The Survival of the Welfare State*

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Abstract

This paper provides an analytical characterization of Markov perfect equilibria in a politico-economic model with repeated voting, where agents vote over distortionary income redistribution. The key feature of the theory is that the future constituency of redistributive policies depends positively on the current level of redistribution, since this affects both private investments and the future distribution of voters. Agents vote rationally and fully anticipate the effects of their political choice on both private incentives and future voting outcomes. The model features multiple equilibria. In "pro-welfare" equilibria, both welfare state policies and their effects on distribution persist forever. In "anti-welfare equilibria", even a majority of beneficiaries of redistributive policies vote strategically so as to induce the formation of a future majority that will vote for zero redistribution.

JEL codes: D72, E62, H11, H31, P16

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1 Introduction

There is now a growing literature bringing politico-economic aspects into macroeconomics. Rather than treating policies as exogenous instruments in the hands of benevolent policy makers, this recent literature describes government policies as endogenous outcomes collectively determined by rational self-interested individuals. While many important issues are dynamic in nature, technical limitations have, however, so far prevented a thorough investigation of dynamic political choices in macroeconomics. This paper takes a step towards overcoming these difficulties.

In particular, we construct a tractable positive theory of the dynamics of income redistribution where policy is set through repeated voting by forward-looking rational agents, and current policy affects future political outcomes through changes in the distribution of voters. So far, this link could only be analyzed by resorting to numerical techniques and therefore, our contribution is partly methodological. Equally important, our theory provides insights on salient aspects of the debate on the determinants of redistributive policies (the "welfare state"). First, it predicts that welfare state policies and their effects on distribution are persistent: shocks to the income distribution that would have transitory effects if policies were exogenous may lead to permanent changes in the demand for redistributive policies. These, in turn, affect private investment behavior and the future dynamics of income distribution. Second, it suggests that welfare state institutions are intrinsically fragile, since even political majorities benefiting from redistribution may want to strategically vote for policies leading to the dismantlement of the welfare state. The latter prediction hinges, in our model, on rational dynamic voting and would be absent if agents voted myopically, ignoring the effect of current political decisions on future political outcomes. Thus, our theory provides an example of how forward-looking political behavior may qualitatively change the predictions of theory.

There are a number of examples, both in the economic literature and the policy debate, where dynamic links between current and future political choices and constituencies are of first-order importance. For instance, in the debate on the optimal speed of transition in post-communist countries, a number of economists have stressed that gradualism in reforms such as restructuring and reorganizing labor markets, privatizing firms and liberalizing prices, may be preferable to a "big bang" approach. The reason is that the latter may give rise to majorities of stakeholders with an interest in blocking or reverting the path of reform at some stage in the process. In contrast, gradual reforms are argued to allow "building constituencies for further reforms" by starting with easier reforms designed to increase future support for more difficult reforms (see Dewatripont and Roland, 1995).[1] Dynamic voting aspects have also been regarded as important in understanding the transition to democracy. Acemoglu and Robinson (2000 and 2001) argue that the political elites extended franchise over the XIXth Century in order to commit to sustained redistributive policies in return for less social unrest. Moreover, land reforms prior to democratization were implemented in order to reduce inequality, thereby ensuring that the poor, once in power, would limit their future demands for redistribution. Such restraints consolidated democracy by reducing the risk of the rich mounting a coup. Finally, more directly related to the analysis of our paper, Lindbeck (1995) argues that the sustained political pressure for redistributive policies and government spending in Sweden is due to that government transfers having become the main source of income for a large share of the Swedish electorate.[2]

Our model is close in spirit to the canonical politico-economic model of Meltzer and Richard (1981), where agents vote over redistribution financed by distortionary taxes in a static setting. Our economy is populated by two-period lived agents who are *ex-ante* identical, but *ex-post* heterogeneous. The overlapping generation structure is intended to capture the idea that, as life goes by, uncertainty about lifetime income is resolved. While young individuals are born identical, the old individuals have heterogeneous preferences for redistribution, since the resolution of uncertainty has turned some of them into high-income ("successful") individuals and others into low-income ("unsuccessful") individuals.

[1] Gradual reforms are also argued to allow for divide and rule-tactics (Dewatripont and Roland, 1992a, 1992b): if it is too expensive to compensate a sufficient constituency of workers today for the costs of massive lay-offs, the government could instead compensate a minority of workers today, so that these will, together with those workers who eventually are retained, secure a constituence in favor of reform in the next period. Aghion and Blanchard (1994) argue that fast reforms would raise the demand for social policies to compensate losers (e.g., unemployment benefits), and the fiscal effects of these policies will slow down the entry of new firms in the reformed sectors.

[2] Strategic voting consideration are often concealed in political debates, arguably for reasons of political correctness. In some cases, however, they have been made explicit. In the debate on the European Monetary Union, for example, Euro-sceptics have argued that a monetary union is intended as a stepping stone towards a more politically integrated federal Europe (see, for instance, Lord Skidelsky's speech in the British House of Lords, January 20 1999 or, on the opposite side, Romano Prodi's speech to the European Parliament, April 13 1999). Among the negative effects, critics see the consolidation of a European bureacraucy that will lobby for further centralization of power in the future.

Another example is the recent debate in Israel on a Knesset bill (passed in November, 2000) increasing the generosity of child allowances to families with five or more children. Some opponents have argued that such policies would destabilize the demographic equilibrium in the Israeli society, by disproportionally increasing the future political influence of groups – e.g., the ultra-orthodox or the Arab – whose fertility decisions are argued to be more responsive to such incentives.

A key assumption is that young individuals can affect their chances of becoming successful by making a private (human capital) investment when young. The optimal investment is negatively affected by the extent of current and future redistribution, which is set period-by-period in political elections. Voters are fully rational, and take into account the effects of policies on current investments and on the future distribution of voters.

The focus of the paper is whether the *ex-post* conflict over redistribution can, on its own, lead to the perpetual survival of the welfare state. To this end, we assume that individuals are risk-neutral, abstracting from a standard alternative motivation for the welfare state, i.e., that a government can deliver the insurance missing markets fail to provide. Our assumption of risk neutrality and the fact that redistribution is distortionary, imply that the welfare state would not survive if the future path of redistribution were set by a utilitarian planner attaching any arbitrary sequence of positive weights on current and future generations. In this sense, the survival of a welfare state would constitute a "political failure", as defined by Besley and Coate (1998). Similarly, there would be no welfare state if young agents could commit to vote in a particular way in the future. However, as such commitments are not feasible in democratic systems, *ex-post* conflicts influence political outcomes.

The theory has two main predictions. First, the political mechanism can sustain the welfare state. In particular, if the economy starts with a pro-redistribution majority, high levels of redistribution will be sustained over time, whereas there will not be a welfare state if the economy starts with an anti-redistribution majority. Moreover, if a one-time shock creates a temporary political majority in favor of redistribution, the model predicts that the support for the welfare state will continue and regenerate a constituency for such policies. This result is due to a self-reinforcing mechanism linking private and collective choices: high current redistribution reduces investments, implying that a larger share of future voters will benefit from redistributive policies.[3]

Second, there exist equilibria where an existing welfare state is irreversibly terminated by forward-looking voters, even when benefit recipients are initially politically decisive. In these equilibria, an initial pro-welfare state majority votes strategically for moderate redistribution so as to induce a future anti-welfare state majority. The expectation that the welfare state will vanish strengthens the incentives of the young to invest, thereby reducing

[3] In related papers, Hassler et al. (1999, 2001a, 2001b), we explore other examples of this mechanism in settings where unemployment interacts with the provision of unemployment insurance. Other related contributions are Lindbeck (1995) and Lindbeck, Nyberg and Weibull (1999) who stress that policy persistence may arise from gradual changes in social norms vis-a-vis recipients of social assistance when a large mass of agents become dependent on such safety nets.

the dependency ratio and current taxes. Furthermore, in an extension, we show that the breakdown of the welfare state becomes more likely when the pre-tax wage inequality is large, since such inequality strengthens the incentives for private investment and reduces, *ceteris paribus*, the constituency of the welfare state.

The first prediction of the theory, i.e., that redistributive programs tend to be persistent is consistent with a number of empirical observations. For instance, a number of welfare state institutions were introduced in the aftermath of the Great Depression and after World War II, when large masses of people were impoverished, thereby creating a demand for public intervention.[4] The size of government programs, redistributional policies and public employment did not diminish after the economies had recovered from the shocks. Instead, these policies persisted and were further expanded in the 1960s. According to our theory, this persistence stems from the fact that once they are in place, redistributive programs, government employment etc., affect private incentives in a way generating a sustained demand for their continuation. Our theory predicts that not only policies, but also their effects on income distribution, are persistent. Such joint persistence is consistent with the dynamics of unemployment and unemployment insurance in European countries after the oil shocks. The sharp increase in unemployment after these shocks was followed by increasing unemployment benefits and larger tax wedges, which contributed to sustaining large unemployment and generating hysteresis.[5] The second prediction of the theory is more difficult to assess empirically. It is, however, broadly consistent with the observation that conservative governments proposing drastic reductions in social policies were elected and re-elected in the 1980s in Anglo-Saxon countries, in times associated with a significant increase in wage inequality. Political changes have instead been insignificant in continental European countries, where changes in wage inequality were less pronounced.

Our theory may contribute to the understanding of various aspects of the dynamics of redistribution, although several important elements are missing in our highly stylized setting. In reality, the political debate is multidimensional, with different issues being salient in different elections. Social groups have conflicting interests on different aspects of the welfare state, and governments have access to more sophisticated policies than our simple rich-to-poor transfers system. Although, in an extension, we allow agents to vote

[4]Between 1929 and 1934, government spending as a fraction of GNP doubled in the US as well as in the major European countries, e.g., France and Italy. A change of the same magnitude occurred in the UK in the 1930s, although most of the expansion took place after 1934.

[5]Unemployment in OECD Europe rose from an average of 2.4% during 1969-73 to 8.0% during 1985-89, while unemployment benefit replacement ratios in OECD Europe rose from 18% to 30% for the same time periods. (Source: OECD Economic Outlook and OECD data base on Benefit Entitlements and Gross Replacement Ratios.)

separately on some inter- and intra-generational redistributional policies, the accumulation of government debt is ruled out throughout the paper. This is as an important limitation for understanding the intergenerational conflict. Finally, we have, for simplicity, abstracted from risk aversion, while fully acknowledging that the insurance motive may be important for understanding the demand for redistribution. While these are all important limitations, we believe that our tractable framework can be enriched and further developed to account for some of these and other aspects of how distributional conflicts are resolved in a dynamic political context.[6]

Several earlier papers have analyzed the political economy of redistribution, but earlier models had to assume either myopic voting behavior, as in Alesina and Rodrik (1994), or that current voters can commit to future policies once and for all, as in Boadway and Wildasin (1989) and Bertola (1993), or, finally, had to limit the attention to environments with no strategic interaction between voters at different dates, as in Benabou (1996 and 2000) and Persson and Tabellini (1994). In contrast, our paper provides an analytical characterization of Markov perfect equilibria without commitment in a politico-economic model where rational voters face a strategic voting incentive.

To the best of our knowledge, the only previous paper that works out an analytical solution to the Markov perfect equilibria of a dynamic political economy model is Grossman and Helpman (1998). They analyze the political determination of intergenerational redistribution in a growth model with overlapping generations, lobbies and an AK technology. In their model, however, agents make no private economic decisions and thus, there is no feedback between public policy and individual behavior, a central mechanism in our analysis. The equilibrium of their linear model features a broad range of indeterminate political choices. Other models have incorporated repeated voting with strategic interactions, but only yielded numerical solutions (e.g., Bassetto (1999), Krusell, Quadrini and Ríos-Rull (1996), Krusell and Ríos-Rull (1996, 1999), and Saint Paul (2001)). Saint Paul (2001) in particular, numerically solves a politico-economic model with dynamic voting, where redistribution may fall after increases in inequality if such inequality is concentrated at the lower tail of the distribution, so that the median voter becomes richer. Saint Paul's paper documents that this may be a realistic description of the political changes that occurred in Anglo-Saxon countries in the 1980s, an episode that is also consistent with our theory, as discussed above.

In our model, expectations of high future redistribution leads to lower investments,

[6] Hassler, Krusell, Storesletten and Zilibotti (2001), for instance, build on the set-up of this paper and consider the choice of redistribution in a more elaborate political model with probabilistic voting and risk averse agents.

which, in turn, increase future demand for redistribution. Such a feed-back mechanism is present in a number of previous papers including Benabou (2000), Glomm and Ravikumar (1995), and Saint Paul and Verdier (1997). Saint Paul and Verdier (1997) show that multiple equilibria can arise in a politico-economic model where agents vote over capital taxation and have access to opportunities of expatriating their savings at costs varying exogenously across individuals. The expectation of the level of future taxation of domestic savings determines the extent to which young individuals exploit these opportunities. Under the assumption that the median voter has better than average access to international capital markets, multiple equilibria may arise. If agents expect high taxes, the median voter will expect a disproportionately large share of her savings and will vote for high taxation of domestic savings. In Glomm and Ravikumar (1995), the endogenous determination of public expenditure creates multiple equilibria in a model where voters are identical. In their model, expectations of high (low) taxes reduce (increase) private educational investments and future income. Under the assumption that taxes are used to finance an inferior public good, high (low) income implies that the homogenous voters prefer low (high) public good provision and taxes, leading to the possibility of multiple equilibria.

In both Glomm and Ravikumar (1995) and Saint Paul and Verdier (1997), as well as in Benabou (2000), the median voter has no stake in future political outcomes and there is no motive for strategic voting. In our model, the utility of the current median voter is instead decreasing in future redistribution, creating such motive. In particular, young agents rationally expect future redistribution to depend on the observed current level of redistribution: if current redistribution is set sufficiently low (high), the young expect no (a continuing) future political majority for the welfare state, and make high (low) investments. The trade-off between taxes and benefits for old voters, who, by assumption, hold the political power, depends on the investment of the young. Old voters therefore face an incentive to strategically moderate their demand for current redistribution so as to induce the expectation of a future majority against redistribution, which would lead to lower current taxes. Multiple equilibria in our model hinge on this strategic voting motive. If voters myopically ignored the effect of their political choice on the future distribution of voters, there would be a unique equilibrium where an initial majority of unsuccessful voters would lead to the perpetual survival of the welfare state. The feed-back between the expectation of future redistribution to the ex-post demand for redistribution would, in contrast to Glomm and Ravikumar (1995) and Saint Paul and Verdier (1997), not alone be sufficient to generate multiple equilibria.

