



Economics Working Paper 96

Competitive Location in Discrete Space*

Daniel Serra[†]
and
Charles ReVelle[‡]

November 1994

Keywords: Competitive Location, Discrete Network Location.

Journal of Economic Literature classification: C61, R30, L11.

* This project was partially funded by DCYGIT grants PB92-1036 and PB92-1037, Ministry of Education, Spain.

[†] Universitat Pompeu Fabra.

[‡] The Johns Hopkins University.

Abstract

The siting of facilities in a competitive environment has recently become a mainstream topic in the field of location-allocation modeling. This paper will focus on the location of facilities in a discrete space. First, an overview of the emerging literature on discrete competitive models will be presented. Second, some formulations of the location of competitive facilities in an oligopolistic market represented by a discrete network will be addressed. We address ways in which entry barriers can be applied to newcomers to the market, and we consider uncertainty in the information used in formulating strategies to maximize market share. Finally, a new model based on early works of ReVelle and Serra will be presented.

1 Introduction

Competitive location has been a mainstream topic in the field of industrial organization, spatial economics, regional science and operations research. The literature is rich in papers that addresses the issue of locating firms that compete for clients in space. Beginning with the seminal work of Hotelling (1929),¹ the most traditional spatial representation of competition has been a continuous line, since it easily yields mathematical results related to price setting, production levels and locational strategies.

In the last decades, new works following the same approach have been developed using other geometrical shapes such as circles or triangles, to try to loosen the strong assumption of a linear market and to allow new strategies such as sequential entry in the market (see, for example, Lerner and Singer (1937), Hay (1976), Rothschild (1976), and Prescott and Visscher (1977)). Although conclusions on the behavior and final strategies of the competitors are relatively easy to obtain, the spatial representation is, in most situations, still distant from reality. On the other hand, the literature on location in Operations Research and in Regional Science has concentrated on non-competitive location-allocation models, even though planar and network representations have been used.

In this sense, the competitive location models can be categorized in three spatial representations. These are (1) continuous planar space -where the potential location of the facilities is anywhere in the plane; (2) a network - where facilities are allowed to locate at the extremities of or at the intermediate points on the arcs; and (3) discrete space -where there is only a finite set of possible locations on the network. Therefore, discrete facility location can be considered as a special case of network facility location. In general, for all these spatial representations consumers are located at specific points, such as the vertices of the network. Obviously, it is not possible to determine which spatial setting is best, since the representation depends on the purpose of the model and the type of facilities to be located. An extensive bibliographic survey with over 100 citations on competitive location can be found in Eiselt

¹it is difficult to find a paper on competitive location that does not cite Hotelling's work in its introduction!

et al. (1993).

Several authors have started to incorporate the spatial representation of non-competitive location models to the strategies and behaviors of competitive siting situations.

Paralell to the spatial definition of the market, it is necessary to define the term "competition". Friesz et al. (1988) define a competitive facility location model as "any facility location model which explicitly recognizes that a firm's location may affect its market share; and, hence, that a location must be chosen so that the objectives of the firm are optimized with respect to market share" (p.48). This wide definition of competition in facility includes models where there is no interaction between firms, and therefore includes, for example, plant location models. For our purposes, a narrower definition will be used stemming from game-theoretical concepts. A competitive facility location model is such that there is more than one firm competing in the spatial market, and that there is interaction between them; the location decision of a firm will affect not only its market share, but also its competitor(s) market share. A similar definition has been offered by Hakimi (1983).

The purpose of this paper is threefold. First, an overview of the emerging literature on discrete competitive models as defined above will be presented. Second, some formulations on the location of competitive facilities in an oligopolistic market represented by a discrete network will be addressed, with the focuss on entry barriers that can be applied to check the entry of newcomers to the market and with focus on the uncertainty in the information used to formulate strategies to maximize market share. Finally, a new model based on early works of ReVelle and Serra will be presented.

2 Discrete Competitive Location Models: an Overview

Hotelling's work on two firms competing in a linear market with consumers distributed uniformly along the line (also known as the ice-cream vendor problem) set the foundations of what is today the burgeoning field of com-

petitive location. Competitive location is not only important from a practical point of view. It is also intriguing because, as Eiselt and Laporte (1989) point out, by modifying the assumptions of the model the results obtained may differ significantly from Hotelling's conclusions: "relatively small changes in the model assumptions result in dramatic changes in the outcome" (p.237).

During the late thirties and early forties, several papers using the same spatial representation as Hotelling but modifying some of the economic assumptions appeared in the economic literature (Hoover (1936), Lerner and Singer (1937), and Smithies (1941)). There followed several decades of stagnation in the contribution of new insights in the field of competitive location in linear markets. Since the late seventies however, a myriad of different models have appeared in the literature of spatial economics and industrial organization. Useful reviews can be found in Ponsard (1983), Graitson (1987), Gabszewicz and Thisse (1991), and Eiselt and Laporte (1989).

Much of the effort in the new evolving literature has aimed at developing insights concerning the equilibrium pattern of competitive location and pricing. Nevertheless, "although these models (linear models) are rich in their theoretical insights about spatial competition and have greatly enhanced our understanding of locational interdependence, they provide very little guidance for developing practical approaches to facility location in competitive environments." (Ghosh and Craig 1984).

Parallel to the development of this body of literature, a new field on location modelling was growing in the late sixties and seventies at a fast rate, namely facility location analysis. This field of research, coming basically from the fields of operations research, regional science and geography, dealt with the problem of locating new facilities in a spatial market in order to optimize one or several geographical and/or economic criteria. These criteria included overall distance minimization and transport and manufacturing cost minimization. The literature in facility location analysis is prolific: good references can be found in Domschke and Drexl (1985), who identified over 1500 references on the subject, Love et al. (1988), and Chhajed et al. (1993). Although these models used more realistic spatial representations, such as networks and planes, most of them dealt exclusively with non-competitive situations, and little attention was paid to the characterization of market

equilibria.

From the late seventies, considerations on the interaction between competing facilities in discrete space have been developed following several different approaches. One of the first of the questions that is addressed by several authors is the existence or (not) of a set of locations in the vertices of a network that will ensure a Nash equilibrium, that is, a position where neither firms have incentives to move. Wendell and McKelvey (1981) considered the location of two competitive firms with one server each and tried to find a situation where a firm would capture at least 50% of the market regardless of the location of its competitor. Results showed that there was not a general strategy for the firm that would ensure this capture if locating at vertices of the network. They did not develop a generic algorithm for finding solutions, but they looked at the possible locational strategies. They also examined the problem in a tree. Hakimi (1986) also analyzed extensively the problem of competitive location on vertices and proved that, under certain mathematical conditions such as concave transportation costs functions, that there exists a set of optimal locations on the vertices of the network.

A similar problem was studied by Lederer and Thisse (1990). Their problem not only looked at the specification of a site but also at the setting of a delivered price. They formulated the problem as a two-stage game, where in the first stage both firms choose locations and in the second stage they simultaneously set delivery prices schedules, and the result is that there is subgame perfect Nash equilibrium. As Hakimi did, they also proved that if firm's transport costs are strictly concave, then the set of locational choices of the firm is reduced to the vertices of the network. As a consequence, the location problem can be reduced to a 2-median problem if social costs are minimized.

A similar result was obtained by Labbe and Hakimi (1991). They developed a two-stage game in which two firms with one server each first select their location and then the quantities they will offer to each market. They proved that a subgame perfect Nash equilibrium exists and that the locations occur on the vertices if transport costs are concave.

The problem of two firms competing in a spatial market has also been

studied in the case where the market is represented by a tree. Eiselt (1992) proved that in such case there is not a sub-game perfect Nash equilibrium if both prices and locations are to be determined. Eiselt and Laporte (1993) extended the problem to the location of 3 facilities in a tree. They found that the existence of equilibria depended on the distribution of weights. In both models, firms were allowed to locate on the edges of the network.

The game-theoretical models presented so far restrict themselves to the location of firms with one facility each that compete against each other. Tobin and Friesz (1986) examined the case of a profit-maximizing firm that entered a market with several plants. They considered price and production effects on the market, since the increase in the overall production level from the opening of new plants in a spatial market stimulates reactions in the competitors. These reactions might affect not only production levels, but also prices and locations.

Tobin and Friesz developed two models: (1) a spatial price equilibrium model which determines equilibria in prices and production levels for a given number of firms, and (2) a Cournot-Nash oligopolistic model in which a few profit maximizing firms compete in spatially separated markets. They used both models to analyze the case of an entering firm that is going to open several new plants in spatially separated markets, and knows that its policy will have impact on market prices. Since profits depend on location and price-levels and these depend on the reaction of the competitors, it is not possible to use a standard plant location model. To tackle the problem, they used sensitivity analysis on variational inequalities to relate changes in production to changes in price to obtain optimal locations. The model was solved using a heuristic procedure where in the first step a spatial competitive equilibrium model was obtained and, in the second step, sensitivity analysis of profit to production changes was done using an integer non-linear program to select locations and production levels likely to maximize total profits. This model was generalized by Friez et al. (1989) to allow the entering firm to determine not only production levels and the site of its plants, but also its shipping patterns, and to examine different market strategies that can occur in the market (Miller et al. 1991). Due to the mathematical complexity of these models, Miller et al (1992) developed several heuristic methods to tackle the problem using the approach of variational inequalities (see, among others,

Harker 1984 and Nagurney 1993).

