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**Basic Research, Product Innovation, and
Growth**

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Abstract

This paper presents a model in which product innovation is the result of the interaction between a social welfare maximizing government who finances basic research and profit maximizing firms who develop new commercial goods. Basic research produces theoretical knowledge as output; theoretical knowledge is a pure public good used as an essential input in the design of new commercial goods. If basic research activities utilize theoretical but not applied knowledge as input, increases in basic research expenditures mean faster growth. However, if applied knowledge is also an essential input to produce new theoretical knowledge, more basic research expenditures imply faster growth only for a large enough economy size.

1 Introduction

Would the invention of new goods in the last two centuries have been possible if the level of theoretical knowledge had been the same as in Galileo's time? Most of the current models of economic growth based on product differentiation [Romer (1990), Segerstrom, Anant & Dinopoulos (1990); Grossman & Helpman (1991); Aghion & Howitt (1992)] consider that economic progress is possible because of the investment in designing new (or better) commercial goods. These models treat R & D as a homogeneous sector that generates a nonrival and partially excludable knowledge good. If we separate this knowledge derived from commercial research from theoretical knowledge (that is considered as a pure public good), a negative answer to the initial question implies that the intervention of some productive agent unconcerned with profit maximization is necessary to obtain unbounded growth.

Basic research has been analyzed as a sector different from commercial research in a surprisingly small number of models of endogenous growth. Following Shell (1967), Grossman & Helpman ((1991), ch. 2) construct a model of endogenous growth in which the public knowledge derived from basic research activities is the source of long run growth. In their model there is no a separate commercial research sector which interacts with the basic research sector. Aghion & Howitt (1994) consider explicitly the difference between basic and applied research. Their distinction is based on the difference between invention and innovation, in a context of a Schumpeterian model of growth. They treat the knowledge generated by invention and innovation as partially excludable knowledge goods, and they do not consider theoretical knowledge.

This paper presents a dynamic general equilibrium model based on three main premises. The first one is that the output of basic research, theoretical knowledge, is a pure public good (nonrival, nonexcludable), whereas the knowledge generated by developing new commercial goods is, as in previous models, a partially excludable knowledge good. Thus, as opposed to the design of new commercial goods, basic research activities cannot be supported in a decentralized equilibrium in which productive decisions are only taken by profit-maximizing firms. In order to obtain a positive provision of theoretical knowledge, it will be assumed that basic research is financed by a government that chooses the allocation of resources to basic research by maximizing a social welfare function.

The second premise is that the stock of theoretical knowledge is used as an essential input in the design of new commercial goods. This implies

that the invention of new goods is possible because of previous developments in theoretical science. For example, it is hard to imagine the existence of a whole modern industry like electronics without the previous discovery of electrons by Thomson and H.A Lorentz. Also, the discovery of transistors would have been impossible without previous developments in ondulatory mechanics or in the theory of electrons in solid. In a similar way, the discovery of electromagnetical waves by Hertz was essential in the development of communications. In fact, it is almost impossible to find any example of technological innovation which is not indebted in this way to basic scientific thought.

The third premise is that both basic and commercial research compete for the same input, skilled labor. In the model, scientists and engineers are indistinctly used to produce new theoretical ideas or to apply these new ideas in the production of blueprints. A person with a doctorate degree in computer science could be employed in a university to do theoretical research or in the laboratory of some firm -I.B.M, Apple, Intel- to design new computers or chips.

From these premises two main conclusions can be derived. First, from the first two assumptions theoretical knowledge is a pure public good provided by the government and essential for the invention of new commercial goods. Hence, it is obtained that long run growth is directly influenced by government decisions on basic research expenditures. Second, as a consequence of the competition for skilled workers, a trade-off between the level of innovation and its rate of growth can arise. More expenditures in basic research implies more skilled workers in this sector and less skilled workers allocated to develop new goods. In the short run, this fact means a lower level of variety in final goods. However, if the rate of growth of theoretical knowledge increases with the number of workers allocated to basic research, this higher theoretical knowledge growth rate means that the productivity of skilled workers in commercial research is augmenting at a higher rate. This commercial research productivity increase means that the economy will eventually be on a path with more innovation and growth.

The paper discusses two basic research technologies. In the first, the only knowledge input for producing new basic knowledge is the stock of theoretical knowledge. Under this technology, long run growth increases proportionally with basic research expenditures. In the second technology, the stock of public knowledge derived from the design of new commercial

goods¹ is also essential for the production of new theoretical knowledge. In this case, increases in basic research expenditures are less effective for speeding up growth because they reduce the number of workers available for commercial research activities, and, thus, the stock of applied knowledge. In fact, I show that increasing basic research expenditures can have a negative effect on growth if the market size is small enough.

The remainder of the paper is organized as follows. Section II presents the model in which basic research uses skilled labor and theoretical (but not applied) knowledge as inputs. Individual decisions about the acquisition of skills are endogenously determined depending on the agent's innate ability base and the ratio between skilled and unskilled labor wages. Section III gives us the equilibrium values for the number of varieties and manufactured output and Section IV analyzes the optimal allocation of resources to basic research. Section V introduces the knowledge generated by developing new commercial goods as a fundamental input to basic research. Section VI concludes resuming the main results and their possible extensions.

2 The Model

Productive activities take place in three sectors: basic research, commercial research and manufacturing of final goods (i.e., consumption goods). There are two inputs: labor, which can be skilled or unskilled; and knowledge, which can be theoretical or derived from learning by doing in commercial research activities. Skilled labor is used in both research sectors but not in manufacturing²; unskilled labor is only used in manufacturing.

Basic research utilizes skilled labor, H_B , and the stock of theoretical knowledge at time t , $I(t)$, to produce new theoretical knowledge. The technology of this sector is represented by

$$dI(t) = \frac{H_B}{b} I(t) dt \quad (1)$$

where $b > 0$ is a productivity parameter. The cost of basic research is given by $w_H H_B = w_H b \frac{dI(t)/dt}{I(t)}$, where w_H is the reward paid to a unit of skilled labor. This cost is financed by the government through lump-sum taxes.

¹This knowledge stock will be indistinctly called applied knowledge or knowledge derived from commercial research activities.

²This assumption is intended to make clearer the analysis of the trade off between both research sectors.

