

Long Run Relationships and Price Rigidity *

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Abstract

I study a repeated buyer-seller relationship for the exchange of a given good. Asymmetric information over the buyer's reservation price, which is subject to random shocks, may lead the seller to use a *rigid* pricing policy despite the possibility of making higher profits through price discrimination across the different states of the buyer's reservation price. The existence of a *flexible* price subgame perfect equilibrium is shown for buyers *sufficiently* locked-in.

When the seller faces a population of buyers whose degree of involvement in the relationship is unknown, the flexible price equilibrium is not necessarily optimal. Thus typically the seller will prefer to use the rigid price strategy. A learning process allowing the seller to screen the population of buyers is derived and the existence of a *switching* point between the two regimes (i.e. price rigidity and price flexibility) is shown.

Keywords: Industrial Organization; Transactional Relationships; Contracts and Reputation; Market Structure and Pricing.

JEL Classification: L10, L14, D4.

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1 Introduction

What are the determinants of a long-lasting bilateral relationship? A very general answer would be *mutual trust*, *reciprocal fairness* and something like *fairplay*. The idea of this work is to relate this insight to the behavior of prices in a buyer-seller relationship. More specifically: to understand whether and to what extent price rigidities can be attributed to the particular characteristics of a given buyer-seller *match*.

Any theory of price rigidities should be compared with the existing literature to understand what, if anything, it adds to the current state of knowledge. Indeed although the issue of the existence of price rigidities has received a great deal of attention in the empirical literature [3], [11], [4] only in relatively recent years, research has focussed on the construction of solid microeconomic foundations to explain this evidence (*menu cost* [8], costly search [9], oligopoly supergames [10], [6]). A common factor in these explanations, is the fact that the relationship between the parties involved in the transaction for the exchange of goods (firms-customers, buyers-sellers) is *given* and does not change over the time. In other words if the parties trade together for long periods of time, it is natural to expect that some aspects of the relationship will change. For instance if a seller faces a buyer that shows to be reliable, it may be the case that he will try to improve their relationship, for example allowing trade credit. Alternatively the improvement could be in the direction of a better pricing policy. Following this insight the idea that is pursued here is that price rigidity has to do with the characteristics of the particular buyer-seller *match*. By this I mean the degree of reciprocal trust that a buyer and a seller share. The intuition is the following. Suppose that a seller faces buyers who purchase repeatedly from him, and may have variable need of the good. As an example one may take a vertical buyer-seller relationship where for instance the seller is the supplier of raw materials that the buyer will transform in some finite product. Buyer's need of the seller's supply may vary according to the strength of the demand he faces or to the availability of different sources of supply. Further, suppose the buyer is not aware of the cost the seller bears to supply the good, this may vary depending on technical conditions as well as on the availability of different alternative buyers. In other words if the exchange price is the result of some bargaining process, then it will depend both on objective, technical conditions *and* on other facts, as for instance the extent to which the two parties depend on each other to carry on their business (as in [7]). Clearly as long as these conditions are

not easily verifiable, one of the parties may take advantage of his superior information to obtain a better deal in the bargaining process. For instance if prices vary according to the buyer's willingness to pay, it is in his interest to claim poor demand conditions and/or the existence of better outside opportunities, in order to obtain more favorable conditions. Equivalently, if the seller has more bargaining power, he could claim high production costs and/or the availability of wealthier buyers to push up the final price. The final result could be mutual distrust leading to a fixed price agreement that insures each party that the other won't exploit him owing to his superior information. Therefore even if gains could be made through negotiation of state dependent prices, lack of mutual trust would prevent their realization. What does a long term relationship add to this situation? Probably, the more the parties deal with each other, the better they get to know the aspects of the relationship that are each one's private knowledge. This has two effects: on the one hand it reduces the power of unverifiable claims since for instance a buyer's request of a price reduction due to a *false* lack of demand would not be trusted by the seller; on the other hand it allows a better pricing strategy, since for example if the seller would be able to know exactly the buyer's willingness to pay, he would be able to price discriminate across the various state of his final demand. Equivalently, knowledge of the technical conditions under which production is carried out, would allow the buyer to match price increases and to (probably) ask for price reductions in the right moments.

