"The gain-loss asymmetry and single-self preferences"

by Antoni Bosch-Domènech and Joaquim Silvestre

Abstract

Kahneman and Tversky asserted a fundamental asymmetry between gains and losses, namely a "reflection effect" which occurs when an individual prefers a sure gain of \$ pz to an uncertain gain of \$ z with probability p, while preferring an uncertain loss of \$$z$ with probability p to a certain loss of \$ pz.

We focus on this class of choices (actuarially fair), and explore the extent to which the reflection effect, understood as occurring at a range of wealth levels, is compatible with single-self preferences.

We decompose the reflection effect into two components, a "probability switch" effect, which is compatible with single-self preferences, and a "translation effect," which is not. To argue the first point, we analyze two classes of single-self, nonexpected utility preferences, which we label "homothetic" and "weakly homothetic." In both cases, we characterize the switch effect as well as the dependence of risk attitudes on wealth.

We also discuss two types of utility functions of a form reminiscent of expected utility but with distorted probabilities. Type I always distorts the probability of the worst outcome downwards, yielding attraction to small risks for all probabilities. Type II distorts low probabilities upwards, and high probabilities downwards, implying risk aversion when the probability of the worst outcome is low. By combining homothetic or weak homothetic preferences with Type I or Type II distortion functions, we present four explicit examples: All four display a switch effect and, hence, a form of reflection effect consistent a single self preferences.

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1. Introduction

Many decisions involve risk. A basic issue is risk attitude: when do people display risk aversion or risk attraction? Daniel Kahneman and Amos Tversky argued that decisions under risk display a fundamental asymmetry between gains and losses: many people prefer a sure gain of $ *pz* to an uncertain gain of $ *z* with probability *p*, while preferring an uncertain loss of $*z* with probability *p* to a certain loss of $ *pz*: they labeled this phenomenon the *reflection effect*. It is generally accepted that gain-loss asymmetries are incompatible with the canonical expected utility model, where the individual maximizes the expectation of final wealth. But, are they compatible with single-self preferences of the non-expected utility variety, or do they necessarily have to appeal to multiple selves, with intersecting families of indifference curves, one for each reference point? This is a more fundamental question than the one concerning expected utility.[1]

If the individual has consistent, single-self preferences, which do not vary with the circumstances in which she makes decisions, then policy recommendations can be based in a nonpaternalistic way on the premise that an individual is the ultimate judge of her welfare. But if her preferences depend on circumstances, then external criteria are needed to evaluate the individual's welfare across circumstance-consumption pairs.

Several regularities have appeared in our experimental work (Bosch-Domènech and Silvestre, 1999, 2004, in press), which has focused on actuarially fair choices, with objective probabilities, between certain and uncertain alternatives involving money.

(a) We systematically find what we call an *amount effect*, i.e., both for choices involving gains and for choices involving losses, people tend to display risk attraction when the amounts at play are small, and risk aversion when they are large.

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[1] See Mark Machina (1982).

(b) We often observe what we call a *probability-switch effect*: increasing the probability of the bad outcome (i. e., the probability of the loss in choices involving losses, or the probability of gaining nothing in choices involving gains) tends to increase the frequency of risk attraction.

(c) We also observe what we call a *translation effect*: moving from gains to losses without changing the probability of the bad outcome tends to increase the frequency of risk attraction.

(d) Finally, when comparing the attitudes of groups with different wealth levels, we observe more frequent risk attraction among the wealthier for choices involving small and moderate amounts of money.

The translation effect turns out to be incompatible with single-self preferences. But Section 5 below develops a family of single-self, nonexpected-utility, *ex ante* preferences that display a switch effect, with various forms of dependence of the willingness to assume fair risks on the person's wealth and on the amount of money at stake. Because Kahneman and Tversky's reflection effect can be decomposed into a translation effect and a switch effect, it follows from our analysis that some forms of reflection effect, namely those that can be totally attributed to a switch effect, are compatible with single-self preferences. Reflections due to a translation effect, on the contrary, are incompatible with single-self preferences.

We should emphasize that here we focus on single-self vs. multiple-selves preferences, rather than on expected vs. nonexpected utility. In fact, there are no differences among the amount, switch and translation effects regarding expected vs. nonexpected utility. [2]

2. Single-self vs. multiple-selves preferences: the case of certainty

Consider, for comparison purposes, the basic model of individual choice under certainty. There is a list of N economic variables, or goods, that affect the individual's welfare: the underlying space of economic goods can thus be modeled as $\Re^N$, and we focus on a subset X of it, called the consumption set, that specifies possible physical constraints, e.g., $X = \Re^N{}_+$. The individual's economic activity involves acquiring or relinquishing various amounts of these goods, as for instance in the process of buying commodities, selling labor, or saving.

Society offers the individual an attainable set, or set of consumption opportunities among which the consumer may choose. In the usual case of price-taking with linear prices, as in price

[2] More precisely, all three violate single-self, expected utility, while all three are compatible with multiple-selves, expected utility. See Section 6 below.

vector P, this set is $\Xi = \{x \in X : P \bullet x \leq W\}$, where W is the wealth of the consumer, which may depend on P and on her property rights (via value of endowments, profit income or other components of wealth). The theory has the positive aim of understanding and predicting the choice of an individual with opportunity set Ξ, as well as the normative aim of judging individual welfare, e. g., whether the individual is better off at Ξ^0 than at Ξ,[1] and thus evaluating economic policies that affect Ξ.

Standard economic theory postulates a well-defined, complete and transitive preference relation $\succsim$ on X, which we will call *single-self preferences*. Given Ξ, the individual chooses $x \in \Xi$ in order to maximize $\succsim$ on the attainable set Ξ. This induces an indirect preference relation $\succsim^*$ on attainable sets expressing whether the individual is better off at Ξ^0 than at Ξ^1 for all possible (Ξ^0, Ξ^1) pairs.

A special case of the standard theory is the model of an exchange economy, where the individual owns an endowment vector $\omega \in \Re^N$ which determines her wealth as $W = P \bullet \omega$. For an individual who maximizes her preferences $\succsim$ on the attainable set, changes in ω will induce changes in the individual's net trade vector $z = x - \omega$, and, thus her net trade vector depends in a sense on her endowment ω. But whenever $P \bullet \omega^0 = P \bullet \omega^1$, both her consumption vector and her welfare levels will be the same at $W^0 = P \bullet \omega^0$ and at $W^1 = P \bullet \omega^1$, and, hence, she will display no "endowment effect" in the sense of the modern literature exemplified by Jack Knetsch (1989), Amos Tversky and Daniel Kahneman (1991) and Kahneman, Knetsch and Richard Thaler (1991).

This literature postulates that the preferences of the individual on X vary with a vector $\bar{x} \in \Re^N$, where perhaps $\bar{x} = \omega$ (endowment effect), or $\bar{x}$ is a "reference vector" determined by "customary consumption," the *status quo*, expectations or aspirations (reference-dependent preferences): the various interpretations yield different insights: see Alistair Munro and Robert Sugden (2003, Sections 7-8), but share a formal similarity: each reference point $\bar{x}$ defines a different self, with a different preference relation on X.

Consider for illustration purposes an individual who consumes two goods: good one is an index of the quality of the environment where she lives, whereas good two is the numeraire. There is an extensive literature that discusses observed discrepancies between the "willingness to pay" (WTP) and the "willingness to accept" (WTA) for the environmental good (see, e.g., Michael

Hahneman, 1991, Jason Shogren et al, 1994, and John Horowitz and K. E. McConnell, 2003). To be precise, consider two levels x^0 and x^1 of environmental quality, and let the individual be endowed with an amount w of the numeraire good: see Figure 1. The WTP for the improvement from x^0 to x^1 is the amount of numeraire that makes the individual indifferent between (x^1, w – WTP) and (x^0, w), whereas the WTA for the deterioration from x^1 to x^0 is the amount of numeraire that makes the individual indifferent between (x^0, w + WTA) and (x^1, w). Many empirical and experimental studies systematically yield measures of WTA that are larger than those of WTP. Both for positive and for normative purposes, it is important to know whether the discrepancy is consistent with single-self preferences, or, on the contrary, is due to an "endowment effect," where the preferences of the individual over (environment, numeraire) pairs change with the "reference" quality of the environment.

It is clear from Figure 1 that some positive difference WTA – WTP is consistent with single-self preferences, represented by the solid indifference curves. Indeed, as long as the environment is a normal good, we must have that WTA > WTP, because writing "$x_2 = f^j(x_1)$" for the equation of the indifference curve that goes through point $\omega^j \equiv (x^j, w)$, $j = 0, 1$, normality implies that $\frac{df^1}{dx_1}(x_1) < \frac{df^0}{dx_1}(x_1) < 0$ (i.e., at a given x_1, the higher indifference curve is steeper than the lower one), and hence the vertical distance between the two indifference curves decreases with x_1, because $\frac{d(f^1(x_1) - f^0(x_1))}{dx_1} = \frac{df^1(x_1)}{dx_1} - \frac{df^0(x_1)}{dx_1} < 0$. As both the WTP and the WTA measure the vertical distance between the two indifference curves, but the WTP does it at a point further to the right, it follows that, under normality WTP < WTA.

Multiple-selves preferences would occur if the solid curves of Figure 1 were indifference curves contingent on the reference point ω^0, i. e., contingent on the individual having the right to the environmental level x^0, whereas if she had rights to the higher level x^1, so that her reference point became ω^1, then the indifference curve through ω^1 would become the steeper dashed curve, crossing the one relevant for the reference point ω^0, and yielding a "willingness to pay" that exceeds the former one by the length AB.

More generally, such reference-dependence or "endowment effect" would lead the individual to choose different consumption points in cases where $W^0 = W^1$. Understanding these choices by preference maximization requires a family $\{\succsim_{\bar{x}} : \bar{x} \in \Omega\}$ of preference relations on X, where Ω is an index set of possible endowment or reference points $\bar{x}$ (again, perhaps $\bar{x} = \omega$, an endowment point), instead of a single preference relation $\succsim$.[3] We then say that preferences are *of the multiple-selves* type if they vary with $\bar{x}$, i.e., if $\succsim_{\bar{x}} \neq \succsim_{\bar{x}'}$ for some $\bar{x}$, $\bar{x}' \in \Omega$. By this definition, multiple selves require the possibility of changes in $\bar{x}$: a single self would be present if $\bar{x}$ never varied, i.e., if Ω were the singleton $\{\bar{x}\}$.[4]

Multiple selves present positive challenges and normative difficulties. A recent literature has developed the positive aspects of the theory by exploring the implications of conditions relating $\succsim_{\bar{x}}$ and $\succsim_{\bar{x}'}$ for different $\bar{x}$, $\bar{x}' \in \Omega$, and developing models where reference points are endogenized by an implicit dynamic process, see Tversky and Kahneman (1991), Ian Bateman *et al*. (1997), and Munro and Sugden (2003).

But, normatively, the family of preference relations $\{\succsim_{\bar{x}} : \bar{x} \in \Omega\}$ is not sufficient to evaluate individual welfare, because, under multiple selves, it is unnatural to assume that the individual has metapreferences on $X \times \Omega$ that induce the family $\{\succsim_{\bar{x}} : \bar{x} \in \Omega\}$ by the equivalence $x \succsim_{\bar{x}} x' \Leftrightarrow (x, \bar{x}) \succsim (x', \bar{x})$). In other words, it is unnatural to assume that the individual can compare the final consumption of vector x when the reference point is $\bar{x}$, denoted $(x, \bar{x})$, with the final consumption of x' when the reference point is $\bar{x}'$, denoted $(x', \bar{x}')$. This basic difficulty remains in the recent positive theories of reference dependence.[5]

Even if unnatural, it is theoretically possible for the individual to have a preference relation on $(x, \bar{x})$ pairs: We may be unable to infer it from her choices, but perhaps we can ask her. [6]

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[3] More generally, the preference relation could conceivably be indexed by both a reference point $\bar{x}$ and by the attainable set Ξ: this is precisely the case for regret theory in a world of uncertainty, see Sugden (1993).
[4] Trivially, if $\Omega = \{\bar{x}\}$, then reference dependence by any definition is irrelevant.
[5] In particular, the "long run" preferences of Munro and Sugden (2003) are preferences on X and not on $X \times \Omega$.
[6] Even when they exist, it may be impossible to derive these metapreferences from observed behavior in the manner in which single self preferences can be deduced, via integrability, from the knowledge of the demand functions. The problem is that we cannot tell from observing the individual's behavior whether the individual is better off at $(\bar{x}^0, \bar{x}^0)$ than at $(\bar{x}^0, \bar{x}^1)$ because the individual never has a chance to choose between $(\bar{x}^0, \bar{x}^0)$ than at $(\bar{x}^0, \bar{x}^1)$. This displays formal similarities with the estimation of preferences for nonmarketed goods, such as quality or the environment, for which ingenious positive results can be obtained by *a priori* postulating particular forms of complementarity or substitutability between marketed and nonmarketed goods (see, e.g., Robert Willig, 1978, and Douglas Larson, 1992). Some of these methods could conceivably be adapted to the present context, but the

Suppose that she asserts to be better off at ($\bar{x}^0$, $\bar{x}^0$) than at ($\bar{x}^0$, $\bar{x}^1$), i.e., she asserts that she would be better off if her endowment or reference point were $\bar{x}^0$ and she stayed at it than if it were $\bar{x}^1$ and she moved to $\bar{x}^0$: this is usually referred to as an "endowment effect." But suppose also ($\bar{x}^0$, $\bar{x}^0$) is socially more costly than ($\bar{x}^0$, $\bar{x}^1$). (Perhaps the implementation of ($\bar{x}^0$, $\bar{x}^0$) requires more bureaucracy.) It is not clear why her preferences should be respected in this case.[7]

At the crux of the matter is the question, why does she prefer ($\bar{x}^0$, $\bar{x}^0$) to ($\bar{x}^0$, $\bar{x}^1$)? If the reference point is an endowment vector, and endowments can change, what distinguishes a change in endowments from a trade? The normative relevance of an expressed preference of ($\bar{x}^0$, $\bar{x}^0$) over ($\bar{x}^0$, $\bar{x}^1$) has to be justified by appealing to basic principles.

More generally, multiple selves appear when the preferences of an individual vary according to the situation in which the individual makes her decision. Besides endowments or reference points, the multiplicity of selves may be defined by circumstances such as time (a present self vs. a future self) or past consumption (addicted self vs. addiction-free selves). In all these cases, any social evaluation of the individual's welfare across different circumstances will to some extent appeal to an external criterion of welfare. Some recent papers (see Colin Camerer *et al*., 2003, and Richard Thaler and Cass Sunstein, 2003) have developed policy recommendations for some such instances of multiplicity of selves.

3. Risk: single-self vs. multiple-selves preferences, and the expected utility hypothesis

3.1. The model: contingent balances and objective probabilities

The model of Section 2 can be extended to decisions under risk, with the interpretation that preferences are *ex ante*, before the uncertainty is resolved. For simplicity, we posit a single *ex post* good, called money. *Ex ante* preferences will depend on the possible amounts of *ex post* money, and are defined on contingent money balances with a finite number of states of the world endowed with probabilities, which we assume objective. We will focus on a simple model of *ex ante* preferences.

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procedure would be justified only if it could be reasonably assumed that the individual does have well-defined preferences on (x, $\bar{x}$) pairs.

[7] If, on the contrary, changing endowments were exactly as costly as trading, then Pareto efficiency in a society where everybody had this type of preferences would require the redistribution of endowments and no trade.

There are $S < \infty$ possible states: the number x_s denotes an amount of money available to the individual in the contingency that state s occurs. A vector $x = (x_1,\ldots, x_S) \in \Re^S_+$ is interpreted as a point of contingent money balances. The individual has an initial position $\omega \in \Re^S_+$, or reference point $\bar{x} \in \Re^S_+$. Thus, in principle the "initial position" may be uncertain. But, for concreteness, here we shall focus on initial positions or reference points that are certain, namely those determined by a certain level w of wealth. Thus, the reference or initial endowment points are of the form $\omega = (w, \ldots, w) \in \Re^S_+$, on the "certainty line." Society offers her some opportunities to trade money balances contingent on the various states: we denote by Z the individual's set of trading opportunities understood as deviations from ω, so that the set of attainable vectors of contingent money balances is, using the notation Ξ of Section 2 above, the attainable set $\Xi = (Z + \{\omega\}) \cap \Re^S_+$. The *ex ante* preferences of the individual depend on both the vector of contingent money balances $(x_1,\ldots, x_S)$ and on the probabilities $(p_1,\ldots, p_S)$, understood as objective, of the various states. We postulate that these preferences are state-independent, i.e., invariant to permutations of the indices $\{1,\ldots, S\}$: the uncertain states just provide a random device, without direct welfare effects.

3.2. Single-self vs. multiple-selves preferences

As in Section 2, *single-self preferences* are modeled as a single *ex ante* preference relation $\succsim$ on the space of contingent money balances cross probabilities. Under standard completeness, transitivity and continuity assumptions, these preferences are representable by a utility function

$$U : \Re^L_+ \times \Delta^{S-1} \to \Re : (x_1,\ldots, x_S; p_1,\ldots, p_S) \mapsto U(x_1,\ldots, x_S; p_1,\ldots, p_S),$$

where $\Delta^{S-1} \equiv \{(p_1,\ldots, p_S) \in \Re^L_+ : \sum_{s=1}^S p_s = 1\}$ is the standard (S-1)-dimensional probability simplex.

A certain outcome x is then represented as a point $(x,\ldots, x; p_1,\ldots, p_S)$ on the "certainty line" of the space of contingent money balances.

Given a certain initial amount of money w, we can express any money magnitude x as its deviation z (positive for a gain, negative for a loss) from w, i. e., so that final wealth is $x = w + z$. Thus, the utility function could equivalently be written as a family of functions U_w with gains or losses z as arguments, defined by $U_w(z_1, \ldots, z_S,\ p_1, \ldots, p_S) \equiv U(w + z_1, \ldots, w + z_S,\ p_1, \ldots,\ p_S)$. In addition, we could as well use x instead of z as arguments, writing

$\hat{U}_w(x_1,\dots,x_S,p_1,\dots,p_S) \equiv U_w(x_1 - w,\dots,x_S - w,p_1,\dots,p_S)$: all this is just alternative notation. The substantive assumption of single-self preferences can be formally expressed in these alternative notations as requiring that, $\forall(\overline{w},\overline{\overline{w}},x_1,\dots,x_S) \geq 0$ and $\forall(p_1,\dots,p_S) \in \Delta^{S-1}$,

$U_{\overline{w}}(x_1 - \overline{w},\dots,x_S - \overline{w},p_1,\dots,p_S) = U_{\overline{\overline{w}}}(x_1 - \overline{\overline{w}},\dots,x_S - \overline{\overline{w}},p_1,\dots,p_S)$, or

$\hat{U}_{\overline{w}}(x_1,\dots,x_S,p_1,\dots,p_S) = \hat{U}_{\overline{\overline{w}}}(x_1,\dots,x_S,p_1,\dots,p_S)$.

On the other hand, and paralleling Section 2 above, we say that *ex ante* preferences are of the *multiple-selves* type if they involve a family $\{\succsim_w: w \in \Re_+\}$ of preference relations instead of a single preference relation $\succsim$, so that $\succsim_w \neq \succsim_{w'}$ for some w and w'.

Again, under continuity such a family of preference relations will be representable by a family of utility functions $\{U_w: w \in \Re_+\}$ (or $\{\hat{U}_w: w \in \Re_+\}$), but now, contrary to the single-self case, $U_{\overline{w}}(x_1 - \overline{w},\dots,x_S - \overline{w},p_1,\dots,p_S) \neq U_{\overline{\overline{w}}}(x_1 - \overline{\overline{w}},\dots,x_S - \overline{\overline{w}},p_1,\dots,p_S)$ (or $\hat{U}_{\overline{w}}(x_1,\dots,x_S,p_1,\dots,p_S) \neq \hat{U}_{\overline{\overline{w}}}(x_1,\dots,x_S,p_1,\dots,p_S)$) for some $(\overline{w},\overline{\overline{w}},x_1,\dots,x_S,p_1,\dots,p_S)$.

3.3. Expected utility

As is well known, under some assumptions, among the utility functions representing the *ex ante*, single-self preference relation $\succsim$ there is at least one of the form

$$U(x_1,\dots,x_S\,;p_1,\dots,p_S) = \sum_{s=1}^{S} p_s u(x_s),$$

where u (which is the same function at all states of the world) is a real-valued function called the individual's *von Neumann-Morgenstern* (vNM) *utility function*, defined over final money balances, in which case we say that the individual has (*state-independent*) *Single-Self Expected Utility (ex ante) Preferences*. This reflects the oldest formulation of decision-making in the face of risk, due to Daniel Bernoulli (1738): he postulated this type of preferences and proposed the function $u(x) = \ln x$: see the top left panel of Figure 2. Of course, understood as a function of the probabilities $(p_1,\dots,p_S)$, the function $\sum_{s=1}^{S} p_s u(x_s)$ is linear, with $u(x_s)$ as the coefficient of p_s.

