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Economics Working Paper 80

**Perishable Medium of Exchange (Can Ice  
Cream be Money?)\***

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Universitat Pompeu Fabra and University of York

May 1994

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*Keywords:* Money, Perishable Commodities, Search Theory.

*Journal of Economic Literature classification:* E40.

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\* I wish to thank Irasema Alonso and Pierre de Trenquallye for many helpful conversations and ideas. I am also grateful to Ramon Faulí-Oller, Gianni de Fraja, Ramon Marimon, Peter Simmons and Randall Wright. All errors are my own.

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## **Abstract**

The aim of this paper is to demonstrate that a perishable good may be used as commodity money, even in economies in which perfectly durable commodities are available. This is shown in the general context of a search theoretical model of a decentralized economy similar to Kiyotaki and Wright (1989). It is shown that the value of holding money is decreasing over time when the medium of exchange is a perishable good.

## 1. INTRODUCTION

The aim of this paper is to demonstrate that a perishable good may be used as medium of exchange, even in economies in which perfectly durable commodities are also available. We present a search theoretical model of a simple decentralized economy with three different goods and three different types of agents who are specialized in production and consumption. The model is close to the ones described in Kiyotaki and Wright (1989) and Cuadras-Morató (1993) (see also Aiyagari and Wallace (1991)), but, in contrast with these models, where goods have respectively different storage costs and quality homogeneity, in our setting goods have different durability (measured as the number of periods during which a good can be consumed with no loss of utility). In particular, we show that a perishable good may play the role of commodity money when the rest of the goods of the economy are perfectly durable.

This extension of the model might be considered relevant because although search theoretical models of money have addressed several issues in monetary economics, they have not explicitly taken into account prices (with the relevant exception of Trejos and Wright (1993) and Shi (1993)) and, consequently, inflation. As a consequence of this, in most of these models the value of money is constant over time. In this paper, however, we prove existence of equilibria in which perishable commodities play the role of medium of exchange and the value of holding a

perishable commodity which is used as medium of exchange is declining over time<sup>1</sup>.

One of the first questions addressed by monetary economists was under what circumstances a particular object may have come to be used as medium of exchange. Jevons (1875) gives a list of requirements that any object should have in order to be suitable to perform the functions of money. Among others, portability, homogeneity, divisibility, stability of value, cognizability and indestructibility are regarded by him as desirable qualities of any commodity performing the role of money. Nevertheless, as Jevons himself was aware, the functions performed by money are of very different nature and they do not have to be necessarily performed by a unique asset with all those characteristics. For instance, in order to be used as store of value, it seems quite obvious that a commodity should have a high degree of durability, although this may not be so important when the role to be played is medium of exchange. Therefore, it may seem intuitively plausible that a perishable object may appear as commodity money playing the role of medium of exchange but, to date, this question has not been explicitly addressed in the literature about monetary exchange. Instead, most models identify very closely money with durable objects.

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<sup>1</sup>Economic historians have also reported historical episodes in which privately issued documents such as bills of exchange, promissory notes, etc. circulated as acceptable media of exchange. Some of these promissory notes had a limit date before which they could be cashed with full legal protection. After this date, to cash the note was still possible, but more difficult (for the case of Catalonia, in Spain, during the industrialization period, there is some research in progress in Cuadras-Morató and Rosés (1994) studying the details of these phenomena).

A large number of models focus on the analysis of fiat money and its role of medium of exchange (see Lucas (1980), Wallace (1980), Townsend (1980), and Kiyotaki and Wright (1991) for different approaches tackling the same issue). In these models, fiat money is an object which has been endowed with all the desirable qualities of money: durability, recognizability, portability, divisibility, etc., and takes on value because of its role as a medium of exchange. In this sense, these models of exchange economies are not particularly interesting for our investigation. For our purposes, we are more interested in the existing models of commodity money, although they do not address our question very directly either. In particular, all models of commodity money we know (among others, King and Plosser (1986), Iwai (1988), Jones (1976), Oh (1989), Harris (1979), and Kiyotaki and Wright (1989)) conclude that commodity money will be a durable good. This conclusion is not, we believe, due to the fundamental nature and roles of money, but to the particular assumptions of these models. We regard the essential nature of money as being strategic. The extrinsic beliefs of agents about the acceptability of the different goods in the economy play an important role in the determination of which goods are going to be used as medium of exchange, together with the intrinsic qualities of those goods. In this context, it is clear that durability may be a desirable quality of money but it is not, by any means, a necessary characteristic of money.

Our modelling strategy is very simple. We take a well known model of commodity money in the literature and introduce the possibility that goods are perishable. Then, we show that a perishable good may well appear as commodity money (even in the case in which the rest of commodities in the economy are durable). The chosen

model (in our opinion the model that best reflects what we think is the true nature of money as medium of exchange) is the search theoretical model of money of Kiyotaki and Wright (1989). In this model, agents choose optimal trading strategies and commodity money appears endogenously as an equilibrium outcome. The goods appearing as commodity money will only be partially determined by the intrinsic qualities of the different goods of the economy (fundamentals). In fact, the extrinsic beliefs held by the agents about acceptability of goods play a major role in the determination of the goods appearing as commodity money. That is, the nature of money is basically that of pure social convention, and its essential characteristic is its acceptability. Other desirable physical characteristics like durability, homogeneity, or storability are not necessary features for the use of money as a medium of exchange.

The structure of the paper is as follows: in section 2 a general model of a decentralized economy with perishable goods is described. As we shall see, this is basically a generalization of the model presented in Kiyotaki and Wright (1989); section 3 examines a particular case of the model. We present several propositions proving the existence of equilibria in which perishable commodities play the role of commodity money. The particular case we study is quite extreme in the sense that there is only one nondurable good in the economy and it has the minimal durability required to appear as commodity money (obviously, a good which perishes immediately after being produced can never be used as medium of exchange). This means that the results contained in the propositions should hold for other less extreme versions of the model with several nondurable goods with longer periods of life.

Section 4 concludes the paper.

## 2. THE ECONOMY

In this section a model of a simple exchange economy with decentralized trade and nondurable goods is specified. The general structure of the model is like the one described in Kiyotaki and Wright (1989) (see also Cuadras-Morató (1993) for a version of the model with goods of heterogeneous quality). The crucial modification here is that we do not assume that goods are perfectly durable and, indeed, we allow for the existence of goods which have different durability over time (durability being defined as the number of periods of time during which a good can be consumed with no loss of utility).

### 2.1. General Environment

Time is discrete. There are three different types of indivisible goods: good 1, good 2, and good 3. There is a continuum of infinitely lived agents who are, in equal proportion, of type I, type II, and type III. Agents of type  $i$  ( $i=1,II,III$ ) are specialized in consumption in such a way that they only consume goods of type  $i$  ( $i=1,2,3$ ). They are also specialized in production with the following pattern: agents of type  $i$  produce only goods of type  $i+1$  (modulo 3). The characteristics of goods of a certain type vary with their age. More precisely, goods of type  $i$  are apt for consumption during a given number of periods,  $n_i$ , ( $n_i > 0$ ) and after that, they go



off and their consumption adds no utility to the agent who consumes them. In general,  $n_i \neq n_j$  ( $\forall i, j = 1, 2, 3$  and  $i \neq j$ ). In order to identify goods of the same type but of different age, the following notation is used to refer to goods of type  $i$ :  $i_0, i_1, \dots, i_t, \dots$  ( $t$  indicating the number of periods that have passed since the good was produced).

Consuming a good  $i_t$  adds  $U_i$  units of utility to agent of type  $i$  if  $r < n_i$ , and no utility at all if  $r \geq n_i$ . After consuming a good of type  $i$ , agents of type  $i$  produce immediately a good  $(i+1)_0$ , with production costs in terms of disutility being denoted by  $D_i$  ( $U_i - D_i > 0$ ). Also, agents of type  $i$  can dispose at any time and at no cost of the good they hold in inventory and produce a new good  $(i+1)_0$  at the same cost  $D_i$ . All goods can be stored by agents at no cost, but only one at a time. Every agent is assumed to be perfectly informed about the type, age, and conditions of consumption of both the good he is holding and the goods held by the other agents in the economy.

The structure of the economy is assumed to be decentralized. No centralized market exists and agents only meet through a random matching process. Every period of time, agents, who always hold a good, meet in pairs and decide about whether to exchange their respective inventories or not (according to trading strategies about which details are provided below) and also about consumption, disposal, and production of commodities. Exchange takes place only by common agreement of the two paired agents and it is always quid pro quo. As it is obvious from the above setting, this economy is such that no agent produces the good he consumes and, also,

there is not double coincidence of wants of the goods produced by any two agents. This means that, in order to consume, agents will have to exchange goods previously, and, also, that this exchange cannot be pure barter between the goods produced by two agents. Some form of monetary exchange pattern must necessarily emerge if there is going to be exchange at all.