Another body of literature on competitive location deals with the siting of retail convenience stores. These type of stores is characterized by (1) a limited and very similar product offering accross outlets, (2) similar store image accross firms, and (3) similar prices. Therefore, location is a major determinant of success.

Ghosh and Craig (1984) considered the location of several retail facilities by two servers. The problem is to locate retail facilities in a competitive market knowing that a competing firm will also enter this market. They used a minimax approach, where the entering firm maximizes its profit given the best location of the competitor. Potential locations were restricted to the vertices of the network. The firm's objective is to maximize the net present value of its investment over a long-term planning horizon. The model did not allow location at the same site for both firms and did not examine the issue of ties. Ghosh and Craig used a heuristic algorithm to obtain solutions. The algorithm is as follows: for each possible set of locations of firm A, the best siting strategy is found for firm B. The final result is the set of locations where Firm A's objective is maximum given the best reactive location strategy of its competitor. A Teitz and Bart hill-climbing heuristic was used to determine the sites for both firms. The model is adapted to examine other strategies such as preemption, i.e., the identification of locations that are robust against competitive action. Other modifications included the relaxation of the number of stores that could be opened by each firm, and collusion by both servers.

In a similar model, Dobson and Karmarkar (1987) introduced the notion of *stability* in the location of retail outlets by two profit maximizing firms in a competitive market. Several integer programming models were developed to identify stable locations such that no competitors can enter the market and have profits given some rules on the competitive strategies. The models were solved using enumeration algorithms.

Most competitive location models assume that consumers patronize the closest shop. Karkazis (1989) considered two criteria that customers may use to decide which shop to patronize: a *level criterion* based on the preferences of a customer on the size of the facility and a *distance criterion* based on

closeness to the store. He developed a model that would determine the location and number of servers to enter the market when there are other firms already operating in the market by maximizing the profit subject to a budget constraint. The problem was solved in a dynamic fashion since there is a trade-off between both criteria.

Another model that examines competition among retail stores in a spatial market was developed by ReVelle 1986. The Maximum Capture Problem (MAXCAP) has formed the foundation of a series of models. These models include issues such as the strategies that competing firms may adopt or the uncertainty that characterizes some situations. In the following sections both the MAXCAP Problem and its extensions will be examined in more detail.

3 A Review of The Maximum Capture Problem

In a excellent review paper on competitive location, Friesz et al. (1988) pointed out that ReVelle's Maximum Capture Problem was one of the three competitive network facility location models that were "likely to serve as foundations for future models" together with the ones of Lederer (1986) and Tobin and Friesz (1986). In fact, The Maximum Capture Problem initiated a series of studies on the location of retail facilities in discrete space. The purpose of this section and the following ones is to review both the original formulation and the extensions of the MAXCAP model and to give new insights on this rich problem.

The MAXCAP model makes the following assumptions: (1) the product sold is homogeneous, in the sense that it is difficult to differentiate it among stores belonging to different firms, (2) the consumer's decision on patronizing the store is based on distance and not on price (i.e., price is considered the same in all stores and does not have any role in the purchasing decision) and (3) unit costs are the same in all stores regardless of ownership. Examples of this type of facilities can be found in the fast food sector, in convenience stores and in the banking sector (bank branches or ATM machines), among others.

In essence, the MAXCAP problem seeks the location of a fixed number of servers (p stores) belonging to a firm in a spatial market where there are other servers from other firms already competing for clients. The spatial market is represented by a network. Each node of the network represents a local market with a fixed demand which is given. The location of the servers is limited to the nodes of the network. Competition is based on distance: a market is "captured" by a given server if there is no other server closer to it. If two servers from competing firms have a local market at the same distance, then they divide in equal part the capture, as in Hotelling's game. The objective of the entering firm is to maximize its market capture. This objective, given the assumptions on the characteristics of the retail stores, is almost equivalent to maximizing profits (Hansen et al., 1987).

In this setting, the concept of market capture by an outlet is very similar to the concept of *security center* proposed by Slater (1975). A security center is a location j which minimizes the maximum for any other location k of the difference between the demand nodes closer to j and closer to k . The MAXCAP problem extends this concept to the location of several servers. Therefore, the p servers of the entering firm located using the MAXCAP problem can be considered as security centers, since they minimize the capture that Firm B can achieve.

The formulation of the MAXCAP problem is based on the Maximal Covering Location Problem (MCLP) of Church and ReVelle (1974) and can also be formulated as a P-median-like model. As mentioned before, the model assumes that a new firm (from now on Firm A) wants to enter a market in order to obtain the maximum capture, given the location of q competitor servers, that can belong to one or more firms. Without loss of generality it is assumed that there is only one competing firm (Firm B) operating in the market. The basic question is to find a set of p locations for Firm A so as to maximize its capture. The formulation of the MAXCAP model is as follows:

$$\text{Max } Z^A = \sum_{i \in I} a_i y_i^A + \sum_{i \in I} \frac{a_i}{2} z_i$$

subject to:

$$y_i^A \leq \sum_{j \in N_i(b_i)} x_j^A \quad \forall i \in I \quad (1)$$

$$z_i \leq \sum_{j \in O_i(b_i)} x_j^A \quad \forall i \in I \quad (2)$$

$$y_i^A + z_i \leq 1 \quad \forall i \in I \quad (3)$$

$$\sum_{j=1}^n x_j^A = p \quad (4)$$

$$y_i^A, z_i, x_j^A = (0, 1) \quad \forall i \in I, \forall j \in J$$

where parameters are:

- i, I = index and set of demand areas
- j, J = index and set of potential locations
- a_i = demand at node i
- d_{ij} = distance from node i to node j
- b_i = Firm B 's server closest to demand node i
- d_{ib_i} = distance from demand node i to the closest B server
- $N_i(b_i) = \{\forall j \in J, d_{ij} < d_{ib_i}\}$
- $O_i(b_i) = \{\forall j \in J, d_{ij} = d_{ib_i}\}$

and variables are defined as follows:

$$y_i^A = \begin{cases} 1, & \text{if Firm } A \text{ captures demand node } i \\ 0, & \text{otherwise} \end{cases}$$

$$z_i = \begin{cases} 1, & \text{if demand node } i \text{ is divided between } A \text{ and } B \\ 0, & \text{otherwise} \end{cases}$$

$$x_j^A = \begin{cases} 1, & \text{if firm } A \text{ locates a server at node } j \\ 0, & \text{otherwise} \end{cases}$$

The first set of constraints (1) depends on the set $N_i(b_i)$, which is known a priori. Each one of the demand nodes i has an associated set $N_i(b_i)$ which contains all the potential nodes of which Firm A can locate a server and capture

node i . Therefore, if one of the variables x_j^A belonging to the corresponding constraint is equal to 1 (i.e., a facility is located within the capture area of node i), then capture variable y_i^A is allowed to be 1, which indicates that node i has been capture by Firm A . The second set of constraints is very similar, but this time the set O_i is used. This set includes all nodes where, if Firm A locates a server, she will divide the demand captured with her competitor ($z_i = 1$). The third group of constraints takes into account the three states that a node can have: either it is fully captured by Firm A ($y_i^A = 1$ and therefore $z_i = 0$), or it is divided between both firms, ($z_i = 1$ and therefore $y_i^A = 0$), or it is lost to Firm B ($y_i^A = 0$ y $z_i = 0$). Finally, the last constraint sets the number of servers that Firm A is going to locate.

The objective defines the total capture that Firm A can achieve with the siting of its p servers. For each node, there is demand a_i to be captured. If $y_i^A = 1$, then a_i is added to the objective. On the other hand, if $z_i = 1$, then only $a_i/2$ is added to the objective, since the other half of the demand goes to Firm B .

The original MAXCAP problem considered that the demand at each node is fixed. If the demand depends on the distance to the server, then the MAXCAP problem can be reformulated using a p -median-like approach. Nevertheless, the number of variables and constraints significantly increases due to the use of assignment variables x_{ij} , where $x_{ij} = 1$ if node i is captured by a facility j belonging to Firm A .

Even though the problem of market capture was shown to be NP-hard for a general network (see Hakimi (1983) on the $(r|X_p)$ -Medianoid problem), ReVelle's mathematical formulation provided an optimal solution method to solve it. ReVelle used linear programming and branch and bound when necessary to solve the problem in a relatively small network (30 nodes). In most cases, the problem needed little or no branch and bound. This is probably due to the strong relationship between the MAXCAP problem and the maximal covering location problem (Church and ReVelle 1974). For large problems, the Algorithm for Market CAPture (AMACA), a one-opt heuristic based on the well known Tietz and Bart method (1968) can be used.

