A New Chance - Constrained Maximum Capture Location Problem[1]

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Abstract

The paper presents a new model based on the basic Maximum Capture model, MAXCAP. The New Chance – Constrained Maximum Capture model introduces a stochastic threshold constraint, which recognises the fact that a facility can be open only if a minimum level of demand is captured. A metaheuristic based on MAX – MIN ANT system and TABU search procedure is presented to solve the model. This is the first time that the MAX – MIN ANT system is adapted to solve a location problem. Computational experience and an application to 55 – node network are also presented.

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1. INTRODUCTION

Up till now, the concept of market threshold has not been used so much in facility location decision models. The threshold concept is particularly relevant to retail location, as it is widely recognized in the retail literature that states "there is a minimum size of a market below which a place will be unable to supply a central good … and is here termed the threshold sales level for the provision of that good from the center" (Berry and Garrison, 1958, p.111, as cited by Shonkwiler and Harris, 1996). In this paper, a new model based on the basic Maximum Capture Model (MAXCAP) formulated by ReVelle (1986) is presented. The MAXCAP model seeks the location of a given number of facilities in a discrete network so as to maximize market share captured. The new model named Chance – Constrained Maximum Capture model introduces two modifications:

- Firstly, the capture is determined by the gravity model proposed by HUFF (1964)[2], and not just based on proximity.
- Secondly, and new, stochastic threshold constraint is introduced. A facility can be open if the probability, that the total demand assigned to that outlet was above the threshold level, is at least a desired probability.

The paper is organized as follows. Section 2 reviews the literature. Section 3 presents the New Chance – Constrained Maximum Capture model. In Section 4, we develop the metaheuristic to solve the problem. Section 5 presents some computational experience on

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[2] Accordingly to this model and using the terminology defined by Drezner (1994), each facility has a known attractiveness level, and the probability that a customer selects a facility is proportional to its attractiveness and inversely proportional to some power of the distance to it.

different sized network. In section 6 an example is presented on a 55-node network. Finally, the conclusions are set out in Section 7.

2. LITERATURE REVIEW

Competitive Location Literature addresses the issue of optimally locating firms that compete for clients in space. The first study of this line was due to Hotelling (1929), where consumers were assumed to patronize the closest facility. Different models based on this assumption of consumer behaviour have been developed. A good review can be found in Drezner (1995).

The key one for this paper is the one developed by ReVelle (1986). ReVelle and his followers have constructed a group of models that examined competition among retail stores in a spatial market. The basic model was the Maximum Capture Problem (MAXCAP, ReVelle (1986)). This model selects the location of servers for an entering firm which wishes to maximizes its market share; the market is one in which competitor servers are already in position. This model has been adapted to different situations. The first modification introduced facilities that are hierarchical in nature and where there is competition at each level of the hierarchy (Serra, et. al. (1992)). A second extension took into account the possible reaction from competitors to the entering firm (Serra and ReVelle (1994)). Finally, another modification of the MAXCAP problem introduced scenarios with different demands and / or competitor locations (Serra, et. al. (1996)). A good review of these models can be found in Serra and ReVelle (1996) and a real application of it in Serra and Marianov (1999).

All these Competitive Location theories find optimal locations assuming that customers patronize the closest shop. **Store – Choice literature** studies the key variables that influences a consumer when deciding where shop as well as the interaction between these variables. Basically, all these models are based on Newton's Law of Gravitation; and for that reason, most researchers refer to them as *Gravity models*. The first study of this line was due to Reilly (1929) and a good review can be found also in Drezner (1995). Literature on the subject reveals that distance is not the only variable consumers take into account when deciding where to make their purchase. Eiselt and Laporte (1989) and Santos – Peñate, et. al. (1996) have introduced these concepts in the basic MAXCAP model. Recently, Colomé and Serra (2000) present an empirical study of the methodology to choose the key store choice attributes and how to introduce them in the basic MAXCAP model.

Another assumption used in the basic MAXCAP model is the possibility to locate an outlet, regardless the level of demand capture. Recently, several authors have recognised that there is a **demand entry threshold** and have introduced this concept in the facility location decision models in different ways.

Balakrishnan and Storbeck (1991) presents the McTHRESH model. This model addressed the issue of locating a given number of outlets so that market coverage was maximised within some predetermined range and the required threshold level of demands were maintained for all sites. In 1994, Current and Storbeck (1994) formulated a multiobjective model that selected franchise locations and identified individual franchise market areas. Constraints in their formulation guarantee that all francise locations were assigned at least a minimal threshold market area with sufficient demand to ensure economic survival.

Recently, Serra, ReVelle and Rosing (1999) presented a decision model for a firm that wished to enter a competitive market where several competitors were already located. The market was such that for each outlet there was a demand threshold level that had to be achieved in order to survive. In this model, the threshold constraint is deterministic and each facility must meet the threshold.

