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Model with Public Capital: Crowding-in
Effects and Stabilization Policy***

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Abstract

In this paper we analyze a stochastic growth model with optimal taxation, where the government expenditure is endogenous and has to finance it by putting a distorting tax on income from capital and the budget is balanced every period.

We found that the correlation of the GNP with the tax rate on income from capital is strongly positive, government spending is pro cyclical and that the GNP components present more volatility in a setting with distorting taxes than in an environment with lump-sum taxes. In this case, our results point out that taking into account the effects of taxes and government spending would imply a different prediction for the variability in the series.

One of the policy implications of our paper is that the effect of government spending on private investment can be evaluated. We find a crowding-in effect, in the sense that public and private investment are positive correlated in the Ramsey allocation and in the full optimum.

We also find that there is a loss in welfare when we compare staying in the Ramsey allocation to being in the lump-sum allocation. In particular, this loss is greater when the elasticity of output with respect to infrastructure increases.

1 INTRODUCTION

One of the oldest issues in macroeconomics is to determine the effect of government spending on the economy, and how this policy variable can be used to control economic fluctuations. In fact, this issue was at the heart of Keynes contribution, and it has been a recurrent theme in the economic literature, with long debates that argued for or against the use of pro or anti cyclical spending policies. Popular macroeconomic concepts like stabilization policy, crowding-out effects, etc. are associated with this debate.

Strangely enough, though, these issues have been ignored by almost all researchers in the last decade. The models popular in modern neo-classical macroeconomics often take no account of government spending: in the cases when it has been studied, government spending has been taken as exogenously given and unproductive. More recently, the importance of these subjects has been recognized (see, for example, Blanchard and Fisher (1989)¹) and some other papers have introduced endogenous government spending.

Our aim is to study the implications of fiscal policy in an uncertain environment where government spending is productive (like public investment on infrastructure, roads, education, etc.) but must be financed by distorting taxes. The government is restricted because it needs to balance its budget period by period, which implies that there is no possibility of running a deficit. In this context, the government chooses not only the optimal level of public investment, but also the optimal level of taxes in order to finance the government spending. Given this optimal fiscal policy, consumers and firms decide

¹In this sense, Blanchard and Fisher (1989) have pointed out the importance of endogenous government spending: "It is also clear, however, that elements of government spending are endogenous. For instance, in a model where public goods are consumption goods (...) and with all goods normal, government spending would normally be pro cyclical. The level of government spending should also be affected by the marginal cost of collecting taxes. If some of government spending is on investment goods (infrastructure), then this too would be expected to be responsive to the shocks hitting the economy. Still, in the equilibrium context some components of government spending may operate as a current input into production. The cyclical pattern of this component of government spending would depend on whether it was a complement or substitute for that factor or factors whose current productivity is affected by current disturbances". page 591, Chapter 11. Also, Jones et al. (1991) point out the problem of assuming that government spending are exogenous in the context of endogenous growth models. Consider a policy that increase the rate of growth of the GNP. If government spending is fixed, it becomes a negligible fraction of output. Jones et al. suggest: "A more realistic approach would include government spending as a productive input" (page 3).

the optimal level of consumption and investment when there is a stochastic shock on productivity.

The net effect of public investment on the economy depends on the form of financing this expenditure. When government spending is financed by a **lump-sum** tax, the only distorting effect is the increase in productivity created by public investment. However, the positive effect of public investment could be reduced when the government uses distorting taxes (like a tax on income from capital) because taxes lower the rate of return on private capital, but, on the other hand, public investment creates more incentives to invest².

In this paper we extend the analysis of optimal taxation when government expenditure is endogenous and productive in a stochastic growth model. We consider a neo-classical economy with capital accumulation subject to a technological shock on productivity. Agents take decisions optimally, including the government. For simplicity we assume that the government can't go into debt, therefore all the income from taxes can be dedicated to public investment. This government is benevolent and its objective is to choose the optimal tax rates and the optimal public investment to maximize the utility of the representative consumer. This is called the **Ramsey problem** and the corresponding solution the **Ramsey allocation**. We use this model to investigate the cyclical properties of the GNP components and to study the importance of public capital in the economy when the government has to finance public investment with a distorting tax on income from capital and compare it with the case in which there is lump-sum taxation. The stochastic component of our model allows us to make predictions about the correlation between the GNP and public investment, while deterministic models cannot (See Jones et al. (1991), Glomm and Ravikumar (1991) and Barro (1990)).

There is an empirical literature supporting the idea that public capital has a strong positive impact on the GNP (See Appendix 2 for an overview of the literature and an analysis of some of the empirical facts). The main conclusion of these papers is that public infrastructure has a positive impact on productivity and that government investment is positively correlated with private investment.

The basic premise of the literature on optimal taxation is that the government chooses optimally how to finance its expenditure by enacting a combination of taxes. This issue has been analyzed in a deterministic setting

²See Barro (1991) and Barro and Sala-i-Marti (1991).

(see for example Chamley (1985,1986)) and in a stochastic environment without capital (see Lucas and Stokey (1983)). Optimal taxation in a stochastic environment with capital accumulation has been considered lately. Chari, Christiano and Kehoe (1990) study the quantitative implications of optimal capital and labor income taxation when the government can run a deficit. They have found that the tax rate on labor income is constant over the business cycle and that the tax rate on income from capital is zero. However, Zhu (1992) in a similar model shows that the zero optimal tax rate on income from capital depends on the structure of the consumer's utility function.

These results on optimal taxation in stochastic growth models with capital accumulation depend on the assumption, on one hand, that government spending is exogenous and unproductive and, on the other, that the government uses many ways of taxation.

In the literature on endogenous growth there are some models of optimal taxation which do not assume that government spending is exogenous and unproductive. For example, Jones, Manuelli and Rossi (1991) make a quantitative assessment of the effects on welfare of drastically changing tax policy. In particular, when they do not assume that government expenditure is exogenously given, they obtain the result that the asymptotic tax rate on capital income is different from zero when distorting tax on income from capital is the only way to finance government spending.

In a similar way, Glomm and Ravikumar (1991) present a deterministic model of endogenous growth where the stock of public capital enters as an input in the production function. The government finances this stock of public capital by using a distorting and proportional tax on output. They show, with a full depreciation rate of private capital and a Cobb-Douglas technology, that in equilibrium the tax rate is positive and public capital grows at the same rate as private capital.³

Our model allow us to take into account the effects of public investment on the GNP, consumption and private investment. We differ from these authors in that we do not allow government debt or labor taxation; these assumptions are made for convenience. Unlike any of the previous papers, we have a stochastic model with endogenous government spending and distorting tax-

³In a similar model, Barro (1990) analyzes the relation between the size of government and the rate of economic growth in the steady state, but with the difference that government is constrained to choose a constant tax rate.

ation, therefore we can study the effect of these fiscal variables on business cycles, welfare and volatility.

In the case of the correlation between the GNP and public investment, our model with taxes on income from capital predicts a pro-cyclical government spending policy (the correlation is strongly positive). On the other hand, the correlation between taxes and the GNP is positive. This result tells us how a *stabilization fiscal policy* should be designed: the government should increase taxes in periods of expansion and let the expanded tax base finance the additional spending on public capital.

One of our findings is that the GNP components present more volatility in a setting with distorting taxes than in an environment with **lump-sum** taxes. Under **lump-sum** taxation, an increase in private investment is followed by an increase in government spending without any other effect. However, when the government finances public investment with a tax on income from capital, an increase in private investment is followed by an increase in public investment and a reduction in the tax rate, leading to a reduction in private investment. Therefore, under endogenous government spending and distorting taxation, private investment becomes more volatile.

This result suggests that, even when the government follows an optimal fiscal policy, endogenous and productive government spending and the distortionary effect of taxes could generate a strong variability. If this is the case, Real Business Cycle models should explicitly take into account how government spending is decided and the ways of taxation that can be used before comparing their results with macro data.

In the case of the correlation between taxes and the GNP, we find that a higher elasticity of output with respect to public infrastructure gives a lower correlation. Also, assuming that government spending is productive results in a different effect on the optimal tax structure. In our model the optimal tax rate on capital is positive and the degree of pro cyclicity depends on the importance of public capital in the economy.

One of the policy implications of our model is that one can analyze the effect of government spending on private investment. We find a *crowding-in effect*, in the sense that public and private investment are positive correlated in the **Ramsey** and in the **lump-sum** allocation. However, given the effects of distorting taxation, it would be suboptimal to pursue policies that caused a higher correlation of public and private investment or policies that generated a

negative correlation between these two variables; this is, therefore, an *optimal crowding-in effect*.

Finally, we find a loss in welfare when we compare staying in the Ramsey allocation to being in the **lump-sum** allocation. In particular, this loss is greater when the elasticity of output with respect to public infrastructure increases.

There are some technical features in the paper which may be of independent interest. First, we present a new approach in dynamic optimization problems in order to find a time inconsistent optimal solution to **Ramsey** problems. We exploit the fact that the Lagrange multipliers associated with the implementability constraint can be interpreted as state variables when we rewrite the problem as a recursive one. Once we reformulate the problem we can use recently developed techniques to solve non-linear dynamic stochastic models. In particular, in this paper we use the **Parameterized Expectations (PEA)** approach, a method developed by Marcet (1989). The basic idea is to parameterize the conditional expectation in the first order conditions with functional forms and iterate on these expectations until we get the best prediction for the series using those functional forms.

In order to apply **PEA** to our problem we have to deal with some crucial details. In order to generate simulations for the model we have to take into account that long run simulations will not, in general, be a good approximation because one of the state variables (the Lagrange multiplier) always starts at a level not in the steady state distribution due to time-inconsistency. In addition, when we compare welfare in **Lump-sum** and **Ramsey** Allocations we have to fix initial conditions and the steady state distribution of capital in the **Lump-sum**, does not overlap with the steady state of the **Ramsey** Allocation. To solve this problem we compute the policy function running many realizations of a short length and starting each realization at the same initial conditions. We select the length of the realization in such a way that our models get the steady state distribution. This procedure proves that Judd's (1992) allegation that **PEA** cannot be used for computing changes in tax policy is wrong: initial conditions can be taken into account correctly when the new tax policy is implemented.

Another important advantages of **PEA** is that we can solve models with many continuous state variables (in the **Ramsey** problem we have 4 continuous state variables). The alternative approach of Coleman (1991)-Judd

(1992) needs to impose a grid on the space of state variables which limits the number of variables that can be introduced. To our knowledge, applications of these methods only have, at most, one continuous state variable (this is the case of Judd's (1992) applications).

The paper is organized as follows. In section 2 we describe the economy and the competitive equilibrium with public capital and distorting taxes; in section 3 and 4 we present the optimal taxation problem with taxes on income from capital and **lump-sum** taxes, respectively. In section 3 we also reformulate the optimization problem in order to use standard recursive techniques; in section 5 we describe the method used to solve the model and how it was implemented; in section 6 we present the functional forms and the parameter values; in section 7 we discuss the results of the simulations and some policy implications; finally, in section 8 we present some conclusions and plans for future research.

2 AN EQUILIBRIUM MODEL WITH PUBLIC CAPITAL AND DISTORTING TAXES

Consider a production economy with one firm and a representative consumer who lives an infinite number of periods. Suppose this consumer is the owner of the capital stock and he rents it to the firm. In each period t , the consumer decides how much to consume and invest in an environment where capital depreciates and there is a stochastic shock in the production function that affects productivity. Public capital is provided by the government, and it affects positively the productivity of private agents. Moreover, this public capital cannot be sold, bought or destroyed by consumers or firms. The timing and action in each period for the consumer is as follows: at the beginning of every period t the current value θ_t of the exogenous shock is realized. So, the consumer knows the values for private and public capital ($k_{p,t-1}$ and $k_{g,t-1}$, respectively) and the realization of the stochastic shock when consumption c_t is decided and end-of period private capital $k_{p,t}$ is accumulated.

We assume that all the government expenditure is financed every period by a flat-rate income tax from capital income. That is, the government can neither finance deficits by issuing debt nor by accumulating assets. The tax

rate can change from period to period, and the government sets the tax rate.

2.1 Consumer

The representative consumer solves the following maximization problem:

$$\max_{\{c_t, l_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

Where c_t is consumption. The utility function $u(c_t): \mathbb{R} \rightarrow \mathbb{R}$, is bounded, continuously differentiable, strictly increasing and concave; $0 < \beta < 1$ is the discount factor and $E_t(\cdot)$ is the expected value conditional on observations of all variables observed at time t or earlier.

The consumer has the following budget constraint:

$$c_t + i_t = r_t k_{p,t-1} (1 - \tau_t^{k_p}) + \omega_t l_t \quad (2)$$

and

$$l_t \leq 1 \quad (3)$$

where i_t is investment, l_t is time dedicated to work, $k_{p,t}$ is private capital, r_t is the rental price of capital, ω_t is the wage rate and $\tau_t^{k_p}$ is the tax on income from capital. The consumer takes the process for $\{r_t, \omega_t \text{ and } \tau_t^{k_p}\}$ as given.

2.2 Firm

Output in the economy is produced according to the technology

$$y_t = f(k_{p,t-1}, k_{g,t-1}, l_t, \theta_t) \quad (4)$$

where $k_{g,t}$ is public capital, l_t is household time dedicated to work and θ_t is a stochastic shock that affects productivity.

Private capital depreciates at a fixed rate $0 < d_p < 1$, and the transition function takes the following form

$$k_{p,t} = (1 - d_p)k_{p,t-1} + i_t \quad (5)$$

In this case the firm solves the following maximization problem:

$$\max \Pi = y_t - r_t k_{p,t-1} - \omega_t l_t \quad (6)$$

where $\{r_t$ and $\omega_t\}$ are given process for the firm.

2.3 Government

Consider a government that can only tax income from capital with a proportional flat-tax rate $\tau_t^{k^p}$. Suppose that public investment is financed with this tax and the government's budget is balanced each period. In this case the budget constraint for the government is

$$g_t = \tau_t^{k^p} r_t k_{p,t-1} \quad (7)$$

Public capital depreciates at a fixed rate $0 < d_g < 1$, and the transition function takes the following form:

$$k_{g,t} = (1 - d_g)k_{g,t-1} + g_t \quad (8)$$

In this economy the resource constraint is given by

$$y_t = c_t + i_t + g_t \quad (9)$$

2.4 Definition: (competitive equilibrium with distortionary taxes)

A competitive equilibrium with distortionary taxes is a stochastic process of prices $\{\omega_t, r_t\}$ and allocations $\{c_t, l_t, k_{p,t}\}$ such that given the policy paths $\{\tau_t^{k^p}, g_t\}$:

a) $\{c_t, l_t, k_{p,t}\}$ maximize the consumer's utility (1) subject to the budget constraint (2), (3) and the transition function for private capital (5), given $\{\omega_t, r_t\}$;

b) $\{k_{p,t}, l_t\}$ maximize the firm's profits (6) subject to (4) given $\{\omega_t, r_t\}$;

c) The government budget constraint (7) and the transition function for public capital (8) are fulfilled at each period;

d) the capital market, labor market and the goods market (9) clear at $\{r_t, \omega_t\}$.

In equilibrium, the following first-order conditions must hold⁴:

$$r_t = f_t^{k_p} \quad (10)$$

$$\omega_t = f_t^l \quad (11)$$

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[r_{t+1} (1 - \tau_{t+1}^{k_p}) + (1 - d_p) \right] \right\} \quad (12)$$

and

$$l_t = 1 \quad (13)$$

Where $f_t^{k_p}$ denote marginal productivity of private capital, f_t^l is marginal productivity of labor and u'_t is marginal utility of consumption. (10) and (11) are the first order conditions for firms, (13) is optimal time dedicated to work and (12) is the *Euler* equation which characterizes consumer's optimal behavior for the intertemporal consumption-investment choice. Note that because leisure is not an argument in the utility function it is optimal to work one unit of time every period. Transition functions for private and public capital ((5) and (8) respectively) and the budget constraint for consumers and government ((2) and (7)) also have to hold.

Using (10) and (11) in (2), (7) and (12) and substituting (5) and (8) in (9), the optimal paths for $\{c_t, k_{p,t}\}$ are given by

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[f_{t+1}^{k_p} (1 - \tau_{t+1}^{k_p}) + (1 - d_p) \right] \right\} \quad (14)$$

$$y_t = c_t + k_{p,t} - (1 - d_p)k_{p,t-1} + k_{g,t} - (1 - d_g)k_{g,t-1} \quad (15)$$

$$k_{g,t} - (1 - d_g)k_{g,t-1} = \tau_t^{k_p} f_t^{k_p} k_{p,t-1} \quad (16)$$

⁴ $f_t^{k_p}$ is the marginal productivity of private capital, f_t^l is the first derivative respect to labor. Second and cross derivatives are indicated in a similar way. The subscript refers to the time period of the shock.

3 OPTIMAL TAXATION WITH DISTORTING TAXES

Consider a benevolent government who wants to maximize the welfare of the individuals by choosing an optimal fiscal policy, taking into account that the consumer maximizes utility and takes prices, taxes and government spending as given. Therefore, the government can choose the tax rate on income from capital and the level of public expenditure, which has the form of public investment in our model.

3.1 Ramsey Allocation

Consider the problem faced by the government. The government chooses policies $\{\tau_t^{k_p}, k_{g_t}\}$ such that the associated allocations $\{c_t, k_{p_t}\}$ chosen by households and firms solve (14), (15) and (16) for a given Markov process θ_t and initial conditions for private and public capital $\{k_{p_{-1}}, k_{g_{-1}}\}$. This is known as the *Ramsey Problem*⁵.

The problem faced by the government is:

$$\max_{\{c_t, i_t, k_{p_t}, k_{g_t}, \tau_t^{k_p}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (17)$$

subject to:

$$c_t + i_t + g_t = y_t$$

$$k_{p_t} = (1 - d_p)k_{p_{t-1}} + i_t$$

$$k_{g_t} = (1 - d_g)k_{g_{t-1}} + g_t$$

$$g_t = \tau_t^{k_p} f_t^{k_p} k_{p_{t-1}}$$

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[f_{t+1}^{k_p} (1 - \tau_{t+1}^{k_p}) + (1 - d_p) \right] \right\}$$

⁵See Lucas (1991).

$$y_t = f(k_{p_{t-1}}, k_{g_{t-1}}, l_t, \theta_t)$$

$$k_{p_{t-1}} = \bar{k}_p$$

$$k_{g_{t-1}} = \bar{k}_g$$

where the *Euler* equation is known as the *Implementability constraint*.

Note that this problem is not a recursive one, because future values of the control variables $\{c_{t+1}, k_{p_t}, k_{g_t}, \tau_{t+1}^{kp}\}$ influence the constraints of government at time t . Because the problem is not recursive, we cannot solve it sequentially using traditional methods of dynamic programming. As a consequence, the optimal fiscal policy is time inconsistent.⁶

The Lagrangean for this problem is:

$$\begin{aligned} \mathfrak{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ & u(c_t) - \lambda_t [c_t + k_{p_t} - (1 - d_p)k_{p_{t-1}} + k_{g_t} - (1 - d_g)k_{g_{t-1}} - f_t] \\ & - \alpha_t [u'_t - \beta E_t \{u'_{t+1} [f_{t+1}^{kp}(1 - \tau_{t+1}^{kp}) + (1 - d_p)]\}] \\ & - \mu_t [k_{g_t} - (1 - d_g)k_{g_{t-1}} - \tau_t^{kp} f_t^{kp} k_{p_{t-1}}] \} \end{aligned} \quad (18)$$

We want to rewrite problem (18) in a way that the objective function and the constraint are recursive.⁷ If we can write this problem as a recursive one,

⁶See Lucas and Stokey with Prescott (1989) for a detailed description of a standard dynamic program. See Kydland and Prescott (1977) and Sargent (1987) for a discussion on time inconsistency.