The AMACA proceeds as follows: Given an initial location of the p Firm

A servers and knowing the location of the competitors, the capture that Firm A achieves in the market can be obtained. Then, from a list of Firm A's servers, the first member of the list is picked and its location is moved to an empty node. The new capture can be computed and compared to the market capture achieved before the one-opt trade. If the objective has improved, the new set of locations is kept as the current solution. On the other hand, if the capture obtained before the trade is higher, this exchange is not considered. These one-opt trades are executed for all pairs of empty nodes and facilities. If at the end of all trades the objective has improved, then the procedure is repeated. The heuristic procedure ends whenever there is no improvement in the objective on completion of all exchanges possible. A more formal statement of the heuristic follows:

1. Locate p Firm A servers using any method.
2. Compute the capture obtained by Firm A.
3. Choose the first Firm A server of a list of its servers and trade its location to an empty node.
4. Compute the new market capture. If this the objective has improved, store the new locational solution. If not, restore the old solution.
5. Repeat steps 4 and 5 until all potential empty locations have been evaluated one at a time for each server.
6. If Firm A has improved its market share to a value greater than in step 2, go to step 3 and restart the procedure.
7. When no improvement is achieved on a complete set of one-at-a-time trades, stop.

The AMACA heuristic procedure is very efficient in terms of computing time but does not guarantee optimality. As it shall be seen later in this section, the AMACA procedure is very useful when the MAXCAP problem has to be solved repeatedly.

4 Extensions of the Maximum Capture Problem

The MAXCAP model is characterized by the following main assumptions:

1. all outlets have equal weights,
2. there is no reaction from the competitors to the entrance of new outlets,
3. there is no uncertainty in the parameters of the model.

The MAXCAP problem has been modified to relax these assumptions. In the following pages these relaxations will be examined.

Eiselt and Laporte (1989) modified the MAXCAP problem to include outlets that have different weights. Their model not only optimally locates an outlet in a competitive region, but also defines its optimal weight, which is loosely defined by the authors as a measure of "the size of the facility, its relative price advantage, the courteousness of its staff, etc." (p.434). Their approach derives from gravity models (Huff, 1964) and closest point models, also known as Voronoi diagrams (see Eiselt and Pederzoli, 1986) or Thiessen polygons. The problem was cast as a non-linear integer program where the objective was to maximize profits by the entering firm. One of the caveats of the formulation is that, even though from a mathematical standpoint the formulation can be adapted to the location of several new servers, the solution method proposed becomes computationally burdensome, since it is necessary to compute for each candidate node the profits that the firm would obtain.

The issue of different weights in the location of outlets in a competitive spatial market has also been examined by Serra et al. (1992). They consider that each firm has different servers organized in a hierarchical fashion. There is competition among outlets belonging to different Firms in each level of the hierarchy. The servers are nested, in the sense that upper-level outlets also sell products offered in the lower-level ones. The sphere of influence that an outlet has also depends on its hierarchical position. The higher in the hierarchy, the larger the sphere of influence. These factors lead to the situation where there

is competition among all levels between each firm. A lower level outlet from Firm A may compete on the basis of distance for a given demand node with an upper level outlet from the competing firm that is located further. Depending on the sphere of influence, an upper level outlet might capture a demand node even if it is located further from the demand node than a lower level outlet from the competing firm. The Maximum-Capture Hierarchical Location Problem is as follows: a Firm wants to enter a market with p_k servers in each hierarchical level k . In the market there are competitor servers that are already located. The main objective is to maximize total market capture. The demand parameter associated with the demand node at each level is defined as potential sales in monetary terms, so the objective is stated as the maximization of total market sales. The model was cast as an integer linear program and was solved using linear programming together with branch and bound when needed. In the examples used, in most cases little or no branch and bound was necessary.

The MAXCAP problem has been modified to take into account the existence of uncertainty in some of the model parameters (Serra and ReVelle, 1994). There might be situations in which the demand or population at a node that the entering firm is setting out to capture is not a known quantity but can assume different values depending on community growth or economic vitality. Furthermore, different number and locations of competitor outlets might occur depending on market expansion and corporate strategies. In this sense, Serra and ReVelle tackle the problem using the classic scenario approach. That is, demands and/or competitor locations are different in each possible scenario. The entering firm can then use at least two different criteria to locate servers despite uncertainty about which scenario will actually occur. These criteria are:

1. To maximize the minimum capture over all scenarios (Maximin criterion)
2. To minimize the worst deviation from the maximum capture that could be obtained in a given scenario if this one would come true (Regret criterion), i.e., to minimize the maximum difference between what is achieved and what might have been achieved.

If the first objective is used, then it is necessary to obtain a set of locations that will give the largest minimum capture possible over all scenarios. It is possible to compute, for a given set of locations for firm A servers, the final capture Z_k that the entering firm would achieve in each scenario k , since

$$m_k = \sum_{i \in I} a_{ik} y_{ik} + \sum_{i \in I} (a_{ik}/2) z_{ik},$$

where a_{ik} is the demand in node i for scenario k and y_{ik} and z_{ik} are set to 1 if node i is captured in scenario k . The model will maximize the capture in the scenario with the lowest m_k . Observe that each scenario will have different sets N_i and O_i , since these sets depend on the location of the competitors. Therefore, there will be one for each scenario (new sets N_{ik} and O_{ik}). This implies that there will be a constraint (4) and (5) for each demand node and each scenario. So the problem is formulated as follows:

$$\max Z = m$$

subject to:

$$\sum_{i \in I} a_{ik} y_{ik} + \sum_{i \in I} (a_{ik}/2) z_{ik} \geq m \quad \forall k = 1, \dots, s \quad (5)$$

$$y_{ik} \leq \sum_{j \in N_{ik}} x_j \quad \forall i \in I, \forall k = 1, \dots, s \quad (6)$$

$$z_{ik} \leq \sum_{j \in O_{ik}} x_j \quad \forall i \in I, \forall k = 1, \dots, s \quad (7)$$

$$y_{ik} + z_{ik} \leq 1 \quad \forall i \in I, \forall k = 1, \dots, s \quad (8)$$

$$\sum_{j \in J} x_j = p \quad (4)$$

$$y_{ik}, z_{ik}, x_j = (0, 1) \quad \forall i \in I, \forall j \in J, \forall k = 1, \dots, s$$

where s is the total number of scenarios.

If the Regret objective is used, then the problem is rewritten as follows:

$$\min Z = U$$

subject to:

$$Z_k - \sum_{i \in I} a_{ik} y_{ik} + \sum_{i \in I} (a_{ik}/2) z_{ik} \leq U \quad \forall k = 1, \dots, s \quad (9)$$

+ constraints (6) (7) (8) (4)

where parameter Z_k represents the maximum capture the the entering firm would obtain when optimally locating p servers in scenario k , i.e., it is necessary to compute s MAXCAP problems, one for each scenario. The left hand side of constraint (9) calculates the deviation from optimality in each scenario given a set of Firm A locations. The objective will try to minimize the largest deviation from optimality.

Observe that if only demands differ in each scenario, then it is not necessary to re-define sets N_i and O_i and capturing variables y_i and z_i . The new formulation is then obtained by adding only constraints (5) or (9) depending on the model to be solved (Maximin or Regret, respectively) and their corresponding objectives.

The use of linear programming relaxation and branch and bound when necessary could require large computer times if the functional form of constraints (5) or (9) does not favor 0,1 solutions. Serra and ReVelle proposed a heuristic algorithm based on the Teitz and Bart one-opt heuristic with two phases. In the first phase the MAXCAP problem is solved for each scenario individually and the the solution of one of the scenarios is chosen as the initial solution depending on the objective used (Maximin or Regret). The second phase tries to improve the solution by relocating one facility at a time to an empty node as in the AMACA heuristic until no improvement is obtained. Both LP+BB and the heuristic were solved on Swain's 55-node network.

The original MAXCAP problem considered that Firm A was new in the market, i.e., did not have any server already positioned. If Firm A is already operating in the market and wants to open new outlets, then it is necessary to exclude from the network those demand nodes that were already under the area of influence of Firm A. But a better capture can be obtained by the expanding Firm A if some of the existing servers are allowed to relocate. The new problem can be stated as follows: suppose that Firm A has p outlets in the spatial market and wants to relocate r outlets ($r \leq p$) and to open s new ones. Which outlets have to be relocated and where to locate them together with the new ones to maximize market capture? This problem - The Maximum Capture Problem with Relocation (MAXRELOC) - was studied by ReVelle and Serra (1991) and further reformulated by Serra et al. (1992). This new problem can be easily formulated by replacing in the MAXCAP model the constraint that fixes to p the number of servers to locate (constraint 4) by the following ones:

$$\sum_{j \in J} x_j^A = p + s$$

$$\sum_{j \in J_A} x_j^A = p - r$$

where J_A is the set of the existing p Firm A locations.

