

# The structural dynamics of US output and inflation: what explains the changes?

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## Abstract

We examine the dynamics of US output and inflation using a structural time varying coefficient VAR. We show that there are changes in the volatility of both variables and in the persistence of inflation. Technology shocks explain changes in output volatility, while a combination of technology, demand and monetary shocks explain variations in the persistence and volatility of inflation. We detect changes over time in the transmission of technology shocks and in the variance of technology and of monetary policy shocks. Hours and labor productivity always increase in response to technology shocks.

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# 1 Introduction

There is considerable evidence suggesting that the US economy has fundamentally changed over the last couple of decades. For example, Blanchard and Simon (2000), McConnell and Perez Quiroz (2001), Sargent and Cogley (2001) and Stock and Watson (2003) have reported a marked decline in the volatility of real activity and inflation since the early 1980s and a reduction in the persistence of inflation over time. What causes these changes? One possibility is that there have been alterations in the mechanism through which exogenous disturbances spread across sectors and propagate over time. Since the transmission mechanism depends on the features of the economy, this means that structural characteristics, such as the behavior of consumers and firms or the preferences of policymakers, have changed over time. The recent literature has paid particular attention to changes in policymakers' preferences. For example, Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) and (2005), Boivin and Giannoni, (2002), Lubik and Schorfheide (2004) have argued that monetary policy was "loose" in fighting inflation in the 1970s but became more aggressive since the early 1980s. Leeper and Zha (2003), Sims and Zha (2004), Primiceri (2004) and Canova and Gambetti (2004) are critical of this view since they estimate a stable policy rule and find the transmission of policy shocks roughly unchanged over time.

There has been a resurgence of interest in the last few years in analyzing the dynamics induced by technology shocks, following the work of Gali (1999), Francis and Ramey (2002), Christiano, Eichenbaum and Vigfusson (2003), Uhlig (2003), Dedola and Neri (2004) and others. However, to the best of our knowledge, the link between structural changes in the US economy and the way technology shocks are transmitted to the economy has not been made. This is a bit surprising given that the trend increase in productivity of the 1990s was to a large extent unexpected (see e.g. Gordon (2003)) and that it may have produced changes in the way firms (see e.g. McConnell and Perez Quiroz (2001)) and consumers responded to economic disturbances. Similarly, the way fiscal policy was conducted in the 1970s and the early 1980s differed considerably from the way it was conducted in the 1990s. For example, large deficits in the early period were turned into surpluses in the 1990s. Furthermore, benign neglect about the size of the public debt has been substituted by a keen awareness of the wealth effects and of the inflation consequences that large debts may have. Studying whether the dynamics induced by technology and fiscal shocks have changed over time may help to clarify which structural feature of the US economy changed and whether the variations in output and inflation reflect changes in the propagation mechanism or to changes in the variance of the exogenous shocks.

This paper provides evidence on these issues investigating the contribution of technology, government expenditure and monetary disturbances to the changes in the volatility and in the persistence of US output and inflation. We employ a time varying coefficients VAR model (TVC-VAR), where coefficients evolve according to a nonlinear transition equation, which puts zero probability on paths associated with explosive roots, and the variance of the forecast errors is allowed to vary over time. As in Cogley and Sargent (2001), (2005) we use Markov Chain Monte Carlo (MCMC) methods to estimate the posterior distributions of the quantities of interest. However, contrary to these authors and as in Canova and Gambetti (2004), we analyze the time evolution of structural relationships. To do so, we identify structural disturbances which are allowed to have different features at different points in time. In particular, we permit time variations in the characteristics of the shocks, in their variance and in their transmission to the economy.

Our analysis is recursive. That is, we can construct posterior distributions for structural statistics, using the information available at that point in time. This complicates the computations significantly - a MCMC routine is needed at each  $t$  where the analysis is conducted - but provides a clearer picture of the time evolution of structural relationships. With this strategy our analysis becomes comparable with the one of Canova (2004), where a small scale DSGE model featuring three types of shocks with similar economic interpretations, is recursively estimated with MCMC methods.

We identify structural disturbances using robust sign restrictions obtained from a DSGE model featuring monopolistic competitive firms, distorting taxes, government expenditure for consumption and investment purposes and rules describing fiscal and monetary policy actions. The model encompasses RBC style and New-Keynesian style models as special cases and features utility yielding government expenditure and private productivity boosting government investments. We construct robust restrictions allowing the parameters to vary within a range which is consistent with statistical evidence and economic considerations. We focus on sign restrictions, as opposed to more standard magnitude or zero restrictions, for several reasons. First, magnitude restrictions typically depend on the parameterization of the model while the sign restrictions we employ are less prone to such problem. Second, our model fails to deliver the full set of zero restrictions needed to identify the three shocks of interest. Third, standard decompositions impose restrictions on the structure of time variations which could bias our view about the evolution of structural dynamics. Finally, with model-based robust sign restrictions, the link between the empirical analysis and the theory is clear therefore making the analysis transparent and inference more credible.

Because time variations in the coefficients induce important non-linearities, standard

response analysis to structural shocks is inappropriate. For example, since at each  $t$  the coefficient vector is perturbed by a shock, assuming that between  $t + 1$  and  $t + k$  no shocks other than the disturbance under consideration hit the system may give misleading conclusions. To trace out the evolution of the economy when perturbed by structural shocks, we define impulse responses as the difference between two conditional expectations, differing in the arguments of their conditioning sets. Such a definition reduces to the standard one when coefficients are constant, allows us to condition on the history of the data and of the parameters, and permits the size and the sign of the shocks to matter for the dynamics of the model (see e.g. Canova and Gambetti (2004)).

Our results are as follows. While there is evidence of structural variations in both the volatility of output and inflation and in the persistence of inflation, our posterior analysis fails to detect significant changes because of the large standard errors associated with posterior estimates at each  $t$ . Technology shocks account for the largest portion of output variability at frequency zero and, on average, across frequencies, while real demand and monetary shocks account for the bulk of inflation variations at frequency zero and, on average, across frequencies. We show that output has become less volatile because the contribution of technology shocks has declined most and that changes in the persistence and volatility of inflation can be only partially accounted for by a combination of the three structural shocks. We detect changes in the transmission of technology shocks and in the variances of technology and monetary policy shocks. Finally, we provide novel evidence on the effects of technology shocks on labor market variables: in our estimated system, technology shocks always imply positive contemporaneous comovements of hours and productivity but the correlation turns negative after a few lags.

All in all, our analysis attributes to variations in the magnitude and the transmission of technology shocks an important role in explaining changes in output volatility. Therefore our results are consistent with the analyses of McConnell and Perez Quiroz (2001) and Gordon (2003). Furthermore, variations in the magnitude of both technology and monetary shocks and the transmission of technology shocks are important in explaining changes in the volatility and in the persistence of inflation. Therefore our results complement with those of Sims and Zha (2004), Primiceri (2004) and Gambetti and Canova (2004), who focused on the role only of monetary policy to explain observed changes.

The rest of the paper is organized as follows. The next section describes the empirical model. Section 3 presents a DSGE model which produces the restrictions used to identify structural shocks. Section 4 briefly deals with estimation - all technical details are confined to the appendix. Section 5 presents the results and section 6 concludes.

## 2 The empirical model

Let  $y_t$  be a  $5 \times 1$  vector of time series including real output, hours, inflation and the federal funds rate and M1 with the representation

$$y_t = A_{0,t} + A_{1,t}t + A_{2,t}y_{t-1} + A_{3,t}y_{t-2} + \dots + A_{p+1,t}y_{t-p} + \varepsilon_t \quad (1)$$

where  $A_{0,t}, A_{1,t}$  are a  $5 \times 1$  vectors;  $A_{i,t}$ , are  $5 \times 5$  matrices,  $i = 2, \dots, p + 1$ , and  $\varepsilon_t$  is a  $5 \times 1$  Gaussian white noise process with zero mean and covariance  $\Sigma_t$ . Letting  $A_t = [A_{0,t}, A_{1,t}, A_{2,t}, \dots, A_{p+1,t}]$ ,  $x'_t = [1_5, 1_5 * t, y'_{t-1}, \dots, y'_{t-p}]$ , where  $1_5$  is a row vector of ones of length 5,  $vec(\cdot)$  denotes the stacking column operator and  $\theta_t = vec(A'_t)$ , rewrite (1) as

$$y_t = X'_t \theta_t + \varepsilon_t \quad (2)$$

where  $X'_t = (I_5 \otimes x'_t)$  is a  $5 \times (5p + 2)5$  matrix,  $I_5$  is a  $5 \times 5$  identity matrix, and  $\theta_t$  is a  $(5p + 2)5 \times 1$  vector. We assume that  $\theta_t$  evolves according to

$$p(\theta_t | \theta_{t-1}, \Omega_t) \propto \mathcal{I}(\theta_t) f(\theta_t | \theta_{t-1}, \Omega_t) \quad (3)$$

where  $\mathcal{I}(\theta_t)$  discards explosive paths of  $y_t$  and  $f(\theta_t | \theta_{t-1}, \Omega_t)$  is represented as

$$\theta_t = \theta_{t-1} + u_t \quad (4)$$

where  $u_t$  is a  $(5p + 2)5 \times 1$  Gaussian white noise process with zero mean and covariance  $\Omega_t$ . We select this specification because more general AR and/or mean reverting structures were always discarded in out-of-sample model selection exercises. We assume that  $corr(u_t, \varepsilon_t) = 0$ , and that  $\Omega_t$  is diagonal. The first assumption imply conditional linear responses to changes in  $\varepsilon_t$ , while the second is made for computational ease - structural coefficients are allowed to change in a correlated fashion. Note that our model implies that the forecast errors are non-normal and heteroschedastic even when  $\Sigma_t = \Sigma$  and  $\Omega_t = \Omega$ . In fact, substituting (4) into (2) we have that  $y_t = X'_t \theta_{t-1} + v_t$  where  $v_t = \varepsilon_t + X'_t u_t$ . Such a structure is appealing since whatever alters coefficients also imparts heteroschedastic movements to the variance of the forecasts errors. Since also  $\Omega_t$  is allowed to vary over time, the model permits various form of stochastic volatility in the forecast errors of the model (see Sims and Zha (2004) and Cogley and Sargent (2005) for alternative specifications).

Let  $S_t$  be a square root of  $\Sigma_t$ , i.e.,  $\Sigma_t = S_t S'_t$  and let  $H_t$  be an orthonormal matrix, independent of  $\varepsilon_t$ , such that  $H_t H'_t = I$  and set  $J_t^{-1} = H'_t S_t^{-1}$ .  $J_t$  is a particular decomposition of  $\Sigma_t$  which transforms (2) in two ways: it produces uncorrelated innovations (via the

matrix  $S_t$ ) and it gives a structural interpretation to the equations of the system (via the matrix  $H_t$ ). Premultiplying  $y_t$  by  $J_t^{-1}$  we obtain

$$J_t^{-1}y_t = J_t^{-1}A_{0,t} + J_t^{-1}A_{1,t}t + \sum_j J_t^{-1}A_{j+1,t}y_{t-j} + e_t \quad (5)$$

where  $e_t = J_t^{-1}\varepsilon_t$  satisfies:  $E(e_t) = 0$ ,  $E(e_t e_t') = I_5$ . Equation (5) represents the class of "structural" representations of interest: for example, a Choleski system is obtained choosing  $S_t = S$  to be lower triangular matrix and  $H_t = I_5$ , and more general patterns, with non-recursive zero restrictions, result choosing  $S_t = S$  to be non-triangular and  $H_t = I_5$ .

In this paper, since we choose  $S_t$  to be an arbitrary square root matrix, identifying structural shocks is equivalent to choosing  $H_t$ . Here we select it so that the sign of the responses at  $t+k$ ,  $k=1, 2, \dots, K_1$ ,  $K_1$  fixed, matches the robust model-based sign restrictions presented in the next section. We choose sign restrictions to identify structural shocks for two reasons. First, the contemporaneous zero restrictions conventionally used are absent from the theoretical (DSGE) model presented in the next section. Second, standard decompositions have an undesirable property whenever  $\Sigma_t = \Sigma, \forall t$ . Take, for example, a Choleski decomposition. If  $\Sigma_t$  is time invariant, its Choleski factor  $S_t$  is time invariant. Hence, since  $H_t = I$ , the contemporaneous effects of structural shocks is restricted to be time-invariant. Our identification approach allows for time variations in both contemporaneous and lagged effects even when  $\Sigma_t$  is time invariant.

