Competition, Innovation and Growth with Limited Commitment*

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Abstract

We study how barriers to business start-up affect the investment in knowledge capital when contracts are not enforceable. Barriers to business start-up lower the competition for knowledge capital and, in absence of commitment, reduce the incentive to accumulate knowledge. As a result, countries with large barriers experience lower income and growth. Our results are consistent with cross-country evidence showing that the cost of business start-up is negatively correlated with the level and growth of income.

1 Introduction

It is widely recognized that sustained levels of income require the adoption of advanced technologies and innovation. A distinguished feature of modern

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technologies such as information and communication technologies, biotechnologies and nanotechnologies, is the importance of *skilled* human capital or *knowledge*. The ability of a society to innovate and grow is then dependent on how human capital is accumulated, organized and the returns shared among the participants in the innovation process. This is especially important when the parties who provide the financial needs—the investors—differ from those who acquire the innovation skills—innovators. For instance, whether a certain innovation project is funded depends on the ability of the investors to recover at least some of the returns of the project. At the same time, workers and managers—who carry out the project and are the actual innovators—must have the right incentives to accumulate the required skills or knowledge. This depends on the type of contractual arrangements that are feasible or enforceable. The goal of this paper is to study how free entry affects the accumulation of innovation skills and the growth of the economy when contracts are not perfectly enforceable.

The limited enforceability of contracts is not only one-sided. On the one hand, the innovator could put low effort in accumulating knowledge or quit the firm. This makes advance payments to the innovator unfeasible. On the other, the investor could replace the innovator and renege promises of payments. This may create an hold-up problem in the accumulation of knowledge. The main result of the paper is to show that, given the limited enforceability of contracts, the hold-up problem in the accumulation of knowledge arises only if there are barriers to entry. Therefore, free entry increases the accumulation of knowledge and enhances income and/or growth.

With free entry, new firms compete for the skills of the innovator. This creates an outside value for knowledge which is used by the innovator as a threat against the investor's attempt to renegotiate the promised payments. Under these conditions, the innovator has an incentive to accumulate a high level of knowledge to keep the threat value high. Because innovations make existing capital obsolete, the higher accumulation of knowledge is inefficient at the firm level. In principle, the firm could prevent the over-accumulation by making advance payments. However, as stressed above, this is not possible if contracts are not enforceable for the innovator. It becomes then crucial whether the firm can commit to future payments, conditional on the accumulation of knowledge. It is in this sense that the double-sided limited commitment plays a crucial role in our framework. It is important to point out that, although the higher accumulation of knowledge could be inefficient at the firm level, it may still be efficient for the whole economy if there are 'externalities' or 'spillovers'.

Our result differs from other papers that also study the accumulation of skills within the firm. In some of these models the investor has full control over the accumulation of skills or human capital. For example, in Acemoglu & Shimer (1999) is the employer that decides the amount of training. In this environment, greater mobility or outside opportunities hold-up the firm from investing in training because the workers capture a larger share of the firm's rents. A similar result is obtained in **?**). In other models, such as the one studied in Acemoglu (1997), workers do control the accumulation of skills, but the main conclusion does not change: greater mobility worsens the hold-up problem and leads to lower accumulation of skills because workers are less likely to benefit from it. In our framework, instead, mobility (enhanced by free entry) increases human capital investment when contracts are not enforceable also for the firm.

Whether free entry enhances innovation has been a major topic of research and debate since Schumpeter's claim that, while product market competition could be detrimental for innovations, competition in the innovation sector encourages innovations. See, for example, Aghion & Howitt (1999) and Aghion & Griffith (2005). Most of the subsequent literature has focused on market structure and product market competition. In particular, on the ability to gain market shares and appropriate the returns to R&D, as in Aghion, Bloom, Blundell, Griffith, & Howitt (2005). More closely related to our work is Aghion, Blundell, Griffith, Howitt, & Prantl (2004). They show—both, theoretically and empirically—that 'firm entry' spurs innovation in technological advanced sectors as firms try to 'escape competition'. Acemoglu, Aghion, & Zilibotti (2002) also study the impact of barriers to entry and show that they are especially costly for economies closer to the technology frontier. Differently from these studies, we focus on the less studied dimension of 'human capital' competition created by free entry, when contracts are not enforceable. With limited enforceability, competition for human capital becomes essential for growth because it keeps the market value of knowledge high and guarantees that innovators are rewarded for their innovation efforts. Thus, free entry enhances the competition for human capital and creates the conditions for greater innovations. Our results are consistent with cross-country evidence showing a negative correlation between the cost of business start-up and the level of per-capita income.

The paper relates to several strands of literature. First, the labor literature that studies the hold-up problem already discussed above (e.g., Acemoglu (1997), Acemoglu & Pischke (1999), Acemoglu & Shimer (1999)). Second, the growth literature, starting with the pioneering work of Romer (1990, 1993), that studies the economics of ideas and the link between competition and innovations (e.g., Greenwood & Jovanovic (1990), Aghion et al. (2004), Aghion et al. (2005)). Third, the growth literature that, building on the work of economic historians (e.g., Mokyr (1990)), emphasizes the role of barriers to riches in slowing growth (Parente & Prescott (1990)). Forth, the literature on dynamic contracts with enforcement constraints such as Marcet & Marimon (1992). Most of the contributions in this literature are not concerned with the issue of technology adoption and innovation (one exception is Kocherlachota (2001)) and they typically assume one-side commitment. Another typical assumption is that default or repudiation leads to market exclusion. In our framework, instead, the value of defaulting is the value of re-entering the market as in Cooley, Marimon, & Quadrini (2004). It is this feature that makes 'free entry' important for economic growth.

The plan of the paper is as follows. In Section 2 we provide cross-country evidence on barriers to business start-up and macroeconomic performance. Section 3 describes the model. To facilitate the intuition for the theoretical results, Section 4 studies a simplified version of the model with only two periods. Section 5 generalizes it to the infinite horizon. Section 7 concludes.

2 Cross-country evidence

A recent publication from the World Bank (2005) provides data on the quality of the business environment for a cross-section of countries. Especially important for this study are the variables that are proxy for the barriers to business start-up. There are three main variables. The first variable is the 'cost to start a new business'. This is the average pecuniary cost needed to set-up a corporation in the country, expressed in percentage of the country per-capita income. The normalization of the cost of business start-up by the level of per-capita income better captures the importance of barriers to business start-up than the absolute dollar cost. What is relevant for the decision to start a business is the comparison between the cost of business start-up and the value of creating a business. Although the dollar cost is on average higher in advanced economies, the value of a new business is also likely to be higher.

The second proxy for the barriers to business start-up is the 'number of bureaucratic procedures' that need to be filed before starting a new business.

The third proxy is the average 'length of time' required to start a new business. Figure 1 plots the level of per-capita GDP in 2004 against these three indicators. All variables are in log. The three panels show a strong negative correlation indicating that the set-up of a new business is more costly and cumbersome in poor countries.

The cost of business start-up is also negatively correlated with economic growth. To show this, we regress the average growth in per-capita GDP from 2000 to 2004 (the five more recent years) to the cost of business start-up. We also include the 1999 per-capita GDP to control for the initial level of development. The estimation results, with t-statistics in parenthesis, are reported in the top section of Table 1. As can be seen from the table, the cost of business start-up is negatively associated with growth even if we control for the level of economic development. Therefore, countries with lower barriers to entry tend to experience faster growth. This finding is robust to the choice of alternative years to compute the average growth rate. The other proxies for the barriers to entry—specifically, the number of procedures and the time required to start a new business—are also negatively correlated with the average growth rate but they are not statistically significant at the conventional levels (1 or 5 percent).

To show that these findings are not an artifact of normalizing the cost of business start-up by the level of per-capita income, the bottom section of Table 1 repeats the same regression estimation but using dollar values, in log, of the cost of business start-up. Again, the cost of business start-up is statistical significant with a negative sign. In this regression, however, the initial per-capita GDP is no longer significant.

To summarize, the general picture portrayed by the data is that the economic development and growth of a country is negatively associated with the cost of starting a business. Of course, correlations do not imply causations. In the following sections we present a model where the correlation is driven by the cost of business start-up, that is, barriers to entry lead to lower income and growth.

