase in the variability of economic activity. At later stages, however, and under the alternative parameter configurations at all stages, the second effect will definitely dominate and higher levels of

more unlikely, will have devastating effects. In contrast, the set-backs observed in the historical development process that were mentioned in the introduction do not have this feature. This feature of the model can be avoided by altering the specification for the structure of returns of the risky assets. An example would be to assume that in every period, a fixed measure $z>0$ of all possible projects have positive return, with each asset having the same probability of success. In this case the probability of a catastrophe in which no risky asset gives a positive returns would be decreasing in the number of active sectors and, tend to zero as n tends to z. But, bad realizations and crises in which a small proportion of the risky assets is successful, can still occur with positive probability until full diversification is reached.

income will correspond to less randomness[8].

Corollary 2: *(i)* If $\gamma > \frac{R}{2R-r}$, then $\frac{\partial Var(\sigma(n(K_t)))}{\partial K_t} \leq 0, \quad \forall K_t$.

(ii) If $\gamma < \frac{R}{2R-r}$, then $\exists\ \tilde{K}\ s.t.\ n^*(\tilde{K}) = \frac{R}{2R-r} < 1$, and

$$\frac{\partial Var(\sigma(n(K_t)))}{\partial K_t} \leq 0, \quad \forall K_t > \tilde{K}\ ; \qquad \frac{\partial Var(\sigma(n(K_t)))}{\partial K_t} \geq 0, \quad \forall K_t < \tilde{K}.$$

Therefore, our model predicts that the variance of the growth rate (conditional on "convergence effects") is either uniformly decreasing with the size of the accumulated capital (case *(i)*) or exhibits an inverse U-shaped relation with respect to it (case *(ii)*). More precisely, if either γ is sufficiently large, or the productivity of risky projects is much higher than that of the safe asset, then the relation will be monotonic. We can recall at this point that the cross-sectional regression presented in section 2 (Figure 1) seems consistent with an inverse U-shape, though the evidence for the increasing region of the "hump" was obtained from relatively fewer observations (the fifteen poorest countries) and is much less solid than that for the decreasing region. Additionally, the evidence reported in section 2 from Quah (1993) and Benhabib and Gali (1994) suggests a pattern where countries in the middle income range have more variability.

Endogenous Growth

It is straightforward to extend our analysis for endogenous growth and the main return from this would bc that the distinction we drew earlier between primitive accumulation, take-off and industrial growth will now fit more nicely since at the last stage the economy will exhibit steady growth. It suffices to note here that all our results carry through to this case and the law of motion of the capital stock takes the form;

[8] Remember that in section 2 we computed the variance of the growth rate after simply removing a linear trend from the time series of the growth rate of each country. This was intended as a rough approximation to conditioning for the presence of convergence effects on variability (which would bias upwards the estimated variability of the growth rate of countries which are farther from their steady state with respect to those which are closer to it). One can make this analysis more precise by either explicitly estimating the "convergence path" or using some more sophisticated approximations which takes into account that the transitional dynamics are non-linear.

$$K_{t+1}=\begin{cases}\dfrac{r(1-n_t^*)}{R-rn_t^*}R\Gamma K_t & prob.\ 1-n_t^* \\ R\Gamma K_t & prob.\ n_t^*\end{cases} \tag{21}$$

It can be seen that as n_t reaches one, the economy achieves a constant and non-random growth rate.

4) Pareto Optimal Allocations and Sources of Inefficiency

The Socially Optimal Portfolio Decision

In this section we will explain why the equilibrium of section 3 is not Pareto optimal, characterize the optimal allocation and discuss when this type of inefficiency may be important. Yet, first, it has to be noted that we are only dealing with the issue of static efficiency. The additional source of inefficiency arising from the overlapping generations aspect will not be discussed in this section, though we will return to it later. For the purpose of the analysis of static efficiency, suppose that instead of decentralized trading in security market, a social planner chooses the allocation of resources in order to maximize the welfare of the current generation. The planner would also solve the program given by (5), with the only difference that n_t will also be a choice variable and (10) will become an additional constraint. The constant saving rule of the individual maximization will again apply due to the simple form of the preferences. Yet, since it no longer follows that the same amount will be invested in all open sectors, (12) changes to

$$\max_{n_t,\phi_t,\{F_t^j\}_{0\le j\le n_t}} \int_0^{n_t}\log(RF_t^j+r\phi_t)dj+(1-n_t)\log(r\phi_t) \tag{22}$$

We now characterize the solution to this problem.

Proposition 2:

Let $n^*(s_t)$ be given by (15), then $n^{FB}(s_t)\ge n^*(s_t)\ \forall s_t$, $n^{FB}(s_t)>n^*(s_t)\ if\ s_t<D$.

The allocation of funds in the first best is as follows;

$$\exists\ j_t^*<n^{FB}(s_t)\ s.t.\ \begin{cases}F_t^j=F_t & if\ j<j_t^* \\ F_t^j=M(j) & if\ n^{FB}(s_t)\ge j_t>j_t^* \\ F_t^j=0 & if\ j>n^{FB}(s_t)\end{cases}$$

FIGURE 5 HERE

Figure 5 gives the diagrammatic form of the first-best allocation (represented by the shaded area). Note that the qualitative properties of the first best are very similar to our dynamic equilibrium. The economy is still characterized by three stages; primitive accumulation, take-off and industrial growth but progress is faster on average. But the transition equation looks considerably more complicated than (16) since the total return is different in each state.

The reason for the failure of the decentralized economy to reach the first best can be seen as a form of *pecuniary externality* due to missing markets. As an additional sector opens, all the existing projects become more attractive relative to the safe asset because they have less risks associated with them (more of the risk is diversified) and as a result, risk-averse agents are more willing to buy the existing securities. However, it is important to note that markets are not assumed to be missing, but this is endogenously determined in equilibrium. This pecuniary externality can be clearly seen from Fig. 4 where the amount that a representative agent wants to invest in each sector is an increasing function of n_t.

In the decentralized equilibrium competition among the intermediate sector firm forces them to charge linear and equal prices, and each consumer then wants to buy an equal amount of all the traded securities which makes the allocation represented in Figure 5 impossible. This inefficiency result is in line with the more general results that Makowski (1980) derives on the Full Walrasian Equilibrium. In particular, Makowski shows that in the presence of non-convexities, the FWE will not be Pareto Efficient because a suboptimal number of commodities will be introduced. In our model, too few projects are undertaken and this slows down the process of industrialization.

The reason for the sub-optimality of the decentralized equilibrium is fundamentally related to the fact that a full Arrow-Debreu equilibrium fails to exist. In other words, there is no price vector at which all excess demands are zero or negative. This can be explained using Figure 6. In the top part, we have the supply and demand of a security for a sector that has no minimum size requirement. The supply is horizontal at $1 because competition forces prices to be equal to marginal cost (as in standard competitive analysis) and the marginal cost of investing $1 in this sector is $1. The bottom panel has the supply and demand schedules for a sector with positive minimum size requirement. The supply is discontinuous because at no price we can supply x units of this security if x is below the minimum size. If as drawn in Figure 6, demand is sufficiently low there is no equilibrium price and the market for this security is missing. If price falls below 1, supply is zero and demand is large so there is excess demand. If the price is larger than 1, there is excess supply. This is why a decentralized equilibrium with linear prices exists only *conditional* on the number of sectors that are open and the auctioneer has to look for a fixed point of a mapping that has n_t in it as well as the prices of securities. It can be seen here that since no two sectors are perfect substitutes, introducing lotteries would not make any difference at all because there will still be separate demand for the securities of each sector, and the equilibrium will still fail to exist (in fact lotteries will never be used).

FIGURE 6 HERE

Inefficiencies Under Alternative Market Structures

It could suspected that the failure of the decentralized mechanism to achieve efficiency can be related to our assumption of no inter-firm share trading. In this subsection, we will first establish that under very weak and plausible assumptions, there is no market structure that can support the efficient allocation as an equilibrium. And secondly, we will show that with a simple refinement of our equilibrium concept that, even in the presence of inter-firm share trading, the equilibrium characterized in Proposition 1 is the unique equilibrium.

First, note that if we allow firms to buy the shares of other firms, a giant "superintermediary" may emerge and buy up all other firms. If further competition were absent, the superintermediary would be able to offer non-linear prices to consumers. In particular he could issue a bond which replicates the structure of returns required by the optimal allocation, and this would destroy our previous equilibrium since agents prefer a portfolio consisting entirely of this bond to the previous equilibrium portfolio. However, this is not sufficient to lead to efficiency and no decentralized equilibrium can achieve the first-best (without intervention). This is established by the following corollary.

Corollary 3:

Suppose intermediary firms can costlessly trade shares and there is free-entry into the intermediate sector. Then the Pareto optimal allocation cannot be sustained as a decentralized equilibrium.

The intuition for this corollary can be given as follows. The Pareto optimal allocation is as shown in Figure 5 and so, each individual needs to hold a "unbalanced" portfolio. But we know that if an individual faces equal prices, he would like to have a balanced portfolio. Thus, it is possible for an entrant to enter the no-minimum-size sectors, charge a price sufficiently close to 1 and sell to the representative individual. Therefore with free-entry, the Pareto optimal allocation cannot be sustained. Therefore overall, despite the fact that our model is quite similar to a convex model, there are serious inefficiencies associated with the interaction of the endogenous commodity space and the indivisibilities; and these inefficiencies cannot be avoided by any market structure.

Now we can establish the stronger result that even when the "ad hoc" assumption of no inter-firm share trading is relaxed, the equilibrium characterized in Proposition 1 remains the unique equilibrium, provided that we refine the equilibrium concept along the lines of Riley's idea of *reactive* equilibrium. According to Riley (1979) to destroy a proposed equilibrium, it must be possible to introduce a contract that (i) is profitable and (ii) does not become unprofitable when still more contracts are added by a new entrant *a posteriori*. The intuitive appeal of this equilibrium concept in our model is clear: it seems little implausible that a firm or intermediary will buy up the whole or a large part of the industrial sector, if it knows that there is no equilibrium at which it can enjoy a profit. In fact, more to the point, using the same argument of the proof of corollary 3, it can be shown that with our original equilibrium concept and interfirm share-trading, there exists no equilibrium.