⁷Our strategy for solving the problem is different to the strategy used by Chari et al. (1991). They put in a single equation (the implementability constraint) all the period-by-period budget constraints faced by the consumer. The Ramsey allocation problem for the government is to maximize the consumer's utility given the feasibility and the implementability constraint. For doing this they have to solve the problem for a given lagrange multiplier associated to the implementability constraint. However, the problem of conditioning on the implementability constraint multiplier is that they can only write the problem as a dynamic recursive program from the first period, while with our strategy we can write completely recursive the Ramsey problem, i.e. from period zero. This means that they have to compute two policy functions: one from time 1 onwards and another from time zero to time equal one. Also, their approach can not be used when the implementability constraints has the form of inequalities and when there are more tax instruments than first

we can compute the numerical solution of the problem because the solution is a time invariant function of the state variables. The general idea is to introduce the expectation constraint in the objective function of (18).⁸

In what follows we describe how to write problem (18) in order to have a recursive one.

First, we write problem (18) term by term:

$$\begin{aligned} \mathfrak{S} = & E_0\beta^0\{u(c_0) - \lambda_0[\cdot] - \mu_0[\cdot] - \alpha_0[u'_0 - \beta E_0\{u'_1[f_1^{k_p}(1 - \tau_1^{k_p}) + (1 - d_p)]\}]\} \\ & + E_0\beta^1\{u(c_1) - \lambda_1[\cdot] - \mu_1[\cdot] - \alpha_1[u'_1 - \beta E_1\{u'_2[f_2^{k_p}(1 - \tau_2^{k_p}) + (1 - d_p)]\}]\} \\ & + E_0\beta^2\{u(c_2) - \lambda_2[\cdot] - \mu_2[\cdot] - \alpha_2[u'_2 - \beta E_2\{u'_3[f_3^{k_p}(1 - \tau_3^{k_p}) + (1 - d_p)]\}]\} \\ & + \dots \end{aligned}$$

The first line is the return function in period 0, the second the return function in period 1, etc. Note that the return function in each period depends on decision variables dated in the next period. For example, in period one appears $\{\tau_2^{k_p}, k_{p2}, k_{g2}, u'_2\}$ in period 2 appears $\{\tau_3^{k_p}, k_{p3}, k_{g3}, u'_3\}$, and so on.

First, we move the term dated in period $t=1$ from the return function in period $t=0$, then the term dated period $t=2$ from the return function in period $t=1$, and so on,

$$\begin{aligned} \mathfrak{S} = & E_0\beta^0\{u(c_0) - \lambda_0[\cdot] - \mu_0[\cdot] - \alpha_0[u'_0]\} + E_0\beta^0\alpha_0\beta u'_1 [f_1^{k_p}(1 - \tau_1^{k_p}) + (1 - d_p)] \\ & + E_0\beta^1\{u(c_1) - \lambda_1[\cdot] - \mu_1[\cdot] - \alpha_1[u'_1]\} + E_1\beta^1\alpha_1\beta u'_2 [f_2^{k_p}(1 - \tau_2^{k_p}) + (1 - d_p)] \end{aligned}$$

order conditions.

⁸See Marcet and Marimon (1992b) for a more complete description of how to solve problems with expectational constraints.

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The first line is the return function in period 0, the second the return function in period 1, etc. Note that the return function in each period depends on decision variables dated in the next period. For example, in period one appears $\{\tau_2^{k_p}, k_{p_2}, k_{g_2}, u'_2\}$ in period 2 appears $\{\tau_3^{k_p}, k_{p_3}, k_{g_3}, u'_3\}$, and so on.

First, we move the term dated in period $t=1$ from the return function in period $t=0$, then the term dated period $t=2$ from the return function in period $t=1$, and so on,

$$\begin{aligned} \mathfrak{S} = & E_0 \beta^0 \{ u(c_0) - \lambda_0 [\cdot] - \mu_0 [\cdot] - \alpha_0 [u'_0] \} + E_0 \beta^0 \alpha_0 \beta u'_1 [f_1^{k_p}(1 - \tau_1^{k_p}) + (1 - d_p)] \\ & + E_0 \beta^1 \{ u(c_1) - \lambda_1 [\cdot] - \mu_1 [\cdot] - \alpha_1 [u'_1] \} + E_1 \beta^1 \alpha_1 \beta u'_2 [f_2^{k_p}(1 - \tau_2^{k_p}) + (1 - d_p)] \end{aligned}$$

order conditions.

⁸See Marcet and Marimon (1992b) for a more complete description of how to solve problems with expectational constraints.

$$+ E_0 \beta^2 \{u(c_2) - \lambda_2 [\cdot] - \mu_2 [\cdot] - \alpha_2 [u'_2]\} + E_2 \beta^2 \alpha_2 \beta u'_3 [f_3^{k_p}(1 - \tau_3^{k_p}) + (1 - d_p)]$$

+ ...

Now we put together all the terms dated in period $t=1$ or earlier, in period $t=2$ or earlier, etc. and using the law of iterated expectations to eliminate E_t we get,

$$\begin{aligned} \mathfrak{S} = & E_0 \beta^0 \{u(c_0) - \lambda_0 [\cdot] - \mu_0 [\cdot] - \alpha_0 [u'_0]\} \\ & + E_0 \beta^1 \{u(c_1) - \lambda_1 [\cdot] - \mu_1 [\cdot] - \alpha_1 [u'_1] + \alpha_0 \beta u'_1 [f_1^{k_p}(1 - \tau_1^{k_p}) + (1 - d_p)]\} \\ & + E_0 \beta^2 \{u(c_2) - \lambda_2 [\cdot] - \mu_2 [\cdot] - \alpha_2 [u'_2] + \alpha_1 \beta^2 u'_2 [f_2^{k_p}(1 - \tau_2^{k_p}) + (1 - d_p)]\} \\ & + E_0 \beta^3 \{u(c_3) - \lambda_3 [\cdot] - \mu_3 [\cdot] - \alpha_3 [u'_3] + \alpha_2 \beta^3 u'_3 [f_3^{k_p}(1 - \tau_3^{k_p}) + (1 - d_p)]\} \\ & + \dots \end{aligned}$$

Observe that the return function is different at $t=0$. In order to write the problem recursive from period zero, we need to put the following term into the return function for period $t=0$:

$$\alpha_{-1} \beta^0 u'_0 [f_0^{k_p}(1 - \tau_0^{k_p}) + (1 - d_p)]$$

and in order to preserve the original objective function (18) we have to add the following constraint⁹:

$$\alpha_{-1} = 0 \tag{19}$$

Taking into account this constraint, we have a recursive dynamic problem (20):

⁹Note that if $\alpha_{-1} \neq 0$ then the problem will not correspond to the original, therefore, $\alpha_{-1} = 0$ must be satisfied.

$$\begin{aligned}
\mathfrak{S} = & E_0 \beta^0 \{ u(c_0) - \lambda_0 [\cdot] - \mu_0 [\cdot] - \alpha_0 [u'_0] + \alpha_{-1} \beta^0 u'_0 [f_0^{k_p} (1 - \tau_0^{k_p}) + (1 - d_p)] \} \\
& + E_0 \beta^1 \{ u(c_1) - \lambda_1 [\cdot] - \mu_1 [\cdot] - \alpha_1 [u'_1] + \alpha_0 \beta u'_1 [f_1^{k_p} (1 - \tau_1^{k_p}) + (1 - d_p)] \} \\
& + E_0 \beta^2 \{ u(c_2) - \lambda_2 [\cdot] - \mu_2 [\cdot] - \alpha_2 [u'_2] + \alpha_1 \beta^2 u'_2 [f_2^{k_p} (1 - \tau_2^{k_p}) + (1 - d_p)] \} \quad (20) \\
& + E_0 \beta^3 \{ u(c_3) - \lambda_3 [\cdot] - \mu_3 [\cdot] - \alpha_3 [u'_3] + \alpha_2 \beta^3 u'_3 [f_3^{k_p} (1 - \tau_3^{k_p}) + (1 - d_p)] \} \\
& + \dots
\end{aligned}$$

Problem (20) with (19) has the form of a standard dynamic program where the feasible set at time t is a known function of the past variables. In this problem the control variables are $\{c_t, k_{p_t}, k_{g_t}, i_t, g_t, \tau_t^{k_p}\}$ and the state variables are $\{k_{p_{t-1}}, k_{g_{t-1}}, \theta_t, \alpha_{t-1}\}$. Notice that the lagrange multiplier is a state variable in this problem.¹⁰

We can interpret α_t as a "reputational state variable" in the following sense: the government takes into account the *Euler* equation (the first order condition for the consumer (14)) every period. At any time t , α_{t-1} summarizes the commitment of government to follow the optimal path for the consumers's decision problem. However, at time $t=0$, when the government solves for the entire path, $\alpha_{-1} = 0$, which means that the commitment of taking into account the *Euler* equation is enacted from time 0 onwards, and the past doesn't matter. It is possible for the government to set $\alpha_t = 0$ at any time t

¹⁰In the Chapter XV, Sargent (1988) has an interpretation of these Lagrange multipliers in a model of dynamic optimal taxation without uncertainty; Hansen, Epple and Roberds (1985) solve a linear-quadratic dominant-competitive game where one of the players (in our model the government) maximizes its objective function taking into account its influence both on prices and the decision rules (Euler equations) of the other player (the consumer in our model). The solution to this problem presents time inconsistency and, in order to have a time-invariant representation, they need to impose that the Lagrange multipliers at time -1 associated to the decision rules be zero; see also Marcet and Marimon (1993) for a similar use of these multipliers as state variables in a stochastic growth model with partial enforcement and full information.

(change the expected return in the *Euler* equation by increasing the tax rate or decreasing the level of public capital) but this is not optimal as it would deviate from the optimum with full commitment.

The first order conditions for problem (20) are:

$$\lambda_t = u'_t - \alpha_t u''_t + \alpha_{t-1} u''_t \left[f_t^{k_p} (1 - \tau_t^{k_p}) + (1 - d_p) \right] \quad (21)$$

$$\begin{aligned} \lambda_t = \beta E_t \{ & \alpha_t u'_{t+1} f_{t+1}^{k_p k_p} (1 - \tau_{t+1}^{k_p}) + \lambda_{t+1} [f_{t+1}^{k_p} + (1 - d_p)] \\ & + \mu_{t+1} [\tau_{t+1}^{k_p} f_{t+1}^{k_p} + \tau_t^{k_p} k_{p_t} f_{t+1}^{k_p k_p}] \} \end{aligned} \quad (22)$$

$$\begin{aligned} \lambda_t + \mu_t = \beta E_t \{ & \alpha_t u'_{t+1} f_{t+1}^{k_p k_g} (1 - \tau_{t+1}^{k_p}) + \lambda_{t+1} [f_{t+1}^{k_g} + (1 - d_g)] \\ & + \mu_{t+1} [\tau_{t+1}^{k_p} f_{t+1}^{k_p k_g} k_{p_t} + (1 - d_g)] \} \end{aligned} \quad (23)$$

$$\mu_t = \frac{\alpha_{t-1} u'_t}{k_{p_{t-1}}} \quad (24)$$

where $(\lambda_t, \alpha_t, \mu_t)$ are the Lagrange multipliers associated to constraints (15), (14) and (16) respectively.

The equations that characterize the optimal path are the transition function for private and public capital ((5) and (8)); the budget constraint of the economy (15); the Euler equation (14); the government budget constraint (16); (19) ($\alpha_{-1} = 0$) and the first order conditions (21)-(24).

4 GOVERNMENT SPENDING FINANCED WITH LUMP-SUM TAXES

Because of the externality implied by public capital and taxes on income from capital, the Ramsey allocation is not Pareto optimal. We want to compare this allocation with those from a social planner that finances public capital with **lump-sum** taxes. This will give us information about what is the cost of distorting taxes and how different is the cyclical behavior of the economy

due to the distorting taxes.

Competitive Equilibrium.

Consumers face the same problem as before, but instead of paying a flat tax rate $\tau_t^{k_p}$, they pay a **lump-sum** tax ψ_t that is given for the consumers. In this case the budget constraint is given by:

$$c_t + i_t + \psi_t = r_t k_{p,t-1} + \omega_t l_t \quad (25)$$

and the government budget constraint is:

$$g_t = \psi_t \quad (26)$$

The firm solves the same problem as before. The consumer faces the following first order conditions:

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[f_{t+1}^{k_p} + (1 - d_p) \right] \right\} \quad (27)$$

$$c_t + k_{p,t} - (1 - d_p)k_{p,t-1} + k_{g,t} - (1 - d_g)k_{g,t-1} = y_t \quad (28)$$

The problem of the social planner in this case is the same as in the *Ramsey* problem: find the taxes and level of public capital $\{\psi_t, k_{g,t}\}$ that maximize the consumer's utility function.

The optimization problem in this case is:

$$\max_{\{c_t, l_t, k_{p,t}, k_{g,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (29)$$

subject to (28):

$$c_t + k_{p,t} - (1 - d_p)k_{p,t-1} + k_{g,t} - (1 - d_g)k_{g,t-1} = y_t \quad (30)$$

Note that because the government spending is set optimally, we use the social planner problem for finding the first best allocation.

In this case we have the following lagrangean

$$\mathfrak{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ u(k_{p,t} - (1 - d_p)k_{p,t-1} + k_{g,t} - (1 - d_g)k_{g,t-1} - f_t) \} \quad (31)$$

First order conditions respect to k_p and k_g are:

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[f_{t+1}^{k_p} + (1 - d_p) \right] \right\} \quad (32)$$

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[f_{t+1}^{k_g} + (1 - d_g) \right] \right\} \quad (33)$$

In this case the *Euler* equations (32) and (33) together with the resource constraint (28) determine the optimal path of $\{c_t, k_{p,t}, k_{g,t}\}$.

5 SOLVING THE MODEL WITH PARAMETERIZING EXPECTATIONS.

The problem in solving the models described before is that the *Euler* equations involve conditional expectations which must be computed as functions of the state.¹¹ We use **Parameterizing Expectations (PEA)**, a method developed by Marcet (1989).¹²

With **PEA** the solution to the problem is calculated iteratively using an approximation to the conditional expectation (in the case of the **Ramsey** problem we have to approximate the expectations in equations (14), (22) and (23) and in the case of the **lump-sum** problem we have to approximate the expectations in (32) and (33)). Once we have a specification of the functional form for these expectations and a "guess" for the parameter vector that characterizes these functions, we obtain a series for the endogenous variables. Because this solution is not consistent with the expectations that we use in the first iteration, we update the expectations using the resulting series to calculate a new estimation of the parameter vector that characterize the functional form. We repeat this procedure until a fixed point in the space of parameters of the expectations is found. (In the Appendix 1 we describe in detail the algorithm and how was implemented). The converged value of the parameter

¹¹See Taylor and Uhlig (1990) and the references therein for a complete discussion of the techniques for solving this kind of problems.

¹²See Den Haan and Marcet (1990) for an application to the simple growth model; Marcet and Singleton (1991) use the method in the context of equilibrium asset prices and savings of heterogeneous agents in the presence of incomplete markets and portfolio constraints; Den Haan (1990) finds the optimal inflation path in a monetary economy.

vector is the approximation to the rational expectations equilibrium.¹³

In order to apply this basic framework to our problem we have to handle some crucial details. In order to generate simulations for the model (Step 2, page 237 in Marcet and Marimon (1992)) we have to take into account that long run simulations may not be a good approximation because one of the state variables starts at a very low level (in this case the lagrange multiplier: $\alpha_{-1} = 0$). Furthermore, long run simulations are not enough for welfare comparison because we need $\{\bar{k}_{p-1}, \bar{k}_{g-1}\}^{lump-sum} = \{\bar{k}_{p-1}, \bar{k}_{g-1}\}^{Ramsey}$; since the support of the steady state distribution is disjoint in our model, if we take, for example, the support of the **Lump-sum** allocation, the solution for the **Ramsey** allocation in the first periods will be inaccurate.

To solve this problem we compute the policy function running many realizations of a short length and starting each realization at $\{\bar{k}_{p-1}, \bar{k}_{g-1}, \alpha_{-1} = 0\}$. We select the length of the realization in such a way that all realizations get the steady state distribution. This procedure proves that Judd's (1992) allegation that **PEA** cannot be used for computing changes in tax policy is wrong: initial conditions are taken into account correctly when the new tax policy is implemented.

Note that some alternative methods, like Linear-Quadratic, could not handle short run simulation: even if the support were not disjoint, the time inconsistency feature of the **Ramsey** problem requires that $\alpha_{-1} = 0$.

6 FUNCTIONAL FORMS AND PARAMETERS

6.1 Functional forms

In order to solve the model we have to specify the parameters and the functional forms for the utility and the production function.

We assume:

$$u(c_t) = \ln(c_t) \tag{34}$$

¹³See Marcet and Marshall (1992) for a description of the set of sufficient conditions under which this fixed point can be made arbitrarily close to the true rational expectations equilibrium.

$$y_t = Ak_{p,t-1}^a k_{g,t-1}^b \theta_t \quad (35)$$

and

$$\log \theta_t = \rho \log \theta_{t-1} + \epsilon_t, \rho < 1 \text{ and } \epsilon_t \text{ i.i.d.} \quad (36)$$

6.2 Parameters

In order to solve the models we need to specify the value of the parameters $\{\beta, \mathbf{a}, \mathbf{b}, d_p, d_g, \rho, \sigma\}$. We can take some of these from the traditional literature in real business cycles.¹⁴ The problem with the parameters for public capital $\{\mathbf{b}$ and $d_g\}$ is that our knowledge about them is particularly sparse. For example, Garcia-Milà (1987) estimates $\{\mathbf{a}, \beta, \mathbf{b}, d_g, d_p\}$ in a stochastic equilibrium model with private and public capital. However, her model is very different because public capital is set exogenously by the government and is financed with lump-sum taxes.¹⁵

We cannot use the set of parameters used by Jones et al. (1992) because the functional forms that they use are different. In particular, they do not have public capital. Instead, they use a CES function for investment $G(i, g) = A(\alpha i^{-\rho} + (1-\alpha)g^{-\rho})^{\frac{1}{\rho}}$, where α is the share of investment output attributed to private investment spending and ϵ (where $\epsilon = \frac{1}{1+\rho}$) is the elasticity of substitution between private and public expenditures on investment.¹⁶ They find that these parameters are the most important for determining the growth and welfare effect. However, they don't have any empirical evidence about the parameter values for public investment.

¹⁴For a discussion of the value of these parameters, see Prescott (1986) and Hansen and Wrigth (1992) among many others.

¹⁵Garcia-Milà (1987) finds that the values for $\{\mathbf{a}, \beta, \mathbf{b}, d_p\}$ are (0.31717, 0.974538, 0.428548 and 0.20953), respectively.