Strategic voting interactions are also absent in the influential paper of Benabou (2000), which constructs a model where redistribution ameliorates capital market imperfections. In his model, political support for redistribution is high when the efficiency-enhancing effect dominates the purely redistributive one. This occurs when inequality is sufficiently small and thus, on the one hand, low inequality induces high redistribution. On the other hand, high redistribution sustains low inequality and hence, multiple steady-states are possible. Thus, Benabou (2000) is consistent with the observation that inequality and redistribution are negatively correlated across developed countries. In contrast, our model shares with earlier politico-economic models *a la* Meltzer and Richard (1981) the prediction that higher inequality creates more demand for redistribution, although only as long as the median voter is poorer than the average agent.[7] This suggests that the effects captured by our theory may have been dominated by other factors – institutional differences across countries (see, for instance, Persson (2002)), different strength of the labor movement, etc. – in shaping cross-country patterns in the size of governments.

Among other related papers, Coate and Morris (1999) construct a model of special interest groups where firms choose their location on the basis of geographical subsidies and have, ex-post (though not ex-ante), an incentive to bribe politicians for the subsidies to be continued. Their model features multiple steady-states and policy persistence, but the political mechanism is very different from ours. In Piketty (1995), social learning about the trade-off between efficiency and incentives gives rise to multiple steady-states with different levels of redistribution. Finally, a series of papers provide positive theories of social security in repeated voting models (Boldrin and Rustichini (2000), Cooley and Soares (1999), Galasso and Conde-Ruiz (1999)). In these papers, intergenerational redistribution is sustained by trigger strategies in infinite horizon games. In contrast, our results would survive in a finite horizon environment.

The plan of the paper is the following. Section 2 describes the model. Section 3 characterizes the political equilibria, and Section 4 explores three extensions of the basic set-up. Section 5 concludes. All proofs are in the appendix.

2 The model

The model economy consists of a continuum of risk-neutral, two-period lived agents. Each generation has a unit mass. All agents are born identical, but their subsequent earnings

[7] In our two-group set-up, the median voter can be richer than the average voter, and the predictions of our theory about the sign of the correlation between inequality and redistribution is ambiguous. For instance, the equilibrium features zero redistribution if inequality is large and the median voter is richer than average, whereas it features positive redistribution if inequality is very low and the median voter is poorer than the average.

are stochastic. "Successful" agents earn a high wage, normalized to unity, in both periods of their life, whereas "unsuccessful" agents earn a low wage, normalized to zero. At birth, each agent undertakes a costly investment, thereby increasing the probability of subsequent success. The cost of investment, which can be interpreted as the disutility of educational effort, is e^2, where e is the probability of success.[8]

The dynamics of redistribution from successful to unsuccessful agents is the focal point of the paper. In each period, a transfer $b \in [0,1]$ to each low-income agent is determined, financed by collecting a lump-sum tax τ.[9] The transfer, and the associated tax rate, are determined before the young agents decide on their investment, and is assumed to be age-independent. Age-dependent taxes and transfers will be discussed as an extension in section 4.3. We shall, however, maintain that the government budget balances in every period.

The expected utility of agents alive at time t is given as follows:

$$\begin{aligned} \tilde{V}^{os}(b_t, b_{t+1}, \tau_t) &= 1 - \tau_t \\ \tilde{V}^{ou}(b_t, b_{t+1}, \tau_t) &= b_t - \tau_t \\ \tilde{V}^{y}(e_t, b_t, b_{t+1}, \tau_t, \tau_{t+1}) &= e_t(1+\beta) + (1-e_t)(b_t + \beta b_{t+1}) - e_t^2 - \tau_t - \beta\tau_{t+1}, \end{aligned} \tag{1}$$

where $\tilde{V}^{os}$, $\tilde{V}^{ou}$, and $\tilde{V}^{y}$ denote the objective of old successful, old unsuccessful, and young agents, respectively. $\tilde{V}^{y}$ is computed prior to individual success or failure and $\beta \in [0,1]$ is the discount factor. It is straightforward to show that the solution to the optimal investment problem of the young, given b_t and b_{t+1}, is $e_t^*(b_t, b_{t+1}) = (1 + \beta - (b_t + \beta b_{t+1}))/2$.

Since agents are *ex-ante* identical, agents of the same cohort choose the same investment, which implies that the proportion of old unsuccessful in period $t+1$ is given by

$$u_{t+1} = 1 - e_t^* = \frac{1 - \beta + b_t + \beta b_{t+1}}{2}. \tag{2}$$

Thus, the future proportion of old unsuccessful depends on benefits in period t and $t+1$. To balance the budget, tax revenues must amount to $2\tau_t = (u_t + u_{t+1})\, b_t$, yielding

$$\tau_t = \frac{1 - \beta + b_t + \beta b_{t+1} + 2u_t}{4} b_t. \tag{3}$$

[8] It is important for the analysis that agents earn income in both periods. The assumption that first and second period income are perfectly correlated is, however, not essential – the qualitative results will be preserved provided that earnings in the two periods are positively correlated.

[9] The assumption that $b \leq 1$ can be motivated by moral hazard considerations. If redistribution were larger than 100%, successful agents would decide not to work and to claim benefits. The assumption that $b \geq 0$ is a useful benchmark, and it can be regarded as the effect of some constitutional principle that public redistribution cannot be regressive. The important consequence of this assumption is that redistribution is bounded from below.

By substituting for τ_t and e_t^* in equation (1), the indirect utility functions can be written as:

$$\begin{aligned} V^{os}(b_t, b_{t+1}, u_t) &= 1 - \frac{(1-\beta) + (b_t + \beta b_{t+1}) + 2u_t}{4} b_t, \\ V^{ou}(b_t, b_{t+1}, u_t) &= b_t - \frac{(1-\beta) + (b_t + \beta b_{t+1}) + 2u_t}{4} b_t, \\ V^{y}(b_t, b_{t+1}, b_{t+2}, u_t) &= \frac{(1+\beta)^2}{4} + \frac{(1-\beta) - 2u_t}{4} b_t - \frac{b_{t+1} + \beta b_{t+2}}{4} \beta b_{t+1}. \end{aligned} \tag{4}$$

Note that taxes per unit of benefits, $\tau_t / b_t = (1 - \beta + b_t + \beta b_{t+1} + 2u_t)/4$, increase in u_t (because higher u_t implies a higher dependency ratio among the old) and in b_t and b_{t+1} (because higher b_t and b_{t+1} reduce investment, implying a higher dependency ratio among the young). Since the old in period t cannot enjoy benefits in period $t+1$, their utility is decreasing in b_{t+1}.

The old successful agents obviously prefer zero benefits, since redistribution implies positive taxes without providing any benefits. In contrast, the old unsuccessful agents are better off with some redistribution, even though their preferences for redistribution may be non-monotonic, as the marginal cost of redistribution is increasing in b_t. Concerning the preferences of the young, note that positive benefits lead to positive (negative) intergenerational redistribution from the old to the young, if the number of old unsuccessful is sufficiently small (large). Holding future benefits constant, the young prefer positive redistribution if and only if $u_t < (1-\beta)/2$.

Before proceeding to the main analysis, we note that any Pareto efficient allocation is characterized by zero redistribution in every period except, possibly, in the first.[10] The reason is that redistribution distorts the effort choice of the young, but has no insurance value as agents are risk-neutral.

3 Political equilibrium

The purpose of this paper is to explore the impact of the *ex-post* conflict of interest between groups on the dynamics of redistribution. More specifically, can an "inefficient" welfare state

[10] More formally, we define the class of Pareto optimal sequences of benefits, $\{b_t\}_{t=1}^{\infty}$, as those which would be chosen by a social planner whose objective function is given by

$$\max_{\{b_t\}_{t=1}^{\infty}} \left\{ \lambda_{0s} (1-u_1) V^{os}(b_1, b_2, u_1) + \lambda_{0u} u_1 V^{ou}(b_1, b_2, u_1) + \sum_{t=1}^{\infty} \lambda_t V^y(b_t, b_{t+1}, b_{t+2}, u_t) \right\},$$

subject to $b_t \in [0,1]$ $\forall t$, where the planner weights λ_{0s}, λ_{0u}, and $\{\lambda_t\}_{t=0}^{\infty}$ are positive and satisfy $\sum_{t=0}^{\infty} \lambda_t + \lambda_{0s} + \lambda_{0u} = 1$. It is straightforward to show that the planner would choose zero benefits after the first period, for any arbitrary sequence of (positive) planner weights. Moreover, a utilitarian planner with equal weights on all initially living individuals would set $b_0 = 0$, for any u_0. The proof is available upon request.

survive over time? Or will dynamic voting decisions make redistribution vanish in the long run?

In answering this question, we restrict the attention to Markov perfect equilibria, where the state of the economy is summarized by the proportion of current unsuccessful old agents (u_t). The political equilibrium is defined as follows.

Definition 1 *A (Markov perfect) political equilibrium is defined as a pair of functions* $\langle B, U \rangle$*, where* $B : [0,1] \to [0,1]$ *is a public policy rule,* $b_t = B(u_t)$, *and* $U : [0,1] \to [0,1]$ *is a private decision rule,* $u_{t+1} = 1 - e_t = U(b_t)$, *such that the following functional equations hold:*

1. $B(u_t) = \arg\max_{b_t} V(b_t, b_{t+1}, b_{t+2}, u_t)$ *subject to* $b_{t+1} = B(U(b_t))$, $b_{t+2} = B(U(B(U(b_t))))$, *and* $b_t \in [0,1]$, *and* $V(b_t, b_{t+1}, b_{t+2}, u_t)$ *is defined as the indirect utility of the current decisive voter.*

2. $U(b_t) = (1 - \beta + b_t + \beta b_{t+1})/2$, *with* $b_{t+1} = B(U(b_t))$.

The first equilibrium condition requires that b_t maximizes the objective function of the decisive (median) voter V, taking into account that future redistribution depends on the current policy choice via the equilibrium private decision rule and future equilibrium public policy rules. Furthermore, it requires $B(u_t)$ to be a fixed point in the functional equation in part 1 of the definition. In other words, suppose that agents believe future benefits to be set according to the function $b_{t+j} = B(u_{t+j})$. Then, we require that the same function $B(u_t)$ defines optimal benefits today.

The second equilibrium condition implies that all young individuals choose their investment optimally, given b_t and b_{t+1}, and that agents hold rational expectations about future benefits and distributions of types. In general, U might be a function of both u_t and b_t. In our model, however, u_t has neither a direct effect on the investment choice of the young, nor, consequently, on the future distribution of voters. Thus, we choose to focus on equilibria where their equilibrium investment choice is fully determined by the current benefit level.

Finally, in order to single out the effects of dynamic rational voting, it is useful to define an alternative *myopic voting equilibrium*, where voters ignore the impact on future political decisions when deciding on current policies. A myopic equilibrium is defined as in Definition 1, but with condition 1 being replaced by

1' $B(u_t) = \arg\max_{b_t} V(b_t, \bar{b}_{t+1}, \bar{b}_{t+2}, u_t)$ *subject to* $b_t \in [0,1]$, *where* $\bar{b}_{t+1}$ *and* $\bar{b}_{t+2}$ *are taken as parametric, subject to rational expectations.*

This condition requires that b_t maximizes the objective function of the decisive (median) voter V, taking future redistribution as given. Voters correctly anticipate future redistribution, but ignore that they could affect its path through their current political choice. In the rest of the analysis, we refer to a Markov perfect equilibrium as an equilibrium, and state explicitly when referring to a myopic voting equilibrium.

3.1 Dictatorship

For expositional reasons it is convenient to start the analysis by describing the equilibrium under the assumption that the political power permanently rests in the hands of one of the two groups of old agents in the society. Then, we extend the analysis to the case of majority voting.

We define "plutocracy" (PL) and "dictatorship of the proletariat" (DP), as the regimes where the level of redistribution is chosen at the beginning of each period by the currently living successful and unsuccessful old agents, respectively. Formally, under DP, $V(b_t, b_{t+1}, b_{t+2}, u_t) \equiv V^{ou}(b_t, b_{t+1}, u_t)$ whereas, under PL, $V(b_t, b_{t+1}, b_{t+2}, u_t) \equiv V^{os}(b_t, b_{t+1}, u_t)$. The equilibrium under dictatorship is characterized in the following Proposition:[11]

Proposition 1 *The PL equilibrium, $\langle B^{pl}, U^{pl} \rangle$, is characterized as follows;*

$$\begin{aligned} B^{pl}(u_t) &= 0 \\ U^{pl}(b_t) &= \frac{1-\beta+b_t}{2}. \end{aligned} \tag{5}$$

Given $u_0 \in [0,1]$, for all $t \geq 1$, $u_t = u^{pl} \equiv \frac{1-\beta}{2}$.
The DP equilibrium, $\langle B^{dp}, U^{dp} \rangle$, is characterized as follows;

$$B^{dp}(u_t) = \begin{cases} \frac{3}{2} - u_t & \text{if } u_t > \bar{u}(\beta) \\ b_{dp} - \frac{2}{2-\beta}(u_t - u_{dp}) & \text{if } u_t \in \left[\frac{3}{2} - \frac{2}{2+\beta}, \bar{u}(\beta)\right] \\ 1 & \text{if } u_t \in \left[0, \frac{3}{2} - \frac{2}{2+\beta}\right] \end{cases} \tag{6}$$

$$U^{dp}(b_t) = \begin{cases} u_{dp} + \frac{2-\beta}{4}(b_t - b_{dp}) & \text{if } b_t \in \left[\frac{2\beta}{2+\beta}, 1\right] \\ \frac{1+b_t}{2} & \text{if } b_t \in \left[0, \frac{2\beta}{2+\beta}\right) \end{cases} \tag{7}$$

[11] The gist of the derivation of the equilibrium functions is the following. Start by assuming B to be linear in u_t, ignoring the constraint that $b \in [0,1]$. Then, as the young are risk-neutral with quadratic effort costs, the function U, satisfying condition 2 in equilibrium definition 1, is linear in b_t. Moreover, the indirect utility is also linear-quadratic in b_t, once B and U have been substituted into (4). It turns out that, in the absence of constraints, the optimal choice of b_t is indeed linear in u_t. Imposing condition 1 in equilibrium definition 1, it is straightforward to solve for the coefficients in B. What remains is to impose the constraints on b_t, and check that no deviations from this constrained linear rule can be optimal. Such deviations may, in some cases, be optimal, as e.g. explained in footnote 12, in which case the policy rule B must be modified in a non-trivial way.

where $u_{dp} \equiv \frac{1}{6}\left(5+\frac{\beta^2}{2+\beta}\right)$, $b_{dp} \equiv \frac{4}{3}\frac{1+\beta}{2+\beta}$ *and* $\bar{u}(\beta) = \frac{\beta+6-\beta\sqrt{4-2\beta}}{2(2+\beta)}$. *The equilibrium law of motion,* $u_{t+1} = U^{dp}\left(B^{dp}(u_t)\right)$, *is as follows;*

$$u_{t+1} = \begin{cases} \frac{5}{4} - \frac{u_t}{2} & \text{if } u_t > \bar{u}(\beta) \\ u_{dp} - \frac{1}{2}(u_t - u_{dp}) & \text{if } u_t \in \left[\frac{3}{2} - \frac{2}{(2+\beta)}, \bar{u}(\beta)\right] \\ \frac{\beta}{4} + \frac{2}{2+\beta} & \text{if } u_t \in \left[0, \frac{3}{2} - \frac{2}{(2+\beta)}\right] \end{cases} \quad (8)$$

Given $u_0 \in [0,1]$, *the economy converges with an oscillatory pattern to a unique steady-state,* $u = u^{dp}$ *and* $b = b^{dp}$.

