

ON OPTIMAL MONETARY AND FISCAL POLICY INTERACTIONS IN OPEN ECONOMIES*

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Abstract

This paper studies monetary and fiscal policy interactions in a two country model, where taxes on firms' sales are optimally chosen and the monetary policy is set cooperatively. It turns out that in a two country setting non-cooperative fiscal policy makers have an incentive to change taxes on sales depending on shocks realizations in order to reduce output production. Therefore whether the fiscal policy is set cooperatively or not matters for optimal monetary policy decisions. Indeed, as already shown in the literature, the cooperative monetary policy maker implements the flexible price allocation only when special conditions on the value of the distortions underlying the economy are met. However, if non-cooperative fiscal policy makers set the taxes on firms' sales depending on shocks realizations, these conditions cannot be satisfied; conversely, when fiscal policy is cooperative, these conditions are fulfilled. We conclude that whether implementing the flexible price allocation is optimal or not depends on the fiscal policy regime.

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1 Introduction

There is a large literature that analyzes the interactions¹ between monetary and fiscal policies in the closed economy environment. This literature usually considers, in a flexible price setting, the problem of the optimal policy choice in presence only of distortionary taxes; yet it has been recently integrated into frameworks with nominal rigidities². It happens otherwise for the open economy set up. At least when a two country framework is examined, the analysis of the interactions between monetary and fiscal policies when prices are rigid has been normally limited to the case in which the fiscal policy instrument is public expenditure financed by lump sum taxes³.

By assuming always a cooperative monetary policy, this paper studies monetary and fiscal policy interactions in the framework of Benigno and Benigno (2003) where there are two countries, prices are set one period in advance and fiscal policy is set either cooperatively or non-cooperatively .

Benigno and Benigno (2003) analysis determines under which conditions implementing the flexible price allocation is the optimal monetary policy both for the cooperative and the non-cooperative case. According to their findings in general⁴, while the non-cooperative policy makers do not implement the flexible price allocation, the cooperative monetary policy makers do it only if the firms revenue tax rates are equal across countries. Our analysis is devoted to examine when this last condition is satisfied once fiscal policy choices on firms' revenue tax rates are endogenized.

The key difference with respect to the open economy literature is that the fiscal policy instrument is not government expenditure. The key difference with respect to the closed economy one is the presence of lump sum taxes. This simplifying assumption allows to keep the model tractable. But, more importantly, it surfaces the following implication of the two country framework: when prices are flexible, non-cooperative fiscal policy authorities have an incentive to use tax rates strategically in order to influence the terms of trade. In fact, under complete markets, being consumption equal across countries, non-cooperative fiscal planners seek to externalize output production by adjusting firm tax rates. Because of this incentive, despite the presence of the lump sum taxes, optimal firm tax rates are state dependent. As a consequence when fiscal policy is set non-cooperatively there is a motive for endogenous movements of the wedge in the marginal rate of substitution between consumption and good production which turn out to matter for monetary policy decisions. If fiscal policy is set non-cooperatively the cooperative optimal monetary policy maker does not implement the flexible price allocation while, when fiscal policy is set cooperatively, implementing the flexible price allocation is always optimal. Which is the optimal coordinated monetary policy depends on the fiscal policy regime.

These conclusions are relevant for the analysis of international policy interactions. First of all at the European level. EMU birth has empathized the question of how

¹See for all Chari and Kehoe (1999).

²See Correia, Nicolini and Teles (2001), Schmitte-Grohé and Uribe (2004), Siu (2004) and Benigno and Woodford (2003).

³See for example Lombardo and Sutherland (2004), Galí and Monacelli (2005) and Beetsma and Jensen (2005).

⁴...i.e. unless either shocks are symmetric or special parameter restrictions are met.

to frame international institutions and regulations for conducting fiscal and monetary policies. The Maastricht Treaty has delegated the control of monetary policy to the ECB and has provided for EMU country fiscal discipline through the Growth Stability Pact. Underlying this institutional design there is the fear that undisciplined fiscal policies may force the ECB to give up the pursuit of price stability. Actually this should be a main objective of the ECB itself. However, according to our results, pursuing price stability is not, in general, the optimal cooperative monetary policy when fiscal policy authorities do not coordinate. Therefore the fact that, while complying the Growth Stability Pact, EMU countries still run autonomously the fiscal policy may be inconsistent with the idea that the ECB optimal policy is to pursue price stability.

Even at the global level there is concern about the spillover effects produced by uncoordinated policies that the last decades increasing interdependence has rendered more relevant. An extensive literature analyzes the need of policy coordination for either monetary or fiscal policy. Our findings suggest that for the analysis of optimal cooperative monetary policy whether fiscal policy is set cooperatively or not matters.

The paper is organized as follows. Section 2 is devoted to present the basic framework. Section 3 analyzes the optimal cooperative and non-cooperative fiscal policies when prices are flexible. Section 4 studies the conditions under which the cooperative monetary policy maker implements the flexible price allocation. Section 5 concludes.

2 The model

The basic framework belongs to the new generation of open economy stochastic general equilibrium models⁵. The world consists of two countries, Home and Foreign which have different currencies but a cooperative monetary policy. Conversely fiscal policy is conducted by single country authorities that may or may not coordinate.

2.1 Preferences

Each country is populated by a continuum of agents: $[0, n)$ and $[n, 1]$ respectively. Agents are, at the same time, consumers and monopolistic producers of a single differentiated good⁶. Preferences of a generic agent j inherent in the home country are defined as:

$$U^j = \sum_{t=0}^{\infty} \beta^t E_0 \left[u(C_t^j) + \chi L \left(\frac{M_t^j}{P_t} \right) - V(y_t(j), z_t) \right] \quad 0 < \beta < 1 \quad (1)$$

where β is the intertemporal discount factor and E is the expectation operator conditional on the information set at time 0; u and L are concave functions respectively increasing in a consumption index C^j and in the money demand $\frac{M^j}{P}$; V is an increasing convex function in agent j produced good $y(j)$ and a country specific shock z . Foreign country agent preferences are represented by a utility function symmetric to (1) where foreign variables are denoted by an asterisk⁷.

⁵See, in particular, Benigno and Benigno (2003) and Benigno (2004).

⁶Therefore $[0, n)$ and $[n, 1]$ indicate the continuum of both agents and goods.

⁷This convention will be used from now on.

Price and consumption indexes

Each agent consumes all the varieties of goods. Actually, the home country consumption index is defined as a CES aggregator of home and foreign consumption bundles. Specifically:

$$C = \left[n^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad \theta > 0 \quad (2)$$

with θ indicating the elasticity of substitution between C_H and C_F . The latter indexes are respectively defined as:

$$C_H = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \quad C_F = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}} \quad \sigma > 1 \quad (3)$$

where $c(h)$ is the differentiated good produced in country H , $c(f)$ the differentiated good produced in country F and σ is the elasticity of substitution between goods produced in the same country.

