Inflation and output dynamics with firm-owned capital.*

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Abstract

We model firm-owned capital in a stochastic dynamic New-Keynesian general equilibrium model à la Calvo. We find that this structure implies equilibrium dynamics which are quantitatively different from the ones associated with a benchmark case where households accumulate capital and rent it to firms. Our findings therefore stress the importance of modeling an investment decision at the firm level – in addition to a meaningful price setting decision. Along the way we argue that the problem of modeling firm-owned capital with Calvo price-setting has not been solved in a correct way in the previous literature.

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1 Introduction

By now there exists a large literature studying inflation and output dynamics in general equilibrium models with sticky prices. These models give important insights about macroeconomic dynamics which are in particular relevant for the conduct of monetary policy. However, it is mostly assumed that labour is the only productive input[1] or alternatively that the aggregate capital stock in the economy is held constant.[2] Woodford (2003) comments on these modeling choices: '[...] while this has kept our analysis of the effects of interest rates on aggregate demand quite simple, one may doubt the accuracy of the conclusions obtained, given the obvious importance of variations in investment spending both in business fluctuations and in the transmission mechanism for monetary policy in particular'.

One way of modeling endogenous capital accumulation in a sticky price model is to use a standard neoclassical assumption, namely that households own the capital stock and rent it to firms. This strategy has been followed by some authors.[3] However, it is unclear *a priori* if this modeling choice is appealing in a New-Keynesian model with staggered price setting. The reason is given by Woodford (2003). He argues that with staggered price setting there exists at any point in time a non-degenerate distribution of prices among the firms in the economy. Opening a rental market for capital could then imply that a substantial part of the aggregate capital stock shifts each period from low demand to high demand producers. This is unrealistic and more importantly it has potentially non-trivial implications for the determination of marginal costs at the firm level, hence for price setting decisions and for inflation dynamics.

Another way of modeling endogenous capital accumulation is to assume that firms own the capital stock. This implies that firms have both a meaningful price setting decision – an appealing feature that has been often stressed in the literature on New-Keynesian models with staggered price setting – and, in addition, that they

[1] See, e.g., Clarida et al. (1999) and Woodford (2003) among others.

[2] Erceg et al. (2000) assume a constant aggregate capital stock together with a rental market for capital in a model where sticky wages are modeled in addition to sticky prices.

[3] See, e.g., Basu (2003), Galí et al. (2003), and Yun (1996).

also have an investment decision. We emphasize the following aspect of the difference between firm-owned and household-owned capital: Firm-owned capital together with the standard assumption that the additional capital resulting from an investment decision becomes only productive with a one period delay implies that a firm's capital stock is a predetermined variable. Hence, a firm cannot choose its current capital stock contingent upon its demand. This way the potentially problematic feature associated with the modeling choice of a rental market for capital is avoided.

In this paper we assess to what extent the modeling of firm-owned capital leads to quantitatively important differences for the implied equilibrium dynamics in a sticky price model à la Calvo (1983) with respect to a benchmark case were we assume a rental market for capital instead. Moreover, we discuss some difficulties associated with the modeling choice of firm-owned capital in a model with staggered price setting.[4] We show that the dynamic structure of the problem is much more complicated than has been considered in the literature. In a nutshell both Woodford (2003) and Casares (2002) do not assess in a correct way over what set of future states of the world an optimizing Calvo price setter will form its expectations in the presence of an investment decision at the firm level.

The paper is organized as follows: Section 2 outlines the model economy. The main novelty associated with having firm-owned capital is that price setters face a simultaneous choice problem: on the one hand, optimal price setting depends on the expected investment policy over the (random) lifetime of the chosen price. The reason is that the resulting capital holdings affect the firm's marginal costs. On the other hand, a firm's expected future price setting decisions are among the determinants of its expected returns to capital and hence are relevant for investment decision making. Section 2 has the following structure: First, we consider households and firms and derive their respective first order conditions. Then we consider the implied linearized equilibrium conditions. In particular, we derive an inflation equation from averaging and aggregating optimizing price setting decisions. At this step we argue why some of the expectations entering a firm's price setting decision

[4]For a sticky price model with firm-owned capital in the more tractable case of Rotemberg pricing, see Dupor (2002).

have been computed in an incorrect way in the previous literature. Finally, we briefly outline the rental market benchmark case. In section 3 we conduct a simulation exercise. We study impulse response functions associated with a shock to the growth rate of money balances both for a model with firm-owned capital and for the benchmark case. We show that the differences in implied equilibrium dynamics are quantitatively important and we suggest a metric which gives a precise meaning to that statement. Among other things we find that the inflation response is smaller in the case of firm-owned capital. The intuition is plain from a comparison between the price setters in the two models. With a rental market each firm produces at the same marginal cost which is independent of the quantity produced by any individual firm. Hence, a firm's price setting decision does not affect its schedule of marginal costs. This is different, however, in our model with firm-owned capital. Since the capital stock at the firm level is predetermined an increase in demand is associated with a higher marginal cost due to a decrease in the marginal product of labour (which is implied by the standard neoclassical production function that we are going to assume). Hence, optimizing price setters adjust their prices by less in response to any given change in the schedule of average marginal costs they face. This mechanism drives the difference since, as we will see, up to the first order approximation to the equilibrium dynamics that we are going to consider, the respective demand blocks in our model and in the benchmark case are identical. Section 4 concludes.