As in Section 3.2, nothing substantial would change if, instead of the single function $u(x)$, we consider a family $\{u_w(z) : w \in \Re_+\}$ or a family $\{\hat{u}_w(x) : w \in \Re_+\}$ of functions: these families would still represent single-self preferences provided that $u(x) = u(w + z) = u_w(z) \equiv \hat{u}_w(x)$.

Consider now the multiple-selves preferences of Section 3.2 above. As a special case, there may be a family of real valued functions $\{u_w: w \in \Re_+\}$ such that $U_w(x_1 - w,..., x_S - w, p_1,..., p_S) = \sum_{s=1}^{S} p_s u_w(x_s - w)$ (or a family $\{\hat{u}_w : w \in \Re_+\}$ such that $\hat{U}_w(x_1,..., x_S, p_1,..., p_S) = \sum_{s=1}^{S} p_s \hat{u}_w(x_s)$). This view can be traced to Harry Markowitz (1952): see the center bottom panel of Figure 2. Again, we have multiple selves if $u_{\overline{w}}(x - \overline{w}) \neq u_{\overline{\overline{w}}}(x - \overline{\overline{w}})$ for some $(x, \overline{w}, \overline{\overline{w}})$.[8]

We will call preferences defined by such a family of utility functions (*state-independent*) *Multiple-Selves* (*ex ante*) *Expected Utility Preferences*, even though the usage of the term "expected utility" in this instance is not universal: some authors, such as Matthew Rabin (2000), would likely refrain from it.

3.4. Risk attitudes: Bernoulli, Friedman-Savage and Markowitz

Let an individual face the choice between the uncertain alternative of adding to her current wealth w the positive or negative amounts of money $(z_1,\dots, z_S)$ with probabilities $(p_1,\dots, p_S)$, and the certain alternative of adding the positive or negative amount of money $\sum_{s=1}^{S} p_s z_s$. (The choice is actuarially fair, because the expected gain or loss is the same in both alternatives.) If the individual prefers the certain alternative, then we say that she displays *risk aversion* in that choice. If, on the contrary, she prefers the uncertain alternative, then she displays *risk attraction*. If she is indifferent between the two, then she displays *risk neutrality*. Aversion, attraction and neutrality are the three possible *risk attitudes*.

Bernoulli (1738) believed that most people display risk aversion in most choices. Indeed, for (single-self or multiple-selves) expected-utility preferences, the strict concavity of the function $u_w(z)$ guarantees risk aversion, and this is certainly the case in Bernoulli's (single-self) $u(w + z)$ $u_w(z) = \ln(w + z)$ (top left panel of Figure 2).

Two centuries after Bernoulli, and in order to accommodate some extent of risk attraction, Milton Friedman and Savage (1948) assumed, again in the single-self context, that u was concave (risk aversion) for low wealth levels, convex (risk attraction) for intermediate ones and concave again for high wealth levels: the center top panel of our Figure 2, which is based on their famous Figure 3. However, justifying its shape is not trivial. Nathaniel Gregory (1980) postulates that

[8] See Sugden (2003) for a recent axiomatization of this type of preferences in the context of subjective probabilities *à la* Leonard Savage (1954).

wealth has two effects on utility: the usual direct effect, and a social rank effect, based on the comparison with the wealth of others, which will depend on the distribution of wealth in society. An alternative justification is provided by Arthur Robson (1992).

Markowitz (1952) criticized the Friedman-Savage view, and proposed what we call multiple-selves, expected-utility preferences, with risk aversion for large gains and small losses, and risk attraction for small gains and large losses. His Figure 5, page 154, is reproduced in the center bottom panel of our Figure 2.

3.5. Kahneman and Tversky's gain-loss asymmetry and their reflection effect

Daniel Kahneman and Amos Tversky (1979), see also Tversky and Kahneman (1992) postulated a basic asymmetry between gains and losses. The right-bottom panel of our Figure 2 is inspired by their Figure 3 (1979, page 279). For positive z's, the function is strictly concave, suggesting risk aversion for gains, whereas, for negative z's the function is strictly convex, suggesting risk attraction for losses.

It should be noted that they did not subscript the function by w: on the contrary, they argued that the level of wealth was unimportant. And they did not call it a "utility function," but a "value function," and denoted it by $v(z)$. More significantly, they did not consider the expectation $\sum_{s=1}^{S} p_s v(z_s)$, but the sum of the $v(z_s)$'s weighted by "decision weights," or "probability distortions," nonlinear in the (true) probabilities. Thus, any utility function representing *ex ante* preferences for one of the selves must be nonlinear in the probabilities: therefore, their theory is of the multiple-selves, nonexpected utility in our terminology.

Kahneman and Tversky's nonlinearity in the probabilities is a major departure from previous literature. Because of it, the strict concavity or convexity of the "value function" does not determine risk attitude: it must be combined with the form of the "decision weight" functions, so that no general implications for risk attitudes can be derived from their assumptions. But based on their observations, they did claim as an empirical regularity that replacing gains by losses through a *reflection*, i.e., the multiplication by minus one of all money amounts, would make the individual move from risk aversion to risk attraction. They called this phenomenon the *reflection effect*, defined as "…the reflection effect implies that risk aversion in the positive domain is accompanied by risk seeking in the negative domain." (Kahneman and Tversky, 1979, page 268.)

4. Fair, binary choices and changes in risk attitudes

4.1. Translation, switch and reflection effects on risk attitudes

Consider the choice between the uncertain alternative of adding to the current wealth w the positive or negative amount of money z with probability p and zero with probability 1- p, and the certain alternative of adding the positive or negative amount of money pz: again, the choice is actuarially fair. Denote such a choice by $< z, p \mid w>$. Note that, in the uncertain alternative of choice $< z, p \mid w >$, if $z > 0$, then the good state is the one where the individual gains z, which occurs with probability p, whereas the bad state is the one where the individual gains nothing, which occurs with probability $1 - p$. If, on the contrary, $z < 0$, then the bad state is the one where the individual loses $|z|$, which occurs with probability p, whereas the good state is the one where the individual loses nothing, which occurs with probability $1 - p$.

Graphically, we can represent the two alternatives of choice $<z, p \mid w>$ in the contingent money balances graph of Figure 3: A point in the graph is a pair (x_1, x_2), where x_1 represents an amount of wealth contingent on State 1, and x_2 represents an amount of wealth contingent on State 2. Note that, for points above the certainty line, State 1 is the bad state, whereas it is the good state for points below the certainty line.

Given $(p_1, p_2) \in \Delta^1$ and $E > 0$, the set of pairs (x_1, x_2) satisfying $p_1x_1 + p_2x_2 = E$ have the same expected value E (geometrically, they constitute the *E-fair-odds line* (or the fair odds line through (E, E), or at level E). For instance, putting the bad state on the horizontal axis, the choice $<100, 0.8 \mid 1000>$ is the choice between the uncertain point $G = \{1000, 1100\}$ and the certain point $C^1 = (1080, 1080)$: both are on the fair-odds line corresponding to the expected money balance 1080.

Suppose that the individual displays a particular risk attitude in the choice $< z, p \mid w>$, say that she displays risk aversion by preferring the certain to the uncertain alternative in choice. We wish to explore possible changes in risk attitude if she instead faces a different choice $<z', p' \mid w'>$ that is related to $< z, p \mid w>$ in a specific fashion.

First we consider a family of transformations of choices that leave w invariant, and either change the sign of z, or switch the probabilities p and $1 - p$, or both.

Define the *probability switch operator s* by $s(< z, p \mid w>) = < z, 1 - p \mid w>$. This operator switches the probabilities of the good and the bad state. In our previous example, s(<100, 0.8

| 1000>) = <100, 0.2 | 1000>, i.e., the uncertain outcome is a gain of 100 with probability 0.2. The choice *s*(<100, 0.8 | 1000>) can be represented in Figure 3 as the choice between the uncertain point *G*'= (1100, 1000) (below the certainty line, because now the state that occurs with probability 0.2 is the good state) and the certain point C^2 = (1020, 1020).

Next, define the *translation operator t* by *t*(< *z*, *p* | *w*>) = < - *z*, 1 – *p* | *w*>. This operator translates the discrete probability density functions along the money axis, but keeps unchanged the probabilities of the good and bad outcomes. If $z > 0$, at the uncertain alternative of choice <*z*, *p* | *w*> the bad event yields the gain of zero, which occurs with probability 1 – *p*, while at the translated choice *t*(< *z*, *p* | *w*>) = < - *z*, 1 – *p* | *w*> the bad event is the loss of | *z* |, which also occurs with probability 1 – *p*. Similarly for the good event. Thus, in this case the translation operator translates the probability distribution leftwards along the money axes. If $z < 0$, at the uncertain alternative of choice <*z*, *p* | *w*> the bad event yields the loss of | *z* |, which occurs with probability *p*, while at the translated choice *t*(< *z*, *p* | *w*>) = < - *z*, 1 – *p* | *w*> the bad event is the gain of zero, which also occurs with probability *p*. Thus, in this case the translation operator translates the probability distribution rightwards along the money axes.

In our previous example, *t*(<100, 0.8 | 1000>) = < -100, 0.2 | 1000>, i.e., the uncertain outcome is a loss of 100 with probability 0.2. It can be represented in Figure 3 by the choice between the uncertain point *L* = (900, 1000) and the certain point C^3 = (980, 980).

Last, define the *reflection operator r* as *r*(< *z*, *p* | *w*>) = < - *z*, *p* | *w*>. This operator transforms a gain of *z* with probability *p* into a loss of *z* also with probability *p*. In our previous example, *r*(<100, 0.8 | 1000>) = <-100, 0.8 | 1000>, i.e., the uncertain outcome is a loss of 100 with probability 0.8. It can be represented in Figure 3 by the choice between the uncertain point *L'* = (1000, 900) and the certain point C^4 = (920, 920).

Kahneman and Tversky's reflection effect asserts a change in risk attitude when a choice is transformed by the reflection operator, no matter what the wealth *w* is.

It is clear that any of the three operators can be obtained by the application of the other two: in fact, the three operators *s*, *t*, and *r* on choices, together with the identity operator *e*, constitute the Klein 4-group, see Table 1. In particular, **Reflection = Translation + Switch.**

	e	***s***	***t***	***r***
e	*e*	*s*	*t*	*r*
s	*s*	*e*	*r*	*t*
t	*t*	*r*	*e*	*s*
r	*r*	*t*	*s*	*e*

Table 1. The group of operators identity, switch, translation and reflection (Klein 4)

Along the lines of Kahneman and Tversky, we say that an individual displays a *switch* (resp. *translation*) effect if she displays risk aversion in choice $< z, p \mid w >$ (where z can be positive or negative), but risk attraction in choice $s(<z, p \mid w >)$ (resp. $t(<z, p \mid w >)$) for a wide range of initial wealth levels w.

Because a reflection can be decomposed into a switch and a translation, a change of risk attitude along a reflection may be due solely to a switch effect, or solely to a translation effect, or to both. The main theme of this paper is the asymmetry between the switch and the translation effect in what concerns the implied preferences: in a nutshell, while the switch effect (and hence the reflection effect) is compatible with single-self preferences, the translation effect is not. This suggests that the switch effect parallels the wealth-effect-induced gap between WTA and WTP (see Section 2 above), and that the translation effect parallels the endowment-effect-induced WTA-WTP gap. However, the parallelism is not exact, because the suggestion of an alternative theoretical model in terms of single-self vs. multiple-selves came, in Section 2 above, from the magnitude of WTA-WTP gap (a large gap suggesting an endowment effect), whereas now we also have a *qualitative* distinction between switch and translation effect.

4.2. Small vs. large risks: the amount effect on risk attitudes

Next, we consider transformations of a choice that leaves probabilities, wealth level and the sign of z unchanged, but change the magnitude of z. Formally, for $\lambda > 0$ define the λ-*scale operator* by $\lambda(<z, p \mid w>) = <\lambda z, p \mid w>$. Given $<z, p \mid w >$, where z can be positive or negative, and where w can be large or small, our experimental work (Bosch-Domènech and Silvestre, 1999, 2004, in press) has consistently evidenced risk attraction for choices $\lambda(<z, p \mid w >)$ when λz is small, and risk aversion when λz is large. We call this an *amount effect*, understood as occurring at a range of values of initial wealth.

Graphically, we have an amount effect at the level of wealth w if, along the fair-odds line with expected money balances of $w + pz$ and on one side of the certainty line, the individual displays risk attraction for choices involving uncertain alternatives close to the certainty line, but risk aversion away from it.

An individual displaying an amount effect takes small risks (of a certain type) but avoids large ones. Let the probability of the gain be $p = 0.8$ and let our individual display risk attraction for $z = 100$, but risk aversion for $z = 200$, both when her initial wealth is 1000 and when her initial wealth is 920. Choosing the risky gain of 100 when her wealth is 1000 means that she chooses the random variable $\tilde{x}^1$, that gives a money balance of 1000 with probability 0.2 and a balance of 1100 with probability 0.8, to the degenerate random variable $\tilde{x}^0$, that gives the certain balance of 1080. Note that the two random variables have the same expectation of 1080, and that $\tilde{x}^0$ second-order stochastically dominates (SOSD) $\tilde{x}^1$. Thus, the individual's choice shows attraction to a pure risk, but one that is relatively small.

On the other hand, by choosing the certain gain of 160 over the 0.8 chance of gaining 200 when her wealth is 920, she chooses $\tilde{x}^0$ over the random variable $\tilde{x}^2$, which results in a money balance of 1120 with probability 0.8 and a balance of 920 with probability 0.2. Again, $\mathrm{E}\tilde{x}^2 = \mathrm{E}\tilde{x}^0 = \mathrm{E}\tilde{x}^1 = 1080$, and $\tilde{x}^0$ SOSD $\tilde{x}^1$ SOSD $\tilde{x}^2$. Thus, she is attracted to the relatively small pure risk of $\tilde{x}^1$, but averse to the larger pure risk of $\tilde{x}^2$.

4.3. The role of wealth on risk attitudes: the wealth effect

The work of Kenneth Arrow (1971) and John Pratt (1964), in the context of expected utility, single self preferences, studies an individual's willingness to bear actuarially favorable risk depending on her wealth level: special interesting cases are those of preferences with constant absolute risk aversion or CARA (vNM utility function $u(x) = -e^{-rx}$, $r > 0$), and those with constant relative risk aversion or CRRA (either $u(x) = \ln x$ or $u(x) = \dfrac{x^{1-r}}{1-r}, r \in (0,1) \cup (1,\infty)$). An individual with any of these preferences facing fair choices will choose the certain alternative, and, therefore, all these preferences display risk aversion.

In order to study the dependence of risk attitudes on the level of wealth, Sections 5.5 to 5.11 below consider single-self preferences that are related to the CARA or CRRA types, but that allow for risk attraction, and, hence, that violate the hence, single-self, expected-utility hypothesis. For $\Delta w > 0$, we define the *Δw-operator* by $\Delta w(<z, p \mid w>) = <z, p \mid w + \Delta w>$. Given $<z, p \mid w>$, where z can be positive or negative, the risk attitudes of the individual are *wealth dependent* if she displays risk aversion for the choice $<z, p \mid w>$ but risk attraction for the choice $<z, p \mid w + \Delta w>$. We call this the *wealth effect*.

5. Single-self preferences and the translation, switch, amount and wealth effects

5.1. Introduction

It is easy to see that translation-dependent risk attitudes imply multiple-selves preferences. Following the discussion in Bosch-Domènech and Silvestre (2004), let $z = 100$ and $p = 0.8$. Assume that, both for initial wealth 1000 and 1100, the individual displays risk aversion for the choice $< 100, 0.8 \mid w >$ but risk attraction for the choice $< -100, 0.2 \mid w >$, i.e., when the individual's wealth is 1000, she prefers a sure gain of 80 to a gain of 100 with probability 0.8, whereas when her wealth is 1100 she prefers a loss of 100 with probability 0.2 to a certain loss of 20. In the graph of contingent money balances of Figure 3, this means that she prefers C^1 to G when her endowment or reference point is ω^1, but G to C^1 if the endowment point is ω^2. Thus, no single set of indifference curves can rationalize her behavior.

Note that the attitude reversal occurs for a range of initial wealth values. There would be no problem if it only occurred for a single w, in which case the expected utility hypothesis could be maintained, with a vNM utility function convex in the interval $(w - z, w)$ and concave in $(w, w + z)$, as in the Friedman-Savage example illustrated in the center bottom of Figure 2.

On the other hand, it is easy to show that the amount and switch effects violate single-self expected utility**.** Let us start with the amount effect. If the reversal of risk attitude occurred at a single level of wealth, then preferences could well be of the single self, expected utility variety, as in those of Friedman and Savage (1948). But single-self, expected utility preferences require the second derivative $u''(x)$ to be positive on the interval where the individual is attracted to small risks, and thus $u(x)$ must be convex on that interval. This contradicts the aversion to large risks involving quantities within this interval. Thus, amount-dependent attitudes are incompatible with single-self, expected utility preferences.

For the switch effect, again there would be no problem if the attitude change only took place for a single w and z, in which case the single self, expected utility hypothesis could be maintained, with a vNM utility function u that is convex in the interval $(w, w + 0.5z)$ and concave in $(w + 0.5 z, w + z)$. But it is not difficult to show that if the switch effect changes the risk attitude over a range of wealth levels, then single-self, expected utility preferences must be ruled out: see Bosch-Domènech and Silvestre (2004), where it is also shown that the amount, switch and translation effects are consistent with multiple-selves, expected utility preferences.

5.2. The amount and switch effects

Recall that, throughout this paper, we consider an individual facing (actuarially fair) choices between an uncertain final wealth of x_1 with probability p_1 and of x_2 with probability p_2, and the certain final wealth of $p_1x_1 + p_2 x_2$, and that we say that the individual displays risk attraction in that choice if she prefers the uncertain alternative, and risk aversion if she prefers the certain alternative. Accordingly, the discussion is limited to $S = 2$.

Assumption 1 below is maintained throughout the paper.

<u>Assumption 1: State independence</u>. The *ex ante*, single-self utility function $U : \Re^2_+ \times \Delta^1 \to \Re$ satisfies $U(y, y', p, p') = U(y', y, p', p), \forall (y, y') \in \Re^2_+, \forall (p, p') \in \Delta^1$.

State independence requires utility to depend only on the outcomes and their probabilities, and not on the state where the outcomes occur, i.e., the utility of a lottery that gives the final wealth y with probability p and the final wealth y' with probability p' can equivalently be written either as $U(y, y', p, p')$ or as $U(y', y, p', p)$.

The following definitions formalize the various effects described above.

<u>Definition</u>. The *ex ante*, single-self utility function $U : \Re^2_+ \times \Delta^1 \to \Re$ displays an <u>amount effect (above the certainty line) for</u> $(p_1, p_2) \in \Delta^1$ <u>and</u> $E > 0$ if there exists an $\varepsilon(p_1, p_2, E) > 0$ such that, defining $F(p_1, p_2, E) \equiv \{(x_1, x_2) \in \Re^2_+ : x_2 > x_1 \ \& \ p_1 x_1 + p_2 x_2 = E\}$

(i) $\text{sgn}(U(x_1, x_2, p_1, p_2) - U(E, E, p_1, p_2)) = \text{sgn}(x_1 + \varepsilon(p_1, p_2, E) - x_2)$,

for all (x_1, x_2) in $F(p_1, p_2, E)$,

(ii) $U(x_1, x_2, p_1, p_2) - U(E, E, p_1, p_2) < 0$, for some $(x_1, x_2) \in F(p_1, p_2, E)$.

The phrase "above the certainty line" will always be left implicit in what follows.

Geometrically, the set $F(p_1, p_2, E)$ is the fair-odds line above the certainty line defined by (p_1, p_2, E). An amount effect occurs at (p_1, p_2, E) if the individual displays risk attraction for uncertain prospects close to the certainty line, and risk aversion for those distant from the certainty line. Point $(E - p_2\varepsilon(p_1, p_2, E), E + p_1\varepsilon(p_1, p_2, E))$ depicts the boundary between these two sets of points (an "attraction-aversion boundary").

Definition. If $U : \Re_+^2 \times \Delta^1 \to \Re$ displays an amount effect for $(p_1, p_2) \in \Delta^1$ and all $E > 0$, then we simply say that U displays an amount effect for (p_1, p_2).

Note that $U(x_1, x_2, p_1, p_2)$ is the utility of the risky alternative, whereas $U(E, E, p_1, p_2)$ is that of the safe alternative. An amount effect occurs if, for a point (x_1, x_2) in the E-fair-odds line for which the good outcome is x_2, the individual displays risk attraction if $x_1 < x_2 < x_1 + \varepsilon(p_1, p_2, E)$, i.e., if x_2 is close to x_1, but risk aversion if $x_2 > x_1 + \varepsilon(p_1, p_2, E)$, i.e., x_2 is far from x_1. The equality "$x_2 = x_1 + \varepsilon(p_1, p_2, E)$" defines the attraction-aversion boundary. Figure 4 illustrates: the individual prefers any point in the segment $\overline{(C, A)}$ to point $C \equiv (E, E)$, thus displaying risk attraction in these choices, but she prefers point C to any point in the segment $\overline{(A, B)}$, therefore displaying risk aversion in these choices.