3 The model

There are two types of agents in the economy: a continuum of 'entrepreneurs' of total mass $m > 1$ and a continuum of 'innovators' of total mass 1. Therefore, innovators are in short supply relatively to the number of entrepreneurs. The lifetime utilities for entrepreneurs and innovators are,

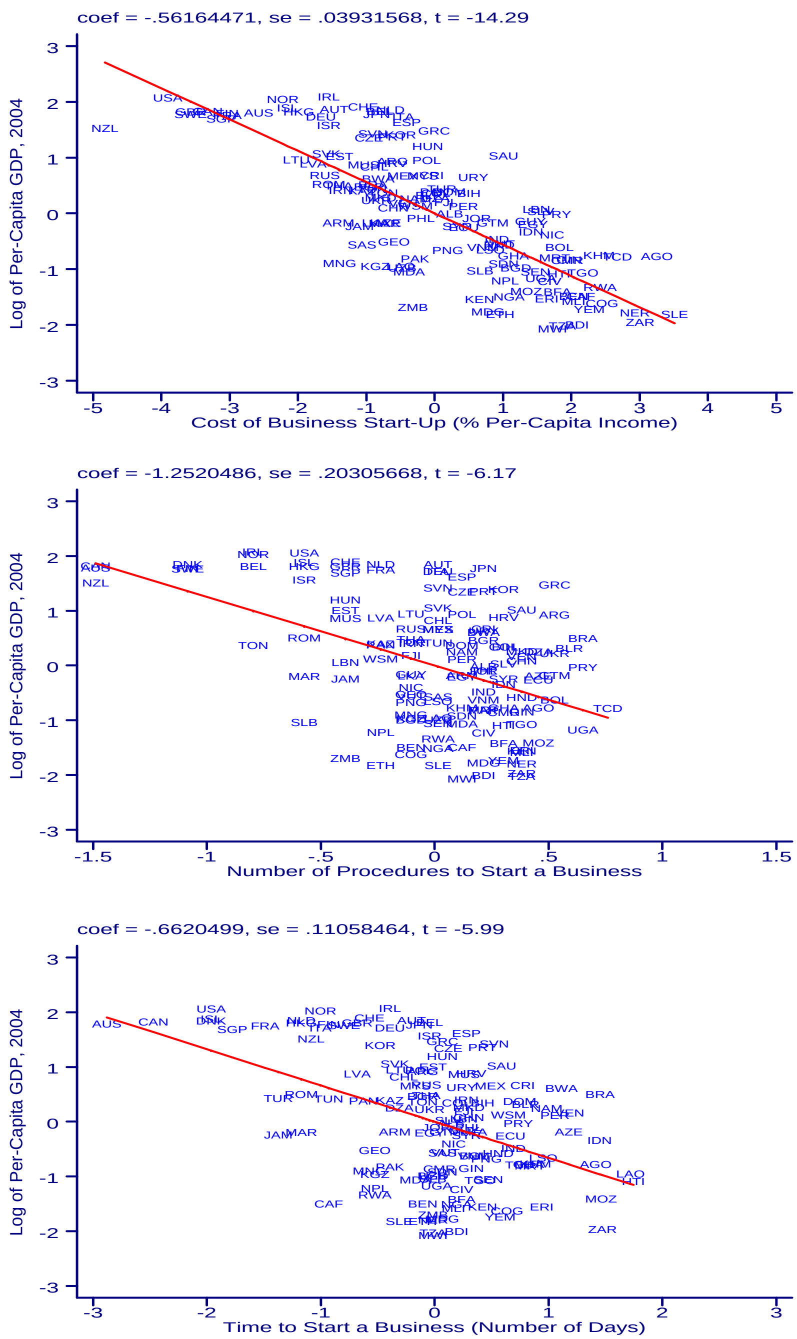

Figure 1: Barriers to business start-up and level of development.

Table 1: Cost of business start-up and growth.

		Constant	*Initial Per-Capita GDP*	*Cost of Business Start-Up*
(a)	Coefficients	15.55	-1.16	-1.04
	t-Statistics	*(5.01)*	*(-3.81)*	*(-4.92)*
	R-square	0.150		
	N. of countries	140		
(b)	Coefficients	6.02	0.20	-0.83
	t-Statistics	*(3.08)*	*(0.94)*	*(-3.75)*
	R-square	0.093		
	N. of countries	140		

NOTES: Dependent variable is the average annual growth rate in per-capita GDP for the five year period 2000-2004. Initial Per-Capita GDP is the log of per-capita GDP in 1999. In panel (a) the costs of business start-up is in percentage of the per-capital Gross National Income as reported in *Doing Business in 2005.* In panel (b) is the dollar value of this cost. Both measures of the cost of business start-up are in logs.

respectively, $\sum_{t=0}^{\infty}\beta^t c_t$ and $\sum_{t=0}^{\infty}\beta^t(c_t - e_t)$, where c_t is also consumption and e_t is the effort to accumulate human capital (knowledge) as specified below.

Innovators do not save. This assumption should be interpreted as an approximation to the case in which innovators discount more heavily than entrepreneurs. The risk neutrality of entrepreneurs implies that the equilibrium interest rate is equal to the intertemporal discount rate, that is, $r = 1/\beta - 1$.

Firms are owned by entrepreneurs who need the management and innovation skills of innovators. The production function is:

$$y_t = z_t^{1-\alpha} k_t^{\alpha}$$

where z_t is the level of technology and k_t is the capital chosen at time $t-1$.

The variable z_t changes over time as the firm adopts new technologies. The key assumption is that the implementation of more advanced technologies requires higher knowledge. An innovator with knowledge h_t has the ability to install or implement any technology $z_t \leq h_t$.

Innovators are needed not only for the introduction and implementation of a new technology but also to run the firm once the technology has been installed. In each period there are two subperiods. The technology is introduced in the first sub-period and run in the second. However, once the technology has been implemented, any innovator can run the firm, independently of his knowledge.

The investment in knowledge, $h_{t+1} - h_t$, requires effort from the innovator. The required effort depends on the economy-wide level of knowledge H_t, due to leakage or spillover effects, according to the following function:

$$e_t = \varphi(H_t; h_t, h_{t+1})$$

The function φ is strictly decreasing in H_t and h_t, strictly increasing in h_{t+1}, and satisfies $\varphi(H_t; h_t, h_t) > 0$. We further assume that the function is homogeneous of degree ρ, where ρ is either 1 or greater than 1. In the first case ($\rho = 1$) the model generates long-term growth differences. In the second ($\rho > 1$) it generates only transitional growth differences. Long-term differences are only in the level of incomes. We remain agnostic whether an endogenous growth model is the best representation of cross-country growth differences. In the analysis that follows we concentrate on the case with $\rho > 1$. This should be interpreted as a detrended model that grows at the exogenous rate dictated by the world-wide level of technology, which is external to an individual country.[1] It should be clear, however, that the analysis of the paper can be easily extended to the case in which φ is homogeneous of degree $\rho = 1$.

Physical capital is technology-specific: when the firm innovates, only part of the physical capital is usable with the new technology. Furthermore, capital obsolescence increases with the degree of innovation. This is formalized by assuming that the depreciation rate increases with the size of the innovation, that is,

$$\delta_t = \delta \cdot \left(\frac{z_{t+1} - z_t}{z_t} \right)$$

Because of capital obsolescence, there is an asymmetry between *incumbent* and *new* firms. Because new firms are still uncommitted to any previous investment, they have greater incentive to innovate.

[1] We could assume that there is a world-wide level of knowledge $\overline{H}$ growing at rate g and the cost to accumulate knowledge takes the form $\tilde{\varphi}(\overline{H}, H_t, h_t, h_{t+1})$ where $\tilde{\varphi}$ is strictly increasing in $\overline{H}$ and homogeneous of degree 1. After normalizing all variables by $\overline{H}$ we would have the stationary model studied here.

The competitive structure of the model is as follows. In each period there is a walrasian market for innovators who can move freely from one firm to the other. The market opens twice: before and after the accumulation of knowledge. Both incumbent and new firms can participate in this market. However, the presence of barriers to entry may limit the effective presence of new firms. There are different ways to model barriers to entry. Here we adopt a simple formulation and assume that new firms incur a deadweight loss proportional to the initial level of knowledge. Given h_{t+1} the initial knowledge chosen by a new firm, the entry cost is $\tau \cdot h_{t+1}$. Our goal is to study how this cost affects the equilibrium when contracts are not enforceable. We would like to point out that the results of the paper are robust to alternative formulations of entry barriers. We have chosen this particular formulation only for its analytical convenience.[2]

The last assumption is that firms remain productive with probability p. Whether a firm survives is revealed after the investment in knowledge. This guarantees that, in the second stage of each period, the mass of innovators is larger than incumbent (surviving) firms. This is important for the characterization of the equilibrium as we will emphasize below. We would like to stress, however, that the value of p is not relevant for the characterization of the equilibrium as long as it is strictly smaller than 1. Given that, we assume that p is very close to 1 and, in the characterization of the individual problems, we will ignore it. The explicit consideration of p will only make the notation more complex but it would not change the qualitative results.

4 Two-period model

To gauge some intuitions about the key properties of the model, it would be convenient to consider first a simplified version with only two periods: period zero and period one. The state variables of the firm at the beginning of period zero are h_0 and k_0. After making the investment decisions, h_1 and k_1, the firm generates output $y_1 = z_1^{1-\alpha} k_1^{\alpha}$ in period one. Because $z_1 = h_1$, the output can also be written as $y_1 = h_1^{1-\alpha} k_1^{\alpha}$. In this simple version of

[2]For example, we could assume that the cost is proportional to the initial capital k_{t+1} or to the initial output $Ah_{t+1}^{1-\alpha} k_{t+1}^{\alpha}$ or to the discounted flows of outputs. None would change the results but would make the analysis more complex. The assumption of proportionality guarantees that the equilibrium impact of τ is continuous while the impact of a fixed cost would be discontinuous, that is, it would have an impact only after it has reached the prohibitive level.

the model we assume that physical capital fully depreciates after production. The innovator receives a payment from the firm (compensation) at the end of period zero, after the choice of h_1. Payments before the choice of h_1 are not feasible because of the limited enforcement of contracts for the innovator, while allowing for additional payments in period 1 does not change the results. For the analysis of this section we also assume that there is no discounting and the effort cost does not depend on the economy-wide knowledge H. The leakage or spillover effect is not relevant when there are only two periods.