2 The model economy

2.1 Households

A representative household maximizes expected discounted utility:

$$E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}, N_{t+k,}(M_{t+k}/P_{t+k})\right), \tag{1}$$

where β is the household's discount factor, C_t is a Dixit-Stiglitz composite consumption index, N_t are hours worked, P_t denotes the price level, and M_t/P_t are real balances. We assume the following: the period utility function is separable in its three arguments, households have access to a complete set of contingent claims and the labour market is perfectly competitive. In Appendix 1 we show that for the utility function we consider this set of assumptions implies the following first order conditions:

$$C_t^{\sigma} N_t^{\phi} = \frac{W_t}{P_t}, \tag{2}$$

$$M_t/P_t = (1 - R_t^{-1})^{-\frac{1}{\nu}} C_t^{\frac{\sigma}{\nu}}, \tag{3}$$

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1}, \tag{4}$$

where W_t is the nominal wage, R_t denotes the gross nominal interest rate, $Q_{t,t+1}$ is the stochastic discount factor for nominal profits, and the price of a risk-less one-period bond is given by $R_t^{-1} = E_t Q_{t,t+1}$. The parameters σ, ϕ, and ν are all positive. σ is the household's relative risk aversion or equivalently the inverse of the household's intertemporal elasticity of substitution. ϕ can be interpreted as the inverse of the Frisch aggregate labour supply elasticity. The inverse of ν is proportional to the semi-elasticity of the household's demand for real balances with respect to the nominal interest rate.[5]

The first and the second equations are the static optimality conditions for labour supply and demand for real balances. The third equation is a standard intertemporal optimality condition.

2.2 Firms

There is a continuum of firms indexed on the unit interval. Firms set prices and make investment decisions with the objective of maximizing the values of their dividend streams. Each firm $i \in [0,1]$ is assumed to have the following Cobb-Douglas production function:

$$Y_t(i) = Z N_t(i)^{1-\alpha} K_t(i)^{\alpha}, \tag{5}$$

where $\alpha \in [0,1]$ is a constant. $K_t(i)$ denotes firm i's capital holdings in period t, and $N_t(i)$ denotes the amount of labour used by that firm in its time t production

[5] The factor of proportionality is a function of the steady state real interest rate.

of output which we denote by $Y_t(i)$. Z is a technology variable which we assume to be constant and common to all firms.

Firms are assumed to act under monopolistic competition.[6] The feature of sticky prices is introduced into the model by invoking the Calvo assumption. This way we capture the fact that firms change prices only infrequently.

Moreover, each firm makes an investment decision at each point in time, with the resulting additional capital becoming productive one period after the investment decision is made. The law of motion of capital at the firm level is given by the following equation:

$$K_{t+1}(i) = (1 - \delta) K_t(i) + I_t(i), \tag{6}$$

where $I_t(i)$ denotes the amount of the composite good purchased in period t by firm i. We don't assume any adjustment cost or other features that would render the investment process more realistic. There are two reasons for this: First, as we have already mentioned, we want to assess to what extent the following aspect of our model is quantitatively important: if capital is owned by firms then price setters take into account the effect of that decision on the current and future expected marginal cost they face. This is a consequence of the decreasing marginal product of labour and the assumption that firms can adjust their capital holdings only with a one period delay. If we were to assume a capital adjustment cost then this effect would become even more important since a less flexible capital stock at the firm level would mean that the price setting decision would have a more important effect on a firm's marginal cost. Second, as we will analyze in section 2.5, the problem of modeling an investment decision at the firm level in a New-Keynesinan model with Calvo price setting has not been solved in a theoretically correct way in the previous literature. Our solution strategy can be outlined most easily in the simple case considered in this paper but, in principle, it can also be used to analyze more realistic cases.

It is natural to consider next how aggregate demand is determined in our model

[6] Monopolistic competition rationalizes the assumption that a firm is willing to satisfy unexpected increases in demand even when a constraint not to change its price is binding. See, e.g., Erceg et al. (2000).

since it affects each firm's demand and therefore the price setting decisions of price setters. Let's start by considering the aggregate capital stock at time t which is given by:

$$K_t = \int_0^1 K_t(i)\, di. \tag{7}$$

As in Woodford (2003) we assume that the Dixit-Stiglitz aggregate for investment purchases has the same constant elasticity of substitution as for consumption purchases. This implies that firms buy the different capital goods in the same proportion as in the consumer aggregate, and therefore total demand is given by:

$$Y_t^d = C_t + K_{t+1} - (1-\delta) K_t. \tag{8}$$

The structure outlined so far implies a joint optimization problem at the firm level: on the one hand, a firm's investment decision takes rationally into account that there might be a chance to choose an optimal relative price in the next period when the resulting additional capital will become productive. On the other hand, a firm's expected capital holdings over the (random) lifetime of a chosen price are among the determinants of the relevant marginal costs affecting the price setting decision. Hence, an optimizing firm forms expectations over the associated relevant investment decisions. As we will see, computing these expectations is an intricate problem and therefore we find is useful to proceed step by step. We start by considering separately a firm's price setting- and investment decision.