Consumption index definitions (2) and (3) allow to determine consistent definitions of price indexes⁸. In particular, P is given by:

$$P = \left[n P_H^{1-\theta} + (1-n) P_F^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (4)$$

while P_H and P_F are specified as:

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}} \quad P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}} \quad (5)$$

with $p(h)$ and $p(f)$ being the prices of single goods produced respectively in country H and F . There are no trading frictions. Thus the law of one price is assumed to hold in all single good markets that is $p(h) = \xi p^*(f)$, being ξ the nominal exchange rate. Consequently, according to consumption and price indexes definitions, even the purchasing power parity holds. In other words $P_H = \xi P_H^*$, $P_F = \xi P_F^*$ and $P = \xi P^*$.

2.2 Consumption, portfolio choices and money demand

The Dixit-Stiglitz structure of preferences and consumption and price indexes allows to solve the consumer problem in three stages. In the first two stages, agents decide how much of real net income to spend for goods produced within a country both at single and aggregate levels. According to the set of optimal conditions, it is possible to

⁸Namely price and consumption indexes are such that expenditures for total consumption, $\int_0^1 p(k)c(k)dk$, and for goods produced within a country, $\int_0^n p(h)c(h)dh$ and $\int_n^1 p(f)c(f)df$, are respectively equal to PC and $P_H C_H$ and $P_F C_F$.

determine agent j demand for single goods, h and f , and for entire bundles, C_H and C_F , as:

$$\begin{aligned} c^j(h) &= \left(\frac{1}{n}\right) \left[\frac{p(h)}{P_H}\right]^{-\sigma} C_H^j & c^j(f) &= \left(\frac{1}{1-n}\right) \left[\frac{p(f)}{P_F}\right]^{-\sigma} C_F^j \quad \text{for } j \in [0, n) \\ c^{*j}(h) &= \left(\frac{1}{n}\right) \left[\frac{p^*(h)}{P_H^*}\right]^{-\sigma} C_H^{*j} & c^{*j}(f) &= \left(\frac{1}{1-n}\right) \left[\frac{p^*(f)}{P_F^*}\right]^{-\sigma} C_F^{*j} \quad \text{for } j \in [n, 1] \end{aligned} \quad (6)$$

$$\begin{aligned} C_H^j &= n \left[\frac{P_H}{P}\right]^{-\theta} C^j & C_F^j &= (1-n) \left[\frac{P_F}{P}\right]^{-\theta} C^j \quad \text{for } j \in [0, n) \\ C_H^{*j} &= n \left[\frac{P_H^*}{P^*}\right]^{-\theta} C^{*j} & C_F^{*j} &= (1-n) \left[\frac{P_F^*}{P^*}\right]^{-\theta} C^{*j} \quad \text{for } j \in [n, 1] \end{aligned} \quad (7)$$

The third stage of the agent j optimization problem coincides with the standard consumer problem. Agents maximize (1) subject to the following sequence of budget constraints⁹:

$$\int_s p_{t+1}^s A_{t+1}^{s,j} ds + M_t^j = A_t^{s,j} + M_{t-1}^j + (1 - \tau_t) p_t(j) y_t(j) - P_t C_t^j + T_t^j \quad (8)$$

which states that nominal saving, net of lump sum transfers and firms' revenue taxes has to equalize the money holding M^j and the nominal value of a state contingent portfolio. In fact $p(j)$ indicates agent j output price, while $[p^s]$ is the pricing vector of a one-period maturity portfolio that pays $A^{s,j}$ when the state of the world s occurs. The assumption underlying (8) is that domestic and international capital markets are complete. Moreover the state contingent wealth at time zero is such that the lifetime discounted budget constraints are identical across agents. These hypotheses jointly with the preferences specification in (1) entail that aggregate consumptions are equal across countries. Indeed, by the first order conditions of the consumer problem:

$$p_{t+1}^s = \beta \pi(s^{t+1}|s^t) \frac{u_c(C_{t+1}^j) P_t}{u_c(C_t^j) P_{t+1}} \quad p_{t+1}^{*s} = \beta \pi(s^{t+1}|s^t) \frac{u_{c^*}(C_{t+1}^{*j}) P_t^*}{u_{c^*}(C_t^{*j}) P_{t+1}^*} \quad (9)$$

where $\pi(s^{t+1}|s^t)$ represents the probability that state s occurs at time $t+1$, given the all past history of states up to period t ¹⁰. Hence, according to (9), the cost of a marginal unit of state s contingent payoff should be equal to the marginal rate of substitution between next period state s contingent consumption and current period consumption. Combining conditions given in (9) it is possible to verify that there is perfect risk sharing that is:

$$u_c(C_t^j) = u_{c^*}(C_t^{*l}) \quad (10)$$

which implicitly states that $C^j = C^{*l}$ for any j and l with $j \in [0, n)$ and $l \in [n, 1]$.

⁹...and subject to the condition that relates single good demand, $y^d(h)$, with the aggregate consumption C which will be determined later on.

¹⁰Implicitly (9) states that $p_{t+1}^s = \pi(s^{t+1}|s^t) Q_{t,t+1}^j$ where $Q_{t,t+1}$ denotes what is usually called the stochastic discount factor.

Moreover the optimality conditions of the consumer problem allow to derive the implicit money demand function. If there are complete markets, then, by *non-arbitrage*, the following condition must hold:

$$\frac{1}{1+i_t} = \int_s p_{t+1}^s ds = E_t(Q_{t,t+1}) \quad (11)$$

Namely the price of a riskless portfolio should be equal to the price of a riskless bond, being i_t the nominal interest rate. Given (11), it can be shown:

$$\chi L_{M/P} \left(\frac{M_t^j}{P_t} \right) = \frac{i_t}{1+i_t} u_c(C_t^j) \quad (12)$$

that is at the optimum the marginal rate of substitution between consumption and real balances is equal to the opportunity cost of keeping one unit more of real balances.

Single good demands and aggregate outputs

The single good total demands can be determined by combining (6) with (7) as follows:

$$y^d(h) = \left[\frac{p(h)}{P_H} \right]^{-\sigma} \left[\frac{P_H}{P} \right]^{-\theta} C^W \quad y^d(f) = \left[\frac{p(f)}{P_F} \right]^{-\sigma} \left[\frac{P_F}{P} \right]^{-\theta} C^W \quad (13)$$

where $C^W \equiv nC + (1-n)C^* = C = C^*$. At the same time, by properly aggregating these demands it is possible to obtain the home and foreign outputs:

$$Y = \left[\frac{P_H}{P} \right]^{-\theta} C \quad Y^* = \left[\frac{P_F}{P} \right]^{-\theta} C \quad (14)$$

$$\text{with } Y = \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n y(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \text{ and } Y^* = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 y(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}.$$

The last conditions make clear that output divergences across countries are explained by movements of the terms of trade¹¹.

2.3 Monetary and fiscal policy makers

Our analysis is focused on fiscal and monetary policy interactions, when monetary policy is run cooperatively. Thus the monetary policy makers main objective is always maximizing the world economy welfare¹². The monetary market equilibrium conditions require that:

$$M_t = \int_0^n M_t^j dj \quad M_t^* = \int_n^1 M_t^{*j} dj \quad (15)$$

On the contrary for fiscal policy we analyze the cooperative and the non-cooperative case. Home country fiscal authority faces a period by period balance budget constraint:

¹¹In our context, the terms of trade are defined as the ratio between P_F and P_H .