The timing of the model can be summarized as follows: The firm starts period zero with initial states h_0 and k_0. At this stage the innovator decides whether to stay or quit the firm. If he quits, he can be hired by an incumbent firm or a new firm (funded by a new entrepreneur). If the innovator decides to stay, he will choose the new level of knowledge h_1 and implement the technology $z_1 = h_1$. The entrepreneur provides the funds to accumulate the new physical capital k_1. After the investment decision has been made, the entrepreneur pays w_0 to the innovator. At this stage the innovator can still quit, but he cannot change the level of knowledge h_1. The entrepreneur is the residual claimant of the firm's output.

4.1 Equilibrium with entrepreneur's commitment

To show the importance of contract enforcement, we will first characterize the equilibrium under the assumption that the entrepreneur commits to the contract. This will facilitate the characterization of the equilibrium without commitment.

When the entrepreneur commits to the long-term contract, all variables are chosen at the beginning of the first period to maximize the total surplus. Let $D(h_0)$ be the repudiation value before choosing h_1 and $\widehat{D}(h_1)$ the repudiation value after choosing h_1. These functions are endogenous and will be derived below as the values that the innovator would get by quitting the firm and re-entering the market. From now on we will use the *hat* sign to denote the functions that are defined *after* the investment in knowledge. The participation of the innovator requires that the value of staying is greater than the repudiation value before and after the knowledge investment, that is,

$$\begin{aligned} w_0 - \varphi(h_0, h_1) &\geq D(h_0) \\ w_0 &\geq \widehat{D}(h_1) \end{aligned}$$

The first is the participation constraint before choosing h_1 and the second is the participation constraint after the choice of h_1. We will show in the next section that, if the first constraint is satisfied, the second is also satisfied. Therefore, in the derivation of the optimal policy we can neglect the second constraint and write the optimization problem as follows:

$$\max_{h_1,k_1,w_0} \left\{ -\varphi(h_0,h_1) - k_1 + \left[1 - \delta \cdot \left(\frac{h_1 - h_0}{h_0}\right)\right] k_0 + Ah_1^{1-\alpha} k_1^{\alpha} \right\} \quad (1)$$

s.t.

$$w_0 - \varphi(h_0,h_1) \geq D(h_0)$$

$$-w_0 - k_1 + \left[1 - \delta \cdot \left(\frac{h_1 - h_0}{h_0}\right)\right] k_0 + Ah_1^{1-\alpha} k_1^{\alpha} \geq 0$$

where the second constraint is the participation constraint for the entrepreneur. In writing the problem we have substituted $z_1 = h_1$ in the production and depreciation function.

From the maximization problem it can be verified that the investment choices (in knowledge and physical capital) are independent of the choice of the innovator's payment w_0. The value of w_0 is determined by the division of the surplus, which we specify below.

To determine the repudiation value before the choice of h_1, we have to solve for the optimal investment when the innovator quits the current firm. The innovator could be hired by an incumbent or a new firm, whoever makes the best offer. However, an incumbent firm will never offer more than a new firm. Therefore, it becomes relevant to determine the offer made by a new firm which is derived by solving the following problem:

$$S(h_0) = \max_{h_1,k_1,w_0} \left\{ -\varphi(h_0,h_1) - \tau h_1 - k_1 + Ah_1^{1-\alpha} k_1^{\alpha} \right\} \quad (2)$$

s.t.

$$w_0 - \varphi(h_0,h_1) \geq D(h_0)$$

$$-w_0 - \tau h_1 - k_1 + Ah_1^{1-\alpha} k_1^{\alpha} \geq 0$$

The problem of a new firm differs from the problem solved by an incumbent firm in two respects. First, a new firm does not have any initial physical capital, and therefore, it does not incur any obsolescence of capital. Second, a new firm has to pay the entry cost τh_1. It is still the case, however, that the choices of h_1 and k_1 are independent of w_0.

Because of competition among potential entrants, the innovator gets the whole surplus $S(h_0)$. This implies that $D(h_0) = S(h_0)$. Therefore, if the innovator stays with the incumbent firm, the payment w_0, net of the effort cost, $\varphi(h_0, h_1)$, must be at least $S(h_0)$. Formally, the participation constraint in problem (1) becomes:

$$w_0 - \varphi(h_0, h_1) \geq S(h_0) \tag{3}$$

Problems (1) and (2) show the different incentive to invest for an incumbent versus a new firm. On the one hand, new firms do not have any physical capital and innovations do not generate capital obsolescence. Hence, they have a greater incentive to innovate than incumbent firms. On the other, they must pay the entry cost τh_1, which discourages knowledge and capital accumulation. This is clearly shown by the first order conditions for h_1 in problems (1) and (2), that is,

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0}\right) \tag{4}$$

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) + \tau \tag{5}$$

where the subscripts denote derivatives.

Condition (4) is for incumbent firms and condition (5) is for new firms. The left-hand-side terms are the marginal productivity of knowledge. The right-hand-side terms are the marginal costs. The marginal cost for an incumbent firm is the effort cost incurred by the innovator plus the obsolescence of physical capital. For a new firm the obsolescence cost is replaced by the entry cost. Using the first order condition for the choice of physical capital, which is $\alpha(k_1/h_1)^{\alpha-1} = 1$ for both incumbent and new firms, the above first order conditions can be rewritten as:

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0}\right) \tag{6}$$

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h_1) + \tau \tag{7}$$

These two conditions clearly show that, when the entry cost is zero (free entry), new firms innovate more than incumbents. However, the innovation incentive of new firms declines as the entry cost increases.

Let h^{Old} be the optimal knowledge investment of an incumbent firm and h^{New} the optimal knowledge investment of a new firm. The following proposition formalizes the above results.

Proposition 1 *The knowledge investment of a new firm h^{New} is strictly decreasing in the entry cost τ and there exists $\bar{\tau} > 0$ such that $h^{New} = h^{Old}$.*

Proof 1 *This follows directly from conditions (6) and (7).* *Q.E.D.*

In the equilibrium with entrepreneur's commitment, there will not be any entrance of new firms at the beginning of the period and the investment in knowledge is $h_1 = h^{Old}$. The potential entrance of new firms only affects the minimum payment received by an innovator. In the second stage of the period there will be new firms entrance because some incumbents become unproductive. However, the initial knowledge of innovators cannot be changed at this stage. The next step is to show that this is not an equilibrium when contracts are not enforceable, that is, there is limited commitment from the entrepreneur.

4.2 Equilibrium with double-side limited commitment

We want to show first that, after the accumulation of knowledge, the entrepreneur has an incentive to renegotiate the optimal contract. Because the innovator can always be hired by a new firm, the value received by an incumbent firm must be at least as large as the surplus generated by a new firm. The surplus of new firms, however, changes before and after the investment in knowledge, which creates the condition for renegotiation.

Let's derive first the surplus generated by a new firm after the investment in knowledge. This is given by:

$$\widehat{S}(h_1) = \max_{k_1, w_0} \Big\{ -\tau h_1 - k_1 + A h_1^{1-\alpha} k_1^{\alpha} \Big\} \tag{8}$$

s.t.

$$w_0 \geq \widehat{D}(h_1)$$

$$-w_0 - \tau h_1 - k_1 + A h_1^{1-\alpha} k_1^{\alpha} \geq 0$$

Because of competition, the innovator gets the whole surplus, that is, $\widehat{D}(h_1) = \widehat{S}(h_1) = w_0$, and the participation constraint, after the investment in knowledge, becomes $w_0 \geq \widehat{S}(h_1)$.

If the innovator stays with the firm and accumulate h^{Old}, the innovator will renegotiate the promised payments if they are greater than $\widehat{S}(h^{Old})$, that is, greater than the payment that the innovator would obtain by quitting the firm with $h_1 = h^{Old}$. In this case the total utility enjoyed by the innovator would be:

$$-\varphi(h_0, h^{Old}) + \widehat{S}(h^{Old})$$

If the innovator quits the firm at the beginning of the period, he will get the whole surplus $S(h_0)$ generated by the new firm, which is equal to:

$$\begin{aligned} S(h_0) &= \max_h \Big\{ -\varphi(h_0, h) + \widehat{S}(h) \Big\} \\ &= -\varphi(h_0, h^{New}) + \widehat{S}(h^{New}) \end{aligned}$$

Because $h^{New} \neq h^{Old}$, the value of quitting at the beginning of the period, $S(h_0)$, is greater than the value of staying.[3] Therefore, the innovator will quit the firm at the beginning of the period unless the firm agrees to the same knowledge investment chosen by a new entrant firm, that is, $h^{Old} = h^{New}$. In this way the innovator keeps the repudiation value high and prevents the entrepreneur from renegotiating. This leads to the following proposition.