¹⁶Jones et al. (1992) use $\alpha = (.5, .6, .7)$ and $\rho = (.66, .99, 1.33, 2, 4)$.

TABLE 1		
PARAMETER VALUES		
Discount Factor	β	0.99
Marginal productivity of private capital	a	0.36
Elasticity of output with respect of public capital	b	0.23
Depreciation rate of private capital	d_p	0.025
Depreciation rate of public capital	d_g	0.025
Constant in the production function	A	1.0
Autocorrelation parameter of $\log[\theta_t]$	ρ	0.95
St. dev. of innovation of $\log[\theta_t]$	σ	0.01

We take as a benchmark case the parameter's values that imply a steady state value of τ^{kp} equal to 31% (see Table 1). In the case of d_g we assume that $d_g = d_p$ and then we modify the importance of public capital in the economy by changing b , the elasticity of output with respect to infrastructure.

7 POLICY IMPLICATIONS

In Tables 2 - 7 we present the results of simulations for **lump-sum** and **Ramsey** allocation for different values of b (Figures 1-4 show the GNP components series for the **lump-sum** and **Ramsey** allocations in the case of $b = 0.23$). The statistics in these tables are computed along the stochastic steady state.

7.1 Stabilization Policy

In Table 3 and 6 we present the correlation between the GNP components and the GNP. The correlation between the GNP and private and public investment is positive and close to one in the **lump-sum** allocation. In an expansion, for example, private investment increases, leading to an increase in public investment, given by the optimal relationship $\frac{a}{b}$. In the **Ramsey** allocation private investment is less correlated with the GNP, although is highly positive.

Our results in Table 6 suggest how an optimal fiscal policy must be designed: in an expansion (recession), public investment must increase (decrease) but the tax rate on income from capital must be increased (reduced). Our intuition is that in an expansion the increase in public investment must be financed with an increase in the tax rate. The stabilization policy calls for

increasing the tax rate in periods of expansion, but not as much that the private income will be shrunk and, therefore, that private investment be under the optimal level.

7.2 Volatility

The results on volatility are in Tables 2 and 5. The first thing to note is that the volatility for the resulting series in the **lump-sum** allocation is lower, for any value of **b**, than in the **Ramsey** allocation. This is because optimal behavior in the **Ramsey** allocation is affected not only by the public capital externality, but also for the distorting tax on income from capital. Under **lump-sum** taxation, an increase in private investment is followed by an increase in government spending and, therefore, **lump-sum** taxes must increase for financing government spending. However, when the government finances public investment with a tax rate on income from capital, an increase in private investment is followed by an increase in public investment and, because tax revenues must finance government spending, the tax rate is increased, leading a higher volatility in private investment.

Table 5 shows the effect on volatility when **b** changes: when **b** is low, the tax rate presents more volatility because it is the variable that has to compensate the variation on the government income.

These results on volatility point out that taking into account the effects of taxes and government spending would imply a strong variability in the series, in particular, investment is almost three times greater under **Ramsey** allocation relative to **lump-sum** allocation. Real Business Cycle models find that the variability in the simulated series is according with the data¹⁷, but if we introduce distorting taxes or endogenous government spending, even in the context of the **Ramsey** problem, we have a higher variability in the simulated series, which will be inconsistent with the data.

7.3 Composition of Output

The values in Tables 4 and 7 are the average values for the GNP components. Both allocations depend strongly of **b**: in the case of the **lump-sum** allocation an increase of **b** in 100% implies an increase of 155% in the GNP. In the case

¹⁷See Prescott (1986).

of **Ramsey** allocation the increase is of 40%¹⁸. The **Ramsey** allocation presents a higher $\tau_t^{k_p}$ for higher values of **b**. This is simply because higher levels of public investment need a higher proportion of income for financing government spending.

7.4 Crowding-In Effects

In our model there is a *crowding-in* effect. In the case of the **lump-sum** allocation, an increase in private investment is followed by an increase in public investment. As mentioned before, this is because there is an optimal relationship between private and public investment. In the case of the **Ramsey** allocation, an increase in private investment also implies an increase in government spending (Table 8 show the highly positive correlation between private and public investment in the **lump-sum** and the **Ramsey** allocation). A classic *crowding out* effect would imply that an increase in government spending shifts out private investment, giving a negative correlation between government spending and private investment. In our model there is a kind of *optimal crowding-in* effect, in the sense that an increase in private investment implies and optimal increase in government spending that is needed in order to reach the optimum. A government spending policy designed to avoid this *crowding-in* effect (for example, if government spending is constant) would imply a sub-optimal solution. Also, given the effects of distorting taxation, it could be suboptimal to pursue policies that caused a lower correlation between private and public investment.

7.5 Welfare

Finally, in Table 9 we present the welfare¹⁹ attained in the **Ramsey** and the **lump-sum** allocation²⁰. The **Ramsey** allocation implies a lower level of

¹⁸Baxter and King (1990) using the highest estimation of **b** obtained by Aschauer (1989) find that the increase of the GNP is about thirteen times the increase in public investment.

¹⁹Private and public capital were fixed in such a way that these values get the steady state distribution in the lump-sum allocation. In the case of the **Ramsey** allocation, we use the same initial conditions, but we run short-run simulations (See Appendix 2).

²⁰Note that the welfare increases if the elasticity of output with respect to public infrastructure is bigger. Jones et al. (1992) find a similar result: there is a lower effect on growth and welfare when α , the share on investment output attributed to private investment spending is high relative to the share on investment attributed to public investment.

welfare even if taxes are fixed optimally because the distortionary taxes are sub-optimal.

We also did the exercise of solving a model where taxes are fixed exogenously by the government. In this case government spending is equal to:

$$g_t = \tau^{k_p} f_t^{k_p} k_{p,t-1}$$

We fixed the tax rate and use the same initial conditions for private and public capital that we use in the Ramsey problem. The loss in welfare relative to the **Ramsey** allocation is bigger when the tax rate departs from the optimal value. (see Table 10 and Figures 5-6)

8 CONCLUSION

In this paper we quantify the implications of optimal taxation on welfare and the cyclical properties of the GNP components in a stochastic growth model where the government can choose taxes and vary government spending accordingly and has to balance its budget every period. This government spending is used as public capital.

We found that the volatility for the series in the **Ramsey** allocation is greater than in an environment with lump-sum taxes. This result could have strong implications for the standard results of Real Business Cycle models, because taking into account decisions on ways of taxation and government spending changes the results of the variability in the series, even when the fiscal policy is set at the optimal level. It's very important to consider these factors because volatility, in the case of private investment, can be almost 3 times higher if we don't. For this reason we suggest that Real Business Cycle models should incorporate these elements before comparing the results with the macro data.

We also found that the correlation of output with the tax rate on income from capital is strongly positive and government spending is pro-cyclical. This implies that a positive shock should be followed by an increase in private and public investment, which would lead to an increase in the GNP, while the positive correlation between the GNP and the tax rate on income from capital implies that the tax rate should be higher, but just enough for the tax base to be able to finance the government spending.

This result suggests how optimal fiscal stabilization policy should be designed: increase the tax rate on income from capital in periods of expansion (because of the positive correlation between the GNP and the tax rate) but not too much, since total tax proceeds should increase and a higher investment in public capital has to take place during expansion.

One of the policy implications of our model is that there is a kind of *crowding-in effect*: public and private investment are positive correlated in the **Ramsey** allocation and in the full optimum. However, given the effects of distorting taxation, it could be suboptimal to design policies that cause a higher correlation of public and private investment or policies that generate a classic *crowding-out effect* (negative correlation between public and private investment). In our model public investment *crowds-in* private investment in an optimal way.

Finally, we compute the welfare loss in the **Ramsey** allocation, in relation to the **lump-sum** allocation. We find a substantial loss in welfare and our results suggest that the loss is very sensitive to the elasticity of output with respect to public infrastructure.

Because our knowledge about the production function parameters for public capital is sparse, we cannot evaluate the size of the effect of public capital on the economy very precisely, but many of our results on welfare losses and cyclical properties hold for a wide range of values for the elasticity of output with respect to public infrastructure (**b**).

Finally, our model ignores many important issues in fiscal policy (like the optimal size of debt and the structure of optimal taxation when there are many ways of taxation in an economy where the government has a productive role) and the way public investment affects private decisions (for example, which sectors of the economy are more sensitive to public investment and the case of public investment taking a long time to become public capital). In an ongoing research we are extending the model taking into account a more complete specification of fiscal policy, including other kind of taxes and, especially, the optimal size of the debt.

TABLE 2:LUMP-SUM ALLOCATION						
VOLATILITY						
b	GNP	c	kp	kg	i	g
.23	3.42	3.58	1.5	1.45	3.3	3.28
.2	3.39	3.53	1.48	1.42	3.28	3.26
.18	3.37	3.5	1.46	1.4	3.27	3.25
.15	3.35	3.45	1.46	1.37	3.27	3.22
.13	3.33	3.43	1.42	1.36	3.24	3.21
.1	3.30	3.38	1.42	1.33	3.23	3.19

Volatility is computed as standard deviation relative to mean.

TABLE 3:LUMP-SUM ALLOCATION					
CORRELATION* WITH OUTPUT					
b	c	kp	kg	i	g
.23	.99	.62	.65	.99	.98
.2	.99	.62	.65	.99	.98
.18	.99	.62	.64	.99	.98
.15	.99	.61	.65	.99	.97
.13	.99	.61	.64	.98	.97
.1	.99	.61	.65	.99	.97

* Contemporaneous correlation.

TABLE 4:LUMP-SUM ALLOCATION						
MEAN*						
b	GNP	c	kp	kg	i	g
.23	22.2	12.9	227.3	145.2	5.7	3.6
.2	14.8	8.9	151.1	84.4	3.8	2.1
.18	11.7	7.2	120.3	60.2	3.0	1.5
.15	8.6	5.5	88.4	36.8	2.2	0.92
.13	7.2	4.7	74.1	26.8	1.9	0.67
.1	5.7	3.9	58.8	16.3	1.5	0.41

* In levels.

TABLE 5:RAMSEY ALLOCATION							
VOLATILITY							
b	GNP	c	kp	kg	i	g	τ^{kp}
.23	5.3	4.5	5.4	4.8	8.8	7.3	2.6
.22	5.1	4.3	5.3	4.5	8.9	7.0	2.6
.18	5.0	4.2	5.2	4.5	8.9	7.1	2.7
.15	4.9	4.0	5.0	4.4	8.8	7.2	3.0
.13	4.8	3.9	4.9	4.4	8.7	7.3	3.1
.1	4.7	3.7	4.7	4.5	8.6	7.8	3.7

Volatility is computed as standard deviation relative to mean.

TABLE 6:RAMSEY ALLOCATION							
CORRELATION* WITH OUTPUT							
b	c	kp	kg	i	g	τ^{kp}	
.23	.97	.88	.84	.92	.97	.67	
.2	.96	.87	.84	.92	.96	.62	
.18	.96	.86	.83	.92	.96	.65	
.15	.95	.86	.82	.92	.95	.67	
.13	.95	.85	.82	.92	.95	.68	
.1	.95	.85	.82	.92	.94	.72	

* Contemporaneous correlation.

TABLE 7:RAMSEY ALLOCATION								
MEAN*								
b	GNP	c	kp	kg	i	g	τ^{kp}	
.23	12.8	9.1	91.0	57.3	2.3	1.4	.31	
.2	9.6	6.9	71.9	37.2	1.8	0.93	.27	
.18	8.1	5.8	62.5	29.1	1.6	0.73	.25	
.15	6.5	4.7	52.4	20.6	1.3	0.51	.22	
.13	5.8	4.2	47.5	16.3	1.2	0.41	.2	
.1	4.9	3.6	42.0	11.6	1.0	0.3	.16	

* In levels.

TABLE 8						
Correlation* between g_t and i_t						
b	.1	.13	.15	.18	.2	.23
Ramsey	.99	.98	.98	.98	.98	.97
Lump-sum	.999	.999	.999	.9987	.9982	.9995

* Contemporaneous correlation.
i: Private investment.
g: Public investment.

TABLE 9: WELFARE				
LUMP-SUM AND RAMSEY ALLOCATIONS				
b	$E(\tau_i^{kp})$	Lump-Sum	Ramsey	%
.23	.31	315615	303130	-4.0
.2	.27	270301	260650	-3.6
.18	.25	244314	236430	-3.2
.15	.22	210590	205007	-2.7
.13	.2	191400	187028	-2.3
.1	.16	166801	164096	-1.6

TABLE 10: TAX RATE FIXED EXOGENOUSLY		
b = 0.23		
τ^{kp}	Welfare	%
.5	294794	-2.7
.45	298096	-1.7
.25	302185	-0.31
.2	300296	-0.93
.15	297043	-2.0

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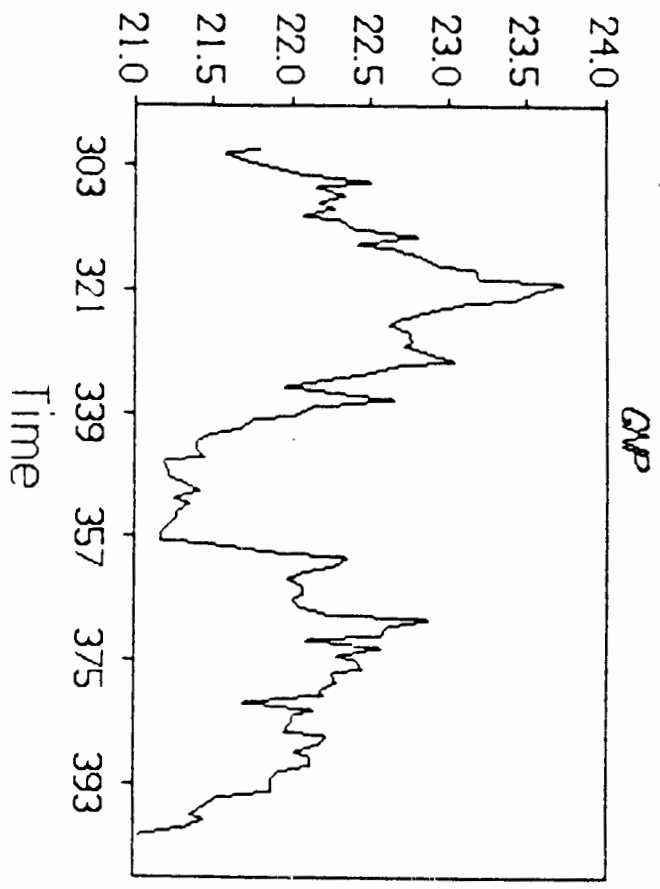
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303
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Time

Lump-sum Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$

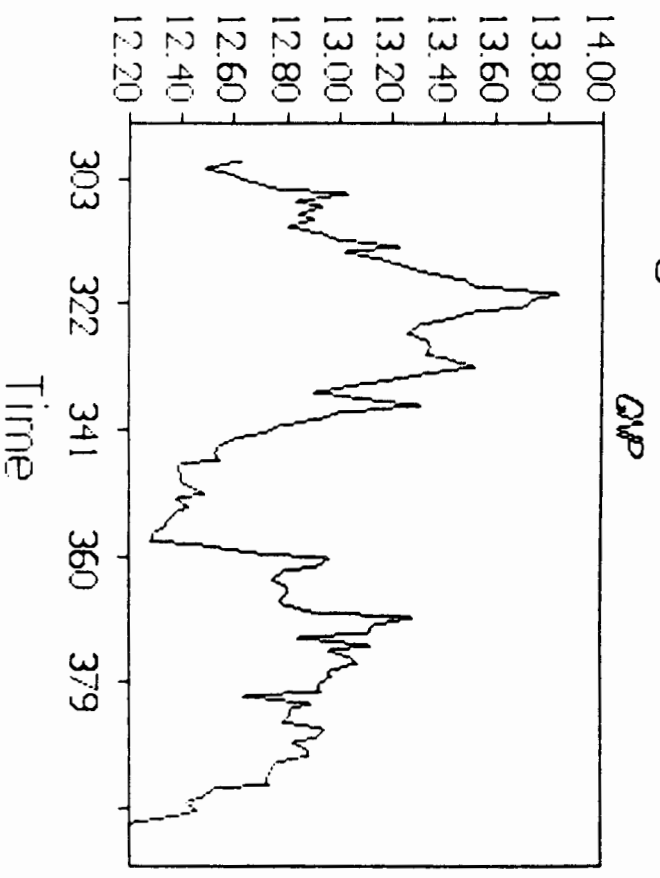
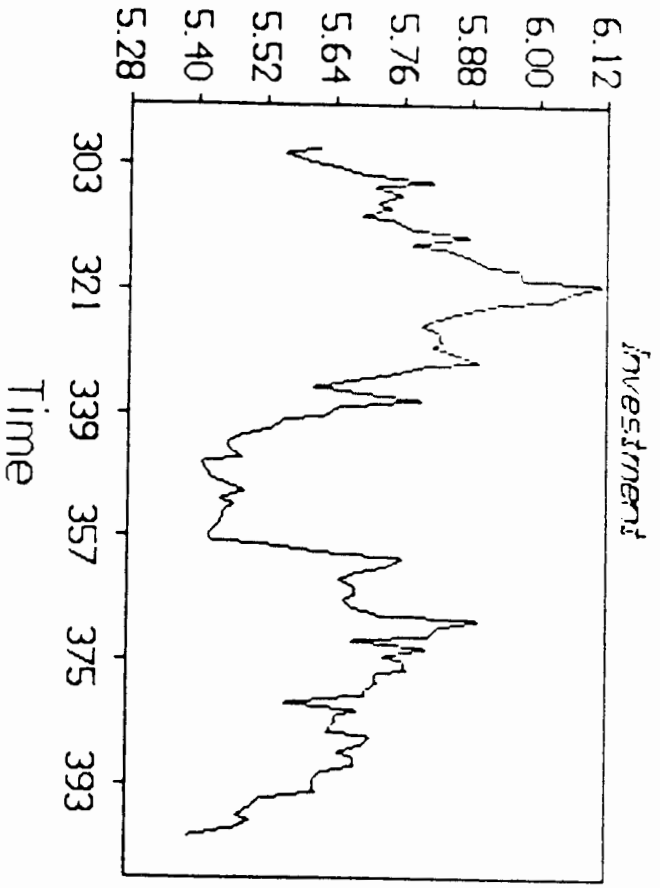