¹²Notice that in our set up, differently from the case of a monetary union, the monetary policy authority relies on two instruments.

$$M_t + n\tau_t P_{H,t} Y_t = M_{t-1} + T_t \quad (16)$$

where $T \equiv \int_0^n T^j dj$, and $n\tau P_H Y = \int_0^n p(j)y(j)dj$ and the money supply is taken as given so that policy makers are instrument-independent¹³. Moreover lump sum transfers are taken as a residual guaranteeing that the governments budget constraint is always satisfied. A symmetric constraint holds for the foreign fiscal authority.

2.4 Flexible price setting

When prices are flexible, monopolist producers in countries H and F choose the optimal price respectively according to:

$$\left(\frac{\sigma-1}{\sigma}\right) (1-\tau_t) u_c(C_t) \frac{P_{H,t}}{P_t} = V_y \left(\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, z_t \right) \quad (17)$$

$$\left(\frac{\sigma-1}{\sigma}\right) (1-\tau_t^*) u_c(C_t) \frac{P_{F,t}}{P_t} = V_y \left(\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t, z_t^* \right) \quad (18)$$

where conditions (17) and (18)¹⁴ are retrieved by taking into account that in the symmetric equilibrium agents inherent in the same country set the same output price. Rewriting (17) and (18) as:

$$\mu_t^{-1} = \left(\frac{P_{H,t}}{P_t}\right)^{-1} \frac{V_y \left([P_{H,t}/P_t]^{-\theta} C_t, z_t \right)}{u_c(C_t)} \quad \mu_t^{*-1} = \left(\frac{P_{F,t}}{P_t}\right)^{-1} \frac{V_y \left([P_{F,t}/P_t]^{-\theta} C_t, z_t^* \right)}{u_c(C_t)} \quad (19)$$

clarifies how real marginal costs are related to the distortions present in the economy. Actually $\mu_t \equiv \frac{\sigma}{\sigma-1} \frac{1}{(1-\tau_t)}$ and $\mu_t^* \equiv \frac{\sigma}{\sigma-1} \frac{1}{(1-\tau_t^*)}$; which is to say that μ and μ^* are determined, on the one hand, by the mark-up charged by monopolistic producers; on the other hand, by the distorting taxes on firms' revenue. Only when $\mu = 1$ and $\mu^* = 1$, the flexible-price allocation is efficient, namely coincides with the competitive equilibrium allocation without distortionary taxes.

2.5 Preference specification and welfare criteria

In our model household preferences are specified as:

$$u(C_t) \equiv \frac{C_t^{1-\rho}}{1-\rho} \quad \rho > 0$$

¹³In fact, being money supply given for fiscal policy authorities, (16) can be rewritten as: $n\tau_t P_{H,t} Y_t = T_t^g$ where $T_t^g = M_{t-1} + T_t - M_t$. In this sense policy authorities do not share the same budget constraint. See Lambertini (2004).

¹⁴These conditions are derived by maximizing agent j utility with respect to $p(j)$ and C_t , subject to the budget constraint and single good demand given in (13).

$$V(y_t(j), z_t) \equiv \frac{z_t(y_t(j))^\nu}{\nu} \quad \text{if } j \in [0, n) \quad V(y_t^*(j), z_t^*) \equiv \frac{z_t^*(y_t^*(j))^\nu}{\nu} \quad \text{if } j \in [n, 1] \quad \nu > 1 \quad (20)$$

where ρ and $\nu - 1$ represent the intertemporal elasticities of substitution in consumption and labor supply respectively.

Policy makers are assumed to be benevolent and to commit credibly, once for all, in the period -1. Therefore they maximize the expected weighted average of consumer utilities defined in (20). Specifically, home and foreign country planner welfare criteria are given by:

$$W \equiv \sum_{t=0}^{\infty} \beta^t E_{-1} \left[u(C_t) - \frac{\int_0^n V(y_t(j), z_t) dj}{n} \right] \quad W^* \equiv \sum_{t=0}^{\infty} \beta^t E_{-1} \left[u(C_t) - \frac{\int_n^1 V(y_t^*(j), z_t^*) dj}{1-n} \right] \quad (21)$$

while the central planner welfare criterium is:

$$nW + (1-n)W^* \quad (22)$$

Notice that (21) and (22) do not include the utility derived from money holdings. Thus we implicitly restrict the analysis to the cashless economy limiting case¹⁵.

3 Optimal fiscal policy with flexible prices

Before analyzing monetary and fiscal policy interactions, we examine the optimal fiscal policy under flexible prices. In fact, as expected, under flexible prices fiscal policy produces real effects while monetary policy does not. The main purpose of this section is to compare non-cooperative fiscal policies with the cooperative one. It turns out that non-cooperative fiscal authorities deviate from the cooperative policy. As a consequence there are potential gains from coordination.

In order to compare cooperative and non-cooperative fiscal policies, first of all we need to retrieve from the appendix the flexible price equilibrium levels of consumption and relative prices that are¹⁶:

$$C_t = \left[\left(\frac{\sigma}{\sigma-1} \right) \frac{z_t}{(1-\tau_t)} \left[\frac{1}{2} \Lambda_t^{\frac{1-\theta}{\gamma}} + \frac{1}{2} \right]^{\frac{\gamma}{1-\theta}} \right]^{\frac{1}{\delta}} \quad (23)$$

$$\Pi_{H,t} = \left[\frac{1}{2} \Lambda_t^{\frac{1-\theta}{\gamma}} + \frac{1}{2} \right]^{\frac{1}{\theta-1}} \quad \Pi_{F,t} = \left[\frac{1}{2} + \frac{1}{2} \Lambda_t^{\frac{\theta-1}{\gamma}} \right]^{\frac{1}{\theta-1}} \quad (24)$$

$$\frac{\Pi_{F,t}}{\Pi_{H,t}} = \Lambda_t^{\frac{1}{\gamma}} \quad (25)$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_t}$, $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_t}$, $\Lambda_t \equiv \left[\frac{z_t^*}{z_t} \frac{(1-\tau_t)}{(1-\tau_t^*)} \right]$, $\gamma \equiv 1 + (\nu - 1)\theta$ and $\delta \equiv 1 - \rho - \nu$.

Note that conditions (23), (24) and (25) give evidence of monetary policy neutrality.

¹⁵See Benigno and Benigno (2003).

¹⁶From now on we do the simplifying assumption that $n = \frac{1}{2}$.