Clearly, (E, E) second-order stochastically dominates any (x_1, x_2) with expected money balances E. Because $\varepsilon(p_1, p_2, E)$ is the supremum of the "$x_2 - x_1$ gaps" (or differences between money balances in the good and bad states) for which the individual displays risk attraction at the constant expected value, $\varepsilon(p_1, p_2, E)$ indicates the largest absolute fair risk that the individual is willing to accept.

Definition. The *ex ante*, single-self utility function $U : \Re_+^2 \times \Delta^1 \to \Re$ displays a switch effect for $(p_1, p_2) \in \Delta^1$ at $(x_1, x_2) \in \Re_+^2$ if

$$\begin{aligned}&\operatorname{sgn}(U(x_1, x_2, p_1, p_2) - U(p_1x_1 + p_2x_2, p_1x_1 + p_2x_2, p_1, p_2))\\ &\quad \times \operatorname{sgn}(U(x_1, x_2, p_2, p_1) - U(p_2x_1 + p_1x_2, p_2x_1 + p_1x_2, p_2, p_1)) = -1\end{aligned}.$$

Note that the sign of any such difference of utilities is plus one if the individual prefers the risky alternative to the safe alternative, and minus one if she prefers the safe alternative. In words, a switch effect occurs at point (x_1, x_2) if the individual displays risk aversion when the probability of the bad outcome is p_1, but risk attraction when the probability of the bad outcome is 1- p_1 (or vice-versa), i.e., switching the probabilities of the good and the bad outcomes reverses the risk attitude.

Figure 5 illustrates. Let $(x_1, x_2) = (\bar{x}, \bar{\bar{x}})$, and let the individual prefer point C^1 to point A when $(p_1, p_2) = (q, 1 - q)$, where $q \in (0,1/2)$, so that $\frac{q}{1-q} < 1$, i.e., the individual displays risk aversion in the choice between the risky prospect that gives $\bar{x}$ with probability q and $\bar{\bar{x}}$ with probability 1- q, and the certain prospect that gives its expected value $q\bar{x} + (1-q)\bar{\bar{x}}$. Note that here the bad outcome is $\bar{x}$ and its probability is relatively low. In addition, let the individual prefer point A to point C^2 when $(p_1, p_2) = (1- q, q)$, i.e., when the probability of the bad outcome is a relatively high $1 - q$, i. e., the individual displays risk attraction in the choice between the risky prospect that gives $\bar{x}$ with probability 1- q, and $\bar{\bar{x}}$ with probability q and its expected value $(1-q)\bar{x} + q\bar{\bar{x}}$. Switching the probabilities of the good and the bad outcomes has led the individual to a reversed risk attitude.

Lemma 1. The *ex ante*, single-self, state-independent utility function $U : \Re^2_+ \times \Delta^1 \to \Re$ displays a switch effect for $(p_1, p_2) \in \Delta^1$ at $(\bar{x}, \bar{\bar{x}}) \in \Re^2_+$ if and only if

$$\begin{aligned} &\operatorname{sgn}(U(\bar{x}, \bar{\bar{x}}, p_1, p_2) - U(p_1\bar{x} + p_2\bar{\bar{x}}, p_1\bar{x} + p_2\bar{\bar{x}}, p_1, p_2)) \\ &\quad \times \operatorname{sgn}(U(\bar{\bar{x}}, \bar{x}, p_1, p_2) - U(p_1\bar{\bar{x}} + p_2\bar{x}, p_1\bar{\bar{x}} + p_2\bar{x}, p_1, p_2)) = -1 \end{aligned}.$$

Proof. By state independence,

$$U(\bar{x}, \bar{\bar{x}}, p_2, p_1) = U(\bar{\bar{x}}, \bar{x}, p_1, p_2) \quad (1)$$

and

$$U(p_1\bar{\bar{x}} + p_2\bar{x}, p_1\bar{\bar{x}} + p_2\bar{x}, p_2, p_1) = U(p_1\bar{\bar{x}} + p_2\bar{x}, p_1\bar{\bar{x}} + p_2\bar{x}, p_1, p_2). \quad (2)$$

By definition, there is a switch effect for $(p_1, p_2) \in \Delta^1$ at $(\bar{x}, \bar{\bar{x}}) \in \Re^2_+$ if and only if

$$\begin{aligned} &\operatorname{sgn}(U(\bar{x}, \bar{\bar{x}}, p_1, p_2) - U(p_1\bar{x} + p_2\bar{\bar{x}}, p_1\bar{x} + p_2\bar{\bar{x}}, p_1, p_2)) \\ &\quad \times \operatorname{sgn}(U(\bar{x}, \bar{\bar{x}}, p_2, p_1) - U(p_2\bar{x} + p_1\bar{\bar{x}}, p_2\bar{x} + p_1\bar{\bar{x}}, p_2, p_1)) = -1 \end{aligned}. \quad (3)$$

But by (1) and (2), $U(\bar{x}, \bar{\bar{x}}, p_2, p_1) - U(p_2\bar{x} + p_1\bar{\bar{x}}, p_2\bar{x} + p_1\bar{\bar{x}}, p_2, p_1)$

$$= U(\bar{\bar{x}}, \bar{x}, p_1, p_2) - U(p_2\bar{x} + p_1\bar{\bar{x}}, p_2\bar{x} + p_1\bar{\bar{x}}, p_1, p_2).$$

Hence, (3) is equivalent to

$$\begin{aligned} &\operatorname{sgn}(U(\bar{x}, \bar{\bar{x}}, p_1, p_2) - U(p_1\bar{x} + p_2\bar{\bar{x}}, p_1\bar{x} + p_2\bar{\bar{x}}, p_1, p_2)) \\ &\quad \times \operatorname{sgn}(U(\bar{\bar{x}}, \bar{x}, p_1, p_2) - U(p_1\bar{\bar{x}} + p_2\bar{x}, p_1\bar{\bar{x}} + p_2\bar{x}, p_1, p_2)) = -1 \end{aligned}. \quad \blacksquare$$

Intuitively, under state independence, the individual is indifferent between switching probabilities and switching outcomes: for instance, point *A* of Figure 5 with the probability 1- *q* of $\bar{x}$ is indifferent to point *A*' with the probability *q* of $\bar{\bar{x}}$. By definition, a switch effect occurs if the individual prefers, say, C^1 to *A* when the probability of $\bar{x}$ is *q* (risk aversion), but *A* to C^2 when the probability of $\bar{x}$ is 1- *q* (risk attraction). Lemma 1 states that, in that case, the individual prefers point *A*' to C^2 at the same probability, *q*, of the outcome on the horizontal axis (i. e., contingent on State 1, which in *A* is the bad state, whereas in *A*' is the good state). Referring to *A*' vs. C^2, instead of *A* vs. C^2, for the choice between the uncertain alternative that gives $\bar{x}$ with probability *q* and $\bar{\bar{x}}$ with probability $1 - q$ and the certain alternative that gives $q\bar{x} + (1 - q)\bar{\bar{x}}$ has the graphical advantage of keeping constant the probability, *q*, of the outcome of the horizontal axis and hence the slope of the fair-odds lines ($-\frac{q}{1-q}$), as well as the map of indifference curves in (x_1, x_2) space.

Assumption 2 below will be postulated on occasion in what follows.

Assumption 2: Strict concavity above the certainty line. For given $(p_1, p_2) \in \Delta^1$, the *ex ante*, single-self utility function $U : \Re_+^2 \times \Delta^1 \to \Re$ is strictly concave with respect to the variables (x_1, x_2) in the subdomain $\{(x_1, x_2) \in \Re_{++}^2 : x_2 \geq x_1\}$.

Note that Assumption 2, even if combined with Assumption 1, allows for failures of concavity with respect to the variables (x_1, x_2) on $\Re_{++}^2$.

5.3. Homothetic, single-self preferences

Definition. The *ex ante*, single-self utility function $U : \Re_+^2 \times \Delta^1 \to \Re$ is homothetic in (x_1, x_2) if

$$\begin{aligned} &\operatorname{sgn}(U(x_1^0, x_2^0, p_1, p_2) - U(x_1^1, x_2^1, p_1, p_2)) \\ &= \operatorname{sgn}(U(tx_1^0, tx_2^0, p_1, p_2) - U(tx_1^1, tx_2^1, p_1, p_2)), \forall t > 0 \end{aligned}.$$

Lemma 2. Let the *ex ante*, single-self utility function $U : \Re_+^2 \times \Delta^1 \to \Re$ be homothetic in (x_1, x_2), and let U display an amount effect for (p_1, p_2) and $\bar{E} > 0$, with associated $\varepsilon(p_1, p_2, \bar{E})$. Then, for all $E > 0$, U displays an amount effect for (p_1, p_2) and E, with

$$\varepsilon(p_1, p_2, E) = \frac{\varepsilon(p_1, p_2, \bar{E})}{\bar{E}} E.$$

Proof. Let U be homothetic in (x_1, x_2) and display an amount effect for $(p_1, p_2) \in \Delta^1$ and $\overline{E} > 0$. Let (x_1, x_2) satisfy $x_2 > x_1$ and $p_1 x_1 + p_2 x_2 = E > 0$. Then

$$\operatorname{sgn}(U(x_1, x_2, p_1, p_2) - U(E, E, p_1, p_2))$$

$$= \operatorname{sgn}(U(\frac{\overline{E}}{E}x_1, \frac{\overline{E}}{E}x_2, p_1, p_2) - U(\frac{\overline{E}}{E}E, \frac{\overline{E}}{E}E, p_1, p_2)) \qquad \text{[by homotheticity]}$$

$$= \operatorname{sgn}(\frac{\overline{E}}{E}x_1 + \varepsilon(p_1, p_2, \overline{E}) - \frac{\overline{E}}{E}x_2) \qquad \text{[because } p_1\frac{\overline{E}}{E}x_1 + p_2\frac{\overline{E}}{E}x_2 = \overline{E} \text{ and } U \text{ displays an amount effect for } \overline{E}\text{]}$$

$$= \operatorname{sgn}(x_1 + \frac{E}{\overline{E}}\varepsilon(p_1, p_2, \overline{E}) - x_2).$$

Thus U displays an amount effect for (p_1, p_2) and E, with $\varepsilon(p_1, p_2, E) = \dfrac{\varepsilon(p_1, p_2, \overline{E})}{\overline{E}}E$.

This proves (i) in the definition of the amount effect. To prove (ii), note that, by assumption, $U(\overline{x}_1, \overline{x}_2, p_1, p_2) - U(\overline{E}, \overline{E}, p_1, p_2) < 0$, for some $(\overline{x}_1, \overline{x}_2) \in F(p_1, p_2, \overline{E})$. Thus, by homotheticity, $U(\frac{E}{\overline{E}}\overline{x}_1, \frac{E}{\overline{E}}\overline{x}_2, p_1, p_2) - U(\frac{E}{\overline{E}}\overline{E}, \frac{E}{\overline{E}}\overline{E}, p_1, p_2) < 0$, with $p_1\frac{E}{\overline{E}}x_1 + p_2\frac{E}{\overline{E}}x_2 = E$, i.e., $(\frac{E}{\overline{E}}\overline{x}_1, \frac{E}{\overline{E}}\overline{x}_2) \in F(p_1, p_2, E)$. ■

Proposition 1. Let the *ex ante*, single-self utility function $U: \Re^2_+ \times \Delta^1 \to \Re$ be homothetic in (x_1, x_2) and satisfy Assumption 2. Then U displays an amount effect for $(p_1, p_2) \in \Delta^1$ if and only if there exists an $\overline{x}_1 \in (0, 1)$ that solves the equation in x_1

$$\text{“}U(x_1, \frac{1 - p_1 x_1}{p_2}, p_1, p_2) = U(1,1, p_1, p_2)\text{,”}$$

in which case such a solution is unique, denoted $x_1^A(p_1, p_2)$, with

(i) $\varepsilon(p_1, p_2, 1) = \dfrac{1 - x_1^A(p_1, p_2)}{p_2}$;

(ii) $\forall E > 0,\ \varepsilon(p_1, p_2, E) = \dfrac{1 - x_1^A(p_1, p_2)}{p_2}E.$

Moreover, in this case we say that $t^A(p_1, p_2) > 1$ exists, where

$$t^A(p_1,p_2) \equiv \frac{1-p_1x_1^A(p_1,p_2)}{p_2x_1^A(p_1,p_2)},$$

with the properties

(iii) $\forall (x_1,x_2,p_1,p_2) \in \Re_+^2$ such that $x_2 > x_1$,

$$\begin{aligned}&\text{sgn}(U(x_1,x_2,p_1,p_2)-U(p_1x_1+p_2x_2,p_1x_1+p_2x_2,p_1,p_2))\\ &=\text{sgn}(t^A(p_1,p_2)x_1-x_2)\end{aligned},$$

(i.e., the attraction-aversion boundary is a ray through the origin given by "$x_2 = t^A(x_1,x_2)x_1$")

(iv) $\forall E>0,$ $\varepsilon(p_1,p_2,E) = \dfrac{t^A(p_1,p_2)-1}{p_1+p_2t^A(p_1,p_2)}E$.

Proof. Consider the function $\varphi:[0,1]\to\Re : \varphi(x_1) \equiv U(x_1,\dfrac{1-p_1x_1}{p_2},p_1,p_2)-U(1,1,p_1,p_2)$.

Clearly, for $x_1 < 1$, $\varphi(x_1) > 0$ (resp. $\varphi(x_1) < 0$) means that the individual displays risk attraction in the choice between the uncertain alternative that gives x_1 with probability p_1 and $\dfrac{1-p_1x_1}{p_2}$ with probability p_2 and the certain alternative that gives its expected value, namely $E = 1$. Note also that $\varphi(1) = 0$.

We want to show that φ is strictly concave. For $\theta \in (0, 1)$, we compute

$$((1-\theta)x_1^0+\theta x_1^1) \equiv U((1-\theta)x_1^0+\theta x_1^1,\frac{1-p_1((1-\theta)x_1^0+\theta x_1^1)}{p_2},p_1,p_2)-U(1,1,p_1,p_2)$$
$$=U((1-\theta)x_1^0+\theta x_1^1,(1-\theta)\frac{1-p_1x_1^0}{p_2}+\theta\frac{1-p_1x_1^1}{p_2},p_1,p_2)-U(1,1,p_1,p_2)$$
$$=U((1-\theta)x_1^0+\theta x_1^1,(1-\theta)x_2^0+\theta x_2^1,p_1,p_2)-U(1,1,p_1,p_2),$$

for $x_2^0 = \dfrac{1-p_1x_1^0}{p_2}$ and $x_2^1 = \dfrac{1-p_1x_1^1}{p_2}$. The strict concavity of U on $\{(x_1,x_2)\in\Re_+^2 : x_2 \geq x_1\}$ guarantees that, when $x_1^0 \neq x_2^0$, the last expression is greater than

$$(1-\theta)(U(x_1^0,x_2^0,p_1,p_2)-U(1,1,p_1,p_2))+\theta(U(x_1^1,x_2^1,p_1,p_2)-U(1,1,p_1,p_2))$$
$$=(1-\theta)(U(x_1^0,\frac{1-p_1x_1^0}{p_2},p_1,p_2)-U(1,1,p_1,p_2))+\theta(U(x_1^1,\frac{1-p_1x_1^1}{p_2},p_1,p_2)-U(1,1,p_1,p_2))$$
$$=(1-\theta)\varphi(x_1^0)+\theta\varphi(x_1^1),$$

proving the strict concavity of φ. Thus, because φ takes the value zero at $x_1 = 1$, it can take the value zero at most once in the interval [0, 1).

If φ does not take the value zero in the interval (0, 1), then φ does not change sign there, and therefore the risk attitude does not change as x_1 ranges over (0, 1) and $x_2 = \frac{1-p_1x_1}{p_2}$: in this case, the individual does not display an amount effect.

If, on the contrary, $\varphi(\bar{x}_1) = 0$ for some $\bar{x}_1 \in (0,1)$, then such a solution is unique, to be written $\bar{x}_1 = x_1^A(p_1, p_2)$, with $\varphi(x_1) > 0, \forall x_1 \in (x_1^A, 1)$, and $\varphi(x_1) < 0, \forall x_1 \in [0, x_1^A)$, i.e., $\text{sgn}(U(x_1, \frac{1-p_1x_1}{p_2}, p_1, p_2) - U(1,1,p_1,p_2)) = \text{sgn}(x_1 - \bar{x}_1)$, hence displaying an amount effect for (p_1, p_2) and $E = 1$, with $\varepsilon(p_1, p_2, 1) = \frac{1-p_1x_1^A}{p_2} - x_1^A = \frac{1-p_1x_1^A - p_2x_1^A}{p_2} = \frac{1-x_1^A}{p_2}$, proving (i).

It follows from Lemma 2 that U displays an amount effect for (p_1, p_2) and any $E > 0$, with $\varepsilon(p_1, p_2, E) = \frac{1-x_1^A(p_1,p_2)}{p_2}E$, proving (ii).

By definition, $t^A(p_1, p_2) = \frac{1-p_1x_1^A}{p_2x_1^A} > \frac{1-p_1}{p_2} = \frac{p_2}{p_2} = 1$, because $x_1^A \in (0,1)$. Using (i) and (ii),

$$\begin{aligned}
&\text{sgn}(U(x_1, x_2, p_1, p_2) - U(p_1x_1 + p_2x_2, p_1x_1 + p_2x_2, p_1, p_2)) \\
&= \text{sgn}(x_1 + \varepsilon(p_1, p_2, p_1x_1 + p_2x_2) - x_2) \\
&= \text{sgn}(x_1 + \frac{1-x_1^A}{p_2}(p_1x_1 + p_2x_2) - x_2) \\
&= \text{sgn}(\left[1 + p_1\frac{1-x_1^A}{p_2}\right]x_1 - \left[1 - p_2\frac{1-x_1^A}{p_2}\right]x_2) \\
&= \text{sgn}(\left[\frac{p_2 + p_1 - p_1x_1^A}{p_2}\right]x_1 - \left[\frac{p_2 - p_2 + p_2x_1^A}{p_2}\right]x_2) \\
&= \text{sgn}(\left[\frac{1-p_1x_1^A}{p_2}\right]x_1 - x_1^A x_2) \\
&= \text{sgn}(\frac{1-p_1x_1^A}{p_2}\frac{x_1}{x_1^A} - x_2) \\
&= \text{sgn}(t^A(p_1, p_2)x_1 - x_2)
\end{aligned}$$

proving (iii). From (ii) and the definition of $t^A(p_1, p_2)$ we obtain (iv). ■

Proposition 2. Let the *ex ante*, single-self utility function $U:\Re^2_+ \times \Delta^1 \to \Re$ be homothetic in (x_1, x_2) and satisfy Assumptions 1 and 2.

(i) If both $t^A(p_1, p_2) > 1$ and $t^A(p_2, p_1) > 1$ exist, as defined in Proposition 1, and $t^A(p_1, p_2) \neq t^A(p_2, p_1)$, then U displays a switch effect for $(p_1, p_2) \in \Delta^1$ at any (x_1, x_2) satisfying

$$\min\{t^A(p_1, p_2), t^A(p_2, p_1)\} < \frac{x_2}{x_1} < \max\{t^A(p_1, p_2), t^A(p_2, p_1)\};$$

(ii) If $t^A(p_1, p_2) > 1$ exists, as defined in Proposition 1, but the equation in x_1

$$U(x_1, \frac{1 - p_2 x_1}{p_1}, p_2, p_1) = U(1,1, p_2, p_1)$$

does not have a solution $\bar{x}_1 < 1$ (i.e., no $t^A(p_2, p_1) > 1$ exists), then U displays a switch effect for $(p_1, p_2) \in \Delta^1$ and either any (x_1, x_2) satisfying $1 < \frac{x_2}{x_1} < t^A(p_1, p_2)$ or any (x_1, x_2) satisfying $\frac{x_2}{x_1} > t^A(p_1, p_2)$.

Proof. (i) Assume that $t^A(p_1, p_2)$ and $t^A(p_2, p_1)$ exist and are greater than one, and let $1 < \min\{t^A(p_1, p_2), t^A(p_2, p_1)\} < \frac{x_2}{x_1} < \max\{t^A(p_1, p_2), t^A(p_2, p_1)\}$. Without loss of generality, let $t^A(p_1, p_2) < t^A(p_2, p_1)$. Because $x_2 > t^A(p_1, p_2)x_1 > x_1$, Proposition 1 (iii) implies that $\operatorname{sgn}(U(x_1, x_2, p_1, p_2) - U(p_1 x_1 + p_2 x_2, p_1 x_1 + p_2 x_2, p_1, p_2)) = -1$. Similarly, because $x_1 < x_2 < t^A(p_2, p_1)$, we have that $\operatorname{sgn}(U(x_1, x_2, p_2, p_1) - U(p_2 x_1 + p_1 x_2, p_2 x_1 + p_1 x_2, p_2, p_1)) = 1$. Thus, the product of the two signs is negative, showing the presence of a switch effect.

(ii). As just argued, if $t^A(p_1, p_2)$ exists and is greater than one, then the sign of $U(x_1, \frac{E - p_1 x_1}{p_2}, p_1, p_2) - U(E, E, p_1, p_2)$ is positive for large x_1 and negative for small x_1. But if the equation "$U(x_1, \frac{1 - p_2 x_1}{p_1}, p_2, p_1) - U(1,1, p_2, p_1) = 0$" does not have a solution in x_1 on the interval (0,1), then the sign of $U(x_1, \frac{1 - p_2 x_1}{p_1}, p_2, p_1) - U(1,1, p_2, p_1)$ is either positive on that interval, or negative on it, which by homotheticity implies that, given $E > 0$, the sign of

$U(x_1, \frac{E - p_2 x_1}{p_1}, p_2, p_1) - U(E, E, p_2, p_1)$ is either positive on (0, E), in which case we have a switch effect for small x_1, or negative on (0, E), in which case we have a switch effect for large x_1. ■

Proposition 3. Let the *ex ante*, single-self utility function $U : \Re_+^2 \times \Delta^1 \to \Re$ be homothetic in (x_1, x_2) and satisfy Assumptions 1 and 2.

If there exists an $\bar{\bar{x}}_1 \in (1, \frac{1}{p_1})$ that solves the equation in x_1

$$\text{“}U(x_1, \frac{1 - p_1 x_1}{p_2}, p_1, p_2) = U(1,1, p_1, p_2)\text{,”}$$

then such a solution is unique, denoted $x_1^B(p_1, p_2)$, and $\frac{1 - x_1^B(p_1, p_2)}{p_2} \in (0,1)$ solves the equation in x_1

$$\text{“}U(x_1, \frac{1 - p_2 x_1}{p_1}, p_2, p_1) = U(1,1, p_2, p_1)\text{,”}$$

i.e., defining $t^B(p_1, p_2) \equiv \frac{1 - p_1 x_1^B(p_1, p_2)}{p_2 x_1^B(p_1, p_2)} < 1$, we have that $t^A(p_2, p_1) = \frac{1}{t^B(p_1, p_2)} > 1$, where $t^A(p_2, p_1)$ satisfies the properties specified in Proposition 1.

Proof. The uniqueness of the solution, denoted $x_1^B(p_1, p_2)$ or simply x_1^B, follows from the concavity argument in the proof of Proposition 1. Accordingly, let $x_1^B \in (1, \frac{1}{p_1})$ satisfy

“$U(x_1^B, \frac{1 - p_1 x_1^B}{p_2}, p_1, p_2) = U(1,1, p_1, p_2)$.” By state independence,

$U(x_1^B, \frac{1 - p_1 x_1^B}{p_2}, p_1, p_2) = U(\frac{1 - p_1 x_1^B}{p_2}, x_1^B, p_2, p_1)$ and $U(1,1, p_1, p_2) = U(1,1, p_2, p_1)$, i.e.,

$U(\frac{1 - p_1 x_1^B}{p_2}, x_1^B, p_2, p_1) = U(1,1, p_2, p_1)$, with $\frac{1 - x_1^B(p_1, p_2)}{p_2} \in (0,1)$ because $x_1^B \in (1, \frac{1}{p_1})$. Thus, writing $t^A(p_2, p_1)$ in accordance with the definition in the statement of Proposition 1 above, we have that $t^A(p_2,p_1) = \dfrac{x_1^B(p_1,p_2)}{\dfrac{1-p_1x_1^B(p_1,p_2)}{p_2}} = \dfrac{p_2x_1^B(p_1,p_2)}{1-p_1x_1^B(p_1,p_2)} = \dfrac{1}{t^B(p_1,p_2)} > 1$, where $t^B(p_1,p_2)$ is

defined as $t^B(p_1,p_2) \equiv \dfrac{1-p_1x_1^B(p_1,p_2)}{p_2x_1^B(p_1,p_2)}$. ■

5.4. Weakly homothetic, single-self preferences

Definition. The *ex ante*, single-self utility function $U:\Re_+^2 \times \Delta^1 \to \Re$ is weakly homothetic[9] in (x_1,x_2) if

$$\begin{aligned}&\operatorname{sgn}(U(x_1^0,x_2^0,p_1,p_2)-U(x_1^1,x_2^1,p_1,p_2)) = \\ &\operatorname{sgn}(U(x_1^0+\delta,x_2^0+\delta,p_1,p_2)-U(x_1^1+\delta,x_2^1+\delta,p_1,p_2)), \forall\delta>0\end{aligned}.$$

Lemma 3. Let the *ex ante*, single-self utility function $U:\Re_+^2 \times \Delta^1 \to \Re$ be weakly homothetic in (x_1,x_2), and let U display an amount effect for (p_1,p_2) and $\overline{E}>0$, with associated $\varepsilon(p_1,p_2,\overline{E})$. Then, for all $E > p_2\varepsilon(p_1,p_2,\overline{E})$, U displays an amount effect for (p_1,p_2) and E, with $\varepsilon(p_1,p_2,E)=\varepsilon(p_1,p_2,\overline{E})$.

Proof. It parallels that of Lemma 2. Let U be weakly homothetic and display an amount effect for $(p_1,p_2)\in\Delta^1$ and $\overline{E}>0$. Let (x_1,x_2) satisfy $x_2>x_1$ and $p_1x_1+p_2x_2=E>0$. Then

$$\begin{aligned}&\operatorname{sgn}(U(x_1,x_2,p_1,p_2)-U(E,E,p_1,p_2))\\ &\quad= \operatorname{sgn}(U(x_1+\overline{E}-E,x_2+\overline{E}-E,p_1,p_2)-U(E+\overline{E}-E,E+\overline{E}-E,p_1,p_2))\\ &\qquad\qquad\text{[by weak homotheticity]}\\ &\quad= \operatorname{sgn}(x_1+\overline{E}-E+\varepsilon(p_1,p_2,\overline{E})-[x_2+\overline{E}-E])\\ &\qquad\qquad\text{[because } p_1[x_1+\overline{E}-E]+p_2[x_2+\overline{E}-E]=\overline{E}\text{, and}\\ &\qquad\qquad U \text{ displays an amount effect for } \overline{E}\text{]}\\ &\quad= \operatorname{sgn}(x_1+\varepsilon(p_1,p_2,\overline{E})-x_2).\end{aligned}$$

This proves (i) in the definition of the amount effect. To prove (ii), note that, by the statement of the lemma, $E > p_2\varepsilon(p_1,p_2,\overline{E})$, i.e., $(0, \dfrac{E}{p_2})$ satisfies: $0 + \varepsilon(p_1,p_2,\overline{E}) - \dfrac{E}{p_2} < 0$, and

[9] This term is inspired by John Chipman (1965, p. 691).

$p_1 \times 0 + p_2 \dfrac{E}{p_2} = E$, i.e., $(0, \dfrac{E}{p_2}) \in F(p_1, p_2, E)$. Thus U displays an amount effect for (p_1, p_2) and E, with $\varepsilon(p_1, p_2, E) = \varepsilon(p_1, p_2, \overline{E})$. ■

Proposition 4. Let the *ex ante*, single-self utility function $U : \Re_+^2 \times \Delta^1 \to \Re$ be weakly homothetic in (x_1, x_2) and satisfy Assumption 2. Then U displays an amount effect for $(p_1, p_2) \in \Delta^1$ and E if and only if the equation in $\overline{\varepsilon}$

$$\text{“}U(0, \overline{\varepsilon}, p_1, p_2) = U(p_2\overline{\varepsilon}, p_2\overline{\varepsilon}, p_1, p_2)\text{”}$$

has a positive solution, in which case the solution is unique and denoted $\varepsilon^A(p_1, p_2) > 0$, and $E > p_2\varepsilon^A(p_1, p_2)$, in which case $\varepsilon(p_1, p_2, E) = \varepsilon^A(p_1, p_2)$.

Proof. To show the uniqueness of the solution under weak homotheticity and Assumption 2, suppose as contradiction hypothesis that two such solutions $\overline{\varepsilon}$ and $\overline{\overline{\varepsilon}}$ exist, with $\overline{\overline{\varepsilon}} > \overline{\varepsilon}$. By weak homotheticity, and noting that $p_2\overline{\varepsilon} + p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}) = p_2\overline{\overline{\varepsilon}}$, we have that $U(p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}), \overline{\varepsilon} + p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}), p_1, p_2) = U(p_2\overline{\overline{\varepsilon}}, p_2\overline{\overline{\varepsilon}}, p_1, p_2)$, and, because $\overline{\overline{\varepsilon}}$ solves the equation, $U(0, \overline{\overline{\varepsilon}}, p_1, p_2)$ also equals $U(p_2\overline{\overline{\varepsilon}}, p_2\overline{\overline{\varepsilon}}, p_1, p_2)$. But

$$\begin{aligned} &p_1\left[p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon})\right] + p_2\left[\overline{\varepsilon} + p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon})\right] = (1 - p_2)p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}) + p_2\overline{\varepsilon} + (p_2)^2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}) \\ &= p_2\overline{\overline{\varepsilon}} - p_2\overline{\varepsilon} - (p_2)^2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}) + p_2\overline{\varepsilon} + (p_2)^2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}) = p_2\overline{\overline{\varepsilon}} \end{aligned},$$

i.e., point $(p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}), \overline{\varepsilon} + p_2(\overline{\overline{\varepsilon}} - \overline{\varepsilon}))$ is on the fair-odds line that goes through points $(0, \overline{\overline{\varepsilon}})$ and $(p_2\overline{\overline{\varepsilon}}, p_2\overline{\overline{\varepsilon}})$, contradicting the strict concavity of U above the certainty line by the argument in the proof of Lemma 3.

To prove the “if” part of the proposition, assume that such an $\varepsilon^A(p_1, p_2) > 0$ exists and choose an $\overline{E} > p_2\varepsilon^A(p_1, p_2)$. We compute

$$\begin{aligned} &\operatorname{sgn}(U(\overline{E} - p_2\varepsilon^A, \overline{E} + p_1\varepsilon^A, p_1, p_2) - U(\overline{E}, \overline{E}, p_1, p_2)) \\ &= \operatorname{sgn}(U(\overline{E} - p_2\varepsilon^A + p_2\varepsilon^A - \overline{E}, \overline{E} + p_1\varepsilon^A + p_2\varepsilon^A - \overline{E}, p_1, p_2) \\ &\qquad - U(\overline{E} + p_2\varepsilon^A - \overline{E}, \overline{E} + p_2\varepsilon^A - \overline{E}, p_1, p_2)) \qquad \text{[by weak homogeneity]} \\ &= \operatorname{sgn}(U(0, \varepsilon^A, p_1, p_2) - U(p_2\varepsilon^A, p_2\varepsilon^A, p_1, p_2)) \\ &= 0. \qquad \text{[by the definition of } \varepsilon^A(p_1, p_2)\text{]} \end{aligned}$$

Moreover, the point $(\overline{E}-p_2\varepsilon^A, \overline{E}+p_1\varepsilon^A)$ is in the segment joining $(\overline{E},\overline{E})$ and $(0,\frac{\overline{E}}{p_2})$, a subset of the fair-odds line through these points, because $p_1(\overline{E}-p_2\varepsilon^A)+p_2(\overline{E}+p_1\varepsilon^A)=\overline{E}$. Hence, the strict concavity of U above the certainty line implies that

$U(x_1,\frac{\overline{E}-p_1x_1}{p_2},p_1,p_2)>U(\overline{E},\overline{E},p_1,p_2)), \forall x_1\in(\overline{E}-p_2\varepsilon^A,\overline{E})$, while

$U(x_1,\frac{\overline{E}-p_1x_1}{p_2},p_1,p_2)<U(\overline{E},\overline{E},p_1,p_2)), \forall x_1\in[0,\overline{E}-p_2\varepsilon^A)\neq\varnothing$, proving the presence of an amount effect for (p_1, p_2) and $\overline{E}$, and, by Lemma 3, for (p_1, p_2) and any $E>p_2\varepsilon^A(p_1,p_2)$, with $\varepsilon(p_1,p_2,E)=\varepsilon^A(p_1,p_2)$.

To prove the "only if" part of the proposition, observe that when the equation in $\overline{\varepsilon}$ "$U(0,\overline{\varepsilon},p_1,p_2)=U(p_2\overline{\varepsilon},p_2\overline{\varepsilon},p_1,p_2)$" has no positive solution, it must be the case that, given $E>0$, the expression $U(x_1,\frac{E-p_1x_1}{p_2},p_1,p_2)-U(E,E,p_1,p_2))$ is either always positive for $\forall x_1\in[0,E)$ or always negative there, ruling out the presence of an amount effect. ■

<u>Proposition 5</u>. <u>Let the *ex ante*, single-self utility function</u> $U:\Re^2_+\times\Delta^1\to\Re$ <u>be weakly homothetic in</u> (x_1,x_2) <u>and satisfy Assumptions 1 and 2.</u>

<u>(i) If both</u> $\varepsilon^A(p_1,p_2)>0$ <u>and</u> $\varepsilon^A(p_2,p_1)>0$ <u>exist, as defined in Proposition 4, and</u> $\varepsilon^A(p_1,p_2)\neq\varepsilon^A(p_2,p_1)$, <u>then</u> U <u>displays a switch effect for</u> $(p_1,p_2)\in\Delta^1$ <u>at any</u> (x_1, x_2) <u>satisfying</u>

$$\min\{\varepsilon^A(p_1,p_2),\varepsilon^A(p_2,p_1)\}<x_2-x_1<\max\{\varepsilon^A(p_1,p_2),\varepsilon^A(p_2,p_1)\};$$

<u>(ii) If</u> $\varepsilon^A(p_1,p_2)>0$ <u>exists, as defined in Proposition 4, but the equation in</u> $\overline{\varepsilon}$

$$U(0,\overline{\varepsilon},p_2,p_1)=U(p_1\overline{\varepsilon},p_1\overline{\varepsilon},p_2,p_1)$$

<u>does not have a positive solution (i.e., no</u> $\varepsilon^A(p_2,p_1)>0$ <u>exists), then</u> U <u>displays a switch effect for</u> $(p_1,p_2)\in\Delta^1$ <u>and either any</u> (x_1, x_2) <u>satisfying</u> $0<x_2<\varepsilon^A(p_1,p_2)+x_1$ <u>or any</u> (x_1, x_2) <u>satisfying</u> $\varepsilon^A(p_1,p_2)+x_1<x_2$.

<u>Proof</u>. Easy adaptation of the proof of Proposition 2, using Proposition 4. ■

<u>Proposition 6</u>. <u>Let the *ex ante*, single-self utility function</u> $U:\Re^2_+\times\Delta^1\to\Re$ <u>be weakly homothetic in</u> (x_1,x_2) <u>and satisfy Assumptions 1 and 2.</u>

If the equation in $\overline{\overline{\varepsilon}}$ "$U(\overline{\overline{\varepsilon}},0,p_1,p_2)=U(p_1\overline{\overline{\varepsilon}},p_1\overline{\overline{\varepsilon}},p_1,p_2)$" has a positive solution, then such solution is unique, to be denoted $\varepsilon^B(p_1,p_2)$, and it satisfies $U(\varepsilon^B(p_1,p_2),0,p_2,p_1)=U(p_2\varepsilon^B(p_1,p_2),p_2\varepsilon^B(p_1,p_2),p_2,p_1)$, i.e., $\varepsilon^B(p_1,p_2)=\varepsilon^A(p_2,p_1)$ as defined in Proposition 4.

Proof. Uniqueness follows from the concavity argument in the proof of Proposition 1. Let $\varepsilon^B(p_1,p_2)$ satisfy $U(\varepsilon^B(p_1,p_2),0,p_1,p_2)=U(p_1\varepsilon^B(p_1,p_2),p_1\varepsilon^B(p_1,p_2),p_1,p_2)$. By state independence, $U(0,\varepsilon^B(p_1,p_2),p_2,p_1)=U(\varepsilon^B(p_1,p_2),0,p_1,p_2)$ and $U(p_2\varepsilon^B(p_1,p_2),p_2\varepsilon^B(p_1,p_2),p_2,p_1)=U(p_2\varepsilon^B(p_1,p_2),p_2\varepsilon^B(p_1,p_2),p_1,p_2)$. Thus, $U(0,\varepsilon^B(p_1,p_2),p_2,p_1)=U(p_2\varepsilon^B(p_1,p_2),p_2\varepsilon^B(p_1,p_2),p_2,p_1)$, i.e., $\varepsilon^A(p_2,p_1)=\varepsilon^B(p_1,p_2)$. ■

Proposition 7. Let the *ex ante*, single-self utility function $U:\Re^2_+\times\Delta^1\rightarrow\Re$ be weakly homothetic in (x_1,x_2) and satisfy Assumptions 1 and 2, and let (z, w) satisfy $w+z\geq 0$, and $w-z\geq 0$. The individual displays risk aversion (resp. attraction) in choice $< z, p \mid w >$ if and only if she displays risk aversion (resp. attraction) in choice $t(< z, p \mid w >) = < -z, 1-p \mid w >$.

Proof. Without loss of generality, let $z < 0$ and let the individual display risk aversion in choice $< z, p \mid w >$, i.e., $U(w+z, w, p, 1-p) < U(w+pz, w+pz, p, 1-p)$. By Proposition 4, this implies that $w-(w+z)<\varepsilon^A(p,1-p)$, i. e., $0<-z<\varepsilon^A(p,1-p)$. But then $w-z-w<\varepsilon^A(p,1-p)=\varepsilon^B(1-p,p)$, by Proposition 6. Thus, $U(w-z, w, 1-p, p) < U(w-(1-p)z, w-(1-p)z, 1-p, p)$, i. e., the individual displays risk aversion in choice $< -z, 1-p \mid w >$. The cases where $z > 0$ and/or the individual displays risk attraction in choice $< z, p \mid w >$ are similarly argued. ■

Remark 1. Proposition 7 shows that a translation never affects risk attitude if preferences are single-self and weakly homothetic. This is a stronger property that the one mentioned in Section 5.1 above, namely that, if preferences are single self, then an attitude reversal due to translation cannot occur over a range of initial wealth values.

5.5. A class of single-self preferences

We illustrate the possibility of single-self *ex ante* preferences displaying the various effects by exhibiting examples of such preferences. Our examples belong to the following class of *ex ante* utility functions

$$U:\Re_+^2\times\Delta^1\to\Re:U(x_1,x_2,p_1,p_2)=\begin{cases}\psi_1^A(p_1)u(x_1)+\psi_2^A(p_2)u(x_2), & for \quad x_2\geq x_1\geq 0,\\ \psi_1^B(p_1)u(x_1)+\psi_2^B(p_2)u(x_2), & for \quad x_1>x_2\geq 0,\end{cases} \qquad (4)$$

where $u'' < 0$, guaranteeing that Assumption 2 is satisfied, and where the superscripts A and B suggest, respectively, "*above*" and "*below*" the certainty line of the contingent-consumption space (x_1, x_2), and where, for $s = 1, 2$, and for $J = A, B$, $\psi_s^J:[0,1]\to[0,1]$. Following tradition, we can think of the ψ_s^J functions as "distortions of probability" or, *à la* Kahneman and Tversky, as "decision weight functions," but any such interpretation is orthogonal to the examples: the essential point is that, in our examples, the ψ_s functions are nonlinear, and hence the function U represents preferences that violate the expected utility hypothesis, yet they are of the single-self type, i.e., well-defined on lotteries with final wealth balances x as prizes.

We restrict ourselves to well-behaved preferences, in the sense that, for $J = A, B$, the functions ψ_1^J and ψ_2^J are continuous on [0, 1] and differentiable on (0, 1), with $\psi_s^J(0)=0$ and $\psi_s^J(1)=1$, and ψ_1^J and ψ_2^J satisfy Assumptions 3-5 below. In some of our examples, $\psi_s^A=\psi_s^B$, implying that U is differentiable at the certainty line, whereas in others, $\psi_s^A\neq\psi_s^B$, in which case U may have a kink at the certainty line.

Assumption 3: Adding-up property. For $J = A, B$, $\psi_1^J(p)+\psi_2^J(1-p)=1, \forall p\in[0,1]$.

The adding-up property ensures that

$$\begin{aligned}&\forall p\in[0,1], \lim_{\substack{(x_1,x_2)\to(x,x)\\ x_1>x_2}} \psi_1^B(p)u(x_1)+\psi_2^B(1-p)u(x_2)\\ &=\psi_1^B(p)u(x_1)+\psi_2^B(1-p)u(x_2)=u(x)=\psi_1^A(p)u(x)+\psi_2^A(1-p)u(x)\end{aligned},$$

which in particular implies that the function U is continuous at the certainty line, even when $\psi_s^A\neq\psi_s^B$, and that $\forall p\in[0,1], U(x,x,p,1-p)=u(x)$, i.e., the probabilities of the states do not matter when the outcomes are the same.

Remark 2. Note the contrast with Kahneman and Tversky (1979, p. 281), who write "…there is evidence to suggest that, for all $0 < p < 1$, $\pi(p) + \pi(1-p) < 1$. We label this property subcertainty." (Their π notation corresponds to our ψ's.) If we write $U(x_1,x_2,p,1-p)=\pi(p)u(x_1)+\pi(1-p)u(x_2)$, then we must have $U(x,x,p,1-p)=[\pi(p)+\pi(1-p)]u(x)$, and the so defined subcertainty yields, for $p\in(0,1)$,

$U(x,x,p,1-p) < u(x)$. But if $\pi(1) = 1$ and $\pi(0) = 0$, then $U(x,x,p,1,0) = \pi(1)u(x) + \pi(0)u(x) = u(x)$, i.e., the utility of x with probability 1 is higher than the utility of a lottery that gives x no matter what! For these reasons, we view Assumption 3 as capturing the well-behavedness of preferences.