In this economy agents must make decisions about trade, consumption, disposal and production of goods. Nevertheless, in order to make things more tractable, we shall restrict the analysis to equilibria with the property that agents use very simple strategies to decide about consumption, production and disposal of goods. This will allow us to concentrate on the trading strategies. In particular, first we look for equilibria in which agents will always accept in exchange, consume immediately, and hence, never hold, their own consumption good whenever they are offered it and provided that it has not perished, producing immediately after a new good to be held in inventory (*consume if possible*). Second, it will be the case that, in equilibrium, agents will never dispose of any good which has not perished yet to replace it producing a new good (*never dispose*). Finally, the information structure of this economy implies that the value of holding a good that has already perished and is not apt for consumption is zero. This is because nobody will be willing to accept a good like this in exchange, for the simple reason that it has no final consumption value to anyone. Nevertheless, we want the equilibria to be such that it will be optimal for an agent in this contingency to dispose of the good which has gone off and produce a new good (*participation constraint*). This means that even in this worst possible case, agents will not drop out of the economy, because they always have the option of

getting rid of the good gone off and produce a new good, these two actions yielding positive value. It will be shown that there exist equilibria with these properties for a large set of the parameter space.

In order to clarify further the structure underlying our setting, it is worthwhile presenting a summary of the sequence of the events in this economy. This will be done by examining what happens to a representative commodity of type  $i$  from the moment it is produced until the moment it is consumed or it perishes and is disposed of by some agent. In order to simplify the exposition, we will assume that  $n_i = 2$ , so that good  $i$  can be consumed with no loss of utility for two periods after it was produced. Figure 1 reproduces graphically the whole process and should help the reader to understand more clearly what is happening from when a good is produced until it is consumed or disposed of.

Good  $i_0$  is produced by an agent  $i-1$  at the end of period  $t$ . Period  $t+1$  starts and agent  $i-1$  is paired randomly with another agent. Both agents recognize mutually their respective holdings and make decisions about trade. Basically, two situations may arise: first, good  $i_0$  is acquired by an agent of type  $i$  who consumes it, which implies its physical destruction, so to speak; alternatively, the good remains in agent  $i-1$ 's hands or is bought by any other agent who does not want it for consumption. In this case, those agents will find themselves holding  $i_1$  at the end of the period  $t+1$ , because one period of time has gone since the good was produced. Period  $t+2$  starts and, again, two situations may accrue: good  $i_1$  may be exchanged and consumed before the end of the period or, alternatively, will be disposed of by the agent who

holds it before the end of the period. This is simply because at the end of period  $t+2$ , two periods of time have passed since the good was produced and the agent holding it would find himself holding  $i_2$ , which is a good with no value for consumption or exchange. That is, whenever an agent is unsuccessful in his search for a trading partner who wishes to take good  $i$  before it perishes, it will be optimal for him to dispose of the good and produce a new one.

Following the notation advanced in Kiyotaki and Wright (1989) (although slightly changed to adapt it to our particular model), let  $V_{ij}$  be the payoff function (optimal value) for an agent of type  $i$  when he walks out of a trade meeting holding good  $j$ . In general, this payoff function is equivalent to the following expression:

$$V_{ijs} = \max \beta E(V_{iht} | j_s)$$

where  $\beta$  is the agents' discount rate and  $0 < \beta < 1$ . This latter expression is a standard Bellman's equation of dynamic programming, where  $E(V_{iht})$  is the expected indirect utility of agent  $i$  at next period random state  $h_t$ , conditional on  $j_s$ , and the maximization is over strategies about exchange, consumption, disposal, and production of commodities. It is worth emphasizing a couple of points that can help to understand better the previous expression. First, the good held by an agent is what characterizes his current state (strategies will define actions to be taken by agents depending upon their states). Second, the random element comes simply from the assumed matching technology.

Before ending this subsection, some more notation is introduced. Let  $p_{i,s}(t)$  be the proportion of type  $i$  agents who are holding good  $j_s$  in inventory ( $\forall i, j$  and  $s \leq n_i$ ) at time  $t$ . By definition,  $0 \leq p_{i,s}(t) \leq 1$  and  $\sum_i \sum_s p_{i,s}(t) = 1$ . Considering that each individual has exactly the same probability of meeting an agent of any type, the probability of being paired with another agent of type  $i$  holding good  $j_s$  at time  $t$  can be simply characterized by the vector  $p(t) = (\dots p_{i,s}(t) \dots)$  which will be called the distribution of inventories at  $t$ .

## 2.2. Trading strategies and equilibrium

The behavior of agents in this economy is determined by their chosen strategies about trade, consumption, production, and disposal of goods. In the previous subsection, we restricted the analysis to equilibria in which agents use simple strategies for consumption, production, and disposal of commodities. Nevertheless, the important strategic elements in this economy occur at the level of the exchange process. Therefore, the keynote for understanding whether monetary exchange can arise and how it is characterized are the trading strategies of agents. A trading strategy is a rule defining the conditions under which an agent of type  $i$  is intending to trade. Specifically, this will depend on the good held by the agent himself and the good being held by the agent with whom he has been matched. The following notation is borrowed from Kiyotaki and Wright (1989) and adapted to our model. Let  $\tau_i(j_r, k_s) = 1$  if agent of type  $i$  wants to trade  $j_r$  for  $k_s$  and  $\tau_i(j_r, k_s) = 0$  otherwise. It follows from this that when type  $i$  with good  $j_r$  meets type  $h$  with good  $k_s$ , they only trade if  $\tau_i(j_r, k_s) \cdot \tau_h(k_s, j_r) = 1$ . A trading strategy for any agent will be a rule that specifies

the actions of the agent (trade denoted by 1, no trade denoted by 0) in all possible states. States are characterized by the goods being held in inventory by the agent and his trading partner. Formally, a trading strategy for an agent of type  $i$  is a vector  $w_i$  of dimension  $(n_1 + n_2 + n_3)^2$  composed of elements 0 and 1 as follows.

$$w_i = (\dots, \tau_i(j_n, h_n), \dots) \quad \forall j, h = 1, 2, 3 \quad n < n_i, \quad n' < n_n$$

This trading strategy completely characterizes all actions of agents in all possible states of the world. It has been specified generally, but it can be simplified recalling the strategies for consumption, disposal and production of goods of the agents in this economy. First, it is never possible for an agent to be holding his consumption good (it is optimal to consume it immediately). This means that it is not necessary to specify the elements  $\tau_i(i_s, j_r)$  of the vector describing the trading strategy of agent  $i$ , because they are only relevant for a hypothetical situation that simply will never arise in our model. It is also known that  $\tau_i(j_r, i_s) = 1$  is always optimal, since agent  $i$  will always be willing to get his consumption good and consume it immediately.

At this point, following Kiyotaki and Wright (1989), we will assume that agents do not randomize between strategies and do not change them over time. Consequently, we are only looking at pure and steady-state strategies. Also, since we only consider symmetric equilibria, we can summarize the strategies of the continuum of agents by simply stating a strategy for each type of agent.

Given an initial distribution, the strategies of the different agents and the realizations of matchings will determine the resulting distribution of inventories at any time  $t$  (i.e.  $p(t) = p(t, w_1, w_2, w_3)$ ). Given a strategy vector  $(w_1, w_2, w_3)$ , we can define a steady-state distribution of inventories  $p(w_1, w_2, w_3)$  as an inventory distribution that satisfies the following condition:

$$p(t, w_1, w_2, w_3) = p(t+1, w_1, w_2, w_3)$$

Finally, let an equilibrium be a vector of strategies  $(w_1^*, w_2^*, w_3^*)$ , a steady-state distribution of inventories,  $p^*$ , and the corresponding optimal value functions  $V_{ijs}(w^*, p^*)$  such that for each agent of type  $i$

1)  $w_i^*$  maximizes individual expected discounted lifetime utility of agent  $i$  given the strategies of the other agents and the steady-state distribution of inventories, or, in other words, it is a best response for agent  $i$  given those strategies and the distribution of inventories,

2)  $p(w_1^*, w_2^*, w_3^*) = p^*$ , and

3)  $U_i - D_i + V_{i,i+1,0} > V_{ijs} \quad \forall i, j=1,2,3 \quad i \neq j \quad s < n_i \quad (\text{consume if possible}) \quad (r.1)$

$$V_{ijs} > -D_i + V_{i,i+1,0} \quad \forall i, j=1,2,3 \quad i \neq j \quad s < n_i \quad (\text{never dispose}) \quad (r.2)$$

$$-D_i + V_{i,i+1,0} > V_{ijs} \quad \forall i, j=1,2,3 \quad i \neq j \quad r \geq n_i \quad (\text{participation constraint}) \quad (r.3)$$

Condition 1 is the usual condition for Nash equilibrium (optimality of trading strategies). Condition 2 is a consistency condition that states that given the vector of strategies  $(w_1^*, w_2^*, w_3^*)$ ,  $p^*$  is a resulting steady-state distribution. The conditions in 3 ensure optimality of the conjectured consumption, disposal and production strategies and basically imply restrictions on the values of the parameters for which equilibria will exist.

