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Probabilistic Maximal Covering Location Models for Congested Systems.

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Abstract

In this paper, we propose several Maximal Covering Location models that include queueing parameters to obtain an efficient and effective service system. The models are based on the fact that, in real life, the number of requests for service is not constant in time, but instead, it is a stochastic process. This stochasticity of the demand is explicitly taken into account in order to derive a capacity-like constraint which, instead of upper-bounding the demand to the capacity of the center, forces a lower bound on the quality of the service at the facility, particularly the waiting time or the number of people in line, awaiting for service. The first model considers that there is only one server at each center. The second one considers that each individual center has a fixed number of servers. In the third model there is a fixed number of servers to be allocated to centers. Finally, solution methods and further research are proposed in the context of integer programming.

1 Introduction

When dealing with limited budgets, Decisions on location of services are usually made with the help of Maximal Covering Location Models (Church and ReVelle 1974) or p-median type models (Hakimi 1964, and ReVelle and Swain 1970). Frequently, these models include a capacity constraint, which forces the demand for service at the center to be smaller than its maximum capacity. In these constraints, the maximum capacity is usually given by either the service load that the center can handle, or by some estimation of the maximum number of users that the center can serve at the same time. On the other hand, in general, as an estimate of the demand for service at any facility, two different figures are typically used. The first one is the total population allocated to the facility, multiplied by some experimental or practical factor, which gives an estimate of the expected number of simultaneous requests for service, or demanded workload, of that center. The second figure is an average of the historical rate of requests originating at the population allocated to the center, if a record of it is available. This demand is then constrained to be smaller than the maximum capacity of the facility.

In the traditional models we described, the demand is implicitly assumed constant in time, equal to an average, which is strictly constrained to be smaller than the capacity of a facility all the time. In some sense, there is a contradiction between the strong, rigurous constraint, and the fact that it is applied to an average. In this paper, we propose a model based on the fact that, in real life, the number of requests for service is not constant in time, but instead, it is a stochastic process. This stochasticity of the demand is explicitly taken into account in order to derive a capacity-like constraint which, instead of upper-bounding the demand to the capacity of the center, forces a lower bound on the quality of the service at the facility, particularly the waiting time or the number of people in line, awaiting for service.

The models presented here are specially useful when locating services that can become easily congested by the users due to limited resources available. In general, this is the case of emergency services, that can have workloads which push them to their limit of their ability to provide effective service.

The optimal location of primary health care centers is an example of such services. These centers must be located in such a way that they could be reached from any demand point within a reasonably short time, and, once a patient has arrived to the center, his/her waiting time should be as short as possible, since waiting time is an important determinant of the perception of service quality. The same reasoning can be applied to emergency rooms and other facilities. Another example of congested systems is the location of distribution centers, where trucks arrive to deliver their load. The presence of several trucks in a facility with only one server can make it easily congested.

The first model addresses the issue of the location of a given number of Single Channel, Single Server queueing system centers in the nodes of a network so as to maximize population coverage within a distance standard, and with the additional probabilistic restriction that the people in queue or the waiting time in line will not exceed a given standard with a probability α. The second model considers that the centers to be located have a given number of servers each ($M/M/m$ queueing system). The last model considers that not only a given number of centers have to be located so as to maximize population coverage given a distance standard, but that there is a given number of servers in the system that has to be allocated among the centers. Finally, several solution methods from the literature are proposed together with some remarks on future research.

2 Development of the M/M/1 Maximal Location Covering Problem

The $M/M/1$ Queueing Maximal Covering Location Problem (QMCLP1) can be stated as:

"Locate p service centers, with 1 server each, and allocate users to them so as to maximize covered population, where coverage is defined as: i) covered population is allocated to a center within a time or distance standard from its home location, and ii) if a user is covered, at his/her arrival to the center, he/she will wait on a line with no more than b other people, with a probability of at least α, or ii) every user will be attended within τ."

