

Money and Prices in Models of Bounded Rationality in High Inflation Economies*

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Abstract

This paper studies the short run correlation of inflation and money growth. We study whether a model of learning can do better than a model of rational expectations, we focus our study on countries of high inflation. We take the money process as an exogenous variable, estimated from the data through a switching regime process. We find that the rational expectations model and the model of learning both offer very good explanations for the joint behavior of money and prices.

Keywords: Inflation and money growth, switching regimes, quasi-rationality.

JEL classification: D83, E17, E31.

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1 Introduction

The purpose of this paper is to explore the empirical implications of departing from rational expectations in very simple money demand models. The objective is to study the joint distribution of money and prices.

It is well known that in the long run money growth and inflation are highly related¹ but that in the short run this relationship is much weaker. It has been long recognized that standard rational expectations monetary models imply a correlation between money and prices in the short run that is way too high relative to the data in low inflation economies. The reason is that velocity (the inverse of real money demand) fluctuates too little in the models relative to the data and so does the ratio of money to prices. Most attempts at reconciling this feature have explored models with price stickiness or with segmented markets.²

Recent monetary models of bounded rationality imply more sluggish adjustment of inflation expectations than the rational expectations versions. A shock to money supply is incorporated more slowly into inflation expectations under learning than under rational expectations. Thus, as long as velocity depends on expected inflation, as it is standard in formulations for money demand, velocity will also exhibit more sluggish movements in a model of learning than under rational expectations. This suggests that a model of learning can break the strong contemporaneous correlation of money growth and inflation in the rational expectations model, and it can perhaps bring the model closer to the data of low inflation countries.

On the other hand, as we show in the main body of the paper, high-inflation economies display a high contemporaneous correlation of money growth and inflation. To the extent that we wish to have a model that can explain the behavior in both high- and low-inflation economies, this seems to pose a challenge for the use of models of learning since, as we explained, models of rational expectations do deliver a high correlation of money growth and inflation. If it turned out that sticky expectations were able to explain the correlation of money and inflation only in low-inflation economies but one had to resort to rational expectations to explain the data

¹This has been documented, for example, in Lucas (1980), Fitzgerald (1999) and McCandless and Weber (1995).

²Some examples of papers explaining the relatively low short run correlation of inflation and money growth observed in the data are Rotemberg (1984), Grossman and Weiss (1983), Alvarez and Atkeson (1997) and Alvarez, Atkeson and Edmond (2001).

in high-inflation countries, our personal conclusion would be that models of learning, overall, fail to explain the correlation of money and inflation. It is not acceptable, in general, to switch conveniently between learning or rational expectations depending on which assumption matches the data for each kind of country, but it would be particularly unacceptable to assume that agents in hyperinflationary countries understand better the behavior of inflation than agents in low-inflation countries. All the evidence suggests that agents in hyperinflationary countries find it harder to understand the working of the economy so, if anything, one has to make sure that a model of learning is consistent with the observations in high-inflation countries.

The literature on models of learning in macroeconomics has been very productive in the last two decades³ but the applications of models of learning to empirical issues is relatively scant. Some references are Chung (1990), Arifovic, Bullard and Duffy (1997), Timmerman (1993), Sargent (1999), Evans and Honkapohja (1993) and Marcet and Nicolini (2003). Most related to our work is the paper by Saint-Paul (2001) who argues that a model of boundedly rational behavior could explain the delayed response of prices to changes in money growth. As we discuss in detail in Marcet and Nicolini (2003) (MN) the main obstacle for using models with bounded rationality to explain actual data has been that there are too many ways of being irrational, leaving room for too many degrees of freedom. A methodological device that we propose in MN to face this "free-parameters" problem is to allow only for expectation-formation mechanisms that depart from the true conditional expectation only by a small distance. We propose three different (lower) bounds to rationality and show that in a model similar to the one we use in this paper they become operative in determining the equilibrium values of the model parameters, therefore solving the free parameters problem. A key feature of the bounds, is that agents are nearly-rational in the sense that the expected value of the difference between expected or perceived inflation and the true conditional expectation of inflation is very small.⁴

We show in MN that a nearly-rational model of learning can have very different implications from a rational expectations model in a setup to explain seigniorage-driven hyperinflations. The learning mechanism we use combines tracking with least squares, two of the most common mechanisms used in the literature. That mechanism works well both in stable as well as in changing

³See Sargent (1993), Marimon (1997) and Evans and Honkapohja (2001) for surveys.

⁴Rational expectations implies that difference to be exactly equal to zero.

environments, so it produces good forecasts both in countries with low and stable inflation and countries with high average inflation that experience, from time to time, recurrent burst of hyperinflations. We show in that paper that, in terms of observations on hyperinflations, the equilibrium outcome can be very different from the rational expectations equilibria *only* when the government follows a high average seigniorage policy, which goes along with high money growth. On the other hand, if the average seigniorage is low, the economy under bounded rationality has similar implications to the rational expectations equilibrium as far as the behavior of inflation is concerned. The reason for this result is that in stable environments, the bounds imply that agents learn the true structure of the model relatively fast and the outcome converges to the rational expectations outcome very quickly. But countries with high average inflation also exhibit very unstable environments and they have recurrent hyperinflations, as the econometric estimates we provide in the appendix of this paper clearly testify. In these changing environments it is much harder to learn the true structure of the model with simple backward looking schemes, and therefore the equilibrium outcome can be very different from the rational expectations equilibrium for a very long transition characterized by recurrent bursts in inflation rates.

In this paper we use the same model of learning as in MN, which is nearly-rational in changing environments, to compare the empirical implications of the bounded rationality hypothesis relative to the rational expectations version in high-inflation countries. In our previous work we were concerned with the issue of whether or not periods of hyperinflations and high money growth could emerge endogenously. Since we are now concerned with the correlation of money growth and inflation, and in order to start with the simplest possible model, here we impose periods of high money growth by assuming a switching regime for exogenous money growth.⁵ So, we attempt

⁵Another recent application of a model with switching regimes in money growth is Andolfatto and Gomme (2003) (AG). They have a cash in advance model with endogenous capital and interest rates. They focus on a model where agents do not observe the underlying state of the economy and follow a Bayesian updating rule to form probabilities of being in each state. This device introduces sluggishness into the expectations formation process, as our learning algorithm does. Our agents in the rational expectations version have more information than agents in AG, since they observe the regime each period, while our agents in the learning version of the model do not see the regime and do not understand the behavior of the economy, so they have less information than the agents in AG. Another difference with AG is that we focus on the implications for the cross-correlogram of money growth and inflation while their focus is on the behavior of interest rates.

to explain the potential role of sticky expectations in explaining the cross-correlations of money and prices in high inflation countries.

The way we model the lower bounds on rationality is also different from our previous work. In MN we computed formally learning parameters that generated good forecasts within the equilibrium, to satisfy a consistency criterion formally defined in that paper. Since the learning scheme that we used in that paper, that combines tracking and OLS, was shown to perform well in hyperinflationary environments we use that learning scheme here. In addition, and given that we computed the equilibrium values for the learning parameters using the same data from Argentina that we use in this paper, we import the equilibrium parameter values that we obtained in that paper. We also show that our main conclusions do not depend much on the exact value used for the learning parameter values. To the extent that the model under learning reproduces the observed cross correlations for most learning parameters we can state that the results are not sensitive to the exact value of the learning parameters. It is in this sense that we are not subject to the free parameters criticism that we stated above.

The model has a single real demand for money equation that is decreasing with expected inflation. The exogenous driving force is the money supply. We fit a Markov switching regime statistical model for the money growth rate for five high inflation countries and solve both the bounded rationality and rational expectations versions of the model. Finally, we compare the joint distribution of money growth and prices on the model and the data. We find that the cross-correlogram of money growth and prices in high inflation countries is consistent both with rational expectations and the bounded rationality hypothesis. Indeed, we find that even though expected inflation under learning adjusts more slowly to money shocks, it turns out that the cross-correlations of money and inflation under RE and learning are similar. This is because money shocks in countries with high inflation are so large that their variability dominates the variability of expectations. Since both RE and learning are able to explain the data, this is encouraging for our longer run research objective of trying to explain the joint behavior of money and inflation both in high and low inflation countries with models of nearly-rational learning.

Section 2 describes the model, it explains the reasons that a model of learning can have different implications than rational expectations, it fits the Markov switching regime to the data, it solves both the rational expectations and learning versions of the model and calibrates all the parameters. Section

3 presents empirical results and compares the models to the data. Section 4 concludes.

2 The Model

The model consists of a demand for cash balances given by

$$P_t = \frac{1}{\phi} M_t + \gamma P_{t+1}^e, \quad (1)$$

where ϕ, γ are positive parameters. P_t, M_t are the nominal price level and the demand for money, and P_{t+1}^e is the forecast of the price level for next period. The driving force of the model is given by the stochastic process followed by the growth rate of money. This well known money demand equation is consistent with utility maximization in general equilibrium in a context of overlapping generations model.

To complete the model one needs to specify the way agents forecast the future price level. In what follows, we compare the rational expectations version with a version in which agents use an ad-hoc algorithm that depends on past information to forecast future prices. There are several ways in which one can restrict the bounded rationality, or learning, version of the model to be "close" to rational expectations. For example, it has been common to study the conditions under which the equilibrium outcome of the learning model converges to the rational expectations model. To be more specific, consider two alternative expectation formation mechanisms in the above money demand:

$$\begin{aligned} P_{t+1}^e &= E_t [P_{t+1}] \\ P_{t+1}^e &= L(P_{t-1}, P_{t-2}, \dots). \end{aligned}$$

for a given function L .⁶ We can then obtain corresponding solutions $\{P_t^{RE}, P_t^L\}_{t=0}^\infty$ and, letting $\rho(S_1, S_2)$ be some distance between two sequences S_1 and S_2 ,

$$\rho(\{P_t^{RE}\}_{t=0}^\infty, \{P_t^L\}_{t=0}^\infty) \quad (2)$$

measures how different two equilibria are. If the model is deterministic, convergence to rational expectations can be written as

$$\lim_{t \rightarrow \infty} (P_t^{RE} - P_t^L) = 0$$

⁶Strictly speaking, L could depend on the time period, as it does if agents use OLS estimates, but we leave this dependence implicit.

When this occurs, the equilibrium outcome under learning "looks like" a rational expectations equilibrium in the limit. Only the transition, if long enough, can then generate different behavior in the rational expectations or in the learning version of the model.

Near-rationality can be interpreted as imposing restrictions on the size of the systematic mistakes the agents make in equilibrium. This amounts to saying that agents' expectations can not be too far away from the actual behavior of the economy or, formally, it amounts to imposing that the distance

$$\rho(\{P_t^L\}_{t=0}^\infty, \{L(P_{t-1}, P_{t-2}, \dots)\}_{t=0}^\infty) \quad (3)$$

can not be too large. In an extreme case, if the function L is such that the learning equilibrium makes zero mistakes we have that

$$\rho(\{P_t^L\}_{t=0}^\infty, \{L(P_{t-1}, P_{t-2}, \dots)\}_{t=0}^\infty) = 0$$

Since, by definition, rational expectations requires that $\rho(\{P_t^{RE}\}_{t=0}^\infty, \{P_{t+1}^e\}_{t=0}^\infty) = 0$, the last equation forces the nearly-rational learning to be the same as rational expectations. If, instead, we impose that the mistakes in (3) need not be zero but that they have to be small, we impose no restriction on the relationship between P_t^{RE} and P_t^L . As a matter of fact, in some models even if L is restricted so that (3) is small, the difference with the rational expectations outcome (i.e., (2)) can be very large. There may be learning equilibria that are near-rational and whose equilibrium is vastly different from RE. In this case, the empirical implications of the rational expectations version and the "small mistakes" version may be different, while the learning equilibrium is still close to being rational, in the proper metric defined by (3).

One example of this behavior is MN. In that paper we impose three bounds that effectively impose different metrics ρ . As we mentioned in the introduction, if inflation is low, then the only learning equilibrium in MN that satisfies the bounds is the rational expectations equilibrium. However, when inflation is high, there are learning equilibrium outcomes that look different from the rational expectations one. In this empirical investigation we focus the analysis on high inflation countries only; at first sight it seems that it is in these countries where it is easier to obtain large differences between the behavior of learning and rational expectations.

2.1 A Regime Switching Model for Money Growth

In order to numerically solve the model, we will use learning and money demand parameter values that are calibrated to the Argentine economy. However, as a robustness check, we will also investigate the evolution of the money supply for other high inflation countries. As we will see, varying either money demand or learning parameters makes little difference in the results. However, the evolution of money supply does make a difference.

In order to fit the process for the nominal money supply, we first look at the evidence for five Latin-American countries that experienced high average inflation and very high volatility in the last decades: Argentina, Bolivia, Brazil, Mexico and Peru. We use data for M1 and consumer prices from the International Financial Statistics published by the International Monetary Fund. To compute growth rates, we used the difference in the log of the variable⁷.

While the high inflation years are concentrated between 1975 and 1995 for most countries, the periods do not match exactly. Thus, we chose, for each country, a sub-period that roughly corresponds to its own unstable years. Figures 1.a to 1.e plot quarterly data on nominal money growth and inflation for the relevant periods in each case.