3.1 The cooperative case

When prices are flexible the cooperative fiscal authority maximizes the expected weighted average of agents' indirect utility functions namely:

$$\sum_{t=0}^{\infty} \beta^t E_{-1} \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{1}{2} \frac{z_t \left(\Pi_{H,t}^{-\theta} C_t \right)^\nu}{\nu} - \frac{1}{2} \frac{z_t^* \left(\Pi_{F,t}^{-\theta} C_t \right)^\nu}{\nu} \right] \quad (26)$$

where consumption and relative prices equilibrium levels are determined according to (23) and (24). The first order conditions with respect to τ_t and to τ_t^* respectively given by:

$$\begin{aligned} \left[C_t^{\nu-1} \left(\frac{1}{2} z_t \Pi_{H,t}^{-\theta\nu} + \frac{1}{2} z_t^* \Pi_{F,t}^{-\theta\nu} \right) - C_t^{-\rho} \right] \frac{\partial C_t}{\partial \tau_t} &= \theta C_t^\nu \left[\frac{1}{2} z_t \Pi_{H,t}^{-\theta\nu-1} \frac{\partial \Pi_{H,t}}{\partial \tau_t} + \frac{1}{2} z_t^* \Pi_{F,t}^{-\theta\nu-1} \frac{\partial \Pi_{F,t}}{\partial \tau_t} \right] \\ \left[C_t^{\nu-1} \left(\frac{1}{2} z_t \Pi_{H,t}^{-\theta\nu} + \frac{1}{2} z_t^* \Pi_{F,t}^{-\theta\nu} \right) - C_t^{-\rho} \right] \frac{\partial C_t}{\partial \tau_t^*} &= \theta C_t^\nu \left[\frac{1}{2} z_t \Pi_{H,t}^{-\theta\nu-1} \frac{\partial \Pi_{H,t}}{\partial \tau_t^*} + \frac{1}{2} z_t^* \Pi_{F,t}^{-\theta\nu-1} \frac{\partial \Pi_{F,t}}{\partial \tau_t^*} \right] \end{aligned} \quad (27)$$

allow to retrieve the optimal tax rates as follows. Substituting the expressions (61), (63) and (64) calculated in the appendix in (27) we obtain:

$$\left[C_t^{-\delta} \left(\frac{1}{2} z_t \Pi_{H,t}^{-\theta\nu} + \frac{1}{2} z_t^* \Pi_{F,t}^{-\theta\nu} \right) - 1 \right] \alpha = (1 - \tau_t) \frac{1}{2} \left[\Pi_{H,t}^{1-\theta} \Lambda_t^{\frac{1-\theta}{\gamma}} - \frac{z_t^*}{z_t} \left(\frac{\Pi_{F,t}}{\Pi_{H,t}} \right)^{-\theta\nu} \Pi_{F,t}^{1-\theta} \Lambda_t^{\frac{\theta-1}{\gamma}} \right] \quad (28)$$

with $\alpha \equiv \frac{\gamma}{\theta\delta} \frac{\sigma}{\sigma-1}$. Notice that the left hand side of (28) and of the foreign country symmetric condition are equal. Therefore:

$$(1 - \tau_t) \left[\Pi_{H,t}^{1-\theta} \Lambda_t^{\frac{1-\theta}{\gamma}} - \frac{z_t^*}{z_t} \Pi_{F,t}^{1-\theta} \Lambda_t^{\frac{\theta(1-\nu)-1}{\gamma}} \right] = (1 - \tau_t^*) \left[\Pi_{F,t}^{1-\theta} \Lambda_t^{\frac{\theta-1}{\gamma}} - \frac{z_t}{z_t^*} \Pi_{H,t}^{\theta-1} \Lambda_t^{\frac{\theta(1-\nu)-1}{\gamma}} \right] \quad (29)$$

Moreover given (25) and rearranging condition (29)

$$(1 - \tau_t) \left[\Lambda_t^{\frac{1-\theta}{\gamma}} + 1 \right] = (1 - \tau_t^*) \Lambda_t^{\frac{1-\theta}{\gamma}} \left[\Lambda_t^{\frac{\theta-1}{\gamma}} + 1 \right]$$

which implies that $\tau_t = \tau_t^*$ for any t , z_t and z_t^* . In other words, in the cooperative case the fiscal authorities choose to equalize firms' revenue tax rates in every period independently of shock realizations. By so doing, output steady state levels across countries are equalized as well and there is no need to try to correct the differences across countries in the expected disutilities of output.

The fact that $\tau_t = \tau_t^*$ allows to determine the optimal tax rate. Indeed if $\tau_t = \tau_t^*$, then, by (28) and being $\Lambda_t = \frac{z_t^*}{z_t}$:

$$\Pi_{H,t}^\gamma \left(\frac{1}{2} \Pi_{H,t}^{-\theta\nu} + \frac{1}{2} \Lambda_t \Pi_{F,t}^{-\theta\nu} \right) (1 - \tau_t) = \frac{\sigma}{\sigma - 1}$$

However (24) entails that $\Pi_{H,t}^\gamma \left(\frac{1}{2} \Pi_{H,t}^{-\theta\nu} + \frac{1}{2} \Lambda_t \Pi_{F,t}^{-\theta\nu} \right) = 1$. Thus we can conclude:

$$(1 - \tau_t) = (1 - \tau_t^*) = \frac{\sigma}{\sigma - 1} \quad (30)$$

for any t , z_t and z_t^* . Not surprisingly, when fiscal policy is set cooperatively, it is optimal to subsidize firm sales in order to exactly offset the monopolistic distortions. By setting $\mu_t = \mu_t^* = 1$ this policy allows to implement the efficient allocation.

The non-cooperative case

In the non-cooperative case the home country planner takes foreign country tax rate as given and maximizes the expected average of home country agent indirect utility functions, that is:

$$\sum_{t=0}^{\infty} \beta^t E_{-1} \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{z_t \left(\Pi_{H,t}^{-\theta} C_t \right)^\nu}{\nu} \right] \quad (31)$$

where consumption and relative price equilibrium levels are given by (23) and (24). According to the first order condition with respect to τ_t :

$$\left[C_t^{\nu-1} z_t \Pi_{H,t}^{-\theta\nu} - C_t^{-\rho} \right] \frac{\partial C_t}{\partial \tau_t} = \theta C_t^\nu z_t \Pi_{H,t}^{-\theta\nu-1} \frac{\partial \Pi_{H,t}}{\partial \tau_t} \quad (32)$$

A symmetric condition can be retrieved for the foreign country. Plugging (63) and (61) in (32) allows to get:

$$\alpha \frac{\sigma-1}{\sigma} \left[z_t C_t^{-\delta} \Pi_{H,t}^{-\theta\nu} - 1 \right] = z_t C_t^{-\delta} \Pi_{H,t}^{-\theta\nu} \Lambda_t^{\frac{1-\theta}{\gamma}} \quad (33)$$

which can be simplified as:

$$(1-\tau_t) \alpha \frac{\sigma-1}{\sigma} \Pi_{H,t}^{1-\theta} - \alpha = (1-\tau_t) \Pi_{H,t}^{1-\theta} \Lambda_t^{\frac{1-\theta}{\gamma}} \quad (34)$$

and leads to the final expression:

$$(1-\tau_t) \left[\alpha \frac{\sigma-1}{\sigma} - \Lambda_t^{\frac{1-\theta}{\gamma}} \right] - \alpha \left[\frac{1}{2} \Lambda_t^{\frac{1-\theta}{\gamma}} + \frac{1}{2} \right] = 0 \quad (35)$$

This expression defines implicitly the home country optimal tax rate as function of exogenous shocks and foreign country strategy¹⁷. According to condition (35) we may expect that, in general, in the non-cooperative case optimal tax rates depend on shock realizations. This last condition allows to prove the next propositions:

Proposition 1 *Under flexible prices, if shocks are symmetric¹⁸, $(1-\tau_t) = (1-\tau_t^*) = \left[\frac{\sigma-1}{\sigma} - \frac{1}{\alpha} \right]^{-1}$ is the optimal non-cooperative fiscal policy in all contingencies and at all times. If shocks are asymmetric, $(1-\tau_t) = (1-\tau_t^*) = \left[\frac{\sigma-1}{\sigma} - \frac{1}{\alpha} \right]^{-1}$ is the optimal non-cooperative fiscal policy in all contingencies and at all times if and only if $\theta = 1$.*

¹⁷In other words (35) represents the home country reaction function.