FIGURE 1

Lump-sum Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$

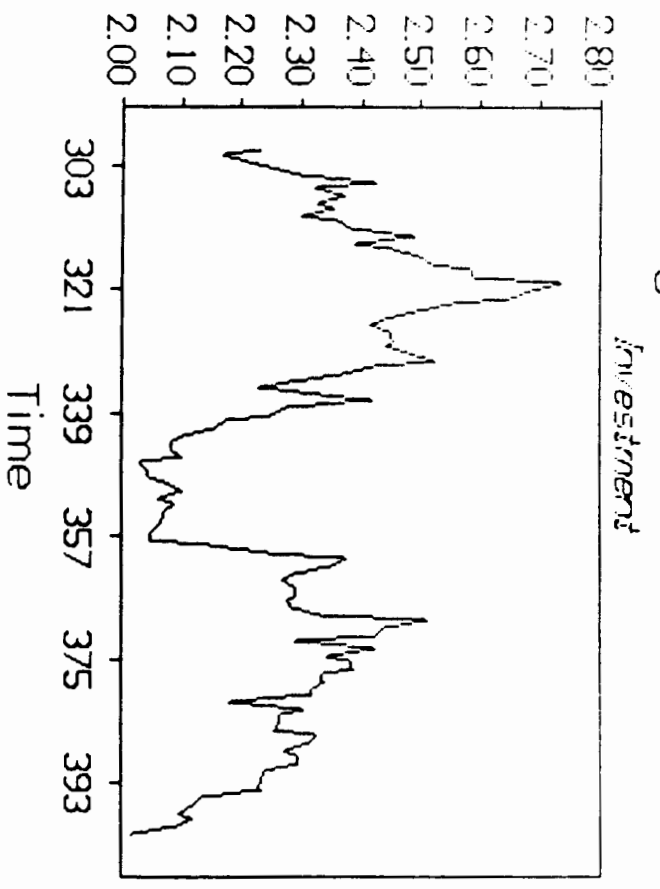
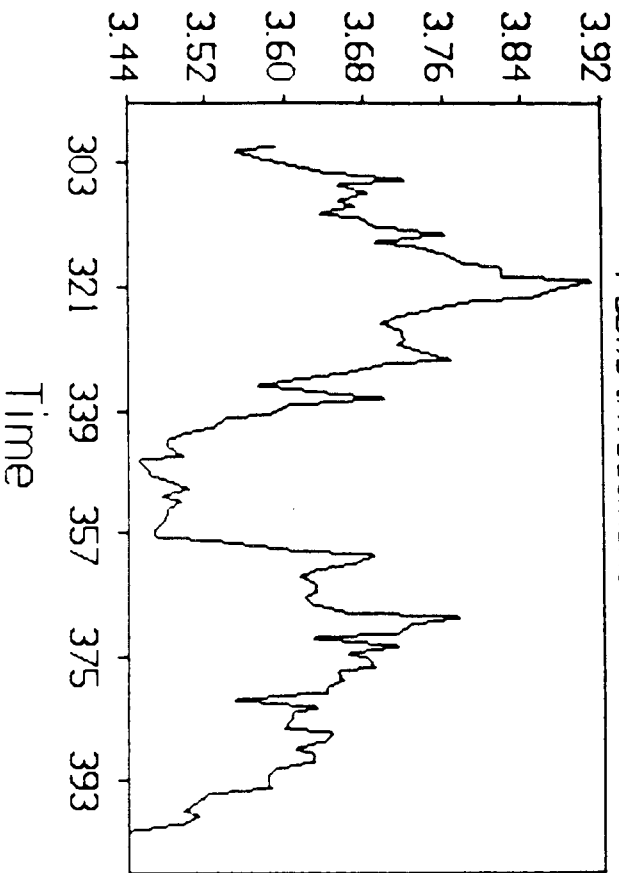


FIGURE 2

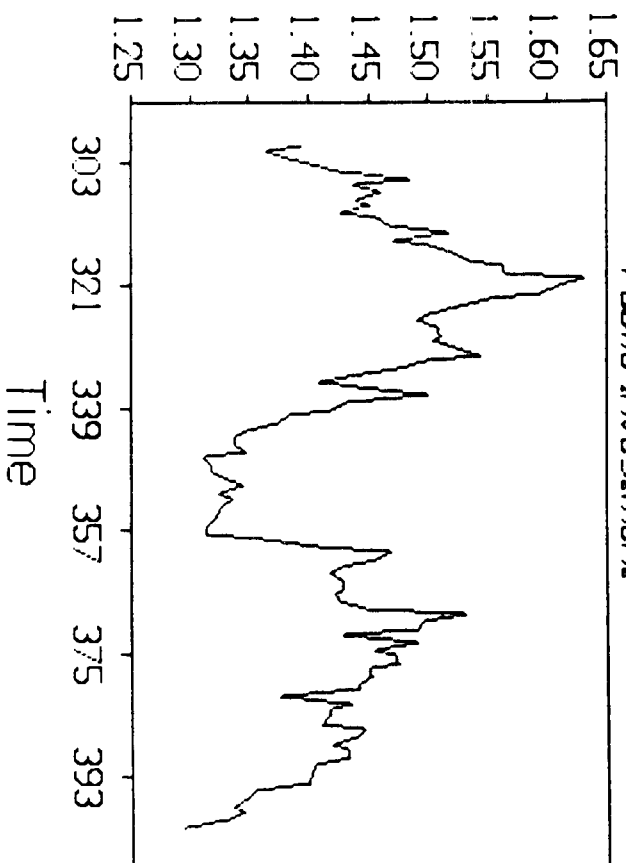
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Lump-sum Allocation: $b = 0.23$
Public Investment

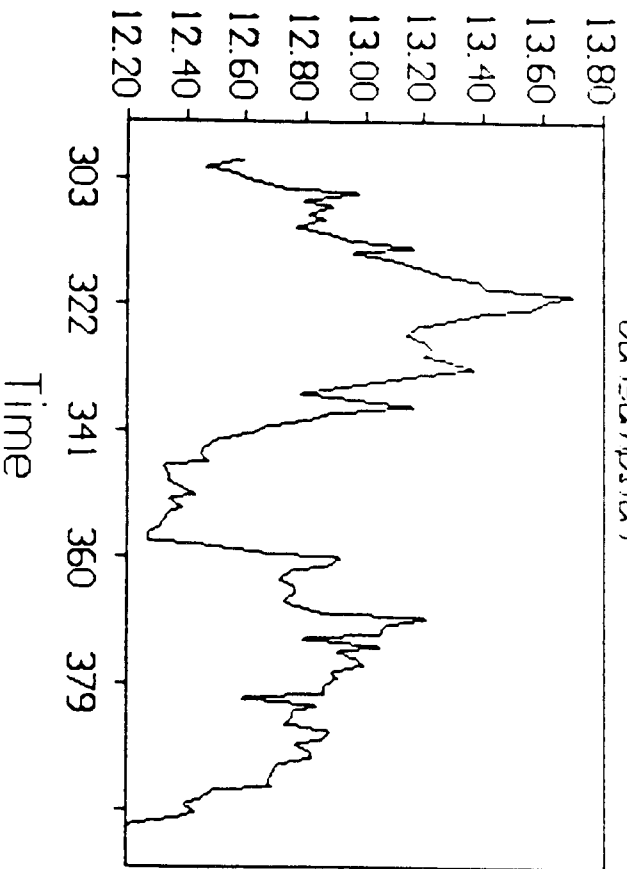


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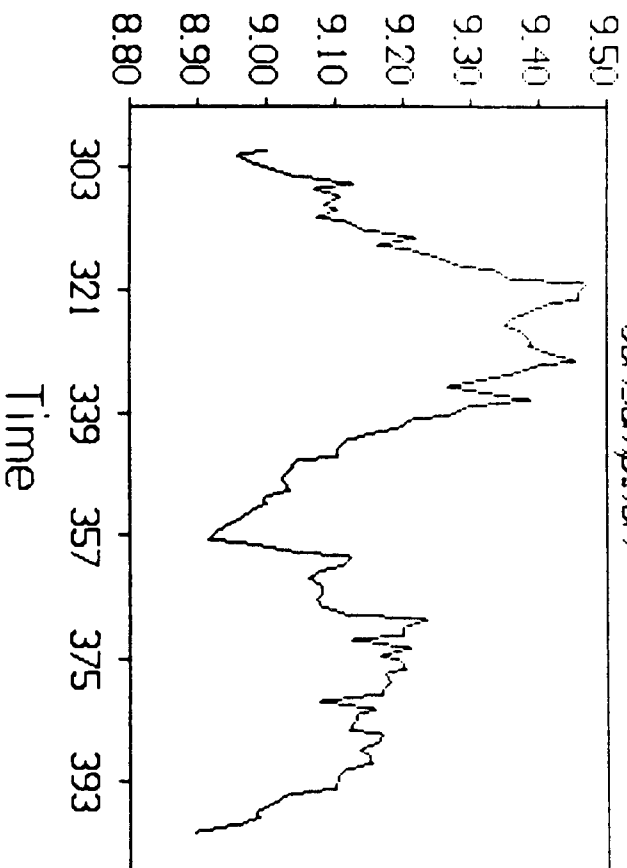
Ramsey Allocation: $b = 0.23$
Public Investment



Lump-sum Allocation: $b = 0.23$
Consumption



Ramsey Allocation: $b = 0.23$
Consumption



Lump-sum Allocation: $b = 0.23$

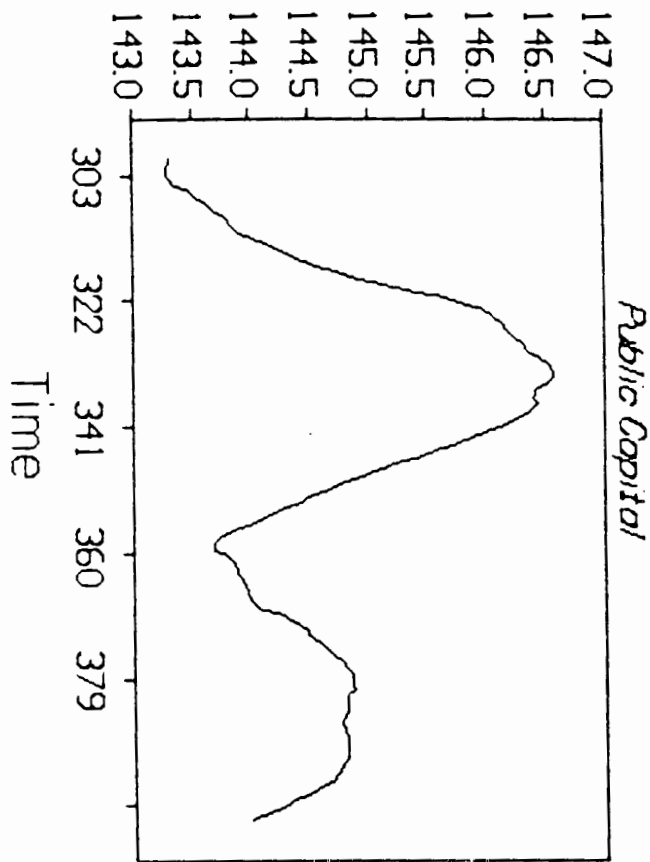
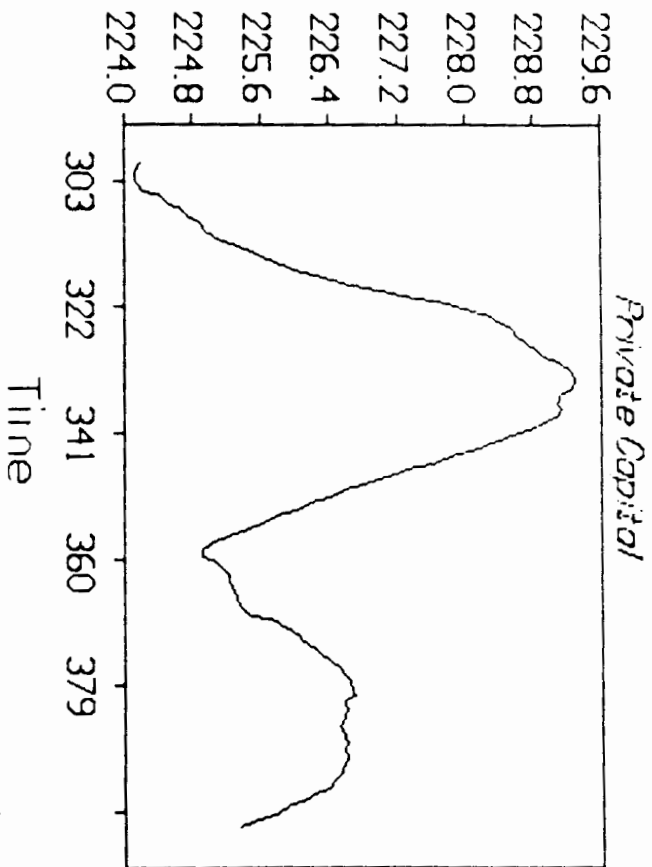
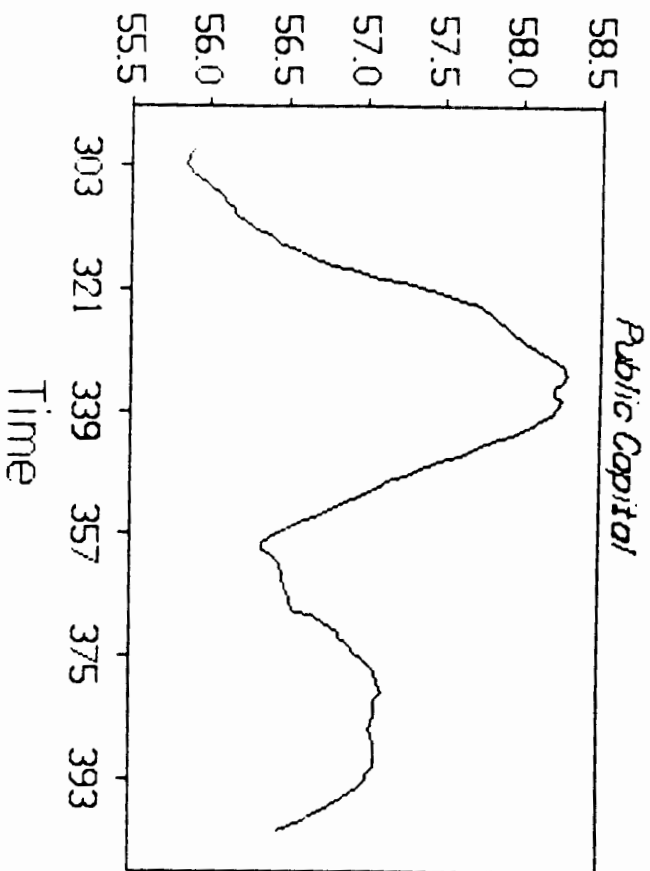


FIGURE 3

Lump-sum Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$

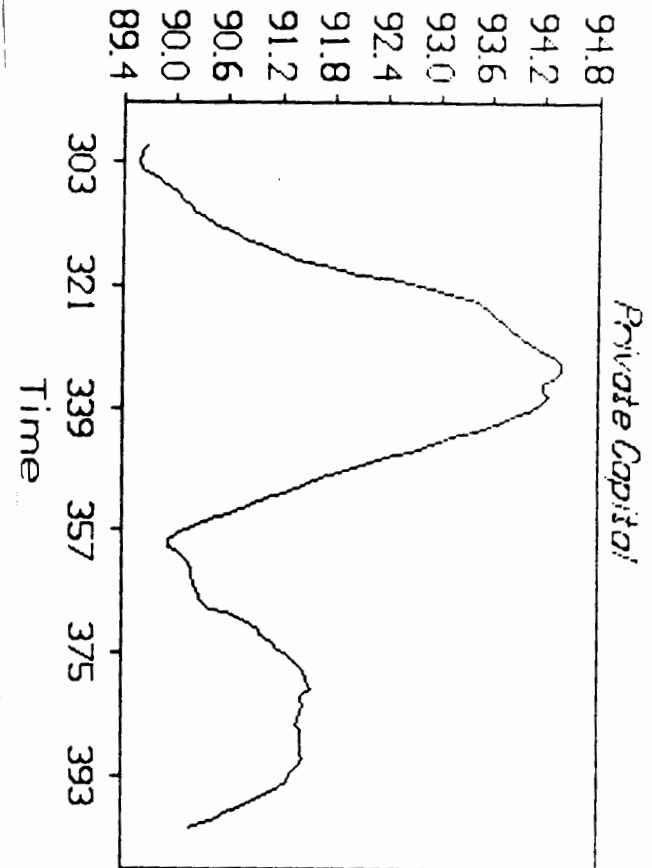
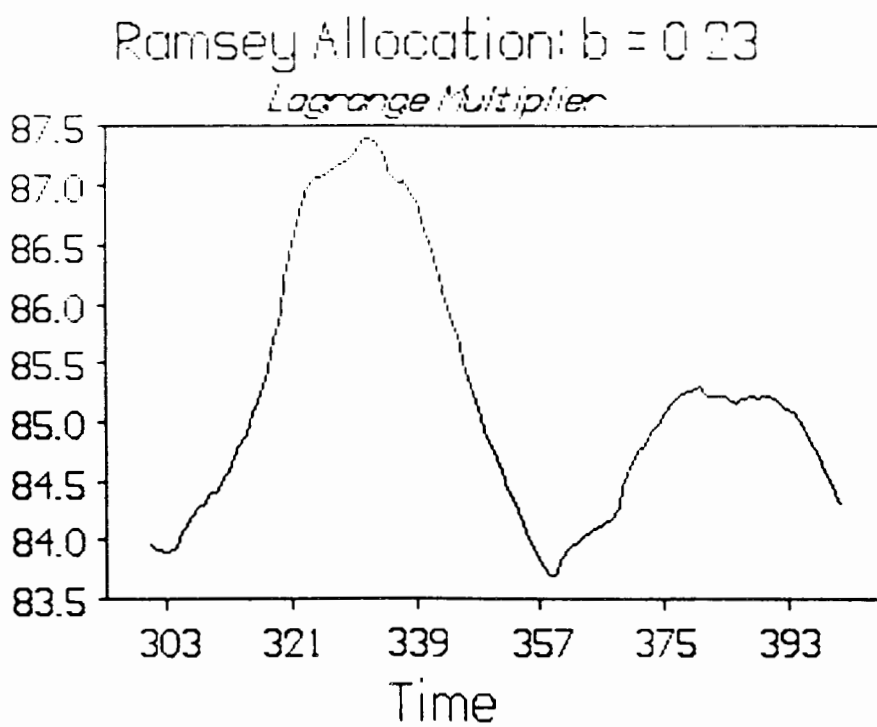
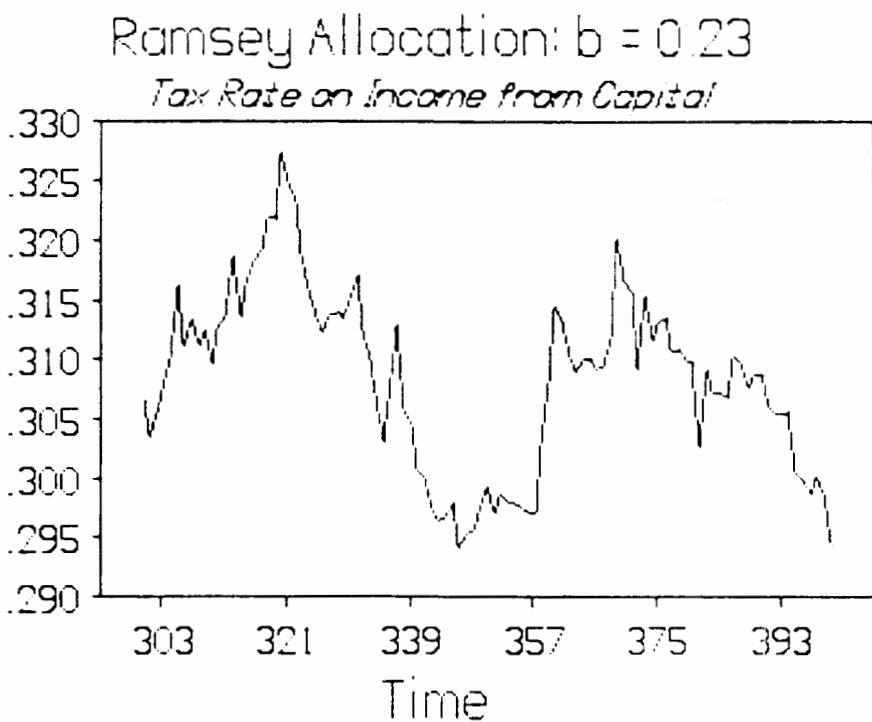