### 3. EQUILIBRIUM RESULTS

The main objective of this section is to present results which prove the existence of equilibria in which a nondurable good appears as commodity money. In order to do that, it will be convenient to analyze a particular case of the economy described in section 2. Specifically, only the case in which goods 1 and 2 are perfectly durable ( $n_1, n_2 = \infty$ ) and good 3 perishes two periods after its production took place ( $n_3 = 2$ ) will be examined. The fact that we only examine a particular case should not cast too much doubt about the generality of our results. This is due to the fact that what we have actually done is to choose a tractable case which is quite extreme in the following sense: there is only one perishable good in our economy, the rest being perfectly durable goods; and it is a commodity of a very short life (the minimum required to be able to appear as commodity money)<sup>1</sup>. Even so, there is a region of

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<sup>1</sup>As a matter of fact, this is the only interesting case for us to study. In Kiyotaki and Wright (1989), the expected "life" of a good 3 (computed as the expected number of periods that goes since the good is produced by an agent of type I until



the parameter space for which an equilibrium in which the perishable good plays the role of commodity money can be found. Hence, there should be a large number of economies of the type described in section 2 where nondurable goods may play the role of commodity money. It is convenient to divide this section in four subsections. First, we describe very briefly the particular economy which is going to be analyzed. Second, we present the equilibrium results. Third, we present further equilibrium results when the assumption of perfect information about the age of goods is somehow relaxed. Finally, we verify that in equilibrium the value of holding perishable commodities as medium of exchange is decreasing over time.

### 3.1. An economy with one perishable good

There will be four different goods in this economy: 1, 2,  $3_0$ , and  $3_1$ . The same notation introduced above is maintained, although for goods 1 and 2 no further information about their age is necessary, because they are perfectly durable and their characteristics do not change over time. Consequently, it will not be necessary to specify any third subscript in the payoff functions and the elements of the distribution of inventories,  $p$ , when  $j=1,2$ . Moreover, it is not possible for agents of type I to hold commodity  $3_0$ . This is because agent I cannot produce good 3 himself and can only get it by trade after at least one period of time has gone. Consequently, the distribution of inventories characterizing this particular economy can be expressed as

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it is eventually consumed by an agent of type III) is three periods of time. This simply means that for a good durable enough ( $n, \geq 3$ ), the results of that model should immediately apply in our setting.

follows

$$P = (P_{12}, P_{131}, P_{230}, P_{231}, P_{21}, P_{31}, P_{32})$$

which, due to the fact that probabilities should add to one, can be reduced to

$$P = (P_{12}, P_{230}, P_{21}, P_{31})$$

### 3.2. Exchange equilibria

The objective of this subsection is to present equilibrium results referred to the simple economy described above and show that there is a region of the parameter space for which there exists an equilibrium in which a perishable good emerges as commodity money. We will also characterize conditions of existence for the rest of pure-strategy equilibria existing in this model and show that, when mixed strategies are allowed, there exist exchange equilibria for almost all values of the parameters  $U_i$  and  $D_i$ .

In this simple economy, optimal exchange strategies for agents can be characterized in the following way. Value functions and equilibrium strategies must satisfy the following incentive compatibility constraints. Agent of type I will play the strategy "use good 3 as money" (i.e.  $\tau_1(2,3_0)=1$ ) iff  $V_{131} > V_{12}$ ; vice versa, he will play the alternative strategy "not use good 3 as money" (i.e.  $\tau_1(2,3_0)=0$ ) iff  $V_{12} \geq V_{131}$ . The first constraint guarantees that it is optimal for agent I to accept good 3 to use it as a medium of exchange, while the second alternative means that the agent prefers to

hold the good he produced until he can swap it for the good he wants to consume<sup>2</sup>. Note the asymmetry between the two equilibrium conditions for the two strategies. In the second case, it is necessary and sufficient for good 2 (which is the good produced by agent I) to be held that it is at least as good as the alternative possibility. Instead, in the first case, we have a strict inequality since to exchange good 2 for good 3 requires that the latter is strictly preferred to the former. The reason for this asymmetry is that mixed strategies are not considered and it is assumed, as in Kiyotaki and Wright (1989), that trade does not take place when one of the agents is indifferent between his good and the good held by his trading partner<sup>3</sup>. This means that, in steady state, whenever agent I is indifferent between good 2 and 3, he will always keep in inventory good 2, which is the good he produced (see Cuadras-Morató (1993) for similar equilibrium conditions in the context of a different model). Equivalently, agent of type II will bring into play strategy "use good 1 as money" (i.e.  $\tau_2(3_0, 1) = 1$ ) iff  $V_{21} > V_{231}$ ; and vice versa, he will play "not use good 1 as money" (i.e.  $\tau_2(3_0, 1) = 0$ ) iff  $V_{231} \geq V_{21}$ <sup>4</sup>. Finally, agent of type III will play to "use

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<sup>2</sup>The situation in which agent of type I holds good  $3_0$  and is offered good 2 (consequently he also must compare  $V_{12}$  and  $V_{131}$ ) will never arise simply because agents of type I can never hold good  $3_0$  (only producers of good 3 can).

<sup>3</sup>This seems the natural assumption to make. You need only to consider explicitly the existence of an arbitrarily small cost of transaction to get it as a result of the model.

<sup>4</sup>A situation in which agent of type II holds good 1 and is offered good  $3_0$  will only arise if his trade partner is also of type II (producer of good 3). Since we know that no mutual benefits from trade can be realized when traders are of the same type,

good 2 as money" (i.e.  $\tau_3(1,2)=1$  and  $\tau_3(2,1)=0$ ) iff  $V_{32} > V_{31}$  and "not use good 2 as money" (i.e.  $\tau_3(1,2)=0$ ) iff  $V_{31} \geq V_{32}$ <sup>5</sup>.

The following notation will be used to denote a vector of strategies (one for each type of agents):  $w = (w_1, w_2, w_3)$  where

$w_i = 1$  iff agent of type  $i$  plays strategy "use good  $i+2$  as money"  
= 0 otherwise

As can be easily seen, the problem of finding the equilibria of this model has become more tractable. The procedure to be implemented to carry out this task is as follows: first, a vector of strategies,  $w$ , is conjectured. Second, the steady-state distribution implied by those strategies of the agents is computed. This is done simply by computing the steady-state of the stochastic Markov process defined by the strategies of the agents and the assumed matching technology in this environment. Third, it has to be checked that the strategies conjectured in the first place effectively satisfy the incentive compatibility conditions of equilibrium. Finally, it has to be verified that, in equilibrium, the conjectured strategies for consumption, disposal and production of goods are optimal for some values of the parameters.

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this is an irrelevant case for our analysis.

<sup>5</sup>If an agent of type III is using this later strategy, it will never happen that he holds good 2 and is offered good 1. This is because it was not optimal to accept good 2 in the first place (agent III produces good 1 and can only get good 2 in the trading process).

The following proposition summarizes the main equilibrium result of our model. Note its close resemblance to the results in Theorem-1 in Kiyotaki and Wright (1989) and Proposition 1 in Cuadras-Morató (1993). There exist also two unique equilibria, one of them involving the use of good 3 (the only nondurable good in our economy) as medium of exchange.

#### Proposition 1

In the economy described above, for values of the parameters such that  $U_1/D_1$  and  $\beta$  are large enough, there exist only the following two pure strategy equilibria: a) in the region of the parameter set for which  $U_1/D_1 > 5.2301$ , there is a unique equilibrium in which goods 1 and 3 are used as commodity money; and b) in the region of the parameter set for which  $U_1/D_1 \leq 5$ , there is a unique equilibrium in which only good 1 is used as commodity money.

Both these equilibria coincide (in the sense that the equilibrium strategies are identical and, consequently, also the media of exchange circulating in the economy) with the equilibria found in Kiyotaki and Wright (1989) (Theorem 1) and Cuadras-Morató (1993) (Proposition 1). In particular, the equilibrium strategies are  $w = (1,1,0)$  for equilibrium a) and  $w = (0,1,0)$  for equilibrium b). The region of the parameter space for which they exist is characterized in Figures 2, 3, and 4. In both equilibria a) and b), the restrictions on the parameters are the incentive compatibility conditions for the conjectured trading strategies to be optimal, while the general restriction on  $U_1/D_1$  and  $\beta$  that ensure that strategies for consumption, disposal and production of goods

are also optimal<sup>6</sup> (that is, condition 3 holds).

*Proof*

To prove existence of equilibria a) and b) we must follow the methodology outlined above: first, conjecture the corresponding strategy; second, work out the probability distribution of inventories; third, check that the conjectured trading strategies satisfy equilibrium conditions; and fourth, check that also the strategies for consumption, disposal and production of goods are optimal for some values of the parameters. To prove uniqueness we simply have to repeat exactly the same procedure with all the rest of possible strategies combinations and discard them as equilibria. Since the number of possible strategy vectors,  $w$ , is finite (there are only eight possible combinations), this is a relatively simple task.

In order to avoid repeating identical arguments several times, we will only give full details of the derivation of the conditions of existence for equilibrium a). The rest of the proof is nothing more than repeating the same procedure for all different possible strategy vectors. Consequently, we first conjecture the following strategy vector,  $w=(1,1,0)$ . Next, the strategies for each type of agent contained in  $w$  plus the

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<sup>6</sup>In equilibrium a),  $U_1/D_1 > (9-6.3816\beta)/(1.5816\beta+0.3456\beta^2) \equiv A_1(\beta)$ ,  $U_2/D_2 > (27-36.9297\beta+9.9297\beta^2)/(8.0703\beta-11.0383\beta^2+2.9680\beta^3) \equiv A_2(\beta)$ , and  $U_3/D_3 > (1-0.80797\beta)/0.19203\beta \equiv A_3(\beta)$  are sufficient conditions for optimality of consumption, disposal and production strategies, while  $U_1/D_1 > (6-5\beta)/\beta \equiv B_1(\beta)$ ,  $U_2/D_2 > (27-18\beta)/(3\beta^2+2\beta^3) \equiv B_2(\beta)$ , and  $U_3/D_3 > (6-5\beta)/\beta \equiv B_3(\beta)$  are the equivalent conditions in equilibrium b) (see the proof for a derivation of these expressions).

assumed matching technology generate a Markov process the steady-state probability distribution of which is equivalent to the steady-state distribution of inventories in our model. In the particular case of the vector of strategies considered here, this is  $p = (.8967, .3456, .5272, 1)$ . (Details of the above computation are provided in an appendix). Thirdly, it has to be checked that the strategies conjectured above satisfy the *equilibrium conditions*. Thus, given the strategies of other agents, the strategy conjectured for an agent of type I would imply that

$$V_{131} = b [-D_1 + V_{12} + P_{21}(-D_1 + V_{12}) + P_{230}(-D_1 + V_{12}) + P_{231}(-D_1 + V_{12}) + P_{31}(U_1 - D_1 + V_{12}) + P_{32}V_{12}] \quad (1)$$

$$V_{12} = b [V_{12} + P_{21}(U_1 - D_1 + V_{12}) + P_{230}V_{131} + P_{231}V_{12} + P_{31}V_{12} + P_{32}V_{12}] \quad (2)$$

where  $b = \beta/3$ . In order to understand better what is happening in the trade meetings taking place at each period of time, an explanation about how expression (1) is derived follows. Agent of type I holding good 3<sub>1</sub> has a payoff (optimal value) function equivalent to the sum of the following terms: 1) whenever he meets another agent of type I no trade takes place, because there cannot be trade mutually beneficial between two agents of the same type. Consequently, agent I would find himself holding good 3<sub>2</sub> (a perished good) at the end of the period and so it is optimal for him to dispose of this good and produce a new good 2 at a cost equivalent to  $D_1$ ; 2) with probability  $p_{21}$  he meets an agent of type II holding good 1. Trade does not take place because it goes against the strategy conjectured for agent of type II. Therefore, agent of type I disposes of the good and produces a new good; 3) with probabilities  $p_{230}$  and  $p_{231}$ , agent I meets an agent of type II holding good 3. Trade does not take

place because both agents are holding the same type of good; 4) with probability  $p_{31}$  agent I meets an agent of type III who is holding good 1. In this case trade takes place and agent I consumes good 1 and produces a new good 2; 5) with probability  $p_{32}$ , agent of type I meets an agent of type III holding good 2. In this case trade takes place because agent III wants to get good 3<sub>1</sub> to consume it and agent I prefers holding good 2 than holding a perished good (which involves having to dispose of it and produce a new good at a disutility cost). The explanation for expression (2) follows similar type of arguments.

From (1) and (2),

$$V_{131} - V_{12} = \frac{(p_{31} - p_{21})(U_1 - D_1) - 2D_1}{1 + bp_{230}}$$

Substituting for the p's and rearranging,

$$V_{131} - V_{12} > 0 \quad \text{iff} \quad \frac{U_1}{D_1} > 5.2301$$

That is, for this region of the parameter space the strategy conjectured for agents of type I is best response given the strategies played by other agents. In similar terms it can be shown that

$$V_{21} - V_{231} = b[p_{12}U_2 + (1 + p_{131})(V_{21} + D_2 - V_{230})] > 0$$



This last expression is always positive because the first term of the sum is obviously positive and so is the second term (recall that it has been conjectured that it is never optimal for an agent to dispose of a good not perished to produce a new good).

Finally, for agents of type III,

$$V_{31} - V_{32} = 0$$

so  $w_3 = 0$  also satisfies the equilibrium condition.

We should find the space of the parameters for which the strategies for consumption, disposal and production are optimal. This is equivalent to find for which values of the parameters the inequalities of Condition 3 in the definition of equilibrium hold. In order to do that, we have to obtain values for the V's as function of the parameters  $U_i$ ,  $D_i$ , and  $\beta$ . This will be done by substituting the values for the p's in the system formed by equations (1) and (2) (analogously for types of agents other than type I) and solving the system for the V's. This gives

$$V_{12} = \frac{U_1 (1.5816\beta + 0.3456\beta^2) - D_1 (1.5816\beta + 1.0368\beta^2)}{9 - 7.9632\beta - 1.0368\beta^2}$$

$$V_{131} = \frac{U_1 (3\beta - 1.0728\beta^2) - D_1 (9\beta - 6.3816\beta^2)}{9 - 7.9632\beta - 1.0368\beta^2}$$

It is a matter of simple algebra to derive the following necessary and sufficient conditions for (r.1), (r.2) and (r.3) in condition 3 of the equilibrium definition to be satisfied,

$$U_1 - D_1 + V_{12} > V_{131} \quad \text{iff} \quad \frac{U_1}{D_1} > \frac{9 - 15.3816\beta + 6.3816\beta^2}{9 - 9.3816\beta + 0.3816\beta^2} \quad (r.1)$$

$$V_{131} > -D_1 + V_{12} \quad \text{iff} \quad \frac{U_1}{D_1} > \frac{-9 + 15.3816\beta - 6.3816\beta^2}{1.4184\beta - 1.4184\beta^2} \quad (r.2)$$

$$-D_1 + V_{12} > 0 \quad \text{iff} \quad \frac{U_1}{D_1} > \frac{9 - 6.3816\beta}{1.5816\beta + 0.3456\beta^2} \quad (r.3)$$

It is easy to check that, when  $U_1/D_1 > 5.2301$ , (r.1) holds for all values of  $\beta$  ( $0 < \beta < 1$ ). This does not happen with (r.2) and (r.3), although (r.2) always holds when (r.3) does so. Following exactly the same procedure for agents of type II and III, it is possible to derive the other constraints to be satisfied by the parameters in equilibrium to make sure that the strategies being used by the agents for consumption, production and disposal are optimal.

In order to show that there is another equilibrium of the model in which good 1 emerges as the only medium of exchange, the same previous procedure would have to be repeated, now for the strategy vector  $w = (0,1,0)$ . It is a matter of simple algebra (available from the author upon request) to repeat the same steps as before to show that  $V_{12} \geq V_{131}$  iff  $U_1/D_1 \leq 5$ , and that  $V_{21} > V_{231}$ , and  $V_{31} = V_{32}$  for all values of the parameters. Equally, to make sure that equilibrium Condition 3) is satisfied in equilibrium b), we need the rest of the constraints on the parameters stated in footnote 3.

Finally, to show that no other equilibria exist in the model, it is just a matter of repeating the procedure for the rest of strategy vectors and check that the equilibrium conditions are not satisfied for all three types of agents. Details are not provided for the sake of brevity, but are available upon request. □

Remark

In this particular model, the inequality (r.1) (*consume if possible*) never binds given the rest of the restrictions on the parameters for which equilibria exist. Consequently, it can be said that in our particular economy, it is always the case that agents choose to accept their consumption good and consume it immediately, producing afterwards a new good.

Focusing the analysis on the parameters  $U_1$  and  $D_1$ , the previous proposition shows that different single equilibria exist for different regions of the parameter space in this model. Also, there are values of the parameters  $U_1$  and  $D_1$  for which there is not pure strategy equilibrium, specifically when  $5.2301 \geq U_1/D_1 > 5$ . The following Proposition 2 proves that, when mixed strategies are considered, there exists a steady-state equilibrium for all parameters  $U_1$  and  $D_1$  of the economy. This is done by constructing a mixed strategy equilibrium which naturally connects equilibria a) and b) in Proposition 1, in a similar fashion to what it was done in Proposition 2 in Cuadras-Morató (1993).

## Proposition 2

In this model, when mixed strategies are taken into consideration, there is a steady-state equilibrium for all values of the parameters  $U_1$  and  $D_1$ .

This proposition fills nicely the gap in Proposition 1, where there was a region of the parameter space formed by  $U_1$  and  $D_1$  for which no pure-strategy equilibrium could be found. Proposition 2 ensures that there is a steady-state equilibrium in which exchange takes place and commodity money emerges for all the values of the parameters  $U_1$  and  $D_1$  of the economy<sup>7</sup>.

(See Appendix for a proof of Proposition 2)

The following lines are intended to provide with an intuition of the results described in Propositions 1 and 2. In equilibrium, it is always optimal for agents of type II and III to play respectively the strategies "use good 1 as money" and "not use good 2 as money". This simply means that agent II always finds optimal to use good 1 (a commodity that neither he produces nor he consumes) as a medium of exchange. Equally, agent III always holds the good he produces until he can exchange it for his consumption good and uses no medium of exchange to carry out his trade. Agent of

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<sup>7</sup>Proposition 2 also ensures that there is an exchange equilibrium for almost all values of the parameters  $U_1$  and  $D_1$ . The only exception would be when  $1.8 > U_1/D_1 > 1$ , as it can be seen from Figure 3. In that case, neither equilibrium a), nor equilibrium b) exist.

type I, however, will find optimal to use a perishable commodity as commodity money only if  $p_{31} (U_1 - D_1) - 2D_1 > p_{21} (U_1 - D_1)$ , that is when the expected utility of holding good 3 is greater than the expected utility of holding good 2. The expected utility of holding good 3 is the level of utility obtained from consuming good 1 plus producing a new good  $(U_1 - D_1)$  times the probability of being matched with an agent of type III holding good 1 ( $p_{31}$ ) (in which case exchange will take place), taking into account the additional cost of being left after the random matching with a useless good that has to be thrown away and replaced at a cost  $D_1$  (something that happens with probability  $p_{131} + p_{12} + p_{21} + p_{230} + p_{231} = 2$ ). In other words, agent I will find optimal to use good 3 as medium of exchange when the liquidity advantage of doing so  $(p_{31} - p_{21})(U_1 - D_1)$  is greater than the expected costs of accepting a perishable good  $2D_1$ .

### 3.3. Exchange equilibria with imperfect information about the age of perishable goods

The objective of this subsection is to check that the results reported in the previous section hold when agents are assumed to have imperfect information about the specific age of perishable goods. This obviously makes the appearance of an equilibrium in which perishable goods appear as medium of exchange more difficult. This is because agents accepting a perishable good to be used as commodity money take, apart from the risk of that good going off before they can trade it for their consumption good, the risk of actually accepting a good which has not perished yet, but will do so at the end of the period (agents not being able to recognize it), which

means that the only optimal thing the agent can do with it is to dispose of it and produce a new good, at the usual cost. Even when this is the case, we shall prove in the following lines that the equilibria characterized in the previous section hold when the model is modified in the way expressed above. Interestingly enough, there now exist at least one pure strategy equilibrium for all values of the parameters  $U_1$  and  $D_1$  and, contrary to the results expressed in Proposition 1, there is a region of the parameter space for which the two equilibria a) and b), coexist (multiple equilibria).

Specifically, we shall assume, as before, that agents can always recognize the difference between a perished and a non-perished good and, consequently, never accept a good already gone off. Nevertheless, we shall now assume that an agent cannot recognize the exact age of a non-perished good when it is offered to him. For instance, in our economy this will mean that agent I cannot make the difference between  $z_0$  and  $z_1$  when he accepts a good of type  $z$ , although the consequences of these two actions are radically different, because while  $z_0$  can be traded for another good before it perishes,  $z_1$  inevitably perishes before it can be exchanged for anything at all, meaning that the agent who accepted it is compelled to produce a new good, at some cost. The rest of the assumptions are unchanged with respect to the model described above.

This change in the assumptions implies a modification on the incentive compatibility conditions which characterize agents of type I's strategies as equilibrium strategies. Specifically now,

$$V_{12} \geq \frac{P_{130}}{P_{130} + P_{131}} V_{131} + \frac{P_{131}}{P_{130} + P_{131}} (-D_1 + V_{12})$$

is the condition for strategy "use good 3 as money" to be part of equilibrium.

Similarly,

$$V_{12} < \frac{P_{130}}{P_{130} + P_{131}} V_{131} + \frac{P_{131}}{P_{130} + P_{131}} (-D_1 + V_{12})$$

is the condition for "not use good 3 as money" to be part of an equilibrium strategy vector.

The following proposition presents the equilibrium results of the model with the modified assumption. The equilibrium set remains basically unmodified, although now we have multiple equilibria for some values of the parameters of the model.

### Proposition 3

Under conditions of imperfect information, for values of the parameters such that  $U_1/D_1$  and  $\beta$  are large enough, there exist the following two pure strategy equilibria:

a) in the region of the parameter set for which  $U_1/D_1 > 5.2301 + 2.3350 (1/\beta)$ , there exists an equilibrium in which goods 1 and 3 are used as commodity money;

and b) in the region of the parameter set for which  $U_1/D_1 \leq 5 + 4 (1/\beta)$ , there exists an equilibrium in which good 1 is used as commodity money.

Again, the restrictions in equilibria a) and b) are conditions for optimality of trading strategies, while the rest of the restrictions, merely ensure that the strategies for consumption, disposal and production are also optimal<sup>8</sup>. The equilibria of this new model are coincident with the previous model (in the sense that the equilibrium strategies and, hence, the goods used as commodity monies are identical). However, there is now a region of the parameter space for which both equilibria a) and b) coexist. Figure 5 shows the region of parameter space  $U_1$ ,  $D_1$  and  $\beta$  for which the different equilibria of the model exist. For many values of  $\beta$ , there is a region for which both equilibria a) and b) exist. Also note that in this model  $\beta$  plays a role in the determination of the optimal trading strategies. This is simply because now agents may have to pay a production cost (necessary to produce a new good in the case they accepted a good which is going to perish in the next period) before starting the new period of time, while in the other model all payments were deferred to future moments of time. In Kiyotaki and Wright (1989) a similar situation occurred due to the fact that storage costs were assumed to be paid at the beginning of each period in which agents hold the goods in inventory. A change of this assumption (e.g. Wright (1993) assumes that storage costs are paid at the end of each period in which agents hold the goods in inventory) simply makes  $\beta$  disappear from the incentive

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<sup>8</sup>In equilibrium a),  $U_1/D_1 > (9-6\beta)/(1.5816\beta+0.3456\beta^2) \equiv C_1(\beta)$ ,  $U_2/D_2 > (27-18.9297\beta+2.9680\beta^2-2.08086\beta^3)/(8.0703\beta-1.09153\beta^3) \equiv C_2(\beta)$ , and  $U_3/D_3 > (1-0.80797\beta)/0.19203\beta \equiv C_3(\beta)$  are sufficient conditions for optimality of consumption, disposal and production strategies, while  $U_1/D_1 > (6-5\beta)/\beta \equiv D_1(\beta)$ ,  $U_2/D_2 > (27-18\beta)/(3\beta^2+2\beta^3) \equiv D_2(\beta)$ , and  $U_3/D_3 > (6-5\beta)/\beta \equiv D_3(\beta)$  are the equivalent conditions in equilibrium b).



compatibility condition for equilibrium.

(The proof of Proposition 3 is not different from the proof of Proposition 1. As a matter of fact, the procedure is the same, and only the details are different. For this reason, the proof of Proposition 3 is relegated to an appendix).

#### 3.4. The decreasing value of holding a perishable good as a medium of exchange

The objective of this subsection is to verify that holding a perishable good as a medium of exchange (something that happens only in equilibrium a) in both our economies presented in subsections 3.2 and 3.3) has a decreasing value when the agent is not able to trade it for his consumption good. This is something that does not happen at all in previous search theoretical models of commodity money. Here, apart from the fact that individuals discount the future, there is the fact that goods perish and, consequently, holding a perishable good for too long may imply the total loss of value of that good. This situation resembles inflation situations when the good being used as medium of exchange is a good that is permanently losing its value and, nevertheless, keeps its economic function.

#### Corollary of Propositions 1 and 3

In equilibrium a), both in conditions of perfect and imperfect information,  $V_{131} > -D_1 + V_{12}$ .

$V_{131}$  is the value of holding good  $z_1$  for agents of type I. The value of holding good  $z_2$ , which is a perished good of no use for consumption, is equivalent to the expression  $-D_1 + V_{12}$ , given that we have assumed that, in equilibrium, it is optimal to dispose of a good that is going to perish and produce a new good.

*Proof*

The proof of this corollary is straightforward for both the cases of Proposition 1 and Proposition 3. It has been already shown in the proof of Proposition 1 that this statement holds for the values of the parameters for which equilibrium a) exists. Similarly, it can be shown that it also holds in the case of Proposition 3.  $\square$

#### 4. CONCLUSIONS

Although it has been included in the catalogue of necessary characteristics of money many times, we have shown that durability is not an indispensable feature for an object to be used as medium of exchange. This is so because, in our model, money has a strategic nature. To a large extent, what determines which good appears as medium of exchange are the extrinsic beliefs of agents about acceptability of goods, more than the intrinsic qualities of those goods. As a result of this, we have found equilibria in which perishable goods may be used as media of exchange, even when other goods perfectly durable are available in the economy. Interestingly, one of the characteristics of these equilibria is that the value of holding a perishable good because of its function as medium of exchange decays over time, that is, we have

commodity money with decreasing value. In many senses, this is what inflation is about: decreasing value of money. In this sense, we regard this as a possible extension of the current search theoretical models of money and monetary exchange. It could be worthwhile to proceed with these ideas to model inflationary processes. This might allow some more extensive discussion about policy issues, which is something still difficult in most current search theoretical models of money as they are.

## APPENDICES

### Computation of steady-state distribution of inventories.

Strategy vector  $w = (1,1,0)$ , together with the assumed matching technology, generates a Markov process characterized as follows. For agents of type I, the distribution of inventories can be defined as the vector  $p_1 = (p_{12}, p_{131})$ , and the matrix of transition probabilities,  $\Pi_1$  as follows

$$\Pi_1 = \begin{pmatrix} \frac{p_{12} + p_{131} + p_{21} + p_{231} + p_{31} + p_{32}}{3} & p_{230} \\ 0 & 0 \end{pmatrix}$$

The resulting steady state distribution of inventories  $p_1^*$  (remember condition for steady state is  $p_1^* \Pi_1 = p_1^*$ ) is characterized by the following equation:

$$p_{12} \ p_{230} = 3(1 - p_{12})$$

Following the same procedure for agents of type II and III, we would get the following system of equations,

$$P_{12} P_{230} = 3(1 - P_{12})$$

$$(1 - P_{21}) P_{31} = P_{21}(1 + P_{12} - P_{31})$$

$$3(1 - P_{21} - P_{230}) = P_{230}(2 - P_{12})$$

$$P_{31} = 1$$

The solution of this system of nonlinear equations gives us the steady state distribution of inventories in the economy, which is what we need to proceed with the proof of the proposition,  $p = (.8967, .3456, .5272, 1)$ .

### Proof of Proposition 2

Let  $r_i$  be the probability that agents of type  $i$  play the strategy  $w_i=1$  ( $0 \leq r_i \leq 1$ ) and let  $r = (r_1, r_2, r_3)$ . With mixed strategies, the assumption of no trade when agents are indifferent between holding their good or the good held by their trading partners will be modified and it will be assumed that agents may randomize between trade and no trade whenever they are indifferent between two goods. Then, best response mixed-strategies will be characterized as follows:

$$\begin{aligned} &\{0\} \quad \text{if } V_{i+1} > V_{i+2} \\ r_i \in [0,1] &\quad \text{if } V_{i+1} = V_{i+2} \\ &\{1\} \quad \text{if } V_{i+1} < V_{i+2} \end{aligned}$$

(note that in this particular model for  $i+1, i+2 = 3$ , the notation concerning the payoff functions will only be complete including the superscript  $s=1$ ).

In order to construct a mixed-strategy equilibrium that connects the pure-strategy equilibrium found in Proposition 1, the following strategy vector is conjectured:  $r_1 \in [0,1]$ ,  $r_2 = 1$ , and  $r_3 = 0$ . This vector of strategies, together with the matching technology, generates a Markov process the steady-state probability distribution of which is equivalent to the steady-state distribution of inventories in our economy. It can be shown that, in this particular case, the steady-state distribution of inventories will be given by the following system of equations:

$$r_1 p_{12} p_{230} = 3(1 - p_{12})$$

$$3(1 - p_{21} - p_{230}) = p_{230}(2 - r_1 p_{12})$$

$$1 - p_{21} = p_{21} p_{12}$$

$$p_{31} = 1$$

In order to simplify notation, let  $p_{12} = x$ . The resolution of the previous system implies finding the roots of the following third order equation,

$$r_1 x^3 - (5 + r_1) x^2 - r_1 x + 5 = 0$$

It can be shown easily that the discriminant of this equation is positive, and consequently, it has three different real roots. It can also be proved that one of these roots has value between zero and one. However, it is not possible to give a general

expression for  $x$  as a function of  $r_1$  (using Cardano's method) because it leads to calculations that require the cube root of an imaginary number -the so-called irreducible case of the cubic. Nevertheless, it is not difficult to compute values of  $x$  between zero and one for the different values of  $r_1$  by using simple numerical methods. An illustration of this follows.

As a matter of example, let  $r_1$  be equal to 0.5. Solving the previous third order equation by simple numerical methods, the value  $p_{12} = 0.9490$  will be obtained, and substituting in the system of equations the following vector  $p$  represents the steady-state inventory distribution,  $p = (0.9490, 0.5131, 0.3224, 1)$ . Computing the payoff functions for agent of type I, and given the equilibrium incentive compatibility constraint, we have the following condition,

$$V_{131} - V_{12} = b[-r_1 p_{230} (V_{231} - V_{12} + (p_{31} - p_{21}) (U_1 - D_1) - 2D_1] = 0$$

Substituting for the  $p$ 's and rearranging, it can be shown that the previous expression only holds iff  $U_1/D_1 = 5.1076$ . Following a similar procedure, we can show that there is a continuum of points in the parameter space for which a mixed strategy for agents of type I with different values of  $r_1$  between zero and one satisfies the equilibrium condition (for instance, for  $r_1 = 0.25$  the equilibrium condition is satisfied iff  $U_1/D_1 = 5.0518$ ). Figure 6 maps the set of best responses for agents of type I (values of  $r_1$ ) with the values of the ratio  $U_1/D_1$ , given the strategies of agents of type II and III,  $w_2 = 1$  and  $w_3 = 0$ .

Showing that the conjectured strategy is best response for agents of type II and III involves repeating exactly the same argument as in Proposition 1, to show that  $V_{21} > V_{231}$  and  $V_{31} = V_{32}$ . Again, it is not difficult to prove that there is a region of the parameter space for which this type of equilibria exists (and for which the strategies for consumption, disposal and production are optimal). This, together with the results of Proposition 1 completes the proof.  $\square$

*Proof of Proposition 3*

The proof of Proposition 3 is similar to the proof of Proposition 1. In particular, the same kind of procedure to prove existence of equilibria will be adopted. Consequently, first the exchange strategy vector  $w = (1,1,0)$  is conjectured. Secondly, the steady state distribution of inventories has to be computed. It could be easily checked that the specific change made on our assumption about information available to agents about the age of perishable goods does not change the particular steady state distribution of inventories. Thus,  $p = (.8967, .3456, .5272, 1)$ . Finally, we must check that the equilibrium incentive compatibility constraints and our initial restrictions on the parameters are satisfied. That is,

$$V_{12} < \frac{P_{130}}{P_{130}+P_{131}} V_{131} + \frac{P_{131}}{P_{130}+P_{131}} (-D_1+V_{12}) , V_{21} > V_{231} , V_{31} \geq V_{32}$$

Substituting for the p's and rearranging, the first of these expressions is

$$V_{131} - V_{12} > 0.3680D_1$$

As before, in order to check that equilibrium conditions are satisfied, it is necessary to compute the value of the payoff functions. For agent of type I,

$$V_{131} = b[-D_1 + V_{12} + (p_{21} + p_{230} + p_{231})(-D_1 + V_{12}) + p_{31}(U_1 - D_1 + V_{12}) + p_{12}V_{12}] \quad (A.1)$$

$$V_{12} = b[V_{12} + p_{21}(U_1 - D_1 + V_{12}) + p_{230}V_{131} + p_{231}(-D_1 + V_{12}) + (p_{31} + p_{32})V_{12}] \quad (A.2)$$

From (A.1) and (A.2),

$$V_{131} - V_{12} = \frac{b[(p_{31} - p_{21})(U_1 - D_1) - (2 - p_{231})D_1]}{1 + bp_{230}}$$

It is a matter of simple algebra to check that

$$V_{131} + V_{12} > 0.3680D_1 \quad \text{iff} \quad \frac{U_1}{D_1} > 5.2301 + 2.3350 \frac{1}{\beta}$$

Equally, for agents of type II and III,

$$V_{21} - V_{231} = b[(1 + p_{131})(V_{21} + D_2 - V_{230})] > 0$$

$$V_{31} = V_{32}$$

Finally, the rest of the constraints on the parameters are conditions that ensure that, in equilibrium, the conjectured strategies for consumption, disposal and production are optimal. It is a matter of simple algebra to show the way these restrictions are derived, just following the procedure outlined in the proof of the Proposition 1.



To specify existence conditions of equilibrium b), the same steps have to be followed, although now the conjectured strategies are defined by the vector  $w = (0,1,0)$ . It is easy to show that equilibrium b) holds when  $U_1/D_1 \leq 5 + 4(1/\beta)$  and strategies for consumption, disposal and production are optimal when the rest of the restrictions on the parameters are met. And further following the same procedure for the rest of possible strategy vectors, and checking that they do not satisfy the equilibrium conditions, it can be proved that there are not equilibria other than a) and b).  $\square$

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FIGURE 1

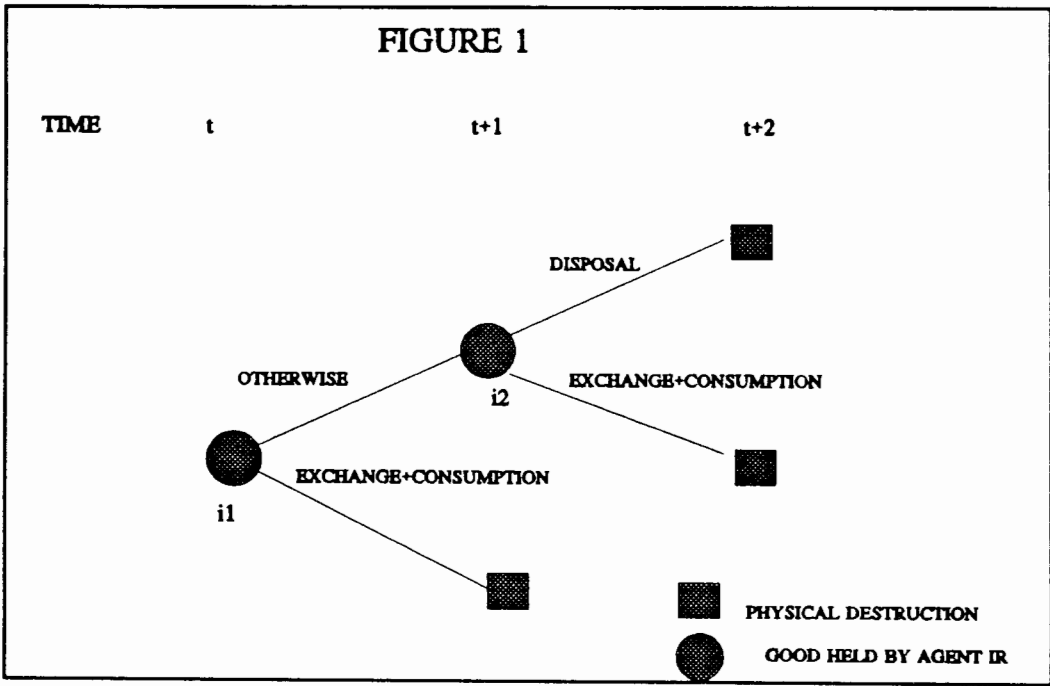


FIGURE 2

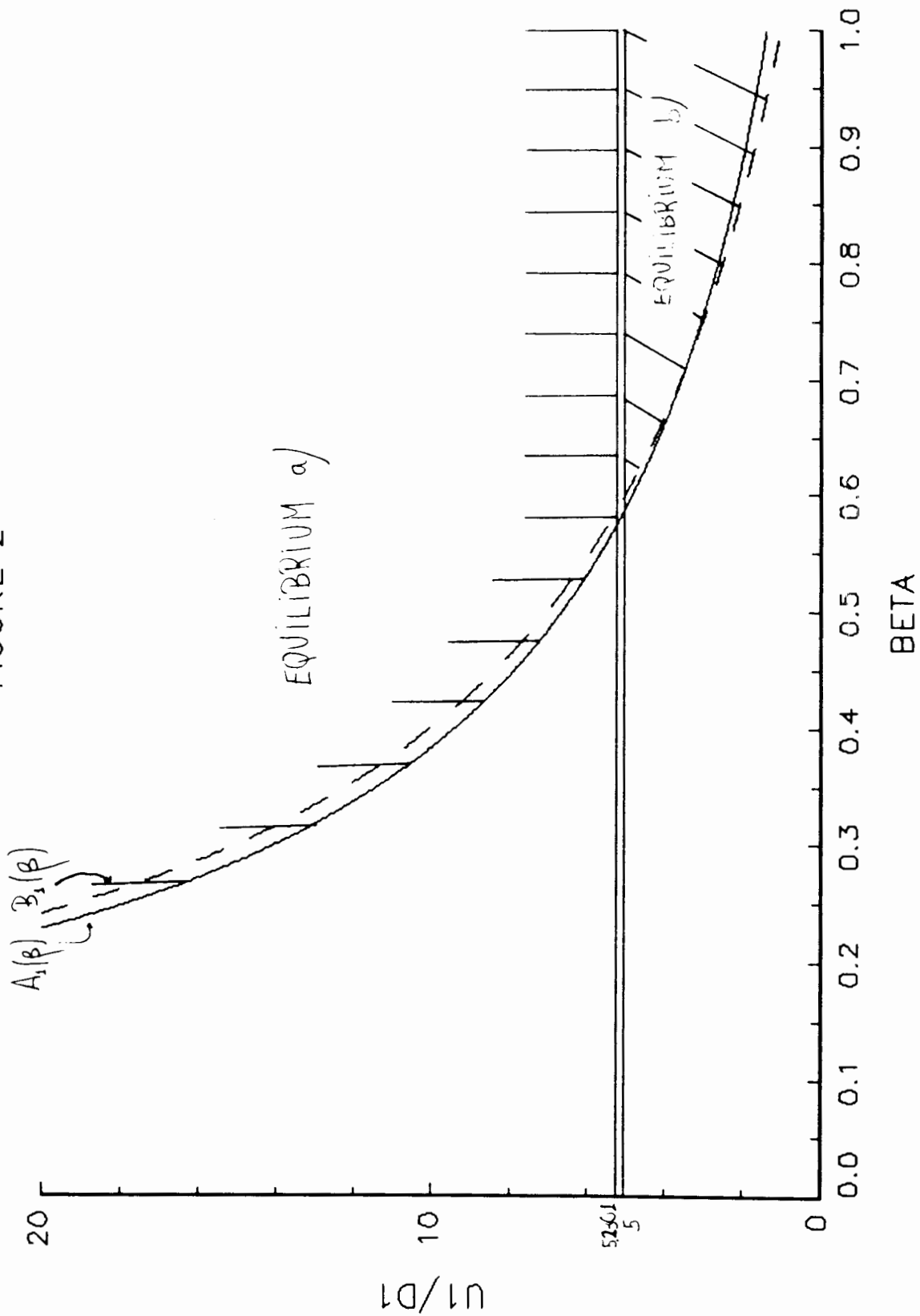


FIGURE 3

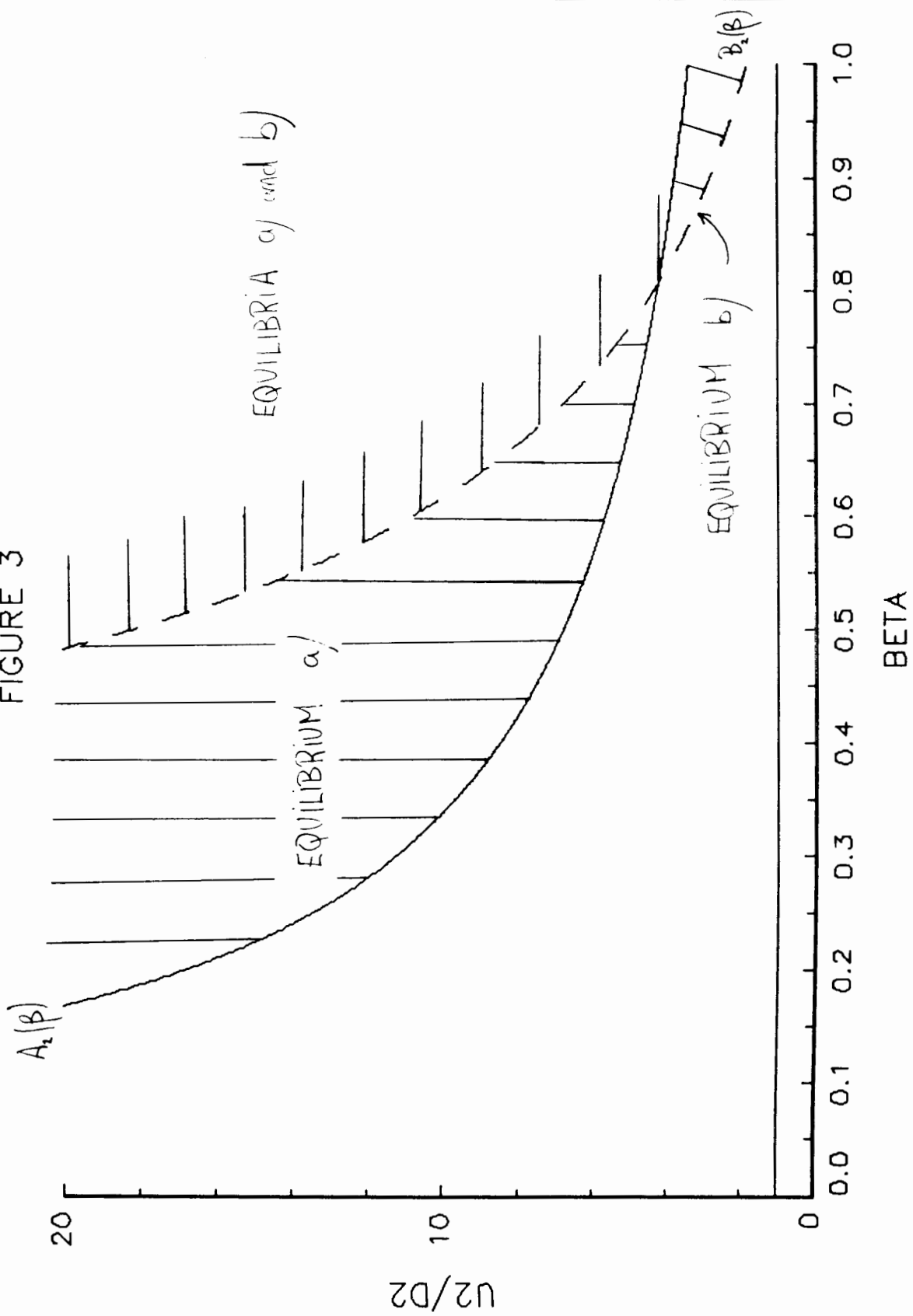


FIGURE 4

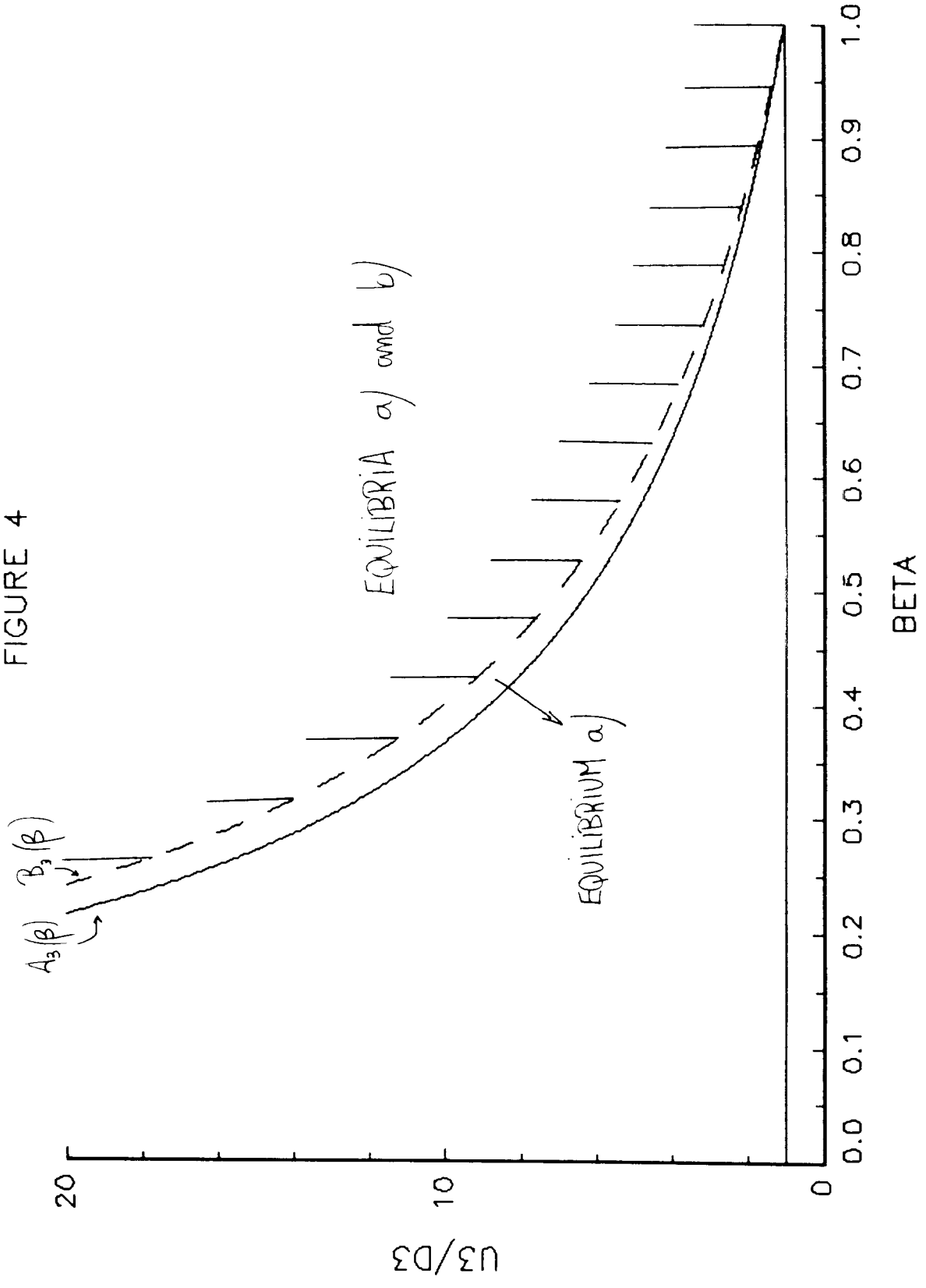


FIGURE 5

||| EQUILIBRIUM b)  
/// EQUILIBRIUM a)

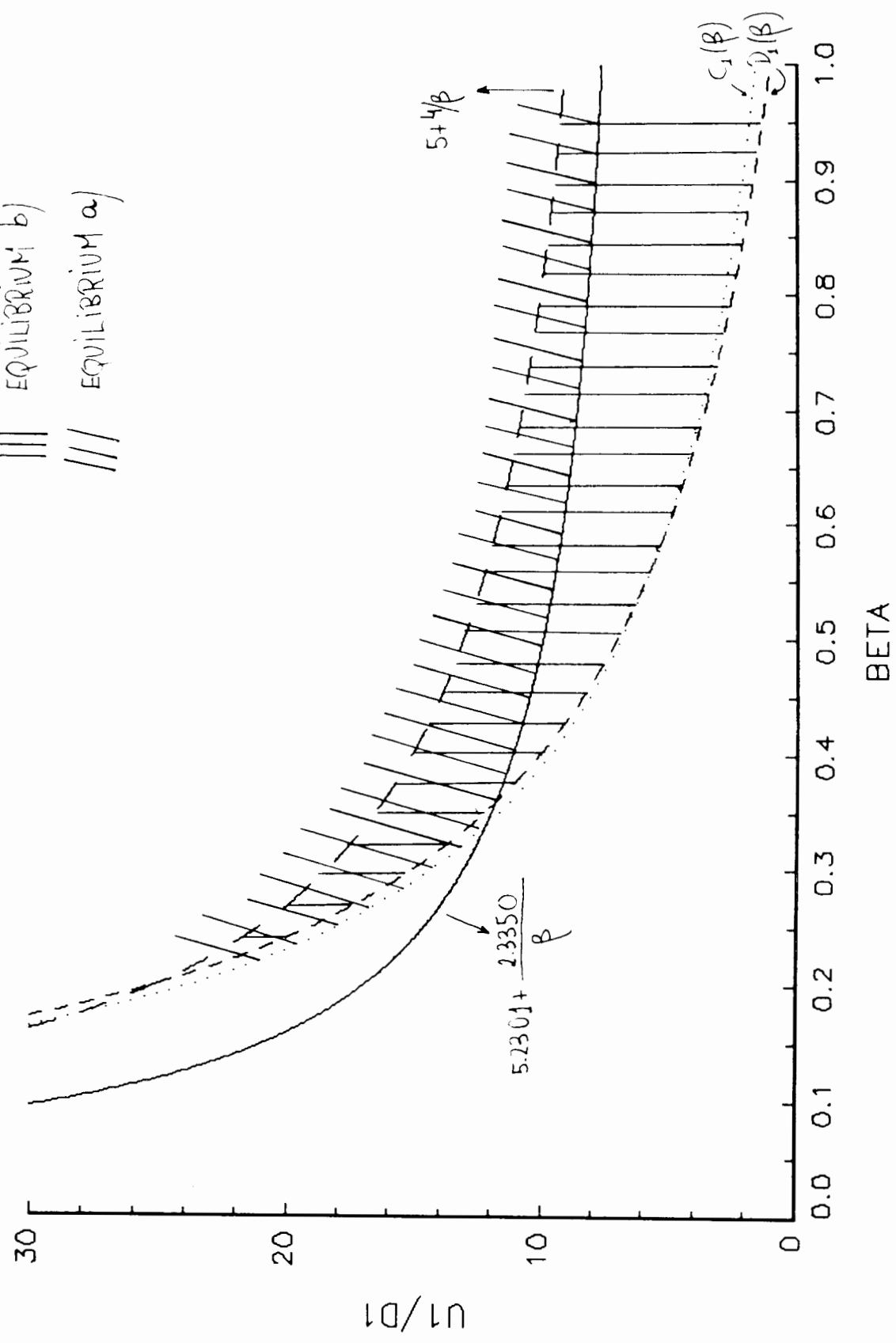




FIGURE 6