The formulation of the model is the following:

$$\max Z = \sum_{i \in I} \sum_{j \in J} a_i x_{ij} \tag{1}$$

subject to:

$$x_{ij} \leq y_j \qquad \forall i \in I, \forall j \in J \qquad (2)$$

$$\sum_{j \in N_i} x_{ij} \leq 1 \qquad \forall i \in I \qquad (3)$$

$$\sum_{j \in J} y_j \leq p \qquad (4)$$

$$\text{P[center } j \text{ has} \leq b \text{ people on queue]} \geq \alpha \qquad \forall j \in J \qquad (5a)$$

$$\text{P[waiting time at fac. } j \leq \tau] \geq \alpha \qquad \forall j \in J \qquad (5b)$$

$$y_j, x_{ij} = (0,1) \qquad \forall i \in I, \forall j \in J$$

where:

i, I = index and set of demand areas
j, J = index and set of potential facilities
S = distance standard
τ = maximum waiting time on line
a_i = Population at node i
d_{ij} = distance between nodes i and j
N_i = $\{j \in J, d_{ij} \leq S\}$

$$y_j = \begin{cases} 1, & \text{if a center locates at node } j \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if node } i \text{ is served by a center located at } j \\ 0, & \text{otherwise} \end{cases}$$

Objective (1) maximizes the population which has been allocated to some center, that is, covered population. Note that there is no constraint forcing every demand node to be covered. Constraint (2) states that it is not possible to allocate demand node i to a site j, unless there is a center at this site. Constraint (3) forces each demand node i to be allocated to at most only one service center j. Constraint (4) limits the number of centers to be located. Constraint ($5a$) forces every facility to have less than, or at most, b people on line with a probability of at least α. This constraint assures that, on his/her arrival to the facility, every user will find a line that is short enough, most of the time. Constraint ($5b$) explicitly makes the waiting time at the facility shorter than or equal to τ with probability of at least α, assuring to every patient a timely attention.

Although the formulation presented is more intuitive, a variation of the model which has less variables and constraints will be used in practice. The variation is very simple: variables x_{ij} are defined only for the pair of indexes (i, j) such that j belongs to N_i, that is, since we are forcing coverage of every demand from a center located within the distance standard, there is no need to define variables that will neve be equal to one. Thus, the number of variables is reduced, the number of constraints (2) is reduced, and constraint (3) becomes:

$$\sum_{j \in J} x_{ij} \leq 1, \qquad \forall i \in I$$

This new formulation would be only used in problems where there is no need to assign uncovered demand to centers. This is not the case when it is expected that everybody in the system will use the facilities, regardless of coverage.

Constraints ($5a$) and ($5b$) could be replaced respectively by the following ones:

$$[\text{Avge. or expected \# of customers in a center } j \leq b] \geq \alpha, \quad \forall j \in J \qquad (5a')$$

$$[\text{Avge. waiting time at center } j \leq \tau] \geq \alpha, \qquad \forall j \in J \qquad (5b')$$

Constraints ($5a'$) force the expected number of users at facility j (the one being attended plus those in queue) to be less than or equal to b. If this constraint holds, on their arrival to the facility, users will find lines shorter than b a 50% of the time (not a 100% percent), as in constraint (4), and the other 50% of the time, they will face lines that are longer than b. Therefore, constraint ($5a'$) is less tight than constraint ($5a$). Finally, constraint ($5b'$), forces the *average* waiting time to be less than or equal to τ. We will use constraints ($5a$) and ($5b$).

Constraints ($5a$) and ($5b$) cannot be written in a usable form unless the underlying probabilistic distributions are known. In general, these constraints will be non linear. If a linear, integer model is to be used, their linear, deterministic equivalents need to be found.

In order to write constraint ($5a$) in a tractable form, we make the reasonable (and customary) assumption that requests for service at each demand node i appear according to a Poisson process with intensity f_i. Since each facility serves a set of demand nodes, the requests for service at that facility are the union of the requests for service of the nodes in the set, and they can be described as another stochastic process, equal to the sum of several Poisson processes. This stochastic process can be easily shown to be also a Poisson process, with an intensity λ_j equal to the sum of the intensities of the processes at the nodes served by the facility. This set of nodes is not known before the solution of the mathematical programming problem is known. However, we can use variables x_{ij} in order to rewrite the parameter λ_j as

$$\lambda_j = \sum_{i \in I} f_i x_{ij}$$

Using this definition, if a particular variable x_{ij} is one, meaning that node i is allocated to facility at j, the corresponding intensity f_i will be included in the computation of λ_j.