FIGURE 4



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FIGURE 5

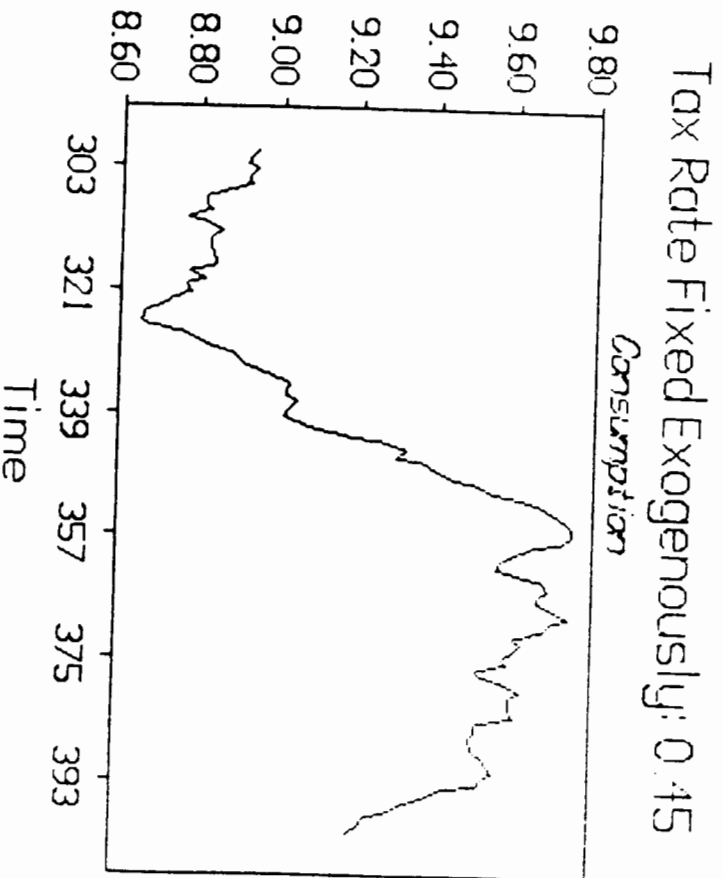
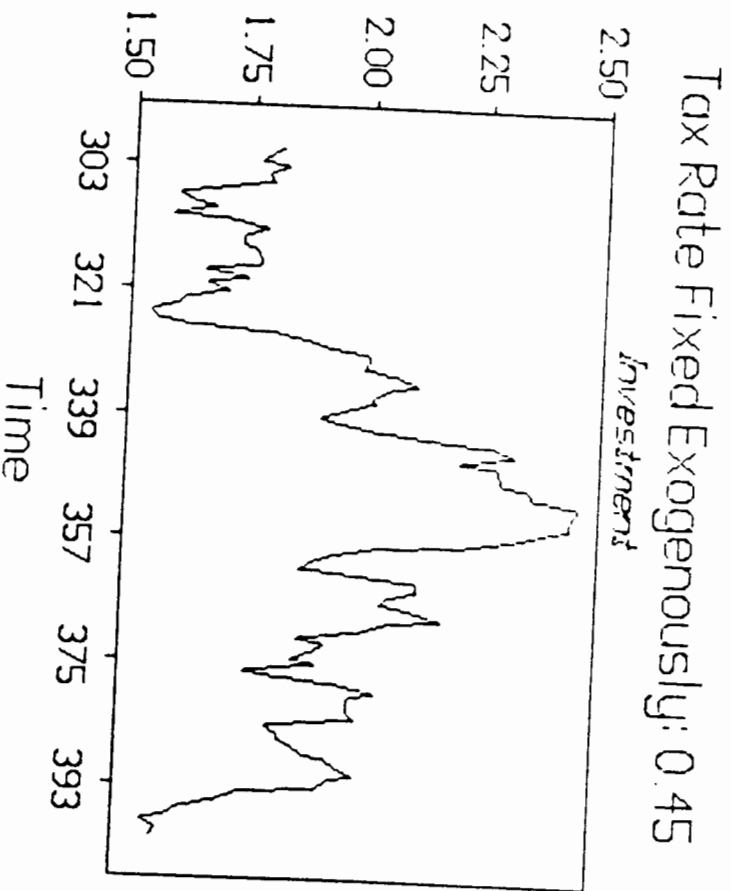
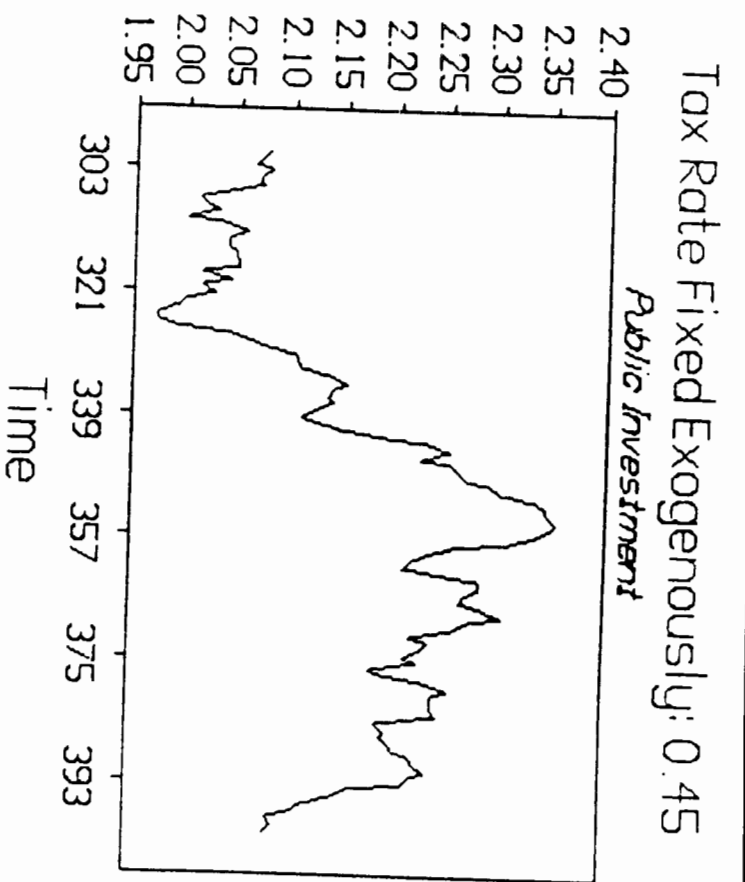
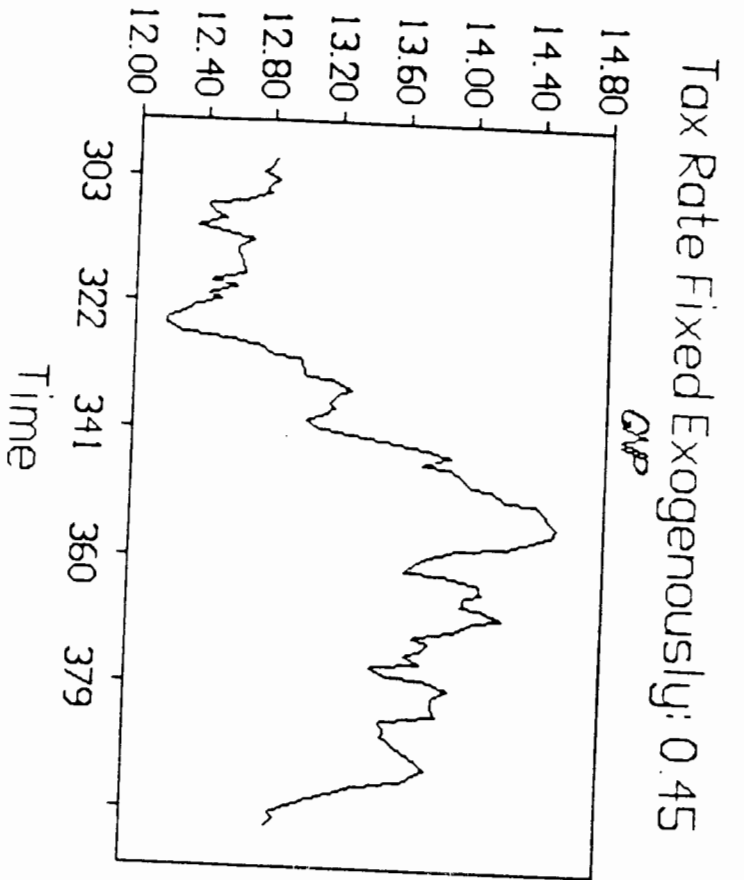
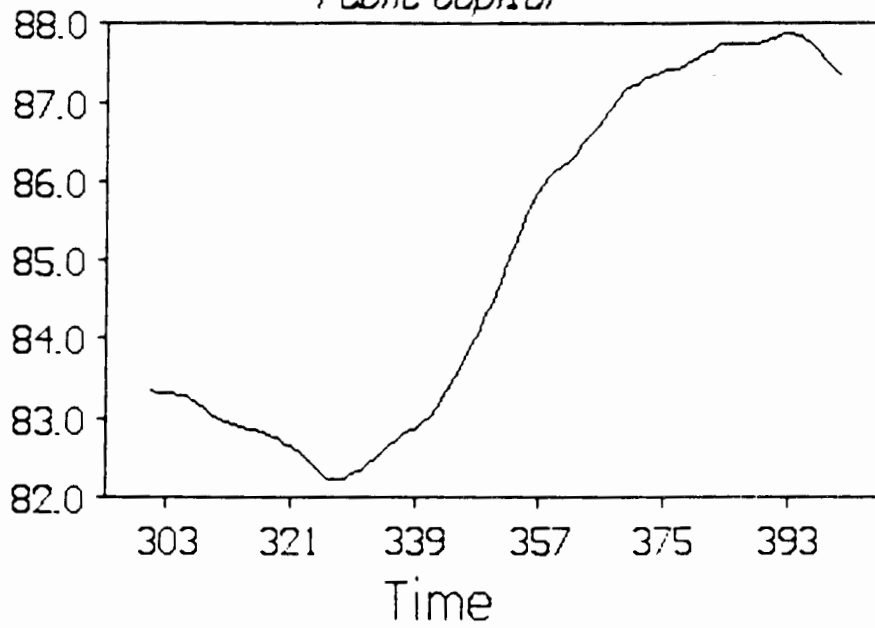


FIGURE 6

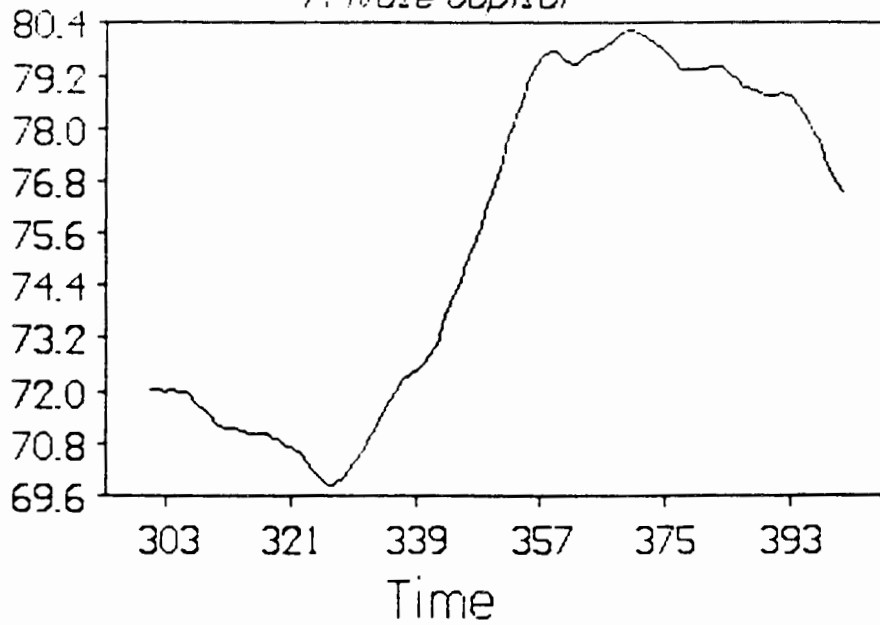
Tax Rate Fixed Exogenously: 0.45

Public Capital



Tax Rate Fixed Exogenously: 0.45

Private Capital



APPENDIX 1: SOLVING THE MODEL WITH PARAMETERIZED EXPECTATIONS

In this appendix we describe in more detail the method for solving the models described in sections 3 and 4.¹ We also provide extensive detail on how the algorithm was applied to our cases.

Ramsey Allocation

The first order conditions for the **Ramsey** allocation are:

$$\lambda_t = u'_t - \alpha_t u''_t + \alpha_{t-1} u''_t \left[f_t^{k_p} (1 - \tau_t^{k_p}) + (1 - d_p) \right] \quad (1)$$

$$\begin{aligned} \lambda_t = \beta E_t \{ & \alpha_t u'_{t+1} f_{t+1}^{k_p k_p} (1 - \tau_{t+1}^{k_p}) + \lambda_{t+1} [f_{t+1}^{k_p} + (1 - d_p)] \\ & + \mu_{t+1} [\tau_{t+1}^{k_p} f_{t+1}^{k_p} + \tau_t^{k_p} k_{p,t} f_{t+1}^{k_p k_p}] \} \end{aligned} \quad (2)$$

$$\begin{aligned} \lambda_t + \mu_t = \beta E_t \{ & \alpha_t u'_{t+1} f_{t+1}^{k_p k_g} (1 - \tau_{t+1}^{k_p}) + \lambda_{t+1} [f_{t+1}^{k_g} + (1 - d_g)] \\ & + \mu_{t+1} [\tau_{t+1}^{k_p} f_{t+1}^{k_p k_g} k_{p,t} + (1 - d_g)] \} \end{aligned} \quad (3)$$

$$\mu_t = \frac{\alpha_{t-1} u'_t}{k_{p,t-1}} \quad (4)$$

$$\alpha_{-1} = 0 \quad (5)$$

First, note that we can write equation (2) and (3) as follows:

¹See Marcet (1988), den Haan and Marcet (1989) and Marcet and Marshall (1992) for a complete description of the method and its properties.

$$\begin{aligned}\lambda_t i_t &= \beta E_t i_t \{ \alpha_t u'_{t+1} f_{t+1}^{k_p k_p} (1 - \tau_{t+1}^{k_p}) + \lambda_{t+1} [f_{t+1}^{k_p} + (1 - d_p)] \\ &\quad + \mu_{t+1} [\tau_{t+1}^{k_p} f_{t+1}^{k_p} + \tau_t^{k_p} k_{p_t} f_{t+1}^{k_p k_p}] \} \end{aligned} \quad (6)$$

$$\begin{aligned}\lambda_t g_t &= \beta E_t g_t \{ \alpha_t u'_{t+1} f_{t+1}^{k_p k_g} (1 - \tau_{t+1}^{k_p}) + \lambda_{t+1} [f_{t+1}^{k_g} + (1 - d_g)] \\ &\quad + \mu_{t+1} [\tau_{t+1}^{k_p} f_{t+1}^{k_p k_g} k_{p_t} + (1 - d_g)] - \frac{\mu_t}{\beta} \} \end{aligned} \quad (7)$$

With the equations (1), (2), (4), (32), (6) (7) and with the following constrains we can solve the **Ramsey** allocation,

$$c_t + k_{p_t} - (1 - d_p)k_{p_{t-1}} + k_{g_t} - (1 - d_g)k_{g_{t-1}} = f(k_{p_{t-1}}, k_{g_{t-1}}, l_t, \theta_t) \quad (8)$$

$$k_{g_t} - (1 - d_g)k_{g_{t-1}} = \tau_t^{k_p} f_t^{k_p} k_{p_{t-1}} \quad (9)$$

$$u'_t = \beta E_t \{ u'_{t+1} [f_{t+1}^{k_p} (1 - \tau_{t+1}^{k_p}) + (1 - d_p)] \} \quad (10)$$

$$k_{p_{t-1}} = k_p \quad (11)$$

$$k_{g_{t-1}} = \bar{k}_g \quad (12)$$

Let $\xi(k_{p_{t-1}}, k_{g_{t-1}}, \alpha_{t-1}, \theta_t)$ stand for a particular function and let Δ_m^l stand for the vector of l initial conditions for each *Euler* equation m . The conditional expectation in the *Euler* equations are parameterized with

$$\xi(\log(k_{p_{t-1}}), \log(k_{g_{t-1}}), \log(\theta_t), \alpha_{t-1}; \Delta_m) \quad (13)$$

Substituting (13) in (6), (1) and (7) we have the following:

$$u'_t = \beta \Delta_1^1 \exp \left[\Delta_1^2 \log(k_{p_{t-1}}) + \Delta_1^3 \log(k_{g_{t-1}}) + \Delta_1^4 \log(\theta_t) + \Delta_1^5 \alpha_{t-1} \right] \quad (14)$$

$$\lambda_t i_t = \beta \Delta_2^1 \exp \left[\Delta_2^2 \log(k_{p_{t-1}}) + \Delta_2^3 \log(k_{g_{t-1}}) + \Delta_2^4 \log(\theta_t) + \Delta_2^5 \alpha_{t-1} \right] \quad (15)$$

$$\lambda_t g_t = \beta \Delta_3^1 \exp \left[\Delta_3^2 \log(k_{p_{t-1}}) + \Delta_3^3 \log(k_{g_{t-1}}) + \Delta_3^4 \log(\theta_t) + \Delta_3^5 \alpha_{t-1} \right] \quad (16)$$

Given an initial guess for the coefficients Δ_m^l , initial conditions for private and public capital, \bar{k}_p and \bar{k}_g , and a realization of θ_t given from

$$\log \theta_t = \rho \log \theta_{t-1} + \epsilon_t \quad (17)$$

we solve a series of length S for the endogenous variables $\{c_t, k_{p_t}, k_{g_t}, \tau_t^{k_p}, \alpha_{t-1}, \lambda_t, \mu_t\}$.

The resulting series will in general not be consistent with the assumed form for the conditional expectation. In this case we have to update the estimation of Δ_m^l . We do this by solving a non-linear squares problem:²

$$\text{Min}_{\Delta} \sum_{t=1}^S \{ [\dots] - \xi(k_{p_{t-1}}, k_{g_{t-1}}, \alpha_{t-1}, \theta_t; \Delta) \}^2 \quad (18)$$

where $[\dots]$ denotes the term inside the conditional expectation in (14), (15) and (16), the *Euler* equations in the **Ramsey** problem.

With the new estimation of Δ_m^l , say $\Delta_m^{(1)}$, we repeat the steps until the value of $\Delta_m^{(n)}$ is equal to $\Delta_m^{(n+1)}$. The converged value of Δ_m^l , say Δ_m^* , is the solution to the problem³. Our claim is that Δ_* is the fixed point that corresponds to the rational expectations equilibrium and is consistent with the assumed form of the conditional expectation.