The adding-up property can be rewritten $\psi_2^J(1-p) = 1 - \psi_1^J(p)$, $J = A, B$, yielding

$$\left.\frac{d\psi_2^J}{dp_2}\right|_{1-p} = \left.\frac{d\psi_1^J}{dp_1}\right|_{p} . \qquad (5)$$

Assumption 4: $\psi_1^B = \psi_2^A$ and $\psi_2^B = \psi_1^A$ equalities. The functions ψ_1^B and ψ_2^A are the same function, i.e., $\psi_1^B(q) = \psi_2^A(q)$, $\forall\, q \in [0,1]$, and, therefore, ψ_1^B transforms p_1 in the same manner as ψ_2^A transforms p_2; similarly, ψ_2^B and ψ_1^A are the same function.

Proposition 8. Let U be of the form (4). Then state independence (Assumption 1) is equivalent to Assumption 4.

Proof. Postulate form (4). Assumption 4 then guarantees Assumption 1, because $U(y,y',p,p') \equiv \psi_1^A(p)u(y) + \psi_2^A(p')u(y') = \psi_1^B(p')u(y') + \psi_2^B(p)u(y) = U(y',y,p',p)$. Conversely, without loss of generality assume that $y' > y$, i.e., $y' = y + \varepsilon$, $\varepsilon > 0$. Assumption 1 guarantees that $U(y, y + \varepsilon, p, p') = U(y + \varepsilon, y, p', p)$, which under (4) can be written $\psi_1^A(p)u(y) + \psi_2^A(p')u(y+\varepsilon) = \psi_1^B(p')u(y+\varepsilon) + \psi_2^B(p)u(y)$, i.e., $[\psi_1^A(p) - \psi_2^B(p)]u(y) = [\psi_1^B(p') - \psi_2^A(p')]u(y+\varepsilon)$. Differentiating both sides with respect to ε yields $0 = [\psi_1^B(p') - \psi_2^A(p')]u'(y+\varepsilon)$, which if evaluated at a point where $u'(y + \varepsilon) \neq 0$ implies $0 = \psi_1^B(p') - \psi_2^A(p')$, and, in turn, $\psi_1^A(p) = \psi_2^B(p)$. ■

It follows from Assumption 4 that ψ_1^A and ψ_1^B are the same function (guaranteeing the differentiability of U at the certainty line) if and only if ψ_1^A and ψ_2^A are also the same function, i.e., they are all the same function.

Assumption 5: Monotonicity. For $J = A, B$, and $p_1 \in (0, 1)$, $\frac{d\psi_1^J}{dp_1} > 0$.

It can be easily shown that, in conjunction with (5), Assumption 5 implies that $\frac{d\psi_2^J}{dp_2} > 0$, $\forall\, p_2 \in (0, 1)$, and that utility increases as the good outcome becomes more likely. Indeed, write $\widetilde{U}(x_1, x_2, p) \equiv U(x_1, x_2, p, 1-p)$, and compute $\frac{\partial \widetilde{U}}{\partial p} = \frac{\partial U}{\partial p_1} - \frac{\partial U}{\partial p_2} = \frac{d\psi_1^J}{dp_1} u(x_1) - \frac{d\psi_2^J}{dp_2} u(x_2) =$

$\frac{d\psi_1^J}{dp_1}(u(x_1) - u(x_2))$, by (5). If $x_1 < x_2$, then x_1 is the *bad* outcome, $J = A$, $u(x_1) - u(x_2) < 0$, and utility *decreases* with the probability (p_1 or p) of x_1. If, on the contrary, $x_1 > x_2$, then x_1 is the *good* outcome, $J = B$, $u(x_1) - u(x_2) > 0$, and utility *increases* with the probability of x_1.

Assumptions 3 and 4 imply that, given one of the four functions ψ_s^J, the other three are determined, e. g., if ψ_1^A, is given, then ψ_2^A is determined by Assumption 3 as $\psi_2^A(q) = 1 - \psi_1^A(1-q)$, ψ_2^B is determined by Assumption 4 as $\psi_2^B(q) = \psi_1^A(q)$, and hence, using Assumption 3 once more, ψ_1^B is determined by $\psi_1^B(q) = 1 - \psi_1^A(1-q)$, see Figures 6 and 7 below.

Let x_1 be the bad outcome ($x_1 < x_2$). If $\psi_1^A(p_1) > p_1$, then $p_1(x_1 - x_2) > \psi_1^A(p_1)(x_1 - x_2)$. Thus,

$$
\begin{aligned}
u(p_1x_1 + (1-p_1)x_2) = u(p_1(x_1 - x_2) + x_2) &> u(\psi_1^A(p_1)(x_1 - x_2) + x_2) \quad \text{[as long as } u \text{ is increasing]} \\
&= u(\psi_1^A(p_1)x_1 + (1 - \psi_1^A(p_1))x_2) \\
&\geq \psi_1^A(p_1)u(x_1) + (1 - \psi_1^A(p_1)u(x_2) \qquad \text{[by concavity]} \\
&= \psi_1^A(p_1)u(x_1) + \psi_2^A(1-p_1)u(x_2) \qquad \text{[by Assumption 3],}
\end{aligned}
$$

i.e., if $\psi_1^A(p_1) > p_1$, then the individual displays risk aversion above the certainty line. If we interpret $\psi_1^A(p_1)$ as a distortion of the true probability p_1, and if $\psi_1^A(p_1) > p_1$ and $x_1 < x_2$, then the probability of the bad outcome x_1 is *distorted upwards* leading to risk *aversion* above the certainty line. If, on the contrary, $\psi_1^A(p_1) < p_1$, then the probability of x_1 is *distorted downwards* when x_1 is the bad outcome, leading to risk *attraction* for small deviations from certainty, i.e., for points close to (and above) the certainty line, because the (one-sided) slope of the indifference curve at the certainty line is $\frac{\psi_1^A(p_1)}{1 - \psi_1^A(p_1)} < \frac{p_1}{1 - p_1}$ there. (The slope is one-sided because the indifference curve may possibly have a kink at the certainty line.) For larger amounts, risk attitude will depend on the relative strengths of the curvature of u and the gap between p_1 and its "distortion," and an amount effect occurs if, in addition, there is risk *aversion* for larger deviations from certainty.

We focus on two special types of ψ functions, which do not exhaust the possibilities for functions satisfying Assumptions 3-5.

Type I. Defined by the inequality $\psi_1^A(p_1) < p_1, \forall p_1 \in (0,1)$.

It follows from Assumption 3 that $\psi_2^A(p_2) > p_2$, and hence, from Assumption 4, that $\psi_1^B(p_1) > p_1$ and $\psi_2^B(p_2) < p_2$. Type I is inspired by John Quiggin (1982, 1993), and Faruk Gul (1991). Intuitively, the individual systematically distorts the probability of the bad event downwards, i.e., $\psi_1^A(p_1) < p_1$ and $\psi_2^B(p_2) < p_2$. This implies that $\psi_s^A \neq \psi_s^B$, leading to kinks of U at certainty line. Consider the numerical example

$$\psi_1^A(p_1) = 1 - \sqrt{1-p_1}\,, \qquad \psi_2^A(p_2) = \sqrt{p_2}\,,$$

$$\psi_1^B(p_1) = \sqrt{p_1}\,, \qquad \psi_2^B(p_2) = 1 - \sqrt{1-p_2}\,, \tag{6}$$

which clearly satisfies Assumptions 3-5. See Figure 6.

Type II. Defined by the following condition: there exists a $\overline{p} \in (0,1)$ such that $\psi_1^A(p) > p, \forall p \in (0, \overline{p})$ and $\psi_1^A(p) < p, \forall p \in (\overline{p}, 1)$.

It follows that from Assumption 3 that $\psi_2^A(p_2) < p_2, \forall p_2 \in (1-\overline{p}, 1)$, and $\psi_2^A(p_2) > p_2, \forall p_2 \in (0, 1-\overline{p})$, with the analogous implications for ψ_1^B and ψ_2^B.

Type II is inspired by Kahneman and Tversky's interpretation of the distortion of probabilities: small probabilities are distorted upwards, and large probabilities downwards, but what is large or small may depend on whether the outcome is good or bad.

For instance, for $a > 0$ and $b > 0$, we may consider functions in the family

$$\psi_1^A(p_1) = \frac{a\sqrt{p_1}}{a\sqrt{p_1} + b\sqrt{1-p_1}}\,, \qquad \psi_2^A(p_2) = \frac{b\sqrt{p_2}}{a\sqrt{1-p_2} + b\sqrt{p_2}}\,,$$

$$\psi_1^B(p_1) = \frac{b\sqrt{p_1}}{a\sqrt{1-p_1} + b\sqrt{p_1}}\,, \qquad \psi_2^B(p_2) = \frac{a\sqrt{p_2}}{a\sqrt{p_2} + b\sqrt{1-p_2}}\,, \tag{7}$$

which clearly satisfy Assumptions 3-5, see Figure 7. Note that, as long as $a \neq b$, the functions ψ_1^A and ψ_1^B are different, and U is not differentiable at the certainty line. Differentiability requires $a = b$, in which case all four functions coincide, i.e., $\psi_1^A(q) = \psi_2^A(q) = \psi_1^B(q) = \psi_2^B(q) = \frac{\sqrt{q}}{\sqrt{q} + \sqrt{1-q}}$.

5.6. A special case of homotheticity

Let $u(x) = \frac{1}{1-r}x^{1-r}, r > 0, r \neq 1$. Then, for U of the form (4), we have that

$U(x_1, x_2, p_1, p_2) = \psi_1^J(p_1)\frac{x_1^{1-r}}{1-r} + \psi_2^J(p_2)\frac{x_2^{1-r}}{1-r}$, where $J = A$ if $x_2 \geq x_1$, and $J = B$ otherwise. Clearly, U is homothetic in (x_1, x_2). Similarly, if $u(x) = \ln x$, then $U(x_1, x_2, p_1, p_2) = \psi_1^J(p_1)\ln x_1 + \psi_2^J(p_2)\ln x_2$, which is also homothetic in (x_1, x_2).

As noted in Section 4.3 above, if $\frac{1}{1-r}x^{1-r}$ (resp. ln (x)) were the vNM utility function of preferences satisfying the expected utility hypothesis, then they would exhibit CRRA, with coefficient of RRA equal to $r \neq 1$ (resp. one). But here the expected utility hypothesis is violated, because the ψ_s^J functions are nonlinear. Yet, Lemma 2 above parallels the fact that, in the CRRA expected utility case, the wealth expansion paths are rays through the origin.[10] Of course, under expected utility and risk aversion, risk taking requires favorable odds, while here we focus on some forms of risk taking under fair odds.

Will our U display an amount effect? Given homotheticity and Lemma 2, the answer depends on whether the equation in x "$\psi_1^A(p_1)u(1) + \psi_2^A(1-p_1)u(t) = u(p_1 + (1-p_1)t)$" has a solution with $t > 1$. Examples 1 and 2 below illustrate the possibility of amount and switch effects.

5.7. Example 1. Homotheticity with Type-I ψ function

We take a specific function of the class discussed in Section 5.6, namely $u(x) = -x^{-1}$ ($r = 2$), and the Type-I ψ function given by (6) above. By Propositions 1-3, we focus on the equation in t^A

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[10] This is in general true for preferences which are homothetic in (x_1, x_2).

$$\psi_1^A(p_1)u(1)+\psi_2^A(1-p_1)u(t^A)=u(p_1+(1-p_1)t^A)\,, \tag{8}$$

and the equation in t^B

$$\psi_1^B(p_1)u(1)+\psi_2^B(1-p_1)u(t^B)=u(p_1+(1-p_1)t^B)\,. \tag{9}$$

Here, (8) becomes $(1-\sqrt{1-p_1})+\sqrt{1-p_1}\,\frac{1}{t^A}=\frac{1}{p_1+(1-p_1)t^A}$,

with solution $t^A(p_1,1-p_1)=\frac{p_1}{p_1-1+\sqrt{1-p_1}}>1$, implying that there is an amount effect at all probabilities.

For (9), we write $\sqrt{p_1}+(1-\sqrt{p_1})\frac{1}{t^B}=\frac{1}{p_1+(1-p_1)t^B}$,

with solution $t^B(p_1,1-p_1)=\frac{\sqrt{p_1}}{1+\sqrt{p_1}}<1$. Moreover, $t^B(p_1,1-p_1)\neq\frac{1}{t^A(p_1,1-p_1)}$, except when $p_1 = 0.5$, where no switch effect may occur by definition. Thus, a switch effect is present for a range of points (x_1, x_2) at all prices, except of course at $p_1 = 0.5$.

Figure 8 depicts the indifference curves in contingent consumption space for $p_1 = 0.2$ and hence $p_2 = 0.8$. A fair-odds line has then slope – ¼. At the certainty line, an indifference curve has the one-sided slope from above of $\frac{\psi_1^A(p_1)}{1-\psi_1^A(p_1)}=\frac{\psi_1^A(0.2)}{1-\psi_1^A(0.2)}=\frac{0.106}{0.894}=0.118<\ 0.25=\frac{p_1}{1-p_1}$.

Thus, the individual of this example will take small fair risks, where x_2 is greater than, and close to, x_1. Similarly, the slope from below of an indifference curve at the certainty line is $\frac{\psi_1^B(p_1)}{1-\psi_1^B(p_1)}=\frac{\sqrt{0.2}}{1-\sqrt{0.2}}=\frac{0.447}{0.553}=0.809>0.25=\frac{p_1}{1-p_1}$, i.e., the individual will take small fair risks, where x_2 is less than, and close to, x_1.

We compute:

$t^A(0.2,0.8)=2.118, t^A(0.8,0.2)=3.236, t^B(0.8,0.2)=0.472, t^B(0.2,0.8)=0.309$, where we can check that $t^B(0.8,0.2)=\frac{1}{t^A(0.2,0.8)}$ and $t^B(0.2,0.8)=\frac{1}{t^A(0.8,0.2)}$, in accordance with Proposition 3. Thus, for $(p_1, p_2) = (0.2, 0.8)$, the individual takes the fair risk, hence displaying risk attraction, if $1<\frac{x_2}{x_1}<t^A(0.2,0.8)=\frac{1}{t^B(0.8,0.2)}=2.118$, whereas she chooses the certain outcome, thus

displaying risk aversion, if $\frac{x_2}{x_1} > 2.118$. In other words, when $(p_1, p_2) = (0.2, 0.8)$, there is an attraction-aversion boundary (AAB) above the certainty line given by the ray "$x_2 = 2.118\, x_1$," see Figure 8. This shows the presence of an amount effect for $(p_1, p_2) = (0.2, 0.8)$.

On the other hand, for $(p_1, p_2) = (0.8, 0.2)$, the individual takes the fair risk, thus displaying risk attraction, if $1 < \frac{x_2}{x_1} < t^A(0.8,0.2) = \frac{1}{t^B(0.2,0.8)} = 3.236$, whereas she chooses the certain outcome, thus displaying risk aversion, if $\frac{x_2}{x_1} > 3.236$. Again, we have an amount effect. Figure 8 also displays, as a dashed ray, the AAB above the certainty line for $(p_1, p_2) = (0.8, 0.2)$, although it should be noted that the indifference curves of Figure 8 are drawn for $(p_1, p_2) = (0.2, 0.8)$, and are not relevant for $(p_1, p_2) = (0.8, 0.2)$.

The lack of coincidence between the AAB for $(p_1, p_2) = (0.2, 0.8)$ and for $(p_1, p_2) = (0.8, 0.2)$ implies a switch effect. Indeed, the uncertain alternative represented by a point in the cone $\{(x_1, x_2) \in \Re^2_+ : 2.118x_1 < x_2 < 3.236x_1\}$ is preferred to its certain expected value when the probabilities are $(p_1, p_2) = (0.8, 0.2)$ (because such point is below the AAB for $(p_1, p_2) = (0.8, 0.2)$), hence displaying risk attraction in that choice, whereas when the probabilities are $(p_1, p_2) = (0.2, 0.8)$ then the point lies above the AAB for $(p_1, p_2) = (0.2, 0.8)$, and corresponds to risk aversion. In other words, for points in that cone, increasing the probability of the bad outcome (which is x_1 for points above the certainty line) from 0.2 to 0.8 leads the individual to switch from risk aversion to risk attraction, in line with our experimental results.

Because of state independence, the graphics below the certainty line exactly correspond to those above it. For $(p_1, p_2) = (0.2, 0.8)$, there is risk attraction if $1 > \frac{x_2}{x_1} > t^B(0.2,0.8) = \frac{1}{t^A(0.8,0.2)} = 0.309$, and risk aversion if $0 < \frac{x_2}{x_1} < 0.309$, whereas for $(p_1, p_2) = (0.8, 0.2)$, there is risk attraction if $1 > \frac{x_2}{x_1} > t^B(0.8,0.2) = \frac{1}{t^A(0.2,0.8)} = 0.472$, and risk aversion if $0 < \frac{x_2}{x_1} < 0.472$. Again, for $0.309 < \frac{x_2}{x_1} < 0.472$, the risk attitude switches from aversion to attraction when the probability p_2 of the bad outcome (which is x_2 for points below the certainty line) increases from 0.2 to 0.8.

If we maintain (p_1, p_2) = (0.2, 0.8), which are the probabilities for which the indifference curves of Figure 8 have been drawn, then risk attraction occurs in the cone $\{(x_1, x_2) \in \Re_+^2 : 2.118x_1 > x_2 > 0.309x_1, x_1 \neq x_2\}$. At these probabilities, all the points in the line going through A, B, C have the same expected value, or, in other words, belong to the same fair-odds line, and points A, B and C are also on the same indifference curve. Restricting our attention to points on that fair-odds line, the individual prefers those inside the cone (i.e., between points A and C or between C and B), to point C, thus displaying risk attraction in these choices, while she prefers point C to points to the left of A or to the right of B, displaying risk aversion there.

Summarizing, Example 1 exhibits the following features.

Amount effect. At all levels of certain outcomes *E*, and both for low (0.2) and for high (0.8) probability of the bad outcome, the individual displays risk attraction for small deviations from certainty and risk aversion for larger deviations,

Switch effect. At all levels of certain outcomes, there are some pairs of outcomes for which the individual displays risk aversion when the probability of the bad outcome is low yet risk attraction when the probability of the bad outcome is high.

The attraction-aversion boundaries are rays through the origin. This is an implication of homotheticity, see Proposition 1. Given a probability pair, the individual displays risk attraction if the ratio of the good outcome to the bad outcome is lower than a certain number, and risk aversion if it is higher.

5.8. Example 2. Homotheticity with Type-II ψ function.

Again, let $u(x) = -x^{-1}$, and let the four functions $\psi_1^A, \psi_2^A, \psi_1^B$ and ψ_2^B be the same function ψ defined by $\psi(p) = \dfrac{\sqrt{p}}{\sqrt{p} + \sqrt{1-p}}$, which is (7) for $a = b$. Because the ψ function is the same above or below the certainty line, the indifference curves are smooth there. Again, it can be easily checked that Assumptions 1-3 above are satisfied.

Now equations (8) and (9) become the same equation

$$\frac{\sqrt{p_1}}{\sqrt{p_1} + \sqrt{1-p_1}} + \frac{\sqrt{1-p_1}}{\sqrt{p_1} + \sqrt{1-p_1}} \frac{1}{t} = \frac{1}{p_1 + (1-p_1)t},$$

with solution $t^A(p_1,1-p_1)=t^B(p_1,1-p_1)=\frac{\sqrt{p_1}}{\sqrt{1-p_1}}\begin{cases}<1 & if \quad p_1<0.5\\ >1 & if \quad p_1>0.5\end{cases}$.

Thus, there is an amount effect (for $x_2 > x_1$) if $p_1 > 0.5$, i.e., if the probability of the bad outcome is high: the attraction-aversion boundary is then given by "$x_2 = \frac{\sqrt{p_1}}{\sqrt{1-p_1}} x_1$." If, for instance, $p_1 = 0.8$, then the AAB is given by $x_2 = 2x_1$, as illustrated in Figure 9, where the indifference curves and the illustrative fair-odds line also take $(p_1, p_2) = (0.8, 0.2)$. Intuitively, because $\psi(0.8) < 0.8$, i. e., when the probability of the bad outcome is a high 0.8, ψ distorts is downwards, attracting to risk: more precisely, the slope of the indifference curve at the certainty line is then $\frac{\psi(0.8)}{\psi(0.2)}$, lower than $\frac{0.8}{0.2}$, the slope of the fair-odds line, implying risk attraction for x_2 above, but close to, x_1.

But if the probability of the bad outcome is low, as in points (x_1, x_2) below the certainty line when $(p_1, p_2) = (0.8, 0.2)$, then the ψ function distorts the probability of the bad outcome upwards, reinforcing the risk aversion favored by the strict concavity of u. Thus, maintaining $(p_1, p_2) = (0.8, 0.2)$, no risk attraction appears below the certainty line: risk attraction only occurs in the cone $\{(x_1, x_2) \in \Re_+^2 : 2x_1 > x_2 > x_1\}$, or, restricting our attention to points on the fair-odds line through points A and C in Figure 9, only the points between A and C are preferred to C.