Proposition 2 *Without commitment from the entrepreneur, the investment in knowledge is $h_1 = h^{New}$.*

Proof 2 *This follows directly from the analysis above.* *Q.E.D.*

According to the proposition, incumbent firms accumulate the same level of knowledge as new firms. Because h^{New} is decreasing in τ (see Proposition

[3]This proves that, in the problem with commitment, if the enforcement constraint is satisfied at the beginning of the period, it is also satisfied after the investment in knowledge, as we assumed.

1), the accumulation of knowledge decreases with the cost of entry. Therefore, higher are the barriers to entry, and lower is the accumulation of knowledge.

There are several points in the proof of the proposition that should be emphasized. First, the renegotiation threat of the firm after the accumulation of knowledge is credible because the firm can always replace the current innovator with other innovators. This could be either an innovator still employed by an incumbent (surviving) firm, or an innovator who separated from an exiting firm. Because in the second stage of the period innovators are in the long side of the market, they only get the reservation value, which is the one received from a new firm.

How would the results change if $p = 1$ which implies that in the second stage of the period the number of innovators is equal to the number of incumbent firms? We would have multiple equilibria. Each firm would renegotiate if all other firms renegotiate. But each firm would not individually renegotiate if all other firms do not renegotiate. The assumption of a positive probability of exit, although small, eliminates this multiplicity.

Another point to clarify is the role played by the assumption that knowledge is only necessary for the introduction of technologies but not for managing a firm after their implementation (although the management still need to be done by an innovator). This assumption eliminates the following problem. Suppose that we are in an equilibrium with $h_1 = h^{New} \neq h^{Old}$. Under this equilibrium, an individual firm may be able to make a credible payment promise to the innovator and convince him to accumulate h^{Old}. Because in the second stage this is the only innovator with knowledge h^{Old} (all others have accumulated h^{New}), there is no innovator in the market that the firm can hire to replace him. Consequently, the firm will be unable to renegotiate.[4] This implies that $h_1 = h^{New}$ cannot be an equilibrium. This problem does not arise if the firm can use any innovator to run the firm once the technology has been adopted.

To summarize, free entry or competition leads to greater investment in knowledge. The greater investment is not individually efficient due to the creative destruction of physical capital. Without externalities, this is also

[4]This is true independently of whether h^{New} is greater or smaller than h^{Old}. If $h^{New} < h^{Old}$, then all the replacements have lower knowledge and are unable to run the more advance technology. If $h^{New} > h^{Old}$, then the deviating firm could use innovators with higher knowledge. But these innovators must be paid more because their reservation value (when employed by a new firm) is higher.

socially inefficient. The presence of splillovers in the accumulation of knowledge, however, may make higher investment desirable. We will re-introduce the splillovers in the analysis of the infinite horizon model to which we now turn.

5 The infinite horizon model

In this section we study the general model with infinitely lived agents. This allows us to derive the level of h endogenously while in the two-period model we were taking this as given.

To use a more compact notation, we define the gross output function, inclusive of undepreciated capital, as follows:

$$\pi(h_t, k_t, h_{t+1}) = \left[1 - \delta \cdot \left(\frac{h_{t+1} - h_t}{h_t}\right)\right] k_t + A h_t^{1-\alpha} k_t^{\alpha} \tag{9}$$

In writing this expression, we assume that the firm uses the best technology adoptable by the innovator, that is, $z_t = h_t$. It is easy to show that the choice of $z_t < h_t$ is never optimal.

We first characterize the equilibrium when the entrepreneur commits to the contract and then we turn to the case of limited enforcement. The comparison between these two environments will clarify how contract enforcement is key for the barriers to entry to affect the accumulation of knowledge.

5.1 Equilibrium with entrepreneur's commitment

Let $D_t(h_t)$ be the repudiation value for the innovator at the beginning of the period, before investing in knowledge. This is the value that an innovator with knowledge h_t would receive from quitting an incumbent firm. Furthermore, let $\widehat{D}_t(h_{t+1})$ be the value of quitting after choosing the knowledge investment, and therefore, after exercising effort. At this point the stock of knowledge is h_{t+1}. These functions also depend on the aggregate states of the economy, which explains the time subscript. For the moment we take these two functions as given.

Consider the optimization problem solved by a new firm (start-up entrepreneur) that hires an innovator with knowledge capital h_t at the beginning of period t. This is before the innovator makes the new investment in knowledge. Because with competition the innovator gets the whole surplus, it will be convenient to characterize the optimal contract by maximizing the value for the innovator, subject to the enforceability and participation constraints.[5] The optimization problem can be written as:

$$V_t(h_t) = \max_{\{w_s, k_{s+1}, h_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t}\Big[w_s - \varphi(H_s; h_s, h_{s+1})\Big] \tag{10}$$

subject to

$$\sum_{j=s}^{\infty} \beta^{j-s}\Big[w_j - \varphi(H_j; h_j, h_{j+1})\Big] \geq D_s(h_s), \qquad \text{for } s \geq t \tag{11}$$

$$w_s + \sum_{j=s+1}^{\infty} \beta^{j-s}\Big[w_j - \varphi(H_j; h_j, h_{j+1})\Big] \geq \widehat{D}_s(h_{s+1}), \quad \text{for } s \geq t \tag{12}$$

$$-\tau h_{t+1} - w_t - k_{t+1} + \sum_{s=t+1}^{\infty} \beta^{s-t}\Big[\pi(h_s, k_s, h_{s+1}) - w_s - k_{s+1}\Big] \geq 0 \tag{13}$$

The objective is the discounted flow of utilities for the innovator. In each period he receives the payment w_s, which is subject to a non-negativity constraint, and faces the disutility from effort $\varphi(H_s; h_s, h_{s+1})$. Constraints (11) and (12) are the enforcement constraints. Starting at time $t+1$, the innovator could quit at the beginning of the period, before choosing the investment in knowledge. In this case the repudiation value is $D_s(h_s)$. After choosing the knowledge investment, the value of quitting becomes $\widehat{D}_s(h_{s+1})$. The last constraint is the participation constraint for the entrepreneur or break-even condition. This simply says that the value of the contract for the entrepreneur cannot be negative.

For an innovator hired *after* investing in knowledge, the value of the contract is:

$$\widehat{V}_t(h_{t+1}) = \max_{\{w_s, k_{s+1}, h_{s+2}\}_{s=t}^{\infty}} \left\{ w_t + \sum_{s=t+1}^{\infty} \beta^{s-t}\Big[w_s - \varphi(H_s; h_s, h_{s+1})\Big] \right\}$$

subject to (11), (12) and (13)

The key difference respect to problem (10) is that the effort to accumulate knowledge has already been exercised by the innovator and h_{t+1} is given at

[5] Alternatively, we could maximize the whole surplus as we did in the analysis of the two-period model. Of course, this would give the same results.

this point. Consequently, the current return for the innovator is only w_t. This also explains why the choice of knowledge starts at $t+2$. We should also point out that the constraint (11) must be satisfied for all $s > t$ while constraints (12) and (13) must be satisfied for all $s \geq t$.

Given the definitions of $V_t(h_t)$ and $\widehat{V}_t(h_{t+1})$, it is easy to see that these two functions are related as follows:

$$V_t(h_t) = \max_{h_{t+1}} \Big\{ -\varphi(H_t; h_t, h_{t+1}) + \widehat{V}_t(h_{t+1}) \Big\} \tag{14}$$

The above optimization problems assume that the repudiation functions $D_t(h_t)$ and $\widehat{D}_t(h_{t+1})$ are known. But these functions are endogenous because they depend on the value functions $V_t(h_t)$ and $\widehat{V}_t(h_{t+1})$. More specifically, the repudiation value is equal to the value of moving to a new firm, that is, $D_s(h_s) = V_s(h_s)$ and $\widehat{D}_s(h_{s+1}) = \widehat{V}_s(h_{s+1})$.

Before proceeding we establish the following property:

Lemma 1 *Constraint (12) is always satisfied if constraint (11) is satisfied.*

Proof 1 *See Appendix A.*

Hence, in characterizing the solution of the optimal contract with entrepreneur's commitment, we can ignore the enforcement constraint (12). This constraint becomes relevant when the entrepreneur does not commit.

5.1.1 Transformed problem and first order conditions

In general, the optimization problem depends on the evolution of the economy-wide knowledge H_t, which in turn depends on the distribution of firms over knowledge and physical capital. In the analysis that follows, however, we concentrate on steady state equilibria where the distribution of firms and the average knowledge remain constant. We can then ignore the time subscript in all value functions. We should emphasize that, although in the steady state all firms will have the same $h = H$, the solution of the contractual problem requires us to solve for the whole transition experienced by a new entrant firm. This is necessary to determine the repudiation value for the innovator.