Lump-sum Allocation

In the **lump-sum** allocation we only have to parameterize two equations:

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[f_{t+1}^{k_p} + (1 - d_p) \right] \right\} \quad (19)$$

²See Pindyck and Rubinfeld (1987)

³These steps define a mapping, L , from Δ_n to Δ_{n+1} where Δ_* is the fixed point of the mapping. We use the updating scheme: $\Delta_{n+1} = eL(\Delta_n) + (1 - e)\Delta_n$ where $e \in (0, 1)$.

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[f_{t+1}^{kg} + (1 - d_g) \right] \right\} \quad (20)$$

Using a polynomial form and substituting in⁴, we have:

$$u'_t = \beta \Delta_1^1 \exp \left[\Delta_1^2 \log(k_{p_{t-1}}) + \Delta_1^3 \log(k_{g_{t-1}}) + \Delta_1^4 \log(\theta_t) \right] \quad (21)$$

$$u'_t = \beta \Delta_2^1 \exp \left[\Delta_2^2 \log(k_{p_{t-1}}) + \Delta_2^3 \log(k_{g_{t-1}}) + \Delta_2^4 \log(\theta_t) \right] \quad (22)$$

Note that this system of equation is undetermined. In order to solve it, we multiply in both sides in equation (21) and (22) by i_t by g_t respectively:⁵

$$u'_t i_t = \beta E_t \left\{ i_t u'_{t+1} \left[f_{t+1}^{kp} + (1 - d_p) \right] \right\} \quad (23)$$

$$u'_t g_t = \beta E_t \left\{ g_t u'_{t+1} \left[f_{t+1}^{kg} + (1 - d_g) \right] \right\} \quad (24)$$

Now we parameterize (23) and (24):

$$u'_t i_t = \beta \Delta_1^1 \exp \left[\Delta_1^2 \log(k_{p_{t-1}}) + \Delta_1^3 \log(k_{g_{t-1}}) + \Delta_1^4 \log(\theta_t) \right] \quad (25)$$

$$u'_t g_t = \beta \Delta_2^1 \exp \left[\Delta_2^2 \log(k_{p_{t-1}}) + \Delta_2^3 \log(k_{g_{t-1}}) + \Delta_2^4 \log(\theta_t) \right] \quad (26)$$

Now, and with the feasibility constraint:

$$c_t + k_{p_t} - (1 - d_p)k_{p_{t-1}} + k_{g_t} - (1 - d_g)k_{g_{t-1}} = y_t \quad (27)$$

with initial conditions for private and public capital, \bar{k}_p and \bar{k}_g , and a realization of the shock, θ_t , we can solve for the endogenous variables $\{c_t, k_{p_t}, k_{g_t}\}$ and then repeat the same steps as before.

⁴However, in this case the lagrange multiplier α_t does not appear as a state variable because there are no tax distortions.

⁵Note that we can solve this problem if we multiply any of the two equations by i_t or g_t . However, when both capitals are equal ($a=b$ and $d_p = d_g$) we must have that the steady state values must be the same. In order to have this property we must have a symmetric solution for both capitals.

Tax Rate Fixed Exogenously

In this case we have to parameterize:

$$u'_t = \beta E_t \left\{ u'_{t+1} \left[(1 - \tau^{k_p}) f_{t+1}^{k_p} + (1 - d_p) \right] \right\} \quad (28)$$

using the functional form mentioned above, we have:

$$u'_t = \beta \Delta_1^1 \exp \left[\Delta_1^2 \log(k_{p,t-1}) + \Delta_1^3 \log(k_{g,t-1}) + \Delta_1^4 \log(\theta_t) \right] \quad (29)$$

In this model government spending is given by:

$$g_t = \tau^{k_p} f_t^{k_p} k_{p,t-1} \quad (30)$$

and with the feasibility constraint 27, initial conditions for private and public capital, \bar{k}_p and \bar{k}_g , a fixed value of τ^{k_p} and a realization of the shock, θ_t , we can solve for the endogenous variables $\{c_t, k_{p,t}, k_{g,t}\}$ and then repeat the same steps as before.

Computation

The models were solved in a Hewlett Packard, Model 720, Work Station Risk01. The algorithms were written in Fortran 77. (All the programs are available upon request).

The fixed point for the vector Δ_m^l was computed using 2500 observations⁶. We stop the iteration when the difference between Δ_m^n and Δ_m^{n+1} was less than 10^{-3} .

For the **Ramsey** allocation the parameters of the policy function are given in Table 1.1. The policy function was computed for a long realization of the series.

⁶We also solve the model using 10,000 observations, but the parameters of the policy function only change in the third digit.

TABLE 1.1:RAMSEY ALLOCATION						
b	.23	.2	.18	.15	.13	.1
Δ_1^1	2.3148	2.4042	2.4851	2.5845	2.6512	2.5878
Δ_1^2	-0.6262	-0.6106	-0.6015	-0.5859	-0.5781	-0.5275
Δ_1^3	-0.0576	-0.0541	-0.0547	-0.0575	-0.0578	-0.0941
Δ_1^4	-0.4614	-0.4332	-0.4224	-0.4059	-0.3936	-0.3783
Δ_1^5	0.0002	0.0003	0.0002	0.0001	0.00005	-0.0006
Δ_2^1	1.4389	1.6725	1.7838	1.9273	2.0331	2.1563
Δ_2^2	-0.4712	-0.4597	-0.4508	-0.4382	-0.4409	-0.4414
Δ_2^3	-0.0683	-0.0883	-0.0961	-0.1073	-0.1061	-0.1068
Δ_2^4	-0.0248	-0.1174	-0.1422	-0.1749	-0.1964	-0.2135
Δ_2^5	-0.0033	-0.0045	-0.0056	-0.0075	-0.0089	-0.0113
Δ_3^1	0.0095	0.0061	0.0019	0.0002	0.00006	0.000002
Δ_3^2	5.9205	7.0196	7.3096	8.1041	8.2690	8.9864
Δ_3^3	-5.0714	-6.4365	-6.6838	-7.4151	-7.6238	-8.0624
Δ_3^4	1.2520	1.2251	1.1961	1.0812	1.0203	1.2083
Δ_3^5	-0.0137	-0.0328	-0.0430	-0.0651	-0.0802	-0.11

b: The elasticity of output with respect to public infrastructure Δ_m^l : where m is the Euler equation and l is the coefficient for each state variable

In table 1.2 we report the parameters of the policy function for the lump-sum allocation.

TABLE 1.2:LUMP-SUM ALLOCATION						
b	.23	.2	.18	.15	.13	.1
Δ_1^1	28.9788	17.8648	15.2034	9.1672	10.3057	7.5046
Δ_1^2	-0.9569	-0.9722	-0.9734	-0.9978	-0.9756	-1.0018
Δ_1^3	0.2047	0.2597	0.2625	0.3757	0.2876	0.3975
Δ_1^4	0.1255	0.1188	0.1176	0.1066	0.1144	0.1026
Δ_2^1	26.146	14.0325	9.4196	5.2241	3.40834	1.8362
Δ_2^2	-0.2166	-0.2396	-0.2426	-0.2844	-2.49	-0.2976
Δ_2^3	-0.6716	-0.6468	-0.6443	-0.5983	.0905	-0.5843
Δ_2^4	0.1517	0.1506	0.1501	0.1505	0.1797	0.148

b: The elasticity of output with respect to public infrastructure Δ_m^l : where m is the Euler equation and l is the coefficient for each state variable

In table 1.3 we present the parameters of the policy function for the model where taxes are fixed exogenously.

TABLE 1.3:TAXES FIXED EXOGENOUSLY						
a=0.36 b=0.23						
τ^{k_p}	.15	.2	.25	.31	.45	.5
Δ_1^1	2.5512	2.4234	2.4661	2.5394	2.1786	2.0164
Δ_1^2	-0.5554	-0.5373	-0.5439	-0.572	-0.5782	-0.5731
Δ_1^3	-0.304	-0.3179	-0.3325	-0.3613	-0.4074	-0.426
Δ_1^4	-0.1436	-0.1572	-0.1563	-0.1372	-0.1104	-0.1042

b: The elasticity of output with respect to public infrastructure
a: The elasticity of output with respect to private capital
 τ^{k_p} : tax rate on income from capital
 Δ_m^l : where m is the Euler equation and l is the coefficient for each state variable

Welfare

For computing welfare we have to simulate:

$$U = E_t \sum_{t=0}^T \beta^t \ln(c_t) \quad (31)$$

The procedure for computing welfare is as follows:

1. Solve the model, which means find Δ_m^l .
2. In order to compare welfare, fix the initial conditions for the private capital, \bar{k}_p , public capital \bar{k}_g and the Lagrange multiplier, α_0 . Table 1.4 contains the initial conditions used for computing welfare.

TABLE 1.4:INITIAL CONDITIONS						
<i>b</i>	.23	.2	.18	.15	.13	.1
k_p	228	152	121	89	75	60
\bar{k}_g	146	85	60.5	37.5	27	16.5
α_0	0	0	0	0	0	0

b: The elasticity of output with respect to public infrastructure

3. For a new realization for the shock, θ_t , generate a series of size T (we use T = 500) for the endogenous variables of the model: $\{c_t, k_{p_t}, k_{g_t}, \tau_t^{k_p}, \alpha_{t-1}\}$

in the case of the **Ramsey** allocation; and $\{c_t, k_{p_t}, k_{g_t}\}$ for the **lump-sum** allocation.

4. Compute $E_t \sum_{t=0}^T \beta^t \ln(c_t)$.

5. Do step 3 and 4 for many realization of the shock (we use 200 realization of the shock).

6. Compute the average of $E_t \sum_{t=0}^T \beta^t \ln(c_t)$ for all the realization of the shock.

(In Figures 7-12 we present the path for the variables in the first 100 periods for each allocation.)

Short-Run Simulations

The algorithm described can approximate the true equilibrium at the steady state distribution arbitrarily well as the length of the simulation go to infinity. However, if one of the endogenous variables starts at a very low level, the Δ_* from long run simulations may not be a good approximation to the conditional expectations during the first periods, as the endogenous variable grows to the steady state distribution.

This could be a problem in our model, because we have the constraint:

$$\alpha_{-1} = 0 \tag{32}$$

and we are interested in computing welfare in the **Ramsey** allocation in the initial periods.

In order to avoid this problem we compute **short-run simulations**⁷. The idea is to compute a different policy function Δ_{sr} for the initial periods and use Δ_{sr} for computing welfare.

We proceed as follows:

1. Solve the model for the long-run simulation and find Δ_* , the steady state policy function;

2. Solve the model for a **short-run simulation** of length T_{sr} , where T_{sr} is selected to be long enough for the economy to get in the range of the steady state distribution (we use $T_{sr} = 200$). In order to stay into this range we compute the last endogenous variables using the steady state policy function Δ_* . (we use $T_{sr} = 200$ and the last 10 observations were computed using the steady state policy function).

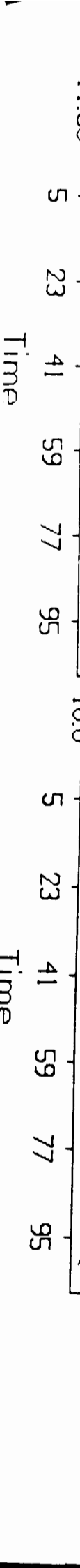
⁷See Marcet and Marimon (1993)

3. Compute welfare as before, but using $T=T_{sr}$.

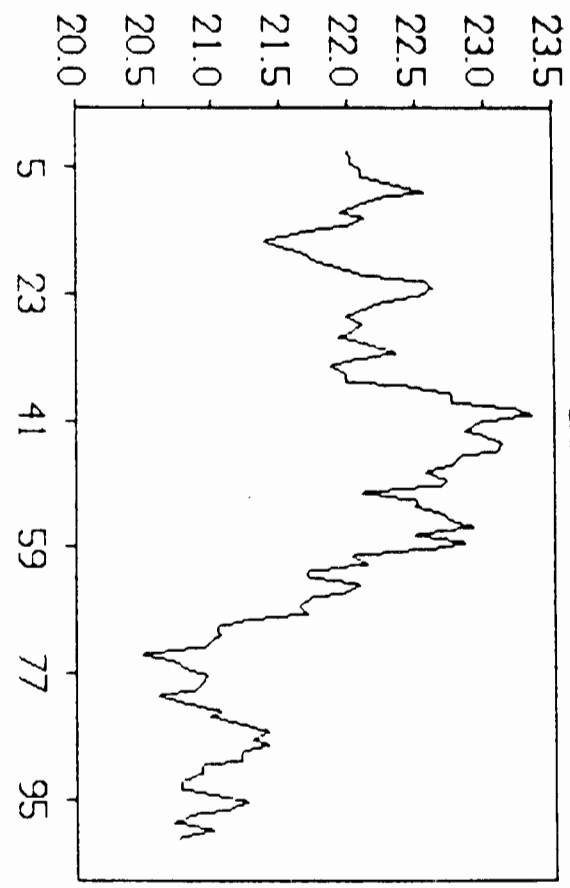
In table 1.5 we report the parameter values of the **short-run** policy function. (in Figure 13 we present the Lagrange multiplier for the **short-run** simulation when $b = 0.23$)

TABLE 1.5: RAMSEY ALLOCATION						
SHORT-RUN SIMULATION						
b	.23	.2	.18	.15	.13	.1
Δ_1^1	2.3618	2.5241	2.5793	2.6029	2.6203	2.5336
Δ_1^2	-0.6095	-0.6173	-0.6071	-0.5825	-0.5678	-0.5162
Δ_1^3	-0.0798	-0.0600	-0.0598	-0.0649	-0.0682	-0.1027
Δ_1^4	-0.4391	-0.4131	-0.4056	-0.3940	-0.3844	-0.3715
Δ_1^5	0.0001	0.0003	0.0003	0.0002	0.0001	-0.0006
Δ_2^1	1.1353	1.3921	1.5105	1.6928	1.8419	2.0790
Δ_2^2	-0.4187	-0.4193	-0.4127	-0.4097	-0.4180	-0.4336
Δ_2^3	-0.0828	-0.0917	-0.0973	-0.1033	-0.1027	-0.1052
Δ_2^4	-0.0734	-0.1432	-0.1608	-0.1847	-0.2011	-0.2151
Δ_2^5	-0.0025	-0.004	-0.0053	-0.0073	-0.0088	-0.0109
Δ_3^1	0.008	0.0043	0.0019	0.0004	0.0001	0.000002
Δ_3^2	5.8193	4.9538	5.1010	5.9688	6.4047	8.9825
Δ_3^3	-4.9213	-3.9747	-4.0336	-4.8515	-5.3388	-8.0349
Δ_3^4	1.2429	1.2812	1.2731	1.2037	1.1705	1.2398
Δ_3^5	-0.0135	-0.0274	-0.0375	-0.0613	-0.0808	-0.1103

b: The elasticity of output with respect to public infrastructure
 Δ_m^l : where m is the Euler equation and l is the coefficient for each state variable



Lump-sum Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$

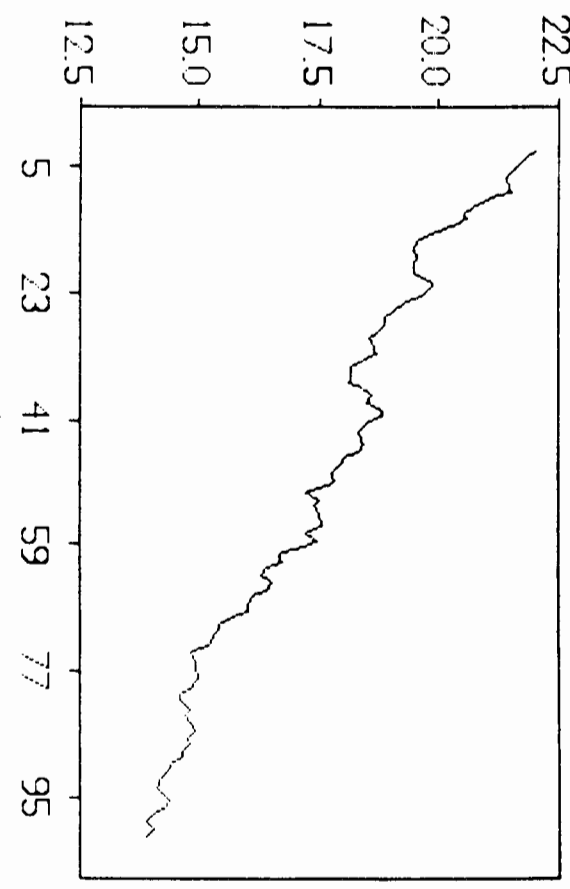
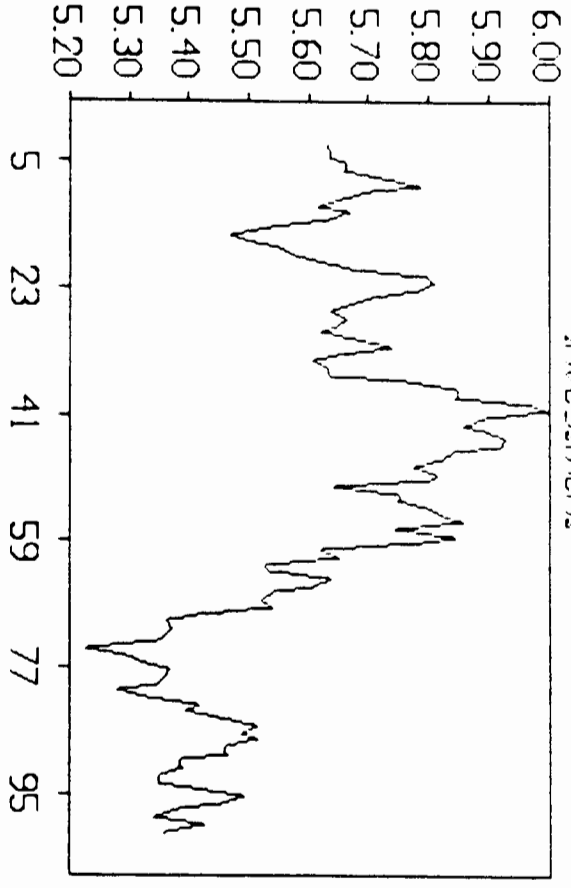