FIGURE 1. Plutocracy and dictatorship of the proletariat

Figure 1 represents the equilibrium public policy rule and private decision rule for the PL and DP equilibrium. In the PL case (upper figures), the policy function is constant at $B(u_t) = 0$, and the private decision rule is upward sloping, reflecting the expectations of the young agents that $b_{t+1} = 0$, irrespective of the choice of b_t. In the DP case (lower figures), the equilibrium redistribution is always strictly positive, and it is 100% for sufficiently low u_t. Furthermore, it is downward sloping, reflecting the fact that the marginal cost of redistribution increases, as the current proportion of old successful agents falls (see equation (3)). The private decision rule is also positively sloped, but less steep than in the PL case, since an increase in b_t negatively affects the choice of b_{t+1}, hence, the current effort choice of the young responds less to an increase in current benefits than if future redistribution were constant.[12]

3.2 Majority voting

We now assume that political decisions are taken through majority voting. Agents vote on the single issue of redistribution. It is straightforward to show that if young agents were pivotal in voting over current benefits, then, for all $t > 0$, benefits would be zero in equilibrium. Intuitively, as the young are still behind the veil of ignorance, they oppose distortionary redistribution.

[12] All figures, including Figure 1, represent a parametric case where $\beta = 0.75$. Note that $\beta < \left(\sqrt{17}-1\right)/4 \approx 0.78$ implies that $\bar{u}(\beta) > 1$ and the range $u_t > \bar{u}(\beta)$ is empty. If, in contrast, $\bar{u}(\beta) \leq 1$, there is a range $u_t \in [\bar{u}, 1]$ such that the unsuccessful old in period t induce a u_{t+1} where the constraint $b_{t+1} \leq 1$ is binding. This creates a downward discontinuity of $B(u_t)$ at $\bar{u}$. The range $[\bar{u}, 1]$ is ephemeral, in the sense that in equilibrium, $u_t < \bar{u}\ \forall t > 0$. Therefore, if the economy starts at $u_0 < \bar{u}$, the policy rule and law of motion are qualitatively identical to those in Figure 1. Otherwise, they differ for one period only. To understand the origin of the downward discontinuity, note that b_{t+1} is negatively related to b_t when the constraint $b_{t+1} \leq 1$ is not binding. Thus, the marginal distortion of current benefits shifts upward at $\bar{u}$, as the constraint on b_{t+1} becomes binding. Therefore, $B(u_t)$ falls discontinuously at $u_t = \bar{u}$.

However, we regard the assumption that voters behind the veil of ignorance are pivotal as unrealistic.[13] Thus, this paper explores the consequences of letting the political decisions be determined by the *ex-post* conflict of interests between individuals who know their type, i.e., the old. Two alternative assumptions can deliver a political preponderance of the old in our model. The first is to assume that young individuals have a lower voting turnout than the old, maintaining that current benefits are set at the beginning of each period. This assumption can be defended empirically as the voting turnout increases with age. For example, Wolfinger and Rosenstone (1980) document that turnout in U.S. elections is sharply increasing in age, rising from 45% for the 20-year old to 75% for the 65-year old.[14] Alternatively, it might be assumed that elections were held at the end of each period, and agents voted over benefits in the next period, after the uncertainty about their individual success had been unraveled. In this case, by not being alive in the next period, the old would have no interests at stake and could be assumed to abstain from voting. Clearly, this is equivalent to assuming that the choice of current benefits is taken at the beginning of each period, but only the old vote. For expositional ease, we maintain in the presentation the interpretation that agents vote over current benefits and only the old vote.[15]

Benefits maximize the indirect utility of the old successful (unsuccessful) if $u_t \leq 1/2$ ($u_t > 1/2$). As we shall see, majority voting can generate persistence in the equilibrium choice of redistribution. If the economy starts with a pro-welfare state majority ($u_t > 1/2$), then there exists an equilibrium where the welfare state and the political majority supporting it is sustained over time. Conversely, if $u_t \leq 1/2$, the welfare state will never arise. The positive feedback mechanism giving rise to the persistence of policies and distributions of voters is that high (low) benefits today affect private incentives so as to induce a large (small) proportion of unsuccessful agents tomorrow, and therefore a broad (narrow) future constituency for redistribution.

An initial majority of unsuccessful individuals does not guarantee, however, the eternal survival of the welfare state. For sufficiently high discount factors, and given an initial

[13] For instance, in a model where agents live for more than two periods and make their investment decision in the first period only, the "young" would constitute a small proportion of the electorate and are not likely to be decisive.

[14] Furthermore, with the aid of an interest group model, Mulligan and Sala-i-Martin (1999) argue that the elderly have a preponderant weight on redistribution policies. In their paper, this arises due to the old having a low opportunity cost of time.

[15] In a previous version of this paper, Hassler et al. (2001c), we also analyzed an intermediate case, when the young vote before knowing their type, albeit with a lower turnout than the old, in elections held at the beginning of each period. In particular, only a share $\varepsilon \in [0,1]$ of the young individuals was assumed to participate in the voting process. The key insight of that extension was that our results when only the old vote ($\varepsilon = 0$) remain unchanged for $\varepsilon > 0$, provided that ε is not too large.

majority of unsuccessful individuals, there exist, in addition, equilibria where any existing welfare state is dismantled in, at most, two periods. The survival of the welfare state is in this case a matter of self-fulfilling expectations.

As we shall see, expectations (beliefs) about the identity of the future median voter play a crucial role in driving such multiplicity. If the agents expect that a majority of successful agents will materialize in the next period, that majority is then expected to implement zero redistribution. Conversely, if they expect a majority of successful agents in the next period, they expect next period redistribution to be strictly positive. Before characterizing equilibria, it is useful to discuss some general properties of the expectations. According to Definition 1, expectations must be rational, which imposes two restrictions. First, it would be irrational to believe that the old successful will be in majority next period if, in the current period, voters set $b_t > \beta$, since the private decision rule would then imply that $u_{t+1} = (1 - \beta + b_t + \beta b_{t+1})/2 > 1/2$. Second, it would be irrational to believe that a majority of unsuccessful individuals will materialize in the next period if the current majority sets $b_t = 0$, since, then, $u_{t+1} = (1 - \beta + \beta b_{t+1})/2 \leq 1/2$. We assume in addition, that expectations about the identity of the median voter are "monotonic": if agents believe that $b_t = x$ induces $u_{t+1} \leq 1/2$, then they must also believe that $b_t < x$ induces $u_{t+1} \leq 1/2$.[16] Finally, we assume throughout that beliefs are stationary.

Given these assumptions, we can summarize beliefs about the identity of the future median voter as a threshold benefit level, denoted by $\theta \in [0, \beta]$, such that all agents expect that zero benefits will be provided at $t+1$, if and only if current redistribution is smaller or equal to θ. Formally, if and only if $b_t \leq \theta$, the equilibrium private decision rule and policy function must feature, respectively, $U(b_t) \leq 1/2$ and $B(U(b_t)) = 0$.[17]

In the rest of this section, we will prove that, conditional on the existence of an initial majority of old unsuccessful agents, equilibria featuring the survival of the welfare state ("pro-welfare equilibria") are sustained if the welfare state is believed to be sufficiently robust, i.e., for sufficiently low θ. Instead, equilibria where the ruling old unsuccessful vote strategically so as to induce a future political majority of successful agents that will vote for zero redistribution ("anti-welfare equilibria") are sustained if the welfare state is believed to be sufficiently "fragile", i.e., for sufficiently high θ. In both cases, the beliefs that determine private investments and political choices are fulfilled in equilibrium.

[16] The assumption of "monotonicity" plays no role in the characterization of the equilibrium path, since, as we shall see, the only essential feature of beliefs is the highest level of b inducing a majority of successful individuals. However, the assumption simplifies the characterization of out-of-equilibrium behavior and reduces the set of observationally equivalent equilibria.

[17] To simplify the notation, they will not be specified as arguments of the equilibrium functions $\langle B, U \rangle$. In particular, with some abuse of notation, we will write $B(u_t)$ and $U(b_t)$ rather than $B(u_t; \theta)$ and $U(b_t; \theta)$.

3.2.1 Pro-welfare equilibria

In this section, we consider pro-welfare equilibria featuring multiple steady-states. If the initial median voter is unsuccessful, the welfare state survives forever. If, instead, the initial median voter is unsuccessful, no welfare state will ever arise. Essentially, the equilibrium functions B^{pw} and U^{pw} are found by splicing together the equivalent functions from the equilibrium under dictatorship in Proposition 1 (i.e., B^{pl} and B^{dp}, and U^{pl} and U^{dp}), and specifying beliefs (θ) such that no switch of political majority ever occurs along the equilibrium path.

Proposition 2 *For all $\beta \in [0,1]$ and $\theta \leq \bar{\theta}(\beta) \in (0,\beta]$, there exists a "pro-welfare equilibrium" (PWE), $\langle B^{pw}, U^{pw} \rangle$, featuring multiple steady-states, with the following characteristics:*

$$B^{pw}(u_t) = \begin{cases} B^{dp}(u_t) > 0 & \text{if } u_t \in \left(\frac{1}{2}, 1\right] \\ B^{pl}(u_t) = 0 & \text{if } u_t \in \left[0, \frac{1}{2}\right] \end{cases} \tag{9}$$

$$U^{pw}(b_t) = \begin{cases} U^{dp}(b_t) > \frac{1}{2} & \text{if } b_t \in (\theta, 1] \\ U^{pl}(b_t) \leq \frac{1}{2} & \text{if if } b_t \in [0, \theta] \end{cases}, \tag{10}$$

where the expression of $\bar{\theta}(\beta)$ is in the proof in the appendix, and $B^{dp}(u_t), B^{pl}(u_t), U^{dp}(b_t)$ and $U^{pl}(b_t)$ are defined in proposition 1.
This implies the following equilibrium law of motion, $u_{t+1} = U^{pw}(B^{pw}(u_t))$,;

$$u_{t+1} = \begin{cases} \frac{5}{4} - \frac{u_t}{2} & \text{if } u_t > \bar{u}(\beta) \\ u_{dp} - \frac{1}{2}(u_t - u_{dp}) & \text{if } u_t \in \left[\frac{3}{2} - \frac{2}{2+\beta}, \bar{u}(\beta)\right] \\ \frac{\beta}{4} + \frac{2}{2+\beta} & \text{if } u_t \in \left(\frac{1}{2}, \frac{3}{2} - \frac{2}{2+\beta}\right] \\ \frac{1-\beta}{2} & \text{if } u_t \in \left[0, \frac{1}{2}\right]. \end{cases} \tag{11}$$

There are two locally stable steady-states. In particular,

1. *if $u_0 \leq 0.5$, the economy converges in one period to a steady-state equilibrium with $\{b, u\} = \{b_{pl}, u_{pl}\}$ as defined in proposition 1.*

2. *if $0.5 < u_0 \leq 1$, the economy converges asymptotically with an oscillatory pattern to an equilibrium with $\{b, u\} = \{b_{dp}, u_{dp}\}$ as defined in proposition 1.*

Figure 2 depicts the equilibrium policy rule and private decision rule (for, again, $\beta = 0.75$, implying $\bar{u}(\beta) > 1$). The left-hand panel shows that, when $u_t \leq 1/2$, then $B(u_t) = 0$ in equilibrium. At $u_t = 1/2$, the policy function increases discontinuously, as the unsuccessful become pivotal. In fact, for an intermediate range of u_t, the equilibrium policy function prescribes 100% redistribution, being downward sloping thereafter. The right-hand panel depicts the private decision rule. A majority of old unsuccessful materializes at $t+1$ if and only if $b_t > \theta$. Since a majority of unsuccessful at $t+1$ would set $b_{t+1} > 0$, whereas a majority of old successful would set $b_{t+1} = 0$, the private decision rule exhibits a downward discontinuity at θ.

FIGURE 2. Pro-welfare equilibrium

The discontinuity in the private decision rule of the young implies a discrete fall in the current tax level at $b_t = \theta$. Thus, the utility of the old unsuccessful is discontinuous at θ. The left hand-side of Figure 3 illustrates this point, by plotting the utility of the old unsuccessful as a function of b, given u and β (the figure depicts $\beta = 0.75$ and $u_t = u_{dp}$) and for sufficiently low θ, as in Proposition 2. Formally, the discontinuity is due to b_{t+1} entering negatively in the utility of the old in equation (4). As a result, the preferences of the median voter are not single peaked with respect to b_t. The old unsuccessful face the temptation to vote strategically for $b_t = \theta$, so as to change the identity of the future median voter, thereby ensuring that the welfare state disappears. A PWE is sustained if, as in the left-hand figure, this strategic voting option is not globally optimal, i.e., the peak in preferences corresponding to $B^{dp}(u)$ yields higher utility than that corresponding to θ. This condition must hold for all $u \in (0.5, 1]$. Note the role of the threshold θ. The lower is θ, the higher the cost in terms of foregone pre-tax earnings (b_t) required to induce the breakdown of the welfare state at $t+1$. Intuitively, a low θ means that agents regard the welfare state as "robust", and think that only very low current redistribution can induce its termination.

