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Adverse Selection and Security Design*

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Abstract

This paper studies the problem of optimal security design by a privately informed entrepreneur. In the context of a simple parametric model, it is shown that the entrepreneur does not find it profitable to float an asset that affords her an informational advantage. In fact, it is optimal simply to issue equity. The reason is that, with rational, uninformed outside investors, the entrepreneur faces adverse selection in the security market, which prevents her from exploiting her position as an insider. This is true whether or not she has market power in trading the asset.

Consider a risk-averse entrepreneur who is contemplating what security to issue. The entrepreneur anticipates that, at the time of trading in the security market, she will have private information on the payoff of her investment, as well as on other risks in the economy. The design of the asset is, therefore, influenced not only by the usual hedging motive, but also by informational considerations involving the potential for insider trading profits.

This paper studies the problem of optimal security design in the context of a simple parametric model with Gaussian risks and CARA utility. It is shown that the entrepreneur (or insider, as she will sometimes be called) does not find it profitable to float an asset that affords her an informational advantage. In fact, it is optimal simply to issue equity,¹ in which case equilibrium is fully revealing. The reason is that, with rational, uninformed outside investors, the entrepreneur faces adverse selection in the security market, which prevents her from exploiting her position as an insider. This is true whether or not she can exercise market power in trading the asset.

The issue of security design when prices have an informational role to play is just beginning to be researched. Closely related to the present paper is the work of Demange and Laroque (1992), who use a rational expectations model with noise traders to analyze the problem posed above. Except for some special cases, however, they are able to get only numerical results. In comparison, the present paper obtains partially revealing equilibria in a model in which all agents are rational. The role of adverse selection in thinning the market is thus brought into clearer focus, resulting in a sharp characterization of the optimal security.

For other perspectives on financial innovation in a partially revealing rational expectations framework, see Paul (1989) for an agency-theoretic model of corporate security design, Boot and Thakor (1993) for a model in which multiple claims are issued to stimulate informed trading by outside investors, and Rahi (1994) and Ohashi (1992, 1993) for models of innovation by futures exchanges. For research on security innovation in a symmetric information setting see, for example, Duffie and Jackson (1989), Hara (1992, 1993), Allen and Gale (1989, 1991), and Pesendorfer (1992). Harris and Raviv (1992) contains a survey of the corporate finance literature on security design that focuses on considerations of agency costs and corporate control. This survey also covers adverse selection models with signalling through choice of capital structure. The present paper does not fall in this category for

¹ The term "equity" is used in this paper to denote a share in the payoff of the enterprise, with unlimited liability.

two reasons. First, it does not take any collection of securities (such as debt and equity) as exogenously given. Rather, it studies the more fundamental problem of which security is optimal from the point of view of the issuer. Second, the asset choice does not serve as a signal, since there is no private information at the design stage.

That adverse selection can have a profound impact on security market equilibrium was first noticed by Hirshleifer (1971). The formulation employed in this paper is adapted from Bhattacharya *et al* (1991, 1994), who derive conditions under which informed trading by a monopolistic insider results in a breakdown of trade. They do not study the security design problem.

The paper is organized as follows. The basic setup is described in the next section. Section 2 analyzes security design by an entrepreneur who is a monopolist in the asset market. Section 3 looks at the case in which she is a price-taker. The final section concludes. Proofs of the technical lemmas are contained in the Appendix.

1. The Model

All random variables are defined on a fixed probability space (Ω, \mathcal{F}, P) . The following notational convention will be used throughout: for random variables g and h , $V_g := \text{Var}(g)$ and $V_{gh} := \text{cov}(g, h)$. Matrices and vectors are distinguished by boldface type. The symbol \top denotes transpose. All normally distributed random variables that appear in the paper belong to a linear space \mathcal{N} of joint normally distributed random variables, endowed with an inner product as follows: for $g, h \in \mathcal{N}$, $(g|h) := \text{cov}(g, h)$.

There are two agents, with von Neumann-Morgenstern utility functions displaying constant absolute risk-aversion (coefficient τ_i for agent i). Agent 1 is a privately informed entrepreneur with a random endowment e , which may be thought of as the outcome of a previous investment decision. Agent 2 is an uninformed outside investor. The economy unfolds in three stages. At the *ex ante* stage the entrepreneur issues an asset with payoff $f \in \mathcal{N}$.² At the *interim* stage she observes an n -vector of signals $\mathbf{s} \in \mathcal{N}^n$, which provides information on the magnitude of her endowment e and on other risks in the economy; and trading occurs on the asset market. Finally, at the *ex post* stage, all uncertainty is resolved, the asset pays off, and consumption takes place.

