On p-Beauty Contest Integer Games[*]

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Abstract

In this paper we provide a full characterization of the pure-strategy Nash Equilibria for the p-Beauty Contest Game when we restrict player's choices to integer numbers. Opposed to the case of real number choices, equilibrium uniqueness may be lost depending on the value of p and the number of players: in particular, as p approaches 1 any symmetric profile constitutes a Nash Equilibrium. We also show that any experimental p-Beauty Contest Game can be associated to a game with the integer restriction and thus multiplicity of equilibria becomes an issue. Finally, we show that in these games the iterated deletion of weakly dominated strategies may not lead to a single outcome while the iterated best-reply process always does (though the outcome obtained depends on the initial conditions).

Keywords: Beuaty Contest, Multiple Equilibria, Iterated Dom inance, Iterated Best-reply.

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1. Introduction

The basic p-Beauty Contest Game[1] (p-BCG), consists of a number $N>1$ of players, a real number $0<p<1$, and a closed interval $[l,h]$ with l and h integers. In such a game, N players have to choose simultaneously real numbers from the given interval. The mean of all chosen numbers is calculated and the winner is the person who chose the closest number to p times the mean and receives a fixed prize (in the case of many winners the prize is equally divided among them), the other players receive nothing.

The game was first introduced by Moulin (1986) as a means to illustrate an equilibrium obtained by iterated deletion of (weakly) dominated strategies. The equilibrium thus obtained was all players playing the lower boundary of the interval, l. Predicting such an equilibrium as an outcome of the game relies in the assumption of rationality of all the players (in the sense that no player is playing a weakly dominated strategy) and that all the players know that the other players are rational and so on ad infinitum.

Starting with the work of Nagel (1994) a variety of experiments on the p-BCG have been conducted to study iterated dominance and learning (for a survey see Nagel(1998)). If we apply the process of iterated best-reply to this game (rather than the iterated deletion of weakly dominated strategies), it turns out that all players playing l is also the unique Nash equilibrium. However, if we allow the players to choose only among integer numbers[2] in the given interval, this is no longer true: although every player playing l continues to be a Nash equilibrium, there could be more. The multiplicity of equilibria makes this game appealing to the empirical issue of equilibrium selection.

The purpose of this paper is first, to characterize the Nash Equilibria of a p-Beauty Contest Integer Game and second to give some insights for further experiments.

The paper is organized as follows: In Section 2 we define a p-Beauty Contest Integer Game and completely characterize its Nash Equilibria in pure strategies for two cases: when the prize a winner earns is fixed and equally divided among the winners and when the prize is

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[1] This name was introduced by Duffy and Nagel (1997) and Ho, Camerer, and Weighel (1998) and is due to a famous analogy by Keynes (1936) between stock market investment and „those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize beeing awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole". Other authors as Nagel (1995) have used the name „guessing game."

[2] Osborne and Rubinstein (1994) pose an exercise with this restriction and p=2/3. Some experiments have made the integer restriction explicit (Thaler(1997)) while in other experiments players actually chose only integer numbers though it was not a restriction in the instructions (e.g., Ho et al (1998)). Nagel (1995) and Nagel (1998), mentions the p-BCIG, however the characterization of equilibria is incomplete.

increasing in the winning number and equally divided among the winners; the results are summarized in Theorem Result 1. The reason for concentrating in these two cases is simply that those are the cases that have been treated in the experiments on p-BCG conducted so far.

In Section 3 we point out a striking fact: every experimental p-BCG can be thought of as a P-Beauty contest integer game (p-BCIG). In Section 4 we show that the equivalence between the iterated dominance and iterated best-reply holding for the p-BCG may fail for the p-BCIG. The results are summarized in Result 2. Finally, Section 5 states the conclusions of our findings.

2. Nash Equilibria of a p-Beauty Contest Integer Game

Definition 1: A p-Beauty Contest Integer Game (p-BCIG) is a p-BCG where players are allowed to choose only among integer numbers from the given interval [l,h].

Let $S = (x_1, x_2,..., x_N)$ be a strategy profile where player i has chosen number x_i

Definition 2: given a strategy profile S, let $pmean$ be the mean of S times p.

Definition 3: we say that x_i is a winning number if $|x_i - pmean| \leq |x_j - pmean| \quad \forall j = 1,..., N$

Definition 4: we say that player i is a winner if x_i is a winning number.