FIGURE 3. Indirect utility of the old unsuccessful ($\beta = 0.75$, $u_t = u_{dp}$)

To further grasp the intuition, it is useful to focus on the extreme belief, $\theta = 0$. In this case, the welfare state is expected to break down only if current redistribution is set equal to zero. But this cannot occur in equilibrium, since this would imply zero net earnings for the old unsuccessful. This explains why, for any β, there exist beliefs (i.e., a range of sufficiently low θ) sustaining the survival of the welfare state.

Finally, we examine the role of rational voting in pro-welfare equilibria. If agents voted myopically, according to our definition in section 3, there would, for all β, exist an equilibrium qualitatively similar to the PWE of Figure 2. It would exhibit multiple steady-states and, if $u_0 > 0.5$, b_t and u_t would converge with an oscillatory pattern to a steady-state with positive redistribution. This steady-state would , however, feature *less* redistribution and a *lower* proportion of unsuccessful in steady-state than under rational voting. The reason is that rational voters understand that, since the policy function $B(u)$ is, in the relevant range, downward sloping, an increase in current redistribution has the desirable effect of reducing future redistribution. Thus, they strategically increase their demand of current redistribution, compared with the myopic voting case.[18]

3.2.2 Anti-welfare equilibria

So far, we have analyzed equilibria where an existing welfare state survives. In this section, we show that, if β is sufficiently large, there exist other rational expectations equilibria, where the welfare state breaks down in finite time. In such equilibria, the old unsuccessful vote strategically so as to change the identity of the future median voter, thereby ensuring the disappearance of the welfare state. We will also show that, as long as $\beta < 1$, no myopic voting equilibrium featuring the breakdown of an existing welfare state exists. Thus, the results of this section hinge on rational forward-looking voting.

Proposition 3 establishes that, if β is sufficiently large, there exist "anti-welfare equilibria" featuring the termination of the welfare state in either one (part 1) or two periods (part 2).

Proposition 3 *Let* $\underline{\beta} \simeq .570$ *be the real solution to the equation* $\left(1+\sqrt{\beta}\right)^{-1} = \beta$, *where* $\underline{\beta} > \bar{\theta}(\beta)$ *for all* $\beta \geq \underline{\beta}$, *and* $\bar{\theta}(\beta)$ *is as in Proposition 11.*
Then, for all $\beta \geq \underline{\beta}$ *and* $\theta \in \left[\underline{\beta}, \beta\right]$, *there exists an "anti-welfare equilibrium" (AWE),* $\langle B^{aw}, U^{aw} \rangle$, *with the following characteristics:*

Part 1 "Switch in one period". If $\beta \geq \left(\sqrt{5}-1\right)/2 \simeq 0.618$, *then, for all* $\theta \in \left[\underline{\theta}(\beta), \beta\right]$:

$$B^{aw}(u_t) = \begin{cases} \theta & \text{if } u_t > 1/2 \\ 0 & \text{if } u_t \in \left[0, \frac{1}{2}\right] \end{cases}$$

$$U^{aw}(b_t) = \begin{cases} \frac{1}{2}(1-\beta+\beta\theta+b_t) > \frac{1}{2} & \text{if } b_t > \theta \\ U^{pl}(b_t) \leq \frac{1}{2} & \text{if } b_t \leq \theta \end{cases},$$

where $\underline{\theta}(\beta) \equiv 1+\beta-\sqrt{\beta(1+\beta)}$ *and* $\underline{\theta}(\beta) \leq \beta$ *for all* $\beta \geq \left(\sqrt{5}-1\right)/2$ *and* $U^{pl}(b_t)$ *is defined in Proposition 1.*

[18] Under myopic voting, the steady-state redistribution is equal to $2(1+\beta)/(3+2\beta) < b_{dp}$. The formal characterization of the myoping voting equilibrium has been omitted and is available upon request.

Part 2 "Switch in two periods". If $\beta \geq \underline{\beta}$, then, for all $\theta \in [\underline{\beta}, \min\{\beta, \underline{\theta}(\beta)\})$:

$$B^{aw}(u_t) = \begin{cases} \theta & \text{if } u_t \geq \hat{u}(\beta,\theta) \\ 1 & \text{if } u_t \in (0.5, \hat{u}(\beta,\theta)) \\ 0 & \text{if } u_t \in \left[0, \frac{1}{2}\right] \end{cases}$$

$$U^{aw}(b_t) = \begin{cases} \frac{1}{2}(1-\beta+\beta\theta+b_t) > \frac{1}{2} & \text{if } b_t > \theta \\ U^{pl}(b_t) \leq \frac{1}{2} & \text{if } b_t \leq \theta \end{cases},$$

where $\underline{\theta}(\beta) > \underline{\beta}$ for all β (see proof in appendix for the exact expression of $\underline{\theta}(\beta)$), and $\hat{u}(\beta,\theta) \equiv \frac{1}{2}(2+\beta-(2\beta+3-\theta)\theta)\cdot(1-\theta)^{-1}$.

The two upper panels in figure 4 illustrate equilibria with switch in one period, as described in part 1 of Proposition 3. These equilibria require high discount factors and sufficiently large θ. If $u_t \leq 1/2$, the AWE policy function is as in a PWE. If $u_t > 1/2$, however, the policy function prescribes $b_t = \theta$, i.e., benefits are set equal to the highest level that can induce $u_{t+1} \leq 1/2$. Accordingly, the private decision rule is discontinuous at θ. As the figure shows, if, at time zero, the old unsuccessful are in majority, then, given the policy function and beliefs, they find it optimal to set $b_0 = \theta$ and induce the switch of majority. The successful then becomes the majority at time one, choosing $b_1 = 0$ and terminating the welfare state. The sequence of equilibrium redistribution is, therefore, $b_0 = \theta$, and $b_t = 0 \ \ \forall t > 0$. The expectation of zero future redistribution induces the young to exert high investment, reduces the dependence ratio and grants low taxes in the first period.

FIGURE 4. Anti-welfare equilibrium

The right hand-side of Figure 3 represents the utility of the old unsuccessful as a function of b, given the same values of u and β as in the left hand-side panel, but for different beliefs, so as to be consistent with part 1 of Proposition 3. In particular, θ is larger, implying that agents regard the welfare state as more "fragile" and believe that a switch of majority will occur for a larger range of current redistribution policies, b. As before, the effort choice of the young falls discontinuously at $b = \theta$. Here, the global maximum occurs at $b = \theta$, however. The AWE with switch in one period is sustained if $b_t = \theta$ is the global maximum for all $u_t \in [0.5, 1]$. Namely, given the private decision rule, $U(b)$, any majority of old unsuccessful will find it optimal to induce the breakdown of the welfare state. The belief that the welfare state is "fragile" (high θ) is crucial in making it attractive for the old unsuccessful to induce the breakdown.

An interesting observation is that, given parameters, AWE with switch in one period Pareto-dominate PWE, at least for economies starting at the steady state of a PWE,

$u_0 = u_{dp}$.[19] Any coordination device inducing the young to believe that the welfare is sufficiently fragile would improve the welfare of all agents in the society, including the ex-ante utility of all future generations. This result can be related to the debate on so-called "fiscal increasing returns". It has been argued (e.g., Blanchard and Summers (1987)) that labor market reforms such as reductions in the bargaining strength of insiders, or lower replacement ratios, may lead, in high unemployment economies, to efficiency gains that are so large that even the groups that suffer a reduction in their pre-tax earnings would benefit from the reforms in after-tax terms. In other words, economies may get stuck in bad equilibria along the declining side of the Laffer curve. In our model, equilibria with inefficiently high taxes are the outcome of expectational traps: under pro-welfare expectations, a system of inefficient redistribution is believed to be very robust, and private and public decisions reinforce each other in sustaining a Pareto-inferior outcome. Changes in expectations (θ) may lead the society to a superior outcome.

It is important to emphasize two effects driving this result. First, current benefits have a direct impact on the government budget that includes a static distortionary effect (τ_t is increasing and convex in b_t, see equation (3)). However, in our model, this standard effect would not be sufficient to induce the median voter (who, recall, is a net recipient of current transfers) to set b to a level low enough to induce a switch of majority. The second factor, which reinforces the previous effect, is the dependence of future redistribution on current redistribution. Rational voters understand that, by voting for lower benefits today, they can trigger a change in the identity of the future median voter, and indirectly reduce future benefits and current taxes. This indirect effect increases the gain from restraining b_t.

The importance of rational dynamic voting in driving multiple equilibria can be appreciated by formally establishing that, if $\beta < 1$, then no myopic voting equilibrium featuring the termination of an existing welfare state would exist. To construct a contradiction, consider a candidate myopic equilibrium where agents at time zero take as parametric,

[19] Consider the AWE sustained by the belief that $\theta = \beta$. Then, for all $\beta \in [(\sqrt{5}-1)/2, 1]$, it is easy to verify that;

$$V^{ou}(\beta, 0|u_{dp}) = \beta \frac{8+4\beta-\beta^2}{12(2+\beta)} > \frac{2(2-\beta)(1+\beta)^2}{(3(2+\beta))^2} = V^{ou}(b_{dp}, b_{dp}|u_{dp})$$

$$V^{os}(\beta, 0|u_{dp}) = \frac{2}{2+\beta} - \frac{\beta(4+8\beta+\beta^2)}{12(2+\beta)} > \frac{(2-\beta)(8+7\beta+2\beta^2)}{(3(2+\beta))^2} = V^{os}(b_{dp}, b_{dp}|u_{dp})$$

$$V^{y}(\beta, 0, 0|u_{dp}) = \frac{(6-\beta)(1+\beta)^2}{12(2+\beta)} > \frac{(7\beta+10)(2-\beta)(1+\beta)^2}{4(3(2+\beta))^2} = V^{y}(b_{dp}, b_{dp}, b_{dp}|u_{dp})$$

where each left-hand side (right-hand side) term of the inequality represents the utility in an AWE (PWE). The result can be generalized to any $\theta \geq \underline{\theta}(\beta)$. For some $u > u^{dp}$, however, counter-examples can be constructed.

$\bar{b}_j = 0$ for all $j > 0$. The old unsuccessful would then choose b_0 so as to maximize $V^{ou}(b_0, \bar{b}_1, u_t) = b_0 - \left((1-\beta) + \left(b_0 + \beta\bar{b}_1\right) + 2u_0\right) b_0/4$, given $\bar{b}_1 = 0$. The solution yields $b_0 = \min\left[\frac{3}{2} + \frac{1}{2}\beta - u_0, 1\right]$. But this level of redistribution is inconsistent with a majority of old unsuccessful at time one, since

$$U\left(b_0|\bar{b}_1 = 0\right) = \frac{1 - \beta + \min\left[\frac{3}{2} + \frac{1}{2}\beta - u_0, 1\right]}{2} > \frac{1}{2}$$

for any $\beta < 1$.[20] Thus, the expectation that $b_1 = 0$ is not rational, and a myopic voting AWE does not exist. Intuitively, myopic voters do not recognize that they need to restrain their demand for current redistribution in order to terminate the welfare state. The expectation of zero future redistribution would, on the contrary, induce them to demand a very high level of redistribution in the current period, so that no switch of majority would materialize.

Consider, finally, the equilibria described in part 2 of Proposition 3, where the switch of majority and the end of the welfare state occurs in two periods. An example is depicted in the two lower panels of Figure 4. The main change, relative to case 1, is the existence of a range of intermediate levels of initial old unsuccessful, $u \in (1/2, \hat{u})$, where the old unsuccessful in majority choose 100% redistribution. In this case, the sequence of equilibrium redistribution is: $b_0 = 1$, $b_1 = \theta$, and $b_t = 0 \; \forall t > 1$.[21]

The intuition for AWE with switch in two periods is the following. Since the cost of redistribution, τ_t/b_t, is increasing with u_t, setting $b_t = \theta$ is optimal, for a range of large u, even for some $\theta < \underline{\theta}(\beta)$. This opens up a new strategic opportunity for economies starting out with an *intermediate* u ($u_0 \in (1/2, \hat{u}(\beta, \theta))$, see Figure 4). Namely, the median voter chooses 100% redistribution and induces, in the next period, a majority of old unsuccessful ($u_1 \in [\hat{u}(\beta, \theta), 1]$) which will set $b = \theta$ and which, in turn, will induce the breakdown of the welfare state two periods ahead.

In summary, Propositions 2 and 3 imply, jointly, that multiple self-fulfilling equilibria

[20] An anti-welfare equilibrium with myopic voting exists in the particular case where $\beta = 1$. In this case, given the expectation that $\bar{b}_1 = 0$, the private decision rule of the myopic voting equilibrium is $U(b_0) = b_0/2$, implying that a majority of old successful materializes in period one, irrespective of the choice of b_0. In fact, the old unsuccesful choose $b_0 = 1$ implying $u_1 = 1/2$ and the end of the welfare state.

[21] Part 2 of Proposition 3 states only sufficient conditions for the existence of AWE with switch in two periods. The necessary and sufficient conditions on the range of parameters (β) and beliefs (θ) that sustain AWE with switch in two periods can also be characterized, but are involved, and are therefore not stated here. Details are available upon request. It is, however, useful to note that there does not exist an AWE for any $\theta < 1/2$. Thus, as $\bar{\theta}(\beta) < 1/2$, there exists a nonempty intermediate range of beliefs for which neither a PWE nor an AWE is sustained.

As a more general remark, we are unfortunately unable to provide a proof that the equilibria characterized exhaust the set of equilibria consistent with definition 1.

exist when $u_t > 0.5$, provided that β is not too small. In one of these equilibria, the welfare state survives, while it is terminated in the other. No AWE exists when β is sufficiently small. The reason is that strategic voting considerations become less important when agents discount the future more highly, since private investments become less sensitive to expectations about future political decisions. In fact, myopic voting and rational voting equilibria coincide for $\beta = 0$.

4 Extensions.

4.1 Stochastic shocks.

Proposition 2 implies that both redistributive policies (b) and their effect on voters' distribution (u) tend to be persistent. But while it shows that an existing welfare state can regenerate its political support and survive in the long run, it also implies that without an initial majority in favor of redistribution, the welfare state would never arise. Government involvement in redistributive programs was, however, very limited in the early XXth Century, suggesting that, at the time, the political majority was opposed to redistribution. Yet, government transfer programs did expand, most notably in the aftermath of the Great Depression. In the introduction, we argued that this may be attributed to a shock that unexpectedly impoverished a large share of the voters and changed the political support to redistributive programs.[22] In this section, we will substantiate this claim by extending the model to incorporate stochastic returns to educational investments.