We also assume an exponentially distributed service time, with a service rate of μ_j ($\lambda_j \leq \mu_j$, otherwise the system does not reach an equilibrium). If we assume steady state, we can use the well known results for a $M/M/1$ queueing system for each facility and its allocated users.

If we define the state k of the system as k users in the system (either being attended or in queue), the state transition diagram of the system is the one shown in Figure 1.

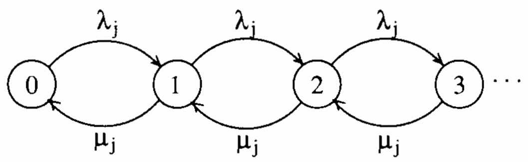

Figure 1: State Transition Diagram

In this figure, state k corresponds to k users at the facility, that is, state zero corresponds to the facility being idle, state 1 to one user being attended at the facility, state 2 to two patients at the facility: one of them getting attention and one in queue, and so on. We want to make the probability of a user being on a line with no more than b other people, at least equal to alpha. If we represent as p_k the steady state probability of being in state k, this requirement is written as:

$$p_0 + p_1 + ... + p_{b+1} \geq \alpha \tag{6}$$

Writing and solving the steady state balance equations of the $M/M/1$ system, we get the following expression for the steady state probabilities (Wolff, 1989):

$$p_k = (1 - \rho_j)\rho_j^k$$

Where $\rho_j = \lambda_j/\mu_j$. Hence, equation (6) becomes:

$$(1 - \rho_j) + (1 - \rho_j)\rho_j + (1 - \rho_j)\rho_j^2 + ... + (1 - \rho_j)\rho_j^{b+1} \geq \alpha$$

or

$$(1 - \rho_j)\sum_{k=0}^{b+1} \rho_j^k \geq \alpha$$

which is equivalent to

$$(1 - \rho_j)[(1 - \rho_j^{b+2})/1 - \rho_j] \geq \alpha,$$

or

$$\rho_j^{b+2} \leq 1 - \alpha,$$

therefore,

$$\rho_j \leq (1 - \alpha)^{1/b+2}.$$

Since $\rho_j = \lambda_j/\mu_j$,

$$\lambda_j \leq \mu_j(1-\alpha)^{1/b+2}. \tag{7}$$

Equation (7) is equivalent to constraint ($5a$). Using the relationship between the intensity at the facility and the intensities at the demand nodes, constraint (5) is rewritten as

$$\sum_{i \in I} f_i x_{ij} \leq \mu_j(1-\alpha)^{1/b+2}, \tag{8}$$

which is a linear, deterministic equivalent of constraint ($5a$).

If we choose to use constraint ($5b$) instead of constraint ($5a$), that is

$$P[\text{waiting time at facility } j \leq \tau] \leq \alpha, \qquad \forall j \in J$$

we may use the probability distribution function of the waiting time in a $M/M/1$ queue, w, which has the following expression (Larson and Odoni, 1981):

$$f_w(w_j) = (\mu_j - \lambda_j)e^{(-\mu_j - \lambda_j)w_j}$$

to derive its cummulative distribution:

$$P(w_j \leq \tau) = F_w(\tau) = 1 - e^{(\mu_j - \lambda_j)\tau} \tag{9}$$

The probability in equation (9) is made greater than or equal to α:

$$1 - e^{\mu_j - \lambda_j)\tau} \geq \alpha, \qquad \forall j \in J,$$

$$e^{-(\mu_j - \lambda_j)\tau} \leq 1 - \alpha, \qquad \forall j \in J,$$

$$-(\mu_j - \lambda_j)\tau \leq ln(1-\alpha), \qquad \forall j \in J,$$

$$\lambda_j \leq \mu_j + 1/\tau ln(1-\alpha) \qquad \forall i \in J.$$

Using equation (8) to rewrite the parameter λ_j , we finally get

$$\sum_{i \in I} f_i x_{ij} \leq \mu_j + 1/\tau ln(1-\alpha) \qquad \forall j \in J. \tag{10}$$

which is the linear deterministic equivalent of equation (5*b*).