2.2.1 Price setting

Each period a measure $1-\theta$ of randomly selected firms change their prices and the remaining firms post their last period's nominal prices. A price setting firm maximizes the current value of its dividend stream taking into account the probability that the chosen price might affect future profits. In Appendix 2 we show that the first order condition associated with the price setting decision is given by:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k}(f) \left[P_t^*(f) - \mu MC_{t+k}(f) \right] \right\} = 0, \tag{9}$$

where $P_t^*(f)$ denotes the newly set nominal price, $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the frictionless markup, and $MC(f)$ denotes the nominal marginal cost of firm f. The latter is given by:

$$MC_t(f) = \frac{W_t}{MPL_t(f)},$$

where $MPL_t(f)$ denotes firm f's marginal product of labour at time t.

Equation (9) takes the form of the standard first order condition for price setting in the Calvo model: When setting a new price a firm takes into account that its choice might affect not only current but also future profits. A price is therefore chosen in such a way that over its expected lifetime a weighted average of current and future expected marginal profits is equalized to zero. However, since a firm's capital stock is among the determinants of its marginal product of labour, we cannot solve the price setting problem unless we consider the firm's investment behavior. We turn to this next.

2.2.2 Investment behavior

In Appendix 2 we show that the first order condition associated with the choice by firm f at time t over its period $t+1$ capital stock is given by:[7]

$$P_t = E_t \left\{ Q_{t,t+1} \left[MS_{t+1}(f) + (1-\delta) P_{t+1} \right] \right\}, \tag{10}$$

where δ is the depreciation rate, and $MS_{t+1}(f)$ denotes the nominal marginal savings in firm f's labour cost at time $t+1$ associated with having one additional unit of capital in place. The latter is given by:

$$MS_{t+1}(f) = W_{t+1} \frac{MPK_{t+1}(f)}{MPL_{t+1}(f)},$$

where $MPK_{t+1}(f)$ denotes the marginal product of capital of firm f at time $t+1$.

Equation (10) takes a standard form: The price P_t of an additional unit of investment at time t is equalized to the expected discounted marginal contribution to the firm's value associated with having one additional unit of capital in place at point in time $t+1$. The latter is given by the nominal marginal return from using that additional unit in production and selling the remaining capital (after depreciation).

[7] The first order condition is the one that can be found in Woodford (2003) particularized for the special case of no adjustment cost in the firm's process of capital accumulation.

As equation (10) shows, the relevant measure of the nominal marginal return to capital is not the marginal revenue product of capital but the marginal savings in a firm's labour cost: firms are demand constrained in our model and hence the return from having an additional unit of capital in place comes from the fact that this allows to produce the quantity that happens to be demanded using less labour. Since a firm's demand depends on its relative price an optimizing investor f when forming expectations over $MS_{t+1}(f)$ has to take into account that its relative price might be readjusted in period $t+1$.

2.3 Some Linearized Equilibrium Conditions

Before we proceed with the full characterization of the equilibrium associated with our model we find it useful to collect some results which can already be obtained from what we have developed so far. We consider a log-linear approximation to the equilibrium dynamics around a zero inflation steady state. Throughout we denote by a hat on a variable the percent deviation of that variable with respect to its steady state value.

Taking conditional expectations on both sides of (4) and log-linearizing gives the household's Euler equation:

$$E_t\widehat{C}_{t+1} = \widehat{C}_t + \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - \rho\right), \tag{11}$$

where i_t denotes the nominal interest rate at time t, and $\rho \equiv -\log\beta$ is the time discount rate. The last equation reflects the household's incentive to smooth consumption.

Log-linearizing the first order condition for investment (10) and averaging over all firms in the economy, invoking the Euler equation and using the fact that the log deviation of the average relative price is zero at any point in time,[8] we obtain the law of motion of the aggregate capital stock:

$$[1-\beta(1-\delta)]E_t\widehat{ms}_{t+1} = \left(i_t - E_t\pi_{t+1} - \rho\right), \tag{12}$$

[8] The latter holds up to the first order of approximation which we consider here.

where ms_{t+1} denotes the average real marginal savings in labour costs at time $t+1$. The latter is given by:

$$ms_{t+1} = \frac{W_{t+1}}{P_{t+1}} \frac{MPK_{t+1}}{MPL_{t+1}},$$

where MPL_{t+1} and MPK_{t+1} denote the average marginal product of labour and capital, respectively, i.e. evaluated at the average capital stock and at the average price at time $t+1$.