To state our idea more formally, let $\pi(C|C \cup C')$ denote the total profit made from a set of contracts C when consumers have access to these contracts and an additional set C'. Then;

Definition 2 (Riley Equilibrium): Consider the vector $(n_t^*, \{C_t\})_{t\geq 0}$ where n_t^* denotes the number of open sectors at time t and C_t is the set of contracts being offered to consumers by the financial intermediaries. This vector is an equilibrium iff

(i) $ed_t^j(n_t^*, C_t^*)=0$ for all $j \leq n_t^*$ (given the set of contracts, supply of security j is equal to demand in all the open (n_t^*) sectors);

(ii) $\nexists$ C' based on a feasible production plan such that $\pi(C'|C^* \cup C')>0$ and $\forall C'': \pi(C''\mid C^* \bigcup C' \bigcup C'')>0,\ \ \pi(C'\mid C^* \bigcup C' \bigcup C'')\geq 0$, and that conditional on C'', C' remains feasible in the sense of $ed_t^j(n_t^*, \{C^* \cup C' \cup C''\}) \leq 0$ for all securities j offered by the set of contracts $C \cup C \cup C''$.

Proposition 3:

Suppose intermediary firms can costlessly trade in shares. Then, the equilibrium characterized in Proposition 1 is the unique Riley equilibrium.

Since we have demonstrated that there is a potentially important market failure and that no decentralized market mechanism can solve this problem, government policy may be required. The necessary government policy is easy to characterize and can be seen to take the form of direct subsidies or regulation that would support high indivisibility projects. It can be noted that this type of policy is in fact not too different from the pattern of industrial policy we observe at early stages of development in some countries (for instance in Germany, with state intervention, there was a large amount of capital invested in heavy industries at the expense of light industries, Gerschenkron (1962, p. 15), see also Cameron (1972)).

5) Attitudes Towards Risk and Underdevelopment Traps

Our results so far have been derived using logarithmic preferences. This has proved to be a very convenient functional form as it induces a constant savings rate. However, these preferences are also very special as they imply that intra-temporal preferences are also logarithmic and thus the rate of relative risk-aversion is constant and equal to one. Yet, risk-aversion plays a key role in our analysis and it is instructive to investigate the implications of more general preferences. We turn to this issue in this section. Most of our results will hold with more general preferences, the only exception is Corollary 2 which showed the uniqueness of the ergodic state of the equilibrium stochastic process. We show that when agents have a rate of relative risk-aversion greater than 1, we may end-up with an equilibrium stochastic process that is non-ergodic. That is underdevelopment traps are possible. An economy that receives a series of unlucky draws may reach a level of capital stock that is sufficiently low that only a limited number of the risky sectors can be opened. In response, the risk-averse consumers of this economy decide to invest only a small proportion of their wealth in risky assets, and the productivity of the capital stock is endogenously low, therefore growth is prevented.

To illustrate these features, we consider the following preferences

$$\log c_t + \beta \log\left(\int c_{t+1}(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} \tag{2'}$$

where $\theta > 0$. . Note that these preferences are not von-Neumann-Morgenstern but of the Kreps-Porteus variety. They are adopted so as to enable us to separate the impact of attitudes towards risk from intertemporal substitution. The form we have adopted still gives rate of intertemporal substitution equal to one, thus a constant savings rate[9] but the rate of relative risk-aversion need no longer equal 1, but is given by θ which can take any positive value.

It can also be noted that Lemma 1 still applies as it was derived only relying on competition among intermediaries and the form of the technology. After some simple algebra, the optimal portfolio can be characterized as

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[9] Generalizing the preferences to allow for other forms of intertemporal substitution would also be interesting, although somewhat cumbersome. If there are precautionary savings, for instance, these will fall as more diversification becomes possible.

$$\max_{\{F_t,\phi_t\}} n_t\frac{(r\phi_t+RF_t)^{1-\theta}}{1-\theta}+(1-n_t)\frac{(r\phi_t)^{1-\theta}}{1-\theta} \tag{24}$$

subject to

$$n_tF_t+\phi_t=s_t \tag{25}$$

where n_t is again taken as given by the representative consumer.

The decision rules that follow are

$$\phi_t=\frac{R(R-rn_t)^{-\frac{1}{\theta}}s_t}{(R-rn_t)^{\frac{\theta-1}{\theta}}+rn_t((1-n_t)r)^{-\frac{1}{\theta}}} \tag{26}$$

$$F_t=\frac{[(r(1-n_t))^{-\frac{1}{\theta}}-(R-rn_t)^{-\frac{1}{\theta}}]rs_t}{(R-rn_t)^{\frac{\theta-1}{\theta}}+rn_t((1-n_t)r)^{-\frac{1}{\theta}}} \tag{27}$$

Notice that in the case in which $\theta\rightarrow 1$ the solution is identical to that obtained in section 3. Yet, in general, it is not possible to find a closed form expression analogous to (15) for the proportion of open sectors. Also, in this case static multiple equilibria may arise, i.e. there exist alternative equilibrium configurations with different numbers of open sectors for a given state of the system. Proposition 4 shows that this can only occur when the relative rate of risk aversion, θ, is greater than one and the economy has a sufficiently high amount of savings to open all sectors.

Proposition 4:

(i) An equilibrium exists for any θ.

(ii) If $\theta>1$, $\exists\ \bar{s} > D$ such that:

(a) if $s_t<D$, then the equilibrium exists and is unique;

(b) if $D\geq s_t\geq\bar{s}$, then there exist two equilibria with n_t^*, n_t^{**} sectors open such that $n_t^*<n_t^{**}=1$;

(c) if $s_t\geq\bar{s}$, then there exists a unique equilibrium with $n_t^*=1$.

(iii) If $\theta<1$ and assumption (A) is satisfied, the equilibrium is unique for $s_t\geq D$.

Let us go back to Figure 3. Since $F(n=1)=s_t$ and $M(n=1)=D$, then $F(n=1)$ is always vertically below M(n=1) when $s_t < D$. This, together with the fact that F(n) is strictly convex when $\theta \geq 1$ rules out the possibility of multiple crossing when the stock of savings is not sufficient to open all sectors. Proposition 1 established that, under assumption (A), in the logarithmic case the equilibrium is also unique when $s_t = D$; geometrically, M(n) is steeper than F(n) at n=1. But this is no longer true for larger risk aversion parameters, and we prove that when $s_t = D$, F(n) necessarily intersects M(n) from below at n=1. Since F(0)>0, however, this implies that F(n) must also intersect M(n) at some lower level of n, and therefore, we have static multiple equilibria. Intuitively, if all agents invest a lot in the risky assets, all sectors can be opened and, because all risks are diversified, the representative agent wants to invest a high proportion of his savings (in our case, all of it) in risky assets. But there is also an equilibrium at which agents invest less in risky assets, and still no profit opportunities are left to individual entrants (unless they can coordinate their decisions). Note that a multiplicity of the same nature also applies to the benchmark logarithmic case if assumption (A) does not hold. Finally, we have checked that in cases with $\theta \leq 1$ the multiplicity does not normally arise, although it has not been possible to rule out formally the possibility of pathological multiple crossing when $s_t < D$.

Although interesting per se, this multiplicity of equilibria only has limited relevance because it can be shown that if we allow interfirm share trading and apply the concept of Riley Equilibrium as in Proposition 3, the multiplicity will disappear and the economy will always choose the Pareto preferred equilibrium which is the one with the highest number of open sectors. In this case of interfirm share trading, the only difference from our baseline analysis would be that the function $n^*(K_t)$ will have a discontinuity at $s_t = D$, so that take-off will be even more abrupt. Corollary 4 states this result;

Corollary 4: Suppose interfirm share-trading is allowed, then for all values of θ, there is a unique Riley equilibrium where n_t^* is equal to the maximum of the static multiple equilibria.

The intuition underlying this result is quite straightforward. Whenever we are in a Pareto inferior equilibrium, a firm can enter buy a sufficient proportion of sectors and offer a larger portfolio. Also since he can do this with a balanced portfolio, no reaction can force it to make negative profits.

Despite the fact that the static multiplicity associated with a high degree of risk-aversion is of limited interest, we will now show that there is an issue of "dynamic multiplicity" or underdevelopment trap, which is more interesting and robust. The stochastic process associated with equilibrium will be unique but will posses more than one ergodic set.

To analyze this issue, consider the law of motion for the capital stock. After some algebra, this can be written as;

$$K_{t+1} = \begin{cases} \left[\dfrac{Rr((1-n_t)r)^{\frac{1}{\theta}}}{(R-rn_t)((1-n_t)r)^{\frac{1}{\theta}} + rn_t(R-rn_t)^{\frac{1}{\theta}}} \right] \Gamma K_t^{\alpha} \equiv \sigma_B(n^*(K_t),r,R,\theta)\Gamma K_t^{\alpha} & \text{with prob } 1-n_t \\ \left[\dfrac{Rr(R-n_t r)^{\frac{1}{\theta}}}{(R-rn_t)((1-n_t)r)^{\frac{1}{\theta}} + rn_t(R-rn_t)^{\frac{1}{\theta}}} \right] \Gamma K_t^{\alpha} \equiv \sigma_G(n^*(K_t),r,R,\theta)\Gamma K_t^{\alpha} & \text{with prob } n_t \end{cases} \tag{27}$$

where Γ is defined by (17) above. The term $\sigma_G(n^*(K_t),R,r,\theta)$ measures the productivity of the capital stock conditional on good news and it can be recalled that in the case of logarithmic preferences, this term turned out to be a constant. In this case we can state;

Lemma 2: $\theta > (<) 1 \Rightarrow \dfrac{\partial \sigma_G}{\partial n} > (<)\ 0$.

Lemma 2 implies that as the number of open sectors increases, the productivity of the investment increases. On the other hand, the neoclassical technology implies decreasing marginal product of capital, thus we have two counteracting effects. Under some parameter configurations, the net effect is such that the marginal product of capital is locally increasing in the level of the capital stock. It will then possible to find a region from where the economy will not be able to escape, in other words, an underdevelopment trap. Once the capital stock is low enough, even in the presence of only good shocks, the economy will not grow beyond a certain point. Also note that in models where the aggregate technology is linear in capital, as in the endogenous growth case discussed above, the marginal product of capital will be everywhere increasing, thus such traps will be more common.