To characterize the solution, it will be convenient to use a transformation of problem (10). Appendix B shows that, in a steady state equilibrium, the optimization problem can be reformulated as follows:

$$\min_{\mu' \geq \mu_0} \max_{w \geq 0, h', k'} \Big\{ -\tau h' - w - k' + \mu' \big[w - \varphi(H; h_0, h') \big] - (\mu' - \mu_0) D(h_0) + \beta W(\mu', h', k') \Big\} \tag{15}$$

where h_0 is the initial level of knowledge and μ_0 is the inverse of the Lagrange multiplier associated with the participation constraint (11). The function W is defined recursively as follows:

$$W(\mu, h, k) = \min_{\mu' \geq \mu} \max_{w \geq 0, h', k'} \Big\{ \pi(h, k, h') - w - k' + \mu' \big[w - \varphi(H; h, h') \big] - (\mu' - \mu) D(h) + \beta W(\mu', h', k') \Big\} \tag{16}$$

Problem (15) is the problem solved by a new firm that hires an innovator with knowledge h_0. After starting the firm and choosing the first period investment, the problem becomes recursive as written in (16). Therefore, problem (16) is the problem solved by an incumbent firm, given the states (μ, h, k).

The variable μ can be interpreted as the weight that a hypothetical planner gives to the innovator. The weight given to the entrepreneur is 1. Over time the planner increases the weight μ to insure that the innovator does not quit the firm, until $\mu = 1$. Higher is the initial weight μ_0 and higher is the initial value of the contract for the innovator, and therefore, lower is the value for the entrepreneur. The value of μ_0 is determined such that the breaking even condition for the entrepreneur, that is, constraint (13) is satisfied with the equality sign. See Marcet & Marimon (1997) for details about the use of the saddle-point formulation.

The solution to problem (15) is characterized by the first order conditions:

$$D(h_0) \leq w - \varphi(H; h_0, h') + \beta D(h') \tag{17}$$

$$\mu' \leq 1 \tag{18}$$

$$\tau + \mu' \varphi_3(H; h_0, h') = \beta W_2(\mu', h', k') \tag{19}$$

$$\beta \pi_2(h', k', h') = 1 \tag{20}$$

and the solution to problem (16) by the first order conditions:

$$D(h) \le w - \varphi(H; h, h') + \beta D(h') \tag{21}$$
$$\mu' \le 1 \tag{22}$$
$$-\pi_3(h, k, h') + \mu' \varphi_3(H; h, h') = \beta W_2(\mu', h', k') \tag{23}$$
$$\beta \pi_2(h', k', h'') = 1 \tag{24}$$

Here we use subscripts to denote the derivative with respect to the particular argument. In conditions (17) and (21) the constraints are strict if $\mu' > \mu$, and conditions (18) and (22) are strict if the innovator's payments are zero, that is, $w = 0$. The envelope term is:

$$W_2(\mu, h, k) \;=\; \pi_1(h, k, h') - \mu' \varphi_2(H; h, h') - (\mu' - \mu) D_1(h)$$

5.1.2 Equilibrium

We can now provide a formal definition of a steady state equilibrium.

Definition 1 *A Balanced Growth Equilibrium with entrepreneur's commitment is defined as:* `(a)` *Decision rules for new firms* $\mu^N(\mu_0, h_0)$, $w^N(\mu_0, h_0)$, $h^N(\mu_0, h_0)$, $k^N(\mu_0, h_0)$*;* `(b)` *Decision rules for incumbent firms* $\mu(\mu, h)$, $d(\mu, h)$, $h(\mu, h)$, $k(\mu, h)$*;* `(c)` *Initial state* $\mu_0(h_0)$ *for new firms;* `(d)` *A repudiation function* $D(h)$ *and a value function* $V(h)$*;* `(e)` *A distribution of firms* $M(\mu, h, k)$*. Such that:* `(i)` *The decision rules* $\mu^N(\mu_0, h)$, $d^N(\mu_0, h_0)$, $h^N(\mu_0, h_0)$, $k^N(\mu_0, h_0)$ *solve problem (15);* `(ii)` *The decision rules* $\mu(\mu, h)$, $d(\mu, h)$, $h(\mu, h)$, $k(\mu, h)$ *solve problem (16);* `(iii)` *The initial state* $\mu_0(h_0)$ *is such that new firms break even;* `(iv)` *The repudiation function is the value of starting a new firm,* $D(h) = V(h)$*;* `(v)` *The distribution* $M(\mu, h, k)$ *remains constant over time.*

The key object of a steady state equilibrium is the constancy of the distribution $M(\mu, h, k)$. With positive spillovers, the accumulation of knowledge is cheaper for agents with $h < H$ and more costly for agents with $h > H$. This implies that in equilibrium all firm will converge to the same level of knowledge. In absence of entry, the steady state distribution of knowledge will be characterized by a unit mass of firms with $h = H$, that is, all innovators have the average knowledge. We state this formally as follows:

Proposition 3 *There is a unique steady state equilibrium in which all firms have the same knowledge H.*

Proof 3 *See Appendix C.*

We can now characterize the steady state equilibrium using the first order conditions derived above. Because all firms have been active for a long period of time, they will have $\mu = \mu' = 1$. Furthermore, their knowledge has converged to H and the capital has converged to K. The steady state value of H and K are determined by conditions (23) and (24), that is:

$$-\pi_3(H,K,H) + \varphi_3(H;H,H) = \beta\Big[\pi_1(H,K,H) - \varphi_2(H;H,H)\Big] \quad (25)$$

$$\beta\pi_2(H,K,H) = 1 \quad (26)$$

After solving for H and K, we can solve for the payment w. This requires us to solve for the whole transition dynamics of new entrant firms as characterized by the first order conditions (17)-(24). Even though we limit the analysis to the steady state, the repudiation value of an innovator is given by the surplus generated by a new firm. But to solve for this surplus, we need to solve for the whole transition experienced by the new firm.

Before turning to the analysis of the next section , we would like to emphasize that the steady state values of capital and knowledge are not affected by the start-up cost τ. This can be easily seen from the fact that τ does not enter conditions (25) and (26). We state this result formally in the following proposition.

Proposition 4 *When the entrepreneur commits to the log-term contract, the steady state values of H and K are independent of the start-up cost τ.*

Proof 4 *The start-up cost τ does not enter the steady state conditions (25) and (26).* *Q.E.D.*

We will see in the next section that this property no longer holds when the entrepreneur is unable to commit to the long-term contract (double-sided limited enforcement). In this case the cost of business start-up τ will have a negative impact on the steady state values of H and K.

5.2 Equilibrium without commitment

Without contract enforcement, the entrepreneur could renegotiate the payments promised to the innovator. In particular, the entrepreneur will renegotiate the contract when the present value of promised payments exceeds the repudiation value for the innovator.

We have shown in the previous section that the contractual problem with entrepreneur's commitment is subject to the following enforcement constraints for the innovator:

$$\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big]\geq D_s(h_s)$$

$$w_s+\sum_{j=s+1}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big]=\widehat{D}_s(h_{s+1})$$

which must hold for any $s\geq t$. The first constraint is before the investment in knowledge while the second is after the investment.

We have also shown that the second constraint is never binding (see Lemma 1), that is, after the innovator has chosen the knowledge investment, the discounted value of promised payments is bigger than the repudiation value. This implies that, after the investment in knowledge, the entrepreneur has an incentive to renegotiate the promised payments. Because of renegotiation, the second enforcement constraint must be satisfied with equality.

Remembering that $D_s(h_s)=V_s(h_s)$ and $\widehat{D}_s(h_{s+1})=\widehat{V}_s(h_{s+1})$, we can rewrite the enforcement constraints as follows:

$$\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big]\geq V_s(h_s) \tag{27}$$

$$\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big]=-\varphi(H_s;h_s,h_{s+1})+\widehat{V}_s(h_{s+1}) \tag{28}$$

In the previous section we have also shown that the two value functions are related as follows (see equation (14)):

$$V_t(h_t)=\max_h\Big\{-\varphi(H_t;h_t,h)+\widehat{V}_t(h)\Big\}$$

The solution to this problem, denoted by h^{New}, is the investment in knowledge chosen by a new firm. We then have the following proposition:

Proposition 5 *Without entrepreneur's commitment, the knowledge investment chosen by an incumbent firm is equal to the knowledge investment chosen by a new firm,* h^{New}.

Proof 5 *The investment chosen by a new firm could be different from the one chosen by an incumbent firm in absence of commitment. Denote this by* h^{Old}. *If this is the case, then* $V_t(h_t) > -\varphi(H_t; h_t, h^{Old}) + \widehat{V}_t(h^{Old})$. *But then constraints (27) and (28) cannot be both satisfied. Therefore, the only feasible solution is the one for which incumbent firms choose the same investment level as new firms, that is,* $h_{t+1} = h^{New}$. *Q.E.D.*

As for the two-period model, this result has a simple intuition. Because the entrepreneur can renegotiate the promised payments after the investment in knowledge, the innovator would not stay with the firm unless the entrepreneur agrees to the same level of knowledge chosen by a new firm. In this way, the innovator keeps the outside value high and prevents the firm from renegotiating.