Finally, Drezner, Drezner and Shiode (2002) presented a location model based on the threshold concept. They assumed that the buying power at each community over the planning horizon was distributed according to some statistical distribution. Assuming that there was a minimum market share threshold to be captured, they introduced the threshold in the objective function. Their location objective become the minimisation of the probability of falling short of the required threshold.

In this paper, we present a decision model for a retail firm with a stochastic threshold, but as a constraint.

3. THE MODEL

The basic model states that a new firm (from now on Firm A) wants to enter with *p* facilities in a market in order to obtain the maximum capture, given that it has to compete with *q* existing outlets[3], and subject to a threshold constraint that is stochastic.

This model studies the location of retail facilities in discrete space. The model takes the following assumptions:

- Demand is totally inelastic.
- The good is homogeneous.

[3] These competitors can belong to one or more firms, but without loss of generality it is assumed that there is only one competing firm (Firm B) operating in the market; as was assumed by ReVelle (1986).

- The threshold level is defined as the minimum amount of demand necessary to cover costs or as the minimum number of customers required[4].

- Price is set exogenously and consumers bear transportation costs.

- The demand a_i at node *i* is a random variable with mean μ_i and standard deviation σ_i. The correlation r_{ij} between demand at any two nodes *i* and *j* is assumed to be non-negative, following Drezner, Drezner and Shiode (2002). We further assume the total demand captured by each facility to be normally distributed. If a facility captures demand from several demand zones and if the demand correlation among these demand zones is low, then the central limit theorem suggest that the total demand captured by the facility will be approximately normally distributed, even if the demand from each demand zone is not. Therefore, this assumption may not be limiting in practice.

- The probability that a customer patronizes a particular shop is independent of the demand a_i at any node.

- We use the simple **gravity model** to define the capture. According to these models and using the terminology defined by Drezner (1994), "the probability that a consumer patronises a shop (or the proportion of demand capture form a node by one

[4] Demand thresholds are usually measured in terms of population required to support one firm (Shonkwiler and Harris, 1996).

shop) is proportional to its attractiveness and inversely proportional to a power of distance to it". In this paper, we used the simple HUFF model[5] (Huff, 1964).

The integer programming formulation of the New Chance – Constrained Maximum Capture Location problem is as follows:

$$\text{MAX } Z = \sum_{i \in I} \sum_{j \in J} \mu_i \ \rho_{ij} \ x_{ij} \qquad (1)$$

Subject to

$$\sum_{j \in J} x_{ij} = p \qquad \forall i \in I \qquad (2)$$

$$x_{ij} \leq y_j \qquad \forall i \in I, \forall j \in J \qquad (3)$$

$$P\left(\sum_{i \in I} a_i \rho_{ij} x_{ij} \geq T \right) \geq \alpha \qquad \forall j \in J \qquad (4)$$

$$\sum_{j \in J} y_j = p \qquad (5)$$

$$x_{ij} = \{0,1\} \quad y_j = \{0,1\} \qquad \forall i \in I, \forall j \in J \qquad (6)$$

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[5] The Huff probability formulation uses distance (or travel time) from consumer's zones to retail centers and the size of retail centers as inputs to find the probability of consumers shopping at a given retail outlet. He was also the first one to introduce the Luce axiom of discrete choice in the gravity model. Using this axiom, consumers may visit more than one store and the probability of visiting a particular store is equal to the ratio of the utility of that store to the sum of utilities of all stores considered by the consumers.

where the parameters are:

$i, I =$ Index and set of consumers' zones or nodes .

$j, J =$ Index and set of potential locations for entering firm.

J^B $(\subseteq J)$ = The set of actual locations of the q outlets of the existing firm.

$p =$ Number of facilities to locate

$d_{ij} =$ The network distances between consumers' zone *i* and a shop in *j*.

ρ_{ij} = The probability that consumers at location *i* will shop at shop *j*. (i.e., The proportion of capture that a shop in *j* will achieve by consumers' zone *i*), based on HUFF model

$$\rho_{ij} = \frac{A_j / d_{ij}^{\beta}}{\sum_{j \in J^B} A_j / d_{ij}^{\beta} + \sum_{j \in J} A_j / d_{ij}^{\beta} * y_j}$$

where A_j = The attractiveness of shop *j* (as in HUFF, the size of the shop)

$\beta =$ Distance decay parameter (as in HUFF, is equal to 2)

$T =$ Threshold demand level

α = Desired probability of satisfying the threshold level

a_i = Demand at consumers' zone *i*.

μ_i = Mean of a_i

σ_i = Standard deviation of a_i

And the variables are defined as follows:

x_{ij} = 1, if consumers' zone *i* is assigned to node *j*; 0, otherwise.

y_j = 1, if a shop of firm's A is opened at node *j*; 0, otherwise.

The constraint set basically that: constraint set (2) states that every consumer zone makes *p* assignments to the *p* new outlets. But for a demand node *i* to be assigned to a facility at *j*, there has to be a facility open at *j*; this is achieved by constraint set (3). Constraints set (4) allows a facility to open at *j* only if the probability that the total demand assigned to node *j* was above than the threshold level, is at least the desired probability of satisfying this required threshold level. Constraint (5) sets the number of outlets to be opened by the entering firm and constraint (6) is the integrality constraint of the decision variables.