As it can be seen from the figures, for all those countries, average inflation was high, but there are some relatively short periods of bursts - the shaded areas - in both money growth and inflation rates, followed, again, by periods of stable but high inflation rates⁸. Note also that the behavior of the money growth rate, the driving force of the model, follows a very similar pattern. Thus, we propose to fit a Markov switching process for the rate of money growth⁹.

⁷This measure underestimates growth rates for high inflation countries. With this scale, it is easier to see the movements of inflation rates in the - relatively - tranquil periods in all the graphs. In what follows, we make the case that the data is best described as a two regime process, so this measure biases the result against us.

⁸This feature of the data has long been recognized in case studies of hyperinflations (see, for example, Bruno, Di Tella, Dornbusch and Fisher (1988)).

⁹Actually, data suggest that during the periods of hyperinflations, both inflation and the rate of money growth increase over time, a fact that is consistent with the model in MN. The statistical model we fit assumes, for simplicity, that during the hyperinflations both have a constant mean. We tried to fit the data to a switching regime model that allowed for an increasing money growth rate in the high state, but the growth element turned out not to be statistically significant and sometimes it took the wrong sign. For this reason we decided to stay with a constant growth rate conditional on each regime. It would

Our empirical analysis is based on quarterly data, since we are interested in high frequency movements in money and prices. Our eyeball inspection of Figures 1.a to 1.e suggests the existence of structural breaks. This is confirmed by the breakpoint Chow Test that we present in the appendix for the five countries, so we model $\Delta \log(M_t) = \log(M_t) - \log(M_{t-1})$ as a discrete time Markov switching regime process. We assume that $\Delta \log(M_t)$ is distributed $N(\mu_{s_t}, \sigma_{s_t}^2)$; where $s_t \in \{0, 1\}$. The state s_t is assumed to follow a first order homogeneous Markov process with $\Pr(s_t = 1 | s_{t-1} = 1) = q$ and $\Pr(s_t = 0 | s_{t-1} = 0) = p$. The evolution of the first difference of the logarithm of the money supply can therefore be written as

$$\Delta \log(M_t) = \mu_0(1 - s_t) + \mu_1 s_t + (\sigma_0(1 - s_t) + \sigma_1 s_t) \varepsilon_t$$

where ε_t is assumed to be i.i.d. and distributed $N(0, 1)$. All empirical results regarding the modelling of the money supply are reported in Appendix A.

One state is always characterized by higher mean and higher volatility of $\Delta \log(M_t)$ in all countries. Both pairs (μ_i, σ_i) are statistically significant, as well as the transition probabilities, p and q , and both states are highly persistent in all countries. These results give very clear evidence that modelling the growth rate of money as having two states is a reasonable assumption. The rate of money growth in the high-mean/high-volatility state (henceforth, the "high state") ranges from three times the rate of money growth in the low state for Argentina to nine times for Bolivia while the volatility of the high state ranges from one and a half times the volatility of the low state for Mexico to eight times for Brazil. The differences across states are gigantic. The high state is always consistent with the existence of high peaks of inflation in each of the countries. These periods are represented as the shaded areas in Figures 1.a to 1.e.

These results clearly demonstrate that the economic environment can be characterized by one in which there are changes in the monetary policy regime.

seem that assuming a constant mean for the growth rate of money in the hyperinflation regime biases the results against our rational expectations model since, in the learning version, agents adapt to whatever data is available, while if the data indeed is generated by a growing inflation in the high inflation regime we are forcing the "rational" agents in our model to use a misspecified process for the money supply. To the extent that our rational expectations model fits the cross-correlogram appropriately, it seems that assuming a constant expected money growth in both states does not affect our results.

2.2 Rational Expectations Equilibrium

Rational expectations (RE) assumes $P_{t+1}^e = E_t(P_{t+1}) \equiv E_t(P_{t+1} | I_t)$ for all t , where E_t is the expectation conditional on information up to time t . Agents observe all the relevant information in the economy, so that $I_t \equiv (M_t, s_t, P_t, M_{t-1}, s_{t-1}, P_{t-1}, \dots)$.

We look for non-bubble equilibria, and we conjecture that in the RE equilibrium

$$E_t(P_{t+1}) = \beta_t^{RE} P_t \quad (4)$$

with β_t^{RE} being state dependent and

$$\beta_t^{RE} = \beta_0(1 - s_t) + \beta_1 s_t.$$

for some constants β_0, β_1 . To solve for an equilibrium, we must find (β_0, β_1) .

Denote the average money growth conditional on each state by $\kappa_j \equiv E\left(\frac{M_{t+1}}{M_t} \middle| s_{t+1} = j, I_t\right)$. Using the fact that $\log \frac{M_{t+1}}{M_t} \sim N(\mu_j, \sigma_j^2)$ we have that

$$\kappa_j = \exp\left(\mu_j + \frac{1}{2}\sigma_j^2\right). \quad (5)$$

and that

$$E\left(\frac{M_{t+1}}{M_t} \middle| s_t = 0, I_t\right) = \kappa_0 p + \kappa_1(1 - p) \quad (6)$$

$$E\left(\frac{M_{t+1}}{M_t} \middle| s_t = 1, I_t\right) = \kappa_0(1 - q) + \kappa_1 q \quad (7)$$

The following lemma characterizes equilibrium prices

Lemma 1

The RE equilibrium in the model of this section is given by

$$\begin{aligned} E\left(\frac{P_{t+1}}{P_t} \middle| s_t = 1, I_t\right) &= \beta_1 = \frac{\kappa_0(1 - q) + \kappa_1 q + \kappa_0 \kappa_1 \gamma(1 - q - p)}{1 + \gamma \kappa_0(1 - p - q)} \\ E\left(\frac{P_{t+1}}{P_t} \middle| s_t = 0, I_t\right) &= \beta_0 = p \kappa_0 + (1 - p) \kappa_1 \frac{1 - \gamma \beta_0}{1 - \gamma \beta_1} \end{aligned}$$

which gives the equilibrium price

$$P_t = \frac{M_t}{\phi(1 - \gamma \beta_t^{RE})}$$

Proof

We have

$$\begin{aligned}
E(P_{t+1}|s_t = 0, I_t) &= pE\left(\frac{M_{t+1}}{\phi(1-\gamma\beta_0)}|s_{t+1} = 0, s_t = 0, I_t\right) + \\
&\quad + (1-p)E\left(\frac{M_{t+1}}{\phi(1-\gamma\beta_1)}|s_{t+1} = 1, s_t = 0, I_t\right), \\
&= \frac{p}{\phi(1-\gamma\beta_0)}E(M_{t+1}|s_{t+1} = 0, s_t = 0, I_t) + \\
&\quad + \frac{(1-p)}{\phi(1-\gamma\beta_1)}E(M_{t+1}|s_{t+1} = 1, s_t = 0, I_t), \\
&= \frac{p\kappa_0}{\phi(1-\gamma\beta_0)}M_t + \frac{(1-p)\kappa_1}{\phi(1-\gamma\beta_1)}M_t, \\
&= p\kappa_0P_t + (1-p)\kappa_1\frac{(1-\gamma\beta_0)}{(1-\gamma\beta_1)}P_t, \\
&= P_t\left(p\kappa_0 + (1-p)\kappa_1\frac{(1-\gamma\beta_0)}{(1-\gamma\beta_1)}\right).
\end{aligned}$$

where the first equality arises from equation (1) and the conjecture that $P_{t+1}^e = E(P_{t+1}|s_t = 0, I_t) = \beta_0P_t$. The third equality comes from (5), and the fourth comes from equation (1). Combining this expression with (4) we get

$$\beta_0 = p\kappa_0 + (1-p)\kappa_1\frac{1-\gamma\beta_0}{1-\gamma\beta_1}. \quad (8)$$

From an analogous derivation conditioning on $s_t = 1$ we get

$$\beta_1 = q\kappa_1 + (1-q)\kappa_0\frac{1-\gamma\beta_1}{1-\gamma\beta_0}. \quad (9)$$

Solve this system of equations for the unknowns (β_0, β_1) to get

$$\beta_1 = \frac{\kappa_0(1-q) + \kappa_1q + \kappa_0\kappa_1\gamma(1-q-p)}{1 + \gamma\kappa_0(1-p-q)}$$

and plugging this expression into (8) we obtain the solution for β_0 .

Plugging this into (4) and (1) we get the equilibrium price. QED

Notice that expected inflation differs from expected money growth in state 0, due to the presence of the factor $\frac{1-\gamma\beta_0}{1-\gamma\beta_1}$ (or its inverse in state 1).

This factor appears because under RE the inflation rate satisfies

$$\frac{P_t}{P_{t-1}} = \frac{1 - \gamma\beta_{t-1}^{RE}}{1 - \gamma\beta_t^{RE}} \frac{M_t}{M_{t-1}}.$$

so that if there is a change of regime from one period to the next, the ratio $\frac{1 - \gamma\beta_{t-1}^{RE}}{1 - \gamma\beta_t^{RE}}$ introduces a wedge between the change in prices relative to the change in the money supply. In this model, therefore, when there is a regime change, the velocity is not constant, due to the fact that expected inflation influences the relationship between money and prices. Notice that the difference between inflation and money growth is larger the larger the difference in expected inflations, and that if $p + q = 1$ the two possible values of inflation are equal to the two possible values of money growth in each state.

2.3 Learning Equilibrium

In the model under learning we assume that agents do not observe the state of the economy, they only observe inflation. We use the same learning mechanism as in MN. Let

$$P_{t+1}^e = \beta_t P_t \tag{10}$$

where

$$\beta_t = \beta_{t-1} + \frac{1}{\alpha_t} \left(\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1} \right), \tag{11}$$

The coefficient α_t is called the "gain" and it affects the sensitivity of expectations to current information.

If we want to produce a model of learning that reproduces the observed switching regimes and where agents are near-rational, we need to use a learning mechanism that produces reasonably low prediction errors when there is a regime switch. Two of the most common specifications for the gain sequence are tracking ($\alpha_t = \alpha$ for all t), which performs well in environments that change often, and least squares ($\alpha_t = \alpha_{t-1} + 1$), that performs well in stationary environments. What is a better learning mechanism in an environment with switching regimes, tracking or OLS?. If agents used pure tracking they would be making large mistakes when a state has been in place for a long time, because the regime is not switching and they adapt too much to money shocks. On the other hand, if agents used pure OLS they would be making large mistakes after a regime switch, because if this occurred when

t is relatively high data on inflation generated by the new regime would be incorporated very slowly into expectations so that they would take a very long time to learn about the new regime.

As we argue in MN a scheme that combines tracking and OLS generates good predictions overall. Since we aim at a model of learning that is nearly-rational, we assume that agents combine both mechanisms so that the gain is assumed to follow OLS as long as the forecast error is not large, but it switches to tracking as soon as some instability is detected. Formally

$$\alpha_t = \begin{cases} \alpha_{t-1} + 1 & \text{if } \left| \frac{\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}}{\beta_{t-1}} \right| \geq \nu, \\ \bar{\alpha} & \text{otherwise} \end{cases}, \quad (12)$$

where $\bar{\alpha}, \nu$ are the learning parameters. Thus, if errors are small, the gain follows a least squares rule, and as long as the regime does not switch, agents soon learn the parameters of the money supply rule. But if a large enough error is detected, the rule switches to a constant gain algorithm, so agents can learn the new parameters of the money supply rule faster.

The solution $\left\{ \frac{P_t}{P_{t-1}}, \beta_t, \alpha_t \right\}$ must satisfy (11), (12) and,

$$\frac{P_t}{P_{t-1}} = \frac{1 - \gamma\beta_{t-1}}{1 - \gamma\beta_t} \frac{M_t}{M_{t-1}}$$

which is obtained by plugging (10) into (1).

2.4 Calibration

As we mentioned before, we calibrate our money demand and learning parameters to data from Argentina.¹⁰ As a first exploration on how the models behave with alternative money supply processes, we also solve the model with the money supplies of other high inflation countries. This turned out to be a very reasonable exercise, since the sensitivity analysis we did by varying the money demand and the learning parameters showed them not to be very important from the quantitative point of view.

¹⁰Note that in the current paper we focus on non-bubble equilibria. The Laffer curve mentioned in this section refers to the fact that in our previous paper, with endogenous money supply, there were two stationary levels of inflation consistent with a given level of seignorage.

Money demand

We borrow the parameter values from our previous paper, in which we use observations from empirical Laffer curves to calibrate them. This is a reasonable choice: since one empirical implication of the original model is that recurrent hyperinflations characterized by two regimes occur when average inflation - driven by average seigniorage - is "high", we need to have a benchmark to discuss what high means. We use quarterly data on inflation rates and seigniorage as a share of GNP for Argentina¹¹ from 1980 to 1990 from Ahumada, Canavese, Sanguinetti y Sosa (1993) to fit an empirical Laffer curve. While there is a lot of dispersion, the maximum observed seigniorage is around 5% of GNP, and the inflation rate that maximizes seigniorage is close to 60%. These figures are roughly consistent with the findings in Kiguel and Neumeyer (1992) and other studies. The parameters of the money demand γ and ϕ , are uniquely determined by the two numbers above. Note that the money demand function implies a stationary Laffer curve equal to

$$\frac{\pi}{1 + \pi} m = \frac{\pi}{1 + \pi} \phi (1 - \gamma(1 + \pi)) \quad (13)$$

where m is the real quantity of money and π is the inflation rate. Thus, the inflation rate that maximizes seigniorage is

$$\pi^* = \sqrt{\frac{1}{\gamma}} - 1$$

which, setting $\pi^* = 60\%$, implies $\gamma = 0.4$. Using this figure in (13), and making the maximum revenue equal to 0.05, we obtain $\phi = 0.37$.