Proposition 5: Suppose (20) is satisfied, and $\theta > 1$.
Then, for a generic subset of parameter values $(\alpha, A, \beta, \gamma, R, r, D)$;
$\exists [K^{+}, K^{++}],\ K^{+} < K^{++} < K^{SS},\ s.t.\ K_t \in (K^{+}, K^{++}) \Rightarrow K_{t+i} \in (K^{+}, K^{++})\ ,\ \forall i \geq 0$.

Formulated differently, if the rate of relative risk aversion is greater than 1, there exists a non-trivial set of economies characterized by a non-ergodic equilibrium stochastic process. The intuition of this proposition can be obtained by noting that the growth rate of this economy conditional on receiving good news is given as $MPK(K_t) \equiv \sigma_G(n^*(K_t),R,r,\theta)\Gamma K_t^{\alpha-1}$. If MPK(K) is locally increasing with K_t, which it can be when $\theta > 1$, then we can have multiple values of the capital stock for which MPK=1 (multiple quasi-steady-state conditional on good news). The corresponding dynamics are described by Figure 7. For K_t in the right neighborhood of K^{++}, the growth rate of the economy, even when it receives only good shocks is negative, thus the economy cannot grow beyond K^{++}. A similar argument using σ_B, the productivity of the capital stock in the presence of bad shocks, will establish the presence of a certain level of the capital stock, K^+ below which the economy cannot fall. $[K^+,K^{++}]$ will be an ergodic set or in other words, an underdevelopment trap. This possibility of underdevelopment trap is different from a static multiplicity, as there is no indeterminacy, and instead captures the possibility of dynamic lock-in in the context of development which were pointed out by David (1985) and Arthur (1989) in different contexts.

FIGURE 7 HERE

It has to be emphasized that the nature of this underdevelopment trap is also different from others in the development literature or elsewhere that are usually driven by increasing returns. In our model, at earlier stages of development agents have access to technologies which are as productive as those available to advanced countries. However, the lack of diversification opportunities makes the adoption of high productivity technologies risky, and agents choose an aggregate technology that is relatively less productive. So, in our model micro level non-convexities only play a role when are associated with risk aversion. For this reason, when risk aversion is not high enough underdevelopment traps can be ruled out. This is stated by the following proposition (note that corollary 1 above is a special case);

Proposition 6: Suppose (20) is satisfied, and $\theta \leq 1$, then $plim_{t\to\infty}(K_t)=K^{SS}$.

This proposition establishes that with low levels of risk-aversion, underdevelopment traps are not possible, thus makes it clear the role that risk-aversion plays in the existence of these traps.

6) International Capital Flows and Development

We have so far dealt with a closed economy and abstracted from the possibility of capital flows. This can be an important omission. Even in the process of industrialization of Western Europe, capital flows were important. The areas that emerged as industrial centers were often attached to (or were themselves) financial centers that attracted capital from other regions and towns and also subsequently provided capital to them (Braudel (1979,1984), Neal (1990)). These observations point out to the importance of analyzing the role of diversification and capital scarcity in the context of a model where inter-country capital flows are possible. Such an analysis will also give us an indication of the importance of the mechanisms we have developed in this paper for the development problems today when international capital flows are relatively more important.

The first point to note is that if the country we are dealing with is a small open economy and capital can flow in and out without any transaction costs or enforceability problems, then all our results disappear. This is of course natural; our main mechanism is derived from capital shortages and the small open economy with perfect capital mobility assumes capital shortages away. This characterization of events does not however seem to accord well with the experience of developing countries today. Many of those try to attract foreign capital but often not very successfully and even try to limit the extent to which domestic capital is allowed to go abroad. Also, capital shortages are often mentioned as an important problem in the context of development (e.g. Viner (1958), Singer (1958), Lewis (1958)). The findings of very imperfect consumption insurance across even OECD countries and almost no insurance across developing economies also give a strong indication that the presence of international capital flows does not correspond to free capital mobility and therefore, small open economy with perfect capital mobility does not seem to be a useful starting point.

In this section we will first look at a two-country model with free capital mobility and then turn to small open economy in the presence of enforcement problems. A two-country model with free capital mobility has interesting implications for the historical process of development and leads to quite rich dynamics. One important implication is that if there are decreasing returns to capital (i.e. a neo-classical production function), a country can act as net exporter capital depending on its own and the world's level of development and then as the world economy develops further, it becomes a net importer. Therefore at early stages, capital flows to the more industrialized regions and countries but then these areas become exporters of capital and ensure convergence across the two countries. This finding is contrary to the prediction of most other growth models that predict either that capital always flows to the poor country (e.g. the standard neo-classical model) or to the rich country (models with increasing returns to capital) and thus accords much better with the development experience of Western Europe and also with that of the relations between the North and the South today. For instance, while in the eighteenth century London attracted a lot funds from Europe, especially from Amsterdam, the flow of funds was reversed in the nineteenth century with Amsterdam attracting funds from London (Neal (1990), Chapter 11). Additionally we also show that in the case of Romer type technology or a small open economy with enforcement problems, the possibility of capital flows can seriously damage the prospects of development in a less developed country.

a) A two-country model. Neo-classical technology ($\delta=0$).

Suppose that there are two countries with identical technologies as described in section 3. That is, both countries have the same production function and access to the same set of investment opportunities. Therefore if the state of nature is j, project j pays a rate of return equal to R in both countries and also the minimum size requirement of this project is M(j) in both countries as given by equation (1)[10].

We also assume that capital flows between the two countries are free: as in the previous sections, intermediaries compete a la Bertrand in the financial market but the assets sold by these intermediaries can be bought by consumers of both countries. Further, since the investors are the old agents who consume all the returns of their investments, they do not care whether the assets they are investing are domestic or foreign.

First, suppose that when both countries are closed, n_1 sectors would open in country 1 and without loss of generality n_1+n_2 in country 2 (with $n_2>0$, this is for conformity with our later notation). Now what will happen when we allow for international capital flows? The answer is not straightforward. On the one hand, capital wants to go to the large economy (country 2) because there are better diversification opportunities there. But on the other, because of the decreasing returns to capital, the rate of return on an additional unit of capital is higher in country 1. These two forces will in general play against each other and will determine the evolution of the two countries. It will also imply that we cannot unambiguously conclude that introducing capital flows will provide better diversification opportunities. Another point can also be noted. Although we implicitly implied that with perfect capital mobility, country 2 will remain larger, this does not necessarily follow. Since capital can costlessly move form one country to the other and they both have the same technological possibilities (and no durable capital beyond one period), the identity of the countries is indeterminate. In what follows, we will avoid this multiplicity problem by calling the country that attracts more funds (has more sectors open) country 2.

For more careful analysis of this economy, let us denote the amount of risky investment in sector j of country 1 by F_1^j and the amount of risky investment in sector j of country 2 by $F_2^{j'}$. Similarly, let ϕ_1 be the investment in the safe asset of country 1 and ϕ_2 the investment level in the safe asset of country 2. These variables should have time-subscripts but these are dropped in this section not to complicate the notation. In fact most of this section will deal with the static problem of fund allocation between the two countries. Our result in the closed economy case was that all open sectors would receive the same amount

[10] It is of course possible to allow the minimum size requirements to vary between the two countries in which case there will be additional gains from capital flows than the ones emphasized. But in this section we prefer to abstract from these in order to maintain the analysis as tractable as possible.

of investment in the decentralized equilibrium. We will start by deriving similar results for our two-country model.

Lemma 3: All risky assets that are traded in equilibrium have the same price.

This result mirrors the one we had in the closed economy case. The prices of traded securities are determined by the technology because of the constant returns aspect. This implies that we can characterize the individual consumer's decision in a simple form.

$$\max \int_0^{n_1} \log[\rho_1^j(r\phi_1 + RF_1^j) + \rho_2^j(r\phi_2 + RF_2^j)]dj + \int_{n_1}^{n_1+n_2} \log[\rho_1^j(r\phi_1) + \rho_2^j(RF_2^j + r\phi_2)]dj$$
$$+ \int_{n_1+n_2}^{1} \log[\rho_1^j(r\phi_1) + \rho_2^j(r\phi_2)]dj \qquad \textbf{(28)}$$

by choosing $\{F_1^j\}$, $\{F_2^j\}$, ϕ_1, ϕ_2 and subject to the constraint

$$\int_0^{n_1} F_1^j dj + \int_0^{n_1+n_2} F_2^j dj + \phi_1 + \phi_2 = S \qquad \textbf{(29)}$$

where ρ_i^j is the marginal product of capital in country i in state j and where we have denoted, following our earlier convention that there are n_1 sectors open in country 1 and $n_1 + n_2$ sectors open in country 2 and as before all the states are equi-probable. Note that the consumer is taking the number of sectors that are open in the two countries as given and also as in the closed economy, the consumer is taking the marginal product of capital in the different states as given. Further, although it is not important for the consumer's maximization problem which sectors will be open, as in the closed economy, sectors with smaller minimum size requirements will open first and we have already incorporated this into the problem.

We can now characterize the solution to the individual maximization problem more closely;

Lemma 4: (i) Suppose $j < j' < n_1$, then $F_1^j = F_1^{j'}$ and $F_2^j = F_2^{j'}$.
(ii) Suppose $n_1 < j < j' < n_1 + n_2$, then $F_2^j = F_2^{j'}$.

The intuition of these results is straightforward. If two sectors j and j' are open in both countries and they trade at the same price, then consumers will never be happy to purchase different amounts of these assets. Similarly, if two sectors are only open in one country, the same will again hold.

Using Lemma 4 we can then write the problem as

$$\max_{F_1,F_2,G,\phi_1,\phi_2} n_1\log[\rho_1(1)(r\phi_1+RF_1)+\rho_2(1)(r\phi_2+RF_2)]+n_2\log[\rho_1(2)(r\phi_1)+\rho_2(2)(r\phi_2+RG)] \quad (30)$$
$$(1-n_1-n_2)\log[\rho_1(3)((r\phi_1)+\rho_2(3)(r\phi_2))]$$

subject to

$$n_1(F_1+nF_2)+n_2G+\phi_1+\phi_2=s \quad (31)$$

where $\rho_i(q)$ denotes the marginal product in country i in state q where q=1 denotes $j \leq n_1$, q=2 denotes $n_1 < j \leq n_1+n_2$ and q=3 denotes $j > n_1+n_2$. Therefore

$$\rho_i(1)=\alpha A(r\phi_i+RF_i)^{\alpha-1} \qquad \rho_1(2)=\rho_1(3)=\alpha A(r\phi_1)^{\alpha-1} \qquad \rho_2(2)=\alpha A(r\phi_2+RG)^{\alpha-1} \quad (32)$$

Next, using the first-order conditions of this maximization problem and substituting for the marginal product of capital, we can establish;

Lemma 5: (i) $RF_1 + r\phi_1 = RF_2 + r\phi_2$.
(ii) $F_2 > F_1$ and $\phi_1 > \phi_2$.
(iii) $G > F_1$.