The relocation of servers is an important component of competitive facility siting because the entrance of competitors after the initial siting of outlets causes a change in the marketing landscape. ReVelle and Serra (1991) used the MAXRELOC formulation to examine different sequential strategies that two firms (a duopoly) can use when competing for clients in a spatial market with several outlets each, based on the classic economic market games suggested by Cournot and Stackelberg. At each time interval, firms are allowed to relocate some of its servers to improve their positioning. The first strategy -the Cournot Strategy - is such that a firm will locate and relocate its servers by maximizing newly captured population, regardless of the location decisions of its competitors in the same time interval. That is, in each period, each firm used the MAXRELOC formulation to relocate some of its servers using the locations obtained by its competitor in the previous period.

The second strategy - the Stackelberg strategy- is based on the assumption that one firm (Firm *A*) acts as a leader and positions its servers using the known locations of its competitor, and Firm *B*, once Firm *A* has relocated its outlets, uses the MAXRELOC model to relocate its servers. Therefore, Firm *B* acts as a follower, and it has an advantage in its strategy since it knows where Firm *A* has relocated its servers. These two strategies were tested repeatedly on a 55-node network and no locational equilibrium was achieved, that is, no Nash equilibrium was obtained, where both firms are not able to improve their market share by relocating some servers.

A similar problem, the Pre-emptive Capture Problem (PRECAP), was studied by Serra and ReVelle (forthcoming). The PRECAP analyzes the situation of a spatial market where there are as yet no competitors. A firm (Firm *A*) wants to locate p servers but knows that *after* their outlets' siting, one or several competitors will enter the market with several outlets too. The only information that Firm *A* has is that there will be q competitor servers being located in the future (say Firm *B* again). Therefore, Firm *A* wishes to pre-empt Firm *B* in its bid to capture market share to the maximum extent possible. The notion of pre-emptive competition has been widely analyzed in the literature of industrial organization (see, for example, Gilbert 1986). Hakimi (1983) defined this problem as the $(r|p)$ -centroid. Pre-emptive competition has also an equivalent concept in voting theory. A *Simpson point* (Simpson 1969) is such that the maximum number of users closer to another point is minimum. The main goal of Firm *A* is to find a set of pre-emptive locations, i.e. Simpson points, that will minimize the maximum possible future capture by Firm *B*.

It has to be noted that if both firms wish to locate the same number of servers, then Firm *B* will be always able to capture at least 50% of the market by locating on top of Firm *A*'s servers. Therefore, the best strategy for Firm *A* would be to obtain a set of locations such that *B* would have no other option than locating its servers in such a manner. As Serra and ReVelle show, in the general case, Firm *B* will obtain more than 50% of the market. This conclusion follows the one of Wendell and McKelvey (1981) for the special case where there is only one server. The locations obtained by Firm *B* once it has entered the market can be considered as *plurality points*, as defined by Wendell and McKelvey (1981) or Condorcet points, in terms of voting theory,

with respect to Firm A locations. The equivalence of this type of points has been proved by Hansen et al. (1986) for a general network.

The mathematical formulation of the PRECAP problem is exactly the same as the one of the MAXCAP problem, except that constraints (1) and (2) cannot be written in extensive form. The problem is that Firm A does not know the future locations of its competitors and therefore the sets $N_i(b_i)$ and $O_i(b_i)$ cannot be defined, since b_i , the closest competitor to node i , is not known a priori, since the sets $N_i(b_i)$ and $O_i(b_i)$ contain all candidate nodes that are closer to i and at the same distance from competitor outlet b_i respectively, and since b_i is unknown, the members of the sets cannot be written down.

In order to tackle the problem, Serra and ReVelle proposed a one opt heuristic that was tested on a 55-node network. Briefly, the PRE-EMptive capture heuristic ALgorithm (PREMAL) starts with the positioning of Firm A's servers using any method (for example, p-median or covering approaches). Then, the MAXCAP problem is used to find Firm B locations. Now, the market shares of Firm A and Firm B are known. Once the initial locations for both competitors are obtained, Firm A moves one of its outlets to an empty node. Again, the MAXCAP problem is solved to obtain Firm B's response. If Firm A's market share improves after the one-opt trade, then it will keep its new outlets' locations as the current solution. Otherwise, Firm A will ignore the relocation and restore the previous solution. The procedure is repeated for all nodes and servers until no improvement is achieved on a complete set of one-at-a-time trades.

This procedure could become computationally burdensome if linear programming relaxation supplemented by branch and bound is used to solve the MAXCAP problem, since a MAXCAP problem has to be solved at each one-opt trade. If large problems were to be solved using the PREMAL algorithm, solutions to the MAXCAP problem can be obtained using the AMACA heuristic. As Serra and ReVelle show in the computational experience of the PRECAP problem, the use of the AMACA heuristic for Firm B yield solutions for Firm A which allowed for Firm B to do slightly better than if the LP version of the MAXCAP problem were solved.

5 Extensions of the Pre-emptive Capture Problem

The Pre-emptive Capture Problem assumes that Firm A does not know the future locations of its competitors, but does know with certainty the number of competitor servers that will enter in the spatial market. In this section the PRECAP model is modified to relax this last assumption. A new formulation is presented, together with a heuristic solution method and an example.

Now the final capture of Firm A depends not only on the location of Firm B 's outlets, but also on the number of servers it throws into the competition. An objective that Firm A might consider is to minimize the average maximum capture that Firm B can achieve. In this case, the problem can be formulated as follows:

$$\max Z^A = \min \sum_{k=1}^q \frac{1}{k} \left(\sum_{i=1}^n a_i y_{ik}^B + \sum_{i=1}^n \frac{a_i}{2} z_{ik} \right)$$

subject to:

$$y_{ik}^B \leq \sum_{j \in N_i} x_{jk}^B \quad \forall i \in I, k = 1, \dots, q \quad (10)$$

$$z_{ik} \leq \sum_{j \in O_i} x_{jk}^B \quad \forall i \in I, k = 1, \dots, q \quad (11)$$

$$y_{ik}^B + z_{ik} \leq 1 \quad \forall i \in I, k = 1, \dots, q \quad (12)$$

$$\sum_{j=1}^n x_{jk}^B = k \quad k = 1, \dots, q \quad (13)$$

$$y_{ik}^B, z_{ik}, x_{jk}^B \in (0, 1) \quad \forall i \in I, \forall j \in J, k = 1, \dots, q$$

where additional notation is:

$$\begin{aligned}
k &= \text{index of the number of servers that Firm B can locate} \\
q &= \text{maximum number of competitor servers} \\
b_i^A &= \text{closest Firm A's server to node } i \\
d_{ib_i^A} &= \text{distance from node } i \text{ to Firm A's closest server} \\
N_i &= \{\forall j \in J, d_{ij} < d_{ib_i^A}\} \\
O_i &= \{\forall j \in J, d_{ij} = d_{ib_i^A}\} \\
y_{ik}^B &= \begin{cases} 1, & \text{if Firm B captures } i \text{ when locating } k \text{ servers} \\ 0, & \text{otherwise} \end{cases} \\
z_{ik} &= \begin{cases} 1, & \text{if node } i\text{'s capture is divided between A and B when B locates } k \text{ servers} \\ 0, & \text{otherwise} \end{cases} \\
x_{jk}^B &= \begin{cases} 1, & \text{if Firm B locates a server at } j \text{ when locating } k \text{ servers} \\ 0, & \text{otherwise} \end{cases}
\end{aligned}$$

The problem has been formulated with respect to the location of Firm B's servers. Since locations and final capture for both Firms depend on the k servers that Firm B might locate, it has been necessary to redefine both the capture variables and location variables of the model. Otherwise, the formulation is very similar to the Pre-emptive Location Problem. Observe that the capture that Firm B will obtain if locating k servers is $Z_k^B = \left(\sum_{i=1}^n a_i y_{ik}^B + \sum_{i=1}^n \frac{a_i}{2} z_{ik} \right)$. Therefore, if Z_k^B is divided by k , then the capture per server is obtained. Total average capture is obtained by adding the average capture for $k = 1, \dots, q$. The objective of Firm A is to locate first its services so as to minimize the total average capture that Firm B can achieve afterwards. If Firm A knows the probability α_k associated with the location of k servers by Firm B, then the objective can be modified by replacing $\frac{1}{k}$ by α_k . In this case, Firm A is minimizing the expected maximum capture. If $(1/k)$ is used the implied assumption is that the number of servers Firm B sites are equally likely. If q is now the maximum number of servers that Firm B might locate, it follows that $\sum_{k=1}^q \alpha_k = 1$.