Summarizing, Example 2 exhibits the following features.

Amount effect. At all levels of certain outcomes, when the probability of the bad outcome is high, then the individual displays risk attraction for small deviations from certainty and risk aversion for larger deviations. But when the probability of the bad outcome is low, then the individual displays risk aversion in all (fair) choices. Thus, an amount effect is present if and only if the probability of the bad outcome is high.

Switch effect. As long as the probabilities are not 50-50, there is a switch effect, because switching from a low probability of the bad outcome to a high probability leads from risk aversion to risk attraction as long as the deviation from certainty is small.

The attraction-aversion boundaries are rays through the origin. Again, because of homotheticity, the attraction-aversion boundary occurs along rays through the origin. In this example, because the indifference curves are smooth at the certainty line, one such boundary is the certainty line itself, and there is only another one, which lies either above or below the certainty line depending on the probabilities.

5.9. A special case of weak homotheticity

Alternatively, let $u(x) = -e^{-rx}, r > 0$. Then $U(x_1, x_2, p_1, p_2) = -\psi_1^J(p_1)e^{-rx_1} - \psi_2^J(p_2)e^{-rx_2}$, where $J = A$ if $x_2 \geq x_1$ and $J = B$ otherwise, which is weakly homothetic in (x_1, x_2). As noted in Section 4.3 above, if u were the vNM utility function of an individual with preferences satisfying the expected utility hypothesis, then she would have CARA, with coefficient of ARA equal to r. Lemma 3 above displays a parallelism with the fact that, in the CARA expected utility case, the wealth expansion paths are straight lines of slope one.

Will our U display an amount effect? Given weak homotheticity and Lemma 3, the answer depends on whether the equation in ε "$\psi_1^A(p_1)u(0) + \psi_2^A(1-p_1)u(\varepsilon) = u((1-p_1)\varepsilon)$" has a solution $\varepsilon^A > 0$. In that case, the individual will display risk attraction for $x_2 \in (x_1, x_1 + \varepsilon^A)$ and risk aversion for $x_2 > x_1 + \varepsilon^A$, as shown in Proposition 4. Propositions 5 and 6, in turn, can be applied to analyze the presence of a switch effect. It is harder to explicitly solve the case $u(x) = -e^{-rx}, r > 0$, with either of our specifications of $\psi_1^A, \psi_2^A, \psi_1^B$ and ψ_2^B, but the following sections offer simple numerical examples for $u(x) = -e^{-x}$ ($r = 1$).

5.10. Example 3. Weak homotheticity with Type-I ψ function

The ψ functions are given by (6), as in Example 1. For $(p_1, p_2) = (0.2, 0.8)$, the indifference curves are depicted in Figure 10.

In accordance with Proposition 4, we compute $\varepsilon^A(0.2,0.8)$ by solving the equation "$-\psi_1^A(0.2)\mathrm{e}^0 - \psi_2^A(0.8)\mathrm{e}^{-\varepsilon} = -\mathrm{e}^{-0.8\varepsilon}$," which yields the solution $\varepsilon^A(0.2,0.8) = 1.404 > 0$, evidencing an amount effect (for $x_2 > x_1$) for $(p_1, p_2) = (0.2, 0.8)$ and for $E > 0.8 \times 1.404$ (as stated in Proposition 4). More specifically, above the certainty line the attraction-aversion boundary for $(p_1, p_2) = (0.2, 0.8)$ is the straight line $x_2 = 1.404 + x_1$, as illustrated in Figure 10, and an amount effect occurs if the fair-odds line hits the vertical axis above $x_2 = 1.404$ (this is the condition $E >$ 0.8×1.404).

Similarly, and in accordance with Proposition 6, we compute $\varepsilon^B(0.2,0.8)$ by solving the equation "$-\psi_1^B(0.2)e^0 - \psi_2^B(0.8)e^{-\varepsilon} = -e^{-0.8\varepsilon}$," which yields the solution $\varepsilon^B(0.2,0.8) = 2.699 > 0$. Thus, below the certainty line the attraction-aversion boundary for $(p_1, p_2) = (0.2, 0.8)$ is the straight line $x_2 = -2.699 + x_1$, as illustrated in Figure 10.

To check that $\varepsilon^A(0.8,0.2) = \varepsilon^B(0.2,0.8)$, as stated in Proposition 6, we compute $\varepsilon^A(0.8,0.2)$ by solving the equation "$-\psi_1^A(0.8)e^0 - \psi_2^A(0.2)e^{-\varepsilon} = -e^{-0.2\varepsilon}$," with solution $\varepsilon^A(0.8,0.2) = 2.699 = \varepsilon^B(0.2,0.8)$. Similarly, by solving "$-\psi_1^B(0.8)e^0 - \psi_2^B(0.2)e^{-\varepsilon} = -e^{-0.2\varepsilon}$" we obtain $\varepsilon^B(0.8,0.2) = 1.404 = \varepsilon^B(0.2,0.8)$. Because $\varepsilon^A(0.2,0.8) \neq \varepsilon^A(0.8,0.2)$, Proposition 5(i) guarantees a switch effect. In order to facilitate the visualization of the bands $\{(x_1,x_2) \in \Re_+^2 : 1.404 + x_1 < x_2 < 2.699 + x_1\}$ (above certainty line) and $\{(x_1,x_2) \in \Re_+^2 : 1.404 + x_2 < x_1 < 2.699 + x_2\}$ }(below certainty line) for which a switch effect occurs, Figure 10 follows Figure 8 by showing as dashed lines the attraction-aversion boundaries for $(p_1, p_2) = (0.8, 0.2)$, even though the indifference curves and the representative fair-odds line are drawn for $(p_1, p_2) = (0.2, 0.8)$. The intuition is similar to that of Example 1 above. The bad outcome is x_1 in the band $\{(x_1,x_2) \in \Re_+^2 : 1.404 + x_1 < x_2 < 2.699 + x_1\}$, whose points are above the attraction-aversion boundary for $(p_1, p_2) = (0.2, 0.8)$ (risk aversion), but below it for $(p_1, p_2) = (0.8, 0.2)$ (risk attraction), i.e., increasing the probability of the bad outcome from 0.2 to 0.8 induces the switch from risk aversion to risk attraction.

If we maintain $(p_1, p_2) = (0.2, 0.8)$, as in the indifference curves and the fair-odds line of Figure 10, then risk attraction occurs in the band $\{(x_1,x_2) \in \Re_+^2 : x_1 - 2.699 < x_2 < x_1 + 1.404, x_1 \neq x_2\}$. All the points in the line going through A, B, C belong to the same fair-odds line, and points A, B and C are also on the same indifference curve. The analysis of risk attitudes along this fair-odds line is essentially that of Figure 8 above.

Summarizing, Example 3 exhibits the following features, which can be compared with those of Example 1.

<u>Amount effect</u>. For sufficiently large levels of certain outcomes *E*, and both for low (0.2) and for high (0.8) probability of the bad outcome, the individual displays risk attraction for small deviations from certainty and risk aversion for larger deviations.

Switch effect. For sufficiently large levels of certain outcomes E, there are some pairs of outcomes with expected value E for which the individual displays risk aversion when the probability of the bad outcome is low yet risk attraction when the probability of the bad outcome is high.

The attraction-aversion boundaries are straight lines of slope one. This is an implication of weak homotheticity, see Proposition 4. Given a probability pair, the individual displays risk attraction if the difference between the good and bad outcomes is lower than a certain number, and risk aversion if it is higher.

5.11. Example 4. Weak homotheticity with Type-II ψ function

Now we combine the function $u(x) = -e^{-x}$ with the ψ functions given by (7) for $a = b$. As in Example 2, an amount effect is present only if the probability of the bad outcome is high, whereas, as in Example 3, the attraction-aversion boundaries, one of which is the certainty line, have a slope of one. Figure 11 illustrates the case where $(p_1, p_2) = (0.8 , 02)$ as in Example 2. The vertical intercept of the higher attraction-aversion boundary is $\varepsilon^A(0.8,0.2) = 1.497$, computed by solving the equation "$-\psi_1^A(0.8)e^0 - \psi_2^A(0.2)e^{-\varepsilon} = -e^{-0.2\varepsilon}$." Now we have the following features.

Amount effect. For sufficiently large levels of certain outcomes E, and for high (0.8) probability of the bad outcome, the individual displays risk attraction for small deviations from certainty and risk aversion for larger deviations. But if the probability of the bad outcome is low, then she displays risk aversion.

Switch effect. For sufficiently large levels of certain outcomes E, switching from a low to a high probability of the bad outcome leads from risk aversion to risk attraction as long as the deviation from certainty is small.

The attraction-aversion boundaries are straight lines of slope one. For a high probability of the bad outcome, the individual displays risk attraction if the difference between the good and bad outcomes is lower than a certain number, and risk aversion if it is higher. But for a low probability of the bad outcome she displays risk aversion and, hence, the certainty line is an attraction-aversion boundary.

Remark 3. In Examples 1-4, it is easy to compute instances of the reflection effect based on the switch effect, rather than on a translation effect (see the discussions in Sections 4.1 and 5.1, and Remark 1 above).

6. Single self vs. expected utility

Our examples in Section 5 imply that the amount and switch effects, as well as some forms of reflection effect, are compatible with single-self preferences. But, as we show in our 2004 paper, the amount, switch and translation effects violate single-self expected utility. First, we note that single-self, expected utility preferences require the vNM utility function $u(x)$ to be locally convex ($u''(x) > 0$) on the interval where the individual is attracted to small risks, and thus $u(x)$ must be convex on that interval. This contradicts the aversion to large risks involving quantities within this interval. Thus, the amount effect is incompatible with single-self, expected utility preferences. Second, it is not difficult to show that if the switch effect changes the risk attitude over a range of wealth levels, then single-self, expected utility (continuous) preferences must be ruled out.[11] Last, as seen in 5.1 above, the translation effect violates single-self preferences and, hence, *a fortiori* single-self, expected-utility preferences.

On the other hand, our 2004 paper also illustrates the consistency of the amount, switch and translation effects with multiple-selves, expected utility preferences by the $u_w(z)$ function reproduced here as Figure 12. First, because the curve is convex close to $z = 0$, and concave away from zero, it entails an amount effect. In addition, there is risk aversion for gains at low probability of the bad state, because $u_w(80) > 0.8\ u_w(100)$. If we switch the probabilities, then we get risk attraction, because $u_w(20) < 0.2\ u_w(100)$. Thus, there is a switch effect for gains. But if we translate gains into losses, at the low probability of the bad state, we get $u_w(-20) < 0.2\ u_w(-100)$, i.e., risk attraction. Thus, there is a translation effect when the probability of the bad state is 0.2.

[11] Assume that, for any $w \in [1000, 1100]$ and any $z \in [0, 100]$, (a) the individual prefers the uncertain gain of z with probability 0.2 to the certain gain of $0.2z$; but (b) she prefers the certain gain of $0.8z$ to the uncertain gain of z with probability 0.8. Under the expected utility hypothesis we can set $u(1000) = 0$, and $u(1100) = 100$. Then (a) implies that $u(1020) < 20$, and (b) that $u(1080) > 80$, which, as long as u is continuous, imply that there is a z' in (20, 80) and a z'' in (80, 100] such that (i) $u(1000 + z') = z'$, (ii) $u(1000 + z'') = z''$, and (iii) $u(1000 + z) > z$, $\forall z \in (z', z'')$. Consider $w \equiv 1000 + z'$ and $z = z'' - z'$. By (a), the individual prefers the uncertain gain of z with probability 0.2 to the certain gain of 0.2 z, i.e., $0.8\ u(1000 + z') + 0.2\ u(1000 + z' + z'' - z') > u(1000 + z' + 0.2(z'' - z'))$, or, using (i)-(ii), $0.8\ z' + 0.2\ z'' > u(1000 + 0.8\ z' + 0.2\ z'')$, contradicting (iii), because $0.8\ z' + 0.2\ z'' \in (z', z'')$.

Thus, (a) and (b) are incompatible with the expected utility hypothesis with single-self, continuous preferences.

To sum up, all three effects contradict single-self, expected utility theory, and none contradicts multiple-selves, expected utility theory. But the translation effect negates the existence of single-self preferences. Table 2 summarizes these results.

	Single-Self Expected Utility (Canonical Eu)	Single-Self Nonexpected Utility	Multiple-Selves "Expected Utility"
Amount Effect	Contradiction	OK	OK
Switch Effect (or reflection due to switch)	Contradiction	OK	OK
Translation Effect (or reflection due to translation)	Contradiction	Contradiction	OK

Table 2. The amount, switch and translation effects vs. single self and expected utility

7. Concluding comments

The core of this paper has focused on single-self, nonexpected utility, convex preferences, that is to say, *ex ante* preferences defined on contingent final money balances (rather than on their changes), representable by a function which is concave in those contingent balances, but which cannot be linear in the probabilities.

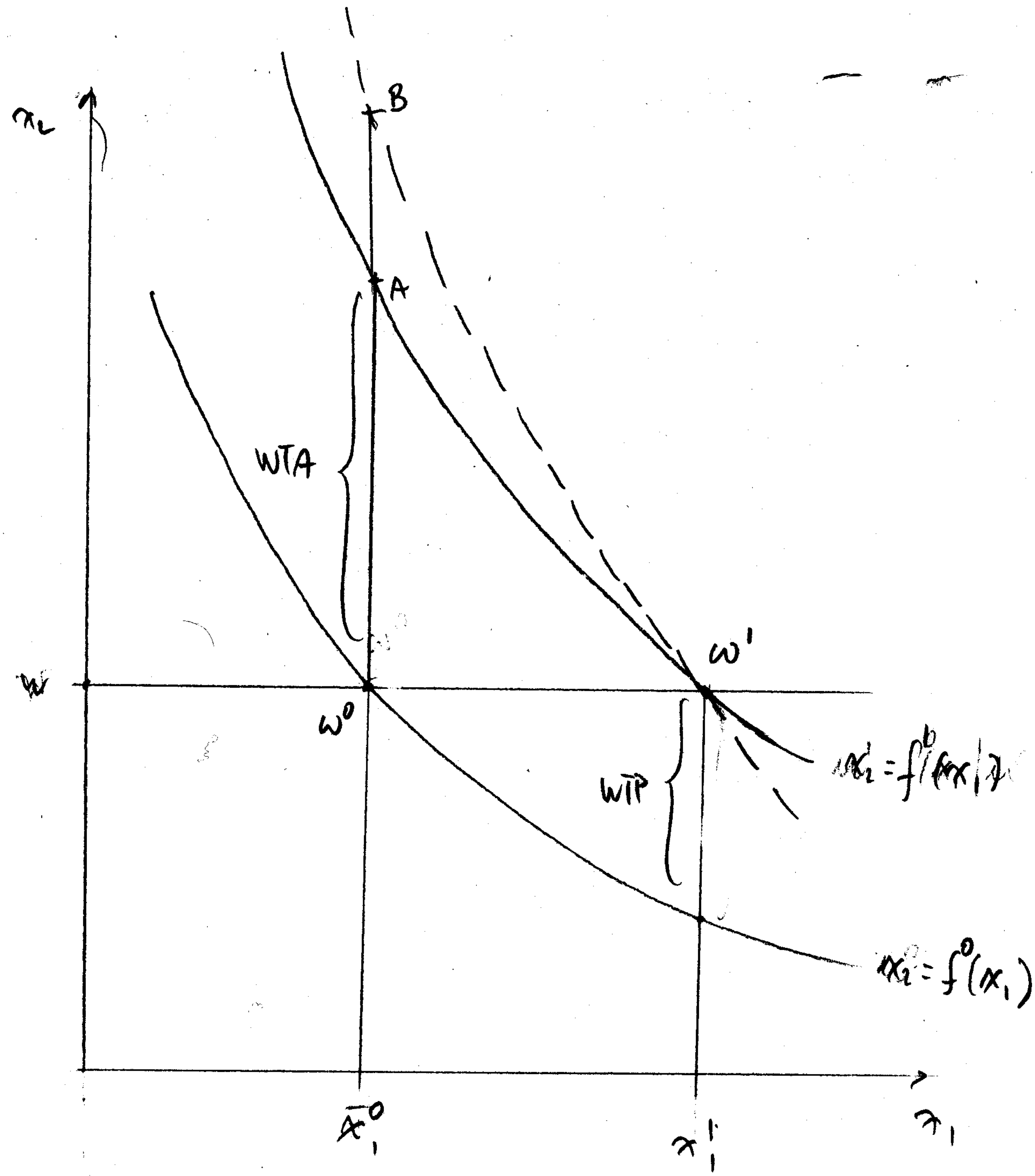
First, we have analyzed two classes of single-self, nonexpected utility preferences that display amount and switch effects, and, therefore, some forms of reflection effects, while allowing for various forms of dependence of risk attitudes on the wealth of the decision maker. We label the two classes homothetic and weakly homothetic: They parallel, respectively, the expected utility cases of Constant Relative Risk Aversion (CRRA) and Constant Absolute Risk Aversion (CARA). It should be emphasized that the preferences discussed in this paper allow for risk attraction, with the individual bearing some amount of fair risk, whereas an individual with expected-utility CRRA or CARA preferences does not take any fair risks, choosing an uncertain prospect only when it is actuarially favorable.

A necessary condition for risk attraction within the preferences described in the paper is that the deviations from certainty be small. Accordingly, when risk attraction is present, two regions appear in the plane of contingent money balances: an attraction region, close to the certainty line, and an aversion region, further away. When preferences are homothetic, the boundary between the attraction and aversion regions is a ray through the origin, and, hence, the maximal fair risk that the individual is willing to accept is proportional to her wealth. But when preferences are weakly homothetic, the boundary is a straight line of unit slope: in other words, past a wealth threshold below which there is only risk aversion, the maximal fair risk that the individual is willing to accept is independent from her wealth.[12]

In addition, both for homothetic and weakly homothetic preferences, we have characterized the presence or absence of a switch effect, understood as a change from aversion to attraction (or vice-versa) when the probabilities of the best and worst outcomes are switched.

Second, we have considered preferences representable by utility functions of a particular form, reminiscent of expected utility but with distorted probabilities, and discussed in detail two types of distortion functions. Type I always distorts the probability of the worst outcome downwards, yielding attraction to small risks for all probabilities. Type II, on the contrary, distorts low probabilities upwards, and high probabilities downwards, implying risk aversion when the probability of the worst outcome is low. Four explicit examples, combining homothetic or weak homothetic preferences with Type I or Type II distortion functions, have been presented: all four display an amount effect and a switch effect. It has also been argued that these switch effects generate a form of reflection effect which is unrelated to any translation of the probabilities.

[12] In principle, both kinds of preferences are possible. In fact, the experimental results in our In Press paper hint at a variety of individual relationships between wealth and the maximal fair risk borne.



The gap between WTA and WTP in single-self and multiple-selves preferences

Figure 1.