Commercial research designs new varieties using H_A units of skilled labor, the stock of knowledge derived from the sector's cumulative experience in developing new goods (which is assumed to be equal to the number of varieties existing at t , $N(t)$)³, and the stock of theoretical knowledge, $I(t)$, to design new varieties. The sector production function is specified as

$$dN(t) = \frac{H_A}{a} N(t)^\eta I(t)^{1-\eta} dt \quad (2)$$

where $0 < \eta < 1$ is a technological parameter and $a > 0$ is a productivity parameter in this sector.

As in previous works, in this model knowledge is a free input that increases the productivity of designing new goods. However, in this paper the stock of the knowledge is decomposed into theoretical knowledge and knowledge derived from learning by doing in commercial research. We can observe two properties in the production function for blueprints (2). First, both kinds of knowledge are essential in the production of blueprints. Also, unbounded growth in the number of varieties is impossible if the theoretical knowledge stock remains fixed. If we take $H_A = H_A^*$ as the equilibrium value for skilled labor allocated to designing new varieties and if theoretical knowledge remains fixed at a level $I(t) = \bar{I}$, the number of available varieties grows at a rate $\frac{\dot{N}(t)}{N(t)} = \frac{H_A^*}{a} [\frac{\bar{I}}{N(t)}]^{1-\eta}$ that converges to zero as the number of varieties grows⁴.

There are many identical perfectly competitive firms producing blueprints. The representative commercial research firm sells infinitely-lived patent rights on the new good n , obtaining a revenue $P(n, t)$. The profits of the representative research firm are $\Pi(n, t) = P(n, t) - \Gamma(t)$, where

$$\Gamma(t) = \frac{aw_H}{N(t)^\eta I(t)^{1-\eta}}$$

is the unit cost implied by the production function (2).

³This assumption implies the existence of knowledge spillovers among firms. Bernstein & Nadiri (1988), (1989), provide empirical evidence on this fact.

⁴We can readily generalize this result to show that if the technology $\dot{N}(t) = H_A F[N(t), I(t)]$ verifies the condition $\lim_{N(t) \rightarrow +\infty} F_N[N(t), \bar{I}(t)] = 0$, then, a fixed stock of theoretical knowledge implies zero long run growth. Notice that this technology implies bounded learning in the sector that develops new commercial goods. An analogous assumption to this one, applied to the manufacturing sector, is made by Young (1993), and has been empirically tested by Epple, Argote & Devadas (1991) and Lehvari & Sheshinski (1973).

New goods are commercialized by identical monopolistically competitive manufacturing firms. The representative manufacturer firm produces a quantity $c(n, t)$ of the consumption variety n at t by using only one input, unskilled labor $L(t)$. The technology is the same for all consumption goods, and is represented by the linear production function

$$c(n, t) = L(t). \quad (3)$$

The amount of profits obtained from the commercialization of a good n invented at t is given by

$$\Pi(n, t) = \int_t^\infty e^{-\int_t^\nu r(\nu) d\nu} \pi(n, s) ds,$$

where $r(\nu)$ is the instantaneous interest rate at time ν and $\pi(n, s)$ represents instantaneous profits at s .

Firms sell the quantity $c(n, s)$ of good n at a price $p[c(n, s)]$. This price is determined by the consumers' demand function. Hence, instantaneous profits $\pi(n, s)$ are

$$\pi(n, s) = p[c(n, s)]c(n, s) - w_L L(s),$$

where w_L is the rental wage paid to a unit of unskilled labor. Firms maximize profits by choosing a price that is a markup on their marginal cost. The linear technology (3) implies that the marginal cost is equal to w_L . Then, from the first order condition of the firms' maximization problem we have the pricing equation

$$p[c(n, t)] = \frac{\epsilon[c(n, t)]}{\epsilon[c(n, t)] - 1} w_L,$$

where $\epsilon[c(n, t)]$ represents the elasticity of the demand for any good n .

On the demand side, the paper assumes the existence of many finitely-lived consumers with the same preferences and the same infinite economic horizon (offsprings' welfare enter into the utility function of present generations). Preferences are represented by the function

$$U(t_0) = \int_{t_0}^\infty e^{-\rho(t-t_0)} \ln \left[\left[\int_0^{N(t)} c(n, t)^\alpha dn \right]^{1/\alpha} \right] dt \quad (4)$$

where $\rho > 0$ is the intertemporal discount factor and $\alpha < 1$ is a parameter of the preferences. This index implies that the elasticity of substitution

between any two goods labeled (n, n') is equal to $\epsilon = 1/(1 - \alpha)$. This elasticity is greater than one but lower than infinity. Therefore, goods are not perfect substitutes, and increases in the available variety imply greater consumers' welfare.

Consumers own all production factors and all shares in firms. The intertemporal aggregate budget constrain is given by

$$\int_{t_0}^{\infty} e^{-\int_{t_0}^t r(s)ds} E(t)dt \leq A(t_0) \quad (5)$$

where $E(t) = \int_0^{N(t)} p(n, t)c(n, t)dn + \tau$ represents the nominal expenditure at t , τ are lump-sum taxes and $A(t_0)$ is the present value of the stream of factor incomes, plus the value of initial asset holdings.

Consumers choose the static demands for the different goods and the stream of nominal expenditure that maximize their welfare. Their problem can be decomposed in two stages (see Appendix I). In the first, given an after-tax expenditure $E_\tau = E(t) - \tau$, households must choose their demands in such a way as to maximize their static welfare. In the second stage, consumers choose the flow of expenditure that maximizes their intertemporal welfare. Since all goods enter symmetrically into the utility function (4), the demand function for every good is the same and equal to

$$c^d(n, t) = \frac{p(n, t)^{-\epsilon} E_\tau(t)}{\int_0^{N(t)} p(n', t)^{1-\epsilon} dn'} \quad (6)$$

The solution of the intertemporal problem implies that the stream of after-tax nominal expenditures follows the differential equation $\frac{\dot{E}_\tau(t)}{E_\tau(t)} = r(t) - \rho$. In this framework is convenient to normalize prizes so that the nominal expenditure remains constant throughout time (see Grossman & Helpman (1991), ch. 2). Using the previous expression, we have

$$\frac{\dot{E}_\tau(t)}{E_\tau(t)} = 0 - r = \rho$$

Particularly, prices will be normalized in such a way that nominal expenditure is always $E_\tau(t) = E(t) - \tau = 1 - \tau$.

Skilled and Unskilled Labor Supply

Individuals choose between participating in the labor market as unskilled or skilled workers. For simplicity, it is assumed here that the only cost of the

acquisition of skills is time devoted to education. If the individuals decide to be skilled, they must allocate a constant period S of their lives to acquire skills. This period is common for all workers that choose to be skilled⁵. If they choose to be unskilled, they work all their life. The potential supply of labor is inelastic, and equal to one, for all the workers in the economy (leisure is not considered).

The decision on specialization depends on the innate parameter of individual ability $i \in [0, i^+]$. This parameter is uniformly distributed on the population, of size M . Therefore, the density of individuals endowed with the productivity parameter i is $m = M/i^+, \forall i$. If an individual endowed with the ability parameter i chooses to be skilled, he obtains a wage income $w_H i^\beta$ between $t+S, t+T$ where $\beta \in (0, 1)$ is a productivity parameter (common for all individuals) that measures the efficiency of the educational system. If the worker does not choose to acquire skills, he obtains a wage income w_L , independent on his innate productivity parameter i . This implies that the productivity parameter is only effective if the worker chooses to acquire skills, and that all unskilled work is homogeneous.

An individual endowed with an ability parameter i chooses to specialize if the discounted wage income obtained in the period $T-S$ is greater than the wage income obtained working the entire period of their life T , that is, if

$$\int_{t+S}^{t+T} e^{-\rho(s-t)} w_H i^\beta ds \geq \int_t^{t+T} e^{-\rho(s-t)} w_L ds. \quad (7)$$

This condition implies that the ability parameter of skilled workers must be greater than i^* , where i^* verifies (see Appendix II)

$$i^* = \left[\frac{1 - e^{-\rho T}}{e^{-\rho S} - e^{-\rho T}} \right]^{1/\beta} \left[\frac{w_L}{w_H} \right]^{1/\beta} \equiv G^{1/\beta} \left[\frac{w_L}{w_H} \right]^{1/\beta}. \quad (8)$$

Given $G \equiv \frac{1 - e^{-\rho T}}{e^{-\rho S} - e^{-\rho T}} > 1$, we can observe that, in equilibrium, a positive supply of skilled labor implies $w_H i^\beta > w_L$. That is, the wage income of skilled workers always must exceed the income of unskilled workers in equilibrium. Notice, however, that some individuals are more productive as unskilled than skilled labor.

We are now ready to determine the supply of skilled labor in this economy. First, we have that the workers that decide to be skilled are those

⁵This analysis differs from Findlay and Kierzkowski (1983) and Stokey (1991), because they focus on the determination of the period of formation for homogeneous workers, while I consider heterogeneous individuals for a given formation period S and endogenously obtain the proportion of workers that choose to be skilled.

endowed with an ability parameter $i \in [i^*, i^+]$, and there is a constant number m of individuals with the ability parameter i . We also know that skilled workers work a fraction $\frac{T-S}{T}$ of their lives. Population is assumed to be uniformly distributed on the age interval $[0, T]$. This implies the nonexistence of population growth, and no changes in life expectancy. Thus, in a stationary equilibrium in which the ratio w_H/w_L does not vary (and neither the interval of abilities of people that choose to be skilled $[i^*, i^+]$) the aggregate supply of skilled labor is given by

$$H^s = \frac{T-S}{T} m \int_{i^*}^{i^+} i^\beta di = M^+ - kmG^{\frac{\beta+1}{\beta}} \left[\frac{w_L}{w_H} \right]^{\frac{\beta+1}{\beta}} \quad (9)$$

where $M^+ = km(i^+)^{\beta+1}$ is the supply of skilled labor if all the workers choose to acquire skills and $k = \frac{T-S}{T(\beta+1)}$ is a positive constant lower than one. Since k , i^+ and β are constants, m is directly related with M^+ , and we can take M^+ as a parameter of the size of the economy.

The supply of unskilled labor is given for all the individuals with an ability parameter lower than i^* , that is, with $i \in [0, i^*]$. Since the population is uniformly distributed in this interval, the aggregate supply of unskilled labor is $L = m[i^* - 0] = mi^*$. Using (16), we have

$$L^s = mi^* = mG^{1/\beta} \left[\frac{w_L}{w_H} \right]^{1/\beta}. \quad (10)$$

As we can observe in Figure 1, the aggregate supply of skilled labor depends positively on the ratio of skilled to unskilled wages w_H/w_L . However, since workers need a period S to specialize, an increase in this ratio at t does not imply an *instantaneous* increase in the supply of skilled labor. The dynamics works as follows: at t , individuals observe the new ratio of wages, and then they choose between being skilled or remaining unskilled workers. At the higher ratio w_H/w_L , the workers that choose to be skilled are those endowed with an ability parameter equal or higher than i^*_1 , where i^*_1 is lower than i^*_0 , that is the minimum ability parameter of workers who decided to be skilled with the previous rate of wages. But workers who decide to be skilled need a period S before participating in the labor market. So between t and $t+S$ the supply of skilled workers is the same as before, with the initial ratio of wages. The only effect of the increase of w_H/w_L between t and $t+S$ in the labor market is the decrease in the supply of unskilled workers, because people endowed with an ability parameter $i \in [i^*_1, i^*_0]$ choose now to devote time to acquire skills, whereas they had previously decided

to participate directly as unskilled workers.

3 The Equilibrium

This section obtains the equilibrium of the model corresponding to the balanced growth path. Thus, the values of the variables will be obtained assuming that all the real variables are always growing at a constant rate⁶. By equation (1), the rate of growth of theoretical knowledge is equal to $\frac{\dot{I}(t)}{I(t)} = H_B/b \equiv \gamma$, where γ is used to denote the skilled labor allocated to basic research given a productivity parameter b . Under the technology expressed in equation (1), γ coincides with the growth rate of the number of varieties along the balanced growth path.⁷ Since in this economy the only government activity is financing basic research, the imposition of a balanced budget condition for the government implies that $w_H H_B = w_H b \gamma = \tau$.

The model is solved in two stages. In the first, the model will be solved assuming that the equilibrium value of the variables is obtained by private agents taking γ as fixed. In the second stage (Section IV) the allocation of resources to basic research γ will be determined as a solution of a government problem (welfare's maximization problem) .

Equilibrium in the Unskilled Labor and Consumption Goods Markets

The unskilled labor market clears if the supply of unskilled labor given by equation (10) is equal to its demand. The linear technology represented by the production function (3) implies that the demand for unskilled labor $L^d(t)$ is equal to the supply of consumption output which, in the market equilibrium, must be equal to the demand for consumption goods. Hence, the equilibrium in the unskilled labor market follows from the equilibrium in the consumption goods market.

From the symmetry between costs and preferences, the price of each variety n must be the same. Substituting the value of the elasticity $\epsilon[c(n,t)] = \frac{1}{1-\alpha}$ into the pricing equation, the common price for all goods is given by $p(n,t) = w_L/\alpha$. Using this result and the demand for consumption goods given by (6), we have that the consumption demand for each variety n is

⁶The analysis of the equilibrium is restricted to the balanced growth path case. Bailei & Rivera-Batiz (1994) show that, out of the balanced growth path, a similar dynamic system can generate endogenous oscillations in innovation and growth.

⁷This implies (see Grossman & Helpman, 1991, ch.3) that the growth rate for GDP and manufactured consumption output is approximately equal to $\frac{1-\alpha}{\alpha}\gamma$.

$c^d(n, t) = \frac{E_\tau}{N(t)p(n, t)} = \frac{\alpha(1-\tau)}{N(t)w_L}$ (recall $E_\tau = 1 - \tau$). The consumption demand for all the varieties is $C^d(t) = N(t)c^d(n, t)$, and is equal to the supply of unskilled labor, given by (10). Thus, the equilibrium in this market is

$$C^d(t) = N(t)\left[\frac{\alpha(1-\tau)}{N(t)w_L}\right] = \frac{\alpha(1-\tau)}{w_L} = mG^{1/\beta}\left[\frac{w_L}{w_H}\right]^{1/\beta} = L^s. \quad (11)$$

Equilibrium in the Patent Market

The firm that designs a new good n sells infinitely-lived patent rights to a manufacturer firm for a price $P(n, t)$. If the patent market is perfectly competitive, the price of a patent must equal the stream of profits of the monopolistically competitive firm. Using this condition and the equalities $r(t) = \rho$ and $\alpha p[c(n, t)] = w_L$, we have

$$P(n, t) = \int_t^\infty e^{-\rho(s-t)}(1-\alpha)p(n, s)c(n, s)ds$$

Competition among research firms drives the economy to an equilibrium in which the price of a patent $P(n, t)$ is equal to the cost of the blueprint. From the discussion surrounding (11), notice that $p(n, s)c(n, s) = \frac{1-\tau}{N(s)}$. Thus, for a positive innovation rate $dN(t)/dt > 0$, it is verified that

$$\int_t^\infty e^{-\rho(s-t)}(1-\alpha)\frac{1-\tau}{N(s)}ds = \frac{aw_H}{N(t)^\eta I(t)^{1-\eta}}.$$

Substituting in this expression the balanced growth path equation for the number of varieties $N(s) = N(t)e^{\gamma(s-t)}$ and calculating the integral yields

$$\frac{(1-\alpha)(1-\tau)}{(\rho+\gamma)N(t)} = \frac{aw_H}{N(t)^\eta I(t)^{1-\eta}}. \quad (12)$$

Equilibrium in the Skilled Labor Market

The demand for skilled labor $H^d(t)$ is the sum of the amounts demanded for basic and commercial research. The technology in both research sectors expressed in equations (1) and (2) give us

$$H^d(t) = H_B(t) + H_A(t) = b \frac{\dot{I}(t)}{I(t)} + a \frac{\dot{N}(t)}{N(t)} \left[\frac{N(t)}{I(t)} \right]^{1-\eta}.$$

The constant growth rate condition ($\frac{\dot{I}(t)}{I(t)} = \frac{\dot{N}(t)}{N(t)} \equiv \gamma$), and the equality (12) between discounted profits and the cost of inventions determine the demand for skilled labor. The skilled labor market clears when the ratio between wages w_L/w_H is such that the supply of skilled labor $H^s(t)$ given by (9) is equal to the demand for skilled labor $H^d(t)$, that is, if

$$\begin{aligned} H^s &= M^+ - kmG \frac{\beta+1}{\beta} \left[\frac{w_L}{w_H} \right]^{\frac{\beta+1}{\beta}} \\ &= b\gamma + \gamma \left[\frac{(1-\alpha)(1-\tau)}{(\rho + \gamma)w_H} \right] = H^d. \end{aligned} \quad (13)$$

Equilibrium Number of Varieties

In this model is possible to obtain endogenously not only the growth rate for the number of varieties but also its initial level. This happens because of the existence of decreasing returns in the technology of the design of new varieties (2) with respect to the number of existing varieties $N(t)$. This means that, for a certain level of theoretical knowledge $I(t)$ and skilled labor to commercial research H_A^* , the number of varieties $N(t)$ can be determined endogenously.

Using the equilibrium conditions (11), (12), and (13) in the unskilled labor, patent and skilled labor markets, plus the condition $I(t) = I(t_0)e^{\gamma(t-t_0)}$ (see Appendix III), we obtain that the equilibrium number of available varieties at t is:

$$\begin{aligned} N(t) &= \left[\frac{(1-\alpha)[M^+ - b\gamma]}{\rho Gk\alpha + \gamma(1-\alpha + Gk\alpha)} a \right]^{\frac{1}{1-\eta}} I(t_0) e^{\gamma(t-t_0)} \\ &= \Lambda(\gamma) I(t_0) e^{\gamma(t-t_0)} \equiv N(t_0) e^{\gamma(t-t_0)} \end{aligned} \quad (14)$$

where $\Lambda(\gamma) \equiv \Lambda(H_B/b)$ is the ratio between the level of variety and the stock of theoretical knowledge, that satisfies $\Lambda'(\gamma) < 0$.

The expression (14) for the level of variety indicates that in this model there is a trade-off between the initial level of varieties $\Lambda(\gamma)I(t_0)$ and its

growth rate γ . Suppose that the government increases the amount of skilled labor in basic research at t_0 H_B . Since the growth rate $\gamma = H_B/b$ predicts a direct relationship between the labor allocated to basic research and the growth of the number of varieties, this increase in H_B means a higher rate of growth of the level of varieties. However, from the equilibrium in the skilled labor market (13), we have that a higher demand for skilled labor from the basic research sector $H_B = b\gamma$ increases the ratio w_H/w_L . This higher ratio w_H/w_L means an increase in the costs $w_H H_A$ faced by firms that design new varieties, and also an increase in the level of lump-sum taxes τ which leads to a decline in manufacturing firms' discounted revenues at t_0 . The increase in research firms' costs and the decrease in discounted profits for manufacturers imply that the equality (12) is verified for a lower initial level of variety, that is, there are fewer goods invented at t_0 .

Consumption Equilibrium Output

Using the equilibrium condition in the unskilled labor and consumption good markets (11), substituting w_L/w_H by its value (see Appendix IV), we obtain that the consumption equilibrium output value is given by

$$C(t) = \left[\frac{m^\beta G \alpha (\rho + \gamma) [M^+ - b\gamma]}{\rho G k \alpha + \gamma (1 - \alpha + G k \alpha)} \right]^{\frac{1}{\beta+1}} \equiv C(\gamma) \quad (15)$$

where $C'(\gamma) < 0$.

Equation (15) incorporates the existence of a trade-off between the resources allocated to basic research γ (and, thus, the growth rate of the level of variety) and the equilibrium output of consumption goods. As previously argued, the greater demand for skilled labor from basic research implies a lower ratio w_L/w_H . This increases the incentives of specialization, decreasing the supply of non-specialized labor and, therefore, the output of consumption goods.

4 Basic Research Policy and Growth

This section introduces government's decisions concerning subsidization of basic research. The government chooses the allocation of resources to basic research $b\gamma \equiv H_B$ that maximizes a social welfare function, taking as given the equilibrium values for the number of varieties $\Lambda(\gamma)$ and consumption output $C(\gamma)$ described by (14) and (15). Since all consumers have the

same preferences, if the government tries to maximize the aggregate welfare, his problem will consist in maximizing ⁸

$$U(\gamma) = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left[\frac{1-\alpha}{\alpha} \ln[\Lambda(\gamma)e^{\gamma(t-t_0)}] + \ln C(\gamma) \right] dt. \quad (16)$$

Developing this equation and differentiating with respect to γ (see Appendix V), we have that the optimal decision must verify the first order condition

$$U'(\gamma) = \frac{1-\alpha}{\alpha\rho} + \frac{1-\alpha}{\alpha} \frac{\Lambda'(\gamma)}{\Lambda(\gamma)} + \frac{C'(\gamma)}{C(\gamma)} = 0$$

or

$$\frac{1-\alpha}{\alpha\rho} = Z(\gamma) \quad (17)$$

where $Z(\gamma)$ represents the marginal loss in social welfare derived from shifting labor to basic research. These costs can be decomposed into a loss caused by the reduction of the initial level of variety $\Lambda(\gamma)$ and a loss given by the reduction in manufactured consumption output $C(\gamma)$. These two losses must equal the marginal increase in social welfare, derived from higher levels of future variety and given by $\frac{1-\alpha}{\alpha\rho}$.

The optimal amount of (skilled) labor to basic research $b\gamma^* = H^*_B$ can be obtained, from equation (17), as an implicit function of the different parameters of the economy. By implicit differentiation, we have $\gamma_G > 0, \gamma_k > 0, \gamma_\eta < 0, \gamma_b < 0, \gamma_\rho < 0, \gamma_{M^+} > 0$. The optimal allocation of resources to basic research depends positively on G and k , which are parameters that measure the productivity of skilled labor versus the productivity of unskilled labor. Analogously, γ^* is negatively correlated with the productivity parameters η and b . An increase in these parameters implies a lower productivity of labor allocated to basic research. As we might expect, the optimal amount of labor allocated to basic research depends negatively on the intertemporal discount parameter ρ . The reason is that a higher discount rate represents a greater valuation of present variety and output with respect to future levels of variety.

Finally, we have that a larger economy size M^+ means greater present variety (by expression (14)), and also an increase in optimal future amounts of variety. As in growth models with a single research sector, in this model

⁸Since all the goods enter symmetrically in the index $D(t)$, we have $D(t) = N(t)^{\frac{1-\alpha}{\alpha}} C(t)$. Substituting $N(t) = \Lambda(\gamma)e^{\gamma(t-t_0)}$, $C(t) = C(\gamma)$, we have the expression (16).

a larger economy size allows amortizing greater research costs. Graphically, (see Figure 2) we can see that an increase in M^+ shift down the curve $Z(\gamma)$, increasing the optimal allocation of resources to basic research γ^* .

This result means that, if there were one single world government trying to maximize social welfare, economic integration between similar economies would speed up the worldwide growth rate. This is similar to the results obtained in models of trade and integration with only one research sector (Rivera-Batiz and Romer (1991)). However, in a multiple government world, the increase in growth depends in an essential way on the kind of behavior adopted by governments after the integration. If governments cooperate between them, maximizing the joint welfare, the growth rate is higher than in the case of non-cooperative behavior. But non-cooperative behavior can be optimal from a single-country perspective, because social benefits derived from investments in basic research are not appropriated only by the country which makes the investment.

5 Applied Knowledge in Basic Research

Up to now, we have neglected the role of knowledge derived from commercial innovation $N(t)$ in basic research. However, the process of development of new commercial goods generates a kind of empirical or applied knowledge that could be essential for the production of new theoretical knowledge. For example, experiments on new materials that can arise in the process of designing a new microprocessor could be very important for the development of some fields of theoretical physics. In this section it is assumed that theoretical and applied knowledge interact between themselves and that both kinds of knowledge are used as essential inputs in the two research sectors.

A workable form for the production function of new theoretical knowledge that considers this assumption is

$$dI(t) = \frac{H_B}{b} I(t)^\theta N(t)^{1-\theta} dt \quad (18)$$

where $0 < \theta < 1$ is a technological parameter.

Under the new basic research technology expressed by (18) the growth rate for the economy is determined both by the government behavior and market incentives. Furthermore, the allocation by the government of skilled labor to basic research has two opposite effects on long run innovation growth. The first one is positive and direct: more skilled labor in basic

research implies a greater rate of growth for the stock of theoretical knowledge. The second effect is negative and comes from the reduction in the stock of knowledge derived from the level of variety. Since basic and commercial research compete for the same input (skilled labor), an increase in basic research expenditures reduces commercial research activity and, consequently, the level of variety in the economy. This means a lower available stock of applied knowledge for basic research activities and, therefore, a reduction in the theoretical knowledge growth rate.

To see this, we solve the model for the parameter restriction $\theta = \eta$. This restriction makes easier the analysis. In Appendix VI, we obtain that, for $\gamma \neq 0$, the number of varieties of the economy is given by

$$N(t) = \Delta(\gamma)^{\frac{1}{1-\alpha}} I(t_0) e^{g(t-t_0)} \quad (19)$$

where g denotes the balanced growth path growth rate (that now differs from the allocation of skilled labor to basic research $\gamma \equiv H_B/b$) and $\Delta(\gamma)$ is

$$\Delta(\gamma) = \left[\frac{(1-\alpha)(M^+ - b\gamma)\gamma}{a(1-\alpha + kG\alpha)} + \left[\frac{kG\alpha\rho}{2(1-\alpha + kG\alpha)} \right]^2 \right]^{1/2} - \frac{kG\alpha\rho}{2(1-\alpha + kG\alpha)}$$

(for $\gamma = 0$, $\Delta = \frac{(1-\alpha)M^+}{kG\alpha\rho}$). It is easy to show that $\Delta'(\gamma) < 0$. Using equation (18), we have that the growth rate for the number of varieties is given by

$$g = \gamma \Delta(\gamma)$$

$$= \left[\frac{(1-\alpha)(M^+ - b\gamma)\gamma}{a(1-\alpha + kG\alpha)} + \left[\frac{kG\alpha\rho}{2(1-\alpha + kG\alpha)} \right]^2 \right]^{1/2} - \frac{kG\alpha\rho}{2(1-\alpha + kG\alpha)}. \quad (20)$$

We can see in this expression for the growth rate, independently of the government policy, an increase in parameters like the economy size have a direct positive effect on growth. Thus, in this framework both the allocation of resources to basic research γ and markets incentives play a direct role in determining long run innovation and growth. Moreover, differentiation in the expression for the growth rate (20) with respect to the allocation of resources to basic research γ yields

$$\frac{dg}{d\gamma} > 0 \Leftrightarrow M^+ > 2b\gamma = 2H_B. \quad (21)$$

Thus, an increase of skilled labor in basic research has positive effects on the growth rate if the economy size is large enough.⁹

Therefore, since basic research expenditures have a negative effect on the short run level of knowledge derived from commercial research and this knowledge input is used to produce new theoretical knowledge, an increase in basic research expenditures will have lower effects on growth than in the previous model in which theoretical knowledge was produced by using theoretical knowledge as the only knowledge input. Notice, however, that basic knowledge continues to be essential in the production of blueprints for new goods. Hence, the previous result of the impossibility of long run innovation and growth without public financing of basic research will not change at all.

6 Conclusion

This paper constructs a model of endogenous product innovation which separates basic research from the design of new commercial goods. Basic research output -theoretical knowledge- is used as an essential input in commercial research, and both research sectors compete for the same input, skilled labor. The path of innovation and growth is obtained as a result of the interaction between the decisions of government and profit-maximizing firms. In particular, if theoretical knowledge is the only knowledge input for the production of new theoretical knowledge, we have that long run growth is determined by government, whereas the static level of variety is determined both by private firms and government. However, if the public knowledge derived from product innovation is also an essential input in the production of new theoretical knowledge, the level of variety and its long run rate of growth are the result of the decision of both private and public agents.

It is possible to extend the model in several ways. First, the assumption of a social welfare maximizing government which finances basic research could be changed by assuming either a different government behavior or by introducing other institutions (private universities or foundations) that would perform this function. Another extension is the study of international cooperation policies in basic research. Since growth depends on government

⁹Alternatively, an equivalent condition to (21) is obtained by differentiating the growth rate expression $\gamma\Delta(\gamma)$ with respect to γ . We obtain that $\frac{d\gamma}{d\gamma}$ is higher than zero if the elasticity of substitution $\epsilon_{(\Delta(\gamma),\gamma)} = \left|\gamma\frac{\Delta'(\gamma)}{\Delta(\gamma)}\right|$ is lower than one. Thus, to obtain a positive relationship between the growth rate and the resources allocated to basic research $b\gamma$ we need a proportional decrease in the number of varieties derived from an increase in skilled labor allocated to basic research lower than one.

decisions on basic research expenditures, a game theoretical approach becomes now a natural way to analyze these cooperation policies, which could be important to determine worldwide growth. Finally, in this context of endogenous supply of skilled labor and growth that depends on basic research expenditures, the study of policies such as domestic income taxation and its international repercussion becomes also essential for explaining innovation and growth in all countries involved.

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Appendix I

Consumers' Problem

A. Static Problem

The representative consumer maximizes (4) subject to (5). The Lagrangian of the problem is

$$L = c^\alpha - \lambda[pc + \tau - E].$$

The first order conditions are

$$\frac{\partial L}{\partial c} = \alpha c^{\alpha-1} - \lambda p = 0 \rightarrow c(n, t) = \left[\frac{\lambda}{\alpha}\right]^{-\epsilon} \quad (22)$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \int_0^{N(t)} p(n, t)c(n, t)dn = E(t) - \tau = E_\tau(t). \quad (23)$$

Substituting (22) into (23)

$$E_\tau(t) = \left[\frac{\lambda}{\alpha}\right]^{-\epsilon} \int_0^{N(t)} p(n', t)^{1-\epsilon} dn' \Rightarrow \left[\frac{\lambda}{\alpha}\right]^{-\epsilon} = \frac{E_\tau(t)}{\int_0^{N(t)} p(n', t)^{1-\epsilon} dn'} \quad (24)$$

and substituting (24) into (22) we obtain the demand function (6).

Dynamic Problem

Let $E_\tau(t) = P_D(t)D(t)$ be the after-tax nominal expenditure, where $P_D(t)$ is a price index correspondent to $D(t) = \left[\int_0^{N(t)} c(n, t)^\alpha dn\right]^{1/\alpha}$. Thus $\ln D(t) = \ln E_\tau(t) - \ln P_D(t)$. Substituting this expression into (4) we determine the indirect utility function. Given the constrain (5), the Lagrangian of the problem is

$$L^* = e^{-\rho(t-t_0)}[\ln E_\tau(t) - \ln P_D(t)] - \lambda(t_0)[e^{-\int_{t_0}^t r(s)ds}(E_\tau(t) + \tau) - A(t_0)].$$

Differentiating w.r.t $E_\tau(t)$ and the multiplier $\lambda(t_0)$, we have the first order conditions

$$\frac{e^{-\rho(t-t_0)}}{E_\tau(t)} = \lambda(t_0)e^{-[R(t)-R(t_0)]}$$

plus the budget constrain (10) ($R(t) - R(t_0)$ is the solution of the interest rate integral). Deriving logarithmically w.r.t time t we have

$$\frac{\dot{E}_\tau(t)}{E_\tau(t)} = \dot{R}(t) - \rho = r(t) - \rho$$

Appendix II

*Determination of i^**

A worker decides to specialize if inequality (7) is verified. The left side of the inequality is

$$\begin{aligned} \int_{t+S}^{t+T} e^{-\rho(s-t)} w_H i^\beta ds &= \frac{e^{-\rho(s-t)}}{\rho} \Big|_{t+S}^{t+T} w_H i^\beta = \\ &= \left[\frac{e^{-\rho S} - e^{-\rho T}}{\rho} w_H i^\beta \right]. \end{aligned}$$

The right side is given by

$$\int_t^{t+T} e^{-\rho(s-t)} w_L ds = \left[\frac{1 - e^{-\rho T}}{\rho} w_L \right].$$

Comparing both expressions and solving for i^* , we have the expression (8).

Appendix III

Equilibrium Number of Varieties $N(t)$

By the equilibrium in the patent market expressed in (12), considering the budget constrain for the government $\tau = w_H b \gamma$, we obtain

$$\frac{(1 - \alpha)(1 - w_H b \gamma)}{(\rho + \gamma)N(t)} = \frac{a w_H}{N(t)^\eta I(t)^{1-\eta}}.$$

Defining $\Lambda = N(t)/I(t)$ and solving for w_H we have

$$w_H = \frac{(1 - \alpha)}{a(\rho + \gamma)\Lambda^{1-\eta} + b\gamma(1 - \alpha)}. \quad (25)$$

The unskilled labor market equilibrium condition (11) give us

$$mG^{1/\beta}[\frac{w_L}{w_H}]^{1/\beta} = \frac{\alpha}{w_L}(1 - w_H b\gamma).$$

Then, the supply of skilled labor is

$$k[m(i^+)^{\beta+1} - mG^{1/\beta}[\frac{w_l}{w_h}]^{1/\beta}G[\frac{w_L}{w_H}]] = k[m(i^+)^{\beta+1} - G\alpha(1/w_H - b\gamma)].$$

This expression must equal the demand for skilled labor (9). Substituting w_H by its value in (25), we have an expression of $N(t)$ and the parameters of the model. Solving for $N(t)$, we determine (14).

Appendix IV

Equilibrium Consumption Output $C(t)$

Given the equilibrium condition for the unskilled labor market (11), solving for w_L we have

$$w_L = \left[\frac{\alpha(1 - w_H b\gamma)w_H^{1/\beta}}{mG^{1/\beta}} \right]^{\frac{\beta}{\beta+1}}.$$

Substituting this expression into (10) gives us

$$L(t) = C(t) = mG^{1/\beta}[\frac{w_L}{w_H}]^{1/\beta} = \dots = [m^\beta G\alpha(1/w_H - b\gamma)]^{\frac{1}{\beta+1}}.$$

Substituting w_H by its value in (25)

$$C(t) = \left[m^\beta G\alpha \frac{(\rho + \gamma)\Lambda(\gamma)^{1-\eta}}{1 - \alpha} \right]^{\frac{1}{\beta+1}}.$$

Using (14), we establish the expression for the equilibrium consumption output (15).

Appendix V

Government's Problem

By (16), $U(\gamma)$ can be expressed as

$$U(\gamma) =$$

$$= \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \frac{1-\alpha}{\alpha} \ln[\Lambda(\gamma)I(t_0)] + \ln C(\gamma) dt + \\ + \frac{1-\alpha}{\alpha} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \gamma(t-t_0) dt.$$

Solving both integrals, we have the equation

$$\rho U(\gamma) = \\ = \frac{1-\alpha}{\alpha} \ln[\Lambda(\gamma)I(t_0)] + \ln C(\gamma) + \gamma \frac{1-\alpha}{\alpha \rho}.$$

Since ρ is a positive constant, the maximization of this expression is equivalent to the maximization of (16). Substituting $\Lambda(\gamma)$ and $C(\gamma)$ (expressions (14) and (15)) and differentiating w.r.t γ we have

$$\frac{1-\alpha}{\alpha \rho} = Z(\gamma) \equiv -\frac{1-\alpha}{\alpha} \frac{\Lambda'(\gamma)}{\Lambda(\gamma)} + \frac{C'(\gamma)}{C(\gamma)} = \\ = \left[\frac{1-\alpha}{\alpha} + \frac{1}{\beta+1} \right] \left[\frac{1}{1-\eta} \right] \left[\frac{b}{M^+ - b\gamma} + \frac{1-\alpha + kG\alpha}{kG\alpha\rho + \gamma(1-\alpha + kG\alpha)} \right] - \\ - \frac{1}{\beta+1} \left[\frac{1}{\rho + \gamma} \right].$$

$Z(\gamma)$ is a polynomial of degree four in γ . In the positive axis, the function is a parabol with the form of U. Their asintotes are $\gamma = 0$, $\gamma = \frac{M^+}{b}$. Thus, the equation $\frac{1-\alpha}{\alpha \rho} = Z(\gamma)$ has two solutions in the positive axis. One of them -when the slope of $Z(\gamma)$ is positive- verifies the second order condition $U''(\gamma) < 0$. This is the solution of the government's problem. The optimal allocation of resources to basic research γ^* can be written as a implicit function of the parameters of the model. Then $\gamma^* = \gamma(G, k, \eta, b, \rho, M^+, \beta, \alpha)$. Applying the Implicit Function's Theorem, we have the results analyzed in Section IV.

Appendix VII

Equilibrium Number of Varieties $N(t)$ (case $\frac{\dot{I}(t)}{I(t)} = \frac{H_B}{b} \left[\frac{N(t)}{I(t)} \right]^{1-\theta}$)

Let g be the common growth rate for all the real variables. The basic and commercial research technologies expressed by (18) and (2) respectively imply

$$g = \frac{\dot{I}(t)}{I(t)} = \frac{H_B}{b} \left[\frac{N(t)}{I(t)} \right]^{1-\theta} = \gamma \Omega^{1-\theta}$$

$$g = \frac{\dot{N}(t)}{N(t)} = \frac{H_A}{a} \left[\frac{N(t)}{I(t)} \right]^{1-\eta} = \frac{H_A}{a} \Omega^{\eta-1}$$

where $\Omega \equiv N(t)/I(t)$, $b\gamma \equiv H_B$ is the allocation of skilled labor to basic research (decided by government) and H_A is the allocation of skilled labor to development of new goods (decided by private firms). Since $N(s) = N(t) \exp[g(s-t)]$ and $\tau = w_H H_B = w_H g b \Omega^{\theta-1}$, the equilibrium in the patent market condition (12) implies now

$$\frac{(1-\alpha)(1-w_H g b \Omega^{\theta-1})}{(\rho+g)N(t)} = \frac{a w_H}{N(t)^\eta I(t)^{1-\eta}}$$