As in the Pre-emptive Capture Problem, this model cannot be solved optimally since the locations of Firm A servers are not known and therefore

constraint sets (10) and (11) cannot be written in extensive form. A modified PREMAL heuristic can be used to tackle the problem. The first step is to locate the p Firm A servers using any method. Then the MAXCAP problem is used q times to locate $1, 2, \dots, k, \dots, q$ Firm B servers. Now the initial objective can be computed. In the second phase, Firm A will try to improve the solution by relocating its servers using a one-opt procedure. At each iteration, Firm A will relocate one of its servers and then, as in the initial phase, use the MAXCAP problem q times to see the average market capture that B can obtain when locating $1, \dots, k, \dots, q$ servers and then compute the objective. If the relocation has provided a set of positions that is better than before the one-opt trade, it will keep the new set of locations as best so far. Otherwise, Firm A will ignore the relocation and will restore the previous solution. The one-opt trade will be done for all nodes and Firm A servers. If, after a complete set of one opt-trades, Firm A has improved its objective, then the procedure is restarted. When no improvement is achieved, the procedure stops and final locations are obtained. As in the PREMAL procedure, the MAXCAP problem can be solved using integer programming or the AMACA heuristic procedure, depending on the computing time and storage available. Of course, multiple starting solutions for the Firm A servers could be used. A more formal statement of the heuristic follows:

1. Locate Firm A's p facilities using any method.
2. Locate 1 to q Firm B servers using the MAXCAP Problem.
3. Compute the average market share per server for each number of servers located by firm B. Sum them. This is our initial solution.
4. Trade the location of one of the p servers of Firm A.
5. Locate 1 to q Firm B servers using the MAXCAP Problem.
6. Compute the average market share per server for each number of servers located by firm B and add them. If the objective is smaller that before, store the new locations of Firm A's servers. If not, restore old locations.
7. Repeat steps 4 to 6 untill all of Firm A's p facilities and nodes have been traded.

8. If Firm A has reduced the objective in steps 4 to 6, go to step 4 and restart the procedure. When no improvement is achieved on a complete set of one-at-a-time trades, stop.

This new heuristic has been tested on Swain's 55-node network (1974, see appendix) using both the AMACA procedure and linear programming and branch and bound when necessary (LP+B&B), to solve the MAXCAP subproblems (heuristic 1 and heuristic 2, respectively). Firm A wishes to locate 5 servers and knows that Firm B is considering to locate after A's initial move from 1 to 9 outlets. In the first phase of the solution method, Firm A's initial locations were found randomly and the MAXCAP problem was solved using both approaches. Results are presented in Table 1. Only two locations, nodes 4 and 13, are obtained in both solutions.

Table 1: Initial locations and final results for Firm A

| | | CPU time |
|--------------------------|---------------|----------|
| Initial Locations | 5,11,12,54,55 | |
| Final Locations | | |
| Heuristic 1 | 4,13,16,31,41 | 20:58 |
| Heuristic 2 | 4,7,13,22,49 | 3:55:23 |

Table 2 presents the initial and final market captures for Firm B using both heuristics if finally locating k servers after Firm A's entrance in the market with p servers. For example, if the decision of Firm B were initially to locate 5 servers, then it would obtain 79.7% of the market. After the the second phase of the heuristic, the one-opt relocation of Firm A's servers, Firm A's final sites would reduce Firm B's market capture to 50.7% if the AMACA procedure were used and to 53.9% if LP+BB were used. It was expected that Firm B would obtain better results using LP+BB since at each time that Firm A would relocate one of its servers, the MAXCAP problem for Firm B would be optimally solved. Nevertheless, the heuristic procedure proposed significantly improves the market capture that Firm A can achieve. Multiple starting positions might correct this anomaly. Finally, initial and final locations for Firm B are presented in Table 3.

As was expected, the heuristic that used the AMACA procedure (Heuristic 1) needed much less CPU time than the heuristic that used LP+BB to solve the MAXCAP problems (see Table 1).

Table 2: Market capture results for Firm B

| # of servers | Initial Locations | | Final Locations | | | |
|--------------------|-------------------|-------|-----------------|-------|-------------|-------|
| | | | Heuristic 1 | | Heuristic 2 | |
| | Capture | | Capture | | Capture | |
| 1 | 1941.0 | 51.8% | 473.0 | 12.6% | 505.0 | 13.5% |
| 2 | 2400.0 | 64.0% | 872.0 | 23.3% | 981.0 | 26.2% |
| 3 | 2615.0 | 69.7% | 1248.0 | 33.3% | 1375.0 | 36.7% |
| 4 | 2818.5 | 75.2% | 1588.0 | 42.3% | 1747.0 | 46.6% |
| 5 | 2988.5 | 79.7% | 1900.5 | 50.7% | 2020.0 | 53.9% |
| 6 | 3066.5 | 81.8% | 2204.5 | 58.8% | 2268.0 | 60.5% |
| 7 | 3126.5 | 83.4% | 2409.5 | 64.3% | 2480.0 | 66.1% |
| 8 | 3179.0 | 84.8% | 2566.0 | 68.4% | 2677.0 | 71.4% |
| 9 | 3223.0 | 85.9% | 2715.0 | 72.4% | 2800.5 | 74.7% |

Table 3: Locations for Firm B

| | Initial Locations | | | | | | | | | Final Locations | | | | | | | | | |
|---|-------------------|----|----|----|----|----|----|----|----|-------------------|----|----|----|----|----|----|----|----|----|
| | | | | | | | | | | Heuristic 1 | | | | | | | | | |
| | Number of servers | | | | | | | | | Number of servers | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| l o c a t i o n s | 4 | 4 | 4 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 9 | 5 | 5 | |
| | | 16 | 16 | 16 | 4 | 4 | 4 | 4 | 4 | 41 | 17 | 11 | 11 | 9 | 9 | 13 | 9 | | |
| | | | | 23 | 16 | 16 | 17 | 17 | 5 | 5 | | 41 | 17 | 13 | 13 | 17 | 13 | 13 | |
| | | | | | 23 | 23 | 27 | 27 | 17 | 17 | | | 41 | 17 | 16 | 31 | 17 | 17 | |
| | | | | | | 47 | 43 | 43 | 27 | 27 | | | | 49 | 29 | 32 | 31 | 27 | |
| | | | | | | | 47 | 47 | 43 | 40 | | | | | 49 | 42 | 32 | 31 | |
| | | | | | | | | 54 | 47 | 43 | | | | | | 49 | 42 | 32 | |
| | | | | | | | | | 54 | 47 | | | | | | | 49 | 42 | |
| | | | | | | | | | | 54 | | | | | | | | 49 | 42 |
| | | | | | | | | | | | | | | | | | | | 49 |
| | | | | | | | | | | Heuristic 2 | | | | | | | | | |
| l o c a t i o n s | | | | | | | | | | 30 | 22 | 1 | 1 | 13 | 5 | 5 | 5 | 5 | |
| | | | | | | | | | | | 42 | 22 | 16 | 16 | 8 | 8 | 8 | 8 | |
| | | | | | | | | | | | | 41 | 29 | 25 | 25 | 16 | 16 | 9 | |
| | | | | | | | | | | | | | 41 | 29 | 29 | 25 | 17 | 13 | |
| | | | | | | | | | | | | | | 42 | 37 | 29 | 25 | 16 | |
| | | | | | | | | | | | | | | | 42 | 37 | 31 | 17 | |
| | | | | | | | | | | | | | | | | 42 | 37 | 20 | |
| | | | | | | | | | | | | | | | | 42 | 25 | | |
| | | | | | | | | | | | | | | | | | | 31 | |

6 Conclusions

We began in this chapter with an overview of the different approaches that have been proposed to analyse locational strategies in a competitive setting. Special focus was on those models where both the demand and the potential locations for servers are discrete. We then reviewed the Maximum Capture problem, a model that has been developed and adapted for different situations - where there is uncertainty in model parameters, where reactions from competitors to the entering firm might occur, and where outlets might have different areas of influence. Finally, a new model was proposed to tackle the Pre-Emptive Capture problem (a variant of the MAXCAP problem) in which there is little information available to the entering firm about the number and locations of future entrants in the spatial market. A heuristic method was proposed and tested on Swain's 55-node network. This network has been the basis for testing most of the models based on the MAXCAP problem, and the heuristics developed are based on the Teitz and Bart one-opt heuristic.

A common characteristic of all MAXCAP-based model is that the entering firm knows exactly the number of servers that it is going to locate. On the other hand, there is no consideration of opening costs that might differ substantially depending on the location chosen. If c_j is defined as the opening cost at node j , then the total opening cost is equal to $\sum_{j \in J} c_j x_j$. In order to obtain a trade-off between total capture and total costs, costs can be included as a second objective which is to be minimized. Then constraint (4), which sets the number of servers to locate, is eliminated. For the bi-objective problem, the weighting method or the constraint method can be used to obtain the trade-off curve between the two objectives. (Cohon 1978).

The bi-objective problem can be transformed into a single objective formulation if a_i is defined as potential sales if node i is captured. Then the problem is similar to the maximization of profits, even though no account is taken of variable costs, which might depend on the size of the area served (the area of influence) by each outlet. This situation can be overcome by defining a_i equal to (unit price) less (unit variable costs) times (number of potential units sold). The new objective would be defined as follows:

$$\max \pi = \sum_{i \in I} a_i y_i + \sum_{i \in I} \frac{a_i}{2} z_i - \sum_{j \in J} c_j x_j$$

These modifications would make it difficult to use any of the one-opt heuristics since to apply them it is necessary to know the number of outlets that the firm wishes to locate. On the other hand, if the opening cost of the outlet depends on the size of the market it will serve, the the problems would need to be reformulated using a P-median like approach, as proposed by ReVelle in the original MAXCAP problem.