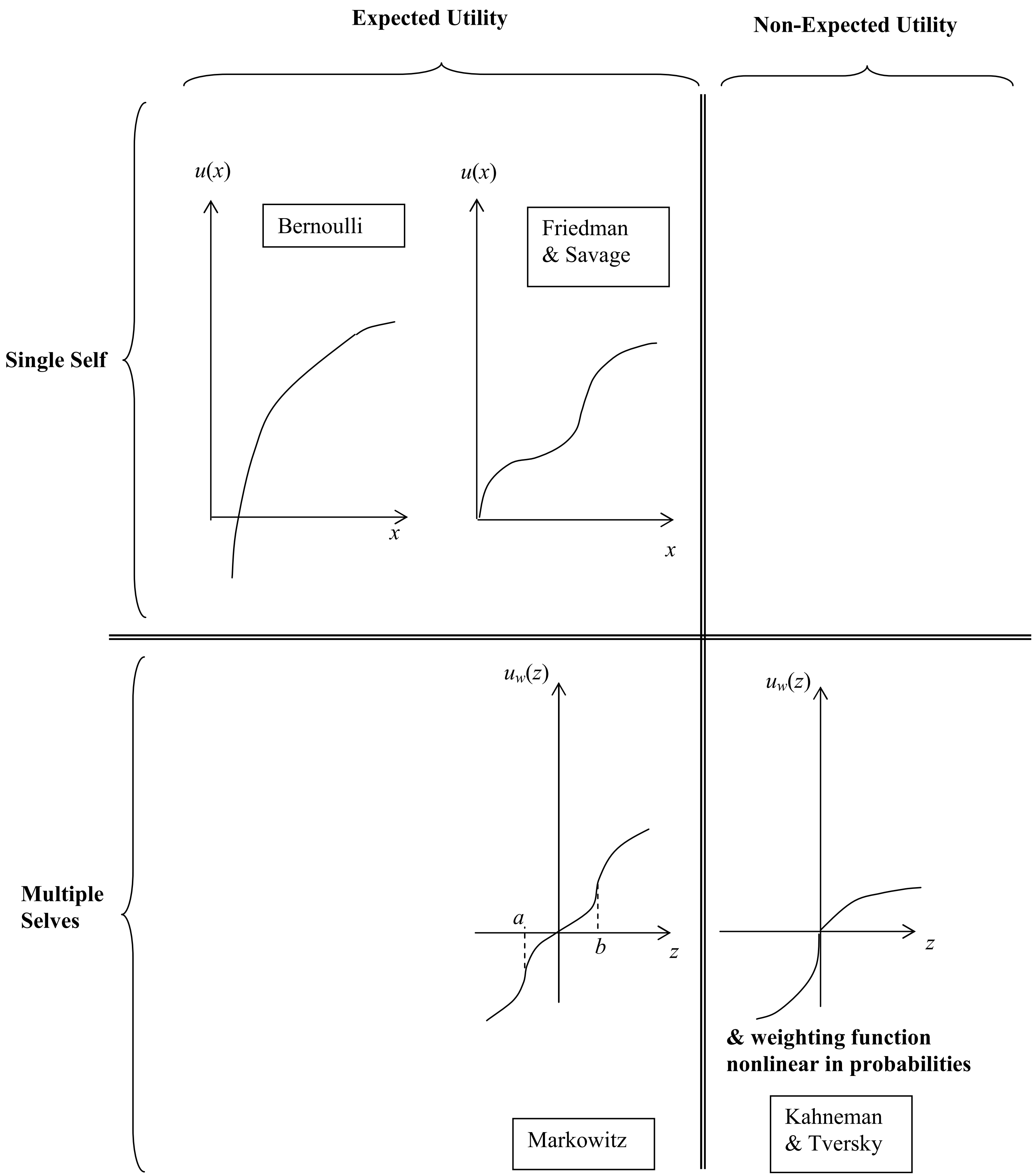


Figure 2. Classification of Various Theories

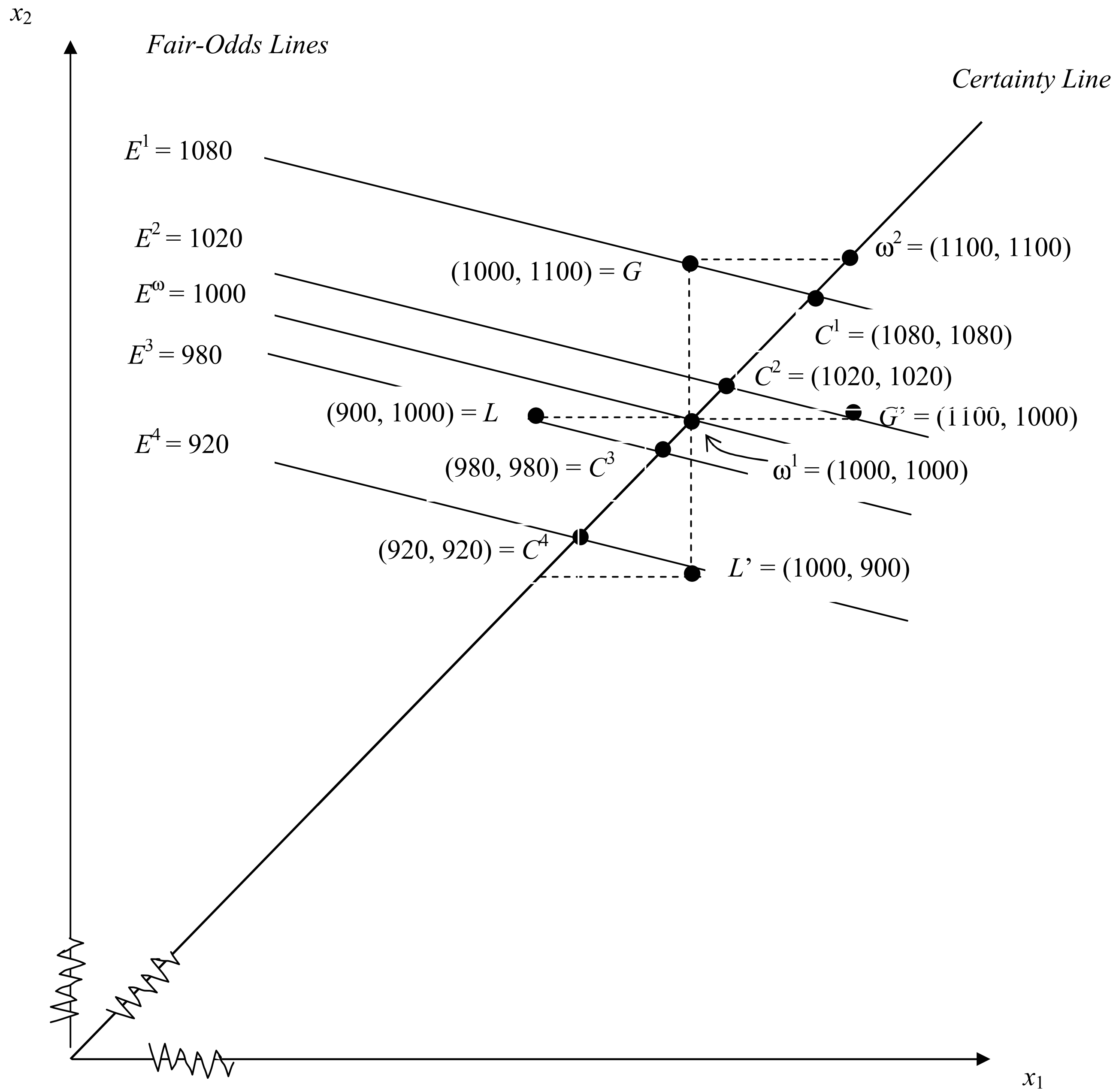
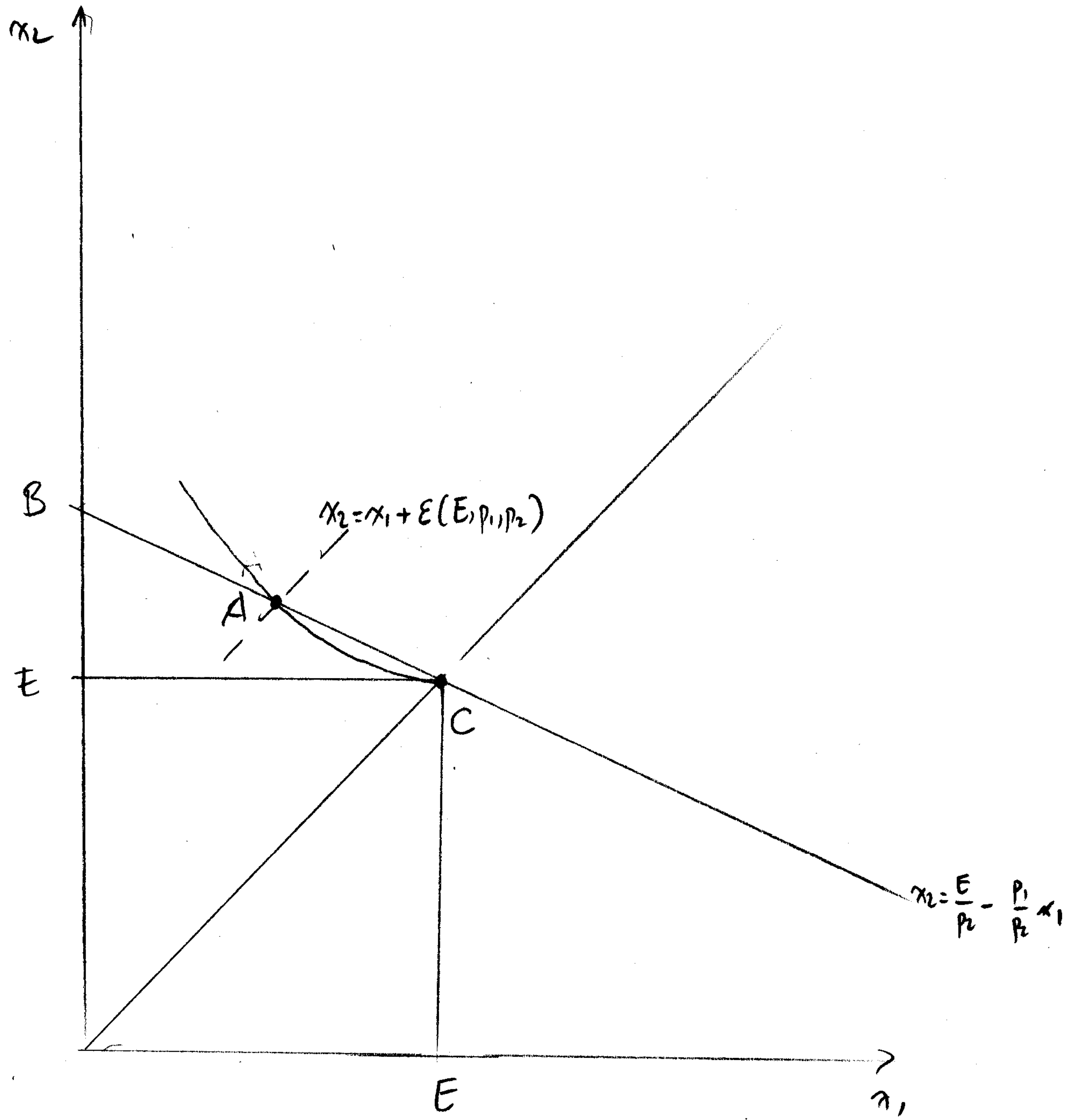
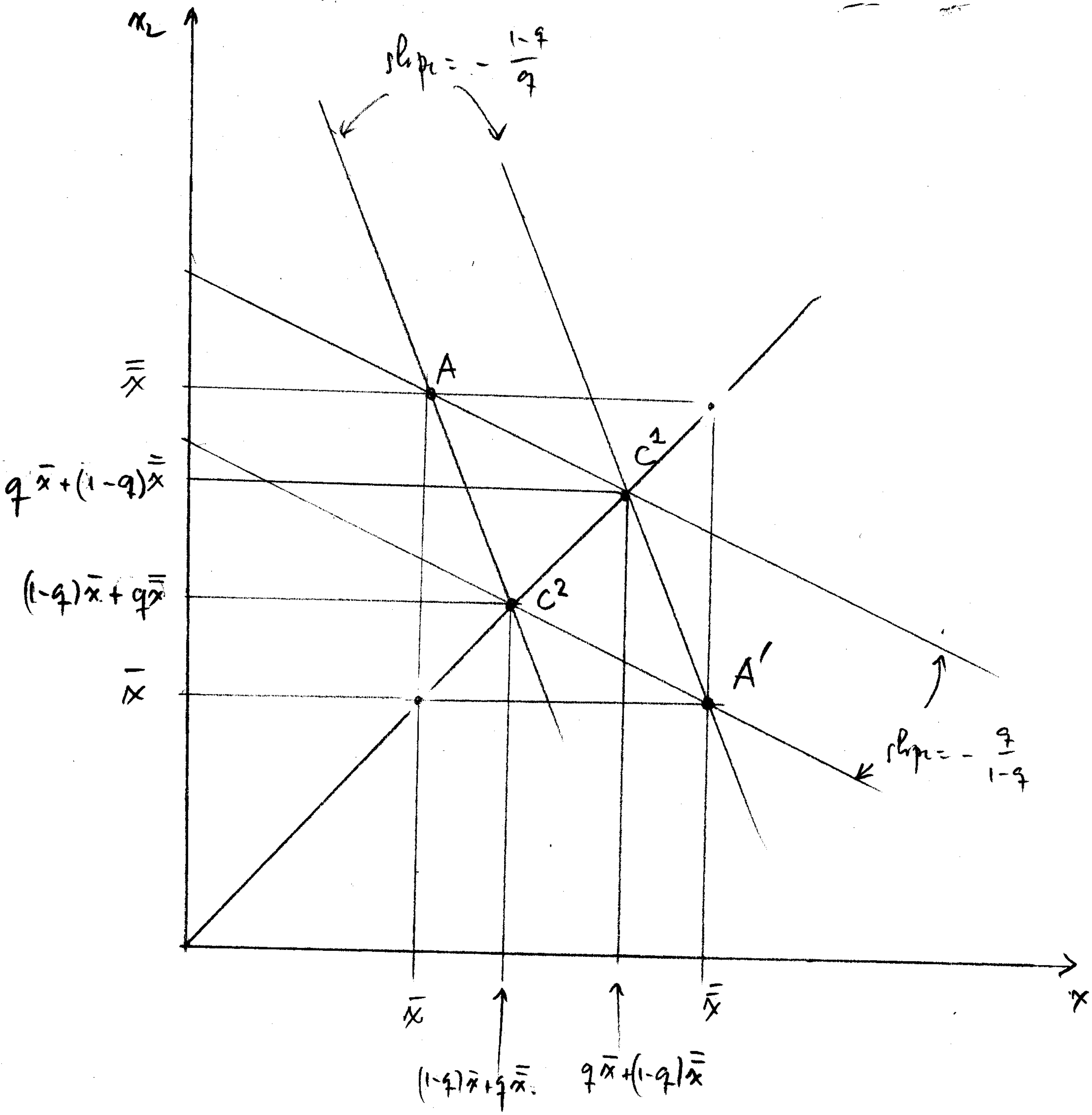


Figure 3. Operators on choices.