Equations (11) and (12) constitute the aggregate demand block of our model. In order to find the aggregate supply equation we have to consider the simultaneous price setting and investment problem at the firm level, which we analyze in the next paragraph. Finally, if we specify how monetary policy is conducted then we can solve for a first order approximation to the equilibrium dynamics implied by our model.

2.4 Linearized Price Setting

Let's go back to the firm's price setting problem. We assume that the government pays a subsidy to each price setter in order to offset the distortions implied by market power. The subsidy is financed by a lump sum tax. Our goal is to derive a linearized inflation equation from averaging and aggregating optimal price setting decisions. A natural starting point is hence to consider the linearized marginal cost at the firm level. Denoting $mc_t(f)$ the real marginal cost of firm f at time t and linearizing gives:

$$\widehat{mc}_t(f) = \widehat{mc}_t - \frac{\varepsilon\alpha}{1-\alpha}\widehat{p}_t(f) - \frac{\alpha}{1-\alpha}\widetilde{K}_t(f), \tag{13}$$

where $p_t(f) \equiv \frac{P_t(f)}{P_t}$ is firm f's relative price, and $\widetilde{K}_t(f) \equiv \widehat{K}_t(f) - \widehat{K}_t$ denotes firm f's linearized relative to average capital holdings, or capital gap, at time t.

The intuition behind equation (13) is the following: For a given capital stock firms that post a higher than average price face a lower than average marginal cost due to the decreasing marginal product of labour. This is reflected in the second term, and it is exactly as in Galí et al. (2001) for a model with decreasing returns to scale without endogenous capital formation. With firm-owned capital there is an extra effect coming from the firm's capital stock which corresponds to the last part. For a given price a firm that has a higher than average capital stock in place faces a lower than average marginal cost. The reason is that the marginal product of labour increases with the capital stock used by the firm.

Using equations (9) and (13) the linearized optimal relative price of firm f can be written as:

$$\widehat{p}_t^*(f) = \sum_{k=1}^{\infty} (\beta\theta)^k E_t \pi_{t+k} + \xi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widehat{mc}_{t+k} - \psi \sum_{k=0}^{\infty} (\beta\theta)^k E_t \widetilde{K}_{t+k}(f), \quad (14)$$

where $\xi \equiv \frac{(1-\beta\theta)(1-\alpha)}{1-\alpha+\varepsilon\alpha}$, and $\psi \equiv \frac{(1-\beta\theta)\alpha}{1-\alpha+\varepsilon\alpha}$. Moreover, $p_t^*(f) \equiv \frac{P_t^*(f)}{P_t}$ is the firm's optimally chosen relative price.[9]

The Calvo assumption implies that optimizing price setters behave in a forward looking manner. In addition to the usual inflation and marginal cost terms a firm's optimal price setting decision does also depend on the current and future expected capital gaps over the (random) lifetime of the chosen price. It is important to emphasize that the relevant capital gaps entering equation (14) are associated with exactly those states of the world where the newly set price is still posted. As we will outline next, the problem of computing these expectations has not been solved in a correct way in the previous literature.

2.5 A short note on the previous literature

Woodford (2003) emphasizes the simultaneous nature of the firm's investment and price setting problems in a Calvo model with firm-owned capital. He notes: 'The capital stock affects a firm's marginal cost, of course, but more subtly, a firm considering how its future profits will be affected by the price it sets must also consider how its capital stock will evolve over the time that its price remains fixed.' Hence, an optimizing price setter has to form expectations over its investment policy as far as this policy affects capital holdings at future points in time and in states of the world where the newly set price is still in place.

[9] The price setting problem is stated in terms of variables that are constant in the steady state.

The way this problem has been analyzed in the literature is not correct.[10] The problem lies in the assessment of what the Calvo assumption implies for evaluating future expected capital holdings as far as they are relevant for price setting decision making. In Woodford (2003) the schedule of linearized expected future relative prices associated with the current price setting decision is used in order to pin down the relevant future expected capital holdings. Woodford (2003) notes that each such expectation can be written as a linear function of the linearized newly set relative price and future expected inflation terms. Specifically, $E_t\{\hat{p}_{t+k}(i)\} = \hat{p}_t^*(i) - \sum_{i=1}^{k} E_t\{\pi_{t+i}\}$, where π_t denotes time t inflation, $p_t^*(i)$ is the optimal relative price chosen by firm i at point in time t, $p_{t+k}(i)$ is the relative price of that firm at point in time $t+k$, and the expectations are taken over the states of the world where $p_t^*(i)$ is still in place. However, this is not consistent with the firm's objective of maximizing the current value of its stream of dividends.[11] The reason why can be seen by looking at the first period: When a time t price setter forms expectations over its marginal cost over those states of the world at time $t+1$ where the newly set price is still posted the firm will *rationally* consider that the capital stock chosen in period t will be in place in period $t+1$ for sure. *In particular*, this capital stock will be in place in those states of the world at time $t+1$ where the newly set price is still in place. But the capital stock for period $t+1$ is *optimally* chosen at time t. Since this is so the investment decision at time t over the capital stock in period $t+1$ will take into account that the firm might be allowed to choose a new price at point in time $t+1$.