Given an asset $f \in \mathcal{N}$, a position θ_1 in the asset market leaves the insider with net

² The restriction to this class is a nontrivial one. It excludes limited liability assets, for example.

wealth

$$w_1 := e + \theta_1(f - p),$$

where p is the asset price.³ Correspondingly, the outsider's net wealth is

$$w_2 := \theta_2(f - p). \quad (1)$$

It is assumed that the entrepreneur's endowment can be written as the product of two independent random variables x and z in \mathcal{N} , each with mean zero. Furthermore, it is assumed that $s \sim N(\mathbf{0}, \mathbf{V}_s)$, where \mathbf{V}_s is positive definite; that $x = \mathbf{r}^\top s$, for some nonzero vector \mathbf{r} in \mathbb{R}^n ; and that z is independent of s . Thus, $e = xz$, where z can be interpreted as the normalized value of the endowment e , about which no information is available at the time of trading; and x can be viewed as a scale parameter whose value can be perfectly inferred given the insider's private information.

Let $\mathbf{y} := (y_1, \dots, y_{n-1})$ be an orthonormal basis for the orthogonal complement of x in the linear subspace of \mathcal{N} spanned by s_1, \dots, s_n . Then, any asset $f \in \mathcal{N}$ can be written in the form:

$$f = \bar{f} + az + bx + c\mathbf{k}^\top \mathbf{y} + d\epsilon, \quad (2)$$

where $(\bar{f}, a, b, c, d) \in \mathbb{R}^5$, $\mathbf{k} \in \mathbb{R}^{n-1}$; and $\epsilon \sim N(0, 1)$ is independent of $(z, \mathbf{x}, \mathbf{y})$ (or, equivalently, of (z, s)). The distributional assumptions can be summarized as follows:

$$(z, \mathbf{x}, \mathbf{y}, \epsilon) \sim N[\mathbf{0}, \text{diag}(V_z, V_x, \mathbf{I}_{n-1}, 1)]. \quad (3)$$

It is convenient to refer to \mathbf{y} as the extraneous private information of the insider (that is, information unrelated to her own endowment), and to ϵ as the extraneous noise in the asset payoff. "Hats" and "tildes" are used to denote moments conditional on s (or, equivalently, on (\mathbf{x}, \mathbf{y})) and p respectively. For example, for random variables g and h , $\hat{E}_g := E(g|s) = E(g|\mathbf{x}, \mathbf{y})$, and $\tilde{V}_{gh} := \text{cov}(g, h|p)$.

In this model, the outsider serves as a representative agent for a large number of uninformed investors (with CARA utility). Hence he is assumed to be a price-taker. He also has rational expectations, that is he knows the "price function" $p : \Omega \rightarrow \mathbb{R}$, and uses his observation of the asset price to update his beliefs. In general, prices are partially revealing.

³ It is implicitly assumed that either f is a futures-type contract, for which the payoff and payment are settled at the same date, or that there exists a riskless asset whose price and payoff are normalized to one.

For any choice of asset at the *ex ante* stage, both agents maximize expected utility of net wealth given their information (the information of the outsider being just what he can infer from prices). For expositional simplicity, it is assumed that whenever an agent is indifferent between a nonzero asset position and not trading at all, the latter is chosen. In the following section, the insider is a monopolist in the asset market. Section 3 studies the case in which both agents behave competitively. In either scenario, the problem is that of calculating the equilibrium *ex ante* expected utility of the entrepreneur for any given $f \in \mathcal{N}$, and then studying the entrepreneur's optimal asset choice. A technical assumption is needed to ensure that this problem is well-defined:

ASSUMPTION 1.1. $1 - r_1^2 V_x V_z > 0$.

This will be a standing assumption for the rest of the paper. It turns out to be a necessary and sufficient condition for the expected utility integral of the insider to converge for each asset $f \in \mathcal{N}$.

2. The Entrepreneur as a Monopolist

Suppose, then, that after a security $f \in \mathcal{N}$ has been designed, the entrepreneur behaves monopolistically, that is she understands the impact that the size of her asset position has on the equilibrium asset price. Attention will be restricted to linear equilibria⁴ of the form:

$$p(\theta_1) = \bar{p} + \delta\theta_1, \quad (\bar{p}, \delta) \in \mathbb{R}^2. \quad (4)$$

The price function must be consistent with market-clearing:

$$\theta_1 + \theta_2 = 0. \quad (5)$$

Since the outsider's asset demand θ_2 depends only on the price p , this condition implies that the coefficient δ is nonzero.