Letting  $C_t = [J_t^{-1}A_{0t}, \dots, J_t^{-1}A_{p+1t}]$ , and  $\gamma_t = \text{vec}(C_t')$ , (5) can be written as

$$J_t^{-1}y_t = X_t' \gamma_t + e_t \quad (6)$$

As in fixed coefficient VARs there is a mapping between the structural coefficients  $\gamma_t$  and the reduced form coefficients  $\theta_t$  since  $\gamma_t = (J_t^{-1} \otimes I_{5p})\theta_t$ . Whenever  $\mathcal{I}(\theta_t) = 1$ , we have

$$\gamma_t = \gamma_{t-1} + \eta_t \quad (7)$$

where  $\eta_t = (J_t^{-1} \otimes I_{5p})u_t$  satisfies  $E(\eta_t) = 0$ ,  $E(\eta_t \eta_t') = E((J_t^{-1} \otimes I_{5p})u_t u_t' (J_t^{-1} \otimes I_{5p})')$ . Hence, the vector of structural shocks  $\xi_t' = [e_t', \eta_t']'$  is a white noise process with zero mean and covariance matrix  $E\xi_t \xi_t' = \begin{bmatrix} I_5 & 0 \\ 0 & E((J_t^{-1} \otimes I_{5p})u_t u_t' (J_t^{-1} \otimes I_{5p})') \end{bmatrix}$ . Since each element of  $\gamma_t$  depends on several  $u_{it}$  via the matrix  $J_t$ , shocks to structural parameters are no longer independent. Note that the model (6)-(7) contains two types of structural shocks: VAR disturbances,  $e_t$ , and structural parameters disturbances,  $\eta_t$ . While, in general, the latters have little interpretation, for the equation representing the monetary policy rule, they capture changes in the preferences of the monetary authorities with respect to developments

in the rest of the economy. Such shocks will not be dealt with here and are analyzed in details in Canova and Gambetti (2004).

To study the transmission of disturbances in a fixed coefficient model one typically employs impulse responses. Impulse responses are generally computed as the difference between two realizations of  $y_{i,t+k}$  which are identical up to time  $t$ , but one assumes that between  $t$  and  $t+k$  a shock in the  $j$ -th component of  $e_{t+k}$  occurs only at time  $t$ , and the other that no shocks take place at all dates between  $t$  and  $t+k$ ,  $k = 1, 2, \dots$ .

In a TVC model, responses computed this way disregard the fact that structural coefficients may also change. Hence, meaningful response functions ought to measure the effects of a shock in  $e_{jt}$  on  $y_{it+k}$ , allowing future shocks to the structural coefficients to be non-zero. The responses we present are obtained as the difference between two conditional expectations of  $y_{it+k}$ . In both cases we condition on the history of the data and of the coefficients, on the structural parameters of the transition equation (which are function of  $J_t$ ) and all future shocks. However, in one case we condition on a draw for the current shock, while in the other the current shock is set to zero.

Formally, let  $y^t$  be a history for  $y_t$ ;  $\theta^t$  be a trajectory for the coefficients up to  $t$ ,  $y_{t+1}^{t+k} = [y'_{t+1}, \dots, y'_{t+k}]'$  a collection of future observations and  $\theta_{t+1}^{t+k} = [\theta'_{t+1}, \dots, \theta'_{t+k}]'$  a collection of future trajectories for  $\theta_t$ . Let  $V_t = (\Sigma_t, \Omega_t)$ ; recall that  $\xi'_t = [e'_t, \eta'_t]$ . Let  $\xi_{j,t+1}^\delta$  be a realization of  $\xi_{j,t+1}$  of size  $\delta$  and let  $\mathcal{F}_t^1 = \{y^t, \theta^t, V_t, J_t, \xi_{j,t}^\delta, \xi_{-j,t}, \xi_{t+1}^{t+\tau}\}$  and  $\mathcal{F}_t^2 = \{y^t, \theta^t, V_t, J_t, \xi_t, \xi_{t+1}^{t+\tau}\}$  be two conditioning sets, where  $\xi_{-j,t}$  indicates all shocks, excluding the one in the  $j$ -th component. Then a response to  $\xi_{j,t}^\delta$ ,  $j = 1, \dots, 5$  is defined as:<sup>1</sup>

$$IR_y^j(t, k) = E(y_{t+k} | \mathcal{F}_t^1) - E(y_{t+k} | \mathcal{F}_t^2) \quad k = 1, 2, \dots \quad (8)$$

While (8) resembles the impulse response function suggested by Gallant et al. (1996), Koop et al. (1996) and Koop (1996), three important differences need to be noted. First, our responses are history dependent but state independent - histories are not random variables. Second, the size and the sign of shocks may, in principle, matter for the dynamics of the system. Third, since  $\theta_{t+1}$  is a random variable,  $IR_y^j(t, k)$  is also random variable.

When  $\xi_{j,t}^\delta = e_{j,t}^\delta$ , which is the case considered in the paper, responses are given by:

$$\begin{aligned} IR_y^j(t, 1) &= J_t^{-1,i} e_{j,t} \\ IR_y^j(t, k) &= \Psi_{t+k,k-1}^j e_{j,t} \quad k = 2, 3, \dots \end{aligned} \quad (9)$$

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<sup>1</sup>One could alternatively average out future shocks. Our definition is preferable for two reasons: it is easier to compute and produces numerically more stable distributions; it produces impulse responses which are similar to those generated by constant coefficient impulse responses when shocks to the measurement equations are considered. Note also that since future shocks are not averaged out, our impulse responses will display larger variability.

where  $\Psi_{t+k,k-1} = \mathcal{S}_{n,n}[(\prod_{h=0}^{k-1} \mathbf{A}_{t+k-h}) \times J_{t+k-(k-1)}]$ ,  $\mathbf{A}_t$  is the companion matrix of the VAR at time  $t$ ;  $\mathcal{S}_{n,n}$  is a selection matrix which extracts the first  $n \times n$  block of  $[(\prod_{h=0}^{k-1} \mathbf{A}_{t+k-h}) \times J_{t+k-(k-1)}]$  and  $\Psi_{t+k,k-1}^j$  is the column of  $\Psi_{t+k,k-1}$  corresponding to the  $j$ -th shock.

When the coefficients are constant,  $\prod_h \mathbf{A}_{t+k-h} = \mathbf{A}^k$  and  $\Psi_{t+k,k-1} = \mathcal{S}_{n,n}(\mathbf{A}^k \times J)$  for all  $k$ , so that (9) collapses to the traditional impulse response function to unitary structural shocks. In general,  $IR_y^j(t, k)$  depends on the identifying matrix  $J_t$ , the history of the data and the dynamics of the reduced form coefficients up to time  $t$ .

### 3 The identification restrictions

The restrictions we use to identify the VAR come from a general equilibrium model that encompasses flexible price RBC and New-Keynesian sticky price setups as special cases. The restrictions we consider are robust, in the sense that they are generated by a wide range of parametrizations, and uncontroversial, in the sense that they are shared by both the RBC and New-Keynesian versions of the model. We use a subset of the large number of model's predictions and, as in Canova (2002), we focus only on qualitative (sign) restrictions, as opposed to quantitative (magnitude) restrictions, to identify shocks. While it is relatively easy to find robust sign restrictions, magnitude restrictions are much more fragile and depend on the exact parametrization of the model.

The economy is the same as in Pappa (2004). It features a representative household, a continuum of firms, a monetary and a fiscal authority. The fiscal authority spends for both consumption and investment purposes. Government consumption may yield utility for the agents and government investment may alter the productive capacity of the economy.

#### 3.1 Households

Households derive utility from private,  $C_t^p$ , and public consumption,  $C_t^g$ , leisure,  $1 - N_t$  and real balances  $\frac{M_t}{p_t}$ . They maximize  $E_0 \sum_{t=0}^{\infty} \beta^t \frac{[(aC_t^p)^{\frac{\sigma-1}{\sigma}} + (1-a)C_t^g]^{\frac{\sigma}{\sigma-1}} (1-N_t)^{1-\theta_n}]^{1-\sigma} - 1}{1-\sigma} + \frac{1}{1-\vartheta_M} \left(\frac{M_t}{p_t}\right)^{1-\vartheta_M}$  choosing sequences for private consumption, hours, private capital to be used next period  $K_{t+1}^p$ , nominal state-contingent bonds,  $D_{t+1}$ , nominal balances and government bonds,  $B_{t+1}$ . Here  $0 < \beta < 1$  is the subjective discount factor and  $\sigma > 0$  a risk aversion parameter. Public consumption is exogenous from the point of view of households. The degree of substitutability between private and public consumption is regulated by  $0 < \varsigma \leq \infty$ . The parameter  $0 < a \leq 1$  controls the share of public and private goods in consumption: when  $a = 1$ , public consumption is useless from private agents' point of view.  $\vartheta_M > 0$  regulates the elasticity of money demand. Time is normalized to one at each  $t$ .



We assume  $C_t^p = \left[ \int_0^1 C_{it}^p(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}}$ ;  $C_t^g = \left[ \int_0^1 C_{it}^g(i)^{\frac{\lambda-1}{\lambda}} di \right]^{\frac{\lambda}{\lambda-1}}$  and  $\lambda > 0$  measures the elasticity of substitution between types of goods. The sequence of budget constraints is:

$$P_t(C_t^p + I_t^p) + E_t\{Q_{t,t+1}D_{t+1}\} + R_t^{-1}B_{t+1} + M_{t+1} \leq (1 - \tau^l)P_t w_t N_t + [r_t - \tau^k(r_t - \delta^p)]P_t K_t^p + D_t + B_t - T_t P_t + M_t + \Xi_t \quad (10)$$

where  $(1 - \tau^l)P_t w_t N_t$ , is the after tax nominal labor income,  $[r_t - \tau^k(r_t - \delta^p)]P_t K_t^p$  is the after tax nominal capital income (allowing for depreciation),  $\Xi_t$  are nominal profits distributed by firms (which are owned by consumers), and  $T_t P_t$  are lump-sum taxes. We assume complete private financial markets:  $D_{t+1}$  are holdings of state-contingent nominal bonds, paying one unit of currency in period  $t + 1$  if a specified state is realized, and  $Q_{t,t+1}$  is their period- $t$  price. Finally,  $R_t$  is the gross return on a one period government bond  $B_t$ . With the disposable income the household purchases consumption goods,  $P_t C_t^p$ , capital goods,  $P_t I_t^p$ , and assets. Private capital accumulates according to:

$$K_{t+1}^p = I_t^p + (1 - \delta^p)K_t^p - \nu \left( \frac{K_{t+1}^p}{K_t^p} \right) K_t^p \quad (11)$$

where  $0 < \delta^p < 1$  is a constant depreciation rate,  $\nu \left( \frac{K_{t+1}^p}{K_t^p} \right) = \frac{b}{2} \left[ \frac{K_{t+1}^p - (1 - \delta^p)K_t^p}{K_t^p} - \delta^p \right]^2$  and  $b \geq 0$  determines the size of the adjustment costs. Since households own and supply private capital to the firms, they bear the adjustment costs.

### 3.2 Firms

A firm  $j$  produces output according to the production function:

$$Y_{tj} = (Z_t N_{tj}^p)^{1-\alpha} (K_{tj}^p)^\alpha (K_t^g)^\mu \quad (12)$$

where  $K_{tj}^p$  and  $N_{tj}^p$  are private capital and labor inputs hired by firm  $j$ ,  $Z_t$  is an aggregate technology shock and  $K_t^g$  is the stock of public capital. The production function displays constant returns to scale with respect to private inputs. Government capital inputs is taken as given by the firm. The parameter  $\mu \geq 0$  regulates how public capital affects private production: when  $\mu$  is zero, government capital is unproductive.