Finally, it would be interesting to modify the MAXCAP problem to account for the location of production plants, where decisions on prices and production levels are also relevant.

We conclude by recalling the theoretical richness of the concept of Hotelling's ice cream vendors on a linear beach. We can observe that the wealth of opportunities that this original problem provided is now being paralleled by a large variety of problems based on the concept of competitive location on a network.

References

- CHHAJED, D., R.L. FRANCIS, AND T.J. LOWE (1993), "Contributions of operations research to location analysis," *Location Science*, 1(4):263–287.
- CHURCH, R. AND C. REVELLE (1974), "The maximal covering location problem," *Papers of the Regional Science Association*, 32:101–118.
- COHON, J. (1978). *Multiobjective Programming and Planning*. Academic Press, New York.
- DOBSON, G. AND U.S. KARMAKAR (1987), "Competitive location on a network," *Operations Research*, 35(4):565–574.
- DOMSCHKE, W. AND A. DREXL (1985). "Location and layout planning: An international bibliography," in *Lecture Notes in Economics and Mathematical Systems*, volume 238. Springer, Berlin.
- EISELT, H. (1992), "Hotelling's duopoly on a tree," *Annals of Operations Research*, 40:195–207.
- EISELT, H. AND G. LAPORTE (1989), "Competitive spatial models," *European Journal of Operational Research*, 39:231–242.
- (1993), "The existence of equilibria in the 3-facility hotelling model in a tree," *Transportation Science*, 27(1):39–43.
- EISELT, H., G. LAPORTE, AND J.F. THISSE (1993), "Competitive location models: A framework and bibliography," *Transportation Science*, 27(1):44–54.
- EISELT, H. AND G. PEDERZOLI (1986), "Voronoi diagrams and their uses - a survey, part i: Theory, part ii: Applications," *Proceedings of the Administrative Sciences Association of Canada*, 7:98–115.
- FRIESZ, T., L. TOBIN, AND T. MILLER (1989), "Existence theory for spatially competitive network facility location models," *Annals of Operations Research*, 18:267–276.
- FRIEZ, T., T. MILLER, AND R. TOBIN (1988), "Competitive network facility location models: A survey," *Papers of the Regional Science Association*, 65:47–57.

- GABSZEWICZ, J. AND J.F. THISSE (1991). "Location, " in Auman, R. and S. Hart, editors, *Handbook of Game Theory with Economic Applications*. North-Holland.
- GHOSH, A. AND C.S. CRAIG (1984), "A location allocation model for facility planning in a competitive environment," *Geographical Analysis*, 16(1):39-51.
- GILBERT, R. (1986). "Preemptive competition, " in Stiglitz, J. and F. Mathewson, editors, *New Developments in the Analysis of Market Structure*, capítulo 3, pages 90-125. MIT press, Cambridge, Mass.
- GRAITSON, D. (1987), "Spatial competition a la hotelling: A selective survey," *Journal of Industrial Economics*, 31:13-25.
- HAKIMI, S. (1983), "On locating new facilities in a competitive environment," *European Journal of Operational Research*, 12:29-35.
- (1986), " p -median theorems for competitive location," *Annals of Operations Research*, 5:79-88.
- HANSEN, P., M. LABBE, D. PEETERS, , AND J.F THISSE (1987). "Facility location analysis, " in Arnott, R., editor, *Systems of Cities and Facility Location*, pages 1-70. Harwood Academic Publishers.
- HANSEN, P., J.F. THISSE, AND R.E. WENDELL (1986), "Equivalence of solutions to network location problems," *Mathematics of Operations Research*, 11(4):672-678.
- HARKER, P. (1984), "A variational inequality approach for the determination of oligopolistic market equilibrium," *Mathematical Programming*, 30:105-111.
- HAY, D. (1976), "Sequential entry and entry-detering strategies," *Oxford Economic Papers*, 28:240-257.
- HOOVER, E. (1936), "Spatial price discrimination," *Review of Economic Studies*, 4:182-191.
- HOTELLING, H. (1929), "Stability in competition," *Economic Journal*, 39:41-57.

- HUFF, D. (1964), "Defining and estimating a trade area," *Journal of marketing*, 28:34-38.
- KARKAZIS, J. (1989), "Facilities location in a competitive environment: A promethee based multiple criteria analysis," *European Journal of Operational Research*, 42:294-304.
- LABBE, M. AND S.L. HAKIMI (1991), "Market and location equilibrium for two competitors," *Operations Research*, 39(5):749-756.
- LEDERER, P. (1986), "Duopoly competition in networks," *Annals of Operations Research*, 6:99-109.
- LEDERER, P. AND J.F. THISSE (1990), "Competitive location on networks under delivered pricing," *Operations Research Letters*, 9:147-153.
- LERNER, A. AND H.W. SINGER (1937), "Some notes on duopoly and spatial competition," *Journal of Political Economy* 39, pages 145-186.
- LOVE, R., J.G. MORRIS, AND G.O. WESOLOVSKY (1988). *Facilities Location: Models and Methods*. North Holland, New York.
- MILLER, T., T.L. FRIESZ, AND R.L. TOBIN (1992), "Heuristic algorithms for delivered price spatially competitive network facility location problems," *Annals of Operations Research*, 34:177-202.
- MILLER, T., R.L. TOBIN, AND T.L. FRIEZ (1991), "Stackelberg games on a network with cournot-nash oligopolistic competitors," *Journal of Regional Science*, 31:435-454.
- NAGURNEY, A. (1993). *Network Economics: A Variational Inequality Approach*. Kluwer Academic Publishers, Boston.
- PONSARD, C. (1983). *A History of Spatial Economic Theory*. Springer-Verlag, Berlin.
- PRESCOTT, E. C. AND M. VISSCHER (1977), "Sequential location among firms with foresight," *Bell Journal of Economics*, 8:378-393.
- REVELLE, C. (1986), "The maximum capture or sphere of influence problem: Hotelling revisited on a network," *Journal of Regional Science*, 26(2):343-357.

- REVELLE, C. AND D. SERRA (1991), "The maximum capture problem including relocation," *Information and Operations Research*, 29(2):130-138.
- ROTHSCHILD, R. (1976), "A note on the effect of sequential entry on choice and location," *Journal of Industrial Economics*, 24:313-320.
- SERRA, D., V. MARIANOV, AND C. REVELLE (1992), "The hierarchical maximum capture problem," *European Journal of Operational Research*, 62(3).
- SERRA, D. AND C. REVELLE (1994), "The maximum capture problem with uncertainty," *Economics Working Paper 74, Universitat Pompeu Fabra*.
- (Forthcoming), "Market capture by two competitors: The preemptive capture problem," *Journal of Regional Science*.
- SIMPSON, P. (1969), "On defining areas of voter choice: Professor Tullock on stable voting," *Quarterly Journal of Economics*, 83:478-487.
- SLATER, P. (1975), "Maximin facility location," *Journal of Research National Bureau of Standards*, 79B:107-115.
- SMITHIES, A. (1941), "Optimum location in spatial competition," *Journal of Political Economy*, 49:423-439.
- SWAIN, R. (1974), "A parametric decomposition algorithm for the solution of uncapacitated location problems," *Management Science*, 21:189-198.
- TIETZ, M. B. AND P. BART (1968), "Heuristic methods for estimating the generalized vertex median of a weighted graph," *Operations Research*, 16:955-965.
- TOBIN, R. AND T.L. FRIESZ (1986), "Spatial competition facility location models: definition, formulation and solution approach," *Annals of Operations Research*, 6:49-74.
- WENDELL, R. AND R.D. MCKELVEY (1981), "New perspectives in competitive location theory," *European Journal of Operational Research*, 6:174-182.

Appendix: 55-node Network demands and coordinates

| node | pop | coord | | node | pop | coord | | node | pop | coord | |
|------|-----|-------|----|------|-----|-------|----|------|-----|-------|----|
| | | x | y | | | x | y | | | x | y |
| 1 | 120 | 32 | 31 | 20 | 77 | 25 | 14 | 39 | 47 | 46 | 51 |
| 2 | 114 | 29 | 32 | 21 | 76 | 29 | 12 | 40 | 44 | 50 | 40 |
| 3 | 110 | 27 | 36 | 22 | 74 | 24 | 48 | 41 | 43 | 23 | 22 |
| 4 | 108 | 29 | 29 | 23 | 72 | 17 | 42 | 42 | 42 | 27 | 30 |
| 5 | 105 | 32 | 29 | 24 | 70 | 6 | 26 | 43 | 41 | 38 | 39 |
| 6 | 103 | 26 | 25 | 25 | 69 | 19 | 21 | 44 | 40 | 36 | 32 |
| 7 | 100 | 24 | 33 | 26 | 69 | 10 | 32 | 45 | 39 | 32 | 41 |
| 8 | 94 | 30 | 35 | 27 | 64 | 34 | 56 | 46 | 37 | 42 | 36 |
| 9 | 91 | 29 | 27 | 28 | 63 | 12 | 47 | 47 | 35 | 36 | 26 |
| 10 | 90 | 29 | 21 | 29 | 62 | 19 | 38 | 48 | 34 | 15 | 19 |
| 11 | 88 | 33 | 28 | 30 | 61 | 27 | 41 | 49 | 33 | 19 | 14 |
| 12 | 87 | 17 | 53 | 31 | 60 | 21 | 35 | 50 | 33 | 45 | 19 |
| 13 | 87 | 34 | 30 | 32 | 58 | 32 | 45 | 51 | 32 | 27 | 5 |
| 14 | 85 | 25 | 60 | 33 | 57 | 27 | 45 | 52 | 26 | 52 | 24 |
| 15 | 83 | 21 | 28 | 34 | 55 | 32 | 38 | 53 | 25 | 40 | 22 |
| 16 | 82 | 30 | 51 | 35 | 54 | 8 | 22 | 54 | 24 | 40 | 52 |
| 17 | 80 | 19 | 47 | 36 | 53 | 15 | 25 | 55 | 21 | 42 | 42 |
| 18 | 79 | 17 | 33 | 37 | 51 | 35 | 16 | | | | |
| 19 | 79 | 22 | 40 | 38 | 49 | 36 | 47 | | | | |