Proposition 1: If for strategy profile S, player i is not a winner then by unilaterally deviating to some strategy profile S' he can become one.

Proof: Define $m = p\left(\frac{1}{N}\sum_{J \neq i} x_j\right)$ and let player i deviate from S to S' by choosing an integer number $x'_i = x$ such that

$$[1] \quad \frac{m - \dfrac{1}{2}}{1 - \dfrac{p}{N}} < x < \frac{m + \dfrac{1}{2}}{1 - \dfrac{p}{N}}$$

Note that such an integer number exists since $\quad \dfrac{m + \dfrac{1}{2}}{1 - \dfrac{p}{N}} - \dfrac{m - \dfrac{1}{2}}{1 - \dfrac{p}{N}} = \dfrac{1}{1 - \dfrac{p}{N}} > 1$

But if [1] holds then we must have

$$[2] \quad \frac{-1}{2} < m + \frac{px}{N} - x < \frac{1}{2}$$

which means that x is the closest integer to the *pmean* of S', therefore player i is a winner.

Corollary 1: In a Nash equilibrium of a p-BCIG, every player must be a winner (thus there can be at most two winning numbers).

Proposition 2: In a p-BCIG, if the prize of the game is fixed and equally divided among the winners (FEDAW) then in a Nash Equilibrium there is only one winning number.

Proof:

Case $N=2$: Trivial since the lowest choice is the winning number.

Case $N>2$: Suppose there is a Nash Equilibrium with two winning numbers x and y. Without loss of generality assume $x<y$. Let m be the number of players choosing x (notice that then, by Corollary 1, the number of players choosing y must be $N-m$).

Suppose $m>1$ and let Z denote the fixed prize so that every winner is receiving Z/N. Consider one of the players choosing x deviating to y, then the *pmean* would be closer to y and therefore only the players choosing y will win receiving $Z/(N-m+1)$. But $N - m + 1 < N$ so that the player deviating has incentives to do so.

Now suppose $m=1$, then one of the players choosing y can improve by deviating to x. Therefore there cannot be a Nash Equilibrium with two winning numbers.

Proposition 3: In a p-BCIG, if the prize of the game is strictly positive, increasing in the winning number and divided by the number of winners (IWND) then in a Nash Equilibria there is only one winning number.

Proof: Case $N=2$: same as Proposition 2.

Case $N>2$: Suppose there is a Nash Equilibrium with two winning numbers x and y. Assume without loss of generality that $x<y$. Let $Z(x)$ and $Z(y)$ be the prize for players choosing x and players choosing y respectively. By assumption of the theorem we must have $Z(x) < Z(y)$.

Also, by Corollary 1, the number of winners is N, thus, a player choosing x is receiving $Z(x)/N$ and a player choosing y is receiving $Z(y)/N$. Consider a player choosing x: if he deviates to y he will drive the *pmean* closer to y and further from x. Therefore the winning number would be y and the player deviating would get no less than $Z(y)/N$ which is greater than $Z(x)/N$. This means that a player choosing x has incentives to deviate. Therefore there cannot be a Nash Equilibrium with two winning numbers.

Now consider a strategy profile S where each player plays the same integer number x. We have the following results:

Proposition 4: $\forall\, 0 < p < 1$, no player has incentives to deviate from S by playing an integer number $y>x$.

Proof: Trivial.

Proposition 5: $\forall\, 0 < p < 1$, if a player has incentives to deviate from S by playing an integer number $y<x$ then he has also incentives to deviate from S by playing x-1.

Proof: Let $y = x - k$ for some integer $k \geq 1$. Since a player has incentives to deviate by playing y it must be true that

$$[3] \quad p\frac{x(N-1)+x-k}{N} - (x-k) < x - \frac{px(N-1)+x-k}{N}$$

where the LHS of [3] is the distance to the *pmean* for the player deviating and the RHS of [3] is the distance to the *pmean* for the players not deviating. Rearranging [3] yields

$$[4] \quad 2x(p-1) + k\left(1 - \frac{2p}{N}\right) < 0$$

Since $1 > \frac{2p}{N}$ (because $p<1$ and $N>1$), then if [4] holds for some $k \geq 1$ it also holds for $k = 1$. Therefore a player has incentives to deviate by playing x-1.

Let $F(p,x) = 2x(p-1) + \left(1 - \frac{2p}{N}\right)$

Proposition 6: In a p-BCIG, a strategy profile S where every player plays the same integer x is a Nash Equilibrium if and only if $F(p,x) \geq 0$ or $x = l$.