We assume that firms are perfectly competitive in the input markets <sup>2</sup>: they minimize costs choosing private inputs and taking wages, the rental rate of capital, and government capital as given. Since firms are identical, they all choose the same amount of private inputs

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<sup>2</sup>The robust restrictions we emphasize below are independent of the presence of frictions in labor markets such as sticky wages or labor unions.

and cost minimization implies

$$\frac{K_{tj}^p}{N_{tj}^p} = \frac{\alpha}{(1-\alpha)} \frac{w_t}{r_t} \quad \forall j \quad (13)$$

Equation (13) and the production function imply that (nominal) marginal costs are:

$$MC_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} Z_t^{\alpha-1} K_t^{g(-\mu)} w_t^{1-\alpha} r_t^\alpha P_t \quad (14)$$

In the goods market firms are monopolistic competitors. The strategy firms use to set prices depends on whether they are sticky or flexible. In the former case we use the standard Calvo setting. That is, at each  $t$ , each domestic producer is allowed to reset her price with a constant probability,  $(1-\gamma)$ , independently of the time elapsed since the last adjustment. When a producer receives a signal to change her price, she chooses her new price,  $P_{tj}^*$ , to maximize  $\max_{P_{tj}^*} E_t \sum_{k=0}^{\infty} \gamma^k Q_{t+k+1,t+k} (P_{tj}^* - MC_{t+kj}) Y_{t+kj}$  subject to the demand curve  $Y_{t+kj} = \frac{P_{tj}^*}{P_{t+k}}^{-\lambda} Y_{t+k}$ . Optimization implies

$$\sum_{k=0}^{\infty} \gamma^k E_t \left\{ Q_{t+k+1,t+k} Y_{t+kj} \left( P_{tj}^* - \frac{\lambda}{\lambda-1} \frac{1}{1-\tau^\lambda} MC_{t+k} \right) \right\} = 0 \quad (15)$$

where  $\tau^\lambda = -(\lambda-1)^{-1}$  is a subsidy that, in equilibrium, eliminates the monopolistic competitive distortion. Given the pricing assumption, the aggregate price index is

$$P_t = [\gamma P_{t-1}^{1-\lambda} + (1-\gamma) P_t^{*1-\lambda}]^{\frac{1}{1-\lambda}} \quad (16)$$

When all firms can reset the price at each  $t$ , prices become flexible and:

$$P_t = \frac{\lambda}{\lambda-1} \frac{1}{1-\tau^\lambda} MC_t, \quad \forall t \quad (17)$$

### 3.3 Fiscal and Monetary Policy

Government's income consists of seigniorage, tax revenues minus subsidies to the firms and proceeds from new debt issue; expenditures consist of consumption and investment purchases and repayment of debt. Government budget constraint is:

$$P_t(C_t^g + I_t^g) + \tau^\lambda P_t Y_t - \tau^l w_t P_t N_t - \tau^k (r_t - \delta^g) P_t K_t^g - P_t T_t + B_t + M_t = R_t^{-1} B_{t+1} + M_{t+1} \quad (18)$$

where  $I_t^g$  is government's investments. Government capital stock evolves according to:

$$K_{t+1}^g = I_t^g + (1-\delta^g) K_t^g - \nu \left( \frac{K_{t+1}^g}{K_t^g} \right) K_t^g \quad (19)$$

where  $0 < \delta^g < 1$  is a constant and  $\nu(\cdot)$  is the same as for the private sector and  $I_t^g$  is stochastic. We treat tax rates on labor and capital income parametrically; assume that the government takes market prices, private hours and private capital as given, and that  $B_t$  endogenously adjusts to ensure that the budget constraint is satisfied. In order to guarantee a non-explosive solution for debt (see e.g., Leeper (1991)), we assume a tax rule of the form:

$$\frac{T_t}{T^{ss}} = \left[ \left( \frac{B_t}{Y_t} \right) / \left( \frac{B^{ss}}{Y^{ss}} \right) \right]^{\phi_b} \quad (20)$$

where the superscript  $ss$  indicates steady states. Finally, there is an independent monetary authority which sets the nominal interest rate according to the rule:

$$\frac{R_t}{R^{ss}} = \frac{\pi_t^{\phi_\pi}}{\pi^{ss}} u_t \quad (21)$$

where  $\pi_t$  is current inflation,  $u_t$  is a monetary policy shock. Given this rule, the authority stands ready to supply nominal balances that the private sector demands.

### 3.4 Closing the model

There are two types of aggregate constraints. First, labor supply must equate labor employed by the private firms. Second, aggregate production must equate the demand for goods from the private and public sector, that is  $Y_t = C_t^p + I_t^p + C_t^g + I_t^g$ .

We assume that the four exogenous processes  $S_t = [Z_t, C_t^g, I_t^g, u_t]'$ , evolve according to

$$\log(S_t) = (I_4 - \boldsymbol{\varrho}) \log(\bar{S}) + \boldsymbol{\varrho} \log(S_{t-1}) + V_t \quad (22)$$

where  $I_4$  is a  $4 \times 4$  identity matrix,  $\boldsymbol{\varrho}$  is a  $4 \times 4$  diagonal matrix with all the roots less than one in modulus,  $\bar{S}$  is the mean of  $S$  and the  $4 \times 1$  innovation vector  $V_t$  is a zero-mean, white noise process. Let  $\mathcal{A} = (\mathcal{A}^1, \mathcal{A}^2)$  represent the vector of parameters of the model.

Figure 1 presents impulse responses produced by the four shocks to the model when the parameters are allowed to vary within the ranges presented in table 1. To be precise, each box reports 68% of the 10000 paths generated randomly drawing  $\mathcal{A}_j$ ,  $j = 1, 2, \dots$  independently from a uniform distribution covering the range appearing in table 1. The first column represents responses to technology shocks, the second responses to government expenditure shocks, the third responses to government investment shocks and the fourth responses to monetary shocks. Since our VAR includes output, hours, inflation, nominal rate and money, figure 1 only plots the responses of these variables to the shocks.

Table 1: Parameter values or ranges

$\beta$	discount factor	0.99
$(B/Y)^{ss}$	steady state debt to output ratio	0.3
$\sigma$	risk aversion coefficient	[0.5,6.0]
$1 - a$	share of public goods in consumption	[0.0,0.15]
$\varsigma$	elasticity of substitution public/private goods	[0.5,3.0]
$\theta_n$	preference parameter	[0.1,0.9]
$b$	adjustment cost parameter	[0.1,10]
$\delta^p$	private capital depreciation rate	[0.013,0.05]
$\delta^g$	public capital depreciation rate	[0.010,0.03]
$\mu$	productivity of public capital	[0,0.05]
$\alpha$	capital share	[0.2,0.4]
$\tau^l$	average labor tax rate	[0,0.3]
$\tau^k$	average capital tax rate	[0,0.2]
$(C^g/Y)^{ss}$	steady state $C^g/Y$ ratio	[0.07,0.12]
$(I^g/Y)^{ss}$	steady state $I^g/Y$ ratio	[0.02,0.04]
$\phi_\pi$	Taylor's coefficient	[0.1,0.4]
$\phi_b$	coefficient on debt rule	[1.05, 2.25]
$\gamma$	degree of price stickiness	[0.0,0.85]
$\lambda$	elasticity of substitution between varieties	[7.0,8.0]
$\vartheta_M$	elasticity of money demand	[1.0,10]
$\rho_{C_g}$	persistence of $C_t^g$ shock	[0.6,0.9]
$\rho_{I_g}$	persistence of $I_t^g$ shock	[0.6,0.9]
$\rho_Z$	persistence of $Z_t$ shock	[0.8,0.95]
$\rho_u$	persistence of $u_t^R$ shock	[0.7,0.9]

Few words regarding the assumed ranges are in order. First, we decompose the parameter vector in two components:  $\mathcal{A}^1$  includes the parameters held fixed to a particular value because of steady state considerations, while in  $\mathcal{A}^2$  are the parameters which are allowed to vary. In  $\mathcal{A}^1$  we have the discount factor, set so that the annual real interest rate equals 4%, and the debt ratio,  $(\frac{B}{Y})^{SS}$ , which is selected to match the average US debt to GDP ratio.

The intervals for the other parameters are centered around standard values and the ranges are selected to contain existing estimates, values assumed in calibration exercises or chosen to satisfy theoretical considerations. For example, the range for the *risk aversion parameter*  $\sigma$  includes the values typically used in RBC ( $\sigma$  from 0.5 to 2), and New-Keynesian models ( $\sigma$  from 1 to 6). Theoretical considerations suggest that the share of public goods in total consumption,  $1 - a$ , should be low (since the private wealth effects following fiscal shocks crucially depend on this parameter) and the chosen range reflect this concern. The range for  $\varsigma$  allows for both complementarities and substitutabilities between private and public goods. The parameter  $\vartheta_n$  regulates the labor supply elasticity and the chosen range [0.1,10] covers well the range of existing estimates. The ranges for the *capital share* in

production,  $\alpha$ , the *capital adjustment costs parameter*,  $b$ , and the *depreciation of private capital*,  $\delta^P$  include standard values and we allow government capital to uniformly depreciate at a slower rate than private one.

The parameter  $\mu$  controls the interactions between public and private goods in production. Depending on its value, an increase in government capital has large or small effects on private output. The range  $[0, 0.05]$  covers both the case of unproductive and productive government capital. The ranges for labor and capital income tax parameters  $(\tau^l, \tau^k)$  cover the values of interest to policymakers and those for the fiscal ratios,  $(\frac{C^g}{Y}, \frac{I^g}{Y})$  match the cross sectional range of values found in US states. The range for the *degree of price stickiness*  $\gamma$  covers cases where prices are very sticky and cases where they are completely flexible.

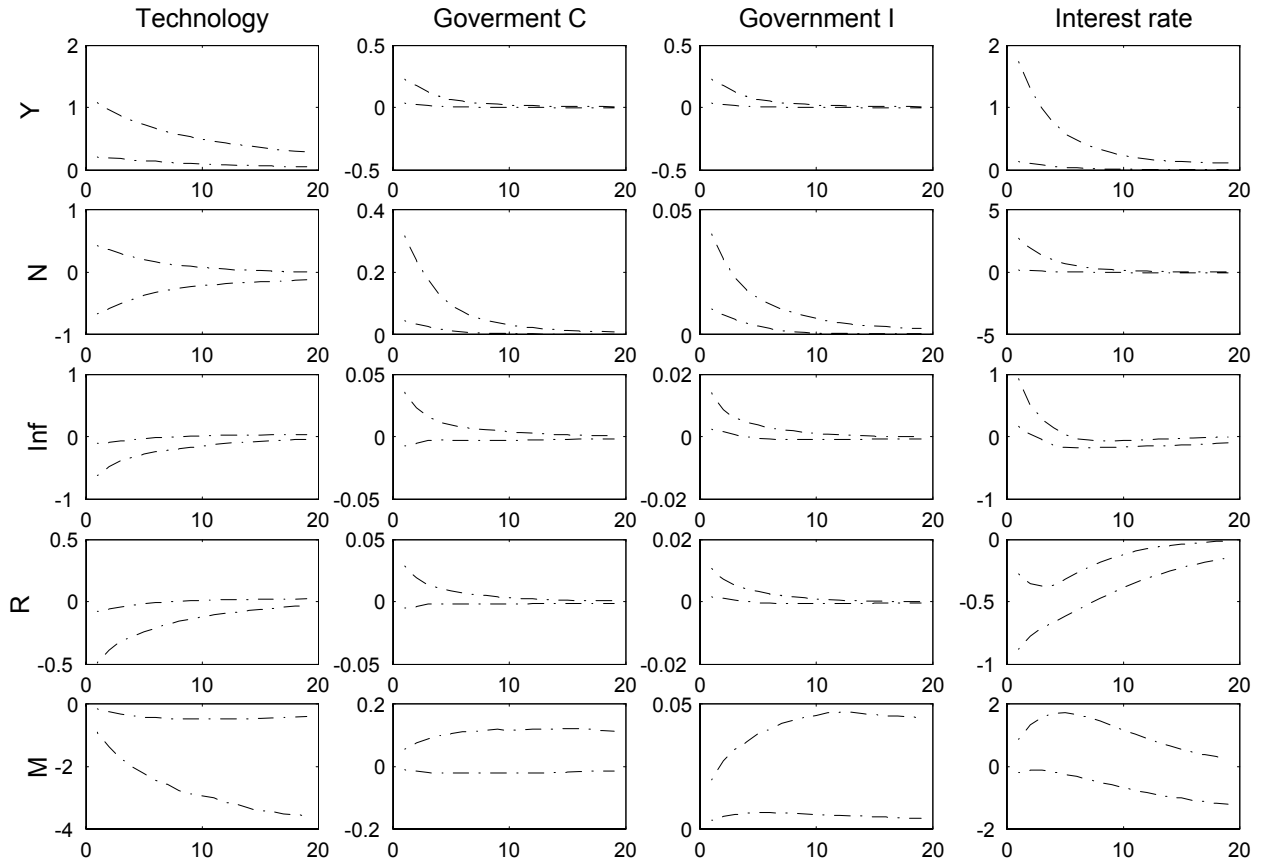


Figure 1: Responses to shocks in the model

Finally, the *coefficient on inflation*  $\phi_\pi$ , and the *coefficient on debt*  $\phi_b$  in the policy rules and the *persistence of the monetary shock*  $\rho_u$  control whether equilibria are determinate or not. The ranges we have selected imply that fiscal policy is active and monetary policy is passive, in the terminology of Leeper (1991); this insure determinacy of the equilibria and implies, among other things, that our analysis will neglect equilibria of the types considered

in Lubik and Schorfheide (2004). Therefore the interpretation of our monetary policy shocks is somewhat different from theirs.