Money supply

For money supply, we use the estimated Markov switching models we discussed above. We fit this process to the observed behavior of money supply to each country. The results of the estimation are reported in Appendix A. We use the estimated parameters and states as the true values of the exogenous process, and we assume that these are known with certainty by the agents. While the money demand parameters are assumed the same for each country, the money supply process is estimated using data from each country.

Learning parameters

¹¹The choice of country is arbitrary. We chose Argentina because we were more familiar with the data.

The parameters described above are sufficient to solve the rational expectations model. However, we still need to be specific regarding our choice of the (still free!) parameters of the learning process, $\bar{\alpha}, \nu$.

In MN, we provide an operational definition of a bound of the type described above. In that paper we search for values of the parameter $\bar{\alpha}$ that satisfy a rational expectations-like, approximate fixed point problem, so that in equilibrium, agents make small systematic mistakes. In this paper we use the equilibrium values we obtained in MN for a model with hyperinflations ($\bar{\alpha}$ between 2 and 4) and we show the robustness of the results we obtain to the choice of these parameter values. For the value of ν , we also follow MN, where we used a value that was roughly equal to two standard deviations of the prediction error.¹²

3 Evaluating the models

The main goal of this paper is to investigate the ability of the simple money demand equation with exogenous money growth to replicate the short-run relationship between the inflation rate and the growth rate of money in high-inflation countries, using both the rational expectations model and the "almost" rational version. We dub LM the equilibrium under the learning mechanism.

Following the RBC tradition, we characterize the data using empirical moments of the joint distribution of money growth and prices. Table 1 in Appendix A.3 presents moments of their marginal distributions. As one should expect, average inflation is very similar to average money growth in each country (see, for example, Lucas (1980) for a discussion of this fact). There are slightly larger differences between the volatility of inflation and money growth rates in each country, without a clear pattern emerging from the table. We will focus our analysis of the joint distribution of money and prices on the cross-correlogram. If velocity were a constant fraction of output, in countries where the volatility of inflation (and money growth) is much larger than the volatility of output growth, the contemporaneous

¹²We also solved the learning model with an alternative specification for v , given the Markov structure of the money growth process: we replace v for v_{st} , where, if $s_t = 1$ is the state with the higher growth, we have $v_l = v$ and $v_1 = (\sigma_h/\sigma_l)v$. With this alternative specification we introduce the switching regime information into the learning mechanism, but it did not make any difference in the results.

correlation between money and prices ought to be close to one, and the leads and lags of the cross-correlation should be equal to the auto-correlogram of the money growth process. Figure 2.a and Figure 2.b present the leads and lags for the cross-correlation of money growth and prices for the five countries. It is interesting to point out that money and prices are highly correlated contemporaneously, contrary to the case of middle and low inflation countries where the correlation is less strong.¹³ In particular, the contemporaneous correlation for Mexico, the country in the sample with lower average inflation, is substantially lower than for the other countries.

We simulate the model using the calibrated parameters for money demand, the estimated values for the money supply process, and we plug in the observed values for the money supply. The results of the simulations are shown in Table 2 in Appendix B and Figures 3 and 4. The columns of Table 2 show the moments for the inflation generated by the model under rational expectations, and under LM. Under LM we have two columns, one for each of the two possible values for $\bar{\alpha} = \{10/4, 10/3\}$.

As we can see, the three simulations (the one under RE and the two under LM) give very similar results for each of the countries. In fact, for Argentina, neither the mean nor the standard deviation of the RE model are statistically different from the ones generated by the two different versions of LM. Most importantly, none of these moments are statistically different from the actual moments of inflation. The same is true for Peru, Bolivia and Mexico. Similar results arise for Brazil, with the exception that the volatility of the inflation rate under learning overestimates the true volatility. It is interesting to point out that while the simulations for Argentina, Bolivia and Peru generate volatilities that underestimate the actual ones, the volatility in the model overestimates the actual volatility for Brazil and Mexico.

Figures 3.a to 3.e and 4.a to 4.e present the leads and lags of the cross-correlogram between $\log(M_t/M_{t-1})$ and the inflation rates generated by RE, LM and the actual one for each of the five countries.¹⁴ We also include an approximation for the confidence band, $(\pm 2/\sqrt{T})$ for the cross-correlogram of the actual series (the dotted lines).

For each country, the cross-correlogram generated with the three models

¹³See Alvarez, Atkenson and Edmond (2001) and references therein for theoretical work that aims at matching these correlations for low inflation countries.

¹⁴More precisely, for each model or data, the entry corresponding to j in the horizontal axis of Figures 3 represents $corr\left(\log\left(\frac{M_t}{M_{t-1}}\right), \log\left(\frac{P_{t-j}}{P_{t-j-1}}\right)\right)$.

are very similar. Again, with the exception of Mexico, none of the cross-correlograms generated by either model is significantly different from the actual one. Learning and rational expectations perform equally well in approximating the actual cross-correlogram. A noticeable fact is that in every country except Mexico the contemporaneous correlation is lower in the simulated series than in the actual ones. This is because, as we pointed out after Lemma 1, the two states of expected inflation do not match the two states of expected money growth, and velocity in the model is not equal to one; since LM looks fairly close to RE in this model, the same occurs in the learning model. The only exception is Mexico, the country with the lowest average inflation, where the simulated contemporaneous correlations are still close to one but the actual correlation is less than .5 (obviously, the contemporaneous correlation is the intercept in Figures 3). Furthermore, Mexico is the only country which presents significant differences between the actual and the simulated cross-correlogram. This is due to the fact that Mexico's actual inflation is not so correlated to $\log(M_t/M_{t-1})$ as it is in the other countries (shown in Figures 1.a and 1.e), and as the simulated inflation are highly correlated to $\log(M_t/M_{t-1})$ they perform worse for this particular case. But even in the case of Mexico the prediction of the money demand model is not very sensitive to the expectation formation mechanism: the three models perform equally less well for Mexico.

The most important conclusion of the paper is that, although sticky expectations seemed a good candidate to obtain different behavior from rational expectations, the models are empirically equivalent if the money supply is forced to behave like in the data.

This is not because we forced the learning model to be artificially close to the rational expectations model by imposing very high rationality requirements in the learning model. In fact, both models do imply different behavior for expected inflation. Figure 5 plots expected inflation for the case of Argentina.¹⁵ *BETA* denotes expected inflation and the labels are the same as previous figures, thus *BETA LM* and *BETA LM1* correspond to the expected inflation in the learning model for $\bar{\alpha}$ equal to $\frac{10}{3}$ and $\frac{10}{4}$, while *BETA RE* stands for the expected inflation under rational expectations. The figure shows that expected inflation under learning exhibits stronger high frequency movements than rational expectations. Thus the real money demand also moves more under learning. However, the impact on the behav-

¹⁵The same happens in the other five countries.

ior of the cross correlogram is quantitatively small. This suggests that, for the calibrated parameter values, the role that high frequency fluctuations on expectations have on the short run dynamics of money and prices is negligible. Money shocks under the calibrated switching regime process are so large that they dominate the cross-correlations of money and inflation under both RE and learning, and the implications for the cross-correlogram are similar for both models despite the differences in expectations.

We can explain this situation in terms of the distances ρ that we defined at the beginning of section 2. In this environment, when the distance between expectations and actual price under learning (3) is small, it turns out that the distance between the series generated under learning and rational expectations (2) is also small, even though the distance between expectations in both models

$$\rho(\{P_t^{e,RE}\}_{t=0}^{\infty}, \{P_t^{e,L}\}_{t=0}^{\infty})$$

is quite large.

The natural exercise to perform then is to see if the results are robust to changes in the calibrated parameters. In particular, it is of interest to simulate the model with higher values for the elasticity. This could give a better chance to expected inflation to influence inflation and to generate a different behavior of the model under learning. This amounts to increasing the value of the slope parameter γ . Note however, that the money demand equation is linear, so we must check that it never becomes negative. For this, we rewrite the money demand as

$$\delta P_t = \frac{1}{\phi} M_t + \gamma P_{t+1}^e.$$

Thus, when increasing the value for γ , we also increased the value for δ such that the money demand was always positive. Note that we are changing the values for the money demand keeping fixed the learning parameters. This is not an equilibrium exercise, since the equilibrium values for the learning parameters do depend on the money demand. The only purpose of this exercise is to amplify the effects of the sluggish expectations and check if they go in the direction of better explaining the data. We solved the model for values of γ between 0.4 and 0.6. The auto-correlograms were already on the target and there was no noticeable difference by changing the values of γ . The following table reports the correlation between the inflation rate and the inflation predicted by the model for different values of the parameter γ .

Correlation between observed and predicted inflation for the two models.

γ	0.4	0.45	0.5	0.55	0.6
Corr(True Inf, Inf RE)	80.85	80.57	80.36	80.19	79.69
Corr(True Inf, Inf LM)	78.12	76.54	73.98	69.92	63.57

The performance of the two models gets poorer as the value of the elasticity is increased. It does worst for the RE model. The reason is that with RE, expected inflation is a step function. Therefore, as the regime changes, the price level makes a larger jump. As observed inflation is a smoother series, the correlation worsens. For the learning model the correlations do not change much. They get (mildly) worse mainly because after the last hyperinflation of Argentina in 1989 the model predicts a sharper drop in inflation in 1990 than the one actually occurred as the elasticity gets bigger.

This exercise reinforces the conclusion that for the high inflation economies, our money demand model puts little role for the behavior of expectations. A possible limitation of our analysis is the linear money demand used, which is not the one that best fits the evidence. However, exploring with log linear specifications is far beyond the scope of this paper.

4 Conclusion

The purpose of this paper is to explore the potential role of “nearly-rational“ expectations in explaining the high frequency movements between money and prices in high inflation countries. Sluggish expectations imply movements on velocity that could potentially explain the observed sluggish response of inflation to money shocks in low inflation countries. But the correlation of inflation and money growth in high inflation countries is quite high. The fact that rational expectations can explain this behavior in high-inflation countries poses a challenge to models of learning.

We use a learning mechanism that produces good forecasts within the model, in a way, imposing an approximate rational expectations requirement. This insures that the learning mechanism introduced in the model is not arbitrarily chosen to match the data and it insures that the agents are not making obvious mistakes in the model. We argue that the learning model we propose is nearly rational in countries where monetary policy exhibits frequent and substantial changes of regime. We fit a Markov switching

process for the exogenous driving force -the money growth rate - to five Latin-American countries. There is ample evidence in favor of the regime switching structure. We calibrate a money demand equation and we study the solutions of the model under the assumptions of both rational expectations and learning.

We find that both learning and rational expectations generate very similar empirical implications that match the observed cross-correlogram of money growth and inflation for the high-inflation countries considered in almost every dimension. This result is robust to increasing the elasticity of money demand. Thus, we conclude, the short run behavior of money and prices in high inflation countries can be explained both by a rational expectations and learning models. This is encouraging, because the high correlation of money growth and inflation observed in high inflation countries does not need the assumption of rational expectations, so it leaves room for models of learning to explain the behavior of both high- and low- inflation countries.

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A APPENDIX, Empirical Results

A.1 Chow Test for Structural Breaks

The Chow Test for the corresponding sub-samples generates the following results

Argentina (1975:01 1992:04)

Sub-samples 1975:01-1988:04, 1989:01-1990:01, 1990:02-1992:04

Chow Breakpoint Test:	1989:1 1990:1	Probability
F-statistic	3.775243	0.002964
Log likelihood ratio	22.09987	0.001161

Bolivia (1975:01 1995:04)

Sub-samples 1975:01-1983:03, 1983:04-1986:04, 1987:01-1995:04

Chow Breakpoint Test:	1983:4 1986:4	Probability
F-statistic	4.205059	0.003498
Log likelihood ratio	16.47067	0.002448

Brazil (1980:01 1995:04)

Sub-samples 1980:01-1987:04, 1988:01-1991:01, 1991:02-1995:04

Chow Breakpoint Test:	1988:1 1991:1	Probability
F-statistic	4.666468	0.001733
Log likelihood ratio	18.12746	0.001165

Mexico (1975:01 1995:04)

Sub-samples 1975:01-1989:04, 1990:01-1992:03, 1992:04-1995:04

Chow Breakpoint Test:	1990:1 1992:3	Probability
F-statistic	4.251409	0.003259
Log likelihood ratio	16.63831	0.002272

Peru (1975:01 1995:04)

Sub-samples 1975:01-1989:04, 1990:01-1991:01, 1991:02-1995:04

Chow Breakpoint Test:	1990:1 1991:1	Probability
F-statistic	16.67572	0.000000
Log likelihood ratio	53.90424	0.000000

A.2 Markov Switching Regime Estimation Results

In this sub-section we present the results of the Markov Switching Regimes estimation. Let $p = \Pr(s_t = 0 | s_{t-1} = 0)$, $q = \Pr(s_t = 1 | s_{t-1} = 1)$, and let μ_i and σ_i be the mean and the standard deviation of the growth rate of money in state i . The following tables summarize the results of the estimation.