¹⁸i.e. $z_t = z_t^*$ in all contingencies and at all times

Proof. Suppose $\tau_t = \tau_t^*$ for any t . Then if shocks are symmetric $\Lambda_t = 1$ for any t . As well if $\theta = 1$ $\Lambda_t^{\frac{1-\theta}{\gamma}} = 1$ for any t . But, if $\Lambda_t^{\frac{1-\theta}{\gamma}} = 1$, condition (35) can be rewritten as:

$$(1 - \tau_t) \left[\alpha \frac{\sigma - 1}{\sigma} - 1 \right] = \alpha \quad (36)$$

An equal condition can be stated for the foreign country. Therefore if shocks are symmetric or $\theta = 1$ $(1 - \tau) = (1 - \tau^*) = \left[\frac{\sigma-1}{\sigma} - \frac{1}{\alpha} \right]^{-1}$ in all contingencies and at all times.

What remains to show is that if $\theta \neq 1$ and shocks are asymmetric $\tau_t = \tau_t^* = \left[\frac{\sigma-1}{\sigma} - \frac{1}{\alpha} \right]^{-1}$ for any t cannot be the non-cooperative optimal fiscal policy.

Suppose $\tau_t = \tau_t^*$ for any t . Then by condition (35) and the correspondent condition for the foreign country:

$$\left[\frac{1}{2} \Lambda_t^{\frac{1-\theta}{\gamma}} + \frac{1}{2} \right] \left[\alpha \frac{\sigma - 1}{\sigma} - \Lambda_t^{\frac{\theta-1}{\gamma}} \right] = \left[\frac{1}{2} \Lambda_t^{\frac{\theta-1}{\gamma}} + \frac{1}{2} \right] \left[\alpha \frac{\sigma - 1}{\sigma} - \Lambda_t^{\frac{1-\theta}{\gamma}} \right] \quad (37)$$

which entails that:

$$\Lambda_t^{\frac{1-\theta}{\gamma}} = \Lambda_t^{\frac{\theta-1}{\gamma}} \quad (38)$$

Provided $\theta \neq 1$ and $\Lambda_t > 0$ for any t , it is easy to see that if $z_t \neq z_t^*$ for some t (38) cannot be satisfied. Then, given that shocks are asymmetric, $\tau_t \neq \tau_t^*$ for some t . As a consequence, when $\theta \neq 1$ and shocks are asymmetric, $\tau_t = \tau_t^* = \left[\frac{\sigma-1}{\sigma} - \frac{1}{\alpha} \right]^{-1}$ cannot be the optimal non-cooperative fiscal policy. ■

Corollary 2 Suppose $\frac{1-\tau_t}{1-\tau_t^*}$ independent of shock realizations namely $\frac{1-\tau_t}{1-\tau_t^*} = k$. Then under flexible prices and if $\theta \neq 1$ $\frac{1-\tau_t}{1-\tau_t^*}$ can be supported as an optimal non-cooperative fiscal policy only if the stochastic process of z_t^* and z_t satisfies the following restriction: $\frac{z_t^*}{z_t} = w_1$ or $\frac{z_t^*}{z_t} = w_2$ where¹⁹

$$w_1^{\frac{1-\theta}{\gamma}} = \frac{-(k-1) \left(1 - \alpha \frac{\sigma-1}{\sigma} \right) + \sqrt{(k-1)^2 \left(1 - \alpha \frac{\sigma-1}{\sigma} \right)^2 + 4k \left(k + \alpha \frac{\sigma-1}{\sigma} \right) \left(1 + k\alpha \frac{\sigma-1}{\sigma} \right)}}{2k^{\frac{1-\theta}{\gamma}} \left(k + \alpha \frac{\sigma-1}{\sigma} \right)} \quad (39)$$

$$w_2^{\frac{1-\theta}{\gamma}} = \frac{-(k-1) \left(1 - \alpha \frac{\sigma-1}{\sigma} \right) - \sqrt{(k-1)^2 \left(1 - \alpha \frac{\sigma-1}{\sigma} \right)^2 + 4k \left(k + \alpha \frac{\sigma-1}{\sigma} \right) \left(1 + k\alpha \frac{\sigma-1}{\sigma} \right)}}{2k^{\frac{1-\theta}{\gamma}} \left(k + \alpha \frac{\sigma-1}{\sigma} \right)} \quad (40)$$

Proof. Taking the ratio between condition (35) and the foreign country correspondent condition and recalling that $\Lambda_t \equiv \left[\frac{z_t^* (1-\tau_t)}{z_t (1-\tau_t^*)} \right]$ we obtain:

$$k \left[\left(k \frac{z_t^*}{z_t} \right)^{\frac{1-\theta}{\gamma}} - \alpha \frac{\sigma-1}{\sigma} \right] \left[\left(k \frac{z_t^*}{z_t} \right)^{\frac{\theta-1}{\gamma}} + 1 \right] = \left[\left(k \frac{z_t^*}{z_t} \right)^{\frac{\theta-1}{\gamma}} - \alpha \frac{\sigma-1}{\sigma} \right] \left[\left(k \frac{z_t^*}{z_t} \right)^{\frac{1-\theta}{\gamma}} + 1 \right]$$

which can be simplified as:

$$k^{\frac{1-\theta}{\gamma}} \left(k + \alpha \frac{\sigma-1}{\sigma} \right) \left(\frac{z_t^*}{z_t} \right)^{2\frac{1-\theta}{\gamma}} + (k-1) \left(1 - \alpha \frac{\sigma-1}{\sigma} \right) \left(\frac{z_t^*}{z_t} \right)^{\frac{1-\theta}{\gamma}} - k^{\frac{\theta-1}{\gamma}} \left(1 + k\alpha \frac{\sigma-1}{\sigma} \right) = 0$$

¹⁹Moreover k should ensure that either w_1 or w_2 or both are real and positive numbers.