The model produces several robust sign implications in responses to various shocks. For example, a persistent technological disturbance increases output, decreases inflation, nominal rates and nominal balances and the sign of the response is independent of the horizon. Note, instead, that the sign of the hours response is not robustly pinned down. This does not depend on the fact that we have allow prices to be flexible: the same pattern is obtained when the lower bound of the range for  $\gamma$  is increased to 0.35.

The model delivers robust implications also in response to the three demand shocks. When government consumption expenditure or government investment expenditure increase, output, hours, inflation, nominal interest rates and nominal balances all increase, while surprise decreases in the interest rate increase output, hours, inflation and nominal balances. Note, in particular, that these patterns obtain for a wide range of values of the elasticity of substitution between private and public goods, the share of capital in the production function, the strength of the reaction of interest rates and taxes to inflation and debt and the degree of price stickiness. In other words, except for monetary policy shocks, responses are independent of whether sticky or flexible prices or whether the RBC or the New-Keynesian versions of the model are considered.

Since the dynamics produced by government consumption and government investment shocks are qualitatively similar - the sign of dynamic responses of the five variables is the same for both shocks - we will identify a technology shock, a monetary shock and only one government shock, without attempting to distinguish between consumption or investment disturbances. The identification restrictions used at each  $t$  are summarized in table 2. Note that the dynamics of hours (and labor productivity) are unrestricted in all cases.

Table 2: Identification restrictions

	Output	Inflation	Interest rate	Money
Technology	$\geq 0$	$\leq 0$		
Government	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$
Monetary	$\geq 0$	$\geq 0$	$\leq 0$	$\geq 0$

There are many ways of implementing the restrictions presented in table 2. The results we present are obtained using an acceptance sampling scheme where draws that jointly satisfy the restrictions for all three shocks are kept and draws that do not are discarded. Tim Cogley pointed out to us that, since the bands presented in figure 1 do not insure that some parameter combinations may fail to satisfy the restrictions, an importance sampling scheme, which gives positive but different weights to different types of draws is, in principle, more

appropriate. Since there are several ways to implement an acceptance sampling scheme, we have tried a few alternatives. First, we have weighted draws in proportion to the number of horizons at which restrictions are satisfied. Thus, if we impose restrictions at three horizons, we give weight  $0.5/n_1$  to draws that satisfy restrictions at all horizons, weight  $0.33/n_2$  to draws that satisfy restrictions at two horizons, and weight  $0.17/n_3$  to draws that satisfy restrictions at one horizon,  $n_1 + n_2 + n_3 + n_4 = n$ , where  $n$  is the total number of draws. Second, we have weighted the draws satisfying all the restrictions by  $0.68/n_1$  and draws which do not satisfy all the restrictions by  $0.32/n_2$ ,  $n_1 + n_2 = n$ . The results we present are qualitatively independent of the scheme used to weight draws even though, quantitatively, some conclusions become more or less significant. An appendix available on request contains the results obtained with these alternatives.

Since the sign restrictions we use are robust to the horizon, we are free to choose how many responses to restrict. However, there is an important trade-off to be considered, since the smaller is the number of restrictions, the larger is the number of draws consistent with the restrictions but, potentially, the weaker is the link between the model and the empirical analysis. Hence, we could obtain more precise estimates of responses which may only be partially related to those of the model. As the number of restricted responses increases, we tight up the empirical analysis to the model more firmly. However, it may be the case that the number of draws satisfying the restrictions drops dramatically, making estimates of standard errors inaccurate. Since the relationship between number of restrictions and number of accepted draws is highly nonlinear, there is no straightforward way to optimize this trade-off. We present results obtained imposing restrictions at two horizons (0 and 1) since this choice seems to account for both concerns.

## 4 Estimation

The model (6)-(7) is estimated using Bayesian methods. We specify prior distributions for  $\theta_0$ ,  $\Sigma_0$ ,  $\Omega_0$ , and  $H_0$  and use data up to  $t$  to compute posterior estimates of the structural parameters and of continuous functions of them. Since our sample goes from 1960:1 to 2003:2, we initially estimate the model for the period 1960:1-1970:2 and then reestimate it 33 times moving the terminal date by one year up to 2003:2

Posterior distributions for the structural parameters are not available in a closed form. MCMC methods are used to simulate posterior sequences consistent with the information available up to time  $t$ . Estimation of reduced form TVC-VAR models with or without time variations in the variance of VAR shocks is now standard (see e.g. Cogley and Sargent

(2005)): it requires treating the parameters which are time varying as a block in a Gibbs sampler algorithm. Hence, at each  $t$  and in each Gibbs sampler cycle, one runs the Kalman filter/smoothing, conditional on the draw of the other time invariant parameters. In our setup the calculations are complicated by the fact that at each cycle, we need to obtain structural estimates of the time varying features of the model. This means that, in each cycle, we discard paths which are explosive and paths which do not satisfy the restrictions. Convergence was checked using a CUMSUM statistic. The results we present are based on 20,000 draws for each  $t$  - of these, after the non-explosive and the identification filters are used, about 200 are kept for inference. The methodology used to construct posterior distributions for the unknowns is contained, together with the prior specifications, in the appendix. The data comes from the FREDII data base of the Federal Reserve Bank of St. Louis and consists of GDP (GDPC1), GDP deflator inflation ( $\Delta$ GD PDEF), the Federal funds rate (FEDFUNDS), hours of all persons in the non-farm business sector (HOANBS) and M1 (M1SL). In parenthesis are the mnemonic used by FREDII.

## 5 The Results

### 5.1 The dynamics of volatility and persistence

We start our analysis presenting in figure 1 the evolution of the structural spectrum of output and inflation from 1970:1 to 2003:2 (first panel) and the 68% central posterior bands for structural persistence (second panel) and for structural volatility (third panel), for the same two variables. The former is measured by the height of the spectrum at frequency zero; the latter by the value of the cumulative spectrum. McConnell and Perez Quiroz (2001), Sargent and Cogley (2001), Stock and Watson (2003), Pivetta and Reis (2003) among others, have documented using reduced form, non-recursive and mostly univariate techniques, that output and inflation volatility and inflation persistence declined over time. Our analysis differs from theirs in the sense that it is multivariate, recursive, and explicitly structural.

Several interesting features emerge from figure 1. First, the spectrum of inflation is relatively stable over time, except for the zero frequency. Therefore, structural changes in inflation volatility are closely associated with changes in its persistence. The spectrum of output is also relatively stable over time at almost all frequencies. However, variations



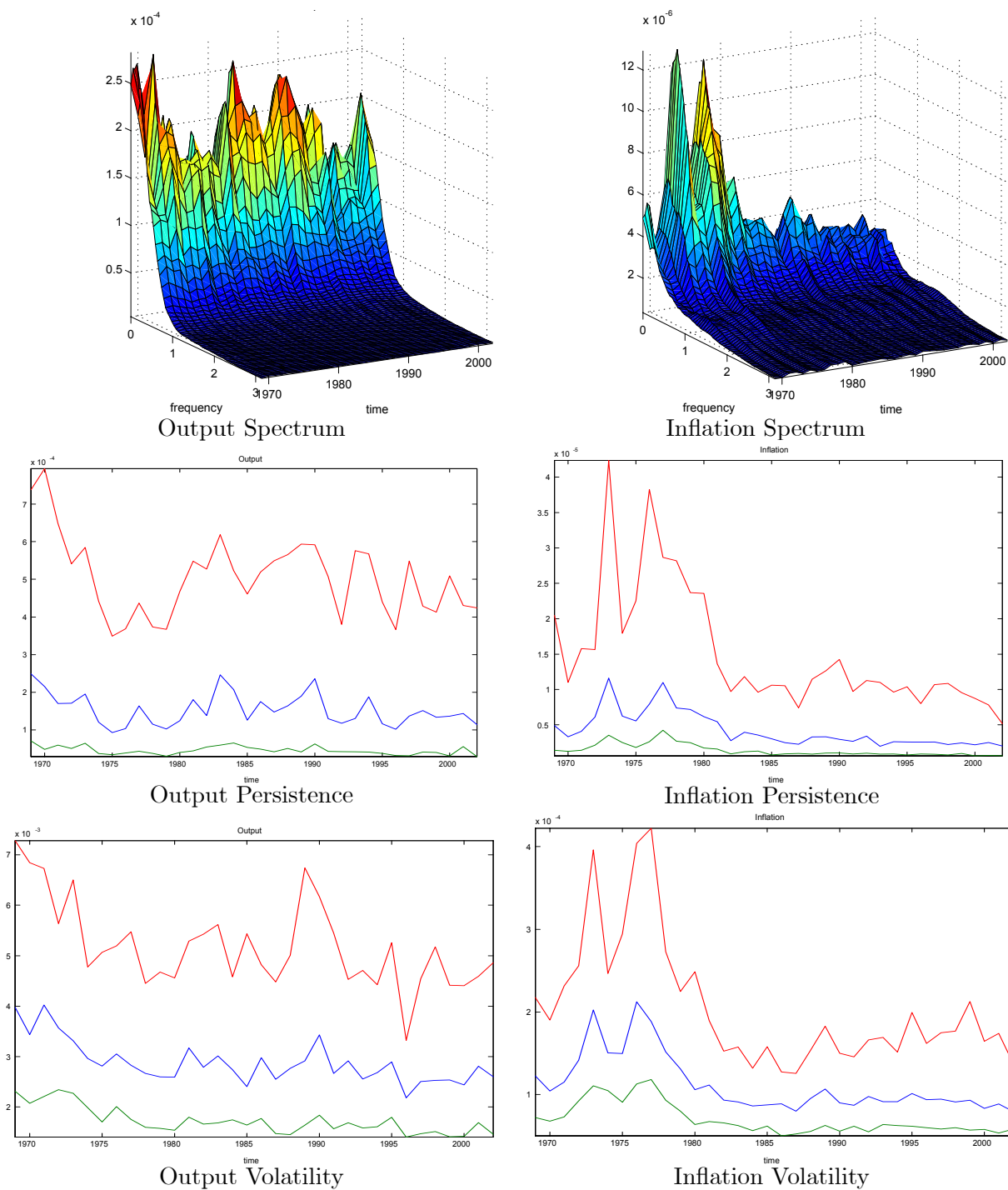


Figure 2: Structural Output and Inflation Dynamics

in structural volatility are not necessarily linked to changes in its persistence. In fact, most of the variations in the spectrum of output are located in the frequencies corresponding to three to five years cycles ( $\omega \in [0.314, 0.52]$ ). Second, inflation persistence shows a marked hump-shaped pattern: it displays a five fold increase in 1973-1974 and then again in 1977-1978, it drops dramatically after that date, and since 1982 the posterior distribution of inflation persistence displays marginal variations. Interestingly, while the mean of the posterior shows a clear declining trend since the mid-1970s - the drop in the mean of the posterior persistence from the peak is about 66 percent - the pattern is not statistically significant because of the large uncertainty associated with the mean increase in the 1970s. Third, variations over time in output persistence are relatively small and there is little posterior evidence that the difference between the mean estimate obtained at any two dates in the sample is significantly different from zero. Interestingly, there seems to be a negative correlation between the time path of the means of the posterior of inflation and of output persistence, and this is particularly evident in the mid-1970s. Fourth, as expected from previous discussion, the dynamics of the posterior 68% band of structural inflation volatility reflect those of structural inflation persistence. Fifth, although output volatility declines by roughly 25 percent from the beginning to the end of the sample, the change is statistically insignificant.