The objective function defines the total expected capture that the entering firm can achieve with the sitting of its p servers.

A deepest analysis of the deterministic equivalent of constraint set (4), following Jobson (1991) shows that this can be rewritten as:

$$\sum_{i \in I} \mu_i \rho_{ij} x_{ij} + K_{1-\alpha} S_j \geq T$$

where, $K_{1-\alpha}$ is the value of a standardized normal distribution above which there is a probability of α

The demand covariance matrix S_j is $\left[\bar{z}_j^T \mathbf{V} \bar{z}_j\right]$, where,

$$V = \begin{bmatrix} \sigma_1^2 & \cdots & r_{1m}\sigma_1\sigma_m \\ \vdots & \ddots & \vdots \\ r_{m1}\sigma_m\sigma_1 & \cdots & \sigma_m^2 \end{bmatrix} \quad \text{and} \quad \bar{z}_j = \begin{pmatrix} \rho_{1j}x_{1j} \\ \ldots \\ \rho_{mj}x_{mj} \end{pmatrix}$$

The formulation of the model is non-linear. The nonlinear condition comes from the fact that ρ_{ij} include the decision variables in the denominator. Moreover, constraint set (4) is, in general, a nonlinear constraint.

4. METAHEURISTIC TO SOLVE THE MODEL

The model presented in the previous section is a combinatorial optimisation problem. Many combinatorial problems are intractable and belong to the class of *NP*-Hard (non-deterministic polynomial-time complete) problems. Kariv and Hakimi (1979) prove that the p-Median problem is a NP-Hard problem on a general graph. Moreover, in this case, the inclusion of a non-linear constraint reinforces the NP-Hard condition of the problem.

The common belief in this field is that no efficient algorithm could ever be found to solve these inherently hard problems. Heuristics, and recently metaheuristics are considered one of the search methods for solving hard combination optimisation problems.

A Metaheuristic is an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space using learning strategies to structure information in order to find efficiently near –optimal solutions (Osman (1995)).

Metaheuristics can be grouped in two classes: *problem – space methods and local search methods*. The problem – space methods are a class of metaheuristics superimposed on fast problem – specific constructive procedure. Its aim is to generate many different starting solutions that can be improved by local search methods. The best representatives are the Greedy Randomized Adaptive Search (GRASP) and recently, Ant System.

The **Ant System** introduced by Colorni, Dorigo and Maniezzo (1991a, 1991b), Dorigo and Di Caro (1999), is a cooperative search algorithm inspired by the behaviour of real ants. Ants deposit an aromatic substance, known as pheromone, on their way to food. An ant chooses a specific path according to the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone is laid down by other ants, therefore the best paths have more intensive pheromone and higher probability of being chosen. The Ant System approach associates pheromone trails to features of the solutions of a combinatorial problem, and can be seen as a kind of adaptive memory of the previous solutions. Solutions are iteratively constructed in a randomized heuristic fashion biased by the pheromone trails left by the previous ants. The pheromone trails, τ_j, are updated after the construction of a solution, ensuring that the best features will have a more intensive pheromone.

Recently, Stützle (1997) proposed an improved version of the Ant System, designated by MAX-MIN Ant System. The MAX-MIN ant system differs from the Ant System in the following way: only the best solution updates the trails in every cycle. To avoid stagnation of the search, i.e. ants always choosing the same path, Stüzle (1998a) proposed a lower and upper limit to the pheromone trail, $\tau_{\min}$ and $\tau_{\max}$, respectively.

Stützle and Hoos(1999), Stützle (1997,1998a) applied this procedure to the Traveling Salesman Problem, the Quadratic Assignment Problem and the Flow-Shop Scheduling Problem; and Lourenço and Serra (2000) applied to the Generalized Assignment Problem.

Local search methods form another class of metaheuristics based on the concept of exploring the vicinity of the current solution. Neighborhood solutions are generated by “a move generation mechanism”. These solutions are selected and accepted according to some pre-defined criteria. The best representatives of this group are *Simulated Annealing, Tabu Search and Genetic Algorithm*. All three local search methods have demonstrated their effectiveness for solving a problem. However, Pirlot (1992) in his tutorial recognized that Tabu Search is generally much faster than Simulated Annealing and Genetic Algorithm.

Tabu Search is a metaheuristic that guides local heuristic search procedures to explore the solution space beyond local optimality. It was introduced by Glover (1989, 1990). In essence, Tabu Search explores a part of the solution space by repeatedly examining all neighborhoods of the current solution, and moving to the best neighborhood even if this leads to a deterioration of the objective function. This approach tries to avoid being trapped in a local optimum. In order to avoid the cycling back to a solution that has recently been examined, nodes are inserted in a tabu list that is constantly updated. Additionally, several criteria of flexibility can be used in the tabu search including aspiration and diversification.