Proof: Suppose a strategy profile S is a Nash Equilibrium, then it must be true that a player has no incentives to deviate by playing x-1, or, if he has them, he cannot do so (because x-1<l), therefore, either $F(p,x) \geq 0$ or $x = l$.

Now suppose $F(p,x) \geq 0$ or $x = l$. If $x = l$ then, by proposition 4, S is a Nash Equilibrium. If $F(p,x) \geq 0$ then a player has no incentives to deviate from S by playing x-1 (since $F(p,x)$ is just the LHS of [4] with $k = 1$). Therefore , by proposition 5, a player has no incentives to deviate from S by playing a lower integer than x, but by proposition 4 a player has no incentives to deviate by playing a higher number than x, therefore S is a Nash Equilibrium.

Remark 1: Notice that when $F(p,x) > 0$ a player is no longer a winner when unilaterally deviating to x-1 while when $F(p,x) = 0$ he remains a winner with the same deviation.

Proposition 7: For $N = 2$ all players playing l is the unique Nash Equilibrium.

Proof: When $N = 2$, $F(p,x) = (1-p)(1-2x) > 0$ only for the integer $x = 0$. Thus both players playing l is the unique Nash Equilibrium.

Proposition 7 completely characterizes the Nash Equilibria for the case $N = 2$ and Proposition 4 implies that every player playing l is always a Nash Equilibrium. We will now focus on the case where $N > 2$ and a strategy S where every player is playing the same integer number $x > l$.

Let $P(x)$ be such that $F(P(x), x) = 0$. Solving for $P(x)$ gives

$$[5] \quad P(x) = \frac{2x - 1}{2x - \dfrac{2}{N}}$$

where clearly $0 < P(x) < 1$ for $N > 2$ and $x > l \geq 0$.

Now notice that $\dfrac{\partial F(p, x)}{\partial p} = 2x - \dfrac{2}{N} > 0$ for $N > 2$ and $x > l \geq 0$. Since F is strictly increasing in p we have that

$$F(p, x) \geq 0 \quad \forall 1 > p \geq P(x)$$

$$F(p, x) < 0 \quad \forall 0 < p < P(x)$$

Corollary 2: For every x integer belonging to $(l, h]$, if $p \geq P(x)$ then S is a Nash Equilibrium.

Proof: If $p \geq P(x)$, then $F(p, x) \geq 0$ and by Proposition 6 S is a Nash Equilibrium.

Now notice that $P(x)$ is strictly increasing in x since $\dfrac{\partial P(x)}{\partial x} = \dfrac{2 - \dfrac{4}{N}}{\left(2x - \dfrac{2}{N}\right)^2} > 0$ for $N > 2$.

Corollary 3: $\forall 0 < p < 1$, if S is a Nash Equilibrium, then the strategy profile S' where each player plays the same integer y satisfying $l \leq y < x$ is also a Nash Equilibrium.

Proof: Take any y satisfying $l \leq y < x$. If S is a Nash Equilibrium we must have $p \geq P(x)$. But $P(x)$ is increasing in x, therefore it is also true that $p \geq P(y)$, but then S' where each player plays y is also a Nash Equilibrium by Corollary 2.

Notice that for a particular p-BCIG, Corollaries 2 and 3 completely characterize those Nash Equilibria where there is a unique winning number: to see this, we just need to solve for x the inequalities $p \geq \dfrac{2x-1}{2x-\dfrac{2}{N}}$ and $x > l$ which yield

$$[6] \quad l < x \leq \frac{1 - \dfrac{2p}{N}}{2(1-p)}$$

Let $B(p,N) = \dfrac{1 - \dfrac{2p}{N}}{2(1-p)}$. Since by Proposition 4 we know that every player playing l is a Nash Equilibrium, we have obtained the following:

Proposition 8: In a p-BCIG, the strategy profile S is a Nash Equilibria with only one winning number if and only if in S every player plays a same integer x in the interval $[l,h]$ satisfying $x \leq B(p,N)$ or $x = l$.

Propositions 2, 3 and 8 yield our first important result:

RESULT 1: In a p-BCIG, if the prize is either FEDAW or IWND then a strategy profile S is a Nash Equilibrium if and only if in S every player plays a same integer x in the interval $[l,h]$ satisfying $x \leq B(p,N)$ or $x = l$.