The intuition underlying this result is instructive. The first part of the lemma follows because if it were not true the marginal product of capital would be higher in one country and by changing, F_1 and F_2, total output in the all states $j \leq n_1$ could be increased. An informal intuition for second part of the lemma is that since G is received in country, the insurance role of safe investments in this country is less important, thus $\phi_1 > \phi_2$. Finally, the last part is due to the fact that in the states $j \in (n_1, n_1+n_2)$, there is lower return in country 1 than in the states $j \leq n_1$ and for insurance reasons, it is beneficial to increase G above F_1. The general form of equilibrium that is implied by this Lemma is also shown in Figure 8.

FIGURE 8 HERE

Next we establish;

Lemma 6: If $n_1+n_2 \to 1$, then $n_1 \to 1$.

In words, one of the countries cannot reach full-diversification before the other. The intuition of this lemma is that when one of the countries is near full diversification, the effect that leads to concentration becomes small relative to the difference between the marginal products of capital in the two countries and the additional value of a dollar there is less than in the country that has fewer sectors open. Therefore, consumers will always want to invest more in the smaller country. As a result, at the aggregate level, it will be impossible to finance the sectors with the highest minimum size in country 2.

Finally to complete the characterization of the equilibrium we need to show that for $n_1 < 1$, $n_2 > 0$. For this we first establish;

Lemma 7: If $n_1 < 1$, then $n_2 > 0$.

Now combining all these lemmas we can summarize the main findings of this section as;

Proposition 7:
The equilibrium of the two-country model always takes the following form;
If $s < 2D$;
(i) One of the countries always attracts more of the capital and open more sectors; $n_2 > 0$.
(ii) No country opens all projects; $n_1 + n_2 < 1$.
(iii) As s approaches 2D, both countries reach full diversification simultaneously.
If $s \geq 2D$, both countries open all their projects and each project receives the same amount of capital.

Our analysis so far has characterized the equilibrium of the static problem of fund allocation, taking the amount of savings, s, as given. To obtain the full dynamic equilibrium, we need to determine the amount of savings and also provide the dynamic transition equations as in the closed economy case. Because of the logarithmic preferences, the aggregate amount of savings will still be a fixed proportion of wage income and the dynamic transition equations of our economy can simply be obtained by determining wage income. However, this extreme simplicity of the two-period overlapping generation structure leads to some artificial results. Consider the world economy starting from a point of capital scarcity and equal wealth in the two countries. Then, the first part of Proposition 7 (or Lemma 6) implies that more capital will go to one of the countries and thus we can claim that capital flows create forces that work in the direction of divergence. However, if a state $j \leq n_1$ occurs, because $r\phi_1 + RF_1 = r\phi_2 + RF_2$, the two countries will have exactly the same output level and the workers will have exactly the same income, thus there will not be any income inequality left in the following period. Then in the following period, again more funds will be invested in one of the countries, thus the capital stocks will not be equalized, but it is still an unattractive feature of our model that for all states $j \leq n_1$, the young in the richer country will have the same income level as those in the poorer one. However, many extensions of the model that take it towards a more realistic direction will avoid this problem. In particular, if there are more intertemporal linkages than allowed here, this problem will not arise. Here we sketch a simple way of allowing for this without complicating our analysis, which is to introduce bequest motives for our agents. More precisely, the utility function (2) can be changed to

$$U(c_t,c_{t+1},b_t)+\log(c_t)+\beta E[\log(c_{t+1})+\mu\log(b_{t+1})] \tag{33}$$

where b_{t+1} is the bequest that generation t agents leave to their off-springs and μ is a parameter that captures the strength of this impure altruism. In this case, the saving decision changes to

$$s_t=\frac{\beta(1+\mu)}{1+\beta(1+\mu)}w_t \tag{34}$$

but since it is still constant and independent of the rate of return expected between t and t+1, all our analysis applies. And, if countries have different income levels at time t, even a state $j \leq n_{1t}$ does not lead to equalization of incomes because the young of the richer country will have received more bequests than those of their poorer neighbor.

With this formulation, the general dynamics of the world economy are straightforward to obtain from Proposition 7. First of all, if there does not exist enough capital in the world for both countries to reach full diversification, neither country will have all its sectors open. Thus bad news will affect both countries, but, in general, not symmetrically. To make another observation let us consider a situation where both countries have initial endowments that are small and also quite close to each other. In this case, the possibility of foreign capital flows first leads to divergence across countries; one of the countries (in our case, country 2), attracts capital from country 1. However, as the world economy receives a series of good shocks, n_1+n_2 and n_1 both approach 1; n_2 becomes arbitrarily small, and the two countries start converging. Therefore, in line with the historical evidence from the industrialization of Western Europe, our two-country model predicts that at the early stages of development, capital flows slow down development in one of the countries while enhancing the other. And at later stages, the relatively backward country receives capital imports from the advanced economy, consequently capital flows contribute to fast (but of course, not immediate) convergence.

It also has to be noted $s_t \geq 2D$ is the necessary condition for the decentralized economy to have all its sectors open. The socially planned economy would open all the sectors in at least one of the countries with less capital, but exactly the same forces that led to inefficiency in the closed economy make such an allocation impossible. In particular, Lemma 4 that parallels the requirement in the closed economy that all open sectors should get the same amount of capital in equilibrium (Lemma 1), introduce inefficiencies in the decentralized economy.

(b) A two-country model. Constant returns to capital ($\delta=1-\alpha$)

In the presence of endogenous growth a la Romer (i.e. the version of the model we analyzed in the end of section 3) the rate of return on capital is constant and capital flows will not influence the productivity of the remaining capital. The implication of this is straightforward: all the capital of the world will go to country 2. This is obviously unrealistic but is also instructive of a set of effects that arise in this type of economy. In the case of constant returns to capital, all the capital of the poor country flowing to the rich country is beneficial for the current old (rentier) generation of both countries and the young of country 2, but as a result of this, there is no capital invested in country 1 and the young of this country are impoverished. Therefore, although capital flows are welfare improving from the viewpoint of the current generation, they lead to important intertemporal inefficiencies and as a result, future generations who are assumed to remain in this country pay a very high price. In our analysis so far, we have not concentrated on the intertemporal aspects of efficiency, nevertheless this case shows that these considerations become very important in the presence of open economies and from the viewpoint of intertemporal efficiency, capital flows can be quite harmful.

(c) Small Open Economy with Enforcement Problems.

Now briefly consider the case of a small open economy. In the absence of transaction costs associated with foreign capital flows and enforceability problems, it is obvious that all the sectors will immediately open. However, it was argued above that this is not an interesting case and does not accord well with the existing observations concerning the experience of developing countries. Instead, suppose that there are enforceability problems. In particular, if the country that we are considering is sovereign, it has the right to repudiate on the debt and often it will choose to do so[11]. To give the main ideas of this case, simply assume that all the foreign investment that comes into a country will not be paid back. Naturally, in this case, there will be no capital inflows. However, the small open economy

[11] Because of the two-period overlapping generations aspect, the old generation will always want to repudiate completely on the debt. It is possible to incorporate these type of enforcement constraints in models with more forward looking behavior and obtain strictly positive but limited foreign direct investment into developing countries (e.g. Bulow and Rogoff).

is still of interest because of the capital outflows. Domestic investors can always put their money in foreign assets and get a fixed rate of return (since the country is small). An interesting implication is that as the productivity of investments abroad increases ϕ_t and F_t will (normally) diminish, therefore, the rate of industrialization will fall. This is the static effect of capital flows out of the country which improves the welfare of the current generation. However, the dynamic effects are more harmful to development. As capital flows out of the country, the wages fall and there is even less capital to invested in the following period. Assuming that enforcement constraints are still limiting the amount of foreign capital that can come in, this process will quickly take the wages in the country to a very low level and future generations will be impoverished and industrialization will be prevented.

How extreme the state of affairs is depends obviously on how strict the enforcement constraints are. However, as in the other case of harmful capital flows, this extension points out that even in the presence of capital scarcities, the possibility of international capital movements may have adverse effects when other market failures are also present.

7) Conclusion

Indivisibilities and non-convexities are often recognized as important factors in the process of development. We argue that the interactions between risk-spreading and indivisible projects will crucially influence the development experience of an economy. In the presence of non-convexities, an economy with limited resources will not be able to invest simultaneously in many sectors, and to the extent that these sectors have imperfectly correlated returns, it will not be able to benefit from diversification. This lack of diversification will often bias the portfolio towards safe but low return projects and slow down the process of accumulation. However, as the economy grows it will have more resources and thus it will be able to undertake more projects and the composition of investment will change towards riskier and more productive projects. This will eventually bring the economy to a take-off stage. Take-off and the associated financial deepening will finally result in steady industrial growth. In this process random events are very important. Since at the early stages of development, the economy only invests in a few sectors, it can easily be unsuccessful and spend too long in the primitive accumulation stage. We also show that when agents in this economy are sufficiently risk-averse, a series of bad draws can condemn a society to permanent "underdevelopment"; at a certain level of the capital stock, the economy will never invest enough in risky projects to take-off from primitive accumulation.

In our model, an interesting inefficiency also arises. Due to sectoral non-convexities, the Arrow-Debreu equilibrium fails to exist and in all equilibria with competition and price-taking, there is inefficiency which takes the form of too few sectors being open thus the economy on average spends too long in primitive accumulation. We show that this source of inefficiency is robust across different market structures and government intervention is necessary to deal with it. The form of required government policy is to subsidize large projects as we observe in the development experience of many economies.