The standard error around the statistics we report are larger than those obtained by other authors. One relevant question is therefore which of feature of our approach is responsible for this outcome. We singled out three possibilities which appear to be relevant. First, it could be that some parameter draws are more consistent than others with the sign restrictions. If these draws imply larger volatility in the coefficients it could be that the estimated variance of the error in the law of motion of the coefficients could be larger for accepted than rejected draws. This turns out not to be the case: the two variances are statistically indistinguishable. Furthermore, similarly large bands obtain when a non-structural Choleski decomposition is used. Second, figure 2 is constructed using recursive analysis. Therefore our estimates are consistent with the information available at each  $t$  and contains less information than those of others which are produced using smoothed estimated of the parameters from the full sample. Although standard errors are reduced when smoothed estimates are considered, the pattern of changes is qualitative unaltered. Third, since our spectral estimates are constructed allowing future coefficients to be random, it could be the case that this source of uncertainty is responsible for the large standard errors we report. We have therefore repeated the analysis averaging out future shocks to the coefficients and found that standard errors are smaller by about 30 percent. Hence, recursive analysis and the methodology use to compute impulse responses appear to be responsible for the larger

standard error we produce.

In summary, three points can be made. First, while there is visual evidence of a decline in the point estimates of output and inflation volatility, the case for evolving volatility is considerably reduced once posterior standard errors are taken into account. This evidence should be contrasted with the one obtained with univariate, in-sample, reduced form methods, which overwhelmingly points to the presence of a significant structural break in the variability of the two series. Second, the case for evolving posterior distributions of persistence measures is far weaker. The posterior mean of inflation persistence shows a declining trend but posterior uncertainty is sufficiently large to make mean differences irrelevant while the posterior distribution of output persistence displays neither breaks nor evolving dynamics. Third, perhaps more importantly, the timing of the changes in persistences and volatilities do not appear to be synchronized. Hence, it is unlikely that we can account for the changes in output and inflation with a single and common explanation.

## 5.2 What drives changes in volatility and in persistence?

Recall that our structural model has implications for three types of disturbances, roughly speaking, supply, real demand and monetary shocks. Therefore, the model allows us to identify at most three of the five structural shocks driving the VAR. The share of the variability in output and inflation explained by these shocks can then be used to gauge the soundness of our analysis.

Given that the spectrum at frequency  $\omega$  is uncorrelated with the spectrum at frequency  $\omega'$ , where both  $\omega$  and  $\omega'$  are Fourier frequencies, it is easy to compute the relative contribution of each of the three structural shocks to changes in the volatility and in the persistence of output and inflation. In fact, the (time varying) structural MA representation of the system is  $y_{it} = \sum_{j=1}^5 \mathcal{B}_{jt}(\ell)e_{jt}$  where  $e_{it}$  is orthogonal to  $e_{i't}$ ,  $i' \neq i$ ,  $i = 1, \dots, 5$ . Since structural shocks are independent, the spectrum of  $y_{it}$  at frequency  $\omega$  can be written as  $S_{y_i}(\omega)(t) = \sum_{j=1}^5 |\mathcal{B}_{jt}(\omega)|^2 S_{e_j}(\omega)(t)$ . Therefore, the fraction of the persistence in  $y_{it}$  due to structural shock  $j$  is  $S_{y_i}^j(\omega = 0)(t) = \frac{|\mathcal{B}_{jt}(\omega=0)|^2 S_{e_j}(\omega=0)(t)}{S_{y_i}(\omega=0)(t)}$  and the fraction of the volatility in  $y_{it}$  due to structural shock  $j$  is  $\sum_{\omega} S_{y_i}^j(\omega)(t)$ . Intuitively, these measures are comparable to variance decomposition shares: while the latter tells us the relative contribution of different shocks at various forecasting horizons, these evaluate the contribution of structural shock  $j$  to the variability of  $y_{it}$  at either one frequency or for all frequencies.

There is considerable stability in the relative contribution of different shocks over time. That is, relatively speaking, sources of fluctuations in output and inflation have been quite similar over time. Interestingly, different shocks dominate at different frequencies. For

example, technology shocks exercise their largest impact on inflation variability at business cycle and high frequencies (mean contribution is about 28%) while their largest impact on output variability is at low frequencies (mean contribution is about 17%). The opposite occurs for the two demand shocks: they tend to explain the largest portion of inflation variability at low frequencies (roughly 20% for real demand shock and 17% for monetary shocks), while they have their largest explanatory power for output fluctuations at business cycle frequencies (roughly 25% for demand shocks and 17% for monetary shocks).

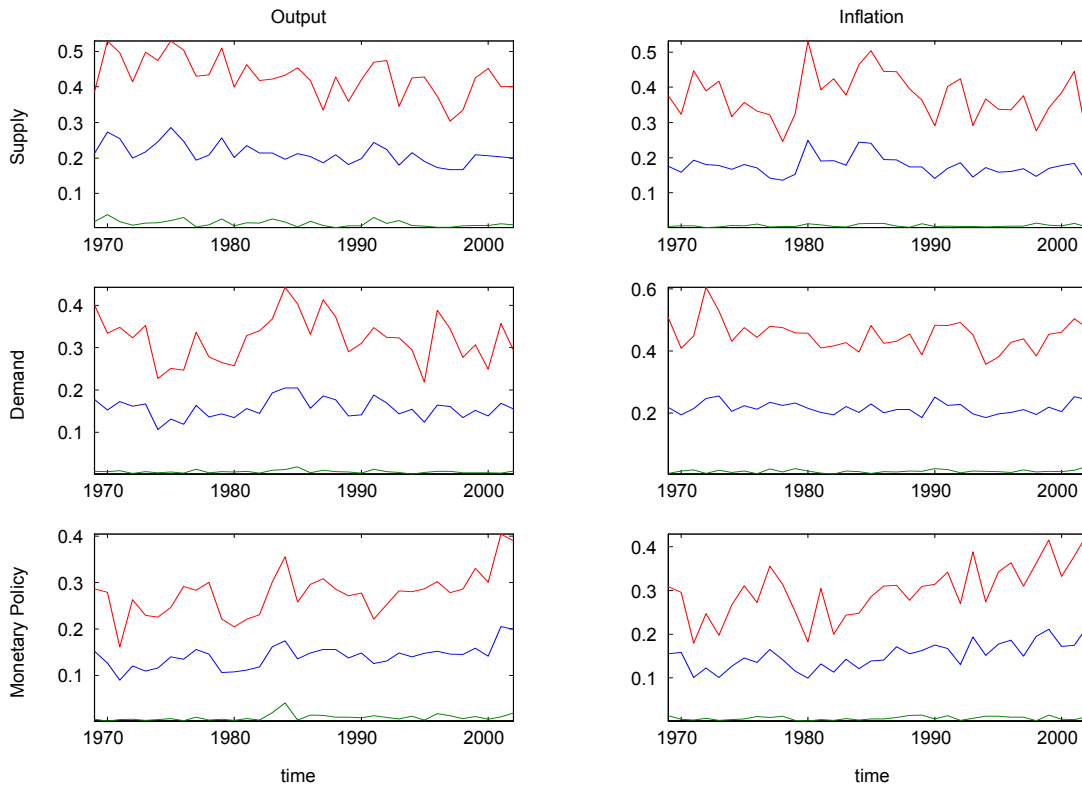


Figure 3: Persistence Shares

On average, over time and frequencies, technology shocks explain 25% of inflation variability and about 15% of output variability, demand shocks about 17% of inflation variability and 25% of output variability, and monetary shocks about 14% of inflation variability and 12% of output variability. We find it remarkable that our three structural shocks are able to explain between 50 and 65 percent of the variability of output and inflation, since we have not tried to identify labor supply or investment specific shocks, which Chang and Schorfheide (2004) and Fisher (2003) have shown to be important to explain output (and

potentially inflation) fluctuations at business cycle and medium run frequencies. On the other hand, and in line with recent evidence (see Gali (1999)), the contribution of technology shocks to output fluctuations is relatively low. Since our technology shocks are assumed to be stationary, this is perhaps not surprising. Moreover, our monetary shocks have little to do with the fluctuations of both variables. In particular, and contrary to the conventional wisdom, their contribution to the low frequencies variability of inflation is estimated to be low.

Figure 3, which reports the 68% posterior bands for the percentage of persistence and volatility of output and inflation due to technology, real demand and monetary shocks, indicates that the relative contribution of technology and demand shocks fluctuates around a constant mean value. On the other hand, the contribution of monetary shocks to both inflation and output persistence shows an increasing trend and, the end of the sample, the mean contribution is about 30 percent larger than at the beginning.

Several authors have attempted to interpret changes in inflation persistence in relationship to changes in the stance of monetary policy (see e.g. Cogley and Sargent (2001) or Benati (2002)). Figure 3 confirms that the relative contribution of monetary policy shocks is changing over time but also suggests an increasing rather than a decreasing share. Therefore, the contribution of some other unexplained sources of variations, different from the shocks we identify, could in part be responsible for the pattern of figure 2. Since the relative contribution of a shock varies because its variance changes or because its transmission mechanism changes, we will attempt to disentangle the two sources of variations in the next subsections.

The decomposition of the estimated posterior mean volatility presented in figure 4 also displays interesting features. Here, the relative contribution of demand and monetary policy shocks is relatively stable over time and their joint share is estimated to be around 35 percent. On the other hand, the mean contribution of technology shocks to output volatility declines and their contribution to inflation volatility shows first downward jump in the mid of the 1970s and then upward trend in the end of the 1970s.

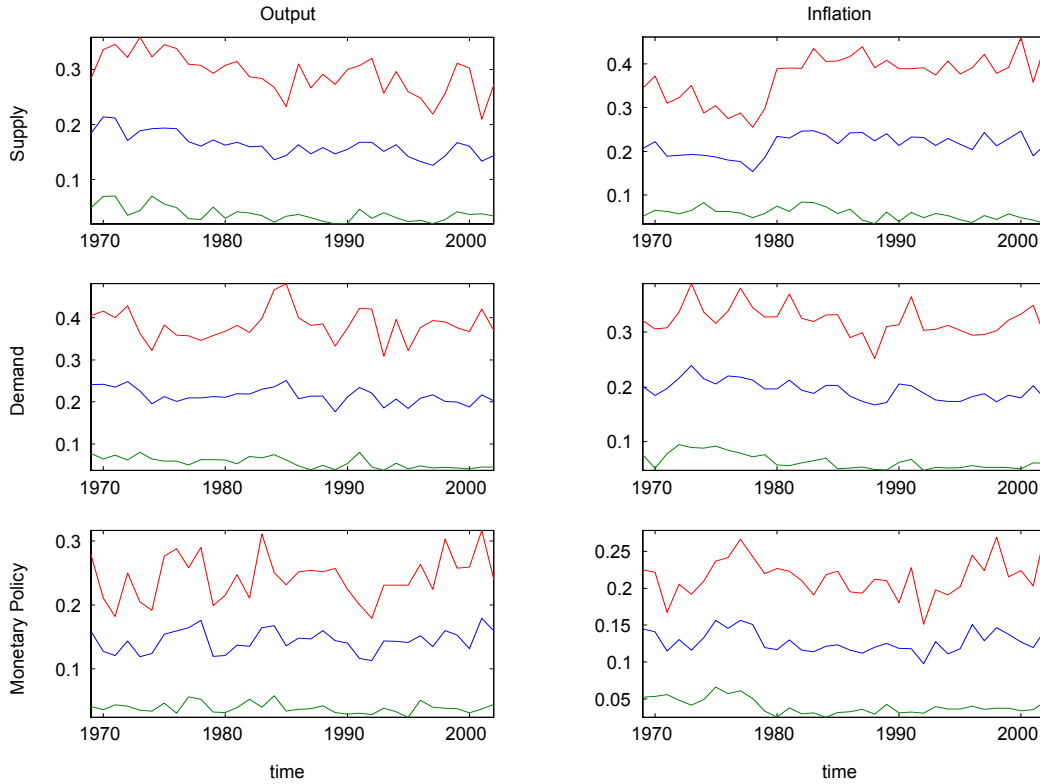
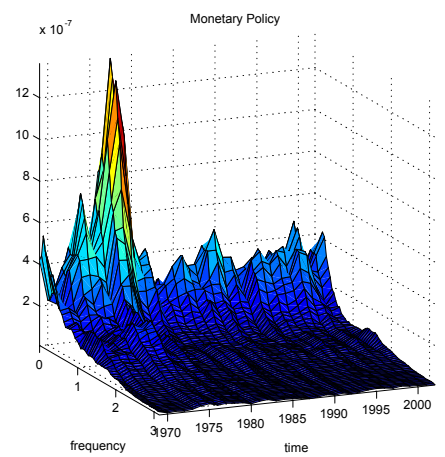
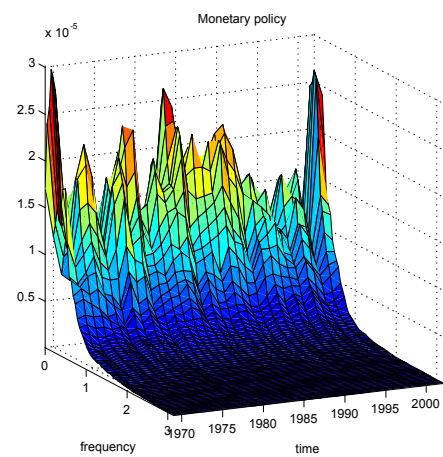
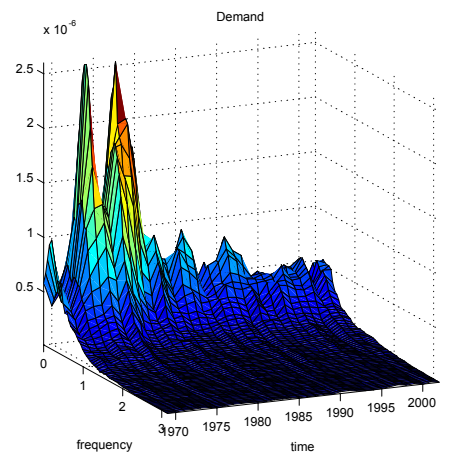
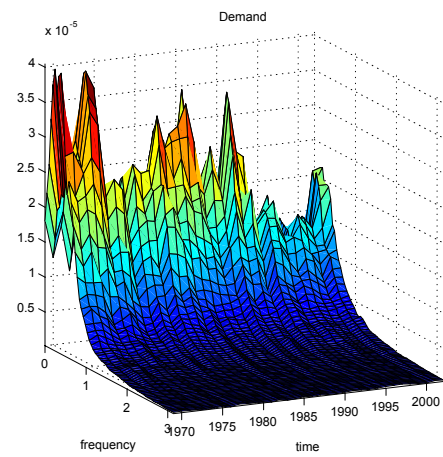
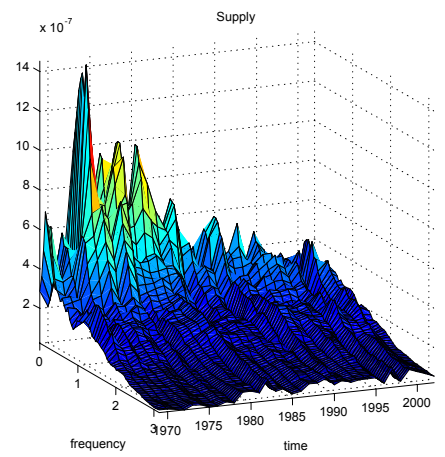
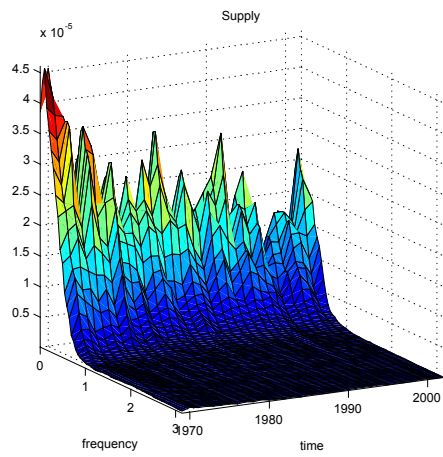