3 Development of the M/M/m QMCLP when the number of servers in each center is fixed

When there is more than one server at center j, the inequality $\mu_j \geq m_j \lambda_j$ must hold, where m_j is the number of servers at center j; otherwise, the system does not reach an equilibrium. By virtue of the new assumptions, the well-known results for a M/M/m queueing system can be used for each center and its allocated users. In this case, the new state transition diagram of the system is as follows.

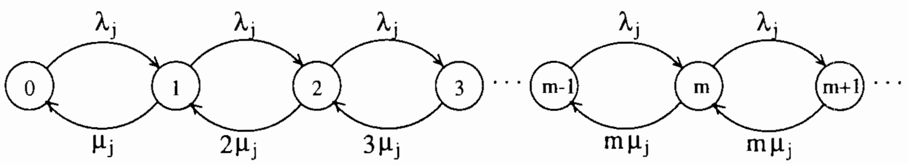

Figure 2: State Transition Diagram, $M/M/m$ system

In this figure, state k corresponds to k users at the center, that is, state zero corresponds to the center being idle, state 1 to one user being attended at the center, state 2 to two users at the center, both of them getting attention, and so on, up to state m, in which all m users in the system are getting attention. In state $m+1$, however, m users are being attended and one in queue; state $m+2$ represents m users in service and 2 in queue, and so on. We want to make the probability of a user finding a line with no more than b other people, at least equal to α. If we represent as p_k the steady state probability of being in state k, this requirement is written as:

$$p_0 + p_1 + \ldots + p_{m+b} \geq \alpha$$

that is, the probability of the queue being shorter that or equal to b users at the arrival of the next request, is greater than α. Also, since $p_0+p_1+...+p_\infty = 1$,

$$p_{m+b+1} + p_{m+b+2} + ... + p_\infty \leq 1 - \alpha, \tag{11}$$

which means that the probability of the queue being longer than b is smaller than $1-\alpha$. Note that the special case $b = 0$ does not mean that the user necessarily finds one server available, because it may happen that all m servers at the center are busy, but there are no users in the queue. In this case, the arriving customer must wait until one of the servers becomes idle. If free server availability is desired, that is, at least one server free with probability α, then $p_0 + p_1 + ... + p_{m-1}$ must be forced to be greater or equal to α.

Writing an solving the steady state balance equations of the M/M/m system, we get the following expression for the steady state probabilities (Wolff, 1989):

$$\begin{aligned} p_k &= p_0\rho^k/k!, & k \leq m \\ p_k &= p_0\rho^k/m!m^{k-m} & k > m \\ p_0 &= \left[\frac{\rho^m}{\left(1-\frac{\rho}{m}\right)m!} + \sum_{j=0}^{m-1}\frac{\rho^j}{j!}\right]^{-1} \end{aligned}$$

Where $\rho = \lambda/\mu$. Although these parameters are specific to each server center, we will not use any subscript for the time being. With these expressions for the steady state probabilities, equation (11) becomes:

$$\sum_{k=m+b+1}^{\infty} \frac{p_0 m^m}{m!}\left(\frac{\rho}{m}\right)^k \leq 1-\alpha$$

or

$$\frac{p_0 m^m}{m!}\left(\sum_{k=0}^{\infty}\left(\frac{\rho}{m}\right)^k - \sum_{k=0}^{m+b}\left(\frac{\rho}{m}\right)^k\right) \leq 1-\alpha.$$

Since $\rho/m \leq 1$, the summations in parentheses converge. Recalling that these summations can be written in a well known, simpler form, we get

$$\frac{m^m}{m!}\left[\frac{\rho^m}{\left(1-\frac{\rho}{m}\right)m!}+\sum_{j=0}^{m-1}\frac{\rho^j}{j!}\right]^{-1}\left(\frac{\left(\frac{\rho}{m}\right)^{m+b+1}}{1-\frac{\rho}{m}}\right)\leq 1-\alpha$$

after some algebreaic manipulation, this equation becomes

$$\sum_{k=0}^{m-1}\frac{(m-k)m!m^b}{k!}\frac{1}{\rho^{m+b+1-k}}\geq\frac{1}{1-\alpha} \tag{12}$$

for the second model.