Notice from $B(p,N)$ that as p goes to 1 then any integer in the interval is a Nash Equilibrium[3]. Notice also that in the case of multiple equilibria, if the prize is IWND, the higher integer x in the interval satisfying $x \leq B(p,N)$ is Pareto dominant.

NOTE 1: MIXED EQUILIBRIA

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[3] If the interval is [0,100], as it has been for most of the experiments on p-BCG, the following are very easy to prove:
i) For $p \geq 3/4$ the p-BCIG has multiple equilibria.
ii) For $p \leq 1/2$ the p-BCIG has a unique equilibrium.
iii) For $1/2 < p < 3/4$ the p-BCIG has multiple equilibria provided the number of players is sufficiently large.

In the p-BCG the unique Nash Equilibria in pure strategies is all players playing the lower bound l and it turns out that this is also the unique mixed equilibrium of the game. Is the set of mixed equilibria equal to the set of pure strategies equilibria in any p-BCIG? The following example shows that the answer is no: consider the FEDAW p-BCIG $(N, p, l, h) = (3, 5/6, 0, 100)$. Since $B(5/6, 3) = 4/3$ we know by Theorem 2 that the pure strategy Nash Equilibria are 0 and 1. It is easy to check that every player playing 0 and 1 each with probability 0.5 is a mixed Nash Equilibrium.

NOTE 2: THE p-BEAUTY CONTEST DECIMAL GAME

Consider now a p-BCG in the interval $[l, h]$ where players are allowed to choose among decimal numbers up to D decimal positions. Let's call this game a p-BCDG. It is easy to see that this game is equivalent to the p-BCIG in the interval $[l^D, h^D]$ where $l^D = l * 10^D$ and $h^D = h * 10^D$ and the equivalence relation is given by $S \Leftrightarrow S^D = S * 10^D$ where S is a strategy in the p-BCDG and S^D is a strategy in p-BCIG in $[l^D, h^D]$.

3. Experimental Implications

The p-BCG has been widely used to test iterated dominance and learning. In most of the experiments it has been assumed that the game has a unique Nash Equilibrium but, in fact, any experimental p-BCG can be thought of as a p-BCIG: the reason for this is that in calculating the *pmean* one must use a decimal approximation which implies that we are facing a p-BCDG which in turn (see Note 2) is equivalent to a p-BCIG. Therefore, all of the results obtained in part I also apply to an experimental p-BCG (E-p-BCG) through its equivalent p-BCIG.

It is easy to see that the exact number of equilibria of a E-p-BCG defined in the interval $[l, h]$ is given by $E(p, N, l, h) = Max(Min(h, \lfloor B(p, N) \rfloor) - l, 0) + 1$.

In Table 1 we find some E-p-BCG for which we have calculated $E(p, N, l, h)$.

Table 1.

Authors	Range	[*l,h*]	Prize	**N**	*p*	*B(p,N)*	*E(p,N,l,h)*
Ho et al. (1998)	[0,100]	Reals	$3.50	7	0.7	1.33	2
				7	0.9	3.71	4
			$1.50	3	0.7	0.89	1
				3	0.9	2.00	3
Nagel(1995)	[0,100]	Reals	$x if x is	12	2/3	1.33	2
			winning number	17	2/3	1.38	2
Bosch & Nagel (1997)	[1,100]	Decimals	$100.000	3696	2/3	1.5	1
Thaler (1997)	[0,100]	Integers	2 NY tickets	1460	2/3	1.5	2
Selten & Nagel (1998)	[0,100]	Decimals	1000 DM	2728	2/3	1.5	2

In order to know the Nash equilibria of a particular E-p-BCG we just need to know the decimal approximation used in the calculations. As an example, for Nagel(1995) the approximation in the calculations used was of one decimal, this means that the Nash equilibria for that game were 0 and 0.1[4]

Now, the aim of all these experiments was to find out whether the players tend to equilibrium or not and, if doing so, establishing the way they did. The last three studies were one-shot games. However, in the first three the game was repeated a number of times to study whether the players „learned" to play the equilibrium or not. Since in all of them a decimal approximation was made the natural question is *which are the equilibria*?