Figure 4: Volatility Shares

The pattern present in figures 3 and 4 suggest a few tentative explanations of the evidence. First, output has become less volatile over time because the relative contribution of technology shocks has declined most. Second, inflation persistence declines because the relative contribution of one of the non-identified shocks has decreased most. Interestingly, as figure 5 shows, the peak in inflation persistence in the mid-late 1970s is attributed by our identification scheme to demand shocks while the one in the beginning of the 1970s is attributed to technology disturbances. Moreover, magnitude changes in inflation persistence are linked to sudden declines in the importance of demand shocks, while the sluggishness in its changes is due to the much slower change in the contribution of technology shocks. Third, the decline in inflation variability at the beginning of the 1980s also occurs in conjunction with a sharp decline in the contribution of one of the non-identified shocks. However, technology shocks appear to account for the sluggish pattern observed after that.



Output
Inflation  
 Figure 5: Contribution of different shocks to spectra

### 5.3 Time Varying Transmission?

Variations over time in the absolute and the relative contribution of shocks to persistence and volatility measures can be generated by two separate mechanisms: changes over time in the transmission of shocks (captured by time variations in  $\mathcal{B}_{jt}(\omega)$ ) and changes over time in the distributions of the shocks (captured by  $S_{e_j}(\omega)(t)$ ). Our structural analysis allows us to separate the two sources of variations and therefore investigate whether structural changes in the economy or structural changes in the shocks are responsible for the observed variations. In Figure 6 we plot median responses of output and inflation to the three structural shocks. Since we normalize the impulse to be the same in every period, the evolution of these responses over time gives us an idea of the changes in the transmission of shocks in isolation from the changes in the posterior distribution of the shocks.

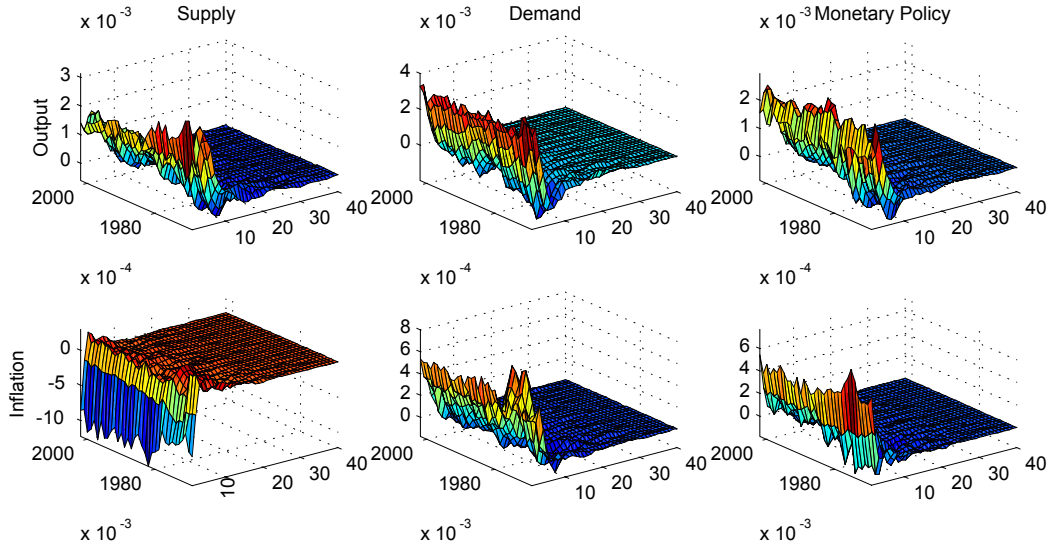


Figure 6: Output and Inflation responses

Few striking features of the figure are worth discussing. First, and qualitatively speaking, the pattern of responses to the three structural shocks is similar over time. Second, there are quantitative changes in the magnitude of some responses. For example, the peak response of output to technology shocks changes location and size. Similarly, the through response of inflation to demand shocks changes location over time. The most stable responses appear to be those to monetary shocks: the shape, the size and the location of output and inflation peak and through responses are very similar over time. Third, real demand shocks appear to



produce the largest displacements of the two variables followed by technology and monetary shocks. Fourth, and relatively speaking, the largest changes in the transmission appear to be associated with output responses to technology shocks. For example, the magnitude of contemporaneous responses is 50% larger in the 1990s than in the 1970s.

Hence, while the qualitative features of the transmission of technology, real demand and monetary shock are similar over time, changes in the quantitative features, involving the magnitude of the responses and, at times, the location of the peak/through are present. Also, while responses to monetary disturbances appear to be similar over time, the transmission of technology disturbances shows important instabilities.

#### 5.4 Time Varying volatility of the structural shocks?

To examine whether there have been significant changes in the distribution of the structural shocks hitting the economy, we plot the time profile of the estimated posterior mean of the volatility of the three structural shocks in figure 7. Real demand shocks are those associated with the first structural equation (normalized on output), supply shocks are those of associated with the second structural equation (normalized on inflation) and the monetary policy shocks are those associated with the third structural equation (normalized on the nominal interest rate).

Overall, it appears that the volatility of supply and of the monetary policy disturbances has declined over time. However, while the decline is smoother for the former, it is much more abrupt for the latter, where a drop of 15% in the late 1970s is evident. The volatility of demand shocks is higher on average than for the other two shocks and, except for late 1980s and the late 1990s, it is relatively similar across time. Interestingly, the decline in the volatility of technology and monetary policy shocks terminates by the early 1980s and since then no changes are detected.

The bottom graph of figure 7 suggests that the decline in volatility of monetary policy shocks occurred in the late 1970s. This decline appear to precede the one found by Sims and Zha (2004). However, differences can be reconciled if one takes into account different estimation techniques and the different ways in which these volatilities are computed (recursive vs. smoothed estimates). Several authors have argued that there is very little evidence that the monetary policy rule and the transmission of monetary policy shocks have changed over time. Instead, they have suggested that drops in the volatility of monetary policy shocks could be responsible for the fall in the variability of output and inflation. Our results are consistent with these view but also suggest that the contribution of technology shocks to the changes observed in the US economy is non-negligible. In fact, the sharp

increase and rapid decline in the variability of reduced form output and inflation forecast errors observed at the end of the 1970s is due, in part, to variations in the distribution from which technology shocks are drawn.

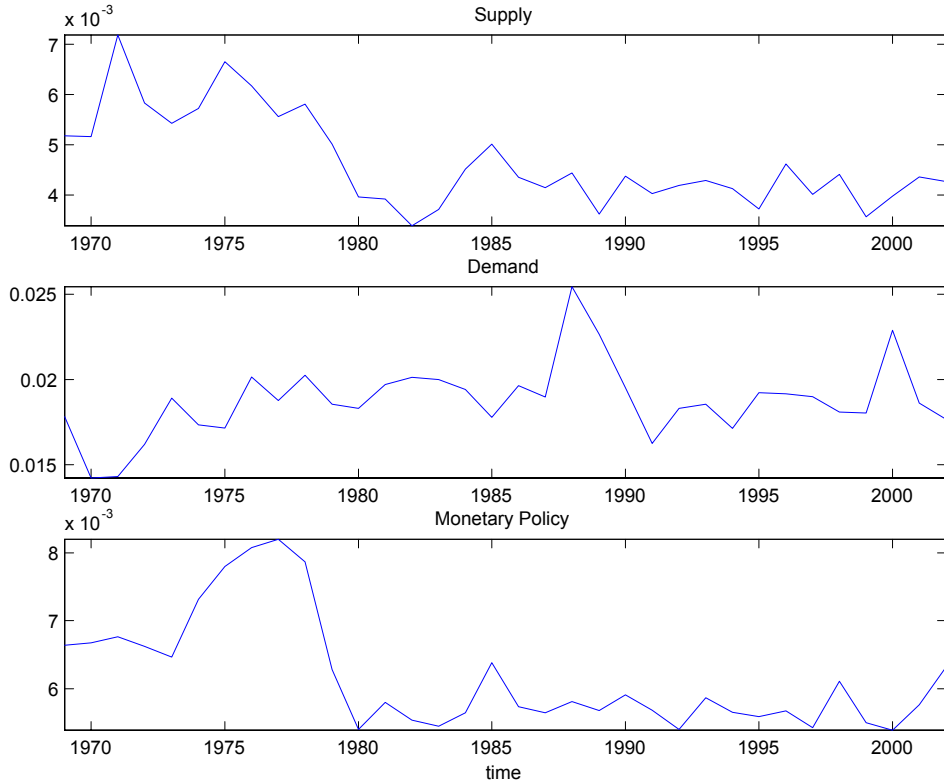


Figure 7: Structural shock variances

## 5.5 The dynamics of hours and labor productivity

Although this paper is primarily interested in studying the structural determinants of changes in output and inflation, our estimated system allows us to also briefly discuss a controversial issue which has been at the center of attention in the macroeconomic literature since work by Gali (1999), Christiano, et. al. (2003), Uhlig (2003), Dedola and Neri (2004) and others: the dynamics of hours and productivity in response to technology shocks. Although the empirical evidence is far from clear, it appears that under some identification and some data transformations (in particular, identification via long run restrictions and variables in the VAR in growth rates) technology disturbances increase labor productivity and decrease hours while with other identifications and other data transformations (in particular, hours quadratically detrended and identification based on short or medium run

restrictions) both labor productivity and hours increase.