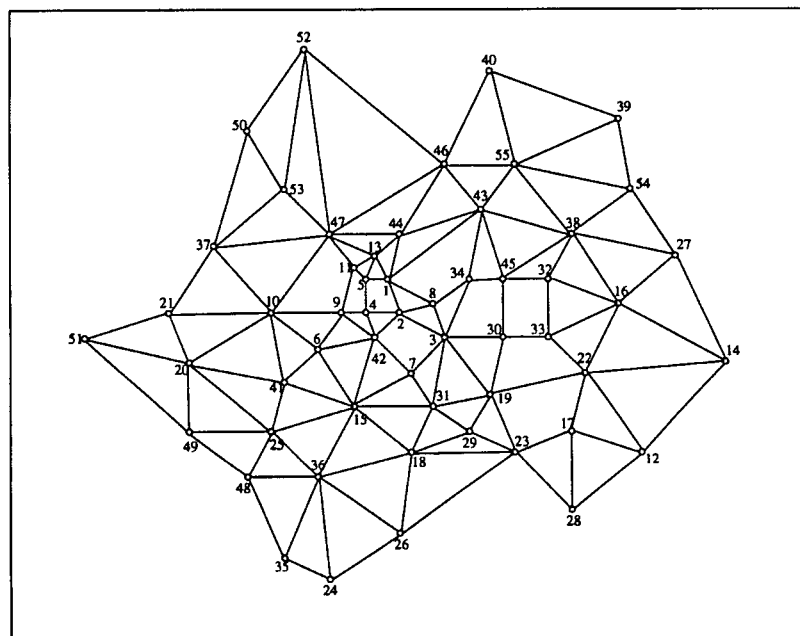


Figure 1: Swain's 55-node network

RECENT WORKING PAPERS

1. Albert Marcet and Ramon Marimon
Communication, Commitment and Growth. (June 1991)
[Published in *Journal of Economic Theory* Vol. 58, no. 2, (December 1992)]
2. Antoni Bosch
Economies of Scale, Location, Age and Sex Discrimination in Household Demand. (June 1991)
[Published in *European Economic Review* 35, (1991) 1589-1595]
3. Albert Satorra
Asymptotic Robust Inferences in the Analysis of Mean and Covariance Structures. (June 1991)
[Published in *Sociological Methodology* (1992), pp. 249-278, P.V. Marsden Edt. Basil Blackwell: Oxford & Cambridge, MA]
4. Javier Andrés and Jaume Garcia
Wage Determination in the Spanish Industry. (June 1991)
[Published as "Factores determinantes de los salarios: evidencia para la industria española" in J.J. Dolado et al. (eds.) *La industria y el comportamiento de las empresas españolas (Ensayos en homenaje a Gonzalo Mato)*, Chapter 6, pp. 171-196, Alianza Economía]
5. Albert Marcet
Solving Non-Linear Stochastic Models by Parameterizing Expectations: An Application to Asset Pricing with Production. (July 1991)
6. Albert Marcet
Simulation Analysis of Dynamic Stochastic Models: Applications to Theory and Estimation. (November 1991), 2d. version (March 1993)
[Published in *Advances in Econometrics* invited symposia of the Sixth World Congress of the Econometric Society (Eds. JJ. Laffont i C.A. Sims). Cambridge University Press (1994)]
7. Xavier Calsamiglia and Alan Kirman
A Unique Informationally Efficient and Decentralized Mechanism with Fair Outcomes. (November 1991)
[Published in *Econometrica*, vol. 61, 5, pp. 1147-1172 (1993)]
8. Albert Satorra
The Variance Matrix of Sample Second-order Moments in Multivariate Linear Relations. (January 1992)
[Published in *Statistics & Probability Letters* Vol. 15, no. 1, (1992), pp. 63-69]
9. Teresa Garcia-Milà and Therese J. McGuire
Industrial Mix as a Factor in the Growth and Variability of States' Economies. (January 1992)
[Forthcoming in *Regional Science and Urban Economics*]
10. Walter Garcia-Fontes and Hugo Hopenhayn
Entry Restrictions and the Determination of Quality. (February 1992)

11. Guillem López and Adam Robert Wagstaff
Indicadores de Eficiencia en el Sector Hospitalario. (March 1992)
[Published in *Moneda y Crédito* Vol. 196]

12. Daniel Serra and Charles ReVelle
The PQ-Median Problem: Location and Districting of Hierarchical Facilities. Part I (April 1992)
[Published in *Location Science*, Vol. 1, no. 4 (1993)]

13. Daniel Serra and Charles ReVelle
The PQ-Median Problem: Location and Districting of Hierarchical Facilities. Part II: Heuristic Solution Methods. (April 1992)
[Forthcoming in *Location Science*]

14. Juan Pablo Nicolini
Ruling out Speculative Hyperinflations: a Game Theoretic Approach. (April 1992)

15. Albert Marcet and Thomas J. Sargent
Speed of Convergence of Recursive Least Squares Learning with ARMA Perceptions. (May 1992)
[Forthcoming in *Learning and Rationality in Economics*]

16. Albert Satorra
Multi-Sample Analysis of Moment-Structures: Asymptotic Validity of Inferences Based on Second-Order Moments. (June 1992)
[Published in *Statistical Modelling and Latent Variables* Elsevier, North Holland. K.Haagen, D.J.Bartholomew and M. Deistler (eds.), pp. 283-298.]

- Special issue Vernon L. Smith
Experimental Methods in Economics. (June 1992)

17. Albert Marcet and David A. Marshall
Convergence of Approximate Model Solutions to Rational Expectation Equilibria Using the Method of Parameterized Expectations.

18. M. Antònia Monés, Rafael Salas and Eva Ventura
Consumption, Real after Tax Interest Rates and Income Innovations. A Panel Data Analysis. (December 1992)

19. Hugo A. Hopenhayn and Ingrid M. Werner
Information, Liquidity and Asset Trading in a Random Matching Game. (February 1993)

20. Daniel Serra
The Coherent Covering Location Problem. (February 1993)
[Forthcoming in *Papers in Regional Science*]

21. Ramon Marimon, Stephen E. Spear and Shyam Sunder
Expectationally-driven Market Volatility: An Experimental Study. (March 1993)
[Forthcoming in *Journal of Economic Theory*]

22. Giorgia Giovannetti, Albert Marcet and Ramon Marimon
Growth, Capital Flows and Enforcement Constraints: The Case of Africa.
(March 1993)
[Published in *European Economic Review* 37, pp. 418-425 (1993)]
23. Ramon Marimon
Adaptive Learning, Evolutionary Dynamics and Equilibrium Selection in
Games. (March 1993)
[Published in *European Economic Review* 37 (1993)]
24. Ramon Marimon and Ellen McGrattan
On Adaptive Learning in Strategic Games. (March 1993)
[Forthcoming in *A. Kirman and M. Salmon eds. "Learning and Rationality in
Economics"* Basil Blackwell]
25. Ramon Marimon and Shyam Sunder
Indeterminacy of Equilibria in a Hyperinflationary World: Experimental
Evidence. (March 1993)
[Forthcoming in *Econometrica*]
26. Jaume Garcia and José M. Labeaga
A Cross-Section Model with Zeros: an Application to the Demand for
Tobacco. (March 1993)
27. Xavier Freixas
Short Term Credit Versus Account Receivable Financing. (March 1993)
28. Massimo Motta and George Norman
Does Economic Integration cause Foreign Direct Investment?
(March 1993)
[Published in *Working Paper University of Edinburgh* 1993:I]
29. Jeffrey Prisbrey
An Experimental Analysis of Two-Person Reciprocity Games.
(February 1993)
[Published in *Social Science Working Paper* 787 (November 1992)]
30. Hugo A. Hopenhayn and Maria E. Muniagurria
Policy Variability and Economic Growth. (February 1993)
31. Eva Ventura Colera
A Note on Measurement Error and Euler Equations: an Alternative to
Log-Linear Approximations. (March 1993)
[Published in *Economics Letters*, 45, pp. 305-308 (1994)]
32. Rafael Crespi Cladera
Protecciones Anti-Opa y Concentración de la Propiedad: el Poder de Voto.
(March 1993)
33. Hugo A. Hopenhayn
The Shakeout. (April 1993)