The entrepreneur faces the following optimization problem:

$$\max_{\theta_1 \in \mathcal{M}} E[-\exp(-r_1 w_1)], \quad (6)$$

⁴ It is worth noting that, in this model, linear equilibria are *focal* in the sense that there exists an equilibrium if and only if there exists a linear equilibrium. See Bhattacharya *et al* (1994).

wealth

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It is assumed that the entrepreneur's endowment can be written as the product of two independent random variables x and z in \mathcal{N} , each with mean zero. Furthermore, it is assumed that $s \sim N(\mathbf{0}, \mathbf{V}_s)$, where \mathbf{V}_s is positive definite; that $x = \mathbf{r}^\top s$, for some nonzero vector \mathbf{r} in \mathbb{R}^n ; and that z is independent of s . Thus, $e = xz$, where z can be interpreted as the normalized value of the endowment e , about which no information is available at the time of trading; and x can be viewed as a scale parameter whose value can be perfectly inferred given the insider's private information.

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$$f = \bar{f} + az + bx + c\mathbf{k}^\top \mathbf{y} + d\epsilon, \quad (2)$$

where $(\bar{f}, a, b, c, d) \in \mathbb{R}^5$, $\mathbf{k} \in \mathbb{R}^{n-1}$; and $\epsilon \sim N(0, 1)$ is independent of $(z, \mathbf{x}, \mathbf{y})$ (or, equivalently, of (z, s)). The distributional assumptions can be summarized as follows:

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In this model, the outsider serves as a representative agent for a large number of uninformed investors (with CARA utility). Hence he is assumed to be a price-taker. He also has rational expectations, that is he knows the "price function" $p : \Omega \rightarrow \mathbb{R}$, and uses his observation of the asset price to update his beliefs. In general, prices are partially revealing.

³ It is implicitly assumed that either f is a futures-type contract, for which the payoff and payment are settled at the same date, or that there exists a riskless asset whose price and payoff are normalized to one.

where

$$w_1 = e + \theta_1 [f - p(\theta_1)],$$

and \mathcal{M} is the space of s -measurable random variables. Conditional on s (and therefore on (x, y)), any choice of θ_1 leaves w_1 normally distributed. The expected utility of the entrepreneur is

$$\begin{aligned} E[-\exp(-r_1 w_1)] &= -E\left[E[\exp(-r_1 w_1)|s]\right] \\ &= -E\left[\exp\left(-r_1 \left[E(w_1|s) - \frac{r_1}{2} \text{Var}(w_1|s)\right]\right)\right]. \end{aligned} \quad (7)$$

The problem (6), therefore, reduces to maximizing

$$E(w_1|s) - \frac{r_1}{2} \text{Var}(w_1|s) = \theta_1 \left(\hat{E}_f - p(\theta_1)\right) - \frac{r_1}{2} \left(\hat{V}_e + \theta_1^2 \hat{V}_f + 2\theta_1 \hat{V}_{ef}\right) \quad (8)$$

pointwise for each realization of s . The first-order condition is:

$$\hat{E}_f - \bar{p} - r_1 \hat{V}_{ef} - \theta_1 (r_1 \hat{V}_f + 2\delta) = 0, \quad (9)$$

and the second-order sufficient condition for a maximum is:

$$r_1 \hat{V}_f + 2\delta > 0. \quad (10)$$

If $r_1 \hat{V}_f + 2\delta < 0$, the entrepreneur's problem (6) has no solution, and as a consequence there does not exist an equilibrium. If $r_1 \hat{V}_f + 2\delta = 0$, (6) has a solution if and only if

$$\hat{E}_f - \bar{p} - r_1 \hat{V}_{ef} = 0, \quad \forall s. \quad (11)$$

In this case, however, it is optimal to set $\theta_1 = 0$. An equilibrium will be called *trivial* if it entails a zero amount of trade. In any nontrivial equilibrium, therefore, (10) must hold, so that the optimal asset position of the entrepreneur is:

$$\theta_1 = \frac{\hat{E}_f - \bar{p} - r_1 \hat{V}_{ef}}{r_1 \hat{V}_f + 2\delta}. \quad (12)$$

Agent 2, the uninformed investor, solves

$$\max_{\theta_2 \in \mathcal{L}} E[-\exp(-r_2 w_2)],$$

where \mathcal{L} is the space of p -measurable random variables, and w_2 is given by (1). His asset demand function is:

$$\theta_2 = \frac{\tilde{E}_f - p}{r_2 \tilde{V}_f}, \quad (13)$$

provided $\tilde{V}_f > 0$, which is the case in any nontrivial equilibrium.