The dynamics of hours and labor productivity are thought to provide important information about sources of business cycle dynamics. In fact, a negative response of hours to technology disturbances is considered by some to be inconsistent with RBC-flexible price based explanations of business cycles (a point disputed e.g. by Francis and Ramey (2002)). In a basic RBC model, in fact, technology shocks act as a supply shifter and therefore have positive effects on hours, output and productivity. On the other hand, in a basic sticky price model, technology shocks act as labor demand shifters. Therefore, firms experience a decline in their marginal costs but given that prices are sticky, aggregate demand increases less than proportionally than the increase in output so that hours decline. These patterns are partially present in the general model we have presented in section 3: when prices are flexible technology disturbances imply robust positive contemporaneous hours responses. When prices are sticky, the contemporaneous response of hours is mostly negative, but there are parameters configurations which produce positive hours responses.

Our estimated structural model allows us to investigate two interesting questions which can shed light on the issue. First, what are the dynamics of hours and labor productivity when sign restrictions derived from a general model are used to identify technology shocks? It is well known, at least since Faust and Leeper (1997), that long run restrictions are only weakly identifying and that the outcome depends on largely unverifiable assumptions about the time series properties of finite stretches of data. Model based robust restrictions can therefore offer a viable and more reliable alternative to identify technology shocks. Second, is there any evidence that the responses of hours to technology shocks displays a time varying pattern? In other words, could it be that the contemporaneous response of hours changes sign as the sample changes?

Figure 8 indicates that the contemporaneous response of hours and productivity to technology shocks is always positive. Interestingly, the response of hours is humped shaped, with the peak occurring after 2 or 3 quarters and this, combined with a smoothly declining output responses, implies that labor productivity becomes negative after some periods. Hence, the results we obtain are fully consistent with a RBC-flexible price explanation of the propagation of technology shocks. Furthermore, while there are quantitative variations in the responses of hours and productivity over time, the sign of the responses is the same at every date in the sample. Therefore, the mixed results found in the literature can not be due to time variations in the response of hours. Note that, consistent with both RBC and sticky price models, we find that hours positively comove with output in response to both demand shocks. However the magnitude of the changes is such that in response to demand

shocks productivity responds positively instantaneously but turns negative afterwards while in response to monetary policy shocks, productivity responses are instantaneously negative and the sign of the response changes with the horizon of the analysis.

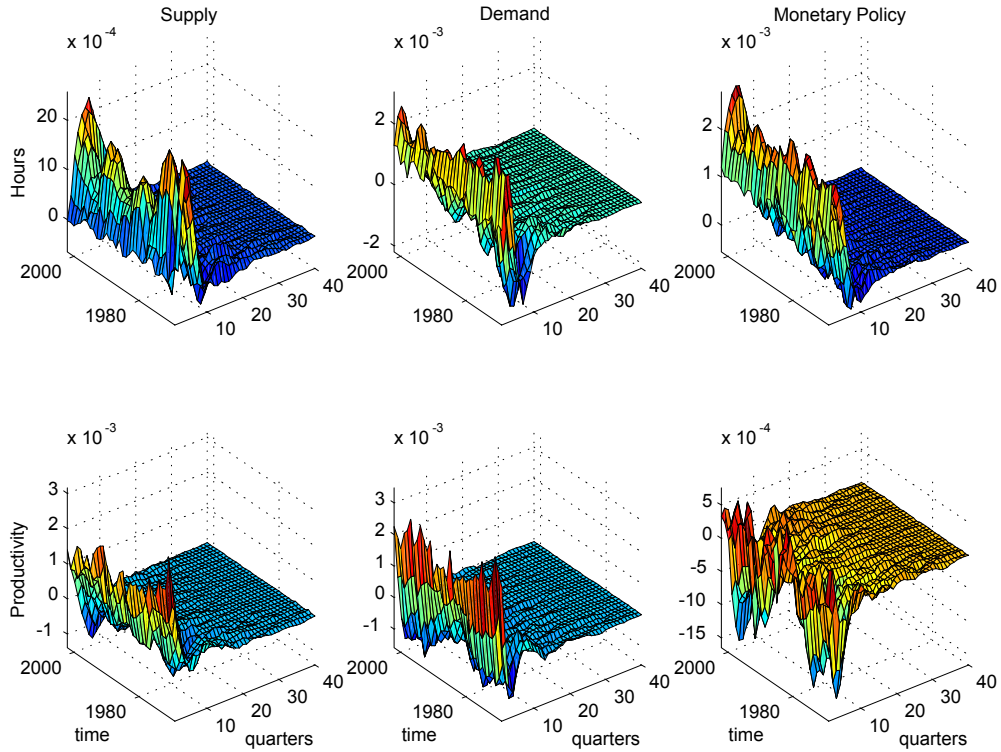


Figure 8: Hours and Productivity Responses

## 6 Conclusions

In this paper we examined structural sources of output and inflation volatility and persistence and attempted to draw some conclusions about the causes of the variations experienced in the US economy over the last 25-30 years. There has been a healthy discussion in the literature on this issue, thanks to the work of Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) (2003), Boivin and Giannoni, (2002), Leeper and Zha (2003), Sims and Zha (2004), Lubik and Schorfheide (2004), Primicieri (2004) and Canova and Gambetti (2004) among others, and although opinions differ, remarkable methodological improvements occurred trying to study questions having to do with time variations in structure of the economy and in the distributions of the shocks.

Here we contribute to advance the technical frontiers estimating a structural time vary-

ing coefficient VAR model; identifying a number of structural shocks using sign restrictions derived from a general DSGE model; providing recursive analysis, consistent with information available at each point in time; and using frequency domain tools to address time variation issues. In our opinion, the paper also contributes to advance our understanding of the cause of the observed variations in output and inflation. In particular, we show that while there are time variations in both the volatility of output and inflation and in the persistence of inflation, differences are statistically insignificant because of the large standard errors associated with posterior estimates at each  $t$ . Standard errors are larger than in other studies for two reasons: our recursive analysis makes them depend on the information available at each  $t$ ; shocks to future parameters are not averaged out.

We show that the output has become less volatile because the contribution of technology shocks has declined over time and that changes in the persistence and the volatility of inflation can be partially explained by changes in the contribution of technology, real demand and monetary policy shocks. Furthermore, we show that there are changes in the transmission of technology shocks and that the variance of both technology and monetary policy shocks has declined over time. We also provide novel evidence on the effects of technology shocks on labor market variables. In our estimated system, they robustly imply positive contemporaneous comovements of hours and labor productivity, even though the correlation between the two variables turns negative after a few lags.

All in all, our analysis indicates that variations in both the magnitude and the transmission of technology shocks are important to explain observed variations in US output. Therefore, our conclusions are consistent with those of McConnell and Perez Quiroz (2001) and Gordon (2003). Our analysis also indicates that both technology and monetary shocks are responsible for the changes in inflation variability and persistence. But while the magnitude and the transmission of technology shocks has changed over time, only changes in the magnitude of monetary policy shocks are evident. Therefore, our results agree with Sims and Zha (2004) and Gambetti and Canova (2004).

Few words of caution are important to put our results in the correct perspective. First, by construction, our analysis excludes the possibility that in one period of history the monetary policy rule produced indeterminate equilibria. Therefore our analysis is not necessarily inconsistent with the one of Lubik and Schorfheide (2004) even though it points out that we can account for a large portion of the observed variations without the need to resort to sunspot explanations. Second, while the fact that the volatility of the shocks has declined is consistent with exogenous explanations of the changes in the properties of output and inflation in the US, such a phenomena is also consistent with an explanations which give

policy actions an important role. For example, if monetary policy had a better control of inflation expectations over the last 20 years and no measure of inflation expectations is included in the VAR, such an effect may show up as a reduction of the variance of the shocks. Therefore, caution should be used to interpret our results one way or another.

Clearly, much work still needs to be done. We think it would be particularly useful to try to identify other structural shocks, for example, labor supply or investment specific shocks, and examine their relative contribution to changes in output and inflation volatility and persistence. It would also be interesting to study in details what are the technology shocks we have extracted, how do they correlate with what economists think are technological sources of disturbances and whether they proxy for missing variables or shocks. Finally, the model has implications for a number of variables. Enlarging the size of our VAR could provide additional evidence on the reasonableness of the structural disturbances we have extracted. Since until there is life, there is time to work, we leave these extensions for future research.

## Appendix

### Priors

We choose prior densities which gives us analytic expressions for the conditional posteriors of subvectors of the unknowns. Let  $T$  be the end of the estimation sample and let  $K_1$  be the number of periods for which the identifying restrictions must be satisfied. Let  $H_T = \rho(\varphi_T)$  be a rotation matrix whose columns represents orthogonal points in the hypersphere and let  $\varphi_T$  be a vector in  $R^6$  whose elements are  $U[0, 1]$  random variables. Let  $\mathcal{M}_T$  be the set of impulse response functions at time  $T$  satisfying the restrictions and let  $F(\mathcal{M}_T)$  be an indicator function which is one if the identifying restrictions are satisfied, that is, if  $(\Psi_{T+1,1}^i, \dots, \Psi_{T+K_1, K_1}^i) \in \mathcal{M}_T$ , and zero otherwise. Let the joint prior for  $\theta^{T+K_1}$ ,  $\Sigma_T$ ,  $\Omega_T$  and  $H_T$  be

$$p(\theta^{T+K_1}, \Sigma_T, \Omega_T, \omega_T) = p(\theta^{T+K} | \Sigma_T, \Omega_T) p(\Sigma_T, \Omega_T) F(\mathcal{M}_T) p(H_T) \quad (23)$$

Assume that  $p(\theta^{T+K} | \Sigma_T, \Omega_T) \propto I(\theta^{T+K}) f(\theta^{T+K} | \Sigma_T, \Omega_T)$  where  $f(\theta^{T+K} | \Sigma_T, \Omega_T) = f(\theta_0) \prod_{t=1}^{T+K} f(\theta_t | \theta_{t-1}, \Sigma_t, \Omega_t)$  and  $I(\theta^{T+K}) = \prod_{t=0}^{T+K} I(\theta_t)$ . Since  $f(\theta^{T+K} | \Sigma_T, \Omega_T)$ , is normal  $p(\theta^{T+K} | \Sigma, \Omega_T)$  is truncated normal.

We assume that  $\Sigma_0$  and  $\Omega_0$  have independent inverse Wishart distributions with scale matrices  $\Sigma_0^{-1}$ ,  $\Omega_0^{-1}$  and degrees of freedom  $\nu_{01}$  and  $\nu_{02}$ , and assume that  $\Sigma_t = \alpha_1 \Sigma_{t-1} + \alpha_2 \Sigma_0$  and  $\Omega_t = \alpha_3 \Omega_{t-1} + \alpha_4 \Omega_0$ ,  $\forall t$ , where  $\alpha_i, i = 1, 2, 3, 4$  are fixed. We also assume that the prior for  $\theta_0$  is truncated Gaussian independent of  $\Sigma_T$  and  $\Omega_T$ , i.e.  $f(\theta_0) \propto I(\theta_0) N(\bar{\theta}, \bar{P})$ . Finally we assume a uniform prior  $p(H_T)$ , since all rotation matrices are a-priori equally likely. Collecting pieces, the joint prior is:

$$p(\theta^{T+K_1}, \Sigma_T, \Omega_T, \omega_T) \propto I(\theta^{T+K}) F(\mathcal{M}_T) [f(\theta_0) \prod_{t=1}^{T+K} f(\theta_t | \theta_{t-1}, \Sigma_t, \Omega_t)] p(\Sigma_t) p(\Omega_t) \quad (24)$$

Note that when  $H_t = I_n$ , the prior reduces to

$$p(\theta^{T+K_1}, \Sigma_T, \Omega_T, H_T) = I(\theta^{T+K}) [f(\theta_0) \prod_{t=1}^{T+K} f(\theta_t | \theta_{t-1}, \Sigma_t, \Omega_t)] p(\Sigma_t) p(\Omega_t) \quad (25)$$

We "calibrate" prior parameters by estimating a fixed coefficients VAR using data from 1960:1 up to 1969:1. We set  $\bar{\theta}$  equal to the point estimates of the coefficients and  $\bar{P}$  to the estimated covariance matrix.  $\Sigma_0$  is equal to the estimated covariance matrix of VAR innovations,  $\Omega_0 = \rho \bar{P}$  and  $\nu_{10} = \nu_{20} = 4$  (so as to make the prior close to non-informative). After some experimentation we select  $\alpha_2 = \alpha_2 = 0, \alpha_2 = \alpha_4 = 1$ . The parameter  $\rho$  measures how much the time variation is allowed in coefficients. Although as  $T$  grows the likelihood

dominates, the choice of  $\varrho$  matters in finite samples. We choose  $\varrho$  as a function of  $T$ . i.e. for the sample 1969:1-1981:2,  $\varrho = 0.0025$ ; for 1969:1-1983:2,  $\varrho = 0.003$ ; for 1969:1-1987:2,  $\varrho = 0.0035$ ; for 1969:1-1989:2,  $\varrho = 0.004$ ; for 1969:1-1995:4,  $\varrho = 0.007$ ; for 1969:1-1999:1,  $\varrho = 0.008$ , and for 1969:1-2003:2,  $\varrho = 0.01$ . This range of values implies a quiet conservative prior coefficient variations: in fact, time variation accounts between 0.35 and a 1 percent of the total coefficients standard deviation.