Denote by $h_{s+1} = g_s(h_s)$ the knowledge investment of a new firm created at time $s \geq t$. The optimization problem for a new firm created at time t can be written as:

$$V_t(h_t) = \max_{h_{t+1}, \{w_s, k_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t}\Big[w_s - \varphi(H_s; h_s, h_{s+1})\Big] \tag{29}$$

subject to

$$\sum_{j=s}^{\infty} \beta^{j-s}\Big[w_j - \varphi(H_j; h_j, h_{j+1})\Big] \geq D_s(h_s), \qquad \text{for } s \geq t$$

$$-\tau h_{t+1} - w_t - k_{t+1} + \sum_{s=t+1}^{\infty} \beta^{s-t}\Big[\pi(h_s, k_s, h_{s+1}) - w_s - k_{s+1}\Big] \geq 0$$

$$h_{s+1} = g_s(h_s), \qquad \text{for } s \geq t+1$$

After the first period, the investment in knowledge is determined by the function $g_s(h_s)$, which is taken as given by the firm.

Following the same steps of Appendix B, we can show that in a steady state equilibrium, problem (29) can be reformulated as follows:

$$\min_{\mu' \geq \mu_0} \max_{w \geq 0, h', k'} \Big\{ -\tau h' - w - k' + \mu' \Big[w - \varphi(H; h_0, h') \Big] - (\mu' - \mu_0) D(h_0) + \beta W(\mu', h', k') \Big\} \quad (30)$$

where h_0 is the initial level of knowledge and μ_0 is the inverse of the Lagrange multiplier associated with the participation constraint for the entrepreneur. The function W is defined recursively as follows:

$$W(\mu, h, k) = \min_{\mu' \geq \mu} \max_{w \geq 0, k'} \Big\{ \pi\Big(h, k, g(h)\Big) - w - k' + \mu' \Big[w - \varphi\Big(H; h, g(h)\Big) \Big] - (\mu' - \mu) D(h) + \beta W\Big(\mu', g(h), k'\Big) \Big\} \quad (31)$$

As in the case with entrepreneur's commitment, the problem can be divided in two parts: the problem solved by new firms and the problem solved by incumbent firms. For new firms the problem is similar to the case of commitment. For incumbent firms, instead, the investment in knowledge is determined by the policy function $g(h)$, which is external to the firm.

The optimal value of h' chosen by a new firm in problem (30) also depends on the investment policy that the firm will follow in the future, that is, $g(h)$. Therefore, we have to solve a non-trivial fixed point problem. This is in addition to the fixed point problem in the determination of the repudiation function $D(h)$.

The first order conditions for problem (30) are given by (17)-(20). For problem (31) they are given by (21), (22) and (24). Notice that condition (23) is no longer relevant because incumbent firms take the policy $h' = g(h)$ as given. We show in Appendix D that the envelope term W_2 is now equal to:

$$W_2(\mu, h, k) = \pi_1\Big(h, k, g(h)\Big) - \mu' \varphi_2\Big(H; h, g(h)\Big) - (\mu' - \mu) D_1(h) + g_1(h) \Big[\pi_3\Big(h, k, g(h)\Big) + \tau \Big] \quad (32)$$

5.2.1 Equilibrium

We can now define a steady state equilibrium for this economy.

Definition 2 *A Steady State Equilibrium without entrepreneur's commitment is defined as:* `(a)` *Decision rules for new firms $\mu^N(\mu_0, h_0)$, $w^N(\mu_0, h_0)$, $h^N(\mu_0, h_0)$, $k^N(\mu_0, h_0)$ for new firms;* `(b)` *Decision rules for incumbent firms $\mu(\mu, h)$, $w(\mu, h)$, $k(\mu, h)$;* `(c)` *An investment rule for incumbent firms $g(h)$;* `(d)` *Initial state $\mu_0(h_0)$ for new firms;* `(e)` *A repudiation function $D(h)$ and a value function $V(h)$;* `(f)` *A distribution of firms $M(\mu, h, k)$. Such that:* `(i)` *The decision rules $\mu^N(\mu_0, h_0)$, $w^N(\mu_0, h_0)$, $h^N(\mu_0, h_0)$, $k^N(\mu_0, h_0)$ solve problem (30);* `(ii)` *The decision rules $\mu(\mu, h)$, $w(\mu, h)$, $k(\mu, h)$ solve problem (31);* `(iii)` *The investment rule of incumbent firms is equal to the investment policy of new firms, $g(h) = h^N(\mu_0(h), h)$;* `(iv)` *The initial state $\mu_0(h_0)$ is such that new firms break even;* `(v)` *The repudiation function is the value of starting a new firm, $D(h) = V(h)$;* `(vi)` *The distribution $M(\mu, h, k)$ remains constant over time.*

The definition is similar to the one provided for the case of commitment, with the exception of condition (iii). This condition imposes that the knowledge investment chosen by an incumbent firm is equal to the investment chosen by a new firm with the same initial states.

The next step is to show that, because of capital obsolescence for incumbent firms, the knowledge investment chosen by a new firm is higher than the level preferred by an incumbent firm and this leads to higher income.

Substituting the envelope term derived in the previous section in equation (19), the first order condition for the investment in knowledge of new firms with $h = H$ can be written as:

$$\begin{aligned} \tau + \mu'\varphi_3(H; H, h') \;&=\; \beta\Big\{\pi_1\Big(h', k', g(h')\Big) - \mu''\varphi_2\Big(H; h', g(h')\Big) \\ &\quad -(\mu'' - \mu')D_1(h') + g_1(h')\Big[\pi_3\Big(h', k', g(h')\Big) + \tau\Big]\Big\} \end{aligned} \tag{33}$$

Because incumbent firms innovate at the same rate as new firms, this is also the condition that determines the investment in knowledge of incumbent firms. Therefore, in order to determine whether the lack of commitment from the entrepreneur leads to higher or lower knowledge, we have to compare this condition to the one determining the optimal investment in knowledge when the entrepreneur is able to commit to the long-term contract. This is condition (23).

Let H^E be the knowledge in the economy with contract enforcement and H^{LE} the knowledge in the economy with limited enforcement. The following proposition characterizes the steady state equilibrium.

Proposition 6 *The steady state knowledge H^{LE} is strictly decreasing in τ and there exists $\bar{\tau} > 0$ such that $H^{LE} > H^E$ for $\tau < \bar{\tau}$ and $H^{LE} < H^E$ for $\tau > \bar{\tau}$.*

Proof 6 *See Appendix E.*

Therefore, when contracts are not enforceable (double-sided limited enforcement), the start-up cost is harmful for the economy. With low barriers the economy would experience an even higher level of income than in the economy with enforceable contracts. Depending on the importance of the externality, this could be welfare improving.

6 Quantitative application

The goal of this section is to study how much of the variation in cross-country incomes can be explained by differences in the cost of business start-up. We start with the assignment of the parameter values and the specification of the functional forms.

The discount factor is set to $\beta = 0.95$, implying an annual interest rate of about 5 percent. The production function takes the form $Ah^{1-\alpha}k^{\alpha}$ with A normalized to 1. The parameter α represents the capital income share and it is set to 0.33. The depreciation of capital is specified as: $\delta_t = \bar{\delta} + \delta \cdot (z_{t+1}/z_t - 1)$. We choose $\delta = 0.1$ but we would like to emphasize that this parameter is not important for the sensitivity of the steady state equilibrium to τ. The parameter $\bar{\delta}$, instead, determines the capital-income ratio independently of the start-up cost. We choose a value of 0.066 which implies a capital-income ratio of 2.8.

The effort cost function is derived from the accumulation equation for the stock of knowledge, which is assumed to take the form:

$$h_{t+1} = \phi h_t + H_t^{\theta} e_t^{\frac{1-\theta}{\rho}}$$

where H_t is the economy-wide level of knowledge, e_t is the effort cost to accumulate knowledge and the parameters satisfy $\phi < 1$, $\theta < 1$ and $\rho > 1$. The parameter θ captures the importance of leakage or spillover effects while ρ captures the importance of world-wide knowledge which is external to the country. By inverting we get the effort cost function:

$$e_t = \varphi(H_t; h_t, h_{t+1}) = \left(\frac{h_{t+1} - \phi h_t}{H_t^{\theta}}\right)^{\frac{\rho}{1-\theta}}$$

which is homogeneous of degree ρ.

Assuming that the economy grows at an exogenous rate of 3 percent, the stock of knowledge must also grow at this rate in the steady state. In the detrended economy this corresponds to a depreciation rate of 3 percent which requires $\phi = 0.97$. The parameter θ is not important for the steady state. It only determines the optimality of the equilibrium. The important parameter is the degree of homogeneity ρ. Unfortunately, there are no direct measures of ρ. Therefore, we will show the results for alternative values of this parameter.

We start by choosing a value of ρ that optimizes the fit of the model with the data. More specifically, we chose ρ to minimize the sum of square deviations between the outputs predicted by the model for each country (given the observed cost of business start up) and the actual per-capita GDP. We limit the sample to countries with a start-up cost smaller than 100 percent to eliminate outliers. This reduces the sample size to 104 countries. We also normalize the model so that it replicates the highest per-capita income with $\tau = 0$. In the 2004 sample the country with the highest per-capita GDP was Ireland with about 40,000 dollars.