This last condition allows to retrieve conditions (39) and (40). ■

According to proposition 1 the uncoordinated fiscal policy authorities deviate from the coordinated optimal choice. Indeed this proposition and its proof entail that (30) cannot be a non-cooperative optimal fiscal policy²⁰. In turn corollary 2 implies that, unless special restrictions are met, the optimal non-cooperative fiscal policy depends on shock realizations.

The interpretation of these results is the following. Uncooperative fiscal policy makers have a conflicting objective: to seek to reduce output disutility. Why this incentive²¹ is present can be made clear by rewriting (17) as:

$$(1 - \tau_t) \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{P_{H,t}}{P_t} \right)^{1-\theta} \frac{u(C_t)}{\nu} = \frac{V(Y_t, z_t)}{1 - \rho} \quad (41)$$

which follows from our preference specification and condition (14). According to condition (41) fiscal policy makers may attempt to increase consumption utility and/or decrease output disutility through two channels. The first channel operates through the indirect impact of firms' sale tax rates on the relative prices and allows to clarify why optimal tax rates may be adjusted to the shock ratio variations. The second channel consists of the direct impact of firm revenue tax on (41) and contribute to explain why uncoordinated fiscal policy is set suboptimally even when $\theta = 1$. Actually in this case, or when shocks are symmetric, the home country optimal tax rate is determined according to (36) independently of shock realizations and, as follows from proposition 1, is different from the one chosen by the cooperative planner. More specifically it is easy to show that, given the parameter restrictions stated in section 2, the non-cooperative optimal tax rate implied by (36) is greater than the cooperative optimal one implied by (30)²². This result substantiates the intuition that the non-cooperative fiscal policy planner key incentive is to seek to externalize the disutility of producing output.

4 Optimal monetary policy with one period in advance price setting

In this section we examine the conditions under which implementing the flexible-price allocation is the optimal cooperative monetary policy. It turns out that whether implementing the flexible price allocation is optimal or not depends on the fiscal policy regime. In fact when fiscal policy is set cooperatively implementing the flexible-price allocation is always optimal. Conversely when fiscal policy is set non-cooperatively, implementing the flexible-price allocation is optimal only when appropriate restrictions are met.

To allow monetary policy to produce real effects, we assume that all prices are

²⁰The proof of corollary 2 implies that when shocks are asymmetric there exists at least a period t such that $\tau_t \neq \tau_t^*$ which is inconsistent with (30).

²¹This incentive is similar to the incentive described in Benigno and Benigno (2003) for uncoordinated monetary policy authorities.

²²Indeed $\left[\frac{\sigma-1}{\sigma} - \frac{1}{\alpha} \right]^{-1} < \left(\frac{\sigma}{\sigma-1} \right)$ for any α .

chosen one period in advance. In that case householder first order conditions lead to:

$$E_{t-1} \left\{ \left[\frac{\sigma-1}{\sigma} (1-\tau_t) u_c(C_t) \frac{P_{H,t}}{P_t} - V_y \left(\left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t, z_t \right) \right] \left(\frac{P_{H,t}}{P_t} \right)^{-\theta} C_t \right\} = 0 \quad (42)$$

$$E_{t-1} \left\{ \left[\frac{\sigma-1}{\sigma} (1-\tau_t^*) u_c(C_t) \frac{P_{F,t}}{P_t} - V_y \left(\left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t, z_t^* \right) \right] \left(\frac{P_{F,t}}{P_t} \right)^{-\theta} C_t \right\} = 0 \quad (43)$$

where the expectation is conditional on the information set at time $t-1$.

Given conditions (42) and (43) it is possible to find the restrictions that a monetary policy must satisfy to be optimal. In fact the common monetary policy authority maximizes:

$$\sum_{t=0}^{\infty} \beta^t E_{-1} \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{1}{2} \frac{z_t \left(\Pi_{H,t}^{-\theta} C_t \right)^\nu}{\nu} - \frac{1}{2} \frac{z_t^* \left(\Pi_{F,t}^{-\theta} C_t \right)^\nu}{\nu} \right] \quad (44)$$

subject to the constraints (42) and (43) for each time t and to the constraint:

$$\frac{1}{2} \Pi_{H,t}^{(1-\theta)} + \frac{1}{2} \Pi_{F,t}^{(1-\theta)} = 1 \quad (45)$$

which holds for each time t and each contingency. Combining the first order conditions (retrieved in the appendix) with respect to $\Pi_{H,t}$ and $\Pi_{F,t}$:

$$\begin{aligned} \theta(1-\Delta\nu) z_t \Pi_{H,t}^{-\theta(\nu-1)-1} C_t^\nu - \Delta(1-\theta)(1-\tau_t) \frac{\sigma-1}{\sigma} C_t^{1-\rho} = \\ \theta(1-\Omega\nu) z_t^* \Pi_{F,t}^{-\theta(\nu-1)-1} C_t^\nu - \Omega(1-\theta)(1-\tau_t^*) \frac{\sigma-1}{\sigma} C_t^{1-\rho} \end{aligned} \quad (46)$$

Using (46) and the derivative with respect to C_t we obtain:

$$\left[1 - (1-\tau_t) \frac{\sigma-1}{\sigma} (1-\rho) \Omega + \frac{1}{2} \Xi_t \Pi_{H,t}^{1-\theta} \right] C_t^{-\rho} \Pi_{F,t} = (1-\Omega\nu) z_t^* \Pi_{F,t}^{-\theta(\nu-1)} C_t^{\nu-1} \quad (47)$$

$$\left[1 - (1-\tau_t^*) \frac{\sigma-1}{\sigma} (1-\rho) \Delta - \frac{1}{2} \Xi_t \Pi_{F,t}^{1-\theta} \right] C_t^{-\rho} \Pi_{H,t} = (1-\Delta\nu) z_t \Pi_{H,t}^{-\theta(\nu-1)} C_t^{\nu-1} \quad (48)$$

where:

$$\Xi_t = \left[(1-\rho) + \frac{1-\theta}{\theta} \right] \left[(1-\tau_t) \frac{\sigma-1}{\sigma} \Delta - (1-\tau_t^*) \frac{\sigma-1}{\sigma} \Omega \right] \quad (49)$$

The latter conditions allow to prove the next proposition:

Proposition 3 *When prices are set one period in advance, if shocks are symmetric, i.e. $z_t = z_t^*$ in all contingencies and at all times, implementing the flexible price allocation in all contingencies and at all times is the optimal cooperative monetary policy; if shocks are asymmetric, implementing the flexible price allocation in all contingencies and at all times is the optimal cooperative monetary policy if fiscal policy is set cooperatively or $\theta = 1$.*

Proof. In order to prove the result, it is sufficient to show that if τ_t and τ_t^* are constant and equal across countries then implementing the flexible price allocation is always the optimal cooperative monetary policy. If $\tau_t = \tau_t^*$ in all contingencies and at all times then:

$$\Delta = \Omega = \frac{\sigma}{\sigma - 1} \frac{1}{(1 - \tau)\delta} - \frac{1}{\delta} \quad (50)$$

is a solution that guarantees that the flexible price allocation is always implemented. Indeed if $\Delta = \Omega$ and $\tau_t = \tau_t^*$, $\Xi_t = 0$. But if $\Xi_t = 0$ and (50) is fulfilled then conditions (47) and (48) can be rewritten as:

$$(1 - \tau^*) \frac{\sigma - 1}{\sigma} C_t^{-\rho} \Pi_{F,t} = z_t^* \Pi_{F,t}^{-\theta(\nu-1)} C_t^{\nu-1} \quad (51)$$

$$(1 - \tau) \frac{\sigma - 1}{\sigma} C_t^{-\rho} \Pi_{H,t} = z_t \Pi_{H,t}^{-\theta(\nu-1)} C_t^{\nu-1} \quad (52)$$

which correspond exactly to the flexible price first order conditions (17) and (18)²³.