In other words, the dynamic structure of the problem is much more complicated than has been analyzed in the previous literature. As we will see, an optimizing Calvo price setter has to consider *all* future states of the world that are consistent with the one that is realized when the price setting decision is made – and not only a subset of them. We turn to this next.

[10] In what follows we discuss some difficulties in Woodford (2003)'s model but mutatis mutandis our argument would also apply to Casares (2002). The reason why we focus on Woodford (2003) is that the structure of his model is more closely related to ours.

[11] The following problem arises even in the special case of Woodford (2003)'s model where adjustment costs in the process of a firm's capital accumulation are zero.

2.6 Inflation dynamics

In order to pin down the relevant capital gaps for price setting decision making we start by invoking the first-order condition for the investment decision given in equation (10). Linearizing the latter equation and combining it with the linearized law of motion for the aggregate capital stock in equation (12) gives:

$$\begin{aligned}\widetilde{K}_{t+1}(f) &= -\varepsilon E_t \widehat{p}_{t+1}(f) \\ &= -\varepsilon \left[\theta \left(\widehat{p}_t^*(f) - E_t \pi_{t+1}\right) + (1-\theta) E_t \widehat{p}_{t+1}^*(f)\right]. \qquad (15)\end{aligned}$$

With probability θ a time t price setter's linearized relative price at point in time $t+1$ will be given by $\widehat{p}_t^*(f) - E_t \pi_{t+1}$. This corresponds to the case where the time t price setter will be restricted to readjust its price at time $t+1$. With probability $1-\theta$ the time t price setter will be allowed to choose a new price at point in time $t+1$. As can be seen from equation (15) the *possibility* of posting a new price at point in time $t+1$ affects a price setter's time t investment decision and hence its time $t+1$ capital gap – *in particular*, in those states of the world where the price chosen at time t is still posted at point in time $t+1$.

In the same way each investment decision depends on the expected relative price that obtains one period after the decision when the resulting additional capital becomes productive. Hence, the relevant future expected capital gaps[12] affecting the price setting decision of firm f at time t are given by:

$$E_t \widetilde{K}_{t+k}(f) = -\varepsilon \left[\theta \left(\widehat{p}_t^*(f) - E_t \widehat{\Pi}_{t,t+k}\right) + (1-\theta) E_t \widehat{p}_{t+k}^*(f)\right], \qquad (16)$$

where $\Pi_{t,t+k}$ denotes gross inflation between time t and time $t+k$.

From the last equation it is plain why the firm's problem is intricate: each price setting decision depends, via the capital gap terms, on expectations over infinitely many future optimal price setting decisions. Moreover, each future optimal price setting decision depends on future expected capital gaps in the same way as this is the case for the time t price setting decision. However, the relevant schedules of future expected capital gaps are different for each optimally chosen price since they

[12] The current capital gap, which is also relevant for a price setting decision, is predetermined.

have to be computed for different states of the world. This can be illustrated by a simple example. If a time t price setter has a chance to post an optimal relative price at time $t+1$ then the time $t+1$ investment decision of that firm will be different from the one that would be made absent the possibility of readjusting the price. The latter investment decision is relevant for the time $t+2$ capital gap affecting the time t price setting decision and the former is relevant for the time $t+2$ capital gap affecting the time $t+1$ price setting decision. This is the reason why an optimizing Calvo price setter has to take into account *all* the states of the world that are consistent with the one realized at the time when the price setting decision is made. We outline next how we solve for the equilibrium inflation dynamics implied by that structure.