Since $\rho=\lambda/\mu$, and since λ is a function of the variables x_{ij}, equation (12) can be also written as a function of variables x_{ij}, becoming the deterministic equivalent of equation ($5a$).

It is intuitively easy to see that, for any fixed value of α, the value of the left hand side of equation (12) can be made large enough to make the equation hold, by making ρ small enough, because its exponent is always positive. The value of variable ρ is decreased by manipulating variables x_{ij} (making as many of them equal to zero as needed). Furthermore, for any value of α there must exist a value ρ_α of ρ which makes the equation (8) hold as an equality, as well as a range of values of ρ such that equation (12) holds as a strict inequality.

Although it is the deterministic equivalent of equation ($5a$), equation (12) cannot be used in a linear model, because of its nonlinearity. However, we show next that its left hand side (LHS) is strictly decreasing with increasing ρ, and we later use this characteristic to find a linear equivalent to it.

The derivative of the LHS of (12) with respect to ρ is

$$\frac{\partial \text{LHS}}{\partial\rho}=\sum_{k=0}^{m-1}\left[-(m+b+1-k)\right]\frac{(m-k)m!m^b}{k!}\frac{1}{\rho^{m+b+2-k}} \tag{13}$$

This derivative is strictly negative because $(m+b+1-k)$ is strictly positive, as well as all the remaining factors of each term of the summation. The entire summation is also strictly negative, and so is the whole derivative. Thus, the LHS of equation (12) is a strictly decreasing function of ρ, which also means that it strictly increases when ρ decreases.

Let ρ_α be the value of ρ which makes the equation (12) hold as an equality. Since the LHS strictly increases when ρ decreases, equation (12) also holds for any value of $\rho\leq\rho_\alpha$. In other words, the inequality $\rho\leq\rho_\alpha$ is a sufficient condition for equation (12) to hold. Furthermore, by virtue of the strictly increasing nature of the LHS of equation (12), for any $\rho > \rho_\alpha$ equation (12) cannot hold. Thus, $\rho \leq \rho_\alpha$ is also a necessary condition for equation (12) to hold. Since $\rho \leq \rho_\alpha$ is a necessary and sufficient condition for equation (12) to hold, we can use it instead of equation (12). Once the value of α is given, the value of ρ_α can be found by using any numeric root-finding technique (Newton methods, for example) on equation (12), written as an equality, and equation

$$\rho_j \leq \rho_{\alpha j}, \qquad \forall j \in J$$

can be used instead of equation (12) for each j.

Since $\rho_j = \lambda_j/\mu_j$,

$$\lambda_j \leq \mu_j \rho_{\alpha j}, \qquad \forall j \in J.$$

Recalling that λ_j is a function of variables x_{ij},

$$\sum_{i \in I} f_i x_{ij} \leq \mu_j \rho_{\alpha j}, \qquad \forall j \in J \tag{14}$$

which is the set of linear, deterministic equivalents of constraint ($5a$).

The second model (QMCLP2) consists of equations (1), (2), (3), (4) and (14). If the problem is solved using its linear relaxation, in this last equation the right hand side may be multiplied by variable y_j in order to improve the integer characteristics of the variables at the solution.

4 Development of the M/M/m QMCLP when the number of servers in each center is *not* fixed

This new model can be stated as follows:

"Locate p^c service centers, and p^s servers, and allocate users to them so to maximize covered population, where coverage is defined as: i) covered population is allocated to a center within a time or distance standard from its home location, and ii) if a user is covered, at his/her arrival to the center, he/she will wait on a line with no more than b other people, with probability of at least α."