We assume that with a positive probability, p, the return to investment is as in the benchmark model, implying that $u_{t+1} = 1 - e_t$, whereas, with probability $1 - p$, the probability of individual success in period t (i.i.d. across agents) is ν_t, where ν_t is drawn from a p.d.f. $f(\nu_t)$ with support on the unit interval and mean $E(\nu_t) = 1/2$. Thus, with probability $1 - p$, the investment in education has no effect on the probability of success of agents. Whether or not effort matters is revealed after benefits have been set and effort is chosen.[23]

To simplify the algebra, we assume that successful agents earn a wage equal to $1/p$ (instead of one) and that consequently, an unsuccessful agent earns b/p (instead of b). This normalization conveniently implies that the return to effort is invariant to the level of p.

[22] Similar arguments can be made for other events having triggered permanent changes in redistribution policies, such as universal suffrage, the European reconstruction after World War II or the sudden increase in the threat of communism in the early 1950's. Or, more recently, the oil shocks arguably caused a sharp increase in unemployment and, at least in Europe, an increase in the demand for unemployment benefits.

[23] Note that, had we modeled uncertainty as an additive shock to u_{t+1}, the interaction of uncertainty with private and political choices would have made the analysis substantially more involved and the model would not remain tractable. We believe, however, the main insights of this section to be robust.

Therefore, the effort choice remains unchanged, i.e. $e_t^*(b_t, b_{t+1}) = (1 + \beta - (b_t + \beta b_{t+1}))/2$. Hence, with probability p, $u_{t+1} = e_t^*(b_t, b_{t+1})$, whereas, with probability $1 - p$, $u_{t+1} = \nu_t \in [0, 1]$. The political choice is affected by the existence of stochastic shocks, though. In particular, while successful agents still prefer $b = 0$, the unsuccessful realize that the smaller p, the smaller is the distortionary cost of redistribution. Thus, uncertainty increases, *ceteris paribus*, the demand for redistribution.

Since we are interested in showing the possibility of a shock giving rise to a welfare state that later persists, we restrict the attention to pro-welfare equilibria. The following Proposition characterizes the equilibrium. The proof is a straightforward extension of the proof of Proposition 2, and is available upon request.

Proposition 4 *For all $\beta \in (0, 1)$, there exists a dense compact subset of parameters and beliefs, $(p, \theta) \in [0, 1] \times [0, \beta]$, including $p = 1$ and $\theta = 0$, that sustains the following "stochastic pro-welfare equilibrium", $\langle B^{spw}, U^{spw} \rangle$;*

$$B^{spw}(u_t) = \begin{cases} \frac{1}{p}\left(\frac{3}{2} - u_t\right) & \text{if } u_t > \bar{u}(\beta, p) \\ b_s - \frac{1}{2p-\beta} 2(u_t - u_s) & \text{if } u_t \in \left[\frac{3}{2} - \frac{2p}{2+\beta}, \bar{u}(\beta, p)\right] \\ 1 & \text{if } u_t \in \left(\frac{1}{2}, \frac{3}{2} - \frac{2p}{2+\beta}\right] \\ 0 & \text{if } u_t \leq 1/2 \end{cases},$$

$$U^{spw}(b_t) = \begin{cases} u_s + \frac{2p-\beta}{4p}(b_t - b_s) & \text{if } b_t \in \left[\frac{2\beta+4(1-p)}{2+\beta}, 1\right] \\ \frac{1+b_t}{2} & \text{if } b_t \in \left(\theta, \frac{2\beta+4(1-p)}{2+\beta}\right) \\ \frac{1-\beta+b_t}{2} & \text{if } b_t \leq \theta \end{cases},$$

where

$$\begin{aligned} u_s &\equiv u_{dp} + (1-p)\frac{4 + \beta(5+\beta)}{3(2+\beta)(1+2p)} \\ b_s &\equiv b_{dp} + (1-p)\frac{2(4+\beta)}{3(2+\beta)(1+2p)} \\ \bar{u}(\beta, p) &= \frac{6 + \beta - 2(1-p)(2p-\beta) - \sqrt{2p(2p-\beta)(\beta + 2(1-p))^2}}{2(2+\beta)}. \end{aligned}$$

The equilibrium law of motion is

$$u_{t+1} = \begin{cases} U^{spw}(B^{spw}(u_t)) & \text{with probability } p \\ \nu_t & \text{with probability } (1-p), \end{cases}$$

where ν_t is i.i.d. *with a p.d.f. $f(\nu_t)$ and $E(\nu_t) = 1/2$.*

Proposition 4 nests the equilibrium of the deterministic economies of Proposition 2. For $p = 1$, the stochastic pro-welfare equilibrium is identical to the PWE of Proposition 2. For $p < 1$, an existing pro-welfare or anti-welfare majority regenerates itself with probability $(1 + p)/2$ each period, whereas a change of majority occurs with probability $(1 - p)/2$.

4.2 Wage inequality and political support for the Welfare State.

In this section, we analyze the effects of changes in pre-tax inequality on the political equilibrium. To this end, we extend the model by allowing the wage of the successful agents, w, to differ from unity. Note that w parameterizes the degree of technological inequality – a large w implies large inequality. The absolute wage level has no effect on the equilibrium of our linear model, which justifies maintaining the normalization that the wage of the unsuccessful agents is equal to zero. Furthermore, as in section 4.1, b_t denotes the *benefit rate*, which implies that unsuccessful agents earn a before-tax income of $b_t w$, and that the constraint $b_t \in [0, 1]$ is maintained.

The optimal effort choice in this extension implies

$$u_{t+1} = 1 - e_t^* = \max\left\{\frac{2 - (1+\beta)\,w + b_t w + \beta b_{t+1} w}{2}, 0\right\},$$

where the constraint $u_{t+1} \geq 0$ is never binding if $w \leq 1$. The tax rate consistent with the balanced government budget constraint is

$$\tau_t = \frac{\max\{2 - (1+\beta)\,w + b_t w + \beta b_{t+1} w, 0\} + 2u_t}{4} w b_t.$$

It can immediately be established that the political equilibrium necessarily features a welfare state as long as $w < 1/(1+\beta)$. In this case, given any initial u_t, $u_{t+1} > 1/2$ irrespective of the redistribution policy chosen by the first generation, and the unsuccessful are always in majority from the second period and onwards. Thus, the equilibrium features a unique steady-state characterized by positive redistribution, irrespective of initial conditions or expectations.

For a range of intermediate values of w, the equilibrium features multiple steady-states, and the welfare state never arises if $u_0 \leq 1/2$, while it survives perpetually if $u_0 > 1/2$. An extension of the argument of Proposition 2 ensures that there does not exist an anti-welfare equilibrium for this intermediate range of wage inequality, i.e. a majority of unsuccessful agents do not strategically hand over power to the next generation of successful individuals. Next, for another range of intermediate, larger values of w, there exist both an equilibrium featuring the breakdown of the welfare state and one featuring its perpetual survival, as analyzed in Section 3.2.2 (recall that if $w = 1$, then there exist multiple equilibria, provided that β is sufficiently large). Finally, for an open set of sufficiently large values of w, there exists no equilibrium featuring a Welfare State for more than one period.[24]

As suggested by this discussion, the model makes predictions about the effect of technology-driven changes in wage inequality. Assume, for instance, that there is an unexpected permanent increase in the premium to education. As a result, agents increase their investment in education and a larger proportion will, *ex-post*, be opposed to redistributional policies. Thus, the initial impact of technological inequality is magnified by reduced support for the welfare state. This prediction of the model is in line with important events characterizing the last quarter of the 20th century. The skill-biased technical change that, as documented by a number of authors, started in the 1970's (see, among others, Katz and Murphy (1992)), was followed by the electoral success of conservative governments, whose political platform included a reduction of the redistributive role of governments, especially in Anglo-Saxon countries. It is interesting to observe that not all industrialized countries went through similar political changes, though. This observation is consistent with the argument of our paper for two reasons. First, we predict that multiple self-fulfilling expectations exist. As a matter of fact, the investment in education increased more in the United States than in continental Europe, which is consistent with the expectation of less future redistribution in the U.S. than in Europe.[25] Second, if other institutions (e.g., unions) compress the wage structure and prevent the productivity differences from giving rise to large wage inequalities, the investment incentives do not change significantly, and the constituency for the welfare state does not dry up in countries where these institutions are established. This can explain why countries experiencing a lower increase in pre-tax inequality also reformed their welfare state institutions less radically.

Finally, in our stylized model, all agents are ex-ante identical and there is no constraint preventing agents from making an educational investment. In reality, agents differ in both

[24] To see why the Welfare State cannot survive when w is sufficiently large, consider the case of $\beta = 0$. This is the easiest case for the survival of the Welfare State, since the old unsuccessful in majority have no strategic motive to induce the break-down of the Welfare State. In this case, if $u_0 > 1/2$, the equilibrium benefit rate is given by

$$\arg\max_b \left[b - \left(\max\left\{2 - w\left(1-b\right), 0\right\} + 2u\right) b/4\right] \geq \frac{1}{2} + \frac{1-u}{w},$$

which is decreasing in w, so that equilibrium b_0 is bounded from below by $1/2$. But, then, $u_1 = \max\left\{1 - w\left(1-b_0\right)/2, 0\right\} < 1/2$ for sufficiently large w. Hence, irrespective of the initial value of u, the majority in the subsequent periods opposes redistribution, provided that w is sufficiently large.

[25] During the period 1975-1990, the average years of secondary or higher education for the population over 25 increased by about one year in the average EU country, and by about 22 months in the United States. In the same period, the average years of higher education in the same population group increased by 2 months in the EU and by almost 8 months in the US. Source: Dataset of Barro and Lee (1993).

ability and the extent to which capital market imperfections restrain their educational choice. If we extend the model in this dimension, it is clear that low-skill or poor agents who cannot increase their educational effort are destined to suffer from both the increasing relative demand for skills and the induced loss of constituency for the welfare state.

4.3 Age-dependent policies.

So far, we have restricted the attention to subsidies to low-income agents financed by lump sum taxes. In reality, governments have access to more sophisticated instruments, allowing for more targeted intra- and inter-generational redistribution.

To address this concern, we modify the benchmark model and allow age-dependent elements in the transfer system. In particular, we assume that all living agents receive a lump sum transfer, denoted by s, financed by age-dependent taxes levied on the earnings of the rich agents.[26] Thus, the tax rate paid by the old successful ($\tau^O \in [0,1]$) and that paid by the young successful ($\tau^Y \in [0,1]$) are allowed to differ. The ex-post earnings of successful and unsuccessful agents are given by $1-\tau^a+s$ and s, respectively, where $a \in \{O,Y\}$. By the same argument used to derive equation (2), it is straightforward to show that the equilibrium investment choice of the young is now given by $e_t^*\left(\tau_t^Y, \tau_{t+1}^O\right) = \left(1+\beta-\left(\tau_t^Y+\beta\tau_{t+1}^O\right)\right)/2$.

We maintain the assumption that the government budget is balanced in each period. Thus, the budget constraint of the government implies that $2s_t = (1-u_t)\,\tau_t^O + e_t^*\left(\tau_t^Y, \tau_{t+1}^O\right)\tau_t^Y$. We assume majority voting, and maintain that only agents whose ex-post type is determined can vote. Our equilibrium definition is unchanged, except that there are two public policy rules, $\tau_t^O = T^O(u_t)$ and $\tau_t^Y = T^Y(u_t)$, and one private decision rule, $u_{t+1} = U\left(\tau_t^Y\right)$, where U is assumed to depend on τ_t^Y only. This assumption essentially maintains that expectations depend on pay-off relevant variables only, as in the benchmark case.

The political objective of the old unsuccessful is to set the two tax rates so as to maximize s_t, given the equilibrium policy function and private decision rules. Thus, if the old unsuccessful are in majority, they will choose taxes so as to solve

$$\max_{\{\tau_t^Y,\tau_t^O\}} s_t = \max_{\{\tau_t^Y,\tau_t^O\}} \left\{ \frac{(1-u_t)\,\tau_t^O}{2} + \frac{\left(1+\beta-\left(\tau_t^Y+\beta\cdot T^O\left(U\left(\tau_t^Y\right)\right)\right)\right)\tau_t^Y}{4} \right\}.$$

[26] There are alternative ways of introducing age-dependent transfers in the model. For instance, we could let agents vote on different benefit rates to the young and the old, financed by lump-sum taxes. Or, alternatively, keep benefits uniform and allow agents to vote on age-dependent lump-sum taxes. Both cases would give rise to simple, but rather uninteresting, extensions of the basic model. In the former case, the old unsuccessful would set the benefits for the young equal to zero whereas, in the latter case, they would impose the highest possible tax on the young.

Similarly, the old successful aim at maximizing $1-\tau^O+s_t$. Hence, if the old successful are in majority, taxes will be chosen so as to solve

$$\max_{\{\tau_t^Y,\tau_t^O\}}\left\{1-\tau_t^O+s_t\right\}=\max_{\{\tau_t^Y,\tau_t^O\}}\left(1-\tau_t^O+\frac{(1-u_t)\,\tau_t^O}{2}+\frac{\left(1+\beta-\left(\tau_t^Y+\beta\cdot T^O\left(U\left(\tau_t^Y\right)\right)\right)\right)\tau_t^Y}{4}\right).$$

It is immediate that since taxing the old is non-distortionary, the old unsuccessful would like to set $\tau_t^O=1$, whereas the old successful would like to set $\tau_t^O=0$. While having opposite interests on intragenerational redistribution, the two groups of old agents fully agree on the extent of intergenerational redistribution, τ_t^Y. Both groups would like to set τ_t^Y so as to be on the top of the Laffer curve, i.e., maximize the net gain from intergenerational redistribution, taking into account its distortionary effect. As we shall see, the shape of the Laffer curve depends on expectations.

The next Proposition shows that there exist self-fulfilling beliefs sustaining, respectively, equilibria with high and low redistribution.

Proposition 5 *Assume $\beta\in(0,1]$ and let $\tilde{\theta}(\beta)\equiv\frac{1}{2}+\frac{1}{2}\beta-\frac{1}{2}\sqrt{\left(2\beta+\beta^2\right)}\in(0,1/2)$. Then,*

1. *for any $\theta\in[0,\tilde{\theta}(\beta))$, there exists a "pro-welfare equilibrium" with the following characteristics:*

$$\begin{aligned}T^O(u_t)&=\begin{cases}1 & \text{if } u_t\in\left(\frac{1}{2},1\right]\\ 0 & \text{if } u_t\in\left[0,\frac{1}{2}\right]\end{cases}\\ T^Y(u_t)&=1/2>\tilde{\theta}(\beta)\\ U\left(\tau_t^Y\right)&=\begin{cases}\frac{1+\tau_t^Y}{2} & \text{if } \tau_t^Y\in(\theta,1]\\ \frac{1-\beta+\tau_t^Y}{2} & \text{if } \tau_t^Y\in[0,\theta]\end{cases}\end{aligned}$$

   This implies, given any $u_0\in[0,1]$, that $u_t=3/4$ and $\tau_t^O=1$, for all $t>0$.