Argentina (1975:01 1992:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.175533792	0.018297495	9.593323735
μ_1	0.454399941	0.098682095	4.604684802
q	0.915099885	0.066209340	13.82131101
p	0.919361013	0.074032079	12.41841415
σ_0	0.072853206	0.011943142	6.100003477
σ_1	0.303486106	0.079758969	3.805040466

Bolivia (1975:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.059627604	0.009754163	6.113041314
μ_1	0.536370837	0.420877046	1.274412187
q	0.935126471	0.066531539	14.05538607
p	0.986617102	0.070142749	14.06584602
σ_0	0.050541181	0.003716448	13.59932350
σ_1	0.373992612	0.159586583	2.343509119

Brazil (1980:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.157931616	0.027645929	5.712653628
μ_1	0.517317340	0.128746245	4.018115930
q	0.926016849	0.125106881	7.401805917
p	0.957205334	0.068582989	13.95689156
σ_0	0.045445418	0.019317040	2.352607738
σ_1	0.389689368	0.089648053	4.346880447

Mexico (1975:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.064677791	0.010181050	6.352762507
μ_1	0.206343338	0.021967776	9.393000814
q	0.733328049	0.174816708	4.194839597
p	0.951845883	0.060318779	15.780257828
σ_0	0.047614481	0.011171478	4.262147003
σ_1	0.061059573	0.025394331	2.404456906

Peru (1975:01 1995:04)

	Coeff.	Std Error.	t-statistic
μ_0	0.119329114	0.015084511	7.910632032
μ_1	0.744060948	0.190436648	3.985303268
q	0.830803948	0.126355006	6.567003388
p	0.971727572	0.048361910	19.86071300
σ_0	0.082091249	0.008656942	10.06842486
σ_1	0.383326249	0.134969389	2.847524436

A.3 First and second moments

Table 1 shows the first (μ) and second (σ) moments of inflation and money growth for every country.

		Table 1			
		Inflation ($\Delta \log P_t$)		Money Growth ($\Delta \log M_t$)	
Sample	Country	μ	σ	μ	σ
1975:01-1992:04	Argentina	0.146870	0.132087	0.138898	0.117783
1975:01-1995:04	Bolivia	0.069557	0.138701	0.072275	0.117080
1980:01-1995:04	Brazil	0.177091	0.131648	0.174454	0.158965
1975:01-1995:04	Mexico	0.036109	0.029010	0.037837	0.031375
1975:01-1995:04	Peru	0.104011	0.140325	0.095859	0.123695

B Simulation Results

Tables 2.a to 2.e show the first and second moments of the simulations.

		Table 2.a		
		Argentina		
Sample:	1975:01-1992:04			
			μ	σ
	<i>True</i>		0.146870	0.132087
	$\bar{\alpha} = \frac{10}{3}$		0.142280	0.137145
	$\bar{\alpha} = \frac{10}{4}$		0.141884	0.152390
	<i>RE</i>		0.144074	0.123287

		Table 2.b		
		Bolivia		
Sample:	1975:01-1995:04			
			μ	σ
	<i>True</i>		0.069557	0.138701
	$\bar{\alpha} = \frac{10}{3}$		0.072237	0.146258
	$\bar{\alpha} = \frac{10}{4}$		0.072189	0.161556
	<i>RE</i>		0.072755	0.127636

		Table 2.c		
		Brazil		
Sample:	1980:01-1995:04			
			μ	σ
	<i>True</i>		0.177091	0.131648
	$\bar{\alpha} = \frac{10}{3}$		0.164544	0.217368
	$\bar{\alpha} = \frac{10}{4}$		0.163226	0.295389
	<i>RE</i>		0.180857	0.168293

		Table 2.d		
		Mexico		
Sample:	1975:01-1995:04			
			μ	σ
	<i>True</i>		0.036109	0.029010
	$\bar{\alpha} = \frac{10}{3}$		0.037844	0.032996
	$\bar{\alpha} = \frac{10}{4}$		0.037805	0.033354
	<i>RE</i>		0.038116	0.033121

		Table 2.e		
		Peru		
Sample:	1975:01-1995:04			
			μ	σ
	<i>True</i>		0.104011	0.140325
	$\bar{\alpha} = \frac{10}{3}$		0.094706	0.146434
	$\bar{\alpha} = \frac{10}{4}$		0.094602	0.161485
	<i>RE</i>		0.096507	0.174697