Empirically, studies concerned with the relationship between the length of the *match* and its effect on the behavior of prices do not abound. The problem is that such studies require a huge amount of information, which is usually not available. Stigler and Kindahl [11] collected a data set containing information on monthly buyer-seller transaction for different categories of intermediate products used in manufacturing obtained through direct interview of 227 buyers chosen among the firms in the Fortune 500 for the period between January 1, 1957 and December 31, 1966. This rich source of information allowed D.W. Carlton [3] to examine more closely the issue of the degree of price rigidity in different industries. Moreover the specific nature of the data set permitted the study of interesting types of correlations such as between price rigidity and the length of buyer-seller association (i.e. the number of periods a given buyer-seller match trades together), or of price changes across different buyers. The result of his study (see Tabel 1) outline the existence of a negative relationship between the length of buyer-seller relationship and

the degree of price rigidity (measured as the size of the frequency of price variation along the relationship). In other words the longer the relationship

Table 1: The relationship between length of buyer-seller association and price rigidity

Product	Length of association and rigidity
Cement	.28 ^a
Chemicals	.16
Glass	-.11
Household appliances ^b	-.87 ^a
Nonferrous metals	.12
Paper	.03
Petroleum	-.25 ^a
Plywood	.10
Rubber tires	-.08
Steel	.03
Truck motors	-.56 ^a
<i>a. Statistically significant at 1%</i>	
<i>b. Based on 11 observations</i>	

the more flexible are the posted prices. To support this empirical evidence Carlton provides some possible theoretical explanations. For instance:

The impediment to changing price may be that the buyer or the seller may feel the other side is taking advantage of him.

and also:

If buyers and sellers know each other well because of their long-standing relationship, this fear of being taken advantage of in the short run will be reduced. In such a case flexible prices may emerge.

The idea, in the present paper, is to develop a formal model to capture these empirical findings.

Consider a repeated buyer-seller relationship with asymmetric information where, at the beginning of each trading period, a buyer privately observes his reservation price (which can be high or low with equal probabilities), and then, given the price set by the seller, decides whether or not to buy *one* unit

of a given good. If the seller would be able to observe the buyer's willingness to pay, he would price in a way to extract all the buyer's surplus. However the lack of this information leads him to choose a constant high price so that in the static Bayesian-Nash equilibrium the buyer purchases only when his willingness to pay is *high*. As a consequence the seller is interested in the possibility of implementing a better pricing strategy in a way to discriminate across the different states of the buyer's willingness to pay. Modeling the degree to which the buyer is *locked-in*, with a probability of termination of the relationship $\delta < 1$, in the first section of the paper the existence of such a pricing strategy is derived as the result of a subgame perfect equilibrium supported by trigger strategies where the threat of reversion to the one shot Bayesian-Nash equilibrium price, induces truthful revelation of the buyer's willingness to pay for buyers with a sufficiently high δ . Intuitively a seller will find worth to allow price reductions to the extent that he is ensured that the buyer will come back to shop with sufficiently high probability. Therefore price rigidity depends here on the existence of asymmetric information (as in [2]).

In the second part of the paper one more level of asymmetric information is introduced. The seller now faces a population of buyers whose δ is unknown. The impossibility of screening different buyers' types induces the seller to use different pricing strategies depending on the length of the relationship with each buyer. The reason is that the probability of meeting *occasional* buyers (i.e. those buyers whose δ is not high enough to make them fear the threat of the reversion to the Bayesian-Nash equilibrium price), makes not necessarily worth the implementation of the flexible price equilibrium derived in the first part. Therefore for each given buyer with whom he is trading with, the seller will observe his behavior in order to *learn* his type and then, after a sufficiently high number of trading periods, decide whether to allow prices to fluctuate or not. Intuitively, buyers who have proved to be sufficiently loyal should be characterized by a δ high *enough* to make the flexible-price subgame perfect equilibrium of the first part implementable.

The paper is organized as follows. In section 2 the model is described and the subgame-perfect equilibrium supporting a flexible price regime, is derived. In section 3 the learning problem faced by the seller is introduced and a result is proven showing that for sufficiently long trading periods the seller may implement the flexible price regime.

2 The Model

A buyer (B) and a seller (S) are engaged in a bilateral relationship for the exchange of a good. In each period B can buy either one or zero units of the good, that S can supply to him at zero marginal cost. B's decision depends on the utility he obtains from the good which is given by a function $v_t = (R_t - p(R_t))$ where R_t is the reservation price he is ready to pay, $p(R_t)$ is the price charged by S. In a one shot setting B will buy whenever $v_t \geq 0$. Assume R_t can take two values with equal probability i.e. $R_t \in \{\bar{R}, \underline{R}\}$ ($(\bar{R}/2) > \underline{R}$), then depending on the amount of information the parties share, two situations can arise.