FIGURE 7

Lump-sum Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$

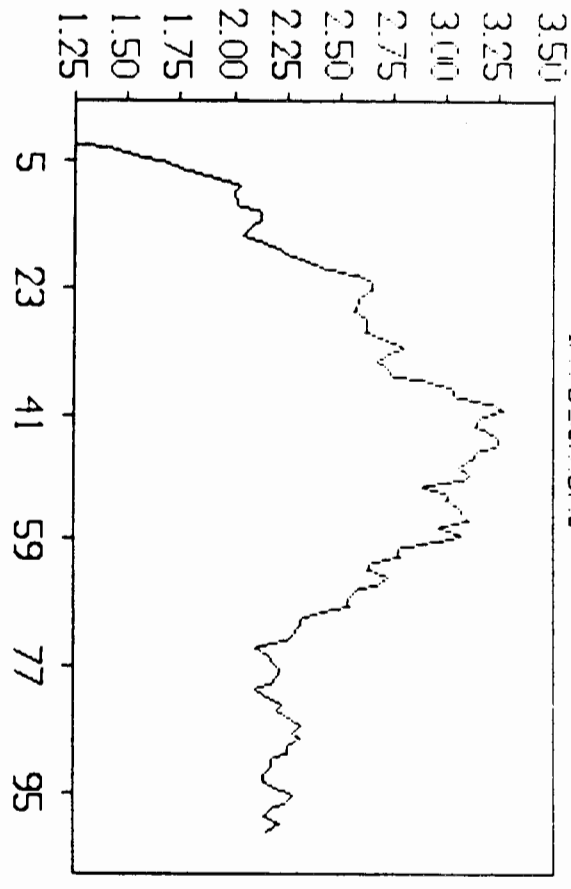
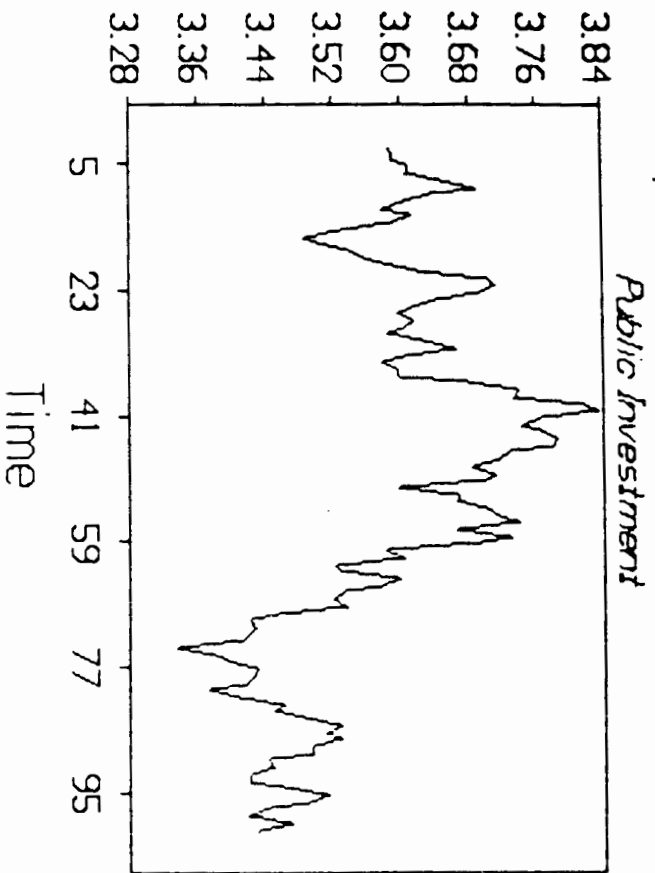
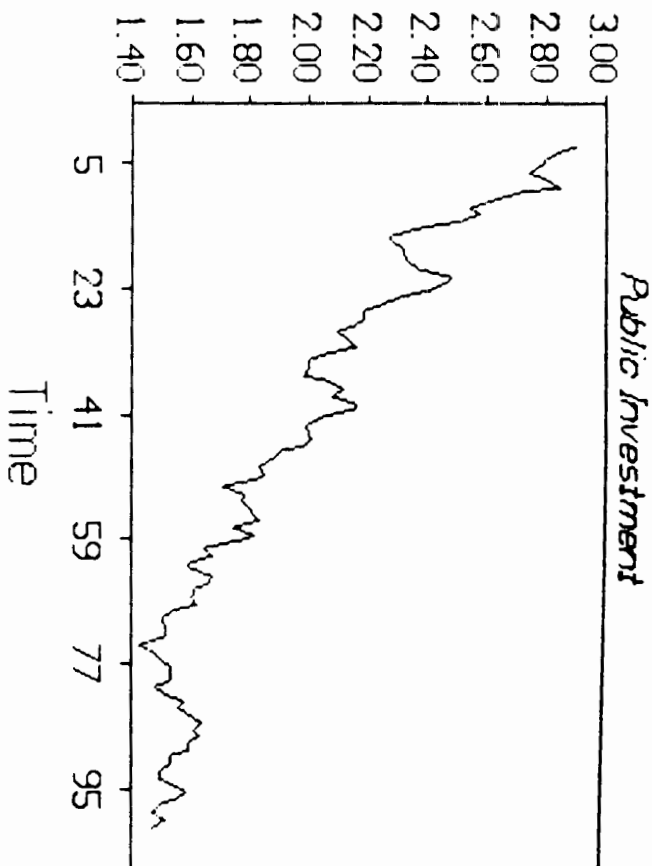


FIGURE 8

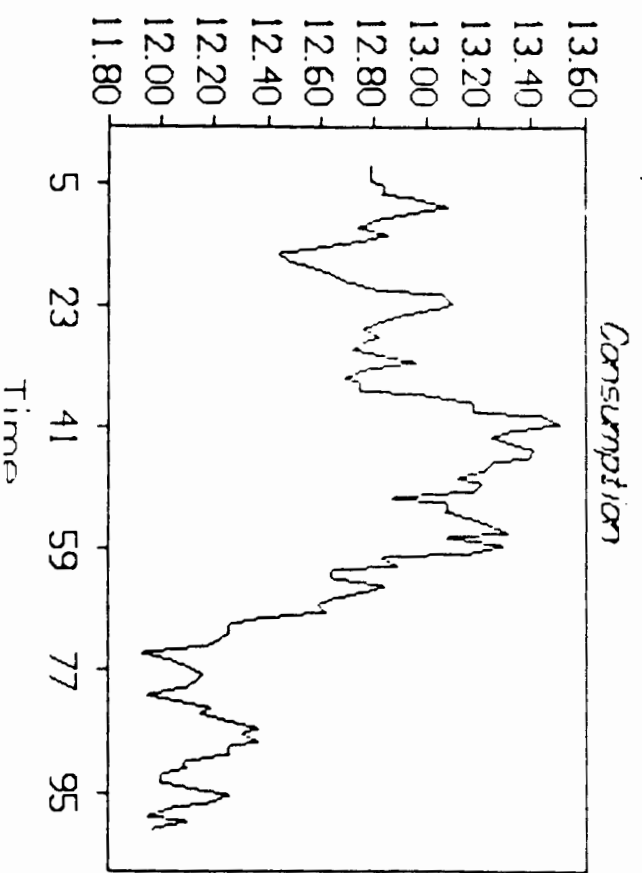
Lump-sum Allocation: $b = 0.23$



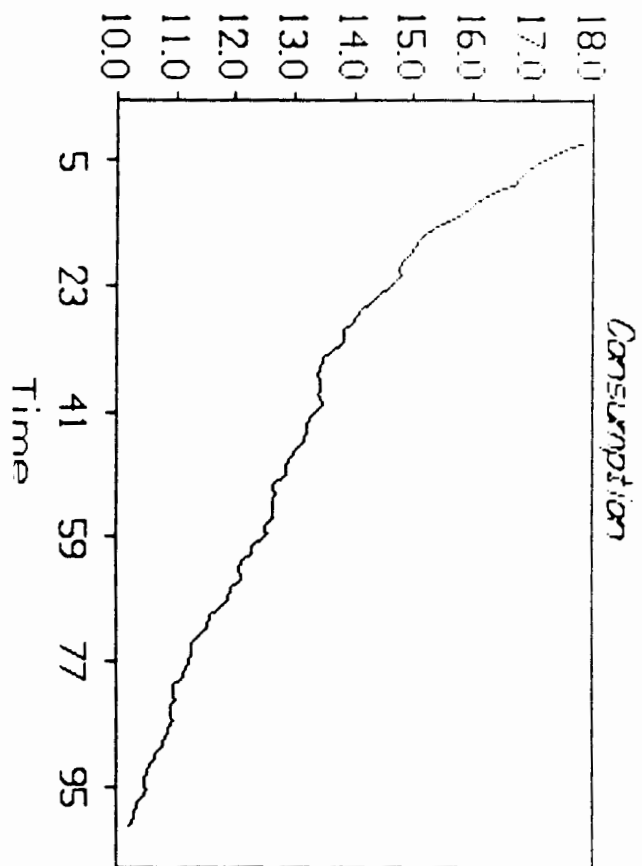
Ramsey Allocation: $b = 0.23$



Lump-sum Allocation: $b = 0.23$



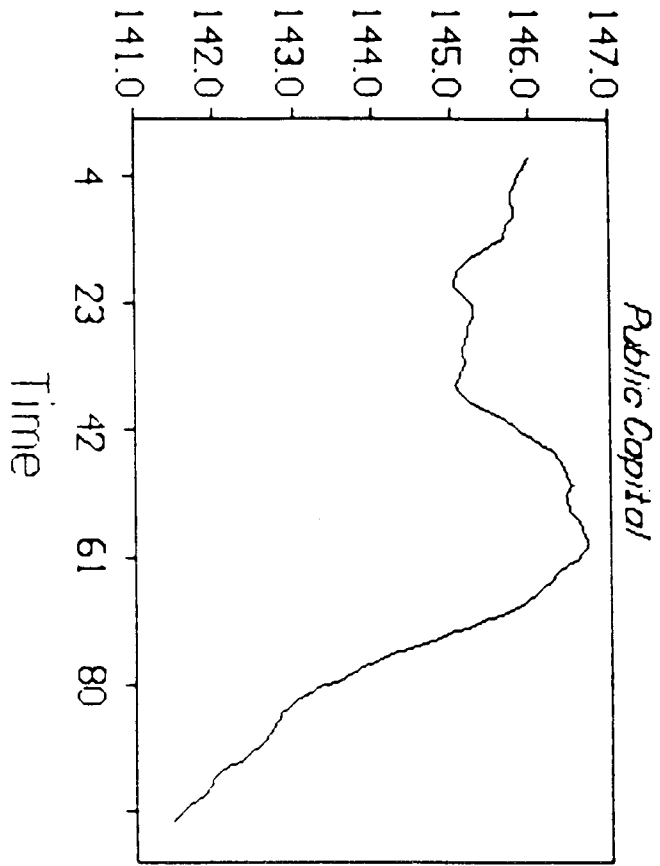
Ramsey Allocation: $b = 0.23$



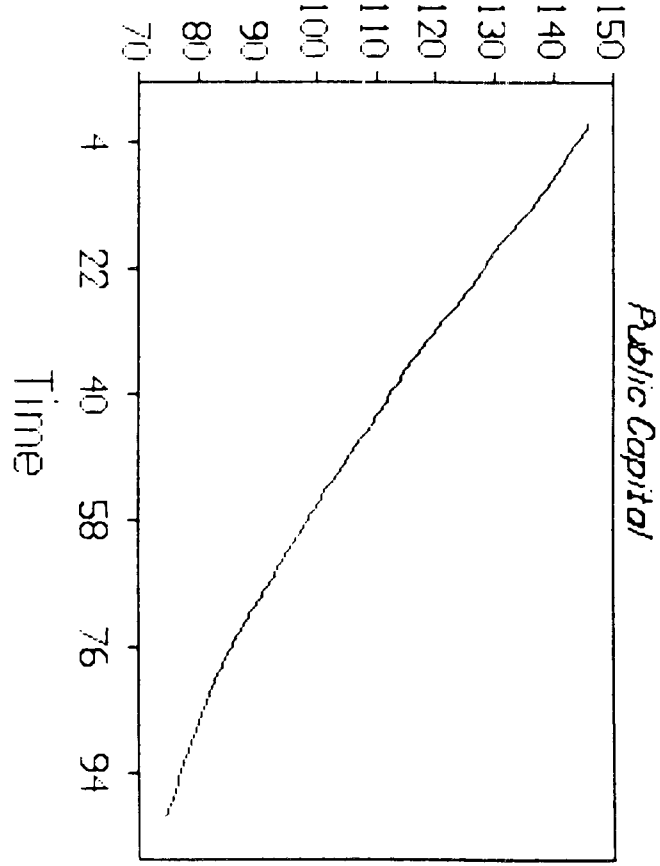
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FIGURE 9

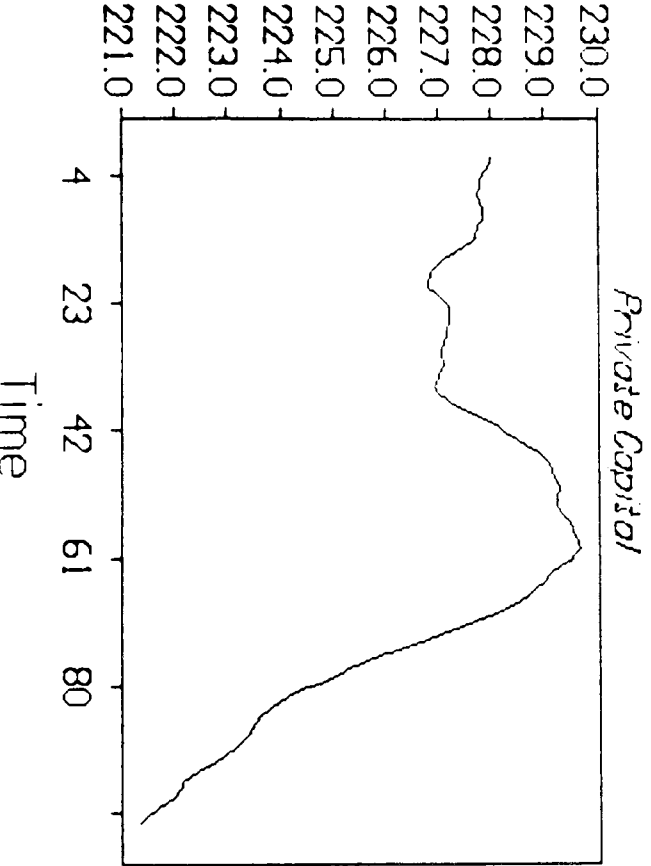
Lump-sum Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$



Lump-sum Allocation: $b = 0.23$



Ramsey Allocation: $b = 0.23$

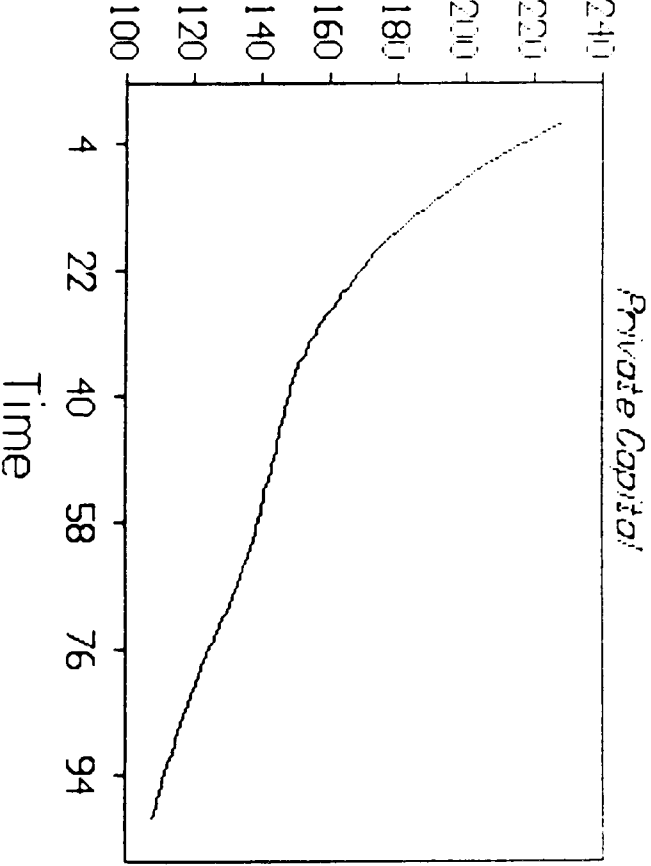
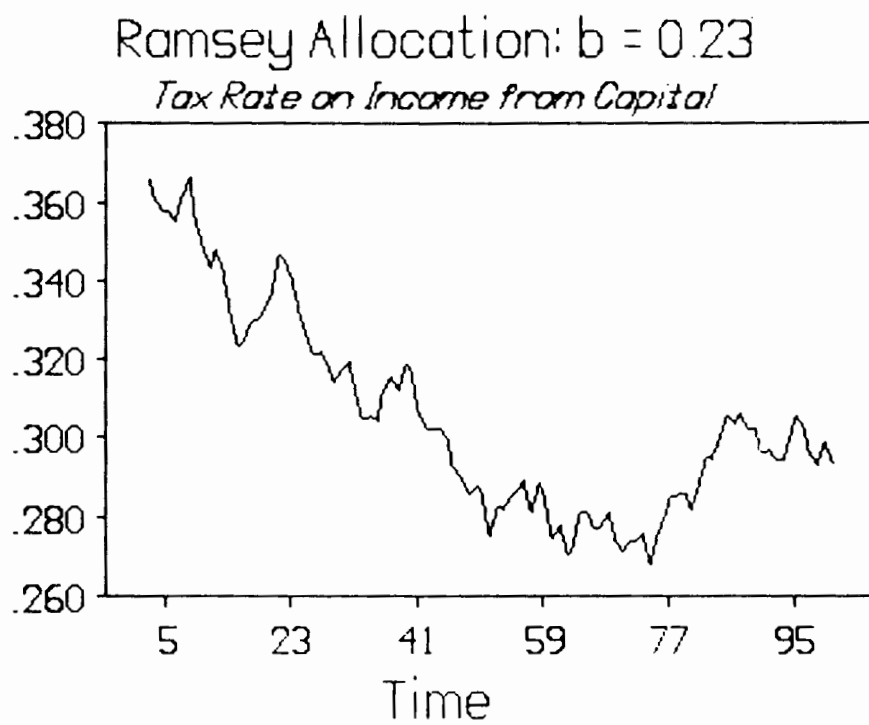