Besides the first order conditions for price setting and for investment we know that the dynamic system has a steady state. In the zero inflation steady state we are considering a firm that is allowed to change its price will optimally choose not to do so. This means that a time t price setter will foresee that it will optimally choose not to deviate from the aggregate price level whenever the firm will have a chance to adjust its price again in the infinitely distant future. Formally: $\lim_{k\to\infty} E_t \widehat{p}^*_{t+k}(f) = 0$. Our strategy is to obtain the inflation equation associated with our model by iterating on the following step: in the first round a price setter behaves in a myopic way, i.e. firm f chooses a price $\widehat{p}^{0,*}_t(f)$ assuming that it will choose a linearized relative price equal to zero by the (random) time when the firm will be allowed again to adjust its price in the future. This way we can solve for the newly set myopic price $\widehat{p}^{0,*}_t(f)$ in terms of aggregate variables only, except for the current predetermined capital gap of firm f. We call this functional relationship a myopic policy function. In the second round a price setter is a bit more rational and chooses $\widehat{p}^{1,*}_t(f)$ assuming that the economy will be in steady state by the time when it will readjust its price for the second time after the initial price setting decision. Of course, all future expected newly set prices as far as they are relevant for the computation of $\widehat{p}^{1,*}_t(f)$ can be found using the myopic policy function obtained in the first round. The newly set price consistent with rational expectations is $\widehat{p}^*_t(f) = \lim_{k\to\infty} E_t \widehat{p}^{k,*}_t(f)$. Our strategy is the following: at each step of the iteration we solve for the average newly set price. Since price setters are randomly selected the current average capital gap in the group of price setters is zero. Hence the average newly set price in the economy is a function of aggregate variables only. Next, we invoke the price index in order to solve for the implied inflation equation. Combing this equation with the equilibrium conditions stated in section 2.3 and specifying how monetary policy is conducted we can solve numerically for the implied first order approximation to the equilibrium dynamics. We assume that the growth rate of nominal money balances follows an AR(1) process and we consider impulse responses associated with a one standard deviation shock to that process. It turns out that already after 3 iterations of our procedure the implied equilibrium dynamics are almost identical.[13] There is a simple intuition for this result: if prices are very flexible then capital gaps are almost irrelevant for the impulse response associated with a monetary policy shock. In the extreme case of fully flexible prices capital gaps are unaffected by this kind of shock. To the extent that prices are sticky the monetary policy shock has an effect on investment decisions and hence on capital gaps. However, as can be seen from equation (16), the stickier are prices the lower is the weight on the expected future newly set price in the computation of any given capital gap. The first two inflation equations which yield almost identical equilibrium dynamics are the ones associated with step 3 and step 4 of the iteration. For step 3 we obtain:

$$\pi_t = \beta_{1,3} E_t \pi_{t+1} - \beta_{2,3} E_t \pi_{t+2} + \beta_{3,3} E_t \pi_{t+3} + \kappa_{0,3} \widehat{mc}_t - \kappa_{1,3} E_t \widehat{mc}_{t+1} + \kappa_{2,3} \widehat{mc}_{t+2}, \quad (17)$$

where the parameters $\beta_{1,3}$, $\beta_{2,3}$, $\beta_{3,3}$, $\kappa_{0,3}$, $\kappa_{1,3}$, and $\kappa_{2,3}$ are functions of the deep parameters of the model.[14] The first subscript of each parameter indicates the lead of the expectation in the variable the parameter refers to and the second subscript indicates the round of the iteration or, more colorfully, the degree of sophistication in price setting that is assumed in deriving the inflation equation. Next, we establish our benchmark case.

[13] All Matlab m-files are available on request.

[14] An additional appendix to this paper with all the details of the derivations of the inflation equations at the different steps of our procedure is available on request.

2.7 The benchmark Economy

In the benchmark case households accumulate the capital stock and rent it to firms. Up to the linear approximation to the equilibrium dynamics which we are considering the set of equilibrium conditions is identical to the one associated with modeling firm-owned capital, except for the inflation equation:[15] with a rental market, all firms face the same marginal cost, and hence the inflation equation associated with the benchmark case is given by:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t, \tag{18}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$, and the average marginal cost is defined in the same way as in the case of firm-owned capital.[16]

3 Simulation Results

3.1 Calibration

The calibration is shown in Table 1. The intertemporal elasticity of substitution is given by $\frac{1}{\sigma}$. Assuming $\sigma = 2$ is consistent with the empirical estimates.[17] It should be emphasized that we are calibrating the intertemporal elasticity of substitution in consumption as opposed to the intertemporal elasticity of substitution in private expenditure as a whole. The reason is that we take investment demand explicitly into account in our model. We assume $\nu = \sigma$. The other parameters take their standard values:[18] We assume a unit labour supply elasticity $\phi = 1$. We set $\beta = 0.99$ implying an average annual real return of about 4 percent. The capital share in the production function α is 0.36. $\epsilon = 11$ implies a frictionless markup of 10 percent. Setting $\theta = 0.75$ means that the average lifetime of a price is equal to one year.

[15] Note in particular that the log-linearized law of motion of aggregate capital is the same as the one associated with modeling firm-owned capital.

[16] See Galí (2003) et al. for a more detailed development of a Calvo type model with a rental market for capital.

[17] See, e.g., Basu (2003) and the references herein.

[18] See, e.g., Galí (2000) and the references herein.

Finally, $\rho_m = 0.5$ and $\sigma^2(e_t) = 0.1$ are consistent with the estimated autoregressive process for M1 in the United States.

3.2 Results

We analyze impulse response functions associated with a one standard deviation shock in the growth rate of money balances for the model with firm-owned capital and, respectively, for the benchmark model where we assume a rental market for capital instead.[19] Both simulations are conducted for the calibration given in Table 1. The following two aspects are noteworthy: First, the inflation response to a shock in the money growth rate is smaller for the model with firm-owned capital. This is shown in Figure 1. With firm-owned capital price setting firms are more reluctant to change their prices in response to the shock. As we have mentioned before, the reason is that a price setting firm takes into account that its marginal cost is affected to some extent by the price it chooses. This effect is absent if a rental market for capital is assumed. In that case each firm produces at the same marginal cost which is independent of the quantity an individual firm supplies. Second, as Figure 2 shows, the output reaction is stronger in the model with firm-owned capital.