Fig 1.a: Argentina

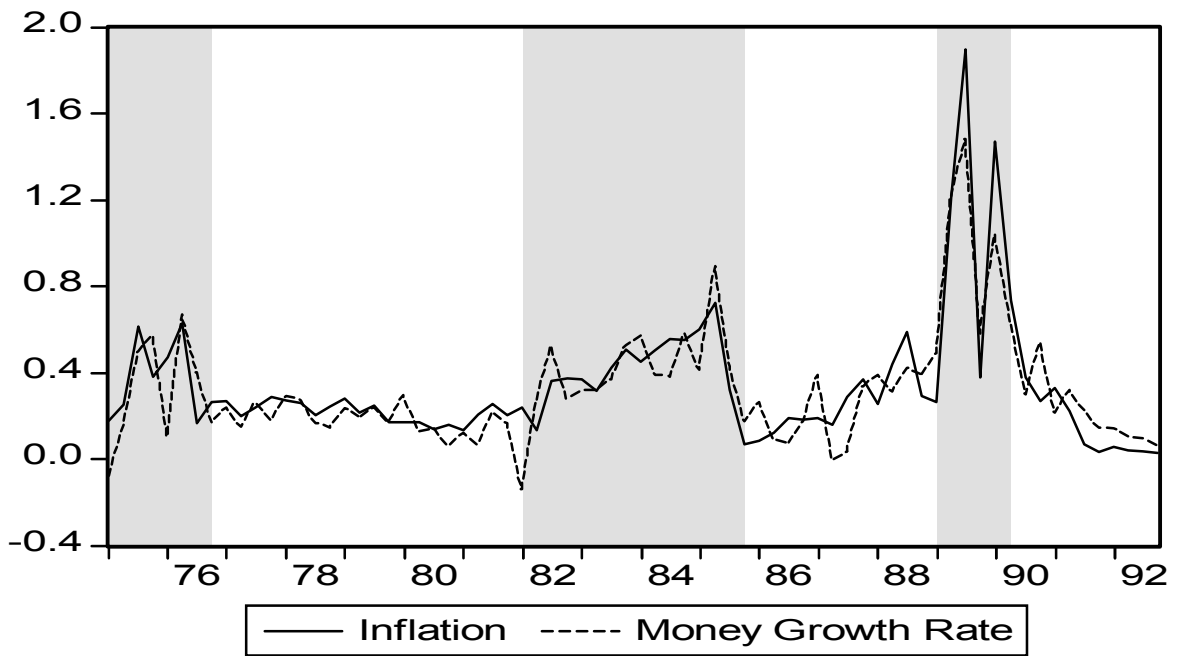


Fig 1.b: Bolivia

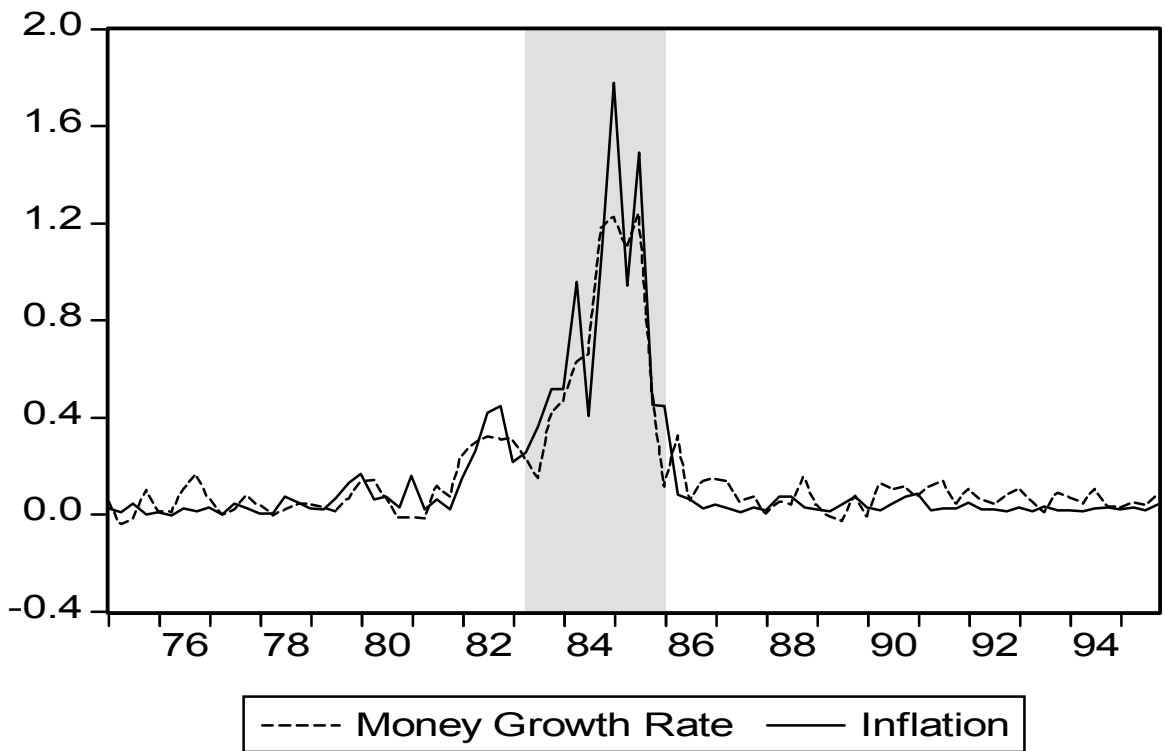


Fig 1.c: Brazil

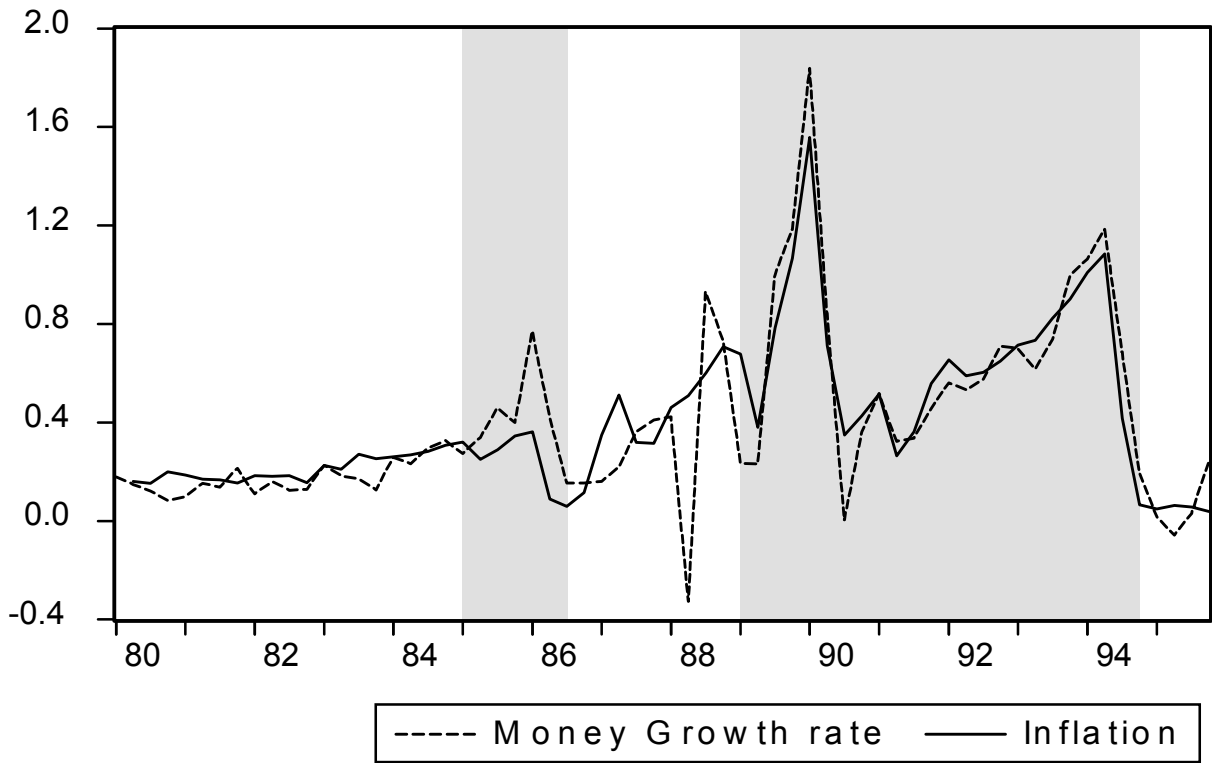


Fig 1.d: Mexico

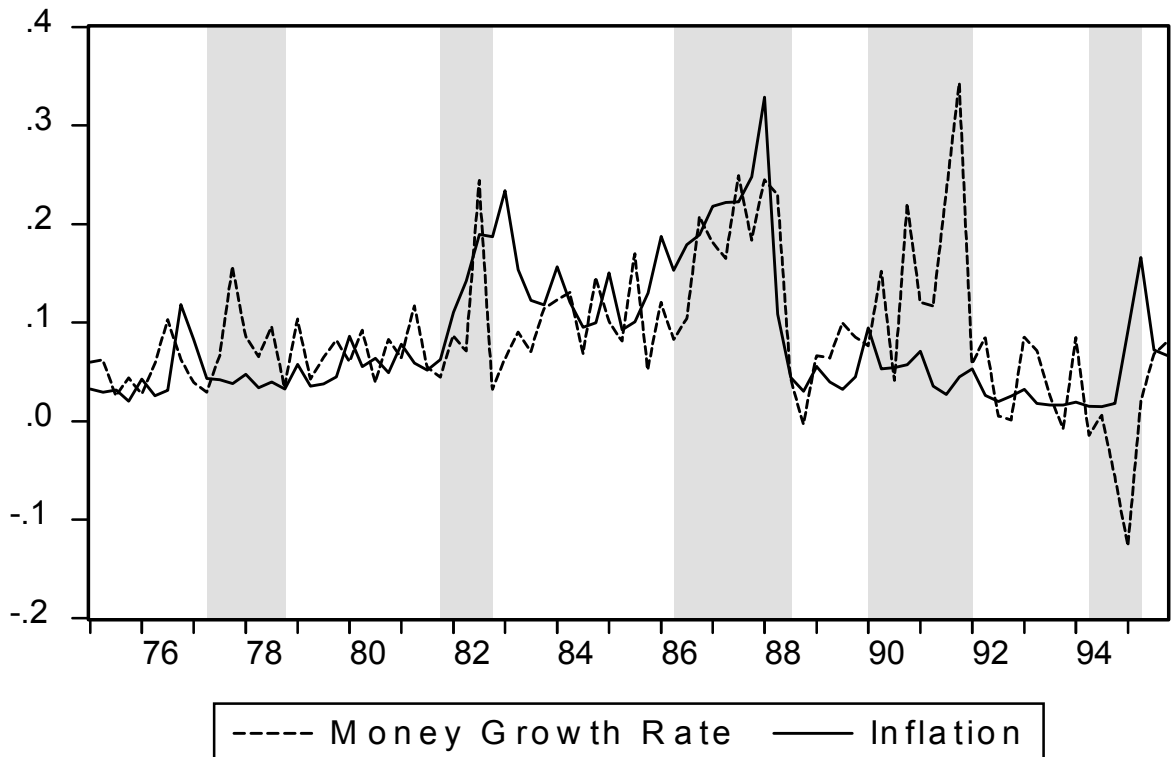


Fig 1.e: Peru

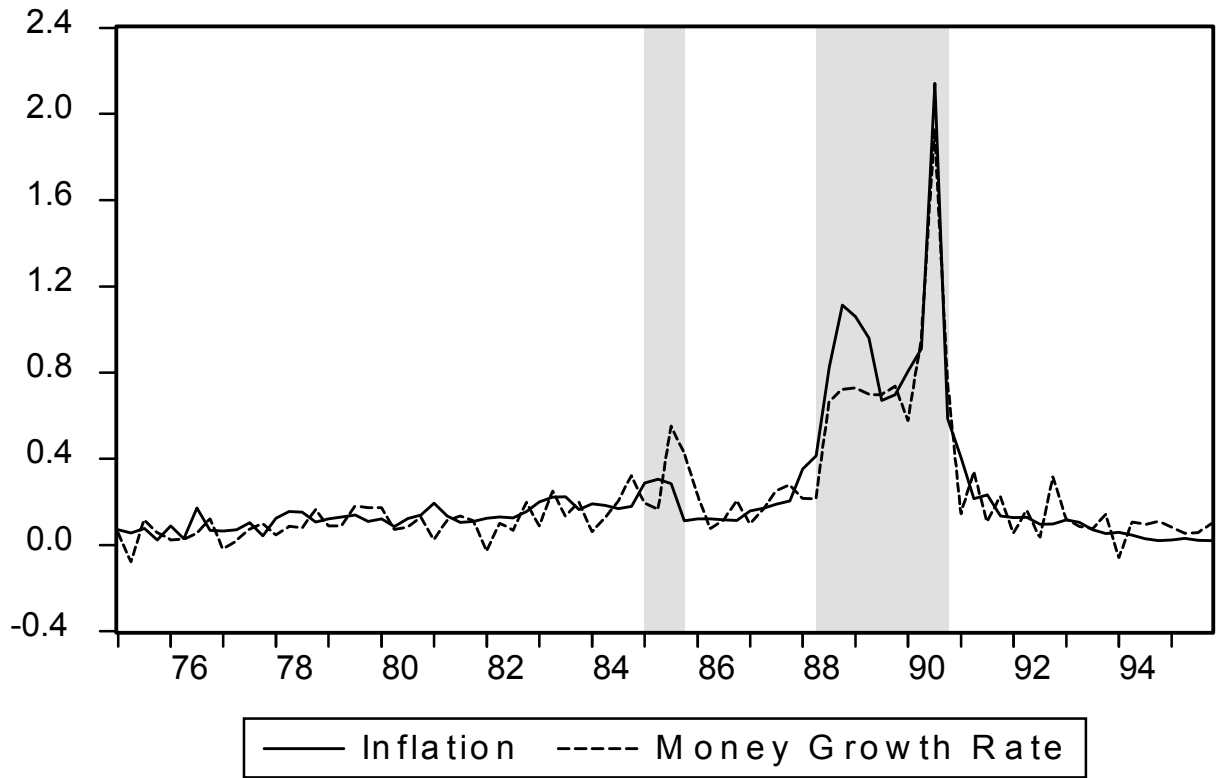


Fig 2.a
Lead of all countries