Although we argue that the investigation of the interactions between risk and indivisibilities will be important in understanding the process of development, a number of issues require further thought and research. First, our model allows perfect risk-pooling and yet, indivisibilities may also limit the access of individual agents to financial instruments and may prevent risk-pooling across agents; it will be interesting to investigate how this interacts with the evolution of risk-diversification at the aggregate level. Second, our model does not grant a role to financial intermediation, and implicitly assume that the process of intermediation is costless and efficient at all stages of development. To the light of the findings of the recent literature, one could try to consider explicitly the role of credit and stock market. Third, though we show in section 6 that international capital flows do not necessarily undo our mechanism, one would like to assess empirically to what extent underdeveloped countries today suffer from capital shortages induced by undiversified risks. An issue which is related to this point is to investigate whether part of the high variability of growth rates of poor countries can be reduced by diversification. Finally, the most pressing, and probably fruitful, extension is to incorporate enforcement problems explicitly in such a model and attempt to determine the size of an accumulation unit together with the degree of indivisibilities that will influence such a unit. Such an extension will enable us to reach a much better assessment of the empirical importance of this mechanism in the context of current development problems.

APPENDIX

Proof of Lemma 1: Suppose $j\epsilon J(t)$, i.e. security j which pays R in state j and nothing otherwise is being traded at time t. If $P_t^j > 1$, then the firm which runs project j (or an intermediary) will make positive profits and a new entrant can enter and also make positive profits. Thus we cannot be in equilibrium. Suppose $P_t^j < 1$, then the firm in question is making losses, thus no equilibrium. Therefore in all equilibria, $P_t^j = 1 = P_t^{j'}$ for all $(j,j')\epsilon J(t)$. All stats of nature j are equally likely and all these securities are selling at the same price, thus $F_t^j = F_t^{j'} = F_t$. QED

Proof of Proposition 1: Consider the allocation characterized by n_t^* in (15); this has $P_t^j = 1$ and $F_t^j = F(n_t^*)$ for all $j \leq n_t^*$. At this point all consumers have an optimal portfolio since their demands are described by F(.) as in (14). Thus we only need to check whether a firm can make positive profit by changing its price or entering. First, $P_t^j < 1$ will make a loss, thus it is ruled out. P_t^j cannot be part of an equilibrium since an entrant can come in and make positive profit. Thus, the firms that are already active are using optimal strategies. Finally we need to check that no entrant can come in. First note that

$$F'(n_t^*) = \frac{r(R-r)}{(R-n_t^*)^2} \tag{A1}$$

and that $F''(.) > 0$. Under assumption (A), $F'(1) = r/(R-r) < D/(1-\gamma)$. Therefore, $F'(n_t^*) < D/(1-\gamma)$ for all values of n_t^*, that is the slope of the F(.) function in Figure 3 is always less than the slope of the M(.) function. Thus when an additional sector opens, the increase in the investment in the risky asset is not sufficient to cover the minimum size requirement of additional sector. Therefore no additional firm can come in. This proves that the characterized allocation is an equilibrium.

For uniqueness first note that $P_t^j = 1$ follows immediately for all $j \in J(t)$ for all allocations. Thus, all equilibria have to be characterized by the intersection of F(n) and M(n) or by

$$n_t=\frac{(R+r\gamma)\pm\sqrt{(R+r\gamma)^2-4r\left[\frac{(R-r)(1-\gamma)}{D}s_t+\gamma R\right]}}{2r} \qquad (A2)$$

and also an equilibrium with $n_t^*=1$ when $s\geq D$.

Denote the solutions to (A2) by n_1 and n_2 where $n_1>n_2$. When assumption (A) is satisfied, $n_1>1$ for all values of $s<D$. But in this case $n=1$ cannot be an equilibrium since not all sectors can be opened. Thus when $s<D$, there is a unique equilibrium given by n_t^* as in (15) which is the smaller root of (A2). Next note that also when assumption (A) is satisfied, the smaller root n_2 is greater than or equal to 1, for all $s\geq D$. Thus when $s\geq D$, there is only a unique equilibrium with all sectors open. This establishes uniqueness. QED

Proof of Corollary 1: The law of motion (16) implies that $0<K_t<K^{SS}\Rightarrow K_{t+1}>K_t$ conditional on favorable realizations. Also, (16) implies that for any K_0 there exists a sequence of good realizations such that K_t reaches K^* in finite time, where K^* is such that $n^*(K^*)=1$. Once this capital level is reached, (16) and (20) ensure that the economy converges deterministically to K^{SS}. Furthermore, K^{SS} is the only absorbing set of the system. Since any finite sequence of "good" realizations occur with positive probability, then convergence to K^{SS} is guaranteed almost surely. QED

Proof of Corollary 2: By using the definition of $\sigma(n(K),r,R)$ and substituting from the optimal decision rule, (14), we can write Var(.) simply as a function of n, the number of open sectors;

$$Var(\sigma(n,.),r,R)=n(1-n)\left[\frac{R(R-r)}{R-rn}\right]^2 \qquad (A3)$$

Therefore,

$$Sign\left\{\frac{\partial Var(\sigma(n,.),r,R)}{\partial n}\right\}=Sign\{(1-n)(R-rn)-n(R-rn)+2r(1-n)n \qquad (A4)$$
$$=Sign\{R-2nR+rn\}$$

Thus if n is greater than $\frac{R}{2R-r}$, the variance is decreasing in n. We also know that n^* will always be as large as γ; therefore, if γ is larger than $\frac{R}{2R-r}$, the variance is always decreasing. Otherwise, it will be non-monotonic (inverse U-shaped, with a maximum at $n^*=\frac{R}{2R-r}$). Since n^* as determined by equation (15) is a monotonic function of the capital stock K, the rest of the proof follows. QED.

Proof of Proposition 2: By substituting constraint (9) into the utility of the individual, let us define

$$V(n_t,\{F_t^j\})=\int_0^{n_t}\log(RF_t^j+s_t-\int_0^{n_t}F_t^i di)dj+(1-n_t)\log(s_t-\int_o^{n_t}F_t^j dj) \tag{A5}$$

Evaluating V(.) at the decentralized equilibrium - i.e. at $n_t=n^*$, $F_t^j=F_t^*$ for all $j\leq n_t$, we obtain

$$\frac{\partial V(n_t^*,F_t^*)}{F_t^j}=0 \quad \forall j\leq n_t^* \tag{A6}$$

and

$$\begin{aligned}\frac{\partial V(n_t^*,F_t^*)}{\partial n}&=\log(RF_t^*+s_t-n_t^*F_t^*)-\log(s_t-n_t^*F_t^*)\\&+\frac{F_t^*(s_t-n_t^*F_t^*+RF_t^*(1-n_t^*))}{(RF_t^*+s_t-n_t^*F_t^*)(s_t-n_t^*F_t^*)}>0\end{aligned} \tag{A7}$$

Since we can reduce F_t^j for some inframarginal sector and increase n_t, V(.) can be increased, thus the decentralized equilibrium is not Pareto Optimal.

To characterize the first-best construct the Lagrangean

$$\begin{aligned}L(n_t,\{F_t^j\})=&\int_0^{n_t}\log(RF_t^j+s_t-\int_0^{n_t}F_t^i di)dj+(1-n_t)\log(s_t-\int_o^{n_t}F_t^j dj)\\&+\lambda_t\int_0^{n_t}F_t^j dj+\phi_t-s_t)+\mu_t^j(F_t^j-M(j))\end{aligned} \tag{A8}$$

and differentiate with respect to F_t^j, we obtain the following first order conditions

$$\frac{R}{RF_t^j+r\phi_t}-\lambda_t-\mu_t^j=0 \tag{A9}$$

Now for all sectors j that do not have binding minimum size requirement, i.e. $F_t^j>M(j)$, the corresponding Lagrangean multiplier is zero, thus $F_t^j = F_t$ for all such j. It is straightforward to see that all sectors with M(j)=0 fall in this category since, by its concavity, V cannot be maximized when no investment is put in these sectors. But for the rest of the sectors (i.e. those with an associated positive multipliers), $F_t^j = M(j)$ must hold true. Thus the form in Proposition 2 follows. We only need to show that j* is strictly smaller than n_t^{FB}. Were this not the case, an equal amount would be invested in all sectors and we would get the decentralized equilibrium which is suboptimal as shown above. QED

Proof of Corollary 3: Let us suppose, there exists a market structure with free-entry that implements the Pareto optimal allocation. Take a sector j that does not have a minimum size requirement. Then by definition $F^j < F^n$. But since all assets are symmetric, this implies that the price that the representative agent is facing for the marginal unit of j, P^j, is higher than the price he is facing for the marginal unit of n. Since no firm in this market structure can make negative profit, this implies that $P^j > 1$. But then since there is no minimum size requirement, an additional firm can come in, set a price below P^j but above 1. If this price is sufficiently near 1, the representative consumer will want to buy further units of this security, thus the firm will make positive profit. As a result with free-entry, the Pareto optimal allocation cannot be sustained. Note also that for this proof it does not matter whether, consumers are buying one combined security or are purchasing each basic Arrow security separately. Only the shadow price matters.

Proof of Proposition 3: We need to show (i) the equilibrium of Proposition 1 is a Riley equilibrium; (ii) there is no other allocation that can be supported as a Riley equilibrium. For this proof, let us also denote by MP^j the maximum price that a consumer would be willing to pay for a marginal unit of security j.

(i) Let C be the set of contracts that maintains the allocation of Proposition 1 as an equilibrium. We need to show that for every C′ that disturbs C, ∀ C″, $\pi(C'' \mid C \cup C' \cup C'') > 0$ and $\pi(C' \mid C \cup C' \cup C'') < 0$. . We know that the equilibrium of proposition 1 cannot be disturbed by a balanced portfolio that has $MP^j = MP$ ∀j. Then consider an unbalanced portfolio to disturb

C, but for $\pi(C'|C \cup C') > 0$, some consumers must be attracted to C' thus some of the prices must be lower than 1, denote these securities by the set J. But then for C' to make positive profit, some prices must be greater than 1, thus $\forall j'$ such that $MP^{j'} > 1$, denote the set of these securities by J'. Now we have to allow for two possibilities; (a) different securities in C' can be bought separately; (b) there is only one combined security in C' (other hybrid cases can be handled similarly). Also denote by n_C the number of sectors open with the set of contracts C (i.e. the number of open sectors in the equilibrium of Proposition 1) and $n_{C'}$ the set of sectors open to support the set of contracts C'. For C' to disturb C and make positive profit we know that $n_{C'}$ must be strictly greater than n_C (since no balanced portfolio can disturb C and if $n_{C'} \leq n_C$, the consumers will hold a balance portfolio).