These differences between the two models are quantitatively important in the following sense: it is possible to reproduce the impulse response in the model with firm-owned capital almost perfectly if we assume a price stickiness parameter $\theta = 0.85$ in the benchmark model. Hence, the differences in the impulse responses shown in Figures 1 and 2 are as important as a change in the average expected lifetime of a price from one year to almost 7 quarters.

Finally, it should be emphasized that both models, i.e. the model with firm-owned capital and the benchmark case with a rental market for capital, generate equilibrium dynamics that are very different from the ones associated with the model by Galí et. al. (2001) where they study the effects of decreasing returns to scale in a Calvo type framework where labour is the only productive input. The reason is that aggregate demand is equal to aggregate consumption in their model which

[19] We use the solution methods described in Söderlind (1999).

is a key difference with respect to both our model with firm-owned capital and the benchmark model with a rental market for capital.

4 Conclusion

We should emphasize the two contributions of our paper and some of the issues that are left for future research. First, we model firm-owned capital in a stochastic dynamic New-Keynesian general equilibrium model à la Calvo and we compare the equilibrium dynamics to the ones associated with a benchmark case where households accumulate capital and rent it to firms. We find that the differences in implied equilibrium dynamics are quantitatively important. Second, we show that the problem of modeling firm-owned capital with Calvo price setting has not been solved in a theoretically correct way in the previous literature.

Clearly, our model with firm-owned capital is very simplistic and lacks many aspects that seem to be relevant for investment decisions by firms in the real economy. One interesting extension is to introduce time to plan into the model developed so far. This will help producing empirically desirable features like a hump shaped output response to a monetary policy shock. The model presented in this paper is not capable of producing this pattern. However, our work shows that the convenience of assuming a rental market for capital is not innocuous in a sticky price model with staggered price setting. In other words, we stress the importance of modeling an investment decision at the firm level in addition to the price setting decision.

Appendix 1: Households

Throughout the appendix we use the notation and the assumptions introduced already in the text.

Consider the representative household's maximization problem:

$$E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}, N_{t+k,} (M_{t+k}/P_{t+k})\right), \tag{19}$$

The period utility function is assumed to be given by:

$$U\left(C_t, N_{t,} (M_t/P_t)\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu}. \tag{20}$$

The consumption aggregate is defined as follows:

$$C_t = \left(\int_0^1 C_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ for } f \in [0,1], \tag{21}$$

and the price index P_t is given by:

$$P_t = \left(\int_0^1 P_t(f)^{1-\varepsilon} df\right)^{\frac{1}{1-\varepsilon}}. \tag{22}$$

The parameter $\varepsilon > 1$ measures the elasticity of substitution between the different types of goods.

The household maximizes expected discounted utility subject to a sequence of budget constraints which take the following form:

$$\int_0^1 \left[P_t(f) C_t(f)\right] df + M_t + E_t\left(Q_{t,t+1} D_{t+1}\right) \leq M_{t-1} + D_t + W_t N_t + T_t, \tag{23}$$

where D_{t+1} is the nominal payoff of the portfolio held at the end of period t (including profits from firms), M_t is the stock of money balances held by the household at the end of period t, and T_t are lump-sum transfers or taxes.

The consumption demand functions for each type of goods are given by:

$$C_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\varepsilon} C_t. \tag{24}$$

The budget constraint can now be written as:

$$P_t C_t + M_t + E_t\left(Q_{t,t+1} D_{t+1}\right) \leq M_{t-1} + D_t + W_t N_t + T_t. \tag{25}$$

The representative consumer's first order conditions stated in the text can be readily obtained once the budget constraint is written in this format, i.e. imposing optimizing behavior.

Appendix 2: Firms

Following Calvo (1983) we assume that each individual firm sets a new price with probability $1-\theta$ each period, independently of the time elapsed since the last price adjustment. This implies that each period a measure $1-\theta$ of randomly selected firms change their prices, while the remaining firms post their last period's nominal prices. Hence, with a probability θ^k a price that was chosen at time t will still be posted at time $t+k$. When setting a new price $P_t^*(i)$ in period t firm i maximizes the current value of its dividend stream over those points in time and states of the world where the newly set price remains effective:

$$\max_{\{P_t^*(i)\}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left[Y_{t+k}(i) \left(P_t^*(i) - MC_{t+k}(i) \right) \right] \right\}$$

s.t.

$$\begin{aligned} Y_{t+k}(i) &\leq \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}^d, \\ Y_{t+k}(i) &= Z K_{t+k}(i)^{\alpha} N_{t+k}(i)^{1-\alpha}, \\ K_{t+k}(i) &= (1-\delta) K_{t+k-1}(i) + I_{t+k-1}(i), \\ & \quad K_t(i) \text{ given.} \end{aligned}$$

Thus, $P_t^*(i)$ must satisfy the first order condition given in equation (9) in the text.