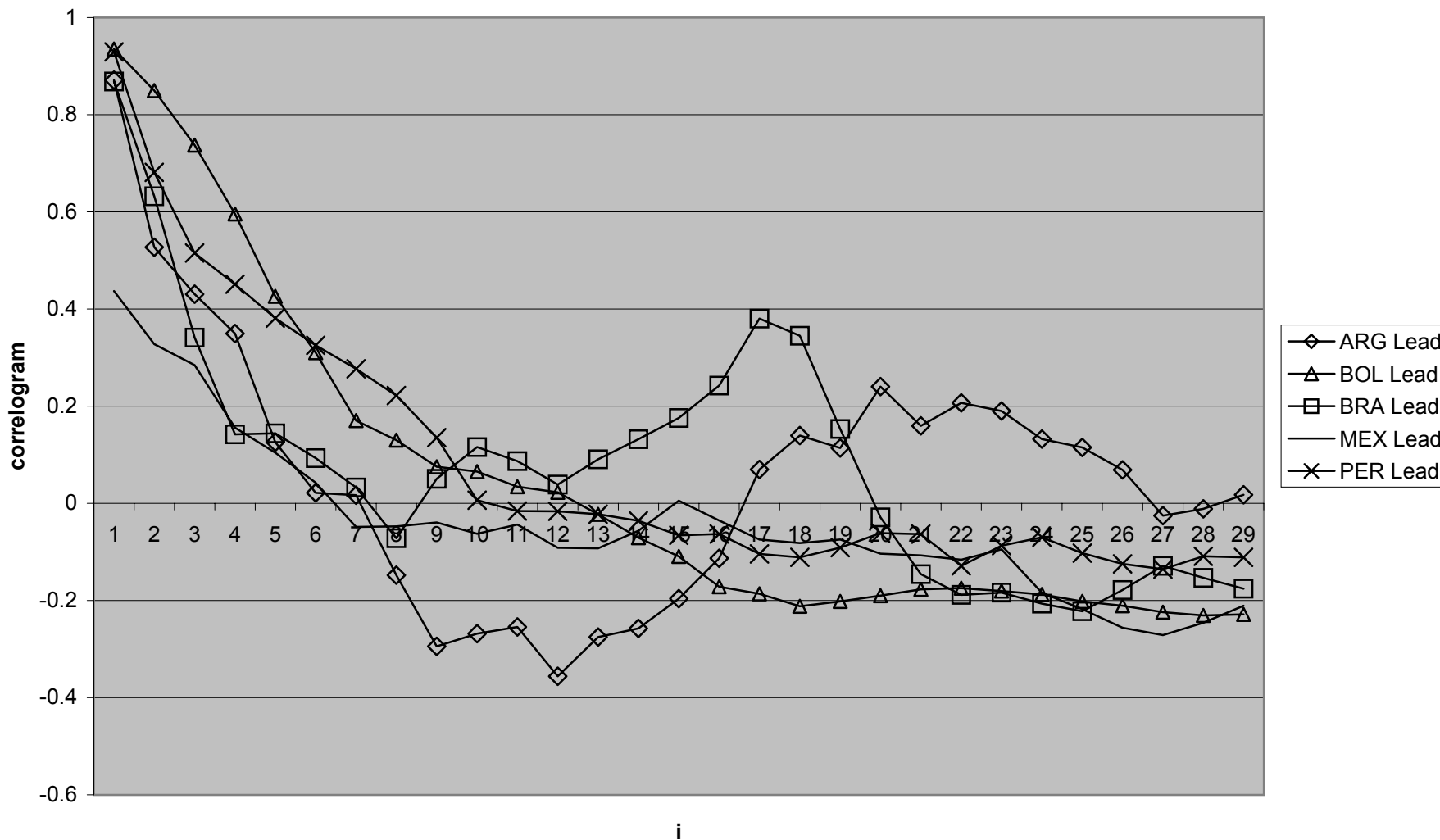


Fig 2.b
Lag of all countries

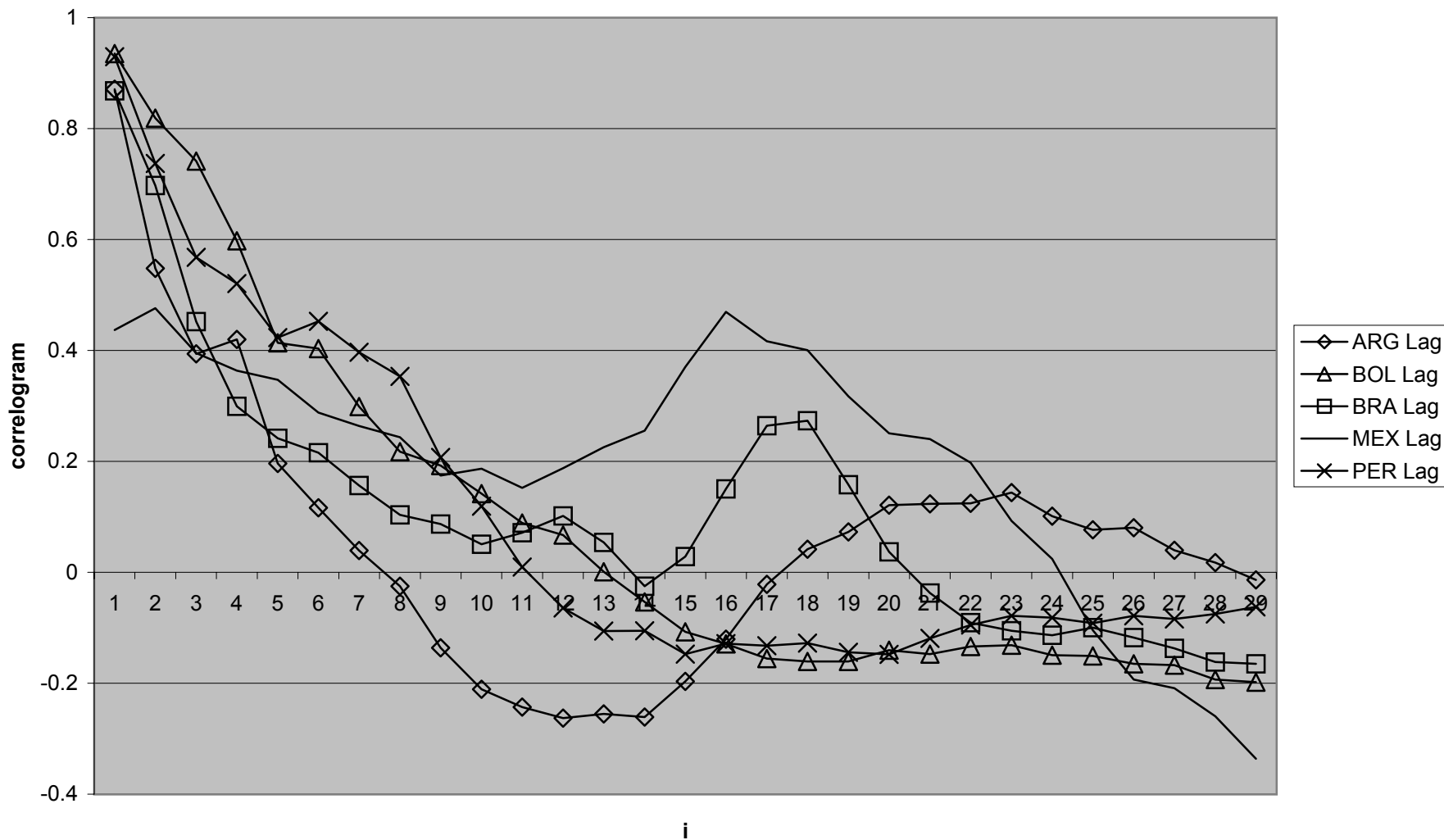


Fig 3.a
Argentina: Lag with confidence band

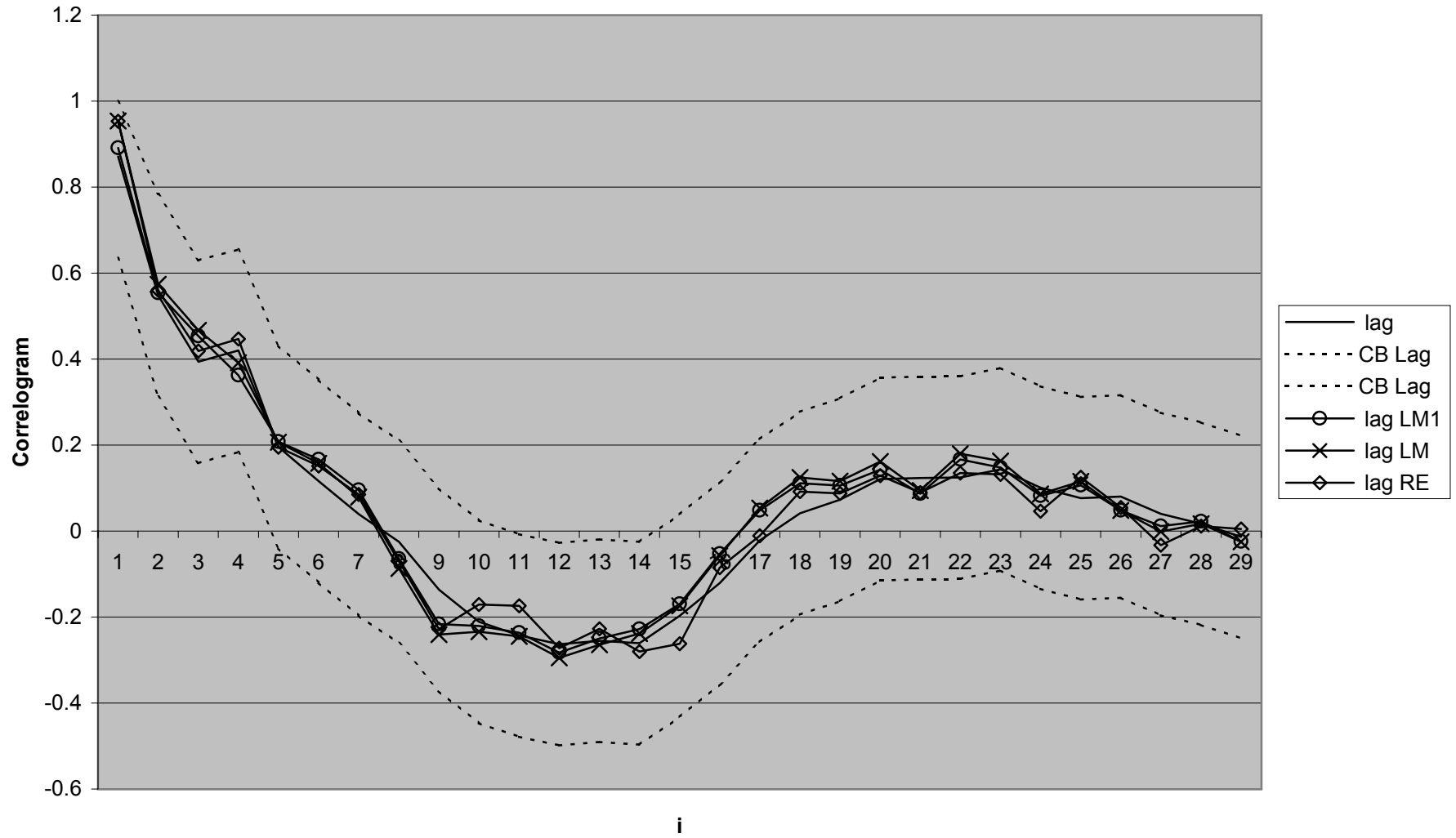


Fig 3.b
Bolivia: Lag with confidence interval

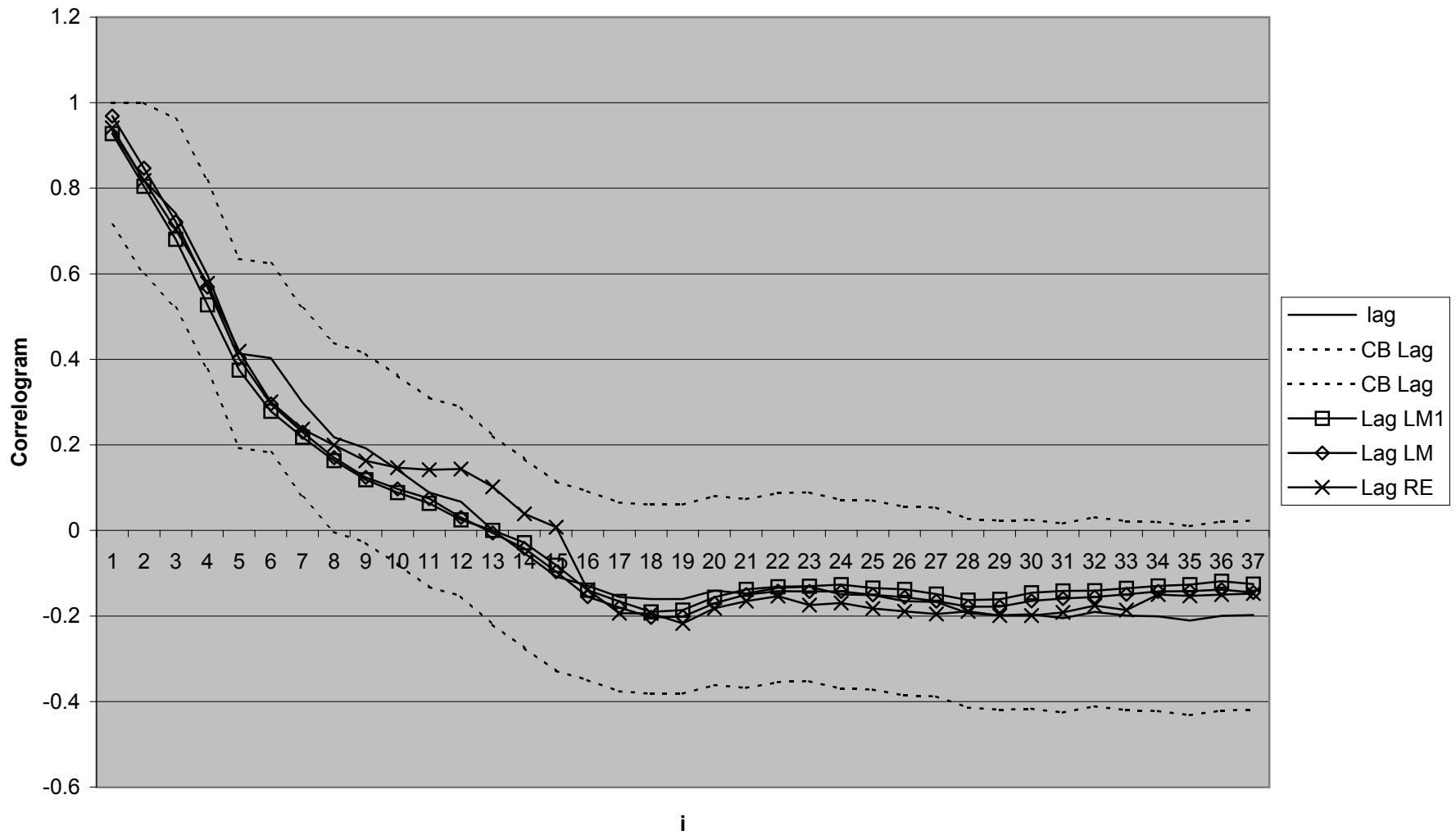


Fig 3.c
Brazil: Lag with confidence interval

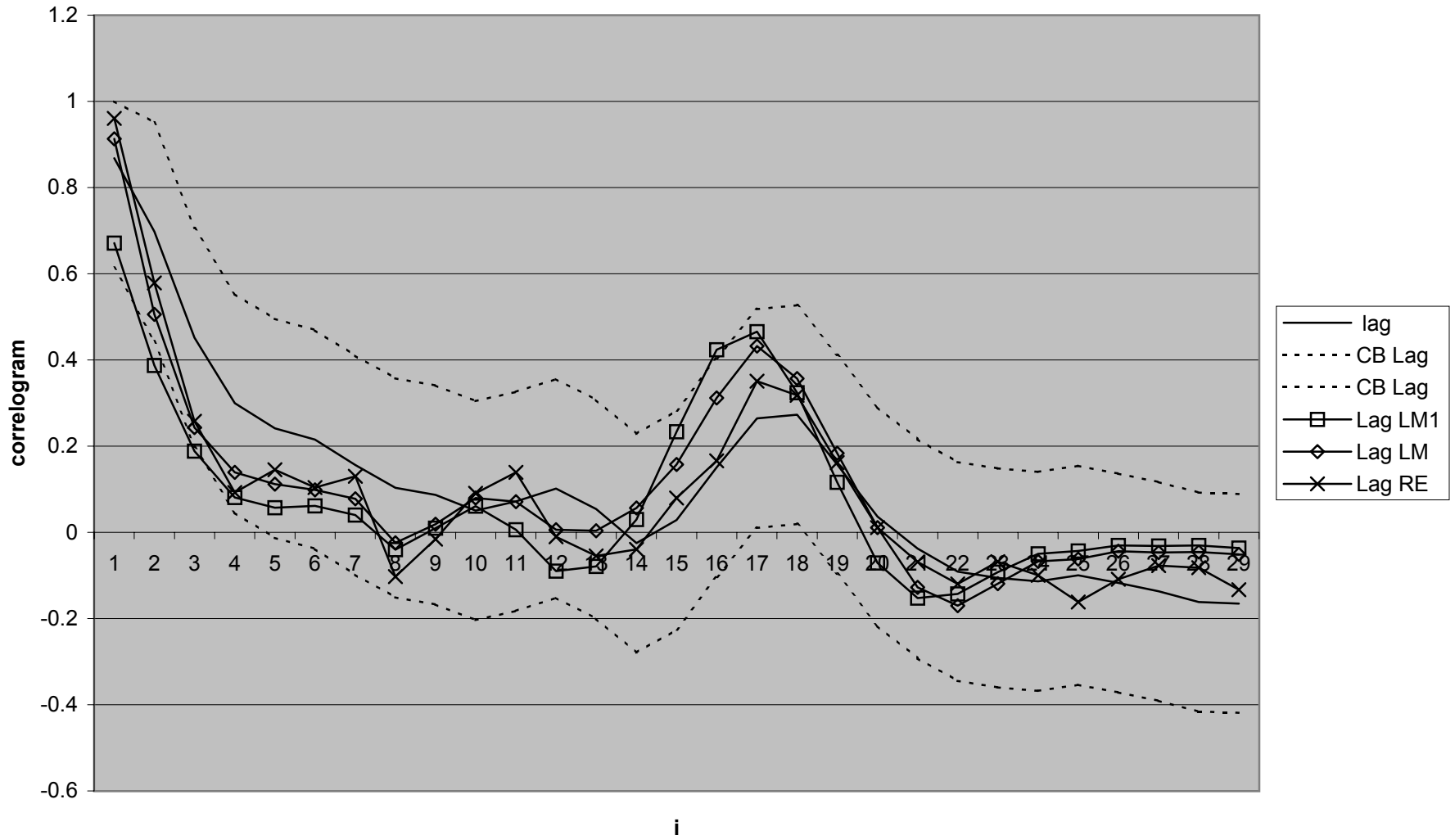


Fig 3.d
Mexico: Lag with confidence interval

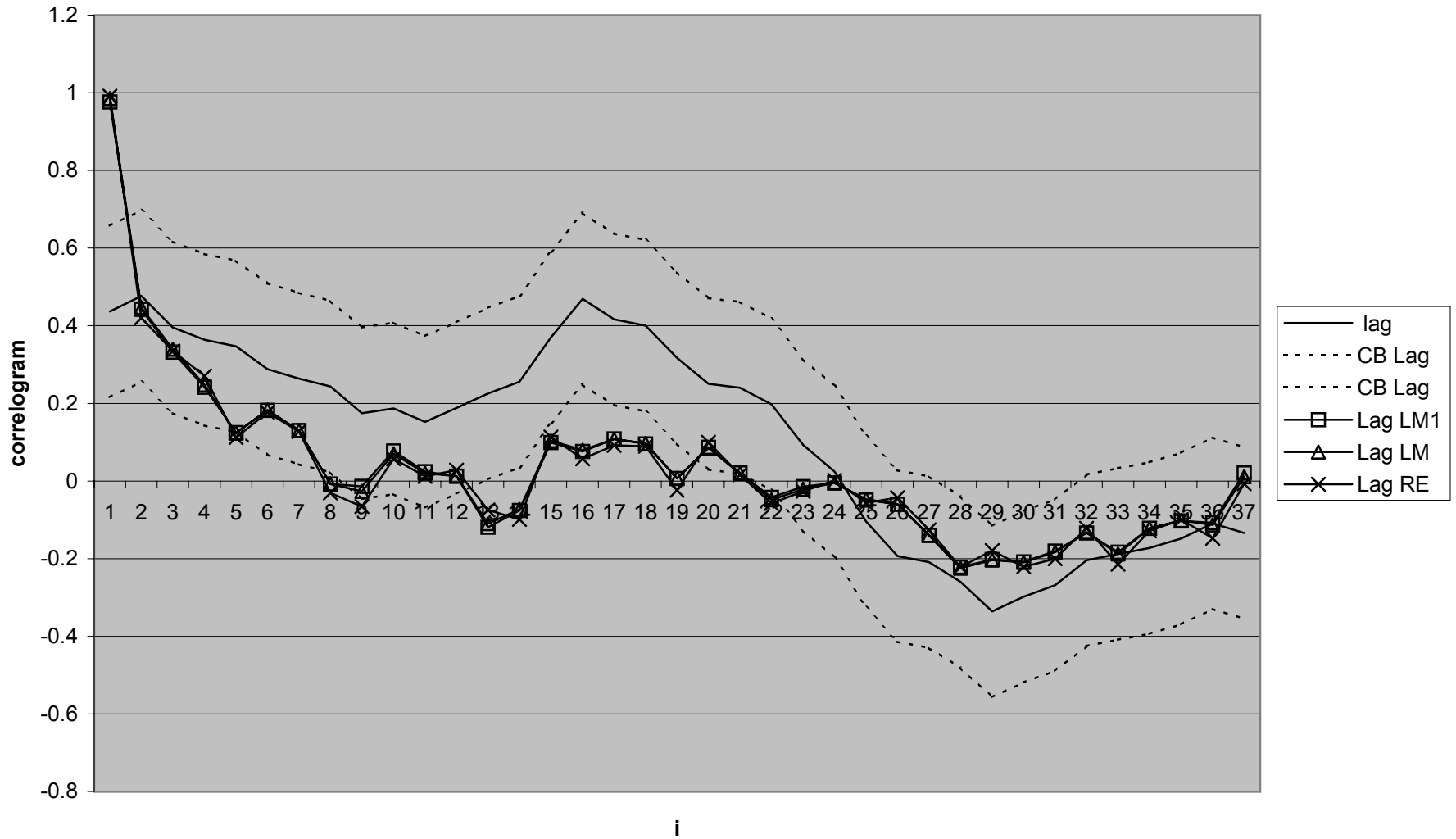


Fig 3.e
Peru: Lag with confidence interval

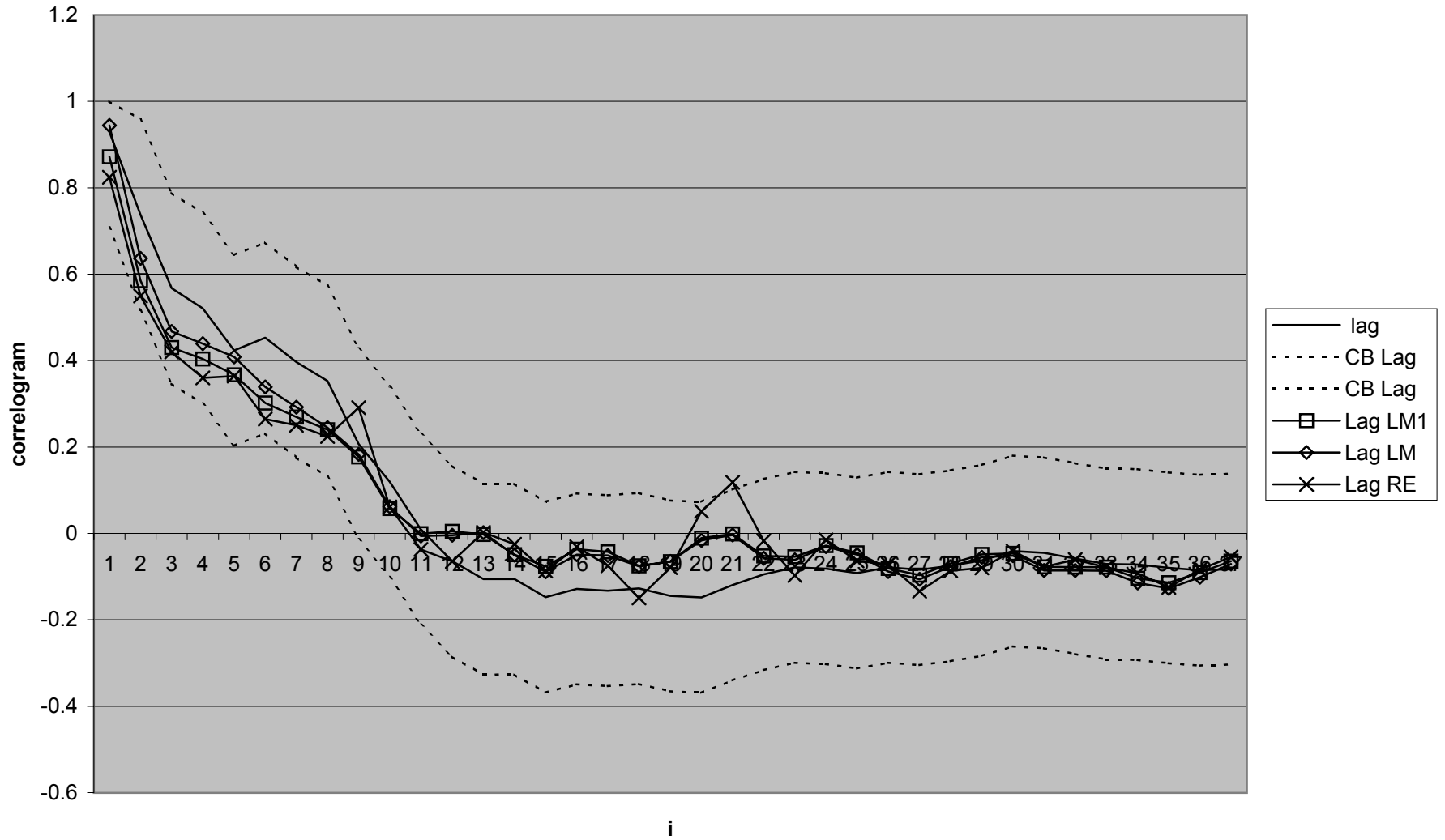


Fig 4.a
Argentina: Lead with confidence interval

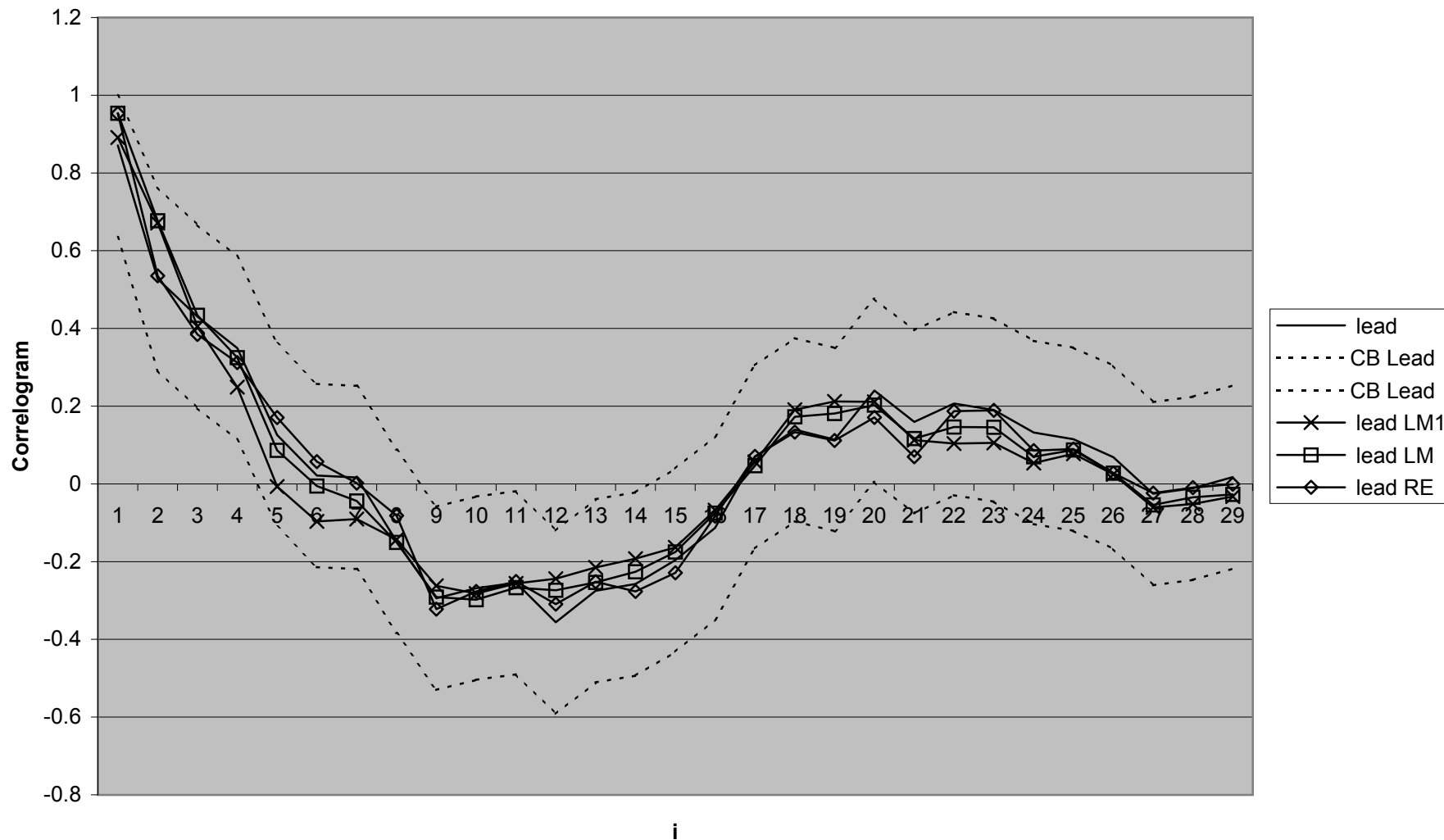


Fig 4.b
Bolivia: Lead with confidence interval