Solving for w_H we have

$$w_H = \frac{(1-\alpha)}{a(\rho+g)\Omega^{1-\eta} + g b \Omega^{\theta-1}(1-\alpha)}. \quad (26)$$

The unskilled labor market equilibrium condition give us

$$m G^{1/\beta} \left[\frac{w_L}{w_H} \right]^{1/\beta} = \frac{\alpha}{w_L} (1 - w_H g b \Omega^{\theta-1}).$$

Substituting this expression into the last expression, the supply of skilled labor is

$$k[m(i^+)^{\beta+1} - m G^{1/\beta} \left[\frac{w_L}{w_H} \right]^{1/\beta} G \left[\frac{w_L}{w_H} \right]] = k[m(i^+)^{\beta+1} - G\alpha(1/w_H - g b \Omega^{\theta-1})].$$

This expression must equal the demand for skilled labor derived from the new technology

$$H^d = H_B + H_A = g[b\Omega^{\theta-1} + a\Omega^{1-\eta}].$$

Substituting w_H by its value in (26)

$$M^+ - \frac{kG\alpha}{1-\alpha} a(\rho+g)\Omega^{1-\eta} = g[b\Omega^{\theta-1} + a\Omega^{1-\eta}].$$

By equation (18), the rate of growth in this economy can be expressed as $g = \gamma\Omega^{1-\theta}$. Substituting this expression we have

$$M^+ - \frac{kG\alpha}{1-\alpha}a(\rho + \gamma\Omega^{1-\theta})\Omega^{1-\eta} = \gamma\Omega^{1-\theta}[b\Omega^{\theta-1} + a\Omega^{1-\eta}].$$

For the parameter restriction $\theta = \eta$ we obtain

$$M^+ - b\gamma = \frac{kG\alpha\rho a}{1-\alpha}\Omega^{1-\theta} + \frac{\gamma a(1-\alpha + kG\alpha)}{1-\alpha}\Omega^{2(1-\theta)}. \quad (27)$$

If we define $\Delta \equiv \Omega^{1-\theta}$, it is possible to obtain the equilibrium number of varieties as a function of the parameters of the model. Making this transformation, equation (27) can be written as

$$\gamma a \frac{1-\alpha + kG\alpha}{1-\alpha} \Delta^2 + \frac{kG\alpha\rho a}{1-\alpha} \Delta - (M^+ - b\gamma) = 0.$$

The positive solution of this equation is given by

$$\Delta = \left[\frac{(1-\alpha)(M^+ - b\gamma)}{\gamma a(1-\alpha + kG\alpha)} + \left\{ \frac{kG\alpha\rho}{2\gamma(1-\alpha + kG\alpha)} \right\}^2 \right]^{1/2} - \frac{kG\alpha\rho}{2\gamma(1-\alpha + kG\alpha)}.$$

Making $\Omega = \frac{N(t)}{I(t)} = \Delta^{1/(1-\theta)}$ we determine the expression for the level of variety (19).

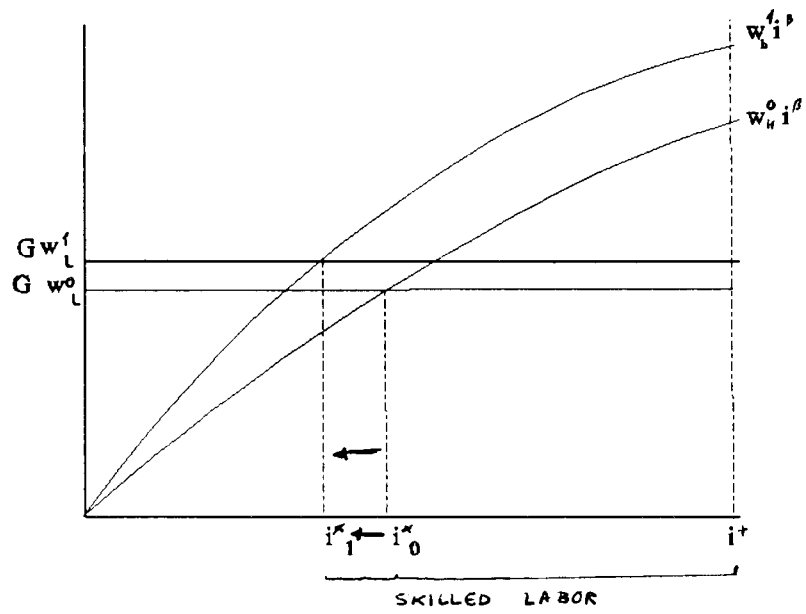


Figure 1: Effects of an increase in (w_h/w_l) on the specialization of labor

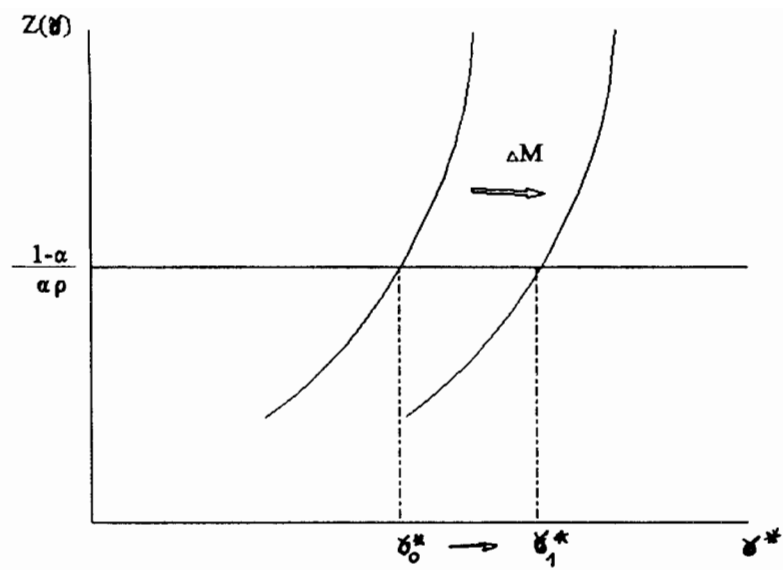


Figure 2: Effects of an increase in the size of the economy (M^+) on optimal growth

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