The aspiration criterion is used as insurance against restricting moves which would have led to finding high quality solutions. In other words, the aspiration criterion determines when a node can be move even if tabu. Usually, this criterion states that if a move produces a solution better than the best known solution (and the resulting solution is feasible), then the tabu status is disregarded and the move is executed.

The diversification criterion is utilized to escape from local optima and is achieved by using a long - term memory function. It allows a broader exploration of the solution space by starting from solutions that have not been well explored.

This method has been successfully applied to a wide variety of location problems: the p-hub Location Problems ((Klincewicz, 1992) and (Marianov, et.al., 1997)), the (r | *Xp)*-Medianoid and the (r | *p*)- Centroid Problems (Benati and Laporte, 1994), the Vehicle Routing Problem (Gendreau, et.al., 1994) and the p-Median Problem (Rolland, et.al., 1996).

Summing up, the Metaheuristic applied to this model has two phases. In the first one, a good initial solution is constructed using MAX-MIN Ant System; and in the second phase, the previous solution found is improved applying the well-known Tabu search heuristic (following Benati and Laporte (1994) application of Tabu Search).

As MAX-MIN ANT SYSTEM has been never applied before to location models, we need to adapt this algorithm to the New Chance – Constrained Maximum Capture Location Problem. To do it, we define τ_j as the desirability of locating a shop in j. Initially, $\tau_j = \sum_{i \in I} \frac{A_j}{d_{ij}^{\lambda}}$. The more attractive the index of a shop in j is, the more desired is the location of an outlet in that node.

The MAX-MIN Ant system is an iteratively procedure with three steps:

- In the first step of the iteratively procedure, a initial solution is constructed. To do this, the nodes are ordered with respect to the probability function defined by $p_j = \frac{\tau_j}{\sum_{l \in J} \tau_l}$. The initial solution is choose randomly, taking into account the probability distribution previously defined.

- The second step of the iteratively procedure tries to improve this initial solution by a local search method; in this case, applying a Teitz and Bart heuristic. In both steps, only feasible solutions are allowed.

- Finally, in the third step of the iteratively procedure, the pheromone trails are updated using the current solution in the following way: $\tau_j^{new} = \rho\tau_j^{old} + \Delta\tau_j$, where ρ, 0 < ρ < 1, is the persistence of the trail, i.e. 1 - ρ, represents the evaporation. The updated amount is $\Delta\tau_j = \tau_{\max} * Q$ if an outlet is located in j; 0, otherwise .

  In this final stage, the MAX-MIN limits are checked and imposed $\tau_{\min} \le \tau_j \le \tau_{\max}, \forall j \in J$, if the updated pheromone falls outside the interval. In this case, the values of the parameters of the metaheuristic are set to Q = 0.05, ρ = 0.75, $\tau_{\max} = p * \max \tau_j$ and $\tau_{\min} = (1/p) * \min \tau_j$ (where, p is the number of outlets to locate).

  The termination condition of this iteratively procedure is the number of total iterations.

A formal description of the metaheuristic procedure is presented.

<u>METAHEURISTIC MAX-MIN Ant System + TABU search</u>

PHASE 1: MAX-MIN Ant System

1. Initialise the parameters of Ant System and compute initial $\tau_j = \sum_{i \in I} \frac{A_j}{d_{ij}^{\lambda}}$, for all nodes j of the network.

2. Let k $\rightarrow$ k + 1 (iterations of Ant System procedure)

First Step: Construct a good initial solution

3. Compute $p_j = \dfrac{\tau_j}{\sum_{l \in J} \tau_l}$, for all nodes j of the network.

4. Choose *p* locations from *J* without replacement, using the probabilities from step 3.

5. Check the threshold constraint.

   - If this is not satisfied, go to step 4.

   - If this is satisfied, go to Second Step.

Second Step: Local search phase; Teitz and Bart**.**

In this second step, the well-known Teitz and Bart heuristic (Teitz and Bart, 1968) is applied. The original heuristic states that, at each iteration, one facility is moved from its current position to another potential facility, if the new objective computed is improved. In this application, the move is done only if the new objective computed is improved and if the threshold constraint is satisfied in the new solution. The one-opt trade is done for all nodes and facilities.

Third Step: Update pheromone trails, using the current solution.

6. Compute the news τ_j for all nodes j of the network, using the best locations found in the previous steps. Check also the max – min limits for all τ_j (i.e. $\tau_{\min} \le \tau_j \le \tau_{\max}, \forall j$).

7. If K < MAXITER (maximum number of iterations), set K $\rightarrow$ K+1 and go to step 2. Updating the Best Solution found Z_{BEST} in each MAX-MIN Ant system iteration.

PHASE 2: TABU SEARCH

In the second phase, the solution found in the previous phase is improved by the application of the Tabu Search metaheuristic. In this case, we applied the Benati and Laporte (1994) version of the Tabu Search. This version uses both the aspiration and the diversification criterion. As in the previous case, the move to a potential facility is done only if the threshold constraint is satisfied in the new solution.