Figure 2 plots the actual values of per-capita GDP and the values predicted by the model when $\rho = 1.014$. This is the value that optimizes the fit of the model.

We have also calculated the 'optimal' steady state level of output. This is the output produced if investment was chosen by a benevolent planner who takes into account the externality in the accumulation of knowledge. The steady state levels of H and K in the planner solution are found by solving the first order conditions:

$$-\pi_3(H,K,H) + \varphi_3(H;H,H) = \beta\Big[\pi_1(H,K,H) - \varphi_2(H;H,H) - \varphi_1(H;H,H)\Big]$$

$$\beta\pi_2(H,K,H) = 1$$

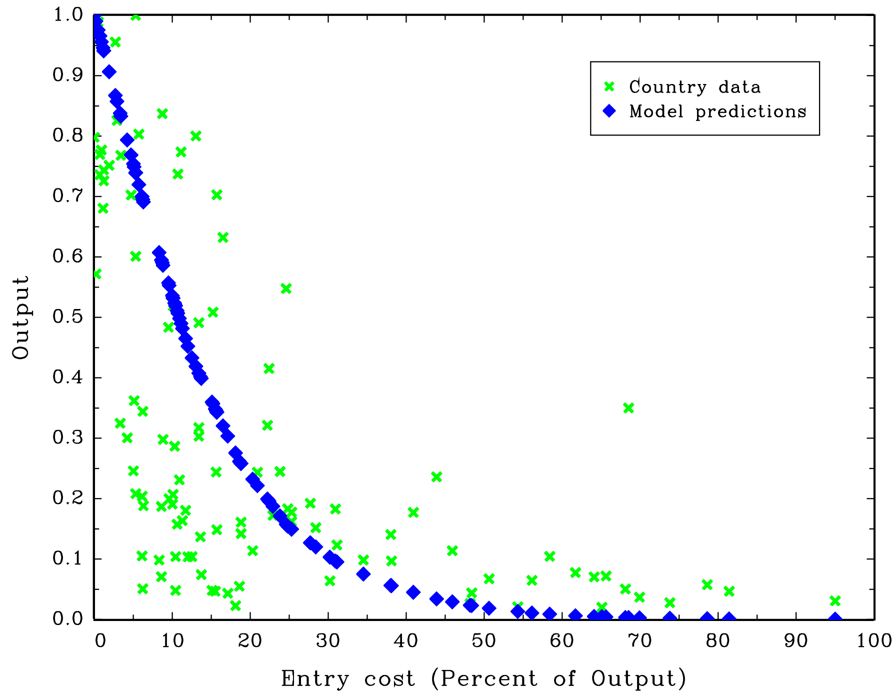

Figure 2: Steady state output for different entry costs.

These are similar to conditions (25) and (26) except for the additional term $\varphi_1(H; H, H)$ in the first equation. This term captures the externality in the accumulation of knowledge which is taken into account by the planner but ignored by the atomistic agents in a competitive economy. In this particular calibration with $\theta = 0.15$, barriers to entry are always inefficient. More specifically, the optimal level of output is about 75 percent above the competitive output when $\tau = 0$. However, this depends on the value of θ. If the externality is small, then moderate barriers to business start-up are optimal. This is because, limited enforcement of contracts leads to over-accumulation of knowledge.

The next Figure 3 adds to the previous graph the predictions of the model for alternative values ρ. As we increase the degree of homogeneity of the cost function, the model is less successful in capturing large differences in per-capita income. A larger value of ρ implies that the economy depends more heavily on the world-wide level of knowledge, which is external to an individual country. Therefore, it becomes more difficult to generate large cross-country differences. On the other hand, when ρ is very small, the economy depends only marginally on the world-wide knowledge. In the limit the long-term growth rates will be different across countries. Therefore, the model could generate any level of cross-country heterogeneity.

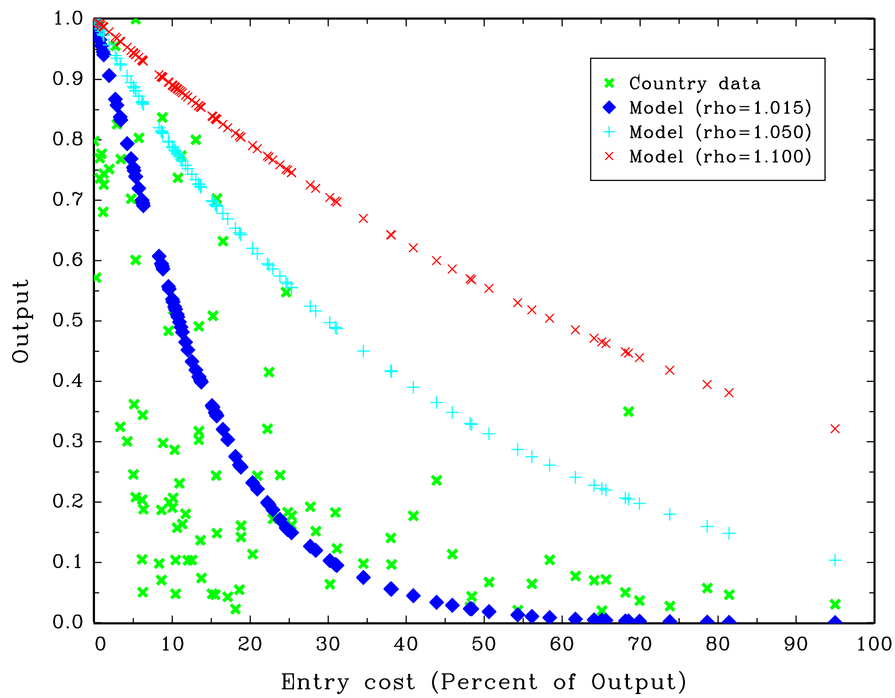

Figure 3: Steady state output for different entry costs.

7 Conclusion

Modern technologies are highly complementary to skilled labor. This implies that the adoption of these technologies requires the accumulation of innovation skills or knowledge from workers and managers. In absence of a commitment devise or enforcement of contracts, under-accumulation of skills may result. However, we have shown that free entry can create an incentive mechanism to accumulate knowledge even if contracts are not enforceable. Free entry increases the demand of knowledge capital and creates an outside opportunity for those accumulating knowledge which can be used against any renegotiation attempts from the current employer. It is this outside opportunity that guarantees the agent the reward for accumulating knowledge when contracts are not enforceable.

Our paper provides a theoretical foundation for the empirical finding that the cost of starting a business is negatively correlated with the level of development and growth of a country.

A Proof of Lemma 1

Conditions (11) and (12) can be rewritten as:

$$\begin{aligned}\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big] &\geq D_s(h_s)\\ \sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big] &\geq -\varphi(H_s;h_s,h_{s+1})+\widehat{D}_s(h_{s+1})\end{aligned}$$

Therefore, to show that the second constraint is satisfied when the first constraint is satisfied, it is enough to show that $D_s(h_s)\geq-\varphi(H_s;h_s,h_{s+1})+\widehat{D}_s(h_{s+1})$ for any value of h_{s+1}. From the definition of the repudiation values we have that $D_s(h_s)=\max_h\{-\varphi(H_s;h_s,h)+\widehat{D}_s(h)\}\geq-\varphi(H_s;h_s,h_{s+1})+\widehat{D}_s(h_{s+1})$ for any h_{s+1}. *Q.E.D.*

B Saddle-point formulation

Consider problem (10). As proved in Lemma 1, constraint (12) is not binding. Therefore, we have to consider only constraints (11) and (13). Let γ_s and λ_t be the Lagrange multipliers associated with these two constraints. The Lagrangian can be written as:

$$\begin{aligned}\mathcal{L} &= \sum_{s=t}^{\infty}\beta^{s-t}\Big[w_s-\varphi(H_s;h_s,h_{s+1})\Big]\\ &+ \sum_{s=t}^{\infty}\beta^{s-t}\gamma_s\left\{\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big]-D_s(h_s)\right\}\\ &+ \lambda_t\left\{-w_t-\tau h_{t+1}-k_{t+1}+\sum_{s=t+1}^{\infty}\beta^{s-t}\Big[\pi(h_s,k_s,h_{s+1})-w_s-k_{s+1}\Big]\right\}\end{aligned}$$

Define $\tilde{\mu}_s$ recursively as follows: $\tilde{\mu}_{s+1}=\tilde{\mu}_s+\gamma_s$, with $\tilde{\mu}_t=0$. Using this variable and rearranging terms, the Lagrangian can be written as:

$$\begin{aligned}\mathcal{L} &= \sum_{s=t}^{\infty}\beta^{s-t}\Big\{(1+\tilde{\mu}_{s+1})\Big[w_s-\varphi(H_s;h_s,h_{s+1})\Big]-(\tilde{\mu}_{s+1}-\tilde{\mu}_s)D_s(h_s)\Big\}\\ &+ \lambda_t\left\{-w_t-\tau h_{t+1}-k_{t+1}+\sum_{s=t+1}^{\infty}\beta^{s-t}\Big[\pi(h_s,k_s,h_{s+1})-w_s-k_{s+1}\Big]\right\}\end{aligned}$$