(a) Consider C'' such that in all $j' \in J'$, a unit of security is sold at the price $1+\epsilon$. This implies that, the firm offering the set C' will not sell any of the securities in J' and since at least some of the other securities in J are sold at a price below 1, it will make a loss.

(b) Let the combined security that constitutes C' have a price P and by definition of the profitability of C', $P > 1$. Then, for all $P^{j'} < P$, the consumer will want to have $F^{j'} \geq F^{j}$ for all $j \in J$ and $j' \in J'$, whereas by construction C' offers more of security J than J'. Thus consumers will reduce their purchase of the combined security in C' to a level no greater than $F(n_{C'})$. But by definition of the equilibrium of Proposition 1, we know that $F(n_C)$ is not sufficient to cover the minimum size requirements of sectors $n_{C'}$-n_C, therefore the supply of these securities is less than the demand, thus C' is not a valid disturbance to C.

(ii) Now consider an allocation supported by a set of contracts that is different from the equilibrium of Proposition 1. Then by definition, MP^j is not constant (otherwise we end up with the allocation of Proposition 1). Then at least for some j, $MP^j > 1$. Then, there exists C' that consists of securities j that sell at $P+\epsilon$ and make positive profits. Consider a reaction C'' that offers contract with $MP^j < P+\epsilon$. However, C' cannot make negative profits, since the worst that can happen is that no units of security j will be sold. Also since at $P+\epsilon$, the supply of security j is equal to its demand however small is ϵ, since F(n) is continuous, there will not be an excess demand after the disturbance. Thus, for all other allocations than that of the Proposition 1, a valid disturbance C' can be found. QED

Proof of Proposition 4: We consider separately the two cases $\theta < 1$ and $\theta > 1$. . To begin with, take the case of a rate of relative risk aversion higher than one. We have to show that F(n) and M(n) cross once and only once for $s_t < D$ (then the argument of Proposition 1 carries over to the general case analyzed here). First, we show that when $\theta > 1$. F(n) is strictly increasing and strictly convex in n. Let us define

$$g(n) \equiv \frac{1}{r\ s\ F(n)} = \frac{(R-rn)^{-\frac{1}{\theta}} R}{\left[(r-nr)^{-\frac{1}{\theta}} - (R-rn_t)^{-\frac{1}{\theta}}\right]} + rn \equiv \tilde{g}(n) + rn \tag{A10}$$

and

$$z(n) \equiv \frac{1}{\tilde{g}(n,.)} = \frac{1}{R}\left(\frac{R-rn}{r-rn}\right)^{\frac{1}{\theta}} - 1 \tag{A11}$$

where, taking first and second derivatives with respect to n:

$$\begin{aligned} z_n(n,.) &= \frac{(R-r)(R-rn)^{-1+1/\theta}}{(r-rn)^{1/\theta}\theta(1-n)R} > 0 \\ z_{nn}(n,.) &= \frac{(R-r)(R-rn)^{1/\theta-2}[R-r+\theta(R+r-2rn)]}{R(1-n)^2\theta^2(r-rn)^{1/\theta}} > 0 \end{aligned} \tag{A12}$$

As z(n,.) is convex in n, it follows that g(n) must concave in n, and F(n) is convex in n. Also, it can be checked that g(n) is everywhere decreasing in n, which implies that F(n) is everywhere increasing in n. Secondly, observe that (i) F(n=1)=s, $\lim_{n\to 1}\frac{\partial F(n)}{\partial n} = +\infty$ and $F(n=0) = \frac{r^{-1/\theta} - R^{-1/\theta}}{R^{-1/\theta}} rs > 0$; (ii), $M(\gamma)=0$, $M(1)=D$, $M'(n) = \frac{D}{1-\gamma}$, $M''(n)=0$.

Now (i) and (ii) together imply $F(n) > M(n)$, $\forall n \le \gamma$, $\forall\ s$; and $F(n=1) < M(1)$, $\forall s < D$. . Then, continuity proves existence. The fact that M(n) and F(n) are, respectively, a linear and a convex function of n implies that they can only cross once over the region of s<D. Next when s=D, F(n)=M(1) and $\lim_{n\to 1} F'(n=1) = +\infty > M'(1) = \frac{D}{1-\gamma}$. . So, F(n)<M(n) for n close to one. But we already know that F(n=0)>M(0), so the two lines must also cross at some n<1 (but, again, no more than once). Also clearly, by the continuity of the functions, when s>D, F(n)=M(n) for two values (n^*, n^+), where $n^* < n^+ < 1$. However, n^+ is not an equilibrium because $F'(n^+) > M'(n^+)$, so once n^+ projects are offered an additional firm can enter, offer a new project conditional on a feasible plan and make a positive profit. So, the only equilibria are n^* and 1. Finally, it is obvious that for any finite relative risk aversion there is a large enough stock of saving such that the only equilibrium is one with all projects open.

Consider now the case of relative risk aversion less than one. Existence if established by showing that $F(n_t)=M(n_t)$ for at least one value of n_t, for all $s_t \in (0,D)$. From equations (1) and (26) it follows that:

$$\left[(r(1-n_t))^{-\frac{1}{\theta}}-(R-rn_t)^{-\frac{1}{\theta}}\right]rs_t=\left[(R-rn_t)^{\frac{\theta-1}{\theta}}+rn_t((1-n_t)r)^{-\frac{1}{\theta}}\right]\frac{D(n-\gamma)}{1-\gamma} \tag{A13}$$

We proceed in three steps. First, observe that both the LHS and the RHS of this expression are monotonically increasing functions of n:

$$\frac{\partial LHS}{\partial n}=\frac{sr^2}{\theta}[(r-rn)^{-(1+1/\theta)}-(R-rn)^{-(1+1/\theta)}]>0$$

$$\frac{\partial RHS}{\partial n}=\frac{Dr(n-\gamma)}{\theta(1-\gamma)}[(1-\theta)(R-rn)^{-1/\theta}+r(r-rn)^{-(1+1/\theta)}]+ \frac{D}{(1-\gamma)}[(R-rn)^{1-1/\theta}+rn(r-rn)^{(-1/\theta)}]>0 \tag{A14}$$

Second, notice that for all $s<D$ (a) LHS>RHS at $n=0$, (b) LHS<RHS at $n=1$. (a) follows from the fact that when $n=0$ LHS>0, whereas RHS is non-positive. (b) follows from the fact that when $s=D$ LHS=RHS, whereas for all $s<D$ LHS<RHS (since both diverge, but $\lim_{n\to 1}\frac{LHS}{RHS}=\frac{s}{D}$). Then, (a), (b) and the continuity of both functions ensure that the two schedules necessarily cross. The proof of (iii) is established by checking that when $\theta<1$, F(n) is bounded from below by the right hand side of expression (14), i.e. the solution to the logarithmic case. Since in that case M(n) lies entirely below F(n) when $s_t>D$, the same must be true when $\theta<1$. QED

Proof of Corollary 4: Suppose there is a unique equilibrium, then Proposition 3 established that this is the unique Riley equilibrium with share-trading. Now suppose there are two equilibria. It must be the case that both equilibria have the same prices by our previous analysis, thus only the number of open sectors can differ. Denote the number of open sectors in the two equilibria respectively, (n_1, n_2) with $n_2>n_1$. Suppose we are in n_1. Then an entrant can buy all sectors

$[n_1,n_2\text{-}\epsilon']$ and offer each security at the price $1+\epsilon$. If ϵ is small enough, the consumers will prefer to switch more of their funds to the risky investments and buy the additional securities $[n_1, n_2\text{-}\epsilon']$ (note that ϵ' is related to ϵ, the mark-up over marginal cost) form the aggregate resource constraint (8). This is a profitable deviation. It is also clear that whatever the reaction, the entrant cannot make negative profits since no security is marketed below marginal cost and also all reactions must undercut the price $1+\epsilon$ for the securities $[n_1,n_2\text{-}\epsilon']$, thus for no sector j, we will have $ed^j>0$. This proves that n_1 cannot be a Riley equilibrium with share trading. QED

Proof of Lemma 2: From

$$\sigma_G(.)=\frac{Rr(R-na)^{\frac{1}{\theta}}}{(R-rn)((1-n)r)^{\frac{1}{\theta}}+rn(R-rn^{\frac{1}{\theta}}} \tag{A15}$$

it follows:

$$sign\left[\frac{\partial\sigma_G(.)}{\partial n}\right]=sign\left[\frac{(R-r)(r-rn)^{\frac{1}{\theta}}}{(r-rn)\left((R-rn)^{\frac{1}{\theta}}+(r-rn)^{\frac{1}{\theta}}\right)}-\theta\right] \tag{A16}$$

By rearranging terms, and calling $z\equiv\frac{1}{\theta}$, one can show that:

$$sign\left[\frac{\partial\sigma_G(.)}{\partial n}\right] = sign\left[z-\frac{r(1-n)}{R-r}\left(\left(\frac{R-rn}{r-rn}\right)^z-1\right)\right] \equiv sign[z-\zeta(z,.)] \tag{A17}$$

where $\zeta_z(z,.)>0$, $\zeta_{zz}(z,.)>0$, $\zeta(1,.)=1$, $\lim_{z\to 0}\zeta(z,.)=0$. Then it follows that:

$$\begin{aligned} z\in(0,1) &\Leftrightarrow \theta>1 \Leftrightarrow \frac{\partial\sigma_G(.)}{\partial n}>0 \\ z\in(1,\infty) &\Leftrightarrow 0<\theta<1 \Leftrightarrow \frac{\partial\sigma_G(.)}{\partial n}>0 \end{aligned} \tag{A18}$$

QED.