2.1 Symmetric Information

In this case both B and S observe the value of R_t and then, assuming that S has all the bargaining power, the price charged will be \bar{R} or \underline{R} depending on the value of R_t and B will always buy in both the states of the world.

2.2 Asymmetric Information

Things change if B has private information about the true value of R_t , the situation can be described by the following Bayesian game:

B,S	0	1
\bar{R}	(0, 0)	(\bar{R} , 0)
\underline{R}	(0, 0)	(\underline{R} , ($\bar{R} - \underline{R}$))

B,S	0	1
\bar{R}	(0, 0)	(\bar{R} , ($\underline{R} - \bar{R}$))
\underline{R}	(0, 0)	(\underline{R} , 0)

B knows on which matrix he is playing and chooses whether to buy (1) or not (0) depending on the price S charges; S knows only that each matrix can happen with probability (1/2) and sets a price \bar{R} or \underline{R} . If the game is played only once, the Bayesian-Nash equilibrium calls for B to buy always in the first matrix independently of the price S decides to set, while in the second matrix he will buy only when $p(R_t) = \underline{R}$. S's strategy will be to charge always the high price \bar{R} since by assumption the expected gain he can make in this way is higher than the one he can get pricing at \underline{R} and selling always given that by assumption $(\bar{R}/2) > \underline{R}$. Is there a way to improve this situation? For instance in a repeated setting S could ask B what is the value of R_t and in exchange charge him a lower average price.

This would change the assumption on the bargaining power: if B has a piece of information that is valuable to S, and this *is* the case since knowledge of R_t would allow S to price discriminate across states of the world, then S may be open to pay something for it; on the other side B has an incentive in using this advantage to the extent that this makes him better off. In brief: both parties can gain from a cooperative agreement.

In order not to complicate the story, I will keep the assumption that all the bargaining power is given to the seller that therefore sets prices transferring to the buyer part of the surplus arising from the transaction. The size of this transfer is likely to depend on the degree to which each party relies on the other.

To fix ideas, let $\delta < 1$ be the probability that once he entered the shop, B will come back to buy again from S, in other words $1 - \delta$ is the per period probability that the relationship between B and S will terminate next period for exogenous reasons. At the beginning of each period B gets to know R_t , then he reveals \tilde{R}_t , given this signal S sets $p(\tilde{R}_t)$ in a way that both he and the buyer are better off, namely S has to find a couple of prices $p(\bar{R}) = \bar{p}$ and $p(\underline{R}) = \underline{p}$ such that the following two conditions are satisfied

$$\frac{\bar{p} + \underline{p}}{2} \geq \frac{\bar{R}}{2} \tag{1}$$

$$\bar{p} \leq \bar{R} \quad \text{and} \quad \underline{p} \leq \underline{R} \tag{2}$$

In words, S has to find a second best welfare improving solution for himself (1) and for B (2).¹ These conditions alone do not guarantee that B won't take advantage of his superior information: since B always buys either zero or one unit of the good, S has no way to infer from B's behavior whether R_t was correctly revealed or not. Inability to screen B's behavior would make the agreement impossible since B would always claim $\tilde{R}_t = \underline{R}$ even when $R_t = \bar{R}$. Assume that S can observe with one period delay the *true* value of R_t ², in this case the following trigger strategy may prove fruitful:

$$p(R_t) = \begin{cases} p(\tilde{R}_t) & \text{if } \tilde{R}_{t-1} = R_{t-1} \\ \bar{R} & \text{if } \tilde{R}_\tau \neq R_\tau \text{ for any } \tau < t \end{cases} \tag{3}$$

¹Intuitively a higher δ denotes a buyer that is strongly tied to the seller and that, therefore, has less strength in the bargaining game over (\bar{p}, \underline{p}) .

²This is equivalent to assume *exp-post* perfect monitoring as in the oligopoly supergames à la Friedman [5]

Therefore given (3) B has the opportunity to *cheat* at most once before S can discover him. If S ever finds B cheating, at any period $\tau > 0$, he reverