NEW TRENDS IN PUBLIC FACILITY LOCATION MODELING[1]

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The past four decades have witnessed an explosive growth in the field of network-based facility location modeling. This is not at all surprising since location policy is one of the most profitable areas of applied systems analysis in regional science and ample theoretical and applied challenges are offered. Location-allocation models seek the location of facilities and/or services (e.g., schools, hospitals, and warehouses) so as to optimize one or several objectives generally related to the efficiency of the system or to the allocation of resources.

This paper concerns the location of facilities or services in discrete space or networks, that are related to the public sector, such as emergency services (ambulances, fire stations, and police units), school systems and postal facilities.
The paper is structured as follows: first, we will focus on public facility location models that use some type of coverage criterion, with special emphasis in emergency services. The second section will examine models based on the P-Median problem and some of the issues faced by planners when implementing this formulation in real world locational decisions. Finally, the last section will examine new trends in public sector facility location modeling

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1 Introduction

The past four decades have witnessed an explosive growth in the field of network-based facility location modeling. As Krarup and Pruzan (1983) point out, this is not at all surprising since location policy is one of the most profitable areas of applied systems analysis and ample theoretical and applied challenges are offered. Location-allocation models seek the location of facilities and/or services (e.g., schools, hospitals, and warehouses) so as to optimize one or several objectives generally related to the efficiency of the system or to the allocation of resources. There are several ways of classifying network-based location models and problems. A good taxonomy of this type of problems can be found in Daskin (1995). The dichotomy between public versus private sector problems is a common way of classification.

This paper concerns the location of facilities or services in discrete space or networks, that are related to the public sector, such as emergency services (ambulances, fire stations, and police units), school systems and postal facilities. This does not mean that these type of services necessarily belong strictly to the public sector, e.g., a medical emergency service may be owned by a private firm but regulated by a public health agency. So the question is, What is the main difference between the location of public facilities and private facilities? The answer lies in the nature of the objective or objectives that decision makers are considering. Public and private sector applications are different, because of the optimization criteria used in both cases. Profit maximization and capture of larger market shares from competitors are the main criteria in private applications, while social cost minimization, universality of service, efficiency and equity are the goals in the public sector. Since these objectives are difficult to measure, they are frequently surrogated by minimization of the locational and operational costs needed for full coverage by the service, or the search for maximal coverage given an amount of available resources. Note that although it is not usual, it is perfectly allowed for a public service planner to use some of the tools that are typical to private investors. For example, a public health service could compete with private providers, and so reduce the subsidy needed from the state for maintaining the service (Marianov and Taborga, 2000).

An additional problem is that in public sector location models there is no one overriding objective, and a variety of responses may be given to a simple question on the “best” locational configuration of some service. For example, when locating ambulances we may be interested in siting them so as to minimize the weighted average response time of the system, or to cover the population at risk within a given time or distance. The first approach corresponds to what is known in location literature as a p-median problem, and the second one is a covering problem (Location Set Covering Problems, LSCP, or Maximal Covering Location Problems, MCLP). Most public facility located models use one of these approaches (or a combination of both) to set the foundations of the formulation at hand. In fact, both p-median and covering problems can be considered benchmarks in the development of location models, and as such, we will classify our examples as belonging to one of these two broad categories.

This paper is structured as follows: In the next section we will focus on public facility location models that use some type of coverage criterion, with special emphasis in emergency services. The third section will examine models based on the P-Median problem and some of the problems faced by planners when implementing this formulation in real world problems. Finally, the last section will examine new trends in public sector facility location problems.

2. Covering Models in the Public Sector

The Notion of Coverage

In this subsection we will refer to public sector applications of covering models. Being proximity (distance or travel time[2]) one of the fundamental aspects of location analysis, many models (as the *p*-median, analyzed in the next subsection) seek to minimize the distance or travel time between a customer and the facility at which she/he receives a service. As opposed to those models, covering models are based on the concept of acceptable proximity. In covering models, a maximum value is preset for either distance or travel time. If a service is provided by a facility located within this maximum, then the service is considered adequate[3] or acceptable; the service is equally good if provided by facilities at different distances, as long as both distances are smaller than this maximum value. Then, a customer is considered covered by the service, or just *covered*, if she/he has a facility sited within the preset distance or time. An example of this is the case in which it is desired that the population in a rural area have access to a health care center within, say, 2 miles. It is said that a customer in this area is *covered* if she/he has a health center within 2 miles of her/his home. Another example appears when dealing with fire fighting services. The Insurance Services Office (ISO), is an organization which rates cities according to their fire protection capability (ISO, 1974). They establish distance standards for fire-fighting response. If the distance standards are not fulfilled in a city, the rating decreases, indicating that the risk of property loss is higher in those cities. Thus, it is reasonable to design fire fighting systems in such a way to assure attention of all calls within the time standard or, equivalently, to have an available server within a standard distance of each and every customer.

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[2] Most of the location models are related to geographical location. In this case, proximity refers to a distance or time metric. However, proximity can be defined also in other spaces; for example, two persons can have proximity in terms of similar opinions.

[3] Note that the distance requirement is one of the necessary conditions for an adequate service; it may not be sufficient to guarantee a good overall service.

Covering models can be classified according to several criteria. One of such criteria is the type of objective, which allows us to distinguish two types of formulations. In the first place, those seeking to minimize the number of facilities needed for full coverage of the population (Set Covering Models) and secondly, those that maximize covered population, given a limited number of facilities or servers (Maximum Covering Models). Covering models can also be classified in formulations for systems with fixed servers and systems with mobile servers. Examples of the former are schools, hospitals, and other systems in which customers travel to the facility to receive service. Examples of systems with mobile servers are emergency services, in which servers are initially located at depots, and whenever a call is received, they travel to the location of the call and back to the depot. In turn, any of these can be classified as capacitated or uncapacitated, depending on the capacity limits of the facilities or servers to be sited. These capacity limits can be for example the number of children that a primary school can accept in a particular year, or the number of customers that can be attended by an ambulance system within a reasonable waiting time.

The notion of coverage can be extended in several ways. For example, a single policeman can not control alone some police emergencies. Coverage, then, must be defined as the response to the emergency by, say, p policemen. If fewer than p policemen attend the call, the emergency is not counted as covered. Similarly, the usual fire emergency puts people *and* property at risk. Then, engine fire companies and ladder fire companies are both needed at the scene of the fire, in order to protect property and people. Furthermore, different numbers of companies are needed in different cases. ISO defines standard response to a fire in medium size cities as response by three engine companies and two ladder companies. In this case, coverage is defined as attendance by three engine companies and two ladder companies, within their respective (response) time standard. Finally, coverage could mean *availability* of a service within certain time limits, as opposed to just *location* of a server within these time limits. For example, in an emergency service, a customer could be considered as covered if all her/his calls find an idle server with probability, say, 98%. Or, a customer might be considered as covered if, whenever she/he arrives at the health care center, there is a waiting line shorter than 5 people, with probability 95%. Note that, in this case, availability is somehow related to the capacity of the servers. Covering models have been used profusely in both private and public sectors (Schilling *et al*, 1993).

Basic Covering Models

There are two basic covering models. The first one is the Location Set Covering Model (LSCP), cast as a linear programming formulation by Toregas *et al* (1971), and Toregas and ReVelle (1973). This model seeks to locate a minimum number of servers needed to obtain mandatory coverage of all demands. In other words, each and every demand point has at least one server located within some distance or time standard *S*. The first application of this model was in the area of emergency services (ReVelle et al. 1976). In this context, the model positions the minimum possible number of emergency vehicles in such a way that the entire population has at least one of these vehicles initially located within the time or distance standard. Note that coverage is not affected by the fact that the servers (vehicles) may be busy at times. The formulation of the model is as follows:

$$\text{Minimize} \quad Z = \sum_{j \in J} x_j \qquad (1)$$

subject to

$$\sum_{j \in N_i} x_j \geq 1 \qquad \forall\, i \in I \qquad (2)$$

$$x_j = 0, 1 \qquad \forall\, j \in J \qquad (3)$$

where

J = set of eligible facility sites (indexed by j) ;

I = set of demand nodes (indexed by i) ;

$$x_j = \begin{cases} 1 \text{ if a facility is located at node } j \\ 0 \text{ otherwise} \end{cases}$$

$N_i = \{ j \mid d_{ji} \leq S \}$; with d_{ji} = shortest distance from potential facility location j to demand node i, and S = distance standard for coverage.

Note that N_i is the set of all those sites that are candidates for potential location of facilities, that are within distance S of the demand node i. If a facility is located in any of them, demand node i becomes covered. The objective (1) minimizes the number of facilities required. Constraints (2) state that the demand at each node i must be covered by at least one server located within the time or distance standard S.

The solution to this model can be easily found solving its linear programming relaxation, with occasional branch and bound applications. Before solving, its size can be reduced by successive row and column reductions, as proposed by Toregas and ReVelle (1973).

Church and ReVelle (1974) and White and Case (1974) formulated the second basic covering model, the Maximal Covering Location Problem (MCLP). Although public services should be available to everybody, as modeled by the LSCP, the MCLP recognizes that mandatory coverage of all people in all occasions and no matter how far they live, could require excessive resources. Thus, MCLP does not force coverage of all demand but, instead, seeks the location of a fixed number of facilities, most probably insufficient to cover all demand within the standards, in such a way that population or demand covered by the service is maximized. The fixed number of facilities is a proxy for a limited budget. Its integer programming formulation is the following:

Maximize $Z = \sum_{i \in I} a_i y_i$ (4)

subject to

$y_i \leq \sum_{j \in N_i} x_j \quad \forall i \in I,$ (5)

$\sum_{j \in J} x_j = p$ (6)

$x_j, y_i = 0,1 \quad \forall j \in J, i \in I,$

where additional notation is

$$y_i = \begin{cases} 1 \text{ if node } i \text{ is covered} \\ 0 \text{ otherwise} \end{cases}$$

p = the number of facilities to be deployed;

a_i = the population at demand node *i*.

and all other variables and parameters are the same as defined for LSCP. The objective (4) maximizes the weighted sum of covered demand nodes. Constraints (5) state that the demand at node *i* is covered whenever at least one facility is located within the time or distance standard *S*. Constraint (6) gives the total number of facilities that can be sited. Church and ReVelle (1974) used relaxed linear programming, supplemented by occasional use of branch and bound, to provide solutions to this problem. Other solving procedures include Greedy or Myopic heuristics (Daskin, 1995, Schilling *et al*, 1993), Lagrangean Relaxation (Daskin, 1995, Galvão and ReVelle, 1996) and Heuristic Concentration (Rosing, 1997, Rosing and ReVelle, 1997).

Applications of these models in the public sector range from emergency services to location of archeological sites (Bell and Church, 1987). The set covering model has been used to allocate bus stops (Gleason, 1975). The maximal covering location model, and different variants of it, has been used for the location of health clinics (Eaton *et al*, 1981), hierarchical health services (Moore and ReVelle, 1982), and many other applications. The MCLP has also been used in several non-locational problems, as for example, the determination of test points in the human eye, to diagnose vision loss in glaucoma suspects (Kolesar, 1980), or for the allocation of marketing resources in journals (Dwyer and Evans, 1981).

Models for Mobile Emergency Services

In the case of most emergency systems, a fundamental issue is the amount of time a customer waits for service. This is the case of any public emergency services, either medical, fire fighting or police related. In the case of medical emergencies, there is a correlation between life loss risk and response time. Thus, it seems to be a good approach to assure medical attention of all calls within a time standard or, equivalently, have an available server within a standard distance of each and every customer. The same happens in the case of fire fighting services. Since it can be expected that loss of property increase with time, each type of company has to respond within its standard time. Police emergencies are not the exception. Again, the best model for these services is a covering model.

Many issues have to be considered in order to determine the performance of an emergency service. Response time is one of them. From the point of view of the geographical design of such a system, an important issue is the location of the depots, that is, the initial location of the emergency vehicles (servers). Another one is the number of servers. A third issue is the *availability* of servers, as opposed to just their initial location within time standard. Availability, in this case, is defined as the actual percentage of time the server is idle, as opposed to being on repair, or attending other calls. Finally, the dispatching policy has also an influence on the efficiency of the system.

Several approaches have been presented to attack the design of systems with mobile servers under congestion (or availability less than 100%). These can be classified in descriptive and prescriptive. Descriptive methods originate from a seminal paper by Larson (1974), in which a descriptive model (Hypercube) is presented for analysis of emergency systems. The hypercube model builds on previous developments by Carter, Chaiken and Ignall (1972) and Larson and Stevenson (1972) for two servers, and describes a spatially distributed queuing system with distinguishable servers. The model can be used, either in its complete version or in approximate versions (Larson, 1975), for testing the responsiveness of an emergency system and all its parameters. Many iterative methods have derived from the hypercube, as Jarvis' (1985) and Burwell, Jarvis and McKnew (1993) . Other descriptive models have been used for location of one mobile server, and can also be used in heuristics that locate multiple servers. Among them, the models and methods by Berman, Larson and Chiu (1985), Batta (1988), Batta, Larson and Odoni (1988), Batta (1989), Berman, Larson and Parkan (1987), Berman and Larson (1985) and Berman and Mandowsky (1986). The interested reader can refer to these papers or a review in Marianov and ReVelle (1995). However, we do not focus on these models, but rather on models derived from the basic coverage formulations.

Prescriptive models are based on optimization, and derive from the basic models outlined above. Good reviews of optimization models presented before 1990 are included in the articles by Daskin, *et al* (1988), and ReVelle (1989). We will include some of them, which are representative of different classes of formulations.

An interesting generalization of the maximal covering model, because it considers the simultaneous location of several types of facilities, is the FLEET (Facility Location and Equipment Emplacement Technique) by Schilling *et al.* (1979). This model was used for locating fire-fighting services in the city of Baltimore. The goal of this formulation is to locate simultaneously two different types of fire-fighting servers (pump or engine brigades and ladder or truck brigades), as well as the depots housing them. The objective of the FLEET model was coverage of the maximum number of people by both an engine company sited within an engine company distance standard *and* a truck company sited within the truck company distance standard. Other objectives included in a multi-objective formulation of the FLEET model were the maximum coverage of fire frequency, maximum coverage of property value and maximum coverage of population at risk.

The maximum population coverage version can be stated mathematically as

$$\text{Maximize } Z = \sum_{i \in I} a_i y_i \qquad (8)$$

subject to

$$y_i \le \sum_{j \in N_i^E} x_j^E \quad \forall i \in I, \qquad (9)$$

$$y_i \le \sum_{j \in N_i^T} x_j^T \quad \forall i \in I, \qquad (10)$$

$$x_j^T \le x_j^S \quad \forall j \in J \qquad (11)$$

$$x_j^E \le x_j^S \quad \forall j \in J \qquad (12)$$

$$\sum_{j \in J} x_j^E + \sum_{j \in J} x_j^T = p^{E+T} \qquad (13)$$

$$\sum_{j \in J} x_j^S = p^S \qquad (14)$$

$$x_j^E, x_j^T, x_j^S, y_i = 0,1 \quad \forall j \in J, i \in I,$$

where

x_j^E = 1, 0; 1 if an engine company positioned in a fire house at site *j*; 0 otherwise

x_j^T = 1, 0; 1 if a truck company positioned in a fire house at site *j*; 0 otherwise

x_j^S = 1, 0; 1 if a fire station or depot is established at site *j*; 0 otherwise

N_i^E = $\{ j \mid t_{ji} \leq E \}$; set of potential engine sites *j* which can cover node *i* by virtue of being within the engine distance standard *E*

N_i^T = $\{ j \mid t_{ji} \leq T$; set of potential truck sites *j* which can cover node *i* by virtue of being within the truck distance standard *T*

p^{E+T} = number of fire companies, and

p^S = number of fire stations or depots.

The first two constraints define coverage as achievable only if *both* one or more engine companies are sited within the engine distance standard *and* one or more truck companies are sited within the truck distance standard. The third and fourth constraints allow housing of companies only at nodes where a depot has been sited. The fifth constraint limits the total number of companies, and the sixth constraint limits the number of stations. Schilling *et al* (1979) solved the linear relaxation of the problem. If two or more servers of each type are needed, because the attendance of only one of each is not enough (as in police or fire emergencies), the model can be modified, as in Marianov and ReVelle, (1991) and (1992). For example, if three engine brigades are needed at the site of the emergency, the second constraint of the preceding model can be changed to:

$$y_i + w_i^E + u_i^E \leq \sum_{j \in N_i^E} \sum_{k=1}^{C_j} x_{kj}^E \qquad \forall i \in I, \qquad (15)$$

and the ordering constraints:

$$y_i \leq w_i^E \qquad \forall i \in I \qquad (16)$$

$$w_i^E \leq u_i^E \qquad \forall i \in I \qquad (17)$$

added. Here, w_i^E, u_i^E = 1 if demand node *i* is covered by a second engine, first engine, respectively, and 0 otherwise. Constraint (15) does not allow variables w_i^E, u_i^E and y_i to be one unless there are three engines located within standard time of node *i*. Constraints (16) and (17) force coverage by one engine before coverage of two engines and coverage by two before coverage by three.

Note that, as opposed to the models in which one server was enough for coverage, when multiple coverage is sought, co-location of servers at the same depots might become convenient. Models which limited the number of stations but allowed more than one server at the same site, up to a certain capacity, were suggested by Bianchi and Church, (1988), for one type of vehicle. Also, Marianov and ReVelle, (1991), propose a tighter set of constraints for siting up to C_j engines plus trucks in a depot:

$$x_{(C_j-k+1)j}^E + x_{kj}^T \leq x_j^S \qquad \forall j \in J, k \leq C_j \qquad (18)$$

$$x_{(k+1)j}^l \leq x_{kj}^l \qquad \forall j \in J, k \leq C_j - 1,\ l = E \text{ or } T \qquad (19)$$

where

x_{kj}^E = 1 if a k^{th} engine company is located at site *j*, 0 otherwise;

x_{kj}^T = 1 if a k^{th} truck company is located at site *j*, 0 otherwise;

These constraints, together, have three effects: first, they allow the siting of servers only at depots. Second, they limit to C_j the number of servers at each depot (no more than $\{C_j - k\}$ engines plus *k* trucks) at a site. Finally, they state that a $(k+1)^{th}$ server must be located at a site after the k^{th} server.

In most cases, a server can not attend more than one call at a time. This means that, when a call arrives, a server can be busy attending other calls or on repair. This leads to congestion, which is the dynamic equivalent to a limited capacity. When there is the possibility of congestion, different approaches can be used. When congestion is not expected to be severe, there is an approach that does not need any analysis of the probabilistic characteristics of the system. This approach consists in seeking redundancy in the servers able to attend calls originating from a demand node. In other words, to allocate more than one server (say, two) to cover each demand within the standard time. The same construct used for attendance of more than one server to an emergency can be used when redundant coverage is needed, as did Hogan and ReVelle (1986) in their BACOP 2 model, which trades off first coverage versus backup coverage. Its formulation is

$$\text{Maximize} \quad Z_1 = \sum_{i \in I} a_i y_i \qquad (20)$$

$$Z_2 = \sum_{i \in I} a_i r_i \qquad (21)$$

subject to

$$r_i + y_i \leq \sum_{j \in N_i} x_j \qquad \forall i \in I \qquad (22)$$

$$r_i \leq y_i \qquad \forall i \in I \qquad (23)$$

$$\sum_{j \in J} x_j = p \qquad (24)$$

$$x_j, r_i = 0,1 \qquad \forall j \in J, i \in I,$$

where r_i is one if a second coverer is sited within standard time of node i. The objectives maximize first and second coverage, respectively. The first constraint says that coverage by a first and second server is not possible unless at least two servers are initially located in the neighborhood. The second constraint reflects the fact that backup coverage can not be fulfilled without first coverage. The next constraint limits the number of servers to be deployed. The authors report that marginal reductions in first coverage improve strongly backup coverage. They solved the linear relaxation of the model, with occasional branch and bound.

Berlin (1972), Daskin and Stern (1981), Benedict (1983) and Eaton et al (1986) used a different approach, based on the LSCP for mandatory first coverage. They solve the LSCP first, and, using a number of servers that is at least the needed for full first coverage, maximized redundant coverage maintaining mandatory first coverage for all demands.

When congestion is expected to be more severe, a frankly probabilistic approach provides safer and more efficient system designs. In this case, a probabilistic modeling of the system is required, as well as the use of this probabilistic models in the developments of objectives or constraints of the optimization model.

Two approaches have been used for probabilistic models: The first consists in maximizing expected coverage of each demand node. The second, in either constraining the probability of at least one server being available (to each demand node) to be greater than or equal to a specified level α, or to count demand nodes as covered if this probability is at least α.

The maximization of expected coverage was proposed by Daskin (1983), who utilized the notion of a server *busy fraction*, or probability of being busy, to formulate the Maximum Expected Covering Location Problem, (MEXCLP). Daskin assumed a single system-wide busy fraction (probability of a server being busy or fraction of the time during which it is busy) q, as well as independence between the probabilities of different servers being busy, which leads to a binomial distribution of the probability of k servers being busy. The MEXCLP maximized the expected value of population coverage within the time standard, given that p facilities are to be located on the network. Daskin computed the increase in the expected coverage of a demand, when a k^{th} server is added to its neighborhood, which turns out to be just $(1 - q)q^{k-1}$. Then, the expected coverage for all possible number of servers k at each neighborhood, and for all demand nodes weighted by their demand, is maximized:

$$\text{Maximize } Z = \sum_{i \in I} \sum_{k=1}^{n_i} a_i (1-q) q^{k-1} y_{ik} \qquad (25)$$

subject to

$$\sum_{k=1}^{n_i} y_{ik} \le \sum_{j \in N_i} x_j \qquad \forall i \in I \qquad (26)$$

$$\sum_{j \in J} x_j = p \qquad (27)$$

$y_{ik} = 0,1 \qquad \forall i,k,$

x_j = integers $\forall j$

where

y_{ik} is one if node i has at least k servers in its neighborhood, zero otherwise,

x_j is the number of servers at site j, and

n_i is the maximum number of servers in N_i.

The first constraint says that the number of servers covering demand i is bounded above by the number of servers sited in the neighborhood. The second constraint limits the number of servers to be deployed. Declining weights $(1 - q)q^{k-1}$ on the variables y_{ik} make unnecessary any ordering constraints for these variables, and help to the integrality of these variables in the solution, if the linear relaxation of the model is solved. Daskin proposed a heuristic method of solution of the MEXCLP, which gives solutions for the system for different ranges of values of q. More details on this important model, can be found in Daskin (1995).

Later, Bianchi and Church (1988), modified MEXCLP to consider location of vehicles and depots as well, referring to their model as the Multiple cover, One-unit Facility Location, Equipment Emplacement Technique (MOFLEET). Recognizing the need for relaxing the assumption of independence between probabilities of servers being busy, Batta *et al* (1989) proposed a modified MEXCLP, in which the factors $(1 - q)q^{k-1}$ are corrected by an approximation to a queuing system, based on the approximated hypercube of Larson (1975). In that model, called AMEXCLP, the busy fraction of servers is still assumed to be the same over the whole system. Another model, by Goldberg and Paz (1991), maximizes the expected number of calls reached within a set time threshold. The model is nonlinear, based on Jarvis' (1975) mean service time computation. The service time depends on call location, and independence is assumed between probabilities of servers being busy. The authors present a heuristic method of solution.

Instead of maximizing expected value of coverage, Chapman and White (1974) formulated a probabilistic version of the LSCP in which the probability that at least one server being available to each demand node was constrained to be greater than or equal to a reliability level a. To compute such a probability, they make use of estimates derived from simulations of the busy fraction q. Again, each server's busy fraction is assumed to be independent of the probability of other servers being busy. Chapman and White's model could not be solved to convergence because busy fractions of individual servers were difficult to estimate. Later, ReVelle and Hogan (1988, 1989a) formulated a new form of Probabilistic LSCP (PLSCP), basically a LSCP with an added constraint on the availability of servers to each demand node, which utilized region-specific (local) estimates of the busy fraction, and binomial distribution. Unfortunately, it is not possible to determine the busy fraction of the servers before knowing the final locations of all of them. To go around this problem, ReVelle and Hogan computed a local estimate of the busy fraction in the neighborhood of demand node i, as the demanded service time in the region, divided by the available service time in the region, that is:

$$q_i = \frac{\bar{t} \sum_{k \in M_i} f_k}{24 \sum_{j \in N_i} x_j} = \frac{r_i}{\sum_{j \in N_i} x_j}$$

where

$\bar{t}$ = average duration of a single call, in hours;

f_k = frequency of calls for service at demand node k, in calls per day;

M_i = set of demand nodes located within S of node i;

$N_i = \{ j \mid t_{ji} \leq S \}$; that is N_i is the set of nodes j located within the time or distance standard of demand node i; and

r_i = utilization ratio.

The probability that at least one server is available within time standard S when node i requests service is 1 minus the probability of all servers within S of node i being busy. Since ReVelle and Hogan assumed the binomial distribution for the probability of one or more servers being busy, this probability is

$$1 - \text{P[all servers of node } i \text{ are busy]} = 1 - \left(\frac{r_i}{\sum_{j \in N_i} x_j} \right)^{\sum_{j \in N_i} x_j}$$

Requiring this probability of at least one server being available to be greater than or equal to a, a nonlinear probabilistic constraint is obtained. This probabilistic constraint does not have an analytical linear deterministic equivalent. However, ReVelle and Hogan found the numerical deterministic equivalent to be

$$\sum_{j \in N_i} x_j \geq b_i \qquad (28)$$

where b_i is the smallest integer which satisfies

$$1-\left(\frac{r_i}{b_i}\right)^{b_i} \geq a \ .$$

The resulting PLSCP has a formulation that is identical to LSCP, with constraint (2) replaced by constraint (28). Marianov and ReVelle (1994), maintaining a neighborhood-specific busy fraction estimate, relaxed the independence assumption in PLSCP, considering each neighborhood as a M/M/s-loss queuing system, and computing the parameter b_i as the smallest integer satisfying

$$\frac{\frac{1}{b_i!} r_i^{b_i}}{1 + r_i + \frac{1}{2!} r_i^2 + \ldots + \frac{1}{b_i!} r_i^{b_i}} \leq 1 - a \ .$$

The probabilistic constraint can be also modified for situations in which full coverage is not mandatory, that is, maximal covering models. In this case, a demand node is counted as covered only if a server is available, within time or distance S, with probability α or more. This was what ReVelle and Hogan (1989b) did with their Maximum Availability Location Problem, MALP. This model sought to maximize the population which had service available within a desired travel time with a stated reliability, given that only p servers are to be located. Using the same reasoning as in PLSCP, ReVelle and Hogan computed the number b_i of servers needed for reliable coverage of node i, and maximized the population in nodes i with b_i or more servers. Their MALP is stated as follows:

$$\text{Maximize} \quad Z = \sum_{i \in I} a_i y_{ib_i} \qquad (29)$$

subject to

$$\sum_{k=1}^{b_i} y_{ik} \leq \sum_{j \in N_i} x_j \qquad \forall i \in I, \qquad (30)$$

$$y_{ik} \le y_{ik-1} \qquad \forall i, k = 2,3,\ldots b_i \qquad (31)$$

$$\sum_{j \in J} x_j = p \qquad (32)$$

$$x_j, y_{ik} = 0,1 \qquad \forall j \in J, i \in I,$$

The variable y_{ik} is one if k servers are potential coverers of node i. The objective maximizes the population in nodes with at least b_i coverers, where this parameter is computed as in ReVelle and Hogan's PLSCP, considering independence between availability of servers. Constraint (30) states that there are at most as many potential coverers as servers sited within the neighborhood of node i. Constraint (31) is an ordering constraint, which forces a node to have k-1 coverers before having k coverers. In the fire protection arena, ReVelle and Marianov (1991), formulated a comparable model, the Probabilistic Facility Location, Equipment Emplacement Technique, PROFLEET. This model considered the deployment of several types of vehicles, simultaneously covering each emergency, as well as the siting of depots or stations. The model considered independence between availabilities of engines and trucks, so Marianov and ReVelle (1992) presented a second version, in which availabilities of engines and trucks were no longer independent. Later, Marianov and ReVelle (1994) formulated a new version of MALP (the QMALP), in which the independence assumption is relaxed through a treatment of each neighborhood as a queuing system, keeping a neighborhood-specific busy fraction.

Meanwhile, Ball and Lin (1993), formulated a new version of the PLSCP, in which a desired level of reliability is mandatory for each demand, condition that is achieved by an upper bound of the "uncoverage probability" of each demand. In their model, Ball and Lin consider the worst case of busy fraction, which occurs when each server is attending all calls from its neighborhood, as if it was alone in the system. In their model, independence is assumed between probabilities of servers being busy.

Models for Fixed Services

Although the LSCP and MCLP models have been used more frequently for locating mobile servers, there are some exceptions. For example, Goodchild and Lee (1989), locate the minimum number of observation points for monitoring an entire geographical region. Meyer and Brill (1988), locate the least number of monitoring wells for detecting contamination in ground water. Bell and Church (1987), locate archeological settlements. Other fixed server application is the location of bus stops that minimize the walking distance for customers (Gleason, 1975). A review of these and other applications can be found in Schilling *et al* (1993). An interesting feature of the LSCP, is that it can be used for solving the p-center problem, which consists on finding the locations of p facilities in such a way as to minimize the maximum distance between a customer and its allocated facility (Daskin, 1995). This problem is adequate for its use in applications in the public sector, because it tends to generate certain equity in the access to facilities by their users.

Besides the applications, it is interesting to give some attention to the variations that have been made to the basic covering models. Set covering models can be formulated for covering arcs, as well as nodes. Also, they can be rewritten for situations in which the demand changes or is uncertain over time. Coverage models can be merged with other models, as Current and Schilling (1989) did with routing, in their covering salesman problem. Coverage, in its usual sense of proximity, can also be reversed, for example when locating obnoxious facilities, which should be located as far as possible from population. Capacity of the facilities is an important issue, and several authors have presented capacitated versions of covering models. Among them, Pirkul and Schilling (1991) propose a Lagrangian Relaxation method for the solution of a maximal covering problem with a capacity constraint.

In some cases, fixed facilities can also suffer from congestion. This is the case of health care services, including hospitals, and, in general, public services of any nature that have fixed offices serving users or customers. The latest developments in capacitated covering models for fixed servers are due to Marianov and Serra (1998 and 2001). In these papers, they develop several probabilistic maximal covering location-allocation models with constrained waiting time for queue length in order to consider service congestion. The first paper addresses the issue of the location of the least number of single-server centers such that all the population is served within a standard distance, and nobody stands in line for a time longer than a given time-limit, or with more than a predetermined number of other clients. They then formulate several maximal coverage models, with one or more servers per service center. In the second paper they address the issue of locating hierarchical facilities in the presence of congestion. Two hierarchical models are presented, where lower level servers attend requests first, and then, some of the served customers are referred to higher level servers. In the first model, the objective minimizes the number of servers and finds their locations so that they will cover a given region with a distance or time standard. The second model is cast as a Maximal Covering Location formulation. In both models they develop a capacity-like constraint to control for congestion in the second level of the hierarchy.

3. P-Median Models in Public Facility Location

The P-Median Problem belongs to a class of formulations called minisum location models. This class of problems was first formulated in its discrete form by Kuehn and Hamburguer (1963), Hakimi (1964), Manne (1964) and Balinski (1965). The problem can be stated as:

> *Find the location of a fixed number of p facilities so as to minimize the weighted average distance of the system*.

The first explicit formulation of the P-Median Problem is attributed to Hakimi (1964). Hakimi not only stated the formulation of the problem but he also proved that in a connected network were the triangle inequality is observed, optimal locations can always be found at the nodes. So it is only necessary to consider as potential locations the nodes of a given network under certain geometric conditions. The model formulated by Hakimi was not applied to a public sector location problem, since was used in the field of telecommunications, more precisely in the location of switching centers on a graph. Four years later, ReVelle and Swain (1970) gave the formulation as an integer linear program and studied its integer properties when solving it with linear programming and branch and bound. As mentioned before, even though the first known proposed application of Hakimi was not for public services and facilities, the P-Median Problem has been since then extensively used as a basis to build problems related to public sector facility location-allocation modeling. A very similar model, the Uncapacitated or Simple Plant Location Problem has been used in private sector location settings[4].

The integer programming formulation of the P-Median Problem is as follows:

$$Min \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} a_i d_{ij} x_{ij} \qquad (33)$$

[4] An excellent presentation of classical Minisum Location Problems can be found in Krarup and Pruzan (1983)

subject to:

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, 2, \cdots, m \qquad (34)$$

$$x_{ij} \le x_{jj} \qquad i = 1,2,\cdots,m \quad j = 1,2,\cdots,n \qquad (35)$$

$$\sum_{j=1}^{n} x_{jj} = p \qquad (36)$$

$$x_{ij} = (0,1) \qquad i = 1,2,\cdots,m \quad j = 1,2,\cdots,n$$

where:

i Index of demand points

m Total number of demand points in the space of interest

j Index of potential facility sites

n Total number of potential facility locations

a_i Weight associated to each demand point.

d_{ij} Distance between demand area *i* and potential facility at *j*.

x_{ij} Variable that is equal to 1 if demand area *i* is assigned to a facility at *j*, and 0 otherwise.

In this formulation it is assumed that all demand points are also potential facility sites (*m*=*n*). The first set of constraints forces each demand point to be assigned to only one facility. The second set of constraints allows demand point *i* to assign to a point *j* only if there is an open facility in this location. Finally, the last constraint sets the number of facilities to be located.

The second set of constraints is known as the “Balinski” constraints, since he was the first to write them in this form in 1965, when studying the Simple Plant Location

Problem. An alternative condensed version of the problem can be formulating by substituting the “Balinski” constraints with the following set:

$$\sum_{j=1}^{n} x_{ij} \leq m x_{jj} \qquad i = 1,2,\cdots,m \qquad (37)$$

This constraint states that no demand node can assign to point j, unless there is a facility open there. While this set of constraints substantially reduces the size of the problem, when solving it using linear programming without any integer requirements will nearly produce all x_{jj} fractional. On the other hand, the “Balinski” set of constraints makes the problem at hand quite large as the number of constraints required together with the number of variables are very large even in relatively small problems. Nevertheless, when solving the P-Median Problem in its extended form using linear programming relaxation, most solutions are integer. ReVelle and Swain (1970) observed that when branch-and-bound was required to resolve fractional variables produced by linear programming, the extent of branching and bounding needed was very small, always less than 6 nodes of a branch-and-bound tree. Therefore, the expanded form of the constraint makes integer solutions far more likely. Morris (1978), solved 600 randomly generated problems of the very similar Simple Plant Location Problem with the extended form of the constraint and found that only 4% did require the use of branch-and-bound to obtain integer solutions. Rosing et al. (1979c) proposed several ways to reduce both the number of variables and constraints in order to make the P-Median Problem more tractable. An extended discussion of “integer friendly” location formulations can be found in ReVelle (1993).

The P-Median Problem, due to its mathematical structure, is NP-hard[5], and therefore cannot be solved in polynomial time. Our experience shows that complete enumeration can be used in a network with up to 50 nodes and 5 facilities in reasonable computer time. Even though the size of the problems that can be solved by using Linear Programming and Branch and Bound (LP+BB), as proposed by

ReVelle and Swain (1970), has been rapidly increasing with the advances in hardware and software technologies and algorithmic sophistication, there is still a strong need for exact and heuristic methods for large and realistic P-Median Problems. Therefore, since its early formulation in 1964, the P-Median Problem has been a fertile ground for innovative approaches and algorithms to obtain solutions. Garfinkel et al. (1974) and Swain (1974) used the Dantzig -Wolfe decomposition to obtain solutions. Another approach, lagrangian relaxation, was used by Gnarl et al. (1977) and Cornuejols et al. (1977) and extended with the use of the linear programming dual by Galvão (1980). See Galvão (1993) for an excellent review on Lagrangian Relaxation applied to uncapacitated facility location problems.

Another class of heuristics, – and most widely used in applications to large problems – are the ones based in interchange methods. Maranzana (1964) presented the first known local search procedure that was extremely fast, but with a weak search strategy. His heuristic begins by finding a feasible solution, that is, locating *p* facilities and then dividing the space into *p* subsets, each one associated with a specific location. Successive relocation within the subset, followed by redivision of the points into clusters, produced stable solutions.

The most widely used heuristic for the problem is the Teitz and Bart (1968). The procedure starts with an initial solution to obtain the initial facility set (for example the Maranzana procedure can be used to obtain the *p* initial locations) and the p-median objective is computed. The second phase of the heuristic seeks the improvement of the initial solution by exchanging members of the facility set for members of the non-facility set. Each exchange is evaluated by computing the new objective value. Trades are only allowed if the objective improves. The heuristic terminates when, after a full cycle of exchanges, no improvement in the objective is found. Rosing et al. (1979a, 1979b) and Cornuejols et al.. (1977), among others, extensively analyzed the performance of the Teitz and Bart heuristic in relatively small networks and obtained

[5] For a discussion of NP hardness, see Krarup and Pruzan (1990)

excellent results. Nevertheless, later on, Rosing (1997) showed that the Teitz and Bart heuristic did not behave in large networks as efficient as in small ones.

Other heuristics with similar search strategy have been proposed by Goodchild and Noronha (1983), with similar results. Whitaker (1983) modified the Teitz and Bart heuristic and developed a greedy stepwise exchange heuristic for which he claims good results. Nevertheless, the heuristic developed cannot be used with different random starts, since they always end in the same solution. That is, either heuristic will produce a single local optimum when applied to any given problem setting (Rosing et al. 1999). Densham and Rushton (1992a, 1992b) developed a very efficient version in computer time of the Teitz and Bart heuristic for very large P-Median Problems, namely, the global/regional interchange algorithm (GRIA). Despite its speed, GRIA was not as good as the Teitz and Bart in finding optimal solutions, since some exchanges are missed (Horn 1996). Rolland et al. (1997) designed a tabu search heuristic to solve the p-median problem that improved both speed and results efficiency over the existing heuristics. In essence, Tabu search is an interchange heuristic that tries to escape from a local optimum, and then continues on towards the global optimum by employing a memory of where it has been already. This memory makes specific, already investigated, interchanges illegal in the hope that a possible short-term degradation of the objective function will lead to an uninvestigated region of the solution space and hence to further improvement of the objective function (Rosing et al. 1998). Full details of this metaheuristic can be found in Glover (1986, 1989, 1990; Glover and Laguna, 1993) and details of its implementation in the p-median context can be found in Rolland et al. (1996). Another very similar Tabu Search approach for Uncapacitated Facility Location Problems (and therefore p-median problems) has been proposed by Al-Sultan and Al-Fawzan (1999).

Another recent heuristic, baptized as Heuristic Concentration, has been developed by Rosing and ReVelle (1997). Basically, this heuristic has two phases. In the first phase, several random trials of an interchange heuristic such as Teitz and Bart are executed. This allows in the second phase the development of a construction set as the union of the sets of facilities (each consisting of p nodes) found in each of the random trials. Then, the best set of facilities is obtained from the concentrated set, by means of LP+BB. In other words, in the second phase the P-Median is solved to optimality using the nodes in the concentrated set as the potential facility locations. Rosing et al. (1998) compared the Heuristic Concentration with the Tabu Search developed by Rosing et al. (1998) and concluded that the first one was superior in finding optimal solutions. On the other hand, it was not clear its efficiency in terms of computer time. Rosing et al. (1999) modified the second phase of the Heuristic Concentration by using a 2-opt algorithm.

Genetic algorithms have also been proposed to solve the problem. A excellent review and a new proposed genetic algorithm can be found in Bozcaya, Zhang and Erkut, 2001.

Since its formulation in the late sixties – early seventies, the p-median problem has been modified to be adapted to specific location problems or to allow a better "real world" implementation in the public sector. Services such as public libraries, schools, pharmacies, primary health care centers have benefited from this model. Nevertheless, in most cases, when implementing the location of such services, it has been necessary to modify the p-median in relation both to its parameters and its basic formulation.

One of the first rigidities of the p-median problem is that it presents a complete inelastic demand with respect to distance. People travel to the closest facility regardless of the distance or time traveled. As early as 1972, Holmes et al. presented a formulation that considered that people would not travel beyond a given a distance or time threshold. In essence the P-Median objective was replaced by the following one:

$$Max \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} a_i (S - d_{ij}) x_{ij} \qquad (38)$$

where S is the threshold distance beyond no one will travel. It is also necessary to re-write constraint number 2 with a " $\leq$ " sign, since not everyone will be assigned to a facility. The model was applied to locate public day care facilities in Columbus, Ohio. In this work, Holmes et al. also introduced the Capacitated P-median problem. In this model, facilities have a limited capacity and therefore the following constraint needs to be added:

$$\sum_{i=1}^{m} a_i x_{ij} \leq C \qquad j = 1, \cdots, n \qquad (6)$$

where C is the maximum capacity level. Computational experience shows that that by adding this constraint the number of fractional variables increases considerably when using linear programming and branch and bound.

Another problem when implementing the p-median problem is related to the distance parameter. The model supposes that distances (or travel times) do not change with time. But, what happens when we want to locate, for example, fire stations in a city? Travel times change during the day and therefore an optimal location during traffic peak hours may be very deficient in valley hours. On the other hand, the demand may also change during the day. CBD areas may be crowded during daytime while residential areas are empty, and vice-versa during night time. Serra and Marianov (1998) introduced the concept of regret and minmax objectives when locating fire stations in Barcelona (Spain) taking into account what they called "changing networks". Basically, uncertainty was treated using the classic scenario approach, in which different patterns of demand or travel times are realized in different scenarios. Frist, over a range of possible demand scenarios, facilities are deployed to site in such a way to minimize the maximum average travel time in a given scenario (minmax approach). Second, over that same range of scenarios, facilities are positioned in such a way as to minimize the maximum regret. Regret is defined as the difference between (1) the optimal average travel time that would be obtained had the decision maker planned its sites for the scenario that actually occurred; and (2) the value of average travel time that was actually obtained (regret approach).

The minmax p-median problem formulation is as follows:

$$\text{Min } M \qquad (39)$$

s.t.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{a_{ik}d_{ij}^{k}}{W_k}x_{ij}^{k} \le M \qquad k = 1,2,\cdots K \qquad (40)$$

$$\sum_{j=1}^{n} x_{ij}^{k} = 1 \qquad i = 1,\ 2,\cdots,m \quad k = 1,2,\cdots K \qquad (41)$$

$$x_{ij}^{k} \le w_j \qquad i = 1,2,\cdots,m \quad j = 1,2,\cdots,n \quad k = 1,2,\cdots K \qquad (42)$$

$$\sum_{j=1}^{n} w_j = p \qquad (43)$$

$$x_{ij}, w_j = (0,1) \qquad i = 1,2,\cdots,m \quad j = 1,2,\cdots,n$$

where additional notation is defined as follows: k and K is the index and number of scenarios respectively, a_{ik} is the population at node i in scenario k, W_k the total population in scenario k and d_{ij}^{k} the travel time between i and j in scenario k. Variables of the model are:

$$x_{ij}^{k} = \begin{cases} 1 & \text{if node } i \text{ is assigned to node } j \text{ in scenario } k, \\ 0 & \text{otherwise} \end{cases}$$

$$w_j = \begin{cases} 1 & \text{if there is a facility at } j, \\ 0 & \text{otherwise} \end{cases}$$

The first constraint is directly related to the objective. Since the maximum average travel time across scenarios is to be maximized, we want to find a set of locations that will give the smallest maximum average travel time possible when evaluated for all scenarios. The left side of each constraint (one for each scenario) represents the demand weighted average travel time that will be achieved in the corresponding scenario. The right-hand side, M, is the same in each constraint. The objective of the model is to minimize M. That is, the model will try to find a set of locations that minimizes the largest total travel time achieved in each scenario. The rest of the constraints are very similar to the constraint set of the p-median problem.

If the regret objective is used, constraint set (40) is replaced by the following:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{a_{ik}d_{ij}^{k}}{W_k}x_{ij}^{k} - Z_k \leq M \qquad k = 1,2,\cdots K \qquad (44)$$

where Z_k is the optimal objective, a known value, found when p facilities are located optimally in each scenario. Its value is found by applying the original p-median formulation to each scenario individually. The unknown variable M represents the largest regret evaluated over all scenarios.

The authors developed a one-opt exchange heuristic to find solutions to both formulations. The regret model was used to locate fire stations in the city of Barcelona.

Another problem related to the p-median is data aggregation. When selecting locations for facilities, the p-median takes (as most location models do) into account the demand for the service provided by the facility. When implementing the discrete problem in a real world setting, it is necessary to identify the demand areas that will be modeled as “nodes” or points”. Therefore, some spatial aggregation of the demand is performed. This is especially true when locating facilities in urban areas. In general, census tracks are aggregated to form demand areas. In the location of fire stations above mentioned, Barcelona has around 1800 census tracks. An aggregation was performed to reduce the problem to 200 demand areas. This aggregation leads to three types of source errors (Hillsman and Rhoda, 1978). Source A errors are a direct result of the loss of locational information. When performing the aggregation, the distance or travel time is modified since it is considered only from the centroid of the area to the potential facility site. Therefore, an over or under estimation of this parameter may occur. Source B errors are a special case of source A errors. If a facility is positioned in a given aggregated demand area, the corresponding weight in the objective function is set to zero since the corresponding distance is set to 0. But in reality it should not be 0, since at the desaggregated level some areas will have to travel to the closest facility. Source B errors always yield a measured weighted travel distance less than the true weighted travel distance. Finally, type C source errors appear when part of an aggregated area is not assigned to its closest facility. Several methods have been proposed to reduce or eliminate these errors (Hillsman and Rhoda 1978; Goodchild 1979; Bach 1981; Current and Schilling 1987; Bowerman et al. 1997 among others). For an excellent overview and new methods of reduction see Erkut and Bozkaya (1999). In this reference, demand point aggregation is examined in detail for the planar p-median problem.

When planning public facilities it may be necessary not only to obtain a good location, but to achieve also a balanced demand assignment level. Sometimes, in order to be efficient, facilities need to have a minimum demand threshold level. An area of application where the concept of threshold is relevant involves to the provision of services that are considered merit goods, but that are services by the private sector. This is specially relevant for merit services that have been publicly owned or controlled in several countries and are being transferred to the private sector, such as postal services, gas stations, fire departments and pharmacies. While the planner seeks to maintain good service quality by keeping a balanced spatial distribution of services, these need to have a minimum service threshold level that will allow them to survive. In the p-median formulation, this is achieved by adding to the original problem the following constraint set:

$$\sum_{i=1}^{m} a_i x_{ij} \geq C \qquad j = 1, \cdots, n \qquad (45)$$

Carreras and Serra (1999) used this formulation to examine the impact of the spatial deregulation of pharmacies in a region of Spain. They showed that by de-regulating the sector, the number of pharmacies would increase by at least 20%.

Sometimes, the location of new facilities is conditioned by the existence of districts in the region of interest. Demand areas within a given district can be assigned only to a facility within the same district. This problem would arise in the allocation of schools in a county, in voting-machine siting among voting districts, or large-scale facility-siting studies for regions encompassing many counties or states. The problem has two aspects. First, to decide how many facilities are assigned to each district. Secondly, where to locate these facilities. ReVelle and Elzinga (1989) developed an algorithm that solved optimally this problem.

The p-median model assumes that facilities are alike or of a single type. Nevertheless, it is widely accepted that many facility systems and institutions are hierarchical in nature, providing several levels of service. More specifically, a hierarchical system is one in which services are organized in a series of levels that are somehow related to one another in the complexity of function/service.

The organizational structure of hierarchical systems may vary considerably. There may be institutional ties between levels, whereby lower levels are administratively subordinate to higher ones (e.g., health care delivery systems, banking systems). On the other hand, there are several hierarchical systems that have no such inter-level linkages, different levels being distinguished solely by the range of goods and/or services they provide (e.g., educational systems, production-distribution systems, waste collection systems) (Hodgson 1986).

The p-median model locates p facilities such that the average distance from the users to their closest facility is minimized. In a hierarchical setting it has been generally used to locate a given number of facilities for each level, one at a time. Several hierarchical models based on the p-median have been formulated.

Calvo and Marks (1973) constructed a multiobjective integer linear model to locate multi-level health care facilities: the model minimized distance (travel time), user costs, and maximized demand or utilization, and utility. It was based on assumptions that (1) users go to the closest appropriate level; (2) there is no referral to higher levels; and (3) all facilities offer lower level services.

Tien et al. (1983) argued that the approach taken by Calvo and Marks resulted in deficient organization across hierarchies. In order to resolve this deficiency, they presented models derived from Calvo and Mark's formulation: nested and non-nested models. They also introduce a new feature whereby a demand cannot assign to a place more than once even if additional service levels may available at that point. Both models, unlike Calvo and Marks', can be solved by standard integer programming solution procedures. Mirchandani (1987) extended the hierarchical p-median formulation of Tien et al. model, allowing various allocation schemes by redefining the cost parameter in the objective.

Harvey et al. (1974) used a p-median formulation to determine the number and optimal locations of intermediate level facilities in a central place hierarchy. The p-median model was used in one-level problem, but consideration was given on the interaction among lower and higher levels.

Narula et al. (1975) developed a nested hierarchical health care facility location model that located on a network a fixed number of facilities. At each level, the objective was to minimize patients' total travel. They considered referrals between levels, based on the proportion of patients treated at each level. Narula and Ogbu (1979) gave some heuristic procedures for the solution of the problem. Later, Narula and Ogbu (1985) solved a two-level mixed-integer p-median problem using the same objective and referral pattern.

Berlin et al. (1976) studied two hospital and ambulance location problems. The first one focused on patient needs by minimizing (1) average ambulance response time from ambulance bases to demand areas and (2) average distance to hospitals from demand areas. The second model added a new objective to take into account the efficiency of the system: minimization of (3) distance from ambulance bases to hospitals. It was named the "dual-facility" location problem: the locations of both hospitals and ambulance depots were basic to determine response times. It is interesting to note that although two levels are defined (stations or depots where ambulances sit, and hospitals), the formulation can be decomposed into independent hospital and ambulance location problems and solved optimally. It is not a clear hierarchical model since relations among levels differ from the traditional regionalized models.

Fisher and Rushton (1979) and Rushton (1984) used the average and maximum distance from any demand area to its closest health care center to study and compare actual and optimal hierarchical location patterns in India. The Teitz and Bart heuristic was used in three ways to determine hierarchies: constructing top-down hierarchical procedure (same as Banergi and Fisher 1974); constructing a bottom-up hierarchical procedure (opposite of top-down); and constructing a hierarchical procedure where the first step was to locate a middle-level of the hierarchy optimally, and then proceed as the bottom heuristic for upper levels, and use the top-down heuristic for lower levels.

Tien and El-Tell (1984) defined a two-level hierarchical LP model consisting of village and regional clinics. It is a top-down formulation in the sense that the flow patterns start at the hospitals. That is, health professionals go from hospitals to village centers. Both village and regional clinics are located using a criterion of minimizing the weighted distance of assigning villages to clinics and village clinics to regional clinics. The model was applied to 31 villages in Jordan.

Hodgson (1984) demonstrated that the use of top-down or bottom-up techniques to locate hierarchical systems generally leads to suboptimal locational patterns. By a top-down (bottom-up) technique is meant the location first of the highest (lowest) level of the hierarchy and then successive location of facilities in the following level.

Hodgson used both the p-median model and a formulation based on Reilly's gravitational law (Reilly, 1929) to compare both techniques with the simultaneous location of all hierarchies.

As mentioned before, sometimes it is necessary to obtain not only good locations but also an efficient district for each facility. Serra and ReVelle (1993) introduced the concept of coherence in hierarchical models. A coherence in a hierarchical system is defined as follows: all areas assigned to a particular facility at one hierarchical level should belong to one and the same district in the next level of the hierarchy. The authors developed the pq-median model. This formulation locates two types of facilities by combining two p-median formulations. Each level has the objective of minimizing the average distance or travel time from the demand areas to the nearest facility whilst ensuring coherence. Hence, a trade off between access to each hierarchical level is expected. The model was used to design the location and districting hierarchical primary health care services in Barcelona (Serra 1996).

The implementation of location-allocation problems in third world areas may present different problems that the ones implemented in developed countries. Perhaps the most notorious problem involves data limitations. An excellent review of application of p-median and covering problems in the real word can be found in Oppong (1996). The work by Oppong also examines the location of hierarchical Primary Health Care Centers in Suhum District, Ghana. This region is affected by strong climate differences during the year. There is a strong seasonal variation in road surface conditions. Therefore, this problem is similar to some extent to the one of locating fire stations in a city: there are different networks according to the season of the year. He developed a decision support tool to improve the solutions given by the p-median formulations.

1.4. Conclusions

The development of models and methodological frameworks to design or reconfigure emergency and non-emergency systems has taken place over the span of a quarter century. It has developed alongside and in concert with the evolution of the modern computer as it transited from a room-filling behemoth to a desk-top associate. And like the computer on which the models must rely, the models and methods are not done evolving. The shape of the next models can be predicted by simply observing how the current generation still falls short of perfectly describing reality. We will focus on live areas.

First, we should begin to see a new generation of models that deal with the issue of co-location of servers from different emergency systems. ReVelle and Snyder (forthcoming) introduce this line of research in the FAST (fire and ambulance siting technique) model that examines the link between ambulance and fire company siting. In the United States, ambulance deployment has traditionally taken place either at hospitals or at fire stations or both, but rarely have ambulances been positioned at free standing ambulance stations. ReVelle and Snyder, in a deterministic covering model, examine the consequences of allowing the ambulances to be sited free of constraints on the location of other services. These models should eventually develop all the probabilistic sophistication and nuances of the models discussed above.

Second, we can expect that the estimate of server or region-specific busy fraction will be refined. Although we have moved from deterministic to redundant to probabilistic models, and although within this last category we have moved from a system-wide busy fraction to a region-specific busy fraction using queuing concepts, we still have not precisely matched the busy fractions estimated by simulation. Unless the challenge exceeds the imaginative powers of investigators, we will soon see server-specific busy fractions or more refined region-specific busy fractions.

Third, we should see focus developing on workload issues, a topic that has largely been ignored till now and one that greatly concerns emergency and non emergency system planners. The issues of busy fraction, workload and threshold levels are tightly connected so progress on the former should bring achievement on the latter as well.

Fourth, we should see a gradual melding of the two lines of evolution, queuing and location. It is hard to predict how this will take place, but certainly the use of heuristics offer the descriptive queuing models an opportunity to compete with the location models in the arena of design. And the introduction of queuing concepts by Marianov and ReVelle, as well as Batta, into location models, suggest movement from the other side as well.

Fifth, most of the models we have examined consider that customers always patronize the closest facility. That is, distance (or travel time) is the only parameter considered by customers. But there may be other decision parameters related to service quality such as service speed, cleanness or efficiency that may influence customers' decisions. New models have been developed to examine this issue (Serra et al. 1999, Colome and Serra 1999).

Sixth, the evolution of public sector deregulation is gradually introducing competition between providers. For example, the deregulation of a health care system may introduce some level of competition among providers at the primary (and secondary) level to attract patients. Therefore, the dichotomy between public and private location modeling is being diffused and Location Capture Models can be adapted to accommodate public sector issues.

Last but not least, this paper has not addressed the development of public sector location-allocation models that consider the siting of undesirable facilities. There is a considerable volume of literature on this topic. A good starting point to interested readers can be found in Murray et al. (1998).

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