Proof of Proposition 5: (A) We show that $f(K_t,.)\equiv\sigma_G(n^*(K_t),R,r,\theta)\Gamma K_t^{\alpha-1}$, which is the growth rate of the capital stock at t (K_{t+1}/K_t) conditional on good news, is increasing over a certain region in K_t. Proposition 4 (ii) established that static multiple equilibria. Suppose that when $s_t \geq D$ all sectors are open (i.e. Pareto superior equilibria are always selected as will be the case when we allow inter-firm share trading as demonstrated in Corollary 4)[1]. Then, at $K_t = \frac{(1+\beta)}{\beta} D \equiv \bar{K}_1$ there will be a discrete increase in the value of $\sigma_G(.)$. On the other hand $\Gamma K^{\alpha-1}$ is everywhere continuous, then

$$\lim_{\epsilon \to 0} f(\bar{K}_1 + \epsilon, .) - f(\bar{K}_1 - \epsilon, .) > 0 \tag{A19}$$

Therefore, there will exist, K^{++} such that at $f(K=K^{++})=1$ and is increasing in K. This however implies that to the left of K^{++}, f(K) which is the growth rate of capital stock conditional on good news is less than 1. Since the growth rate conditional on bad news is by definition less, the capital stock will never grow beyond K^{++}.

(B) Inada conditions

$$\lim_{K \to 0} f(K,.) = +\infty \ \textit{and} \ \lim_{K \to \infty} f(K,.) = 0 \tag{A20}$$

the marginal product of capital near zero becomes very high that there must exist K^{+} the growth rate is positive even conditional on bad news.

(A) and (B) establish the result. QED

Proof of Proposition 6: The proof follows from Lemma 2, and the concavity of the final sector technology. The reasoning is exactly the same as that of the proof of Corollary 2. QED

Proof of Lemma 3: Suppose $P^j > 1$. Then a new firm can enter and also offer this security at a

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[1] The same argument can be established even when the Pareto inferior sequence of equilibria is selected when $s_t > D$, though this case has limited interested as it would only apply when share trading is restricted. In this case $\lim_{n \to 1^-} \frac{\partial \sigma_G}{\partial n}(n,.) = \infty$ and this implies $\exists \ \bar{K}_2 \ s.t. \ \lim_{K \to \bar{K}_2^-} f(K,.) = \infty > 0$. For some parameters such that f(K,.) will be increasing at $\bar{K} \ s.t. \ f(\bar{K},.) = 1$.

price higher than 1, making positive profits. Also, $P^j < 1$ leads to losses, thus $P^j = 1$ for all traded assets. QED

Proof of Lemma 4: Follows from the same logic of the proof of Lemma 1. QED

Proof of Lemma 5: We first write the first order conditions of the maximization problem (28) and denote the Lagrangean multiplier associated with the budget constraint (29), λ;

$$\frac{n_1 R(RF_1+r\phi_1)^{\alpha-1}}{(RF_1+r\phi_1)^{\alpha}+(RF_2+r\phi_2)^{\alpha}}=\lambda n_1 \tag{A21}$$

$$\frac{n_1 R(RF_2+r\phi_2)^{\alpha-1}}{(RF_1+r\phi_1)^{\alpha}+(RF_2+r\phi_2)^{\alpha}}=\lambda n_1 \tag{A22}$$

$$\frac{\alpha n_1 R(RF_1+r\phi_1)^{\alpha-1}}{(RF_1+r\phi_1)^{\alpha}+(RF_2+r\phi_2)^{\alpha}}+\frac{\alpha n_2 r(\phi_1)^{\alpha-1}}{(r\phi_1)^{\alpha}+(r\phi_2+RG)^{\alpha-1}}+\frac{\alpha(1-n_1-n_2)r(r\phi_1)^{\alpha-1}}{(r\phi_1)^{\alpha}+(r\phi_2)^{\alpha}}=\lambda \tag{A23}$$

$$\frac{n_1 R(RF_2+r\phi_2)^{\alpha-1}}{(RF_1+r\phi_1)^{\alpha}+(RF_2+r\phi_2)^{\alpha}}+\frac{n_2 r(RG+r\phi_2)^{\alpha-1}}{(r\phi_1)^{\alpha}+(r\phi_2+RG)^{\alpha-1}}+\frac{(1-n_1-n_2)r(r\phi_2)^{\alpha-1}}{(r\phi_1)^{\alpha}+(r\phi_2)^{\alpha}}=\lambda \tag{A24}$$

$$\frac{n_2 R(RG+r\phi_2)^{\alpha-1}}{(r\phi)^{\alpha}+(r\phi_2+RG)^{\alpha}}=\lambda n_2 \tag{A25}$$

Combining the two first-order conditions, gives (i); $RF_1+r\phi_1=RF_2+r\phi_2$.

To obtain (ii) combine the third (A23) and fourth (A24) first-order conditions and use (i). Since the second term has RG additional in the fourth condition, for $G>0$, it follows that $\phi_2>\phi_1$ and the rest follows from (i). Thus conditional on $G>0$ we have also established (ii).

Next we establish (iii) which also ensures that $G>0$. Combine (A21) and (A25) and set them to common denominator. This gives

$$(r\phi_1)^\alpha = 2(RG+\phi_2)^{\alpha-1}(RF_2+r\phi_2)-(RG+r\phi_2)^\alpha \tag{A26}$$

Suppose now that $G=F_2$, then it follows that $r\phi_1=RG+r\phi_2$ or that $G=F_2-F_1$ which is a contradiction since F_1 cannot be equal to zero. Next suppose that $r\phi_1 > RG+r\phi_2$ but as long as $G>0$, this also gives a contradiction if we combine (A23) and (A24), which would imply that

$$b_0(r\phi_1)^{\alpha-1}+c_0(r\phi_1)^{\alpha-1}=b_0(r\phi_2+RG)^{\alpha-1}+c_0(r\phi_2)^{\alpha-1} \tag{A27}$$

where b_0 and c_0 are constants that readily follows from the first-order conditions. If $r\phi_1 > RG+r\phi_2$, this cannot be true. Thus if $G > 0$, it must be true that $r\phi_1 < RG+r\phi_2$. But then, (A26) implies that $G > F_2$.

To complete the proof we just need to show that $G = 0$ gives a contradiction too. Take (A26), substitute for $G = 0$ and we end up with $r\phi_1=RF_2+r\phi_2$ which is impossible since F_2 is positive. QED.

Proof of Lemma 6: Let $n_1 + n_2 \to 1$ and $n_2 > 0$. The third term in both (A23) and (A24) vanish and combining these conditions we get $r\phi_1 = RG + r\phi_2$ and $n_1 + n_2 \to 1$, $\phi_2 \to 0$. Thus $G=r\phi_1/R$. Now also as $n_1 + n_2 \to 1$, G needs to tend to D to satisfy the feasibility constraint. Thus $\phi_1 \to RD/r > D$. But this cannot be feasible because it would imply $s > 2D$ which cannot the case (otherwise all the sectors in both countries would be open). QED

Proof of Lemma 7: From the proof of Lemma 6 we know that consumers prefer to invest $G > F_2$ in the sectors that are only open in country 2. This implies that when $n_2 = 0$, an additional firm can enter in country 2 and raise enough funds to finance a marginal project. Since consumers strictly prefer this situation, such a firm can also make positive profits. Thus $n_2 = 0$ cannot be an equilibrium for $n_1 < 1$. QED

Proof of Proposition 7: Follows from Lemmas 3 to 7. QED

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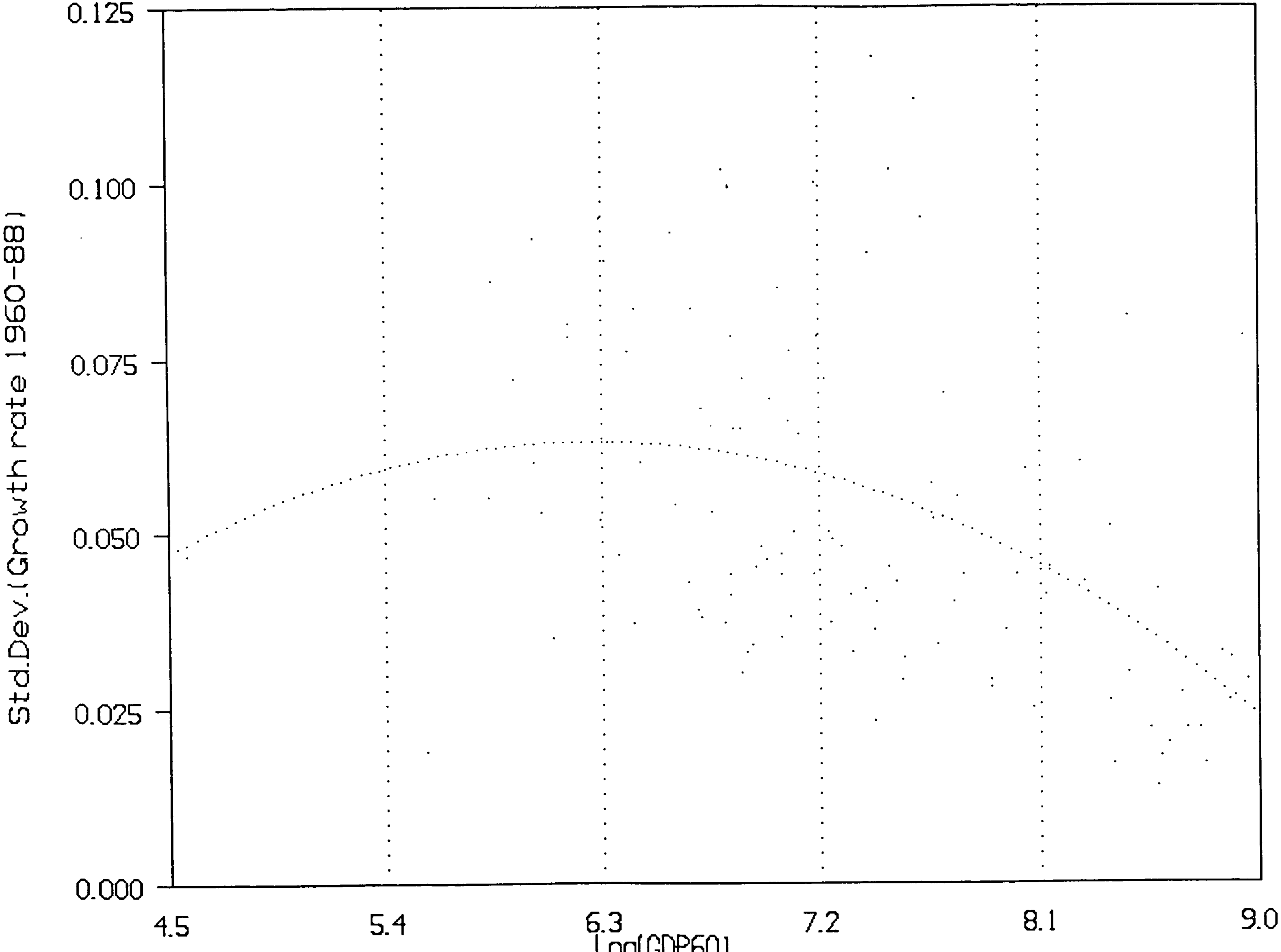
Figure 1