In this model, the total number of servers is given, and it is distributed among the centers in the best possible way given by the solution of the mathematical program, as opposed to the former model, in which a fixed number of m servers per center is sited. In order to formulate this model, which builds on the former one, a new variable z_{jk} is defined. This variable is one if at least k servers are located at service center j, and zero otherwise. As many variables are defined for each center as the maximum number of servers that may be sited at that center. The set of ordering constraints

$$z_{jk} \leq z_{jk-1}, \qquad \forall j, k = 2, 3, ..., C_j, \tag{15}$$

where C_j is the maximum capacity of a center located at node j are added, to indicate that a k^{th} server cannot be located at node j without first locating the $(k-1)^{th}$ server. Constraint (14) becomes

$$\sum_{i \in I} f_i x_{ij} \leq \mu_j \left[z_{j1}\rho_{\alpha j1} + \sum_{k=2}^{C_j} z_{jk}(\rho_{\alpha jk} - \rho_{\alpha j(k-1)}) \right]. \qquad \forall j \in J \tag{16}$$

In equation (16), the parameter $\rho_{\alpha jk}$ represents the value of $\rho_{\alpha j}$ which makes equation (12) hold as an equality if the number of servers located at j is k. This value can be computed previous to the solution of the mathematical program, using $m = 1$, $m = 2$, up to $m = C_j$ in equation (12) Note that, if the solution of the mathematical program indicates that there are four servers at the center located at node j, the four first variables z_{jk} will be equal to one, and the right hand side of equation (16) will be equal to $\mu_j \rho_{\alpha j4}$.

Note also that adding an extra server to center j, we add service capacity to it. Although this is intituively true, it can be shown, proving that $\rho_{\alpha jk} - \rho_{\alpha j(k-1)}$ in last equation is positive, which we next do, proving that $\rho_{\alpha jk}$ is greater than $\rho_{\alpha j(k-1)}$.

Recall that $\rho_{\alpha jk}$ is the value of ρ which makes equation (12) hold as equality, when m servers are located at node j. We compute the left hand side of equation (12) for the same value of ρ, and $m+1$ servers, LHS$(m+1)$:

$$\text{LHS}(m+1) = \sum_{k=0}^{m} \frac{(m+1-k)(m+1)!(m+1)^b}{k!} \frac{1}{\rho^{m+1+b+1-k}}$$

Changing summation variables, $j = k+1$,

$$\begin{aligned}\text{LHS}(m+1) &= \sum_{j=-1}^{m-1} \frac{(m-j)(m+1)!(m+1)^b}{(j+1)!} \frac{1}{\rho^{m+b+1-j}} \\ &= \frac{(m+1)!(m+1)^{b+1}}{\rho^{m+b+2}} + \sum_{j=0}^{m-1} \frac{(m-j)(m+1)!(m+1)^b}{(j+1)!} \frac{1}{\rho^{m+b+1-j}}.\end{aligned}$$

Renaming j as k again, we get

$$\text{LHS}(m+1) = \frac{(m+1)!(m+1)^{b+1}}{\rho^{m+b+2}} + \sum_{k=0}^{m-1} \frac{(m+1)}{(k+1)} \frac{(m-k)m!(m+1)^b}{k!} \frac{1}{\rho^{m+b+1-k}}.$$

The first term of this expression is strictly positive, and clearly, each term of the summation is also strictly greater than its equivalent term in the expression for LHS(m). Thus, we can conclude that, for the same value of ρ, LHS$(m+1)$ is strictly greater than LHS(m). Let $\rho = \rho_{\alpha jm}$. Since this value of ρ makes LHS(m) equal to $1/(1-\alpha)$, it makes LHS$(m+1)$ strictly greater than $1/(1-\alpha)$. Recall that LHS is a decreasing function of ρ. Thus, in order to decrease LHS$(m+1)$ to make it equal to $1/(1-\alpha)$, and find $\rho_{\alpha j(m+1)}$, we must decrease ρ. Hence, $\rho_{\alpha j(m+1)}$ is strictly greater than $\rho_{\alpha jm}$, and the term $\rho_{\alpha j(m+1)} - \rho_{\alpha jm}$ is positive for all m.

The reasoning shows that adding a server to a center, the service capacity is increased, and that structure of constraint (15) represents adequately this characteristic.