2. *for any $\theta\in[\tilde{\theta}(\beta),\beta]$, there exists an "anti-welfare equilibrium" with the following characteristics:*

$$\begin{aligned}T^O(u_t)&=\begin{cases}1 & \text{if } u_t\in\left(\frac{1}{2},1\right]\\ 0 & \text{if } u_t\in\left[0,\frac{1}{2}\right]\end{cases}\\ T^Y(u_t)&=\theta\\ U\left(\tau_t^Y\right)&=\begin{cases}\frac{1+\tau_t^Y}{2} & \text{if } \tau_t^Y\in(\theta,1]\\ \frac{1-\beta+\tau_t^Y}{2} & \text{if } \tau_t^Y\in[0,\theta]\end{cases}\end{aligned}$$

   This implies, given any $u_0\in[0,1]$, that $u_t=(1-\beta+\theta)/2\leq 1/2$ and $\tau_t^O=0$, for all $t>0$.

As in the benchmark case with age-independent taxes, the equilibrium described by Proposition 5 features multiple equilibria. Unlike the benchmark case, however, there are multiple equilibria for *any* initial level of u. The reason is twofold. First, the current intragenerational conflict among the old has no effect on the private decision rules of the young. Thus, the equilibrium is entirely forward-looking, and it is of no importance whether the current majority is for or against intragenerational redistribution. Second, with age-dependent taxes, there is no interaction between the dependency ratio of the old and the distortionary effect of taxation. Hence, the policy function, $T^O(u_t)$, is piece-wise horizontal in u_t, as opposed to the PWE of section 3.2.1.

As before, dynamic voting is crucial. If agents voted myopically, the pro-welfare equilibrium would be the unique equilibrium. To see why, consider a candidate equilibrium where the old anticipate zero intragenerational redistribution in the future ($\tau_t^O = 0$ for $t > 0$). The myopic old would, in this case, set $\tau_t^Y = (1 + \beta)/2 > 1/2$, and the resulting majority of unsuccessful would then set $\tau_1^O = 1$. Thus, myopic agents cannot rationally expect zero future intragenerational redistribution, and the unique myopic equilibrium is, in fact, identical to the "pro-welfare" equilibrium of Proposition 5.

5 Conclusion

In this paper, we have analyzed the dynamics of redistribution under repeated voting, assuming agents to be fully rational and forward-looking. Following previous research, we have restricted the attention to Markov perfect equilibria. In contrast to most previous papers, however, we have provided analytical solutions. The key assumption delivering analytical solutions is that agents have linear utility in income and quadratic disutility of effort. Under this assumption, piecewise linear equilibrium policy functions can be found by a standard guess-and-verify technique.

In our model, redistribution from rich to poor agents has a distortionary effect and agents do not attach any *ex-ante* value to redistribution. Nevertheless, some agents want redistribution *ex-post*, and this can sustain welfare state institutions. Our theory therefore provides a complement to the standard explanation of the existence of a welfare state – that a government can deliver the insurance missing markets fail to provide.

We have analyzed two types of equilibria. In the former, if a pro-welfare majority initially exists, the policy distorts private investment decisions in a way that regenerates political support for redistribution. In the latter case, even an initial pro-welfare majority induces the termination of subsidy programs in finite time.

This paper has focused on simple majority voting as the political mechanism delivering persistent political failure. In this setting, all political decision power rests in the hands of the majority. In some related work in progress (Hassler et al., 2001), alternative political mechanisms, featuring more political influence for minorities, are explored. In addition, agents are risk-averse, allowing for an insurance motive in redistribution programs. In future work, we plan to extend our model to infinite horizon settings and investigate whether the political mechanism can amplify technological shocks and generate persistence at business cycles frequencies.

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Appendix. Proofs.

For notational convenience, the indirect utilities are, in this section, rewritten as follows;

$$\begin{aligned}\hat{V}^j(b_t,u_t) &\equiv V^j(b_t,B(U(b_t)),u_t),\ \text{for } j\in\{os,ou\},\\ \hat{V}^y(b_t,u_t) &\equiv V^y(b_t,B(U(b_t)),B(U(B(U(b_t)))),u_t).\end{aligned}$$

Proposition 1

Proof. We must show that the pair $\langle B^i, U^i\rangle$, for $i\in\{pl,dp\}$, satisfies the equilibrium conditions

1) $B^i(u_t)=\arg\max_{b_t}\hat{V}^d(b_t,u_t)$ subject to $b_t\in[0,1]$; and

2) $U^i(b_t)=\left(1-\beta+b_t+\beta B^i\left(U^i(b_t)\right)\right)/2$,

where $d=os$ if $i=pl$ and $d=ou$ if $i=dp$.

If $i=pl$, it is straightforward to see that $\hat{V}^{os}$ is maximized by setting $b_t=0$ in every period and that, consequently, $u_t=\frac{1-\beta}{2}$ for all $t\geq 1$.

Next, consider the DP equilibrium.

First, we note that

$$B^{dp}\left(U^{dp}(b_t)\right)=\begin{cases}\frac{3b_{dp}-b_t}{2} & \text{if}\quad b_t\geq\frac{2\beta}{2+\beta}\\ 1 & \text{else}\end{cases},\tag{12}$$

where we used the fact that for any $b_t\in[0,1]$, $U^{dp}(b_t)<\bar{u}$. Then, $\hat{V}_t^{ou}$ can be expressed as;

$$\hat{V}_t^{ou}(b_t,u_t)=\begin{cases}b_t-\frac{1}{4}\left(1-\beta+b_t+\beta\frac{3b_{dp}-b_t}{2}+2u_t\right)b_t & \text{if}\quad b_t\geq\frac{2\beta}{2+\beta}\\ b_t-\frac{1}{4}(1+b_t+2u_t)\,b_t & \text{else}\end{cases}.\tag{13}$$

Maximizing $\hat{V}_t^{ou}$ over b_t yields;

$$b_t=\begin{cases}\frac{3}{2}-u_t & \text{if } u_t>\bar{u}(\beta)\\ b_{dp}-\frac{2}{2-\beta}(u_t-u_{dp}) & \text{if } u_t\in\left[\frac{3}{2}-\frac{2}{2+\beta},\bar{u}(\beta)\right]\\ 1 & \text{if } u_t\in\left[0,\frac{3}{2}-\frac{2}{2+\beta}\right]\end{cases}=B^{dp}(u_t).$$

This proves that equilibrium condition 1 is satisfied.

To prove that the second condition is satisfied, we use (12) to substitute for b_{t+1} in the optimal investment expression, giving,

$$\left(1-\beta+b_t+\beta B^{dp}\left(U^{dp}(b_t)\right)\right)/2=\begin{cases}u_{dp}+\frac{2-\beta}{4}(b_t-b_{dp}) & \text{if}\quad b_t\in\left[\frac{2\beta}{2+\beta},1\right]\\ \frac{1+b_t}{2} & \text{else}\end{cases}=U^{dp}(b_t).$$

To see the steps of the first part of the proof in more detail, define $\hat{V}^a(u_t)$ and $\hat{V}^b(u_t)$ as follows;

$$\begin{aligned}\hat{V}^a(u_t) &\equiv \max_{b_t\in\left[0,\frac{2\beta}{2+\beta}\right]} \hat{V}_t^{ou}(b_t,u_t)\\ &= \begin{cases} \frac{9}{16}-\frac{3}{4}u_t+\frac{1}{4}u_t^2 \equiv \hat{V}^{a,int}(u_t) & \text{if} \quad u_t > \frac{6-\beta}{2(2+\beta)}\\ \beta\frac{6+\beta-2u_t(2+\beta)}{2(2+\beta)^2} \equiv \hat{V}^{a,cor}(u_t) & \text{else}\end{cases}\\ \hat{V}^b(u_t) &\equiv \max_{b_t\in\left[\frac{2\beta}{2+\beta},1\right]} \hat{V}_t^{ou}(b_t,u_t)\\ &= \begin{cases} \frac{1}{8}\frac{\left(\beta^2-3\beta+2u_t(2+\beta)-6\right)^2}{(2-\beta)(2+\beta)^2} \equiv \hat{V}^{b,int}(u_t) & \text{if} \quad u_t \geq \frac{3}{2}-\frac{2}{(2+\beta)}\\ \frac{1}{8}\frac{8+\beta(6-\beta)}{2+\beta}-\frac{u_t}{2} \equiv \hat{V}^{b,cor}(u_t) & \text{else,}\end{cases}\end{aligned}$$

where $\hat{V}^{a,cor}(u_t)$ and $\hat{V}^{b,cor}(u_t)$ result from corner solutions in the respective ranges (the corners being $b_t=\frac{2\beta}{2+\beta}$ and $b_t=1$, respectively) while $\hat{V}^{a,int}(u_t)$ and $\hat{V}^{b,int}(u_t)$ result from the interior solutions $b_t=3/2-u_t$ and $b_t=b_{dp}-\frac{2}{2-\beta}(u_t-u_{dp})$, respectively.

First, standard algebra establishes that $\hat{V}^{b,int}(u_t)-\hat{V}^{a,cor}(u_t)=\frac{1}{8}\frac{\left(\beta^2-2\beta u_t-\beta+6-4u_t\right)^2}{(2-\beta)(2+\beta)^2}>0$ and that, in the range where $u_t\leq\frac{3}{2}-\frac{2}{(2+\beta)}$, $\hat{V}^{b,cor}(u_t)-\hat{V}^{a,cor}(u_t)>\frac{1}{8}(2-\beta)\frac{4(1-\beta)+\beta^2}{(2+\beta)^2}>0$. Thus, whenever $\hat{V}^a(u_t)=\hat{V}^{a,cor}(u_t)$, then $\hat{V}^b(u_t)>\hat{V}^a(u_t)$.

Second, if $\beta<\frac{2}{3}$, then $\frac{6-\beta}{2(2+\beta)}>1$ and $\hat{V}^a(u_t)=\hat{V}^{a,cor}(u_t)$ for all u_t. Thus, $\hat{V}^b(u_t)>\hat{V}^a(u_t)$ if $\beta<2/3$.

Third, note that if $\beta\geq 2/3$, then there exists a range of u_t, where $\hat{V}^a(u_t)=\hat{V}^{a,int}(u_t)$. In this range, standard algebra establishes that $\hat{V}^{b,int}(u_t)>\hat{V}^{a,int}(u_t)$ for all u_t provided that $\beta<\left(\sqrt{17}-1\right)/4$. Thus, $\beta<\left(\sqrt{17}-1\right)/4$ implies that $\hat{V}^b(u_t)>\hat{V}^a(u_t)$ for all $u_t\in[0,1]$.

Finally, consider the range of parameters such that $\beta\geq\left(\sqrt{17}-1\right)/4$, implying $\bar{u}\leq 1$. In this case, for all $1\geq u_t>\bar{u}(\beta)$, $\hat{V}^b(u_t)<\hat{V}^a(u_t)=\hat{V}^{a,int}(u_t)$. ■

Proposition 2

Proof. We must show that for all t and u_t, $\langle B^{pw},U^{pw}\rangle$ satisfies the two equilibrium conditions

1) $B^{pw}(u_t)=\arg\max_{b_t}\left\{\hat{V}^{pw}(b_t,u_t)\right\}$, subject to $b_t\in[0,1]$; and

2) $U^{pw}(b_t)=(1-\beta+b_t+\beta B^{pw}(U^{pw}(b_t)))/2$,

where $\hat{V}^{pw}(b_t,u_t)=\hat{V}^{os}(b_t,u_t)$ if $u_t\leq 1/2$ and $\hat{V}^{pw}(b_t,u_t)=\hat{V}^{ou}(b_t,u_t)$ otherwise.

We start from condition 1. Consider first $u_t\leq 1/2$. Then, $V_t^{pw}(b_t,u_t)=V_t^{os}(b_t,u_t)$, which is maximized by setting $b_t=0$.

Then, consider $u_t > 1/2$. First, we define

$$\bar{\theta}(\beta) \equiv \min\left\{\beta, \tilde{\theta}(\beta)\right\},$$

where

$$\tilde{\theta}(\beta) \equiv \begin{cases} \frac{1+\beta}{2} - \frac{\sqrt{\beta(2+\beta)}}{2}, \text{ if } \beta > \frac{\sqrt{17}-4}{4}, \\ \frac{1+\beta}{2} - \frac{\sqrt{\beta(6+\beta)}}{2}\frac{1+\beta}{2+\beta} \text{ if } \beta \leq \frac{\sqrt{17}-4}{4}. \end{cases} \tag{14}$$

Now, note that $\theta \leq \bar{\theta}(\beta) \leq \beta$, implying that $U^{pw}(b_t) \leq (>) 1/2$ if $b_t \leq (>)\ \theta$. Thus,

$$B^{pw}(U^{pw}(b_t)) = \begin{cases} \frac{3b_{dp}-b_t}{2} & \text{if} \quad b_t \geq \frac{2\beta}{2+\beta} \\ 1 & b_t \in \left(\theta, \frac{2\beta}{2+\beta}\right) \\ 0 & b_t \leq \theta \end{cases}, \tag{15}$$

and

$$\begin{aligned} &\hat{V}_t^{ou}(b_t, u_t) \\ = &\begin{cases} b_t - \frac{1}{4}\left(1 - \beta + b_t + \beta\frac{3b_{dp}-b_t}{2} + 2u_t\right)b_t & \text{if} \quad b_t \geq \frac{2\beta}{2+\beta} \\ b_t - \frac{1}{4}(1 + b_t + 2u_t)\, b_t & b_t \in \left(\theta, \frac{2\beta}{2+\beta}\right) \\ b_t - \frac{1}{4}(1 - \beta + b_t + 2u_t)\, b_t & b_t \leq \theta \end{cases}. \end{aligned}$$

Comparing this to (13), we see that utility in the PWE is identical to utility under DP for $b_t > \theta$. Thus, a sufficient condition for equilibrium condition 1 to be satisfied for $u_t > 1/2$ is that in this range, $\max_{b_{t>\theta}} \hat{V}_t^{ou}(b_t, u_t) \geq \max_{b_{t\leq\theta}} \hat{V}_t^{ou}(b_t, u_t)$. It is straightforward to verify that this is the case for sufficiently low θ, in particular when $\theta \leq \tilde{\theta}(\beta)$, which is implied by $\theta \leq \bar{\theta}(\beta)$.