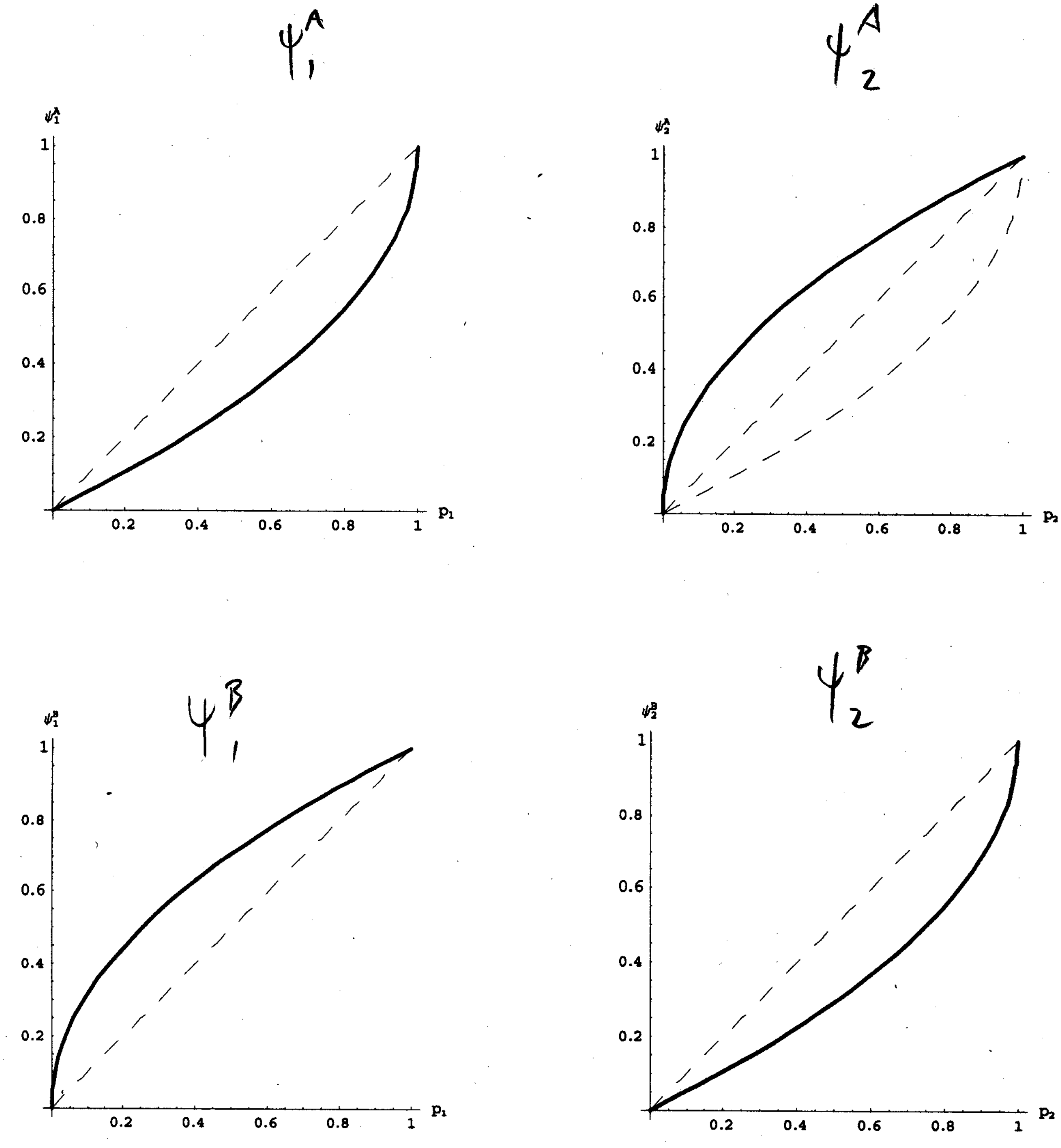
Amount Effect

Figure 4



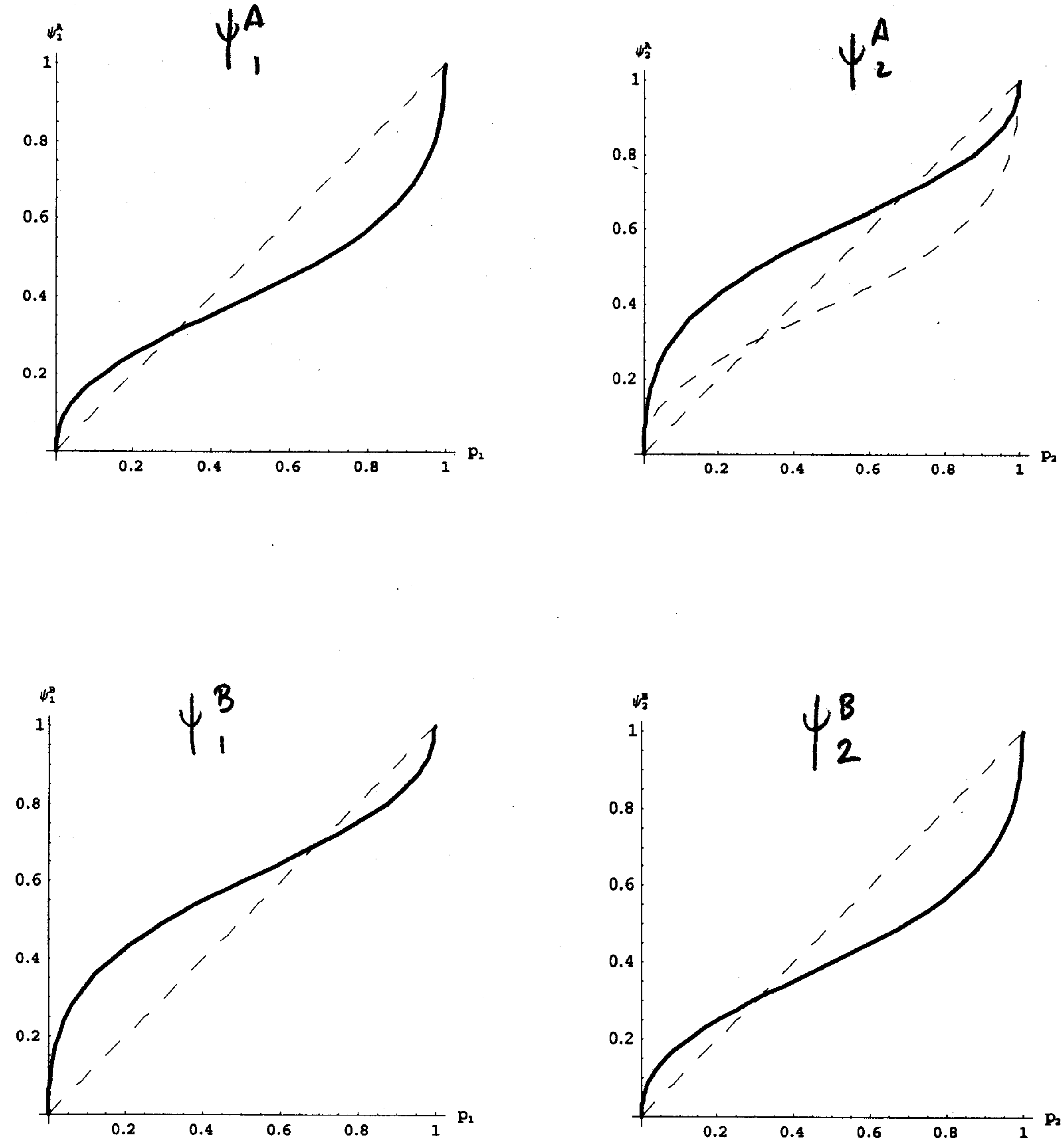
Switch Effect

Figure 5



Type I ψ functions

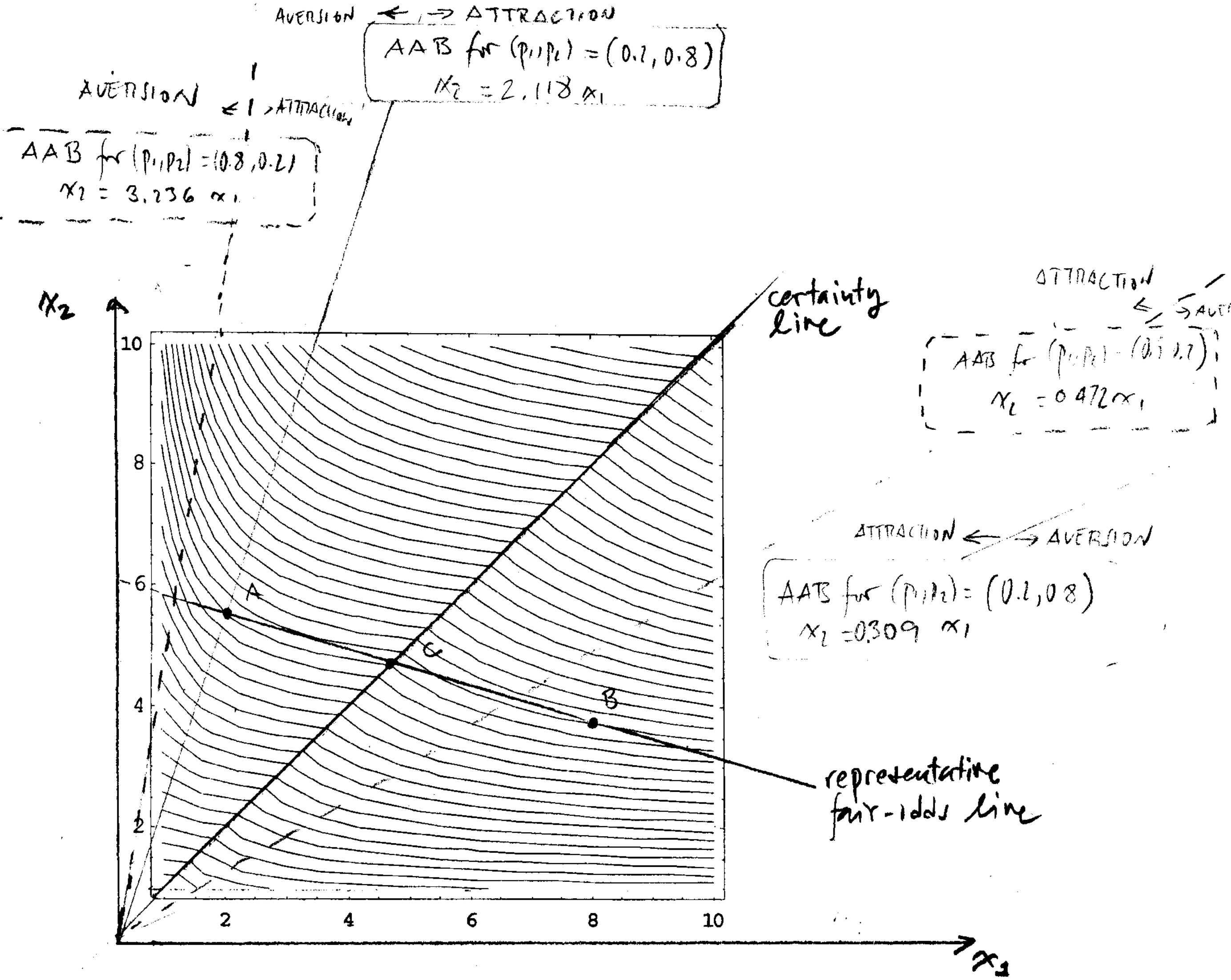
Figure 6



Type II ψ functions

$a=1$, $b=1.5$

Figure 7



<u>Example 1</u>

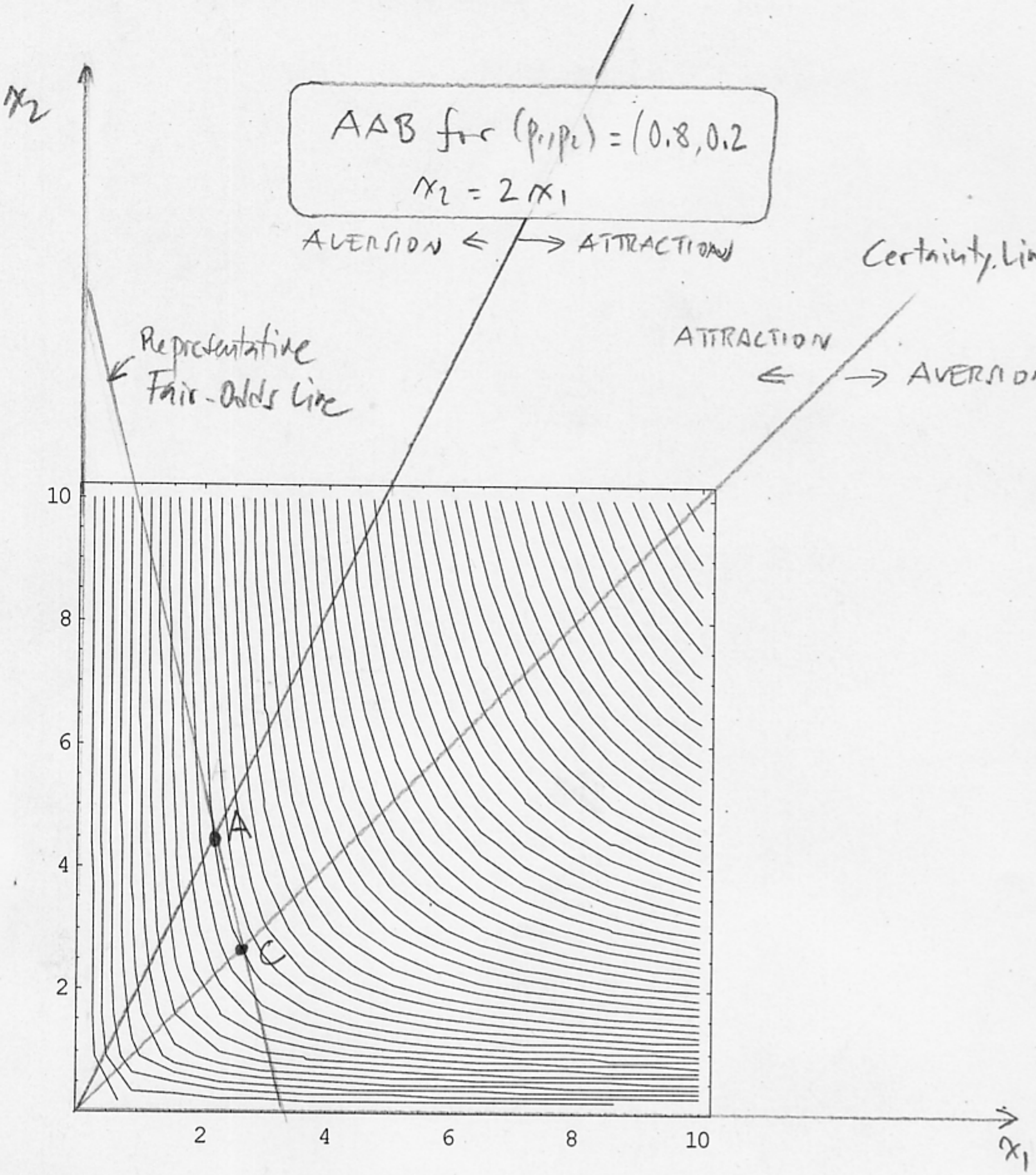
$$u(x) = -x^{-1}$$

$\psi_1^A(p_1) = 1 - \sqrt{1 - p_1}$, $\psi_2^A(p_2) = \sqrt{p_2}$,

$\psi_1^B(p_1) = \sqrt{p_1}$, $\psi_2^B(p_2) = 1 - \sqrt{1 - p_2}$.

Indifference Curves and Representative Fair-Odds Line for $(p_1, p_2) = (0.2, 0.8)$
Attraction/Aversion Boundaries (AAB) for $(p_1, p_2) = (0.2, 0.8)$ (solid)
Attraction/Aversion Boundaries (AAB) for $(p_1, p_2) = (0.8, 0.2)$ (dashed)

Figure 8



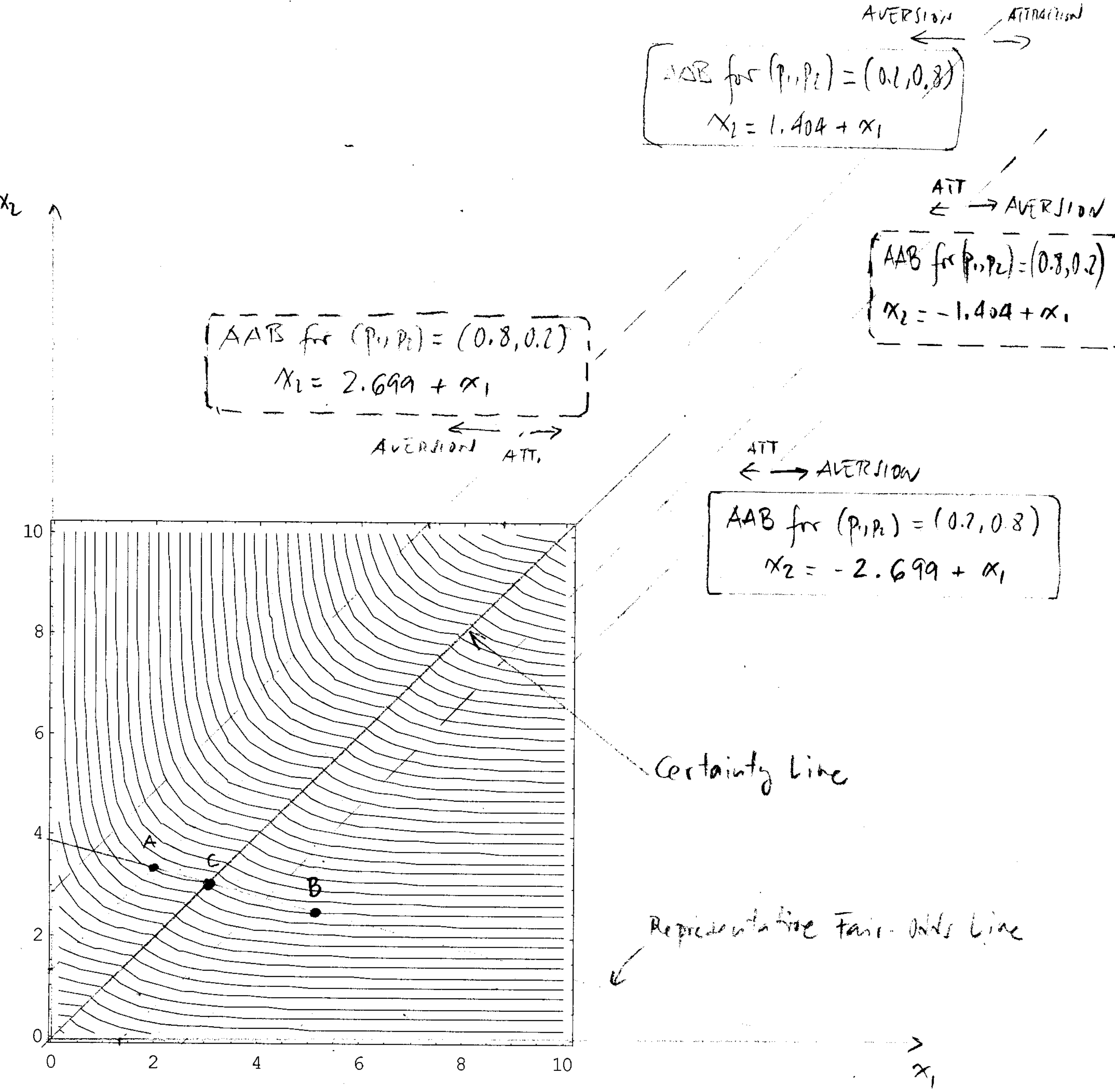
Example 2

$$u(x) = -x^{-1}$$

$$\psi_1^A(p_1) = \psi_1^B(p_1)\frac{\sqrt{p_1}}{\sqrt{p_1}+\sqrt{1-p_1}}, \qquad \psi_2^A(p_2) = \psi_2^B(p_2) = \frac{\sqrt{p_2}}{\sqrt{1-p_2}+\sqrt{p_2}}$$

Indifference Curves, Representative Fair-Odds Line
and Attraction/Aversion Boundary (AAB) for $(p_1,p_2)=(0.8,0.2)$

Figure 9



<u>Example 3</u>

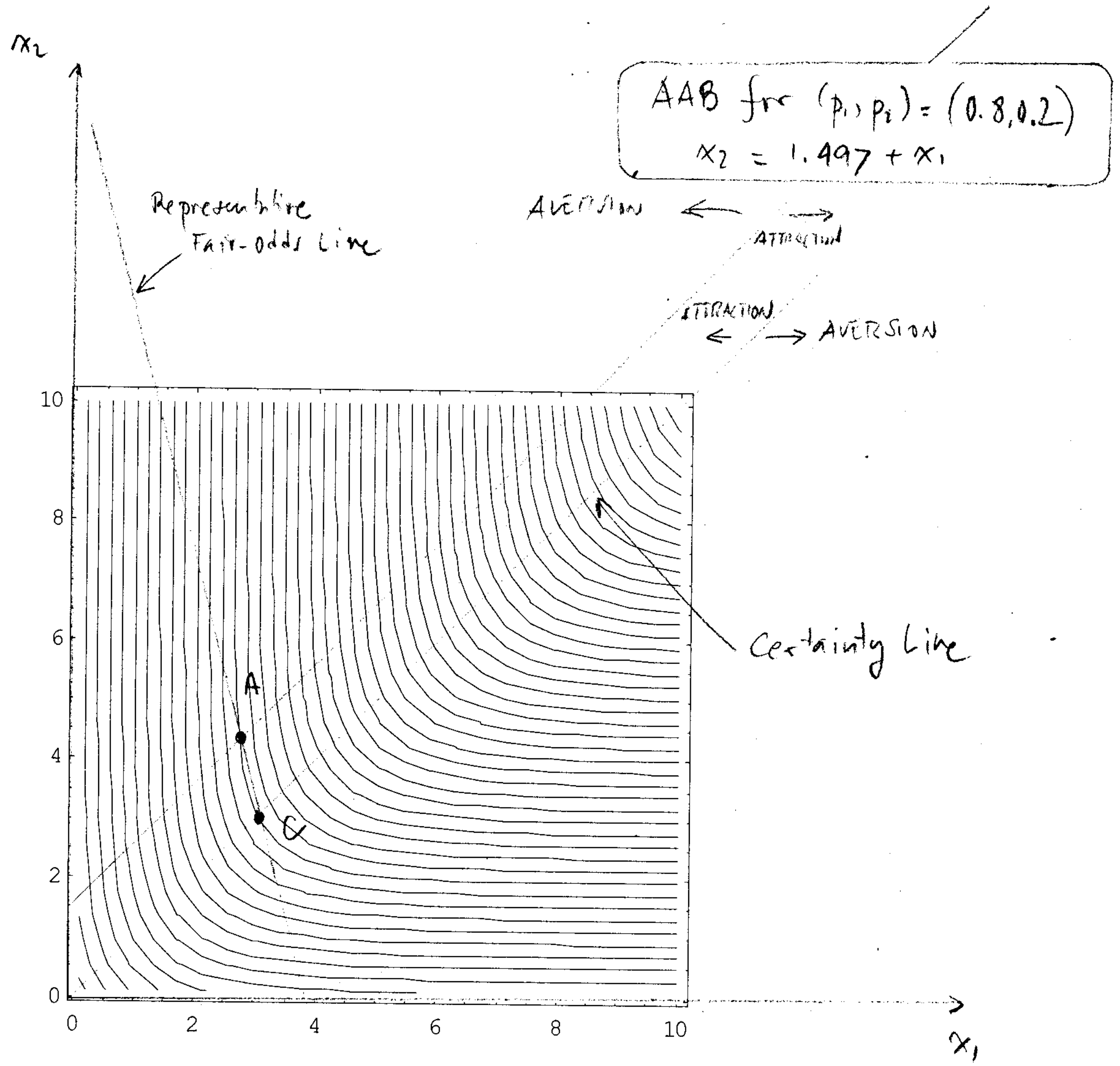
$$u(x) = -\mathrm{e}^{-x}$$

$\psi_1^A(p_1) = 1 - \sqrt{1-p_1}$, $\qquad \psi_2^A(p_2) = \sqrt{p_2}$,

$\psi_1^B(p_1) = \sqrt{p_1}$, $\qquad \psi_2^B(p_2) = 1 - \sqrt{1-p_2}$.

Indifference Curves and Representative Fair-Odds Line for $(p_1, p_2) = (0.2, 0.8)$
Attraction/Aversion Boundaries (AAB) for $(p_1, p_2) = (0.2, 0.8)$ (solid)
Attraction/Aversion Boundaries (AAB) for $(p_1, p_2) = (0.8, 0.2)$ (dashed)

Figure 10



<u>Example 4</u>

$$u(x) = -e^{-x}$$

$$\psi_1^A(p_1) = \psi_1^B(p_1)\frac{\sqrt{p_1}}{\sqrt{p_1} + \sqrt{1-p_1}}, \qquad \psi_2^A(p_2) = \psi_2^B(p_2) = \frac{\sqrt{p_2}}{\sqrt{1-p_2} + \sqrt{p_2}}$$

Indifference Curves, Representative Fair-Odds Line
and Attraction/Aversion Boundary (AAB) for $(p_1, p_2) = (0.8, 0.2)$

Figure 11

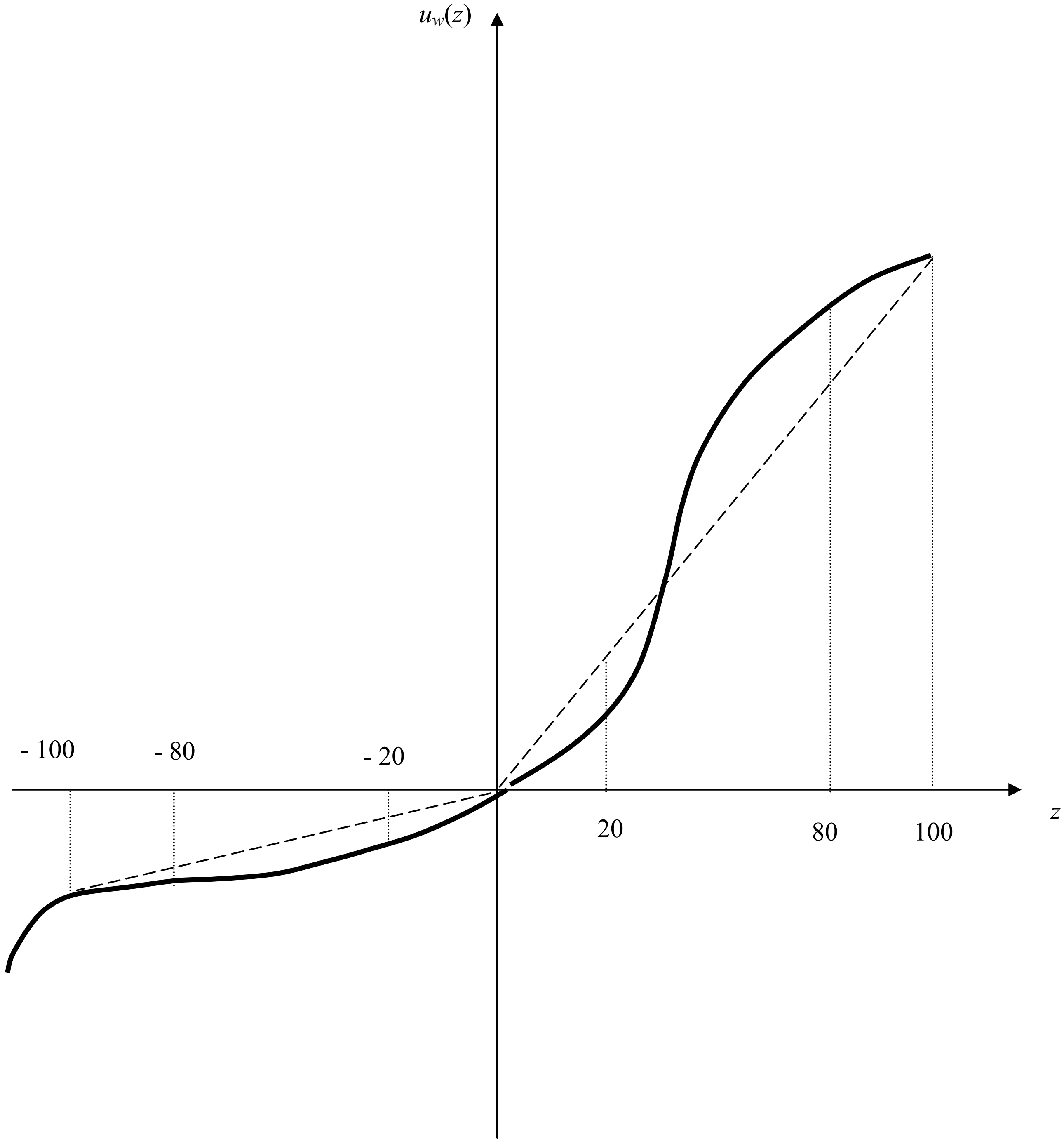


Figure 12.

The amount, switch and translation effects are consistent with multiple-selves, “expected utility” preferences.

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