LEMMA 2.1. *With a monopolistic insider, a nontrivial linear equilibrium exists if and only if*

$$J := r_1^2 a^2 V_x V_z^2 - b^2 V_x - c^2 \mathbf{k}^\top \mathbf{k} > 0. \quad (14)$$

Such an equilibrium is unique, and the price function is given by

$$p = \bar{f} + \frac{K}{J} \theta_1, \quad (15)$$

where

$$\begin{aligned} K := & r_1(a^2 V_z + d^2) [-bV_x(r_1 aV_z - b) + c^2 \mathbf{k}^\top \mathbf{k}] + r_2(a^2 V_z + d^2) [V_x(r_1 aV_z - b)^2 + c^2 \mathbf{k}^\top \mathbf{k}] \\ & + r_1^2 r_2 a^2 c^2 \mathbf{k}^\top \mathbf{k} V_x V_z^2. \end{aligned} \quad (16)$$

A situation in which no trade takes place, either because an equilibrium does not exist or because the equilibrium is trivial, will be referred to as a *market breakdown*, following Bhattacharya and Spiegel (1991). The above lemma shows that there is a market breakdown if and only if $J \leq 0$ (condition (14) is violated). This happens when the informational motive of the insider is too strong relative to her hedging motive.

The next lemma shows how the equilibrium *ex ante* expected utility of the entrepreneur (which will henceforth be referred to simply as the entrepreneur's equilibrium utility) depends on the asset he designs.

LEMMA 2.2. *The equilibrium utility of the monopolistic entrepreneur is given by*

$$\mathcal{U}_m := - [(1 - r_1^2 V_x V_z) + \mathcal{G}_m(f)]^{-\frac{1}{2}},$$

where

$$\mathcal{G}_m := J \cdot \frac{r_1 [V_x(r_1 aV_z - b)^2 + c^2 \mathbf{k}^\top \mathbf{k}(1 - r_1^2 V_x V_z)]}{(r_1 + 2r_2)(a^2 V_z + d^2) [V_x(r_1 aV_z - b)^2 + c^2 \mathbf{k}^\top \mathbf{k}] + 2r_1^2 r_2 a^2 c^2 \mathbf{k}^\top \mathbf{k} V_x V_z^2},$$

unless there is a market breakdown ($J \leq 0$), in which case $\mathcal{G}_m = 0$.

The problem of the insider at the *ex ante* stage is to design an asset that maximizes her equilibrium utility. Since she can always design a security such that \mathcal{G}_m is strictly positive, it is not optimal for her to violate (14). There is clearly no point in issuing a security that is not going to be traded. Furthermore, we have

PROPOSITION 2.3. *The monopolistic entrepreneur's equilibrium utility is monotonically decreasing in the weights that the asset payoff assigns to extraneous private information and extraneous noise. Specifically,*

$$\frac{\partial \mathcal{U}_m}{\partial (c^2)} \leq 0 \quad \text{and} \quad \frac{\partial \mathcal{U}_m}{\partial (d^2)} \leq 0.$$

The derivatives are zero if and only if there is a market breakdown ($J \leq 0$).

Proof. From Lemma 2.2, \mathcal{G}_m is a monotonic transformation of \mathcal{U}_m . The sign of the derivative of \mathcal{G}_m with respect to d^2 is obvious from inspection (using Assumption 1.1). After some tedious algebra it can be shown that

$$\begin{aligned} \text{sgn} \left(\frac{\partial \mathcal{G}_m}{\partial (c^2)} \right) = & - \text{sgn} \left((r_1 + 2r_2)(1 - r_1^2 V_x V_z)(a^2 V_z + d^2) [V_x(r_1 a V_z - b)^2 + c^2 \mathbf{k}^\top \mathbf{k}]^2 \right. \\ & + 2r_1^2 r_2 a^2 V_x V_z^2 (1 - r_1^2 V_x V_z) c^4 (\mathbf{k}^\top \mathbf{k})^2 \\ & + 2r_1^2 r_2 a^2 V_x^3 V_z^2 (r_1 a V_z - b)^2 (r_1^2 a^2 V_z^2 - b^2) \\ & + r_1^2 (r_1 + 2r_2) V_x^3 V_z (a^2 V_z + d^2) (r_1 a V_z - b)^2 \\ & \left. \cdot [(r_1 a V_z - b)^2 + r_1^2 a^2 V_z^2 - b^2] \right). \end{aligned}$$

Each of the terms in the parenthesis is positive (from Assumption 1.1 and (14)). ■

And now the main result:

THEOREM 2.4. *The class of assets that are optimal for the monopolistic entrepreneur is given by the set*

$$S_m := \{f = \bar{f} + az \mid (\bar{f}, a) \in \mathbb{R}^2, a \neq 0\}.$$

Designing an asset in this class is equivalent to issuing equity in the entrepreneur's payoff e . In equilibrium, a constant proportion $2r_2/(r_1 + 2r_2)$ of e is retained by the entrepreneur, the rest being held by the outsider.