To prove that the second condition is satisfied, we use (15) to substitute for b_{t+1} in the optimal investment expression, giving,

$$\begin{aligned} &(1 - \beta + b_t + \beta B^{pw}(U^{pw}(b_t)))/2 \\ = &\begin{cases} \left(1 - \beta + b_t + \beta\frac{3b_{dp}-b_t}{2}\right)/2 & \text{if} \quad b_t \geq \frac{2\beta}{2+\beta} \\ (1 + b_t)/2, & b_t \in \left(\theta, \frac{2\beta}{2+\beta}\right) \\ \frac{1-\beta+b_t}{2} & b_t \leq \theta \end{cases}, \\ = &\ U^{pw}(b_t), \end{aligned}$$

where we used $\theta \leq \bar{\theta}(\beta) \leq \beta$.

To see the steps in the proof of the first condition in more detail, we first note that $\theta \leq \beta$, implies that $\max_{b_{t\leq\theta}} \hat{V}_t^{ou}(b_t, u_t)$ is a corner solution with $b_t = \theta$, and utility equal to

$\frac{3+\beta-\theta-2u_t}{4}\theta$. It is easily verified that the difference $\max_{b_t>\theta}\hat{V}_t^{ou}(b_t,u_t)-\max_{b_t\leq\theta}\hat{V}_t^{ou}(b_t,u_t)$ is minimized at $u_t=1$, where it is given by

$$\begin{cases} \frac{1}{16}-\frac{1}{4}\theta\left(3+\beta-(2+\theta)\right) & \text{if } \beta>\frac{\sqrt{17}-4}{4} \\ \frac{1}{8}\left(1+\beta\right)\frac{2+\beta(1-\beta)}{(2+\beta)^2}-\frac{1}{4}\theta\left(3+\beta-(2+\theta)\right) & if\ \beta\leq\frac{\sqrt{17}-4}{4}. \end{cases}$$

Solving $\max_{b_t>\theta}\hat{V}_t^{ou}(b_t,u_t)-\max_{b_t\leq\theta}\hat{V}_t^{ou}(b_t,u_t)=0$ for θ, yields (14). ∎

Proposition 3

Proof. As in the proof of Proposition 1, we must show that, for all t and u_t, $\langle B^{aw},U^{aw}\rangle$ satisfies

1) $B^{aw}(u_t)=\arg\max_{b_t}\left\{\hat{V}^{aw}(b_t,u_t)\right\}$, subject to $u_{t+1}=U^{aw}(b_t)$, $b_t\in[0,1]$ and $b_{t+1}=B^{aw}(u_{t+1})$; and

2) $U^{aw}(b_t)=\left(1-\beta+b_t+\beta B^{aw}\left(U^{aw}(b_t)\right)\right)/2$.

1. Consider first the case when $\beta\geq\frac{\sqrt{5}-1}{2}$ and $\theta\in\left[\underline{\theta}(\beta),\beta\right]$. Note first that in the range $\beta\geq\frac{\sqrt{5}-1}{2}$, $\underline{\theta}(\beta)\leq\beta$, ensuring that the set of beliefs under consideration is non-empty. To prove (1), consider, first, the range where $u_t>1/2$.

$$\hat{V}^{aw}(b_t,u_t)=\hat{V}^{ou}(b_t,u_t)=\begin{cases} b_t-\frac{1}{4}\left(1-\beta+b_t+\beta\theta+2u_t\right)b_t & \text{if}\quad b_t>\theta \\ b_t-\frac{1}{4}\left(1-\beta+b_t+2u_t\right)b_t & \text{if}\quad b_t\leq\theta \end{cases}.$$

Standard differentiation shows that $\hat{V}^{aw}(b_t,u_t)$ is increasing in b_t for all $b_t\leq\theta$ (since $\theta\leq\beta$) and that, as long as $\theta\geq\underline{\theta}(\beta)$,

$$\hat{V}^{aw}(\theta,u_t)=\theta-\frac{1}{4}\left(1-\beta+\theta+2u_t\right)\theta>b_t-\frac{1}{4}\left(1-\beta+b_t+\beta\theta+2u_t\right)b_t=\hat{V}^{aw}(b_t>\theta,u_t),$$

for all $b_t>\theta$ and $u_t>1/2$. Thus, $B^{aw}(u_t)=\theta$ for $u_t>1/2$.
If $u_t\leq1/2$, $\hat{V}^{aw}(b_t,u_t)=\hat{V}^{os}(b_t,u_t)$, which is decreasing in b_t. Hence, $B^{aw}(u_t)=0$, for $u_t\leq1/2$.
To prove part (2), observe that

$$\begin{aligned} &\left(\left(1-\beta+b_t+\beta B^{aw}\left(U^{aw}(b_t)\right)\right)\right)/2 \\ =&\begin{cases} \left(1-\beta+b_t+\beta\theta\right)/2 & \text{if}\quad b_t>\theta \\ \left(1-\beta+b_t\right)/2 & \text{if}\quad b_t\leq\theta \end{cases}=U^{aw}(b_t), \end{aligned}$$

where the equality follows from the facts that, for all $b_t\leq\theta\leq\beta$, $\left(1-\beta+b_t\right)/2\leq1/2$, and, for all $b_t>\theta\geq\underline{\theta}(\beta)$, $\frac{1}{2}\left(1-\beta+\beta\theta+b_t\right)>\frac{1}{2}$. The latter inequality can be checked by inserting the definition of $\underline{\theta}(\beta)$ in the left hand-side of the inequality.

2. Consider now the case when $\beta \geq \underline{\beta}$ and $\theta \in \left[\underline{\beta}, \min\left\{\beta, \underline{\theta}(\beta)\right\}\right)$. Note first that $\underline{\beta} < \underline{\theta}(\beta)$ for all $\beta \geq \underline{\beta}$, implying that the set of beliefs under consideration is non-empty.

As to part (1), consider, first, the range where $u_t > 1/2$.

$$\begin{aligned}\hat{V}^{aw}(b_t, u_t) &= \hat{V}^{ou}(b_t, u_t) \\ &= \begin{cases} b_t - \frac{1}{4}(1 - \beta + b_t + \beta\theta + 2u_t)\, b_t & \text{if } b_t > \theta \\ b_t - \frac{1}{4}(1 - \beta + b_t + 2u_t)\, b_t & \text{if } b_t \leq \theta \end{cases}.\end{aligned}$$

It is immediate to see that the value function has a discontinuous fall at $b_t = \theta$. Moreover, standard differentiation shows that $\hat{V}^{aw}(b_t, u_t)$ is increasing in b_t, throughout in the region $b_t \leq \theta$, and provided that $u_t \geq \hat{u}(\beta, \theta)$ in the region $b_t > \theta$. Furthermore,

$$\hat{V}^{aw}(1, u_t) - \hat{V}^{aw}(\theta, u_t) = \left(1 - \frac{1}{4}(1 - \beta + 2u_t)\right)(1 - \theta) - \frac{1}{4}\left(1 + \beta\theta - \theta^2\right) \quad (16)$$

is a decreasing function of u_t, strictly positive for $u_t \in (0.5, \hat{u}(\beta, \theta))$, equal to zero when $u_t = \hat{u}(\beta, \theta)$, and strictly positive for $u_t \in (\hat{u}(\beta, \theta), 1]$. Thus, in the range $u_t \in (0.5, \hat{u}(\beta, \theta)]$, $\hat{V}^{aw}(1, u_t) \geq \hat{V}^{aw}(\theta, u_t)$, with equality holding if and only if $u_t = \hat{u}(\beta, \theta)$. This shows that setting $b_t = 1$ is optimal for the old unsuccessful in the range $u_t \in (0.5, \hat{u}(\beta, \theta)]$ and, hence, $B^{aw}(u_t) = 1$ in that range.

Finally, we need to show that setting $b_t = \theta$ is optimal for the old unsuccessful in the range $u_t \in (\hat{u}(\beta, \theta), 1]$. Since, as already noted, $\hat{V}^{aw}(b_t, u_t)$ is increasing in b_t for all $b_t \leq \theta$, it remains to be shown that $\hat{V}^{aw}(\theta, u_t) > \max_{b \in (\theta, 1]}\left\{\hat{V}^{aw}(b, u_t)\right\}$ when $u_t \in (\hat{u}(\beta, \theta), 1]$. This can be shown as follows.

(a) First, note that if $u_t \in \left(\hat{u}(\beta, \theta), \frac{1}{2} + \frac{\beta(1-\theta)}{2}\right]$, then, $\arg\max_{b \in (\theta, 1]}\left\{\hat{V}^{aw}(b, u_t)\right\} = 1$. In this case, as pointed out above (see equation (16) and the following discussion), $\hat{V}^{aw}(\theta, u_t) > \hat{V}^{aw}(1, u_t)$, establishing the claim.

(b) Next, if $u_t \in \left[\frac{1}{2} + \frac{\beta(1-\theta)}{2}, 1\right]$, then $b^*(\theta, u_t) \equiv \arg\max_{b \in (\theta, 1]}\left\{\hat{V}^{aw}(b, u_t)\right\} = \frac{3}{2} + \frac{\beta(1-\theta)}{2} - u_t < 1$. Define, then,

$$\begin{aligned}\Delta\hat{V}(u_t, \beta, \theta) &\equiv \hat{V}^{aw}(\theta, u_t) - \hat{V}^{aw}(b^*(\theta, u_t), u_t) \\ &= \left(-\frac{1}{2}\theta + \frac{1}{4}\beta + \frac{3}{4} - \frac{1}{4}\beta\theta\right)u_t - \frac{1}{4}u_t^2 \\ &\quad + \frac{3}{4}\theta + \frac{5}{8}\beta\theta - \frac{1}{4}\theta^2 - \frac{1}{16}\beta^2 - \frac{9}{16} - \frac{3}{8}\beta - \frac{1}{16}\beta^2\theta^2 + \frac{1}{8}\beta^2\theta.\end{aligned}$$

Since $\Delta\hat{V}(u_t, \beta, \theta)$ is hump-shaped in u_t, it must attain its minimum in the range

$u_t \in \left[\frac{1}{2}+\frac{\beta(1-\theta)}{2},1\right]$ at either $u_t=\frac{1}{2}+\frac{\beta(1-\theta)}{2}$ or $u_t=1$. It turns out that

$$\begin{aligned}
\Delta\hat{V}\left(\frac{1}{2}+\frac{\beta(1-\theta)}{2},\beta,\theta\right) &= \frac{1}{4}\left(-1+2\theta-\theta^2(1-\beta)\right)\geq\frac{1}{4}\left(-1+2\underline{\beta}-\underline{\beta}^2(1-\beta)\right)\geq 0\\
\Delta\hat{V}(1,\beta,\theta) &= \frac{1}{4}\left(-\left(1+\frac{1}{4}\beta^2\right)\theta^2+\frac{1}{2}(2+\beta)(1+\beta)\theta-\frac{1}{4}(1+\beta)^2\right)\\
&\geq \frac{1}{4}\left(-\left(1+\frac{1}{4}\beta^2\right)\underline{\beta}^2+\frac{1}{2}(2+\beta)(1+\beta)\underline{\beta}-\frac{1}{4}(1+\beta)^2\right)\geq 0.
\end{aligned}$$

Thus, $B^{aw}(u_t)=\theta$ for $u_t>\hat{u}(\beta,\theta)$.

The proof for the range $u_t\leq 1/2$ is identical to the first case and thus, part (1) is proved.

To prove part (2), observe that

$$\begin{aligned}
&\left(1-\beta+b_t+\beta B^{aw}\left(U^{aw}(b_t)\right)\right)/2\\
=&\begin{cases}(1-\beta+b_t+\beta\theta)/2 & \text{if} \quad b_t>\theta\\ (1-\beta+b_t)/2 & \text{else}\end{cases} = U^{aw}(b_t)
\end{aligned}$$

where the equality follows from the fact that $(1-\beta+b_t)/2<1/2$ for all $b_t<\theta$, and that $(1-\beta+b_t+\beta\theta)/2\geq\hat{u}(\beta,\theta)$ for all $b_t\in(\theta,1]$. The latter can be checked as follows. Recall that, in the range under consideration, $\beta\geq\underline{\beta}$ and $\theta\geq\underline{\beta}$. Then:

$$\begin{aligned}
(1-\beta+b_t+\beta\theta)/2-\hat{u}(\beta,\theta) &> (1-\beta+\theta+\beta\theta)/2-\hat{u}(\beta,\theta)\\
&\geq \left(-1-2\beta+(3+4\beta)\underline{\beta}-(2+\beta)\underline{\beta}^2\right)\frac{1}{2(1-\theta)}\\
&\geq \left(-1-2\underline{\beta}+\left(3+4\underline{\beta}\right)\underline{\beta}-\left(2+\underline{\beta}\right)\underline{\beta}^2\right)\frac{1}{2(1-\theta)}>0
\end{aligned}$$

■

Proposition 5

Proof. The characterization of $T^O(u_t)$ is proved in the text. To determine $T^Y(u_t)$, observe that both the old successful and the old unsuccessful wish to set τ_t^Y so as to maximize net transfers from the young, $R\left(\tau_t^Y\right)\equiv\left(1+\beta-\left(\tau_t^Y+\beta\cdot T^O\left(U\left(\tau_t^Y\right)\right)\right)\right)\tau_t^Y/4$. Conditional on $T^O\left(U\left(\tau_t^Y\right)\right)=1$, the optimal choice is $\tau_t^Y=1/2$. This choice induces $u_{t+1}=3/4$ and the expectation of $\tau_t^O=1$ is, therefore, fulfilled. The net transfers from the young is then given by $R(1/2)=\left(1+\beta-\left(\frac{1}{2}+\beta\right)\right)/8$. Consider, next, the alternative of setting $\tau_t^Y=h\in[0,\theta]$, where rational expectations impose that $\theta\leq\beta$. This choice induces $u_{t+1}=(1-\beta+h)/2\leq 1/2$, and the expectation of $\tau_t^O=0$ is, therefore, also fulfilled.