5. COMPUTATIONAL EXPERIENCE

The algorithm has been applied to several randomly generated networks, having the number of nodes n equal to 35, 50 and 70. For each n, three different threshold level T were set using the following formula: $T = \gamma\left[{pop}\big/{(p+q)}\right]$, where *pop* is the total expected amount of demand to be served, defined as $pop = \sum_i \mu_i$; and γ as a threshold factor that was set to 0.1, 0.2 and 0.3. For the threshold constraint, we assume α = 95% $\Rightarrow K_{1-\alpha} = -1.645$.

We assume that there are five existing outlets. For each generated network, the location of the five existing outlets are found using the Teitz and Bart heuristic with a weighted total distance objective (i.e., minimised weighted by the population / demand of each node).

For each n, and each T, three different numbers of outlets of the entering firm are used; p = 2,3,4.

In this case, to generate the networks, the distributions of the demand nodes need to be established. We assume that the demand nodes follow a multivariate normal distribution $a_i \sim N(\mu_i, \sigma_i^2)$. This distribution will be established in the following way: $\mu_i \sim$ *uniform*

(50-100) and $\sigma_i^2 \sim \frac{\mu_i}{4}(uniform\ (0-1))$. We also give a priori value to the correlation between different demand nodes. This can be either unrelated or positively related (i.e., $r_{ij} = r = 0$ **or 0.1**, as in Drezner, Drezner and Shiode, 2002)

Finally, we also need to pre-establish the value of the attractiveness of each shop $A_j \sim uniform\ (60,100)$. It can be assumed that the attractiveness level represents the size of the shops .

Summing up, for each n, each γ , each p and each r; ten networks are randomly generated. Therefore, a total of 540 networks are generated.

Optimal solutions are obtained using complete enumeration. The heuristic was programmed in FORTRAN and executed in Pentium III 450 Mhz with 128 mb of RAM. Results of heuristic performance are shown in table 1 and 2.

Table 1. Heuristic Performance (r = 0)

n	*ϒ*	*(p,q)*	*Infeasible*	*Optimal Solution found*	*Average percent deviation at end first phase**	*Average percent deviation at end of second phase**
35	**0.1**	**(2,5)**		100%	5.10%	0.00%
		(3,5)		100%	6.91%	0.00%
		(4,5)		70%	8.52%	1.29%
	0.2	**(2,5)**		90%	4.57%	1.05%
		(3,5)		80%	8.13%	2.02%
		(4,5)		80%	7.82%	1.13%
	0.3	**(2,5)**		100%	2.26%	0.00%
		(3,5)		90%	6.30%	0.42%
		(4,5)		70%	9.16%	2.90%
50	**0.1**	**(2,5)**		100%	4.29%	0.00%
		(3,5)		100%	4.29%	0.00%
		(4,5)		70%	7.41%	2.15%
	0.2	**(2,5)**		90%	4.05%	1.70%
		(3,5)		80%	4.47%	1.46%
		(4,5)		70%	6.80%	2.04%
	0.3	**(2,5)**		100%	1.54%	0.00%
		(3,5)		90%	6.05%	1.40%
		(4,5)		70%	6.23%	1.06%
70	**0.1**	**(2,5)**		90%	1.50%	0.10%
		(3,5)		80%	1.71%	1.20%
		(4,5)		70%	4.99%	0.49%
	0.2	**(2,5)**		90%	1.01%	3.30%
		(3,5)		90%	1.58%	1.90%
		(4,5)		70%	5.90%	0.49%
	0.3	**(2,5)**		90%	1.28%	1.27%
		(3,5)		90%	2.59%	1.72%
		(4,5)		70%	3.65%	0.91%

* For non-optimal solutions

Table 2. Heuristic Performance (r = 0.1)

n	*ϒ*	*(p,q)*	*Infeasible cases*	*Optimal Solutions found**	*Average percent deviation at end first phase***	*Average percent deviation at end of second phase***
35	**0.1**	**(2,5)**		100%	5.41%	0%
		(3,5)		80%	5.67%	2.36%
		(4,5)		80%	8.56%	1.11%
	0.2	**(2,5)**		100%	4.85%	0%
		(3,5)		100%	8.27%	0%
		(4,5)		70%	5.87%	1.66%
	0.3	**(2,5)**		90%	4.58%	3.39%
		(3,5)		100%	6.34%	0%
		(4,5)		80%	8.92%	0.86%
50	**0.1**	**(2,5)**		100%	1.91%	0%
		(3,5)		90%	6.03%	0.42%
		(4,5)		70%	5.81%	1.24%
	0.2	**(2,5)**		90%	3.15%	0.78%
		(3,5)		80%	4.94%	3.81%
		(4,5)		70%	5.44%	2.23%
	0.3	**(2,5)**	20%	100%	21.94%	0%
		(3,5)	30%	90%	1.88%	0.36%
		(4,5)	20%	80%	23.36%	0.62%
70	**0.1**	**(2,5)**		100%	0.36%	0%
		(3,5)	10%	80%	2.89%	1.23%
		(4,5)	20%	70%	2.77%	2.31%
	0.2	**(2,5)**	40%	100%	0.21%	0%
		(3,5)	70%	90%	1.99%	0.73%
		(4,5)	100%	100%	0%	0%
	0.3	**(2,5)**	100%	100%	0%	0%
		(3,5)	100%	100%	0%	0%
		(4,5)	100%	100%	0%	0%

* For the feasible cases. * For non-optimal solutions in the feasible cases.