Proof. It is not optimal for the entrepreneur to design an asset for which there is a market breakdown. From Proposition 2.3, therefore, an optimal asset is of the form (2), with $c = d = 0$. Lemma 2.2 then gives:

$$\mathcal{G}_m = \frac{r_1 V_x (r_1^2 a^2 V_z^2 - b^2)}{(r_1 + 2r_2) a^2 V_z},$$

which is decreasing in b^2 , and independent of \bar{f} . With $b = 0$, it is also independent of a . However, $a \neq 0$, for otherwise there is a market breakdown (see (14)).

Now, consider the equilibrium for an asset in \mathcal{S}_m . It is straightforward to calculate the price function and optimal asset position of the insider, using Lemma 2.1 and (21), and plugging the relevant expressions into (4) and (12):

$$\begin{aligned} p &= \bar{f} + \tau_2 a^2 V_z \theta_1 \\ \theta_1 &= -\frac{\tau_1}{a(\tau_1 + 2\tau_2)} x. \end{aligned} \quad (17)$$

Clearly, the constant \bar{f} is irrelevant since it is completely reflected in the price. Let ζ_m be the retained equity of the entrepreneur. Using (17):

$$\begin{aligned} \zeta_m &:= e + \theta_1 (f - \bar{f}) \\ &= e \left(\frac{2\tau_2}{\tau_1 + 2\tau_2} \right). \quad \blacksquare \end{aligned}$$

Note that, while the price of an optimal asset does depend on the entrepreneur's private information, the equilibrium is fully revealing. In fact, if we think of the case in which there is no private information at all (fix the scale parameter x to be one, for example), an optimal asset is one that has the highest correlation with the insider's endowment, that is, precisely of the form above. It is easy to verify that the proportion of retained equity is also the same.

3. The Entrepreneur as a Price-taker

This section analyzes optimal security design when the entrepreneur is a price-taker in the asset market. She solves the same problem as in the previous section, except that she assumes that $\delta = 0$. In any nontrivial equilibrium $\tilde{V}_f \geq \hat{V}_f > 0$, and the insider's asset demand function is:

$$\theta_1 = \frac{\hat{E}_f - p - \tau_1 \hat{V}_{ef}}{\tau_1 \hat{V}_f}. \quad (18)$$

The demand function of agent 2 is given by (13) as before. Now, an equilibrium price function of the form (4) can be determined by using the market-clearing condition (5).

LEMMA 3.1. *When both the insider and the outsider are price-takers, a nontrivial linear equilibrium exists if and only if*

$$L := \tau_1 a V_x V_z (\tau_1 a V_z - b) \neq 0. \quad (19)$$

Such an equilibrium is unique, and the price function is given by

$$p = \bar{f} + \frac{K}{L} \theta_1, \quad (20)$$

where K is defined by (16).

For a price-taking insider a market breakdown occurs if and only if $L = 0$. Comparing (14) and (19), it can be seen that the adverse selection problem is more severe in the monopolistic case. The set of assets for which there is a market breakdown for the competitive entrepreneur is a proper subset of the corresponding set for the monopolist.

LEMMA 3.2. *The equilibrium utility of the competitive entrepreneur is given by*

$$\mathcal{U}_c := - \left[(1 - r_1^2 V_x V_z) + \mathcal{G}_c(f) \right]^{-\frac{1}{2}},$$

where

$$\mathcal{G}_c := L^2 \cdot \frac{r_1^2 (a^2 V_z + d^2) [V_x (r_1 a V_z - b)^2 + c^2 \mathbf{k}^\top \mathbf{k} (1 - r_1^2 V_x V_z)]}{\left[(r_1 + r_2) (a^2 V_z + d^2) [V_x (r_1 a V_z - b)^2 + c^2 \mathbf{k}^\top \mathbf{k}] + r_1^2 r_2 a^2 c^2 \mathbf{k}^\top \mathbf{k} V_x V_z \right]^2},$$

unless there is a market breakdown ($L = 0$), in which case $\mathcal{G}_c = 0$.