■

Corollary 4 *When prices are set one period in advance, implementing the flexible price allocation in all contingencies and at all times is the optimal cooperative monetary policy only if, at all times, $\frac{(1-\tau_t)}{(1-\tau_t^*)}$ is independent of shock realizations.*

Proof. The result can be proven by noting that, when (17) and (18) are satisfied, from condition (46) it follows:

$$\frac{(1 - \tau_t^*)}{(1 - \tau_t)} = \frac{[\theta(1 - \Delta\nu) - \Delta(1 - \theta)]}{[\theta(1 - \Omega\nu) - \Omega(1 - \theta)]} \quad (53)$$

■

Proposition 3 may be interpreted as follows. If fiscal policy is set cooperatively implementing the flexible price allocation is always optimal for the cooperative monetary policy. In that case, the flexible price allocation is efficient all the distortions present in the economy being eliminated: the one due to the monopolistic competition through the cooperative fiscal policy, the other due to the presence of nominal rigidities through the cooperative monetary policy. However even when shocks are symmetric or $\theta = 1$ the flexible price allocation is constrained efficient being firm tax rates constant and equal across countries. In these cases the condition indicated in Benigno and Benigno (2003) according to which implementing the flexible price allocation is the optimal cooperative monetary policy is satisfied.

Corollary 4 jointly with corollary 2 entails instead the following implication: in general, unless special conditions on structural parameters or on the shock stochastic process are met, if fiscal policy is set non-cooperatively, implementing the flexible price allocation is not optimal for the cooperative monetary authority. Indeed when prices are flexible, the non-cooperative fiscal authorities have an incentive to react to shock ratio variations by strategically using firms' sale tax rates. These tax rate movements force the cooperative policy maker to depart from the flexible price allocation in order to try to stabilize the variations of the wedge in the marginal rate of substitution between consumption and good production.

²³Moreover if (17) and (18) are satisfied even (42) and (43) are satisfied as well.

5 Conclusion

Our analysis adds some new insights on the interactions between optimal monetary and fiscal policy in an open economy context. It clarifies that when prices are flexible, in general the non-cooperative fiscal authorities behavior generates endogenous variations of firms' sale tax rates. Conversely the cooperative fiscal policy maker chooses constant tax rates that allow to exactly offset the monopolistic distortions. As a result unless special restrictions are met, implementing the flexible price allocation is the optimal cooperative monetary policy only when fiscal policy is set cooperatively.

Our analysis can be extended in different directions. First of all by considering the case of a monetary union. In general, in a monetary union, the common central bank cannot implement the flexible price allocation because, being the currency common to all countries, there are not enough instruments to correct the nominal rigidities present in the economy. However, as made clear by Benigno (2004), when the monopolistic distortions are eliminated by appropriate subsidies the common central bank seeks to approximate the flexible price allocation through a suitable inflation targeting policy. Our results suggest that this policy may be not optimal when fiscal policy is set non-cooperatively because in that case not only the flexible price allocation is not efficient, but also the endogenous variations of firms' tax rates generate an additional distortion the common central bank tries to cope with.

Secondly by using a more general framework. Abstracting from the presence of lump sum taxes, considering a price setting *a la* Calvo, introducing the public expenditure may render the set up more appropriate to investigate the interdependence between optimal cooperative monetary policy and fiscal policy regime.

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APPENDIX

Flexible price setting

Consumption and relative prices

Given our preference specification, we can rewrite conditions (17) and (18) as:

$$\left(\frac{\sigma-1}{\sigma}\right)(1-\tau_t)C_t^{-\rho}\Pi_{H,t} = z_t \left(\Pi_{H,t}^{-\theta}C_t\right)^{(\nu-1)} \quad (54)$$

$$\left(\frac{\sigma-1}{\sigma}\right)(1-\tau_t^*)C_t^{-\rho}\Pi_{F,t} = z_t^* \left(\Pi_{F,t}^{-\theta}C_t\right)^{(\nu-1)} \quad (55)$$

with $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_t}$ and $\Pi_{F,t} \equiv \frac{P_{F,t}}{P_t}$ ²⁴. Taking the ratio between (55) and (54) it is possible to show that:

$$\frac{\Pi_{F,t}}{\Pi_{H,t}} = \left[\frac{z_t^*(1-\tau_t)}{z_t(1-\tau_t^*)}\right]^{\frac{1}{1+(\nu-1)\theta}} \quad (56)$$

In order to simplify the notation we define $\Lambda_t \equiv \left[\frac{z_t^*(1-\tau_t)}{z_t(1-\tau_t^*)}\right]$ and $\gamma \equiv 1 + (\nu - 1)\theta$ which is to say $\frac{\Pi_{F,t}}{\Pi_{H,t}} = \Lambda_t^{\frac{1}{\gamma}}$. The price index specification stated in (4) entails:

$$\Pi_{F,t} = \left[2 - \Pi_{H,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \quad (57)$$

and

$$\frac{\Pi_{F,t}}{\Pi_{H,t}} = \left[2\Pi_{H,t}^{\theta-1} - 1\right]^{\frac{1}{1-\theta}} \quad (58)$$

From combining (56) with (57) and (58) it follows:

$$\Pi_{H,t} = \left[\frac{1}{2}\Lambda_t^{\frac{1-\theta}{\gamma}} + \frac{1}{2}\right]^{\frac{1}{\theta-1}} \quad \Pi_{F,t} = \left[\frac{1}{2} + \frac{1}{2}\Lambda_t^{\frac{\theta-1}{\gamma}}\right]^{\frac{1}{\theta-1}} \quad (59)$$

Finally plugging (59) in (54) we can retrieve flexible-price equilibrium level of consumption:

$$C_t = \left[\left(\frac{\sigma}{\sigma-1}\right)\frac{z_t}{(1-\tau_t)}\left[\frac{1}{2}\Lambda_t^{\frac{1-\theta}{\gamma}} + \frac{1}{2}\right]^{\frac{\gamma}{1-\theta}}\right]^{\frac{1}{\delta}} \quad (60)$$

where $\delta \equiv 1 - \rho - \nu$.

²⁴Note that by (54) $C_t = \left[\left(\frac{\sigma}{\sigma-1}\right)\frac{z_t}{(1-\tau_t)}\Pi_{H,t}^{-\gamma}\right]^{\frac{1}{1-\rho-\nu}}$.