The percentages of optimal solutions for the feasible cases are presented in the column labelled "optimal solutions found". If at least a no optimal solution is found among the ten runs, the average deviation from optimality in both stages of the metaheuristic are presented at the two last columns. In this case, it can be noticed that the stochastic condition of the model arises the difficulty to find the optimal solutions. With the metaheuristic, a near-optimal solutions were found with a minimal deviation.

- **r = 0**. 41 out of 270 runs were non-optimal based on our comparison with complete enumeration. The maximum average deviation from optimality did not exceed 3.3 %.
- **r = 0.1**. 29 out of 270 runs were non-optimal based on our comparison with complete enumeration. The maximum average deviation from optimality did not exceed 3.9 %.

In both tables, an additional column has been included. The column "infeasible cases" represents the percentages of cases without a feasible solution; in other words, a network where the entering firm cannot find a solution that satisfied all the constraints; included the threshold constraint. It can be noticed that this lack of solution appears in table 2 with an r = 0.1. We can deduce, from previous models without an stochastic threshold constraint, that this constraint is the one no satisfied in these cases. A statistical interpretation of this reality can be that a greater correlation means a greater S_j and, as $K_{1-\alpha}$ is negative, the threshold constraint is more difficult to achieve.

Tables 3 and 4 show the average execution time in seconds spend per phases by global metaheuristic and per enumeration procedure.

Table 3. Time Performance (r = 0)

N	*ϒ*	*(p,q)*	*Max-Min Average Time*	*TABU Average Time*	*TOTAL Average Time*	*Enumeration Average Time*
35	**0.1**	**(2,5)**	0.20	1.34	1.55	0.44
		(3,5)	0.34	3.08	3.42	7.80
		(4,5)	0.36	5.51	5.87	86.58
	0.2	**(2,5)**	0.36	1.34	1.70	0.46
		(3,5)	0.32	3.11	3.43	7.78
		(4,5)	0.33	5.60	5.93	86.59
	0.3	**(2,5)**	0.19	1.33	1.52	0.46
		(3,5)	0.35	3.07	3.42	7.79
		(4,5)	0.38	5.61	5.99	86.60
50	**0.1**	**(2,5)**	1.14	2.31	3.45	1.62
		(3,5)	1.16	5.44	6.60	40.40
		(4,5)	1.42	10.00	11.41	659.86
	0.2	**(2,5)**	0.60	2.29	2.88	1.60
		(3,5)	0.91	5.38	6.29	40.39
		(4,5)	1.77	9.86	11.63	659.84
	0.3	**(2,5)**	0.74	2.30	3.04	1.61
		(3,5)	1.01	5.38	6.39	40.39
		(4,5)	1.08	9.70	10.77	659.78
70	**0.1**	**(2,5)**	4.02	3.96	7.98	5.41
		(3,5)	8.33	9.18	17.50	192.03
		(4,5)	8.52	16.65	25.17	4458.96
	0.2	**(2,5)**	2.66	3.93	6.59	5.40
		(3,5)	5.11	9.14	14.25	191.70
		(4,5)	5.03	16.41	21.44	4451.67
	0.3	**(2,5)**	3.44	3.96	7.40	5.40
		(3,5)	4.29	9.25	13.53	191.76
		(4,5)	11.82	16.88	28.70	4452.62

Table 4. Time Performance (r = 0.1)

n	*ϒ*	*(p,q)*	*Max-Min Average Time*	*TABU Average Time*	*TOTAL Average Time*	*Enumeration Average Time*
35	**0.1**	**(2,5)**	0.23	1.33	1.55	0.46
		(3,5)	0.57	3.08	3.64	7.80
		(4,5)	0.37	5.62	5.99	86.58
	0.2	**(2,5)**	0.28	1.31	1.60	0.47
		(3,5)	0.32	3.13	3.45	7.80
		(4,5)	0.51	5.55	6.06	86.59
	0.3	**(2,5)**	0.22	1.32	1.55	0.45
		(3,5)	0.45	3.08	3.53	7.80
		(4,5)	0.57	5.64	6.21	86.64
50	**0.1**	**(2,5)**	1.66	2.31	3.97	1.61
		(3,5)	1.11	5.31	6.42	40.39
		(4,5)	1.50	9.83	11.33	660.78
	0.2	**(2,5)**	1.17	2.31	3.48	1.62
		(3,5)	1.74	5.16	6.90	40.39
		(4,5)	3.35	9.67	13.01	659.72
	0.3	**(2,5)**	2.42	2.16	4.58	1.61
		(3,5)	2.85	4.99	7.84	40.40
		(4,5)	4.16	8.97	13.13	659.98
70	**0.1**	**(2,5)**	5.70	3.76	9.46	5.39
		(3,5)	9.75	8.47	18.22	191.98
		(4,5)	14.12	16.56	30.68	4463.64
	0.2	**(2,5)**	8.96	3.53	12.49	5.40
		(3,5)	12.30	7.74	20.05	191.66
		(4,5)	18.12	14.02	32.13	4459.10
	0.3	**(2,5)**	8.12	3.40	11.52	5.42
		(3,5)	13.66	7.74	21.40	191.67
		(4,5)	17.87	13.98	31.85	4459.01