As in the monopolistic case, the equilibrium utility of the entrepreneur is monotonic in the informational parameters of the asset:

PROPOSITION 3.3. *The competitive entrepreneur's equilibrium utility is monotonically decreasing in the weights that the asset payoff assigns to extraneous private information and extraneous noise. Specifically,*

$$\frac{\partial \mathcal{U}_c}{\partial (c^2)} \leq 0 \quad \text{and} \quad \frac{\partial \mathcal{U}_c}{\partial (d^2)} \leq 0.$$

The derivatives are zero if and only if there is a market breakdown ($L = 0$).

Proof. From Lemma 3.2, \mathcal{G}_c is a monotonic transformation of \mathcal{U}_c . The result is straightforward to show by differentiating \mathcal{G}_c , and using Assumption 1.1. ■

The following is now immediate:

THEOREM 3.4. *The class of assets that are optimal for the competitive entrepreneur is given by the set*

$$S_c := \{f = \bar{f} + az + bx \mid (\bar{f}, a, b) \in \mathbb{R}^3, a \neq 0, b \neq r_1 a V_z\}.$$

Designing an asset in this class is equivalent to issuing equity in the entrepreneur's payoff e . In equilibrium, a constant proportion $r_2/(r_1 + r_2)$ of e is retained by the entrepreneur, the rest being held by the outsider.

Proof. It is never optimal for the insider to choose the parameters of the asset payoff (2) such that $L = 0$, because then $\mathcal{G}_c = 0$ (Lemma 3.2), and there clearly are assets for which $\mathcal{G}_c > 0$. From Proposition 3.3, it is optimal to set $c = d = 0$. Now, from Lemma 3.2,

$$\mathcal{G}_c = \frac{r_1^4 V_x V_z}{(r_1 + r_2)^2},$$

which is independent of \bar{f} , a , and b . However, $L \neq 0$ if and only if $a \neq 0$, and $b \neq r_1 a V_z$.

The rest of the proof mimics that of Theorem 2.4. ■

It can be shown that the situation here is identical to the no-information case. As one would expect, the price-taking insider withholds a smaller proportion of equity than does her monopolistic counterpart. However, the characteristics of an optimal asset are essentially independent of the ability of the entrepreneur to affect asset prices.

4. Concluding Remarks

This paper has shown that the nature of a market with rational traders is such that, in designing a security, an entrepreneur is forced to disregard her privileged position as an informed agent. Even if she can design a complicated asset whose payoff is sensitive to her information, she does no better than just issuing equity. The resulting equilibrium is fully revealing, with all private information being transmitted through prices.

APPENDIX

It is useful first to calculate some conditional moments. Using (2) and (3), and the standard theory of the multivariate normal distribution (see, for example, Anderson (1984), Ch. 1):

$$\begin{aligned}
 \hat{E}_f &= \bar{f} + b\mathbf{x} + c\mathbf{k}^\top \mathbf{y} \\
 \hat{V}_f &= a^2 V_z + d^2 \\
 \hat{V}_e &= \mathbf{x}^2 V_z \\
 \hat{V}_{ef} &= a\mathbf{x} V_z.
 \end{aligned} \tag{21}$$

Let

$$\tau := \hat{E}_f - r_1 \hat{V}_{ef} - \bar{f}. \tag{22}$$

Using (21),

$$\tau = -(r_1 a V_z - b)\mathbf{x} + c\mathbf{k}^\top \mathbf{y}.$$

Therefore,

$$\begin{aligned}
 E_\tau &= 0 \\
 V_\tau &= V_x (r_1 a V_z - b)^2 + c^2 \mathbf{k}^\top \mathbf{k} \\
 V_{f\tau} &= -b V_x (r_1 a V_z - b) + c^2 \mathbf{k}^\top \mathbf{k}.
 \end{aligned} \tag{23}$$

Proof of Lemma 2.1. Suppose there exists an equilibrium. Then the equilibrium is trivial if and only if (11) holds and $p = \bar{f}$. Using (21), this is equivalent to the following condition:

$$b = r_1 a V_z \quad \text{and} \quad c\mathbf{k} = 0. \tag{24}$$

Now, suppose there exists a nontrivial equilibrium. Then (10) holds. Since $\delta \neq 0$, (4) implies that observing p is equivalent to observing θ_1 . Furthermore, from (9) and (22),

$$\tau = \bar{p} - \bar{f} + \theta_1 (r_1 \hat{V}_f + 2\delta), \tag{25}$$

so that observing p is, in fact, equivalent to observing τ . From (23) and (24), $V_\tau = 0$ if and only if the equilibrium is trivial. Therefore $V_\tau > 0$, and

$$\begin{aligned}
 \tilde{E}_f &= \bar{f} + \frac{V_{f\tau}}{V_\tau} \tau \\
 \tilde{V}_f &= V_f - \frac{V_{f\tau}^2}{V_\tau}.
 \end{aligned} \tag{26}$$