When making its investment decision $I_t(j)$ in period t firm j has the same objective of maximizing the current value of its dividend stream. However, the relevant part of the maximand takes now a different form since each firm j makes an investment decision in each period. Moreover, since we don't assume any adjustment cost an optimizing firm looks only ahead one period. The reason for the latter is that it takes one period until the additional capital resulting from an investment decision becomes productive.

Hence, the firm has the following investment problem:

$$\max_{\{I_t(j)\}} E_t \left\{ Q_{t,t+1} \left[Y_{t+1}(j) P_{t+1}(j) - W_{t+1} N_{t+1}(j) + (1-\delta) P_{t+1} I_t(j) \right] \right\} - P_t I_t(j)$$

s.t.

$$\begin{aligned} Y_{t+1}(j) &\leq \left(\frac{P_{t+1}(j)}{P_{t+1}} \right)^{-\varepsilon} Y_{t+1}^d, \\ K_{t+1}(j) &= (1-\delta) K_t(j) + I_t(j), \\ Y_{t+1}(j) &= Z K_{t+1}(j)^{\alpha} N_{t+1}(j)^{1-\alpha}, \\ & \quad K_t(j) \text{ given.} \end{aligned}$$

Therefore, $I_t(j)$ must satisfy the first order condition given in equation (10) in the text.

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Table 1

The period length:	one quarter
Preference parameters:	$\sigma = \nu = 2$, $\phi = 1$, $\beta = 0.99$
Production Function:	$\alpha = 0.36$
Elasticity of substitution between goods	$\epsilon = 11$
Depreciation:	$\delta = 0.025$
Price stickiness:	$\theta = 0.75$
Monetary Policy:	$\rho_m = 0.5$, $\sigma^2(e_t) = 0.1$

Figure 1: Shock to the Money Growth Rate (ρ_m = 0.5)

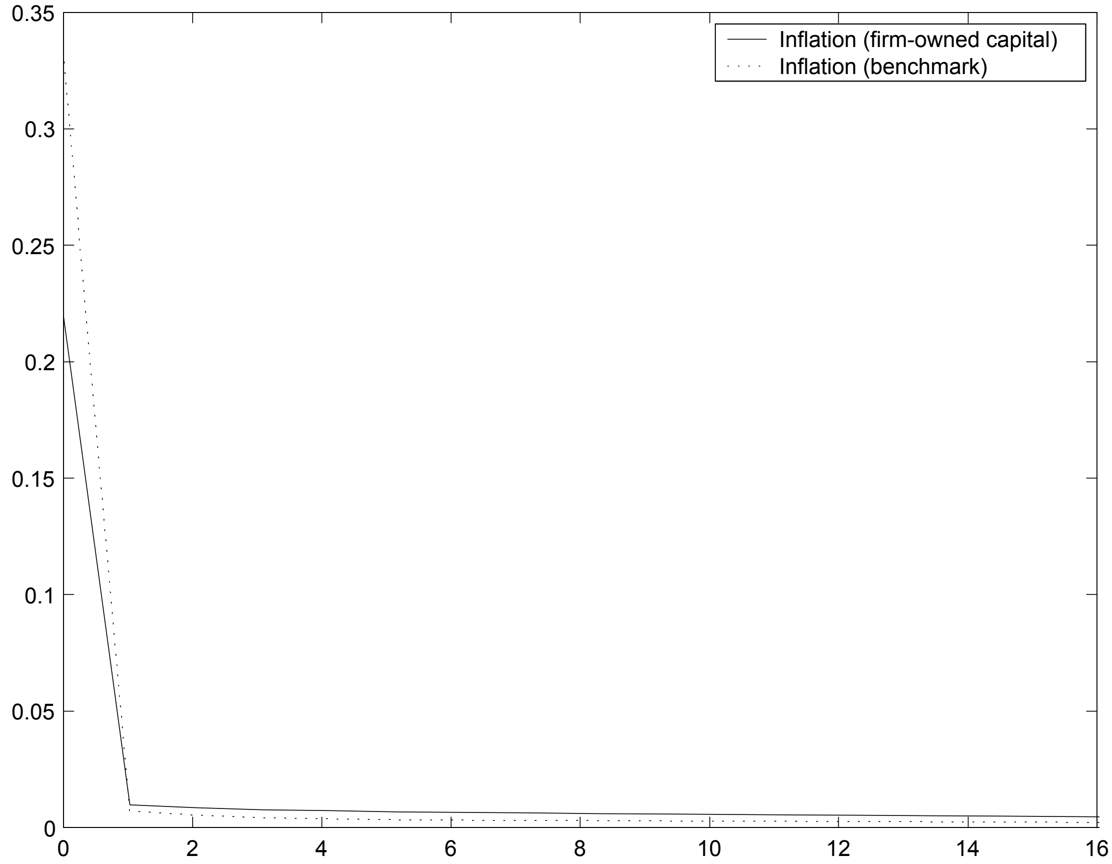

Figure 2: Shock to the Money Growth Rate (ρ_m = 0.5)