Effects of tax rates marginal changes on equilibrium levels

Conditions (59) and (60) allow to derive the impact of marginal changes in home and foreign tax rates on equilibrium consumption and relative prices. More precisely:

$$\frac{\partial \Pi_{H,t}}{\partial \tau_t} = \frac{1}{2\gamma(1-\tau_t^*)} \frac{z_t^*}{z_t} \Pi_{H,t}^{2-\theta} \Lambda_t^{\frac{1-\theta-\gamma}{\gamma}}$$

which can be simplified by using the definition of Λ_t as:

$$\frac{\partial \Pi_{H,t}}{\partial \tau_t} = \frac{1}{2\gamma(1-\tau_t)} \Pi_{H,t}^{2-\theta} \Lambda_t^{\frac{1-\theta}{\gamma}} \quad (61)$$

Similarly it can be shown that:

$$\frac{\partial \Pi_{H,t}}{\partial \tau_t^*} = -\frac{1}{2\gamma(1-\tau_t^*)} \Pi_{H,t}^{2-\theta} \Lambda_t^{\frac{1-\theta}{\gamma}} \quad (62)$$

Given (61), we can easily recover from (60) the effect of a marginal change in home tax rate:

$$\frac{\partial C_t}{\partial \tau_t} = \frac{1}{\delta} C_t^{1-\delta} \left(\frac{\sigma}{\sigma-1} \right) \frac{z_t}{(1-\tau_t)^2} \Pi_{H,t}^{1-\gamma-\theta} \left[\Pi_{H,t}^{\theta-1} - \frac{1}{2} \Lambda_t^{\frac{1-\theta}{\gamma}} \right]$$

which considering (59) and (60) implies that:

$$\frac{\partial C_t}{\partial \tau_t} = \frac{1}{2\delta(1-\tau_t)} C_t \Pi_{H,t}^{1-\theta} \quad (63)$$

Symmetric conditions can be stated for the foreign country. Hence:

$$\frac{\partial \Pi_{F,t}}{\partial \tau_t^*} = \frac{1}{2\gamma(1-\tau_t^*)} \Pi_{F,t}^{2-\theta} \Lambda_t^{\frac{\theta-1}{\gamma}} \quad \frac{\partial \Pi_{F,t}}{\partial \tau_t} = -\frac{1}{2\gamma(1-\tau_t)} \Pi_{F,t}^{2-\theta} \Lambda_t^{\frac{\theta-1}{\gamma}} \quad (64)$$

and

$$\frac{\partial C_t}{\partial \tau_t^*} = \frac{(1-n)}{\delta(1-\tau_t^*)} C_t \Pi_{F,t}^{1-\theta} \quad (65)$$

The cooperative monetary policy problem

By taking as given fiscal policy variables the cooperative monetary authorities maximize:

$$\sum_{t=0}^{\infty} \beta^t E_{-1} \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{1}{2} \frac{z_t \Pi_{H,t}^{-\theta\nu} C_t^\nu}{\nu} - \frac{1}{2} \frac{z_t^* \Pi_{F,t}^{-\theta\nu} C_t^\nu}{\nu} \right] \quad (66)$$

subject to the constraints for each time t :

$$E_{t-1} \left\{ \left[\frac{\sigma-1}{\sigma} (1-\tau_t) C_t^{-\rho} \Pi_{H,t} - z_t \Pi_{H,t}^{-\theta(\nu-1)} C_t^{\nu-1} \right] \Pi_{H,t}^{-\theta} C_t \right\} = 0 \quad (67)$$

$$E_{t-1} \left\{ \left[\frac{\sigma-1}{\sigma} (1-\tau_t^*) C_t^{-\rho} \Pi_{F,t} - z_t^* \Pi_{F,t}^{-\theta(\nu-1)} C_t^{\nu-1} \right] \Pi_{F,t}^{-\theta} C_t \right\} = 0 \quad (68)$$

and

$$\frac{1}{2}\Pi_{H,t}^{1-\theta} + \frac{1}{2}\Pi_{F,t}^{1-\theta} = 1 \quad (69)$$

for each time t and each contingency. According to the first order conditions with respect to C_t :

$$\begin{aligned} & C_t^{-\rho} - \frac{1}{2}z_t\Pi_{H,t}^{-\theta\nu}C_t^{\nu-1} - \frac{1}{2}z_t^*\Pi_{F,t}^{-\theta\nu}C_t^{\nu-1} \\ & - \frac{1}{2}\Delta(1-\tau_t)\frac{\sigma-1}{\sigma}(\rho-1)\Pi_{H,t}^{1-\theta}C_t^{-\rho} + \frac{1}{2}\Delta(1-\tau_t)\frac{\sigma-1}{\sigma}\nu z_t\Pi_{H,t}^{-\theta\nu}C_t^{\nu-1} \\ & - \frac{1}{2}\Omega(1-\tau_t^*)\frac{\sigma-1}{\sigma}(\rho-1)\Pi_{F,t}^{1-\theta}C_t^{-\rho} + \frac{1}{2}\Omega(1-\tau_t^*)\frac{\sigma-1}{\sigma}\nu\Pi_{F,t}^{-\theta\nu}C_t^{\nu-1} = 0 \end{aligned} \quad (70)$$

and with respect to $\Pi_{H,t}$ and $\Pi_{F,t}$:

$$\theta z_t\Pi_{H,t}^{-\theta(\nu-1)-1}C_t^\nu - \Delta(1-\theta)(1-\tau_t)\frac{\sigma-1}{\sigma}C_t^{1-\rho} - \Delta\theta\nu z_t\Pi_{H,t}^{-\theta(\nu-1)-1}C_t^\nu = \varphi_t(1-\theta) \quad (71)$$

$$\theta z_t^*\Pi_{F,t}^{-\theta(\nu-1)-1}C_t^\nu - \Omega(1-\theta)(1-\tau_t^*)\frac{\sigma-1}{\sigma}C_t^{1-\rho} - \Omega\theta\nu z_t^*\Pi_{F,t}^{-\theta(\nu-1)-1}C_t^\nu = \varphi_t(1-\theta) \quad (72)$$

where Δ , Ω and φ_t are respectively the lagrange multipliers of constraints (67), (68) and (69)²⁵. Condition (70) can be rewritten as:

$$\begin{aligned} & C_t^{-\rho} \left[1 - \frac{1}{2}(1-\tau_t)\frac{\sigma-1}{\sigma}(1-\rho)\Delta\Pi_{H,t}^{1-\theta} - \frac{1}{2}(1-\tau_t^*)\frac{\sigma-1}{\sigma}(1-\rho)\Omega\Pi_{F,t}^{1-\theta} \right] = \\ & \frac{1}{2}z_t\Pi_{H,t}^{-\theta\nu}C_t^{\nu-1}(1-\Delta\nu) + \frac{1}{2}z_t^*\Pi_{F,t}^{-\theta\nu}C_t^{\nu-1}(1-\Omega\nu) \end{aligned} \quad (73)$$

²⁵Note that while the lagrange multiplier of constraint (69) is state dependent the ones of constraints (67) and (67) are not.