34. Walter Garcia-Fontes
Price Competition in Segmented Industries. (April 1993)
35. Albert Satorra i Brucart
On the Asymptotic Optimality of Alternative Minimum-Distance Estimators in Linear Latent-Variable Models. (February 1993)
[Published in *Econometric Theory*, 10, pp. 867-883]
36. Teresa Garcia-Milà, Therese J. McGuire and Robert H. Porter
The Effect of Public Capital in State-Level Production Functions Reconsidered. (February 1993)
37. Ramon Marimon and Shyam Sunder
Expectations and Learning Under Alternative Monetary Regimes: an Experimental Approach. (May 1993)
38. José M. Labeaga and Angel López
Tax Simulations for Spain with a Flexible Demand System. (May 1993)
39. Daniel Serra and Charles ReVelle
Market Capture by Two Competitors: The Pre-Emptive Location Problem. (May 1993)
[Forthcoming in *Journal of Regional Science*]
40. Xavier Cuadras-Morató
Commodity Money in the Presence of Goods of Heterogenous Quality. (July 1993)
[Published in *Economic Theory* 4 (1994)]
41. M. Antònia Monés and Eva Ventura
Saving Decisions and Fiscal Incentives: A Spanish Panel Based Analysis. (July 1993)
42. Wouter J. den Haan and Albert Marcet
Accuracy in Simulations. (September 1993)
[Published in *Review of Economic Studies*, (1994)]
43. Jordi Galí
Local Externalities, Convex Adjustment Costs and Sunspot Equilibria. (September 1993)
[Forthcoming in *Journal of Economic Theory*]
44. Jordi Galí
Monopolistic Competition, Endogenous Markups, and Growth. (September 1993)
[Forthcoming in *European Economic Review*]
45. Jordi Galí
Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand. (October 1993)
[Forthcoming in *Journal of Economic Theory*]

46. Oriol Amat
The Relationship between Tax Regulations and Financial Accounting: a Comparison of Germany, Spain and the United Kingdom. (November 1993)
[Forthcoming in *European Management Journal*]
47. Diego Rodríguez and Dimitri Vayanos
Decentralization and the Management of Competition. (November 1993)
48. Diego Rodríguez and Thomas M. Stoker
A Regression Test of Semiparametric Index Model Specification. (November 1993)
49. Oriol Amat and John Blake
Control of the Costs of Quality Management: a Review of Current Practice in Spain. (November 1993)
50. Jeffrey E. Prisbrey
A Bounded Rationality, Evolutionary Model for Behavior in Two Person Reciprocity Games. (November 1993)
51. Lisa Beth Tilis
Economic Applications of Genetic Algorithms as a Markov Process. (November 1993)
52. Ángel López
The Command for Private Transport in Spain: A Microeconomic Approach. (December 1993)
53. Ángel López
An Assessment of the Encuesta Continua de Presupuestos Familiares (1985-89) as a Source of Information for Applied Research. (December 1993)
54. Antonio Cabrales
Stochastic Replicator Dynamics. (December 1993)
55. Antonio Cabrales and Takeo Hoshi
Heterogeneous Beliefs, Wealth Accumulation, and Asset Price Dynamics. (February 1993, Revised: June 1993)
56. Juan Pablo Nicolini
More on the Time Inconsistency of Optimal Monetary Policy. (November 1993)
57. Lisa B. Tilis
Income Distribution and Growth: A Re-examination. (December 1993)
58. José María Marín Viguera and Shinichi Suda
A Model of Financial Markets with Default and The Role of "Ex-ante" Redundant Assets. (January 1994)
59. Angel de la Fuente and José María Marín Viguera
Innovation, "Bank" Monitoring and Endogenous Financial Development. (January 1994)

60. Jordi Galí
Expectations-Driven Spatial Fluctuations. (January 1994)
61. Josep M. Argilés
Survey on Commercial and Economic Collaboration Between Companies in the EEC and Former Eastern Bloc Countries. (February 1994)
62. German Rojas
Optimal Taxation in a Stochastic Growth Model with Public Capital: Crowding-in Effects and Stabilization Policy. (September 1993)
63. Irasema Alonso
Patterns of Exchange, Fiat Money, and the Welfare Costs of Inflation. (September 1993)
64. Rohit Rahi
Adverse Selection and Security Design. (February 1994)
65. Jordi Galí and Fabrizio Zilibotti
Endogenous Growth and Poverty Traps in a Cournotian Model. (November 1993)
66. Jordi Galí and Richard Clarida
Sources of Real Exchange Rate Fluctuations: How Important are Nominal Shocks?. (October 1993, Revised: January 1994)
[Forthcoming in *Carnegie-Rochester Conference in Public Policy*]
67. John Ireland
A DPP Evaluation of Efficiency Gains from Channel-Manufacturer Cooperation on Case Counts. (February 1994)
68. John Ireland
How Products' Case Volumes Influence Supermarket Shelf Space Allocations and Profits. (February 1994)
69. Fabrizio Zilibotti
Foreign Investments, Enforcement Constraints and Human Capital Accumulation. (February 1994)
70. Vladimir Marianov and Daniel Serra
Probabilistic Maximal Covering Location Models for Congested Systems. (March 1994)
71. Giorgia Giovannetti.
Import Pricing, Domestic Pricing and Market Structure. (August 1993, Revised: January 1994)
72. Raffaella Giordano.
A Model of Inflation and Reputation with Wage Bargaining. (November 1992, Revised March 1994)
73. Jaume Puig i Junoy.
Aspectos Macroeconómicos del Gasto Sanitario en el Proceso de Convergencia Europea. (Enero 1994)

74. Daniel Serra, Samuel Ratick and Charles ReVelle.
The Maximum Capture Problem with Uncertainty (March 1994)
75. Oriol Amat, John Blake and Jack Dowds.
Issues in the Use of the Cash Flow Statement-Experience in some Other Countries
(March 1994)
76. Albert Marcet and David A. Marshall.
Solving Nonlinear Rational Expectations Models by Parameterized Expectations:
Convergence to Stationary Solutions (March 1994)
77. Xavier Sala-i-Martin.
Lecture Notes on Economic Growth (I): Introduction to the Literature and
Neoclassical Models (May 1994)
78. Xavier Sala-i-Martin.
Lecture Notes on Economic Growth (II): Five Prototype Models of Endogenous
Growth (May 1994)
79. Xavier Sala-i-Martin.
Cross-Sectional Regressions and the Empirics of Economic Growth (May 1994)
80. Xavier Cuadras-Morató.
Perishable Medium of Exchange (Can Ice Cream be Money?) (May 1994)
81. Esther Martínez García.
Progresividad y Gastos Fiscales en la Imposición Personal sobre la Renta (Mayo
1994)
82. Robert J. Barro, N. Gregory Mankiw and Xavier Sala-i-Martin.
Capital Mobility in Neoclassical Models of Growth (May 1994)
83. Sergi Jiménez-Martin.
The Wage Setting Process in Spain. Is it Really only about Wages? (April 1993,
Revised: May 1994)
84. Robert J. Barro and Xavier Sala-i-Martin.
Quality Improvements in Models of Growth (June 1994)
85. Francesco Drudi and Raffaella Giordano.
Optimal Wage Indexation in a Reputational Model of Monetary Policy Credibility
(February 1994)
86. Christian Helmenstein and Yury Yegorov.
The Dynamics of Migration in the Presence of Chains (June 1994)
87. Walter García-Fontes and Massimo Motta.
Quality of Professional Services under Price Floors. (June 1994)
88. Jose M. Bailen.
Basic Research, Product Innovation, and Growth. (September 1994)

89. Oriol Amat and John Blake and Julia Clarke.
Bank Financial Analyst's Response to Lease Capitalization in Spain (September 1994)
90. John Blake and Oriol Amat and Julia Clarke.
Management's Response to Finance Lease Capitalization in Spain (September 1994)
91. Antoni Bosch and Shyam Sunder.
Tracking the Invisible Hand: Convergence of Double Auctions to Competitive Equilibrium. (July 1994)
92. Sergi Jiménez-Martin.
The Wage Effect of an Indexation Clause: Evidence from Spanish Manufacturing Firms. (September 1994)
93. Albert Carreras and Xavier Tafunell.
National Enterprise. Spanish Big Manufacturing Firms (1917-1990), between State and Market (September 1994)
94. Ramon Faulf-Oller and Massimo Motta.
Why do Owners let their Managers Pay too much for their Acquisitions? (October 1994)
95. Marc Sáez Zafra and Jorge V. Pérez-Rodríguez.
Modelos Autorregresivos para la Varianza Condicionada Heteroscedástica (ARCH) (October 1994)
96. Daniel Serra and Charles ReVelle.
Competitive Location in Discrete Space (November 1994)