The value of choosing h is given by $R(h) = (1+\beta-h)\,h/4$. Further, since $R'(h) > 0$ for all $h < \beta$, and $\theta \leq \beta$, then, $R(\theta) > R(h)$ for all $h \in [0,\theta)$. The utility of choosing $\tau_t^Y = \theta$ is $R(\theta) = (1+\beta-\theta)\,\theta/4$. Finally, the median voter compares $R(1/2)$ with $R(\theta)$, and chooses $\tau_t^Y = 1/2$ if and only if

$$R(1/2) = \frac{\left(1+\beta-\left(\frac{1}{2}+\beta\right)\right)}{8} > \frac{(1+\beta-\theta)\,\theta}{4} = R(\theta) \text{ and } \theta \leq \beta,$$

which is satisfied if and only if $\theta < \frac{1}{2}+\frac{1}{2}\beta-\frac{1}{2}\sqrt{(2\beta+\beta^2)} \equiv \tilde{\theta}(\beta)$. Else, she chooses $\tau_t^Y = \theta$.

■

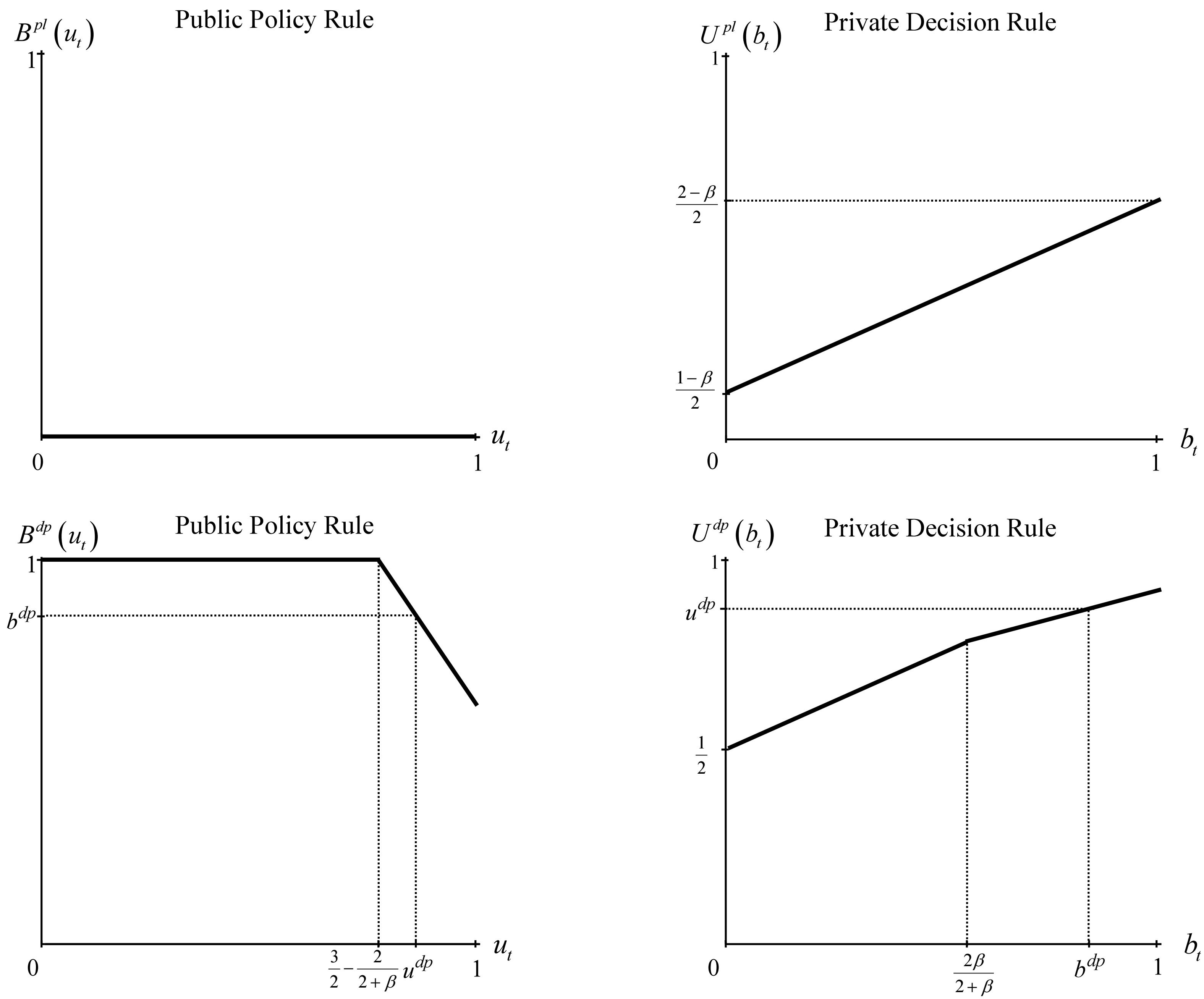

Figure 1. Public policy rule and private decision rule under Plutocracy (upper panels) and Dictatorship of Proletariat (lower panels)

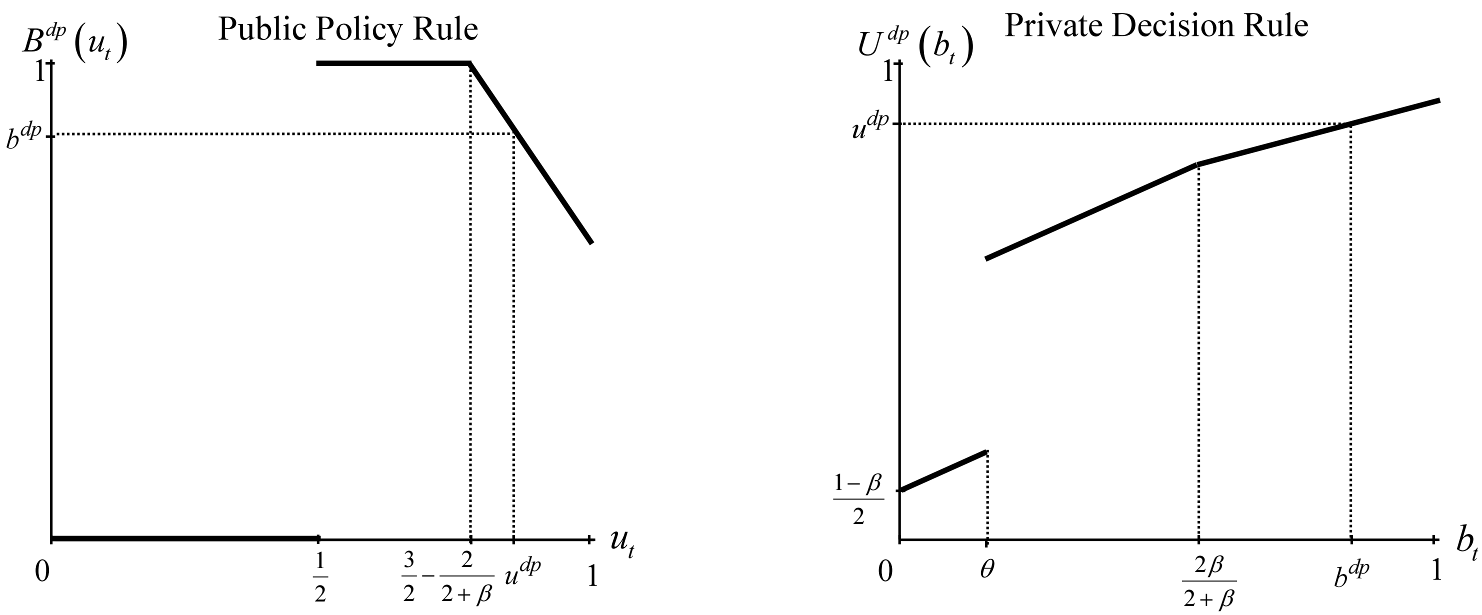

Figure 2. Public policy rule and private decision rule under majority voting: Pro-Welfare Equilibrium. $\beta = 0.75$.

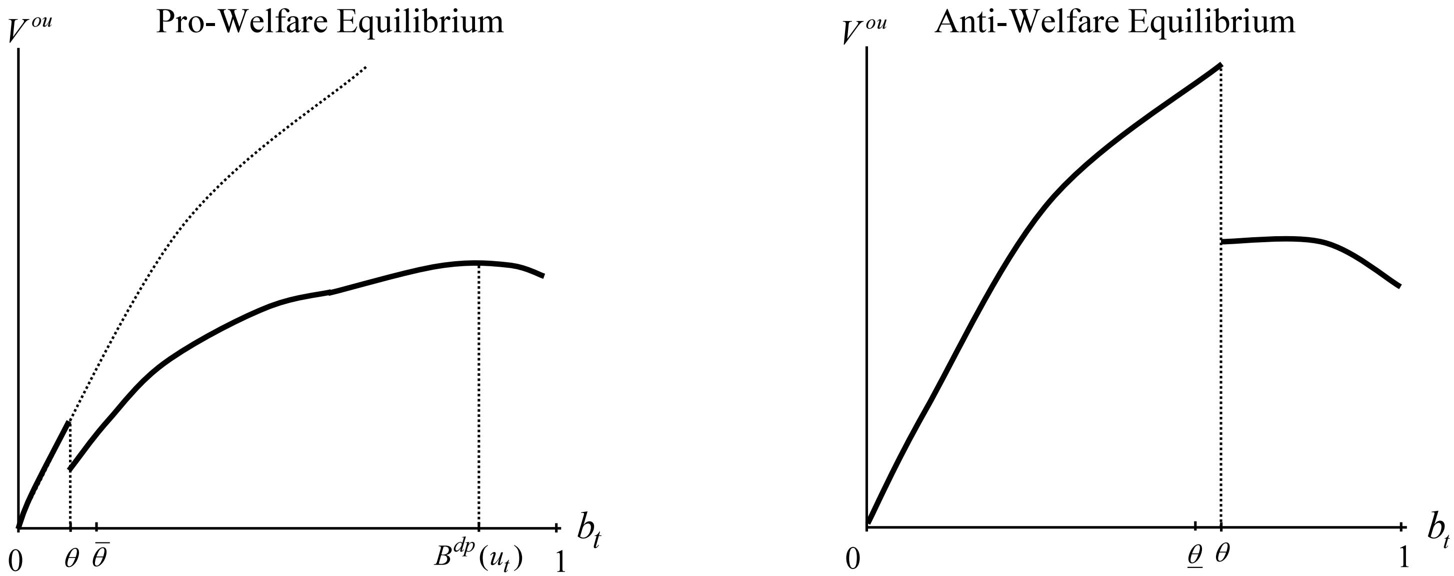

Figure 3. Indirect utility of the old unsuccessful, $V^{ou} = V^{ou}(b_t, B(U(b_t)), u_t)$ under PWE and AWE. $\beta = 0.75$ and $u_t = u^{dp}$.

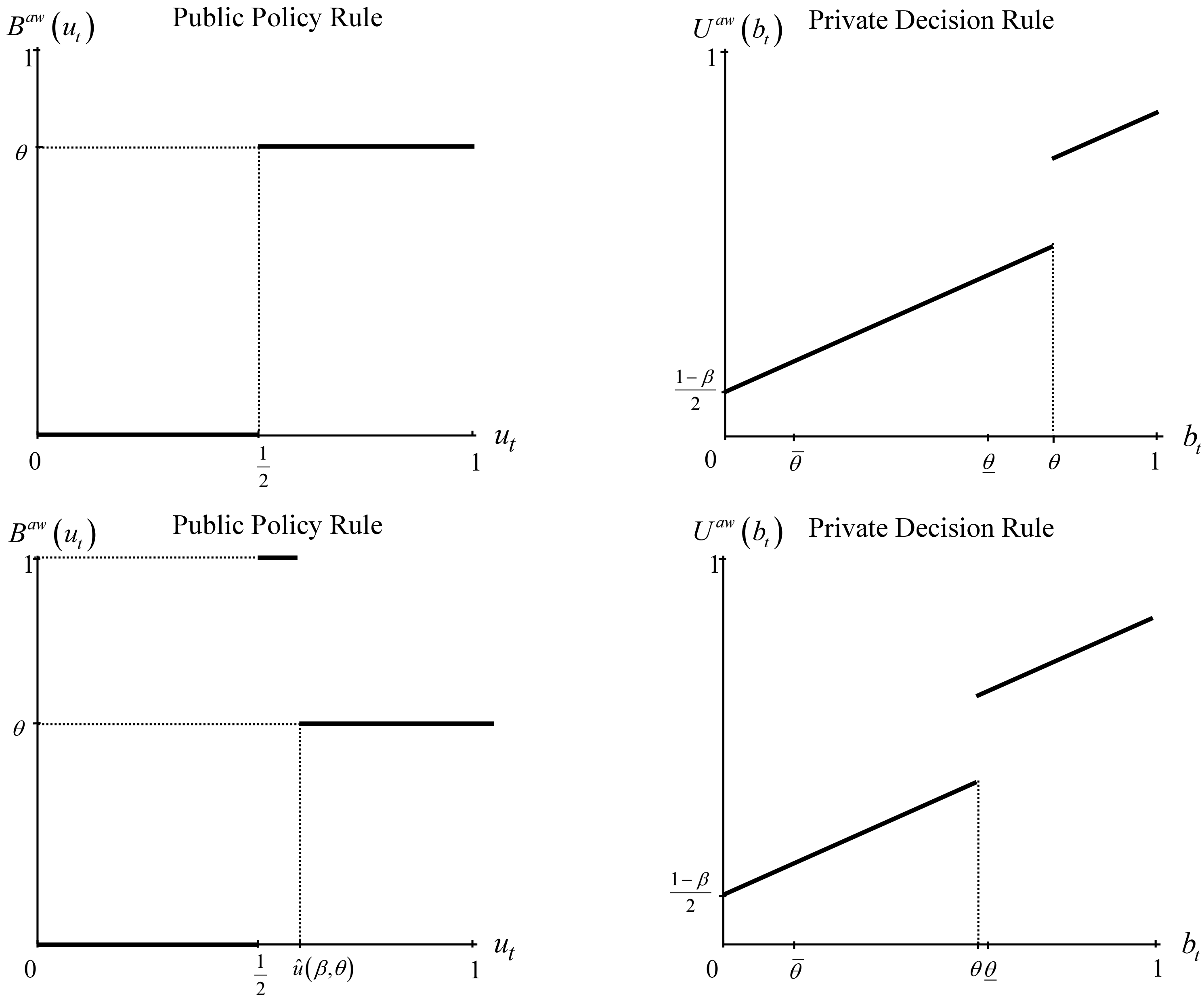

Figure 4. Public policy rule and private decision rule under majority voting: Anti-Welfare Equilibrium with switch in one period (upper panels, $\theta = 0.75$) and with switch in two periods (lower panels, $\theta = 0.58$). $\beta = 0.75$.