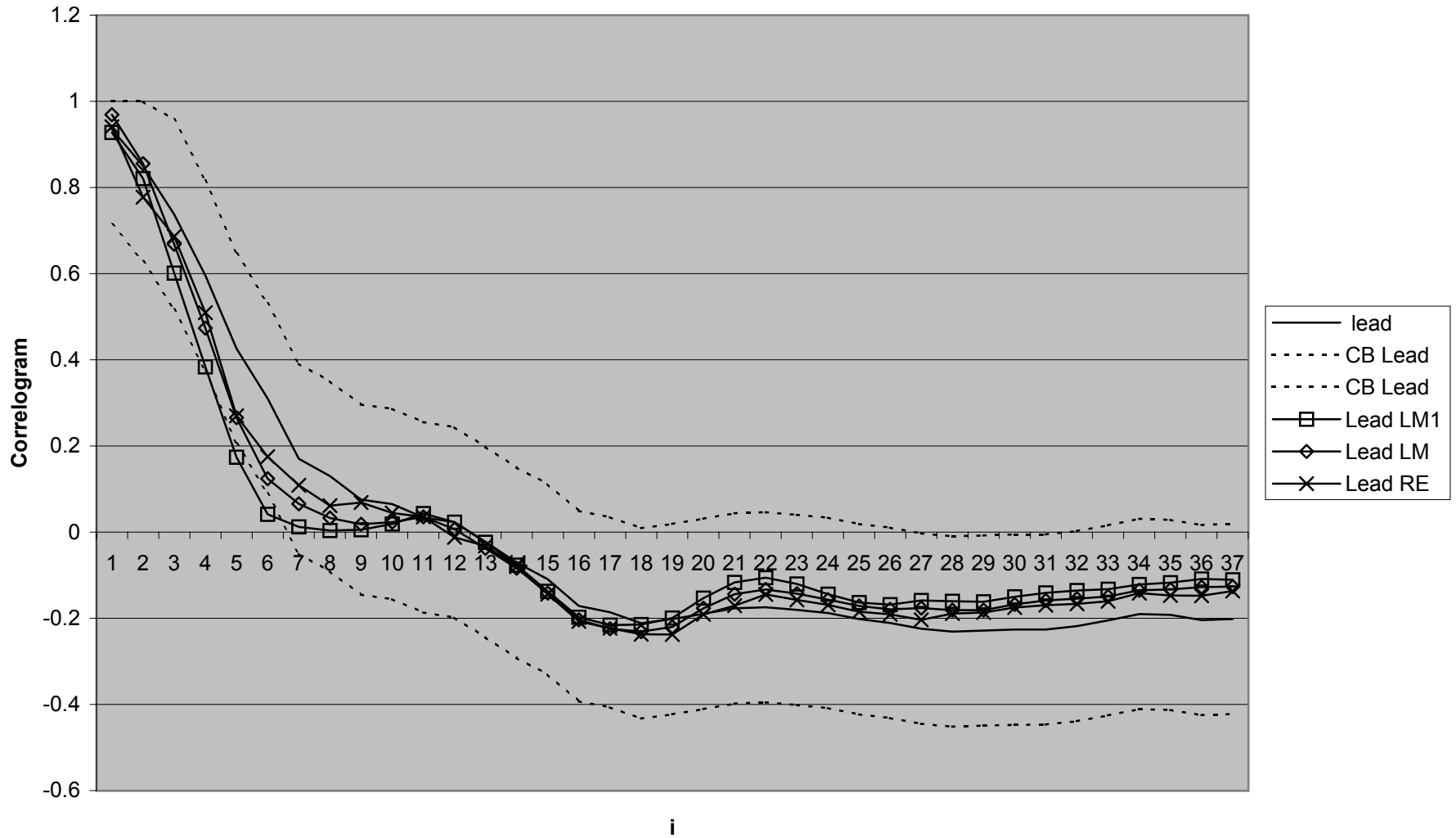


Fig 4.c
Brazil: Lead with confidence interval

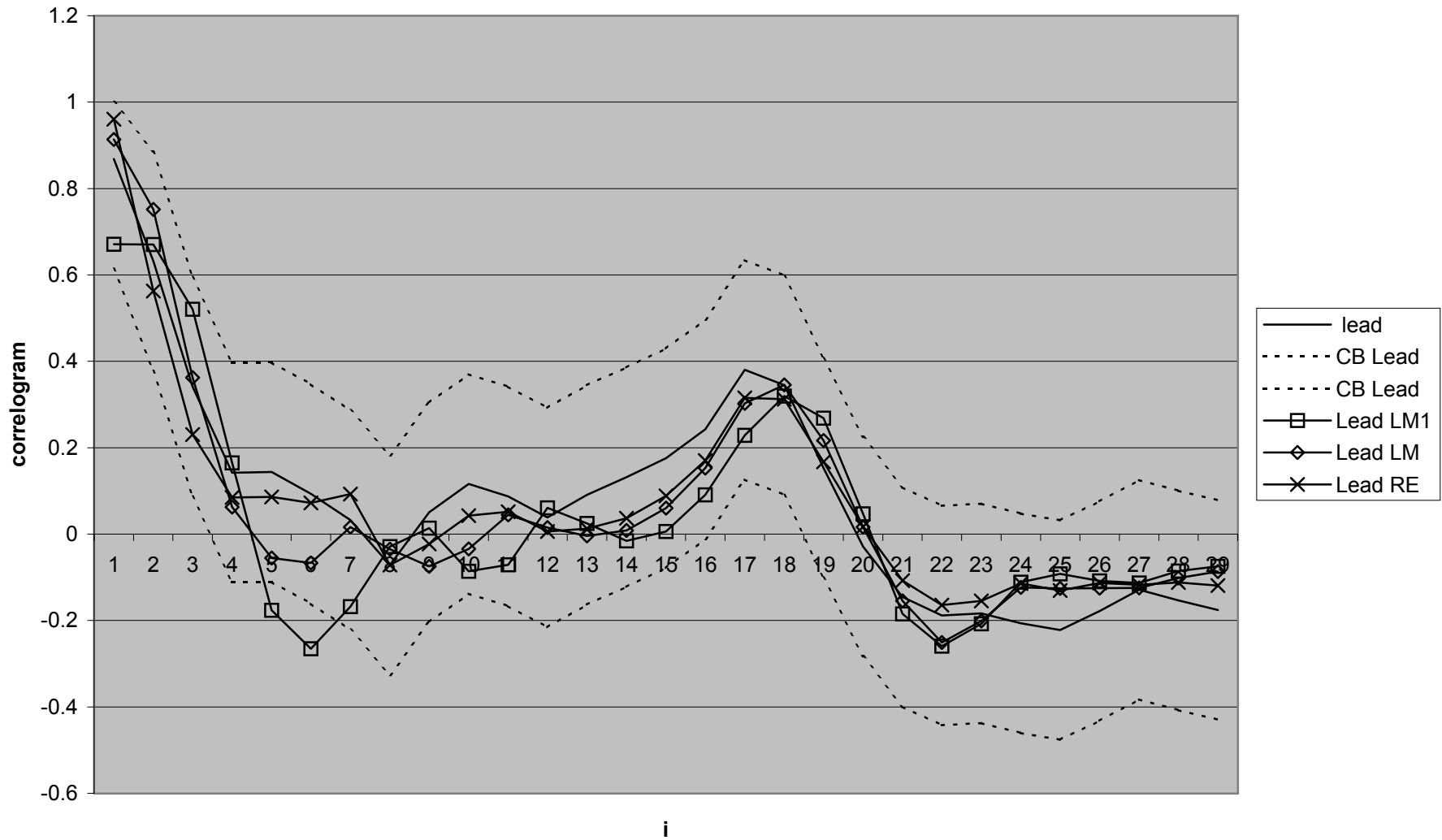


Fig 4.d
Mexico: Lead with confidence interval

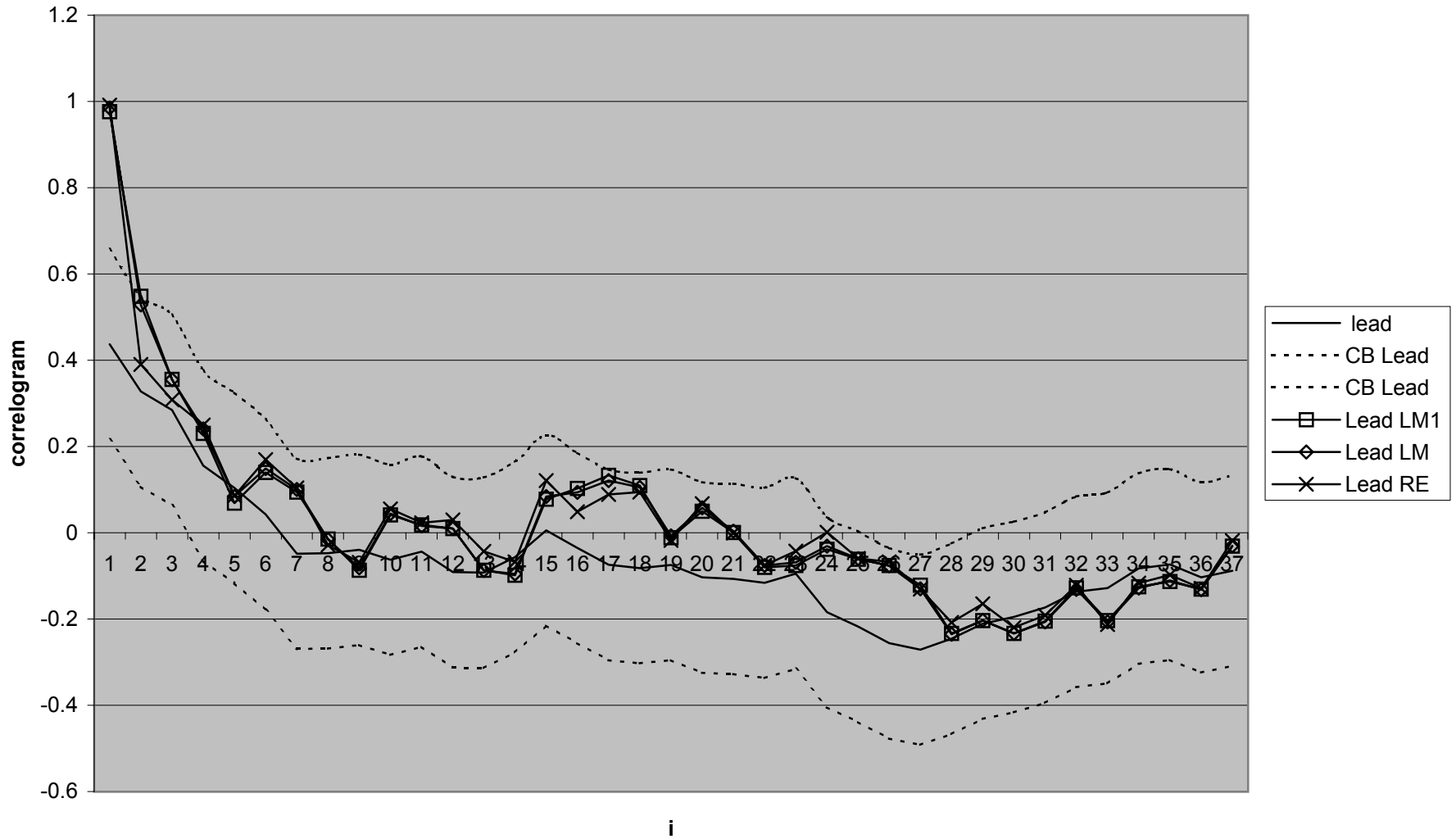


Fig 4.e
Peru: Lead with Confidence interval

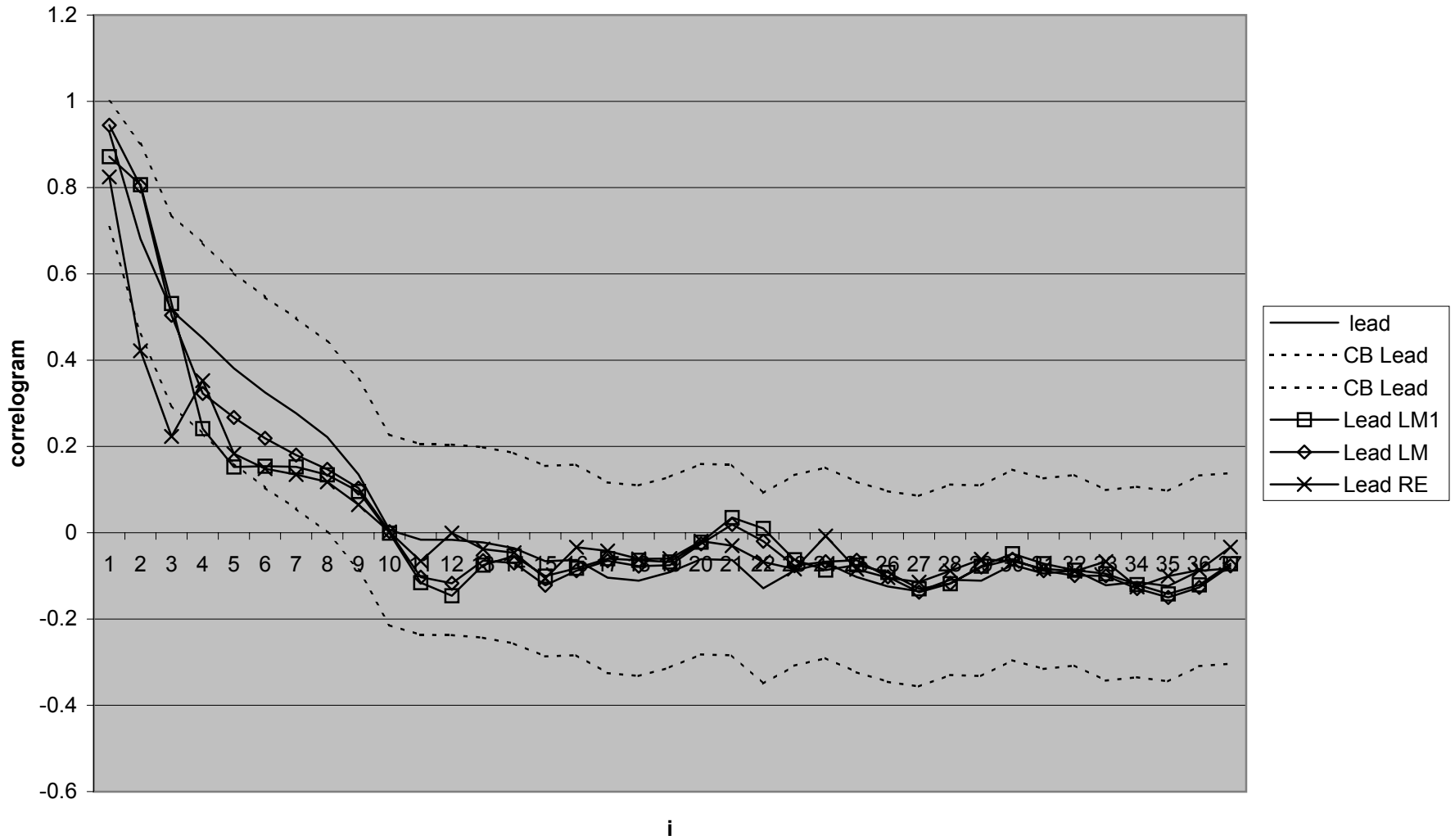


Fig 5
Argentina: Inflation generated by LM, LM1 and RE