4. Theoretical Predictions for the p-BCIG.

Predicting the outcome of a game constitutes one of the main issues of game theory. On simultaneous-move games the concepts of strict dominance and rationalizable strategies (see Fudenberg and Tirole (1991) or Mas-Colell, Whinston and Green (1995)) are useful to restrict the set of possible outcomes relying solely in the assumption of rationality: a rational player should never play a strictly dominated strategy nor a strategy that is never a best-response; therefore, the iterated deletion of these strategies is justified. It is easy to see that in

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[4] For this same example if the aproximation were of D decimal positions the equilibria would be 0 and $1*10^{-D}$.

the p-BCG there are no strictly dominated strategies which implies that every strategy might be a best-response. Therefore, the concepts of strict dominance or rationalizable strategies are of no use for narrowing down the set of possible outcomes of this game. The most used reasoning processes to refine the theoretical predictions of this game have been the iterated deletion of weakly dominated strategies (IDWDS), and the iterated best-reply[5] (IBR). In the first one, it is assumed that players iteratively eliminate weakly dominated strategies, the process ending when no player has a weakly dominated strategy left. In the second one, players act *à la Cournot*: starting from a hypothetical strategy profile they iteratively best-reply to the previous profile, the process ending when a fixed-point is reached.

It is easy to see that for the p-BCG, both processes lead to the unique prediction of all players playing *l* (which is actually the unique Nash Equilibrium), independently of the order of deletion and the initial strategy profile for the IDWDS and the IBR respectively. However, the situation changes dramatically in the p-BCIG: we will show that under very mild conditions the IDWDS will not lead to a single prediction, while depending on the initial strategy profile, the IBR process will lead to one. Therefore, the equivalence between the IDWDS and the IBR processes that we had in the case of a unique Nash Equilibrium fails under multiple equilibria. This is worth noticing since the experimental results show that individuals use rather IBR than IDWDS (see e.g., Nagel (1995), Stahl (1996), Ho et al. (1998)).

Proposition 9: Let $l < k \leq h$ be the highest Nash Equilibria of a p-BCIG. Let $S(t) = (s_1(t), s_2(t), \cdots, s_N(t))$ be the strategy profile at iteration t in a IBR process, with $S(0)$ the initial strategy profile. If $\forall i \quad s_i(0) \geq k$ then the IBR process leads to the unique prediction of all players playing k.

Proof: Since $s_i(0) \geq k$ for all i then the *pmean* of $S(0)$ is closer to k than to k-1. Therefore, in the next iteration we have that $s_i(1) \geq k$ for all i, and so on: $s_i(t) \geq k$ $\forall i \quad \forall t \geq 1$. Suppose that at some iteration t there are some players choosing higher integers than k. Let $h(t)$ be the highest of those integers. We claim that $s_i(t+1) < h(t)$ for all i: the *pmean* of $S(t)$ is lower (or equal) than $p^* h(t)$. But $p^* h(t)$ is closer to $h(t)$ -1 than to $h(t)$ so a

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best reply at time $t+1$ must be a lower number than $h(t)$. Therefore the IBR process leads to every player choosing k.

Proposition 10: Let k be an integer such that in the p-BCIG all players playing k constitutes a strict[6] Nash Equilibrium then no player can eliminate k by IDWDS.

Proof: Suppose that at some iteration t player i is the first to eliminate k. Let $S_i(t)$ denote the set of possible strategies for player i at iteration t. Then it must be true that some strategy $s' \neq k$ and $s' \in S_i(t)$ weakly dominates k for player i at iteration t. But then, by definition of weak domination we must have: $U_i(s', s_{-i}) \geq U_i(k, s_{-i}) \quad \forall s_{-i} \in S_{-i}(t)$ and with strict inequality for at least one s_{-i}, where s_{-i} denotes a strategy profile for all players except i and $S_{-i}(t)$ is the set of possible strategy profiles, at iteration t, for all players except i. Let k_{-i} be the strategy profile where all players but i play k. Since by assumption at iteration t no player except i has eliminated k, we have $k_{-i} \in S_{-i}(t)$. But by strictness of the Nash Equilibrium $U_i(s', k_{-i}) < U_i(k, k_{-i})$ so that s' does not weakly dominate k for player i at iteration t. Therefore no player can eliminate k by IDWDS.

Proposition 11: In a IWND p-BCIG every Nash Equilibrium is strict.

Proof: Consider any strategy profile S constituting a Nash Equilibrium. By Result 1 we know that S must be of the form where all players play the same integer k. Since the game is IWND we know that the payoff for player i if not deviating must be $U_i(k, k_{-i}) = Z(k)/N > 0$. Clearly, if deviating to $k' > k$ player i obtains 0. If deviating to $k' < k$ player i obtains at most $Z(k')/N < Z(k)/N$. Therefore the unique best reply to k_{-i} is k.

Proposition 12: In a FEDAW p-BCIG with $M \geq 2$ Nash Equilibria, a Nash Equilibrium k is strict if and only if $k < B(p, N)$.

Proof: Immediate by Remark 1 and the fact that $F(p, k) > 0$.

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[6] Following Harsanyi (1973), we say that a Nash Equilibrium is *strict* if each player has a unique best reply to his rival's strategies.

Notice that by Proposition 12 and Result 1 the only way to have a not strict Nash Equilibrium k in a multiple equilibria FEDAW p-BCIG is if $k = B(p,N)$. Therefore an immediate implication of Proposition 12 is that if there are $M \geq 2$ Nash Equilibria at least M-1 are strict. Therefore, the propositions of this section imply our second important result:

RESULT 2: In a multiple Nash-Equilibria p-BCIG

I) The IBR process will lead to the highest Nash Equilibrium when starting with a high initial strategy profile

II) The IDWDS process does not lead to a unique prediction if any of the following
i) The game is IWND.
ii) The game is FEDAW and the highest Nash Equilibrium k satisfies $k < B(p,N)$.
iii)The game is FEDAW and there are at least 3 Nash Equilibria.

5. Conclusions.

We completely characterized the Nash Equilibria of a p-BCIG. We have found three new results: 1. We showed that in the p-BCIG the number of equilibria depends on all the parameters of the game ($p,N,[l,h]$) while for the p-BCG the unique Nash equilibrium is l. 2. We also showed that any experimental p-BCG is in fact a p-BCIG because of the approximation needed to do the calculations. 3. We proved that under very soft conditions the iterated deletion of weakly dominated strategies (IDWDS) does not lead to a unique prediction of the game while the iterated best reply (IBR) might do. This is worth noticing since experimental results show that subjects use IBR rather than IDWDS.

Because of the multiplicity of equilibria in the p-BCIG it might be interesting to do further experiments with explicit integer restrictions in order to get more insight in the long-standing problem of equilibrium selection.

Finally, with the explicit introduction of the p-BCIG and the characterization of the equilibria we have closed the gap between the coordination game with $p = 1$ where any number can be an equilibrium and the p-BCG where only one equilibrium exists.

6. References.

Bosch, A., Nagel, R. (1997, June 16). El juego de adivinar el número X: una explicación y la proclamación del vencedor [The game of guessing a number: An explanation and the announcement of the winner]. *Expansion*, 42-43

Cournot, A. (1838). *Recherches sur les Principes Mathématiques de la Théorie des Richesses.* [English edition: *Researches into the Mathematical Principles of the Theory of Wealth.* New York: Macmillan, 1897.]

Fudenberg, D., Tirole, J. (1991). *Game Theory.* Cambridge, Mass.: MIT Press.

Harsanyi, J. (1973). Oddness of the number of equilibrium points: A new proof. *International Journal of Game Theory* 2: 235-250.

Ho, T., Camerer, C., Weigtel, K. (1998). Iterated dominance and iterated best-response in experimental "p-beauty contests". *American Economic Review*, 88, 947-969.

Mas-Colell, A., Whinston, M., Green, J. (1995). *Microeconomic Theory.* New York, NY: Oxford University Press.

Moulin, H.(1986). *Game theory for Social Sciences.* New York, NY: New York Press.

Nagel, R. (1995). Unraveling in guessing games: An experimental study. *American Economic Review,* 85, 1313-1326.

Nagel, R. (1998) A Survey on Experimental Beauty Contest Games: Bounded Rationality and Learning. *Games and Human Behavior, Essays in Honor of Amnon Rapoport.* Eds. D. Budescum, I. Erev and R. Zwick. Lawrence Erlbaum Associates Inc. New Yersey. p.105-142

Stahl, D. O. (1996) "Rule Learning in a Guessing Game", *Games and Economic Behavior*, 1996, 16(2), 303-330.

Osborne, M., Rubinstein, A. (1994). *A Course in Game Theory.* Cambridge, Mass.: MIT Press

Thaler, R. (1997, June 16). Giving markets a human dimension. *Financial Times*, Mastering Finance section 6, 2-5.