Define $\mu_s = (1+\tilde{\mu}_s)/\lambda_t$ for $s \geq t+1$ with $\mu_t = 1/\lambda_t$. Substituting we get:

$$\begin{aligned}
\mathcal{L} &= \lambda_t \Bigg\{ - w_t - \tau h_{t+1} - k_{t+1} + \mu_{t+1}\Big[w_t - \varphi(H_t; h_t, h_{t+1})\Big] - (\mu_{t+1}-\mu_t) D_t(h_t) \\
&+ \sum_{s=t+1}^{\infty} \beta^{s-t}\Big[\pi(h_s, k_s, h_{s+1}) - w_s - k_{s+1} + \mu_{s+1}\Big[w_s - \varphi(H_s; h_s, h_{s+1})\Big] \\
&\qquad - (\mu_{s+1}-\mu_s) D_s(h_s)\Big]\Bigg\}
\end{aligned}$$

We now look at the special case of a steady state equilibrium in which the stock of aggregate knowledge is constant. Dividing by λ_t, the problem can be rewritten as:

$$\begin{aligned}
\min_{\mu_{t+1}\geq \mu_t} \max_{\substack{w_t \geq 0,\\ h_{t+1}, k_{t+1}}} &\Bigg\{ - w_t - \tau h_{t+1} - k_{t+1} + \mu_{t+1}\Big[w_t - \varphi(H_t; h_t, h_{t+1})\Big] \\
&- (\mu_{t+1}-\mu_t) D(h_t) + \beta W(\mu_{t+1}, h_{t+1}, k_{t+1})\Bigg\}
\end{aligned}$$

for given μ_t and with the function W defined recursively as follows:

$$\begin{aligned}
W(\mu, h, k) &= \min_{\mu' \geq \mu} \max_{\substack{w\geq 0,\\ h', k'}} \Bigg\{ \pi(h, k, h') - w - k' + \mu'\Big[w - \varphi(H; h, h')\Big] \\
&\qquad - (\mu' - \mu) D(h) + \beta W(\mu', h', k')\Bigg\}
\end{aligned}$$

The initial state μ_t is determined such that the participation constraint for the entrepreneur is satisfied. *Q.E.D.*

C Proof of Proposition 3

The accumulation of knowledge for an incumbent firm is dictated by the first order condition (23). After substituting the envelope condition, it reads:

$$-\pi_3(h, k, h') + \mu' \varphi_3(H; h, h') = \beta\Big[\pi_1(h, k, h') - \mu' \varphi_2(H; h, h') - (\mu' - \mu) D_1(h)\Big]$$

We know that the state μ will eventually converge to 1 for an incumbent firm. Therefore, without new entry, all firms will have $\mu = 1$ in a steady state equilibrium. Furthermore, $h = H$ in a steady state equilibrium. Therefore, the above condition can be rewritten as:

$$\varphi_3(H; H, H) + \beta \varphi_2(H; H, H) = \pi_3(H, K, H) + \beta \pi_1(H, K, H) \qquad (34)$$

The right-hand-side terms of this expression remain constant in a steady state. In fact, taking into account the functional form of π (see equation (9)) and imposing the steady state conditions $h = H$ and $k = K$, we have that $\pi_3(H, K, H) = -\delta(K/H)$ and $\pi_1(H, K, H) = \delta(K/H) + (1 - \alpha)A(K/H)^{\alpha}$. These two terms only depend on the ratio K/H. But condition (24)—that is, $\pi_2(H, K, H) = 1 + \alpha A(K/H)^{\alpha-1} = 1$—requires the constancy of K/H.

Let's look now at the left-hand-side terms. Because φ is homogenous of degree $\rho > 1$, the derivatives φ_2 and φ_3 are homogeneous of degree $\rho - 1$. Therefore, the left-hand-side can be written as

$$\varphi_3(H; H, H) + \beta\varphi_2(H; H, H) = \Big[\varphi_3(1; 1, 1) + \beta\varphi_2(1; 1, 1)\Big] H^{\rho-1}$$

Because $\rho - 1$, this term is strictly increasing in H, converges to zero as $H \to 0$ and to infinity as $H \to \infty$. Therefore, there exists a unique value of H that solves condition (34). *Q.E.D.*

D Derivation of the envelope condition (32)

Differentiating equation (31) with respect to h we get:

$$\begin{aligned} W_2(\mu, h, k) &= \pi_1\Big(h, k, g(h)\Big) + \pi_3\Big(h, k, g(h)\Big)g_1(h) - \mu'\varphi_2\Big(H; h, g(h)\Big) \\ &- \mu'\varphi_3\Big(H; h, g(h)\Big)g_1(h) - (\mu' - \mu)D_1(h) + \beta W_2(\mu', h', k')g_1(h) \end{aligned} \quad (35)$$

From condition (19) we have

$$\tau + \mu'\varphi_3(H; h_0, h') = \beta W_2(\mu', h', k')$$

This is the condition for new firms which, with limited enforcement, must be satisfied at any point in time. Substituting this condition in (35), we get:

$$\begin{aligned} W_2(\mu, h, k) &= \pi_1\Big(h, k, g(h)\Big) - \mu'\varphi_2\Big(H; h, g(h)\Big) - (\mu' - \mu)D_1(h) \\ &+ g_1(h)\Big[\pi_3\Big(h, k, g(h)\Big) + \tau\Big] \end{aligned}$$

Q.E.D.

E Proof of Proposition 6

All firms are alike in the steady state. Therefore, $h = H$. In a steady state equilibrium μ_0 must be equal to 1. In fact, because potential new firms start with the same H as incumbents, μ_0 must be equal to the μ of incumbents firms, which is 1. Imposing $\mu_0 = 1$, the first order condition for the accumulation of knowledge (equation (33)) can be written as:

$$\tau + \varphi_3(H; H, H) = \beta\Big[\pi_1(H, K, H) - \varphi_2(H; H, H)\Big] + \beta g_1(H)\Big[\pi_3(H, K, H) + \tau\Big]$$

With commitment, the first order condition for the accumulation of knowledge is given by equation (23), which in the steady state becomes:

$$\varphi_3(H; H, H) = \beta\Big[\pi_1(H, K, H) - \varphi_2(H; H, H)\Big] + \pi_3(H, K, H)$$

Let's notice first that, given the structure of function π defined in (9), conditions (20) and (24) will uniquely determine the capital-knowledge ration K/H, which are the same in both economies independently of τ. Second, the derivatives $\pi_1(H, K, H)$ and $\pi_3(H, K, H)$ only depend on the ratio K/H which, as observed above, is uniquely determined by conditions (20) and (24).

The homogeneity of degree ρ of the cost function φ implies that the derivatives are homogeneous of degree $\rho - 1$. Therefore, we can rewrite the above two conditions as follows

$$\begin{aligned}\Big[\varphi_3(1;1,1) + \beta\varphi_2(1;1,1)\Big]H^{\rho-1} &= \beta\pi_1(H, K, H) + \beta g_1(H)\pi_3(H, K, H) \\ &- \tau\Big[1 - \beta g_1(H)\Big]\end{aligned} \quad (36)$$

$$\Big[\varphi_3(1;1,1) + \beta\varphi_2(1;1,1)\Big]H^{\rho-1} = \beta\pi_1(H, K, H) + \pi_3(H, K, H) \quad (37)$$

Because $\rho - 1$, the left-hand-side terms are strictly increasing in H, converge to zero as $H \to 0$ and to infinity as $H \to \infty$. We further observe that, as shown in the proof of Proposition 3, the terms π_1 and π_3 only depend on the ratio K/H which is uniquely pinned down by conditions (20) and (24).

Consider first the case in which the start-up cost is zero, that is, $\tau = 0$. We start by assuming that the term $\beta g_1(H) < 1$, which we will prove below. Because $\pi_3(H, K, H) < 0$, the assumption that $\beta g_1(H) < 1$ implies that the right-hand-side of (36) is bigger than the right-hand-side of (37). Therefore, the left-hand-side of (36) must also be bigger. This implies a higher H. Therefore, $H^{LE} > H^E$. Notice that, without capital obsolescence, $\pi_3(H, K, H) = 0$. Therefore, conditions (36) and (37) are indistinguishable. This implies that the commitment of the entrepreneur does not affect the long-term growth as long as $\tau = 0$.

Let's consider now the case in which $\tau > 0$. This variable only affects condition (36). If the term $\beta g_1(H) < 1$, then an increase in τ reduces the right-hand-side of (36). The reduction in the left-hand-side term then requires a lower value of H.

For a sufficiently large τ, the steady state level of knowledge declines to the point in which $H^{LE} < H^{E}$.

To complete the proof we have to show that $\beta g_1(H,H) < 1$. *Q.E.D.*

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