The outsider's optimal asset position is given by (13). Substituting from (25) and (26):

$$\theta_2 = \frac{\bar{f} + \frac{V_{f\tau}}{V_\tau} \left[\bar{p} - \bar{f} + \theta_1(r_1 \hat{V}_f + 2\delta) \right] - p}{r_2 \left(V_f - \frac{V_{f\tau}^2}{V_\tau} \right)}. \quad (27)$$

Now, using the market-clearing condition (5), we can solve for the price function:

$$p = \bar{f} + \frac{V_{f\tau}}{V_\tau} (\bar{p} - \bar{f}) + \theta_1 \left[\frac{V_{f\tau}}{V_\tau} (r_1 \hat{V}_f + 2\delta) + r_2 \left(V_f - \frac{V_{f\tau}^2}{V_\tau} \right) \right]. \quad (28)$$

Comparing coefficients with (4):

$$\bar{p} - \bar{f} = \frac{V_{f\tau}}{V_\tau} (\bar{p} - \bar{f}) \quad (29)$$

$$\delta(V_\tau - 2V_{f\tau}) = r_1 \hat{V}_f V_{f\tau} + r_2 (V_f V_\tau - V_{f\tau}^2). \quad (30)$$

Substituting from (21) and (23), (30) can be written as

$$\delta J = K,$$

where J and K are defined in the statement of the lemma. Some algebra yields:

$$(r_1 \hat{V}_f + 2\delta)J = r_1 \hat{V}_f V_\tau + 2r_2 (V_f V_\tau - V_{f\tau}^2).$$

It is easy to show that the right hand side is positive in a nontrivial equilibrium. Then (10) implies that $J > 0$. This, in turn, means that, $V_{f\tau} \neq V_\tau$, so that, from (29), $\bar{p} = \bar{f}$. Hence the equilibrium price function is given by (15).

To prove the converse, suppose $J > 0$. Then an equilibrium exists since we can actually compute it in the manner just described. This equilibrium is nontrivial since $J > 0$ implies that the triviality condition (24) is violated. ■

Proof of Lemma 2.2. First, we take the case in which $J > 0$. Substituting into (8) from (4), and using (12), (21), and the fact that $\bar{p} = \bar{f}$ (from Lemma 2.1), we get:

$$\begin{aligned} \mathcal{E}_m &:= E(w_1 | \mathbf{s}) - \frac{r_1}{2} \text{Var}(w_1 | \mathbf{s}) \\ &= -\frac{r_1}{2} \hat{V}_e + \frac{\theta_1^2}{2} (r_1 \hat{V}_f + 2\delta) \\ &= -\frac{r_1}{2} \mathbf{x}^2 V_z + \frac{[(r_1 a V_z - b)\mathbf{x} - c\mathbf{k}^\top \mathbf{y}]^2}{2(r_1 \hat{V}_f + 2\delta)} \\ &= \left[\frac{(r_1 a V_z - b)^2}{2(r_1 \hat{V}_f + 2\delta)} - \frac{r_1 V_z}{2} \right] \mathbf{x}^2 + \left[\frac{c^2}{2(r_1 \hat{V}_f + 2\delta)} \right] (\mathbf{k}^\top \mathbf{y})^2 \\ &\quad - \left[\frac{c(r_1 a V_z - b)}{r_1 \hat{V}_f + 2\delta} \right] \mathbf{x} (\mathbf{k}^\top \mathbf{y}). \end{aligned}$$

From (7),

$$\mathcal{U}_m = -E[\exp(-\tau_1 \mathcal{E}_m)].$$

The desired expression for \mathcal{U}_m is now derived by noting that \mathcal{E}_m is a quadratic form in $(\mathbf{x}, \mathbf{k}^\top \mathbf{y})$, applying the methods in Bray (1981, Appendix) to evaluate the expectation (Assumption 1.1 is used here), and substituting $\delta = \frac{K}{J}$ (Lemma 2.1).