Since impulse response functions depend on  $\Phi_{T+k,k}$ ,  $S$  and  $H_T$ , we first characterize the posterior of  $\theta^{T+K}$ ,  $\Sigma_T$ ,  $\Omega_T$ , which are used to construct  $\Phi_{T+k,k}$  and  $S$ , and then describe an approach to sample from them.

## Posteriors

To draw posterior sequences we need  $p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T | y^T)$ , which is analytically intractable. However, we can decompose it into simpler tractable conditional components.

First, note that

$$\begin{aligned} p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T | y^T) &\equiv p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T | y^T) \\ &\propto p(y^T | H_T, \theta^{T+K}, \Sigma_T, \Omega_T) p(H_T, \theta^{T+K}, \Sigma, \Omega_T) \end{aligned} \quad (26)$$

Second, since the likelihood is invariant to any orthogonal rotation  $p(y^T | H_T, \theta^{T+K}, \Sigma_T, \Omega_T) = p(y^T | \theta^{T+K}, \Sigma_T, \Omega_T)$ . Third,  $p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T) = p(\theta^{T+K}, \Sigma_T, \Omega_T) F(\mathcal{M}_T) p(H_T)$ . Thus

$$p(H_T, \theta^{T+K}, \Sigma_T, \Omega_T | y^T) \propto p(\theta^{T+K}, \Sigma_T, \Omega_T | y^T) F(\mathcal{M}_T) p(H_T) \quad (27)$$

where  $p(\theta^{T+K}, \Sigma_T, \Omega_T | y^T)$  is the posterior distribution for the reduced form parameters, which, in turn can be factored as

$$p(\theta^{T+K}, \Sigma_T, \Omega_T | y^T) = p(\theta_{T+1}^{T+K} | y^T, \theta^T, \Sigma_T, \Omega_T) p(\theta^T, \Sigma_T, \Omega_T | y^T) \quad (28)$$

The first term on the right hand side of (29) represents beliefs about the future and the second term the posterior density for states and hyperparameters. Note that  $p(\theta_{T+1}^{T+K} | y^T, \theta^T, \Sigma_T, \Omega_T) = p(\theta_{T+1}^{T+K} | \theta^T, \Sigma_T, \Omega_T) = \prod_{k=1}^K p(\theta_{T+k} | \theta_{T+k-1}, \Sigma_T, \Omega_T)$  because the states are Markov. Finally, since  $\theta_{T+k}$  is conditionally truncated normal with mean  $\theta_{T+k-1}$  and variance  $\Omega_T$ ,

$$\begin{aligned} p(\theta_{T+1}^{T+K} | \theta^T, \Sigma_T, \Omega_T) &= I(\theta_{T+1}^{T+K}) \prod_{k=1}^K f(\theta_{T+k} | \theta_{T+k-1}, \Sigma_T, \Omega_T) \\ &= I(\theta_{T+1}^{T+K}) f(\theta_{T+1}^{T+K} | \theta^T, \Sigma_T, \Omega_T) \end{aligned} \quad (29)$$

The second term in (29) can be factored as

$$p(\theta^T, \Sigma_T, \Omega_T | y^T) \propto p(y^T | \theta^T, \Sigma_T, \Omega_T) p(\theta^T, \Sigma_T, \Omega_T) \quad (30)$$



The first term in (31) is the likelihood function which, given the states, has a Gaussian shape so that  $p(y^T|\theta^T, \Sigma_T, \Omega_T) = f(y^T|\theta^T, \Sigma_T, \Omega_T)$ . The second term is the joint posterior for states and hyperparameters. Hence:

$$p(\theta^T, \Sigma_T, \Omega_T|y^T) \propto f(y^T|\theta^T, \Sigma_T, \Omega_T)p(\theta^T|\Sigma_T, \Omega_T)p(\Sigma_T, \Omega_T) \quad (31)$$

Furthermore, since  $p(\theta^T|\Sigma_T, \Omega_T) \propto I(\theta^T)f(\theta^T|\Sigma_T, \Omega_T)$  where  $f(\theta^T|\Sigma_T, \Omega_T) = f(\theta_0|\Sigma_T, \Omega_0) \prod_{t=1}^T f(\theta_t|\theta_{t-1}, \Sigma_t, \Omega_t)$  and  $I(\theta^T) = \prod_{t=0}^T I(\theta_t)$ , we have

$$p(\theta^T, \Sigma, \Omega_T|y^T) \propto I(\theta^T)f(y^T|\theta^T, \Sigma_T, \Omega_T)f(\theta^T|\Sigma_T, \Omega_T)p(\Sigma_T, \Omega_T) = I(\theta^T)p_u(\theta^T, \Sigma_T, \Omega_T|y^T) \quad (32)$$

where  $p_u(\theta_T, \Sigma_T, \Omega_T|y^T) \equiv f(y^T|\theta^T, \Sigma_T, \Omega_T)f(\theta^T|\Sigma_T, \Omega_T)p(\Sigma_T, \Omega_T)$  is the posterior density obtained if no restrictions are imposed. Collecting pieces we finally have

$$p(H_T, \theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T|y^T) \propto \left[ \prod_{t=0}^T I(\theta_t)f(\theta_{T+1}^{T+K}|\theta^T, \Sigma_T, \Omega_T)I(\theta^T)p_u(\theta^T, \Sigma_T, \Omega_T|y^T) \right] F(\mathcal{M}_T)p(H_T) \quad (33)$$

Note that for  $H_t = I$ ,  $p(\theta_{T+1}^{T+K}, \theta^T, \Sigma_T, \Omega_T|y^T) = \prod_{t=0}^T I(\theta_t)f(\theta_{T+1}^{T+K}|\theta^T, \Sigma_T, \Omega_T)p_u(\theta^T, \Sigma_T, \Omega_T|y^T)$ .

## Drawing structural parameters

Given (34) draws for the structural parameters can be obtained as follows

1. Draw  $(\theta^T, \Sigma_T, \Omega_T)$  from the unrestricted posterior  $p_u(\theta^T, \sigma_T, \Omega_T|y^T)$  via the Gibbs sampler (see below). Apply the filter  $I(\theta^T)$ .
2. Given  $(\theta^T, \Sigma_T, \Omega_T)$ , draw future states  $\theta_{T+1}^{T+K}$ , i.e. obtain draws of  $u_{T+k}$  from  $N(0, \Omega_T)$  and iterate in  $\theta_{T+k} = \theta_{T+k-1} + u_{T+k}$ ,  $K$  times. Apply the filter  $I(\theta^{T+K})$ .
3. Draw  $\varphi_{i,T}$  for  $i = 1, \dots, 6$  from a  $U[0, 1]$ . Draw  $H_T = \rho(\varphi_T)$ .
4. Given  $\Sigma$ , find the matrix  $S_T$ , such that  $\Sigma_T = S_T S_T'$ . Construct  $J_T^{-1}$ .
5. Compute  $(\Psi_{T+1,1}^{i,\ell}, \dots, \Psi_{T+K,K}^{i,\ell})$  for each replication  $\ell$ . Apply the filter  $F(\mathcal{M}_T)^\ell$  and keep the draw if the identification restrictions are satisfied.

## Drawing reduced form parameters

The Gibbs sampler we use to compute the posterior for the reduced form parameters iterate on two steps. The implementation is identical to Cogley and Sargent (2001).

- Step 1: States given hyperparameters

Conditional on  $(y^T, \Sigma_T, \Omega_T)$ , the unrestricted posterior of the states is normal and  $p_u(\theta^T|y^T, \Sigma_T, \Omega_T) = f(\theta_T|y^T, \Sigma_T, \Omega_T) \prod_{t=1}^{T-1} f(\theta_t|\theta_{t+1}, y^t, \Sigma_t, \Omega_t)$ . All densities on the right end side are Gaussian they their conditional means and variances can be computed using the Kalman smoother. Let  $\theta_{t|t} \equiv E(\theta_t|y^t, \Sigma_t, \Omega_t)$ ;  $P_{t|t-1} \equiv Var(\theta_t|y^{t-1}, \Sigma_t, \Omega_t)$ ;  $P_{t|t} \equiv Var(\theta_t|y^t, \Sigma_t, \Omega_t)$ . Given  $P_{0|0}$ ,  $\theta_{0|0}$ ,  $\Omega_0$  and  $\Sigma_0$ , we compute Kalman filter recursions

$$\begin{aligned}
P_{t|t-1} &= P_{t-1|t-1} + \Sigma_t \\
\mathcal{K}_t &= (P_{t|t-1}X_t)(X_t'P_{t|t-1}X_t + \Omega_t)^{-1} \\
\theta_{t|t} &= \theta_{t-1|t-1} + \mathcal{K}_t(y_t - X_t'\theta_{t-1|t-1}) \\
P_{t|t} &= P_{t|t-1} - \mathcal{K}_t(X_t'P_{t|t-1})
\end{aligned} \tag{34}$$

The last iteration gives  $\theta_{T|T}$  and  $P_{T|T}$  which are the conditional means and variance of  $f(\theta_t|y^T, \Sigma, \Omega_T)$ . Hence  $f(\theta_T|y^T, \Sigma, \Omega_T) = N(\theta_{T|T}, P_{T|T})$ .

- Step 2: Hyperparameters given states

Conditional on the states and the data  $\varepsilon_t$  and  $u_t$  are observable and Gaussian. Combining a Gaussian likelihood with an inverse-Wishart prior results in an inverse-Wishart posterior, so that  $p(\Sigma_t|\theta^T, y^T) = IW(\Sigma_{1t}^{-1}, \nu_{11})$ ;  $p(\Omega_t|\theta^T, y^T) = IW(\Omega_{1t}^{-1}, \nu_{12})$  where  $\Sigma_{1t} = \Sigma_0 + \Sigma_T$ ,  $\Omega_{1t} = \Omega_0 + \Omega_T$ ,  $\nu_{11} = \nu_{01} + T$ ,  $\nu_{12} = \nu_{02} + T$  and  $\Sigma_T$  and  $\Omega_T$  are proportional to the covariance estimator  $\frac{1}{T}\Sigma_T = \frac{1}{T}\sum_{t=1}^T \varepsilon_t \varepsilon_t'$ ;  $\frac{1}{T}\Omega_T = \frac{1}{T}\sum_{t=1}^T u_t u_t'$ . Under regularity conditions and after a burn-in period, iterations on these two steps produce draw from  $p_u(\theta^T, \Sigma, \Omega|y^T)$ .

In our exercises  $T$  varies from 1970:2 to 2003:2. For each of these  $T$ , 20000 iterations of the Gibbs sampler are made. CUMSUM graphs are used to check for convergence and we found that the chain had converged roughly after 2000 draws for each date in the sample. The densities for the parameters obtained with the remaining draws are well behaved and none is multimodal. We keeping one every four of the remaining 8000 draws and discard all the draws generating explosive paths. The autocorrelation function of the 2000 draws which are left is somewhat persistent but this is not a problem since only about 10% of these draws satisfy the identification restrictions in each sample.

## Computing structural impulse responses and spectra

Given a draw from the posterior of the structural parameters, calculation of impulse responses to VAR shocks is straightforward. In fact, given a draw for  $(\theta^{T+K}, \Sigma, \Omega_T, H_{T+1})$  we calculate  $\Psi_{T+k,k}$ , compute the posterior median and the 68% central credible set at each horizon  $k$  across draws. then, spectra are computed as described in section 5.2.

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