FIGURE 10



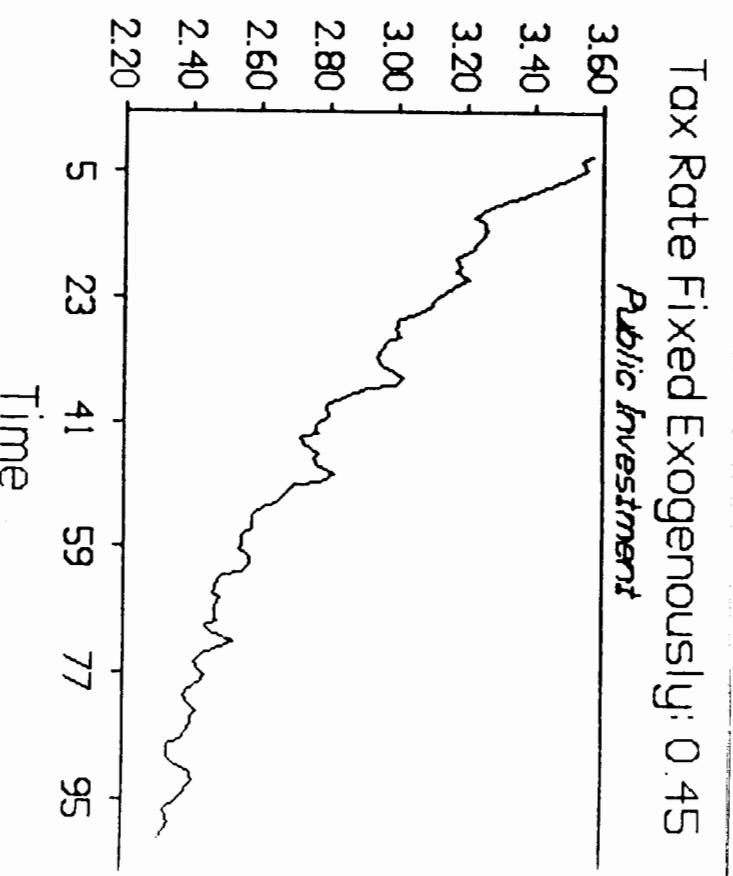
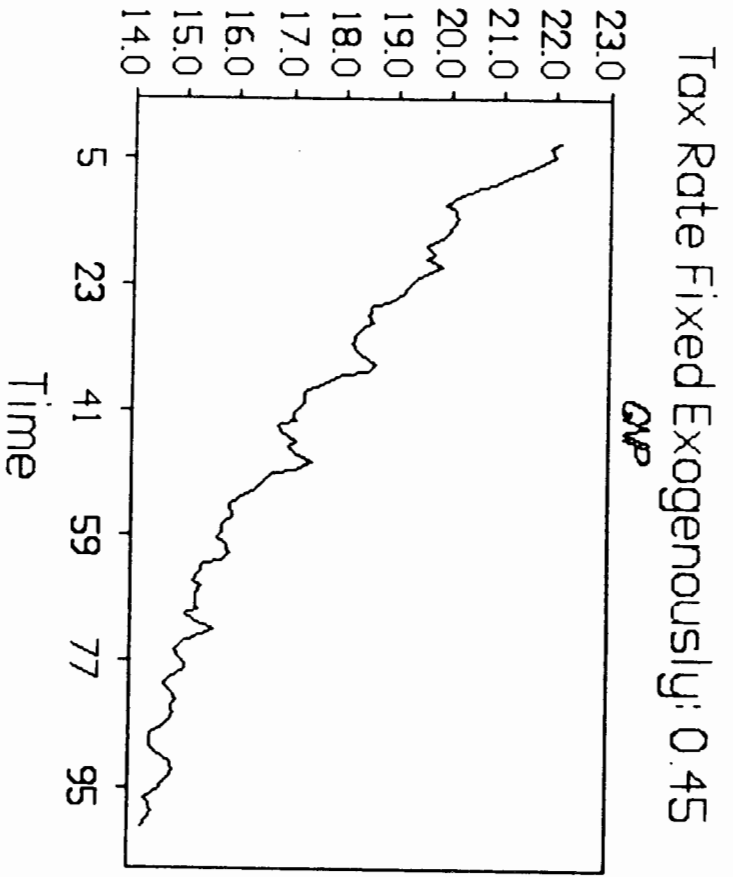


FIGURE 11

Tax Rate Fixed Exogenously: 0.45

Tax Rate Fixed Exogenously: 0.45

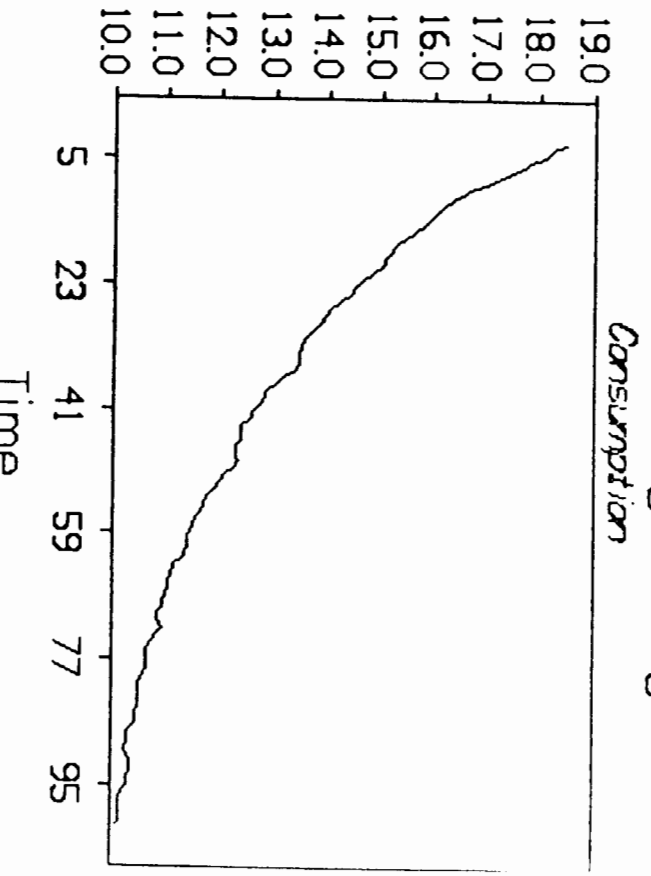
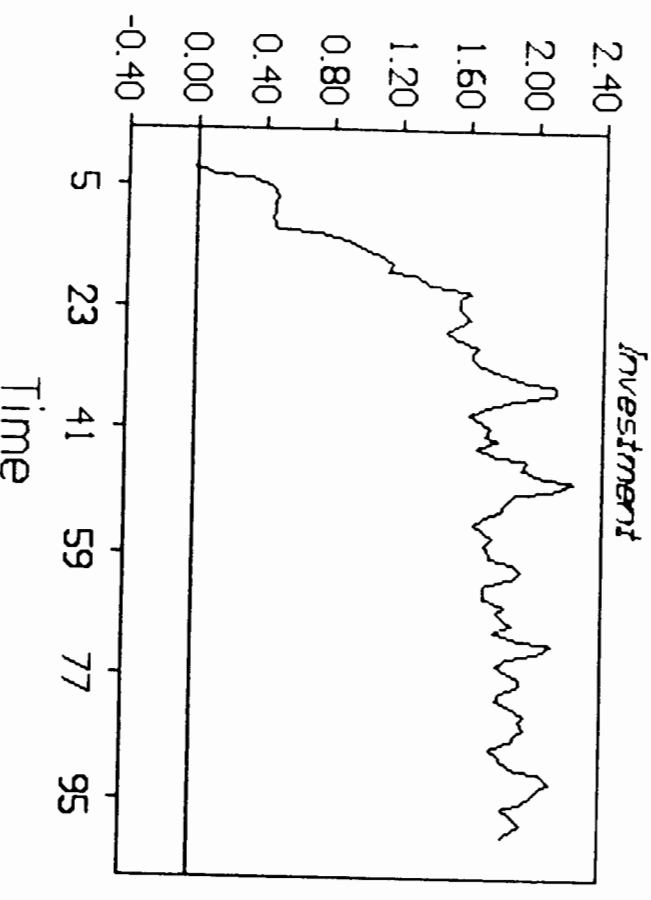
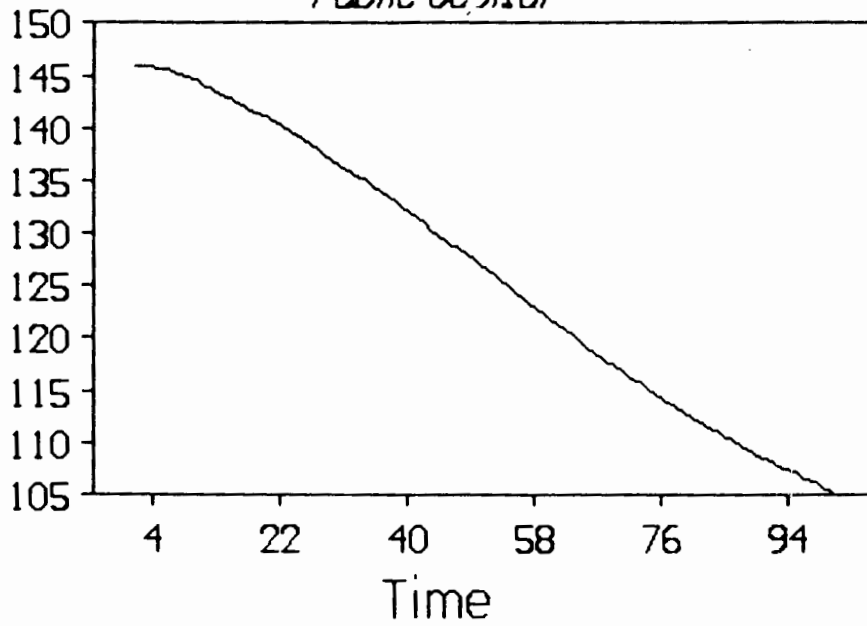


FIGURE 12

Tax Rate Fixed Exogenously: 0.45

Public Capital



Tax Rate Fixed Exogenously: 0.45

Private Capital

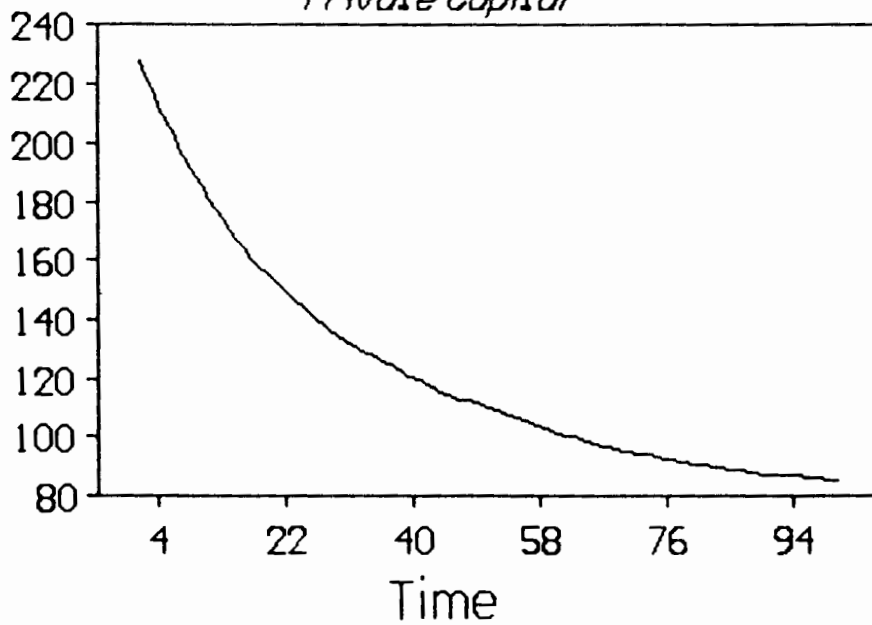
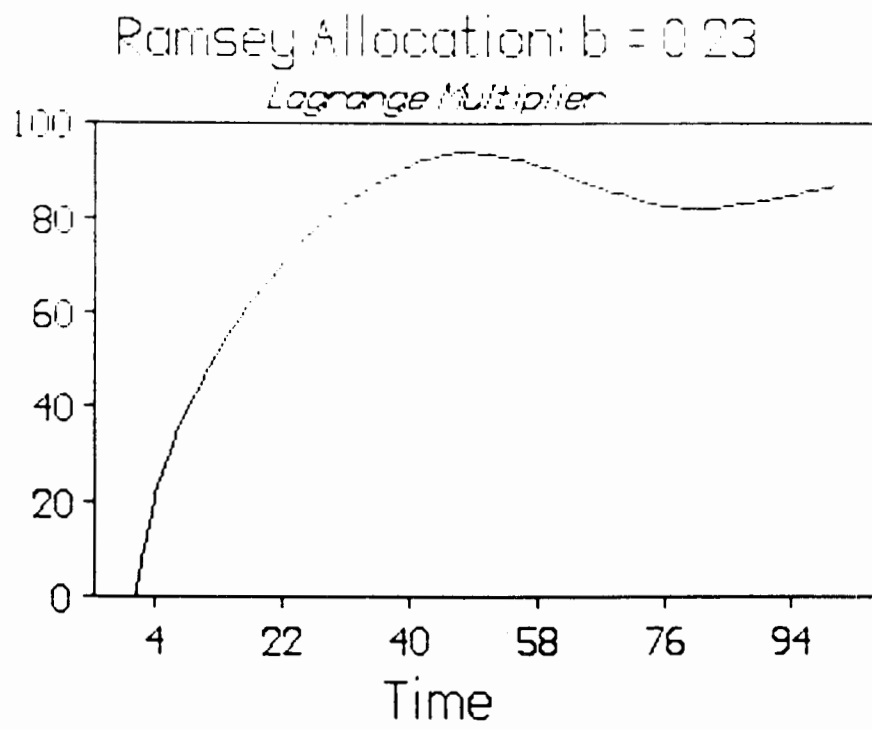


FIGURE 13:
Short-run Simulation



APPENDIX 2: SOME EMPIRICAL FACTS

There is an empirical literature supporting the idea that public capital has a strong positive impact on the GNP. For example, Aschauer (1989a, 1989b) found a statistically significant relation between productivity and public capital in the USA for post-war data; in the case of Garcia-Mila (1987), she estimated a 9 dimensional VAR of real the GNP components using quarterly post-war USA data, (3 of these components were government purchases). Her result is that the response of the GNP to an innovation in state and local purchases is positive and persistent; Kocherlakota and Yi (1992) also found that there is evidence that changes in the share of the GNP allocated to government non military structural investment lead to a permanent increase in the growth rate of the GNP; Morrison and Schwartz (1991) using annual data from 48 states in the USA over 1970-1987 report that the stock of public infrastructure has a positive impact on productivity. Finally, Easterly and Rebelo (1992), using data since 1960 on public investment for 28 individual countries, find that the consolidated general government investment is consistently positively correlated with growth and private investment (they report a contemporaneous correlation of 0.3 and near 1, respectively).

Contrary to this evidence, Kydland and Prescott (1990) report that government purchases are not correlated with the GNP for the USA in the period 1954-1989. In the case of the output components, their finding is that consumption in nondurables and investment is highly procyclical. However, government purchases do not present a systematic evidence of cyclicity. According to Kydland and Prescott "Some of the interesting features of the other components are that government purchases has no consistent pro or counter cyclical pattern..." (page 14). The result comes from the fact that the current cross correlation of real GNP with government purchases is 0.05 (if we divide this purchases in federal and state & local we have correlations of -0.02 and .25, respectively).

This result suggest that is not very intuitive to consider models where public goods are consumption goods (if all goods are normal) or models where government spending enters as an input in the production function, because in these cases government spending would be pro cyclical and would not match the observed data¹. Moreover, one of the most important is-

¹See Blanchard and Fisher (1989), page 591.

sues in macroeconomic policy is how government spending could influence the economic activity and some recent papers assume that there is a clear relationship between these variables.

An example of these kind of models are the papers by Barro (1990) and Jones et al. (1991). In the case of Barro, the economy is on the optimal rate of growth when government spending grows at the same rate as the GNP and the optimal government spending path is, obviously, pro cyclical (because he considers that government purchases are inputs to private production). Jones et al. study an optimal taxation model where government spending increases the productivity of private investment. In their model high investment implies, as an optimal decision by the government, more government spending. In these two models, the government spending has a positive relationship with private investment and the GNP, leading a contradiction with the real data.

In this note we want to show that the result of Kydland and Prescott depends on the sample periods chosen and the kind of government purchases considered. Specially, we think we have to consider separately some components of government spending that could have a pro-cyclical pattern, like state and local purchases.

Table 2.1 present the cross correlations between the components of government purchases and the GNP when we consider four different samples. In the case of the federal government expenditure (and the defense spending component) there is a very different pattern in the four samples considered. Note that only after 1960 the federal government expenditure does not present a well defined cyclical behavior. Nondefense federal expenditure only present some evidence of pro cyclically in the sixties, but in general there is no a clear cyclical pattern.

However, state and local purchases present a very different pattern. Cross correlation in Table 2.1 show that this type of expenditure exhibits counter cyclical behavior between 1947-1954 and pro cyclical behavior after that period.² Therefore, the conclusion is that not all the components of government expenditure exhibit the same pattern and that it depends on the sample under consideration.

²If we adopt the standard division into cycles determined by the NBER chronology (see Blanchard and Watson (1986)), the cross correlation for the GNP and state and local expenditures are positive after the sixties (with the exception for of the cycle 5, where the current cross correlation is close to zero:

These findings show that the result of Kydland and Prescott depend on the sample and the type of government considered. The presence of significant correlation between state and local expenditures support the idea that government expenditure in the form of inputs in private production could be pro cyclical, at least in the last 30 years, and supporting the idea that some type of government spending (in particular productive public investment) could have a strong influence in the economic activity.

Cross Correlation of the GNP with GGSE						
Cycle	Period	(t-2)	(t-1)	(t)	(t+1)	(t+2)
1	1949:4 to 1954:2	0.8	-0.7	-0.4	0.04	0.3
2	1954:4 to 1958:2	-0.4	-0.4	-0.3	0	0.3
3	1958:2 to 1961:1	-0.7	-0.7	-0.4	0	0.3
4	1961:1 to 1970:4	0.3	0.5	0.5	0.6	0.6
5	1970:4 to 1975:1	0.4	0.2	-0.1	-0.2	-0.5
6	1975:1 to 1980:2	0.3	0.3	0.3	0.2	0.1
7	1980:2 to 1982:4	0	0.3	0.6	0.8	0.8
*	1982:4 to 1986:1	0.6	0.7	0.6	0.4	0.1
* For this period we don't have information about the cycle						

TABLE 2.1:CYCLICAL BEHAVIOR OF U.S.A GOVERNMENT PURCHASES					
Deviations from trend*. Quarterly:1947-1986					
CORRELATION OF THE GNP WITH X					
<i>var(x)</i>	(t-2)	(t-1)	(t)	(t+1)	(t+2)
GGE (Government Purchases)					
47:1-54:4	0.88	0.79	0.61	0.35	0.1
55:1-61:4	-0.59	-0.58	-0.47	-0.38	-0.29
62:1-70:4	0.45	0.44	0.41	0.25	0.01
71:1-86:3	0.15	0.16	0.12	0.03	0.0
GGFE (Federal Government Purchases)					
47:1-54:4	0.91	0.82	0.64	0.37	0.1
55:1-61:4	-0.5	-0.5	-0.41	-0.37	-0.34
62:1-70:4	0.41	0.35	0.29	0.1	-0.1
71:1-86:3	-0.06	-0.04	-0.09	-0.15	- 0.11
GGFEN (National Defense Government Purchases)					
47:1-54:4	0.89	0.86	0.72	0.49	0.24
55:1-61:4	-0.4	-0.42	-0.35	-0.32	-0.35
62:1-70:4	0.37	0.29	0.18	0.01	-0.16
71:1-86:3	-0.32	-0.23	-0.1	-0.1	0.0
GGFEO (Nondefense Government Purchases)					
47:1-54:4	0.18	-0.06	-0.27	-0.45	-0.58
55:1-61:4	-0.3	-0.3	-0.22	-0.13	-0.01
62:1-70:4	0.12	0.26	0.45	0.4	0.41
71:1-86:3	0.12	0.08	-0.06	-0.15	-0.15
GGSE (State and Local Government Purchases)					
47:1-54:4	-0.78	-0.79	-0.7	-0.51	-0.22
55:1-61:4	-0.65	-0.57	-0.35	-0.09	0.25
62:1-70:4	0.34	0.53	0.66	0.7	0.7
71:1-86:3	0.41	0.4	0.4	0.3	0.19
*All series were detrended with the Hodrick-Prescott filter					
Source of data: Citicorp's database					

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