In case of a market breakdown, $\theta_1 = 0$, so that

$$\mathcal{E}_m = -\frac{\tau_1}{2} \mathbf{x}^2 V_z,$$

and the result is obtained using the same methods as above. ■

Proof of Lemma 3.1. Suppose there exists an equilibrium. Then it is trivial if and only if

$$\hat{E}_f - p - \tau_1 \hat{V}_{ef} = 0, \quad \forall s, \quad \text{and} \quad p = \bar{f}.$$

As in Lemma 2.1 this condition is equivalent to (24).

Now, if there exists a nontrivial equilibrium, $\hat{V}_f \geq \hat{V}_f > 0$, and the demand functions for the insider and outsider are given by (18) and (13) respectively. From (4), (18), and (22),

$$\tau = \bar{p} - \bar{f} + \theta_1(\tau_1 \hat{V}_f + \delta).$$

It must be the case that

$$\tau_1 \hat{V}_f + \delta \neq 0, \tag{31}$$

for otherwise $V_\tau = 0$, implying the triviality condition (24). Hence, just as in the monopolistic case, observing p is equivalent to observing τ . Therefore, analogous to (27) and (28):

$$\theta_2 = \frac{\bar{f} + \frac{V_{f\tau}}{V_\tau} [\bar{p} - \bar{f} + \theta_1(\tau_1 \hat{V}_f + \delta)] - p}{r_2 \left(V_f - \frac{V_{f\tau}^2}{V_\tau} \right)}$$

and

$$p = \bar{f} + \frac{V_{f\tau}}{V_\tau} (\bar{p} - \bar{f}) + \theta_1 \left[\frac{V_{f\tau}}{V_\tau} (\tau_1 \hat{V}_f + \delta) + r_2 \left(V_f - \frac{V_{f\tau}^2}{V_\tau} \right) \right].$$

Comparing coefficients with (4):

$$\bar{p} - \bar{f} = \frac{V_{f\tau}}{V_\tau} (\bar{p} - \bar{f}) \tag{32}$$

$$\delta(V_\tau - V_{f\tau}) = \tau_1 \hat{V}_f V_{f\tau} + r_2 (V_f V_\tau - V_{f\tau}^2). \tag{33}$$

Substituting from (21) and (23), (33) can be written as

$$\delta L = K.$$

After some algebraic manipulations, we get

$$(\tau_1 \hat{V}_f + \delta)L = \tau_1 \hat{V}_f V_\tau + 2r_2(V_f V_\tau - V_{f\tau}^2).$$

In a nontrivial equilibrium \hat{V}_f and V_τ are positive, so that the right hand side of the above equation is positive. Hence, using (31), $L \neq 0$. This implies that $V_{f\tau} \neq V_\tau$, so that, from (32), $\bar{p} = \bar{f}$. The equilibrium price function is, therefore, given by (20).

To prove the converse, suppose $L \neq 0$. Then we can calculate an equilibrium as above. This equilibrium is nontrivial since the triviality condition (24) holds only if $L = 0$. ■

Proof of Lemma 3.2. The steps of the proof are the same as in Lemma 2.2. If $L \neq 0$,

$$\begin{aligned} \mathcal{E}_c &:= E(w_1|\mathbf{s}) - \frac{r_1}{2} \text{Var}(w_1|\mathbf{s}) \\ &= -\frac{r_1}{2} \hat{V}_e + \frac{\theta_1^2}{2} \cdot r_1 \hat{V}_f \\ &= -\frac{r_1}{2} \mathbf{x}^2 V_z + \frac{r_1 \hat{V}_f [(r_1 a V_z - b)\mathbf{x} - c\mathbf{k}^\top \mathbf{y}]^2}{2(\tau_1 \hat{V}_f + \delta)^2} \\ &= \left[\frac{r_1 \hat{V}_f (r_1 a V_z - b)^2}{2(\tau_1 \hat{V}_f + \delta)^2} - \frac{r_1 V_z}{2} \right] \mathbf{x}^2 + \left[\frac{r_1 \hat{V}_f c^2}{2(\tau_1 \hat{V}_f + \delta)^2} \right] (\mathbf{k}^\top \mathbf{y})^2 \\ &\quad - \left[\frac{r_1 \hat{V}_f c (r_1 a V_z - b)}{(\tau_1 \hat{V}_f + \delta)^2} \right] \mathbf{x} (\mathbf{k}^\top \mathbf{y}). \end{aligned}$$

Now \mathcal{U}_m can be calculated as in Lemma 2.2, substituting $\delta = \frac{K}{L}$ (see Lemma 3.1).

In case of a market breakdown, $\theta_1 = 0$, so that:

$$\mathcal{E}_c = -\frac{r_1}{2} \mathbf{x}^2 V_z,$$

and the result follows as in Lemma 2.2. ■

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