The average computing time of the heuristic is similar, maintaining the others parameters equal, for a network assuming r = 0 and r =0.1. The average computing time of the heuristic increases with the number of nodes and the number of outlets, as expected. Notice that the algorithm becomes very useful when we have to locate 3 or more entering outlets, regardless of the constraint level. In these cases, the time spent by the algorithm is less than the one for the enumeration procedure. For example, in n = 70, p = 4 and Υ=0.2, the time spent by the algorithm is 21.44 seconds while the enumeration procedure spent 4451.67 seconds to find the same solution.

6. AN EXAMPLE

The model was also tested in the well-known Swain's (1974) 55-node network (Figure A1 in the appendix). The demand at each node follows a multivariate normal distribution, considering: μ_i is the original demand of the Swain's network indicated in Table A1 of the appendix, $\sigma_i^2 \not\sim \frac{\mu_i}{4}(uniform\ \mathbf{(0-1)})$ and $r_{ij} = r = 0$. In this case, the total amount of demand to be captured is not always equal to 3.575.

We also need to pre-establish the value of the attractiveness of each shop. In this case, we assume that all the shops have the same attractiveness, ($A_j = 100$), regardless of node and ownership.

The model is solved to optimality by using complete enumeration. As in the previous section, the location of the five existing outlets are found using the Tetiz and Bart heuristic with the weighted total distance objective.

For the example, different scenarios are examined; which varies with respect to the number of outlets to be located by Firm A (p = 2, 3 and 4), and to the threshold level T:

$T = \gamma\left[{pop}/{(p+q)} \right]$ (where pop is the total expected amount of demand to be served; i.e., $pop = \sum_i \mu_i$ and $\gamma = 0.3$, 0.5 and 0.7).

Results are shown in table 5, 6 and 7. In these tables, the locations and percentage of demand captured by Firm B are computed before and after the entering of Firm A locates its outlets (using as objective function the one of the New Chance – Constrained Maximum Capture Location Problem). Firm's A optimal locations and its percentage of demand capture are also computed. Finally, the following values are also computed for each scenario:

- $\% \; Capture > T = \dfrac{(Capture \text{ - Threshold level})}{\text{Threshold level}} * 100$; the percentage of capture above the threshold level achieve by each Firm's A location.

- $\% \; Constraint \; A. = \dfrac{\left(\sum_{i=1}^{m} \mu_i \rho_{ij} x_{ij} + K_{1-\alpha} S_j - Threshold \text{ Level} \right)}{Thershold \text{ Level}} * 100$; the percentage of threshold constraint accomplishment.

Note that the percentage of capture above the threshold level measures the accomplishment of the threshold constraint in the actual event. While the percentage of threshold constraint accomplishment measures this value in the general characteristics of the scenario defined.

Table 5. 55-nodes example (r = 0 and Υ=0.3)

Υ	(p,q)	Firm's B Location	Initial Capture	Final Capture	Firm's A Location	Capture	% Capture > T	% Constraint A.
0.3	**(2,5)**	17	16%	14%	4	18%	311%	240%
		41	22%	13%	5	12%	178%	228%
		38	13%	12%				
		31	18%	16%				
		5	30%	15%				
		Total Capture	***100%***	***70%***		***30%***		
	(3,5)	12	12%	10%	4	15%	316%	284%
		41	22%	12%	5	13%	243%	243%
		38	14%	11%	31	11%	210%	209%
		31	21%	13%				
		5	31%	15%				
		Total Capture	***100%***	***61%***		***39%***		
	(4,5)	17	13%	11%	3	11%	234%	245%
		25	25%	10%	4	14%	327%	266%
		38	12%	9%	5	11%	233%	238%
		3	25%	14%	25	8%	137%	150%
		5	25%	14%				
		Total Capture	***100%***	***57%***		***43%***		

Table 6. 55-nodes example (r = 0 and Υ=0.5)