0.125
0.100
0.075
0.050
0.025
0.000
Std.Dev.(Growth rate 1960-88)
4.5
5.4
6.3
7.2
8.1
9.0
Log(GDP60)

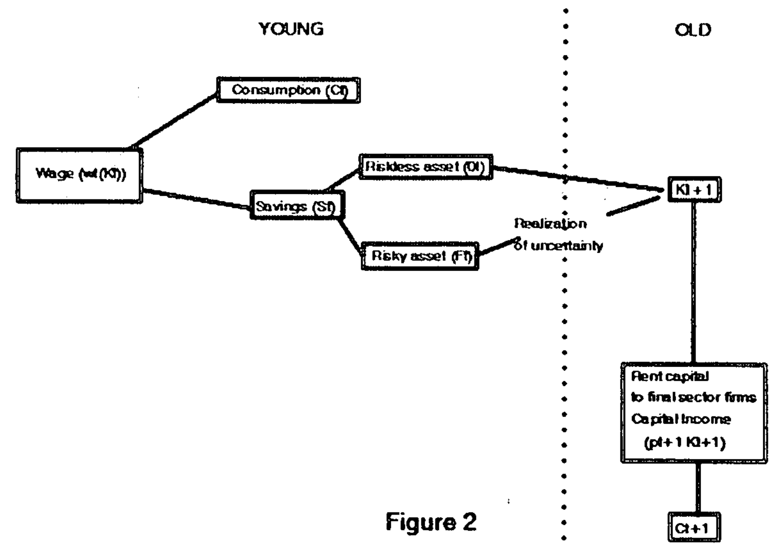

YOUNG
OLD
Consumption (Ct)
Wage (wt(Kt))
Riskless asset (Ot)
Savings (St)
Risky asset (Ft)
Realization
of uncertainty
Kt+1
Rent capital
to final sector firms
Capital Income
(ρt+1 Kt+1)
Ct+1

Figure 2

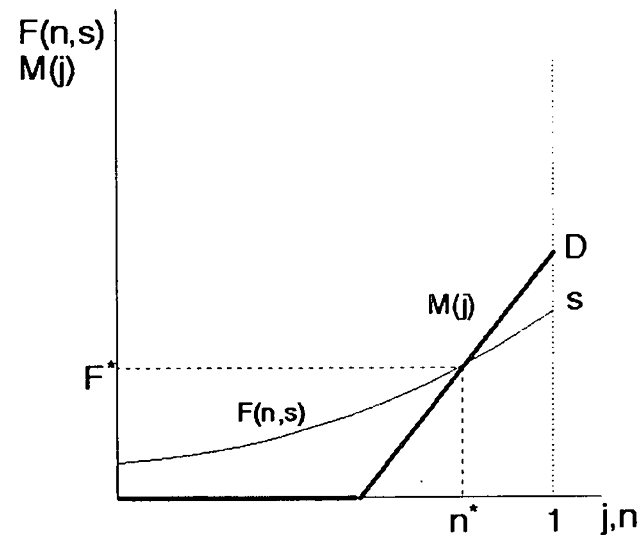
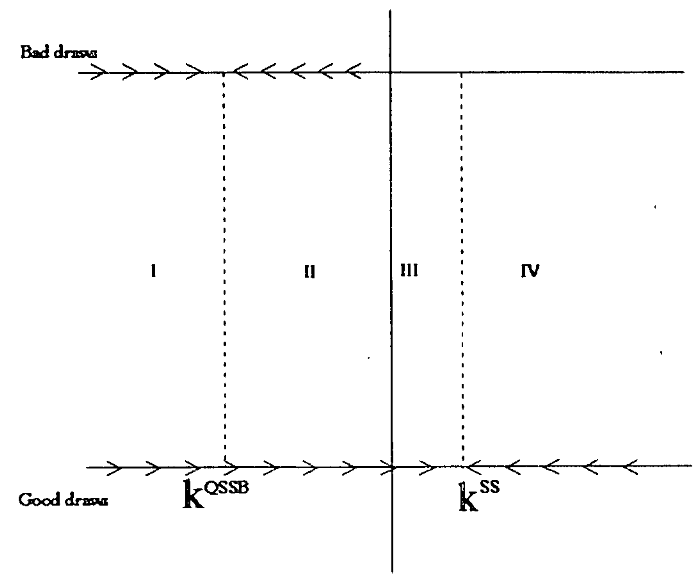


Figure 3

$n^{\circ} = 1$



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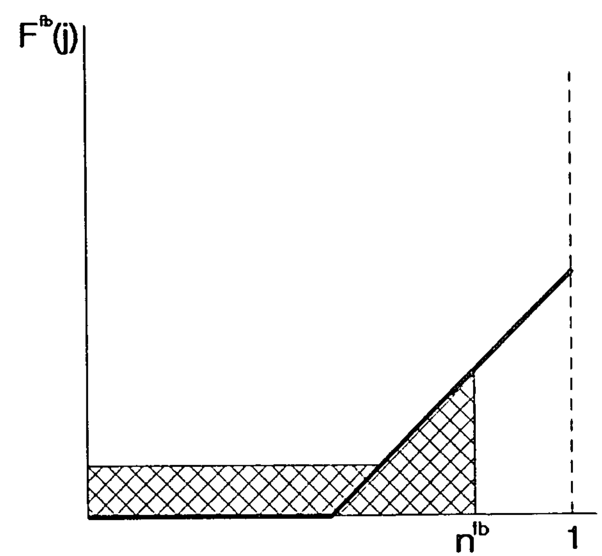
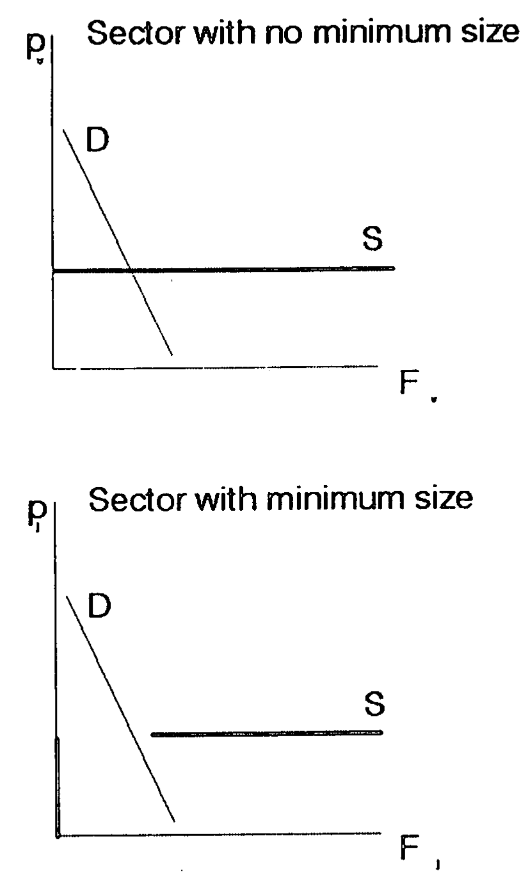


Figure 5



Sector with no minimum size
p
D
S
F
Sector with minimum size
p
D
S
F

Figure 6

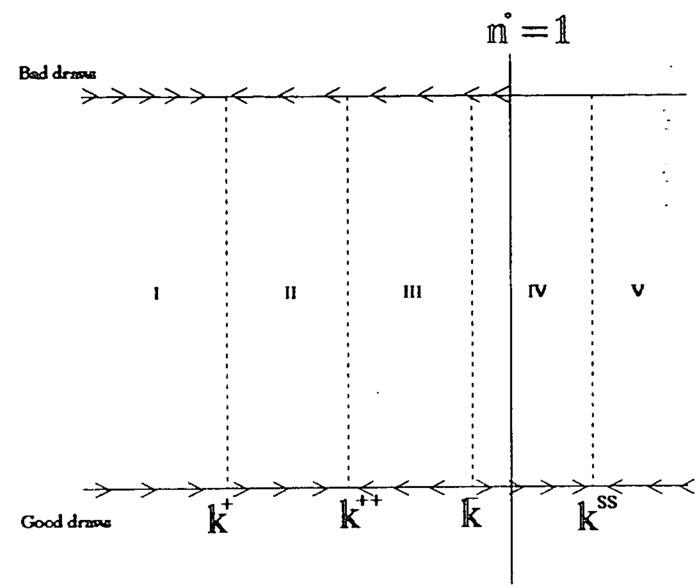
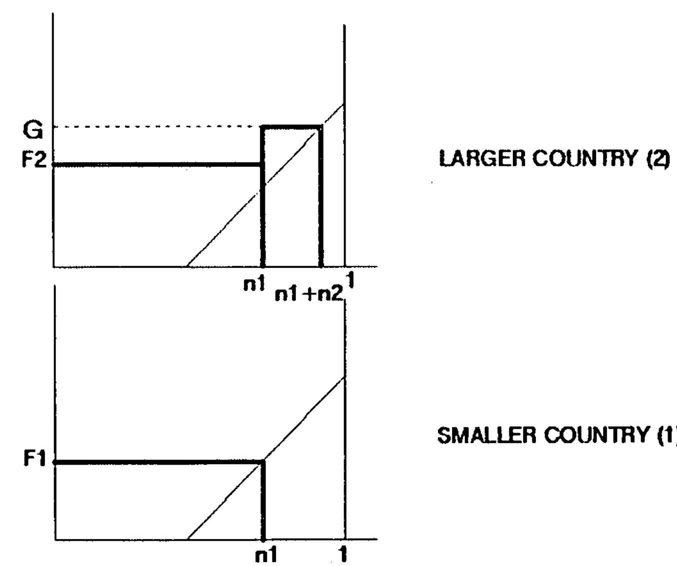


Figure 7



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