Finally, the third model (QMLCP3) is the following:

$$\min Z = \sum_{i \in I} \sum_{j \in N_i} a_i x_{ij} \tag{17}$$

subject to:

$$z_{jk} \leq z_{j(k-1)} \qquad \forall j, k = 2, 3, \ldots, C_j \quad (18)$$

$$x_{ij} \leq z_{j1} \qquad \forall i \in I, \forall j \in N_i \quad (19)$$

$$\sum_{j \in N_i} x_{ij} = 1 \qquad \forall i \in I \quad (20)$$

$$\sum_{j,k=1}^{k=C_j} z_{jk} = p^s \quad (21)$$

$$\sum_j z_{j1} = p^c \quad (22)$$

$$\sum_{i \in I} f_i x_{ij} \leq b_j \qquad \forall j \in J \quad (23)$$

$$z_{jk}, x_{ij} = (0,1) \quad \forall i \in I, \forall j \in N_i, \forall k \in K$$

where

$$b_j = \mu_j \left[z_{j1}\rho_{\alpha j1} + \sum_{k=2}^{C_j} z_{jk}(\rho_{\alpha jk} - \rho_{\alpha j(k-1)}) \right]$$

5 Conclusions

The discipline of location-allocation theory has mainly focussed on the issue of the optimal siting of services and facilities, but little attention has been paid to the issue of service congestion, i.e, service quality. In this paper several models that address the issue of queueing and location have been presented. The models are based on the well-known Maximal Covering Location Problem, introducing a set of constraints that restrict the number of users at a given time to be less than a standard with an α probability. While these constraints might seem highly non-linear, deterministic equivalents have been found that allow the problems to be casted as an integer programs (IP).

In fact, the IP mathematical formulations presented for QMCLP1 and QMCLP2 are exactly the same as the one corresponding the Capacitated Maximal Covering Problem (CMCP) developed by Chung et al. (1983) and Current and Storbeck (1988), and developed by Pirkul and Shilling . As these authors point out, these problems "belong the to the notably difficult

NP-Complete class of problems" and therefore, general-purpose integer programming codes become highly inefficient for reasonable size problems. To overcome this problem, Pirkul and Shilling developed a Lagrangean relaxation procedure to obtain solutions that proved to be very efficient, with lower bound gaps typically well below two percent. Both QMCLP1 and QMCLP2 can be solved using their approach.

The mathematical structure of the third model, QMLCLP3, differs significantly from the previous models QMCLP1 and QMCLP2. While the methodology presented here provides globally optimal solutions using a general-purpose IP code, it can become very expensive in terms of computing time due to both the large number of variables and constraints, and to the mathematical structure of constraint set (23) for medium-size problems. As Current and Storbeck (1988) point out " although many real-world location problems fall in the small to moderately sized range, others do not". Because the widespread applicability of QMCLP models, the developement of efficient heuristics based on Lagrangean relaxation for large QMCLP3 problems is an important area of future research.

References

CHUNG, C., A. SHILLING, AND R. CARBONE (1983), "The capacitated covering problem: A heuristic," *Proceedings of the Fourteenth Annual Pitsburg Conference on Modelling and Simulation.*

CHURCH, R. AND C. REVELLE (1974), "The maximal covering location problem," *Papers of the Regional Science Association*, 32:101–118.

CURRENT, J. AND J. STORBECK (1988), "Capacitated covering models," *Environment and Planning B*, 15:153–164.

HAKIMI, S. (1964), "Optimum location of switching centers in a communications network and some related graph theoretic problems," *Operations Research*, 12:450–459.

LARSON, R. AND A. ODONI (1981). *Urban Operations Research.* Prentice Hall, Englewood Cliffs, NJ.

PIRKUL, H. AND D. SHILLING (1991), "The maximal covering location problem with capacities on total workload," *Management Science*, 37(2):233–248.

REVELLE, C. AND R. SWAIN (1970), "Central facilities location," *Geographical Analysis*, 2:30–42.

WOLFF, R. (1989). "Stochastic modeling and the theory of queues, " in Fabrycky, W. and J.H. Mize, editors, *International Series in Industrial and Systems Engineering.* Prentice All.

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