Υ	(p,q)	Firm's B Location	Initial Capture	Final Capture	Firm's A Location	Capture	% Capture > T	% Constraint A.
0.5	**(2,5)**	22	16%	13%	3	17%	136%	94%
		25	21%	14%	5	15%	109%	124%
		38	11%	10%				
		31	21%	14%				
		6	31%	18%				
		Total Capture	***100%***	***69%***		***31%***		
	(3,5)	16	16%	11%	3	11%	75%	91%
		41	25%	12%	4	14%	141%	110%
		23	15%	11%	5	14%	130%	100%
		3	18%	13%				
		2	25%	14%				
		Total Capture	***100%***	***61%***		***39%***		
	(4,5)	22	17%	11%	3	13%	127%	96%
		20	17%	10%	4	15%	160%	124%
		31	24%	11%	5	10%	69%	107%
		38	11%	9%	31	9%	64%	74%
		5	32%	13%				
		Total Capture	***100%***	***54%***		***46%***		

Table 7. 55-nodes example (r = 0 and Υ=0.7)

Υ	(p,q)	Firm's B Location	Initial Capture	Final Capture	Firm's A Location	Capture	% Capture > T	% Constraint A.
0.7	**(2,5)**	12	10%	9%	5	18%	79%	77%
		25	20%	14%	31	12%	24%	29%
		31	24%	15%				
		38	14%	12%				
		5	32%	20%				
		Total Capture	***100%***	***70%***		***30%***		
	(3,5)	22	19%	13%	4	16%	79%	62%
		25	21%	12%	5	11%	28%	51%
		43	11%	10%	31	11%	22%	25%
		31	20%	13%				
		5	28%	14%				
		Total Capture	***100%***	***62%***		***38%***		
	(4,5)	22	19%	12%	3	14%	83%	53%
		20	15%	10%	4	14%	82%	65%
		38	11%	9%	5	11%	44%	54%
		18	19%	10%	18	8%	4%	19%
		5	36%	14%				
		Total Capture	***100%***	***54%***		***46%***		

From the previous tables, we can point out the following:

- The percentage of total demand achieved by the entering firm is the same for a given number of outlets located, regardless threshold level. For example, when the entering firm locates 3 outlets, it captures the 39%, 39% and 38% of total demand, with Υ=0.3, 0.5 and 0.7 respectively.

- Obviously, the percentage of capture above the threshold level and the percentage of threshold constraint accomplishment achieve by each Firm's A, decrease with an increase of Υ value.

Finally, the robustness of the model is checked in the example. The model is solved several times in a specific scenario[6]. In each simulation of this scenario, demand nodes are randomly chosen following the fixed normal distribution defined and the model is solved to optimality by using complete enumeration.

Given the stochastic condition of the model, we want to check if the optimal locations vary in these different events of the demand nodes following a fixed normal distribution. The model was solved 200 times, and in all the cases, the optimal solution found was the same[7]. Therefore, we can conclude that the model is quite robust.

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[6] The existing firm has five outlets located in nodes 5, 22, 25, 31, 38; and the entering firm wants to locate 3 new outlets. The threshold level is defined as: $T = \gamma\left[pop \big/ (p+q)\right]$ (where $pop = \sum_i \mu_i$ and $\gamma = 0.5$). The rest of conditions are the ones defined for the example (page 23).

[7] The entering firm has to locate its outlets in nodes 1, 4 and 31.

7. CONCLUSIONS

In this paper, a new location model have been presented to study the issue of minimum requirements to survive in a given spatial setting. The threshold requirement has been introduced as a stochastic constraint. A metaheuristic based on MAX – MIN Ant System and TABU system has been used to solve the new model. It is the first time that the MAX – MIN Ant system is adapted to solve a location problem.

The model is particularly relevant to private retail sector setting because it takes into account two real characteristics of the market. First of all, the capture is determined by a gravity model, which is a revealed preference model. And secondly, the model includes a threshold constraint which reflects the fact that a facility cannot be open if the demand captured is below a threshold level.

Future research will focus on the effect of *e-commerce* in the retail location decisions. This can be analyzed from two perspectives. First of all, models can be modified to take into account that consumer store-choice behavior can be changed with the possibility of shopping from their computer. Secondly, up till now, all the Store Location models were based on the assumption that consumers go to the shop. With the e-commerce, this assumption is not absolutely true because, in this new business environment, part of the business is done in the reverse way; i.e. the stores go to the consumer's houses.

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APPENDIX

Table A1. 55-Node Demand of Swain's (1974) Network.

Node	Demand	Node	Demand	Node	Demand
1	120	20	77	39	47
2	114	21	76	40	44
3	110	22	74	41	43
4	108	23	72	42	42
5	105	24	70	43	41
6	103	25	69	44	40
7	100	26	69	45	39
8	94	27	64	46	37
9	91	28	63	47	35
10	90	29	62	48	34
11	88	30	61	49	33
12	87	31	60	50	33
13	87	32	58	51	32
14	85	33	57	52	26
15	83	34	55	53	25
16	82	35	54	54	24
17	80	36	53	55	21
18	79	37	51		
19	79	38	49		

Figure A1. 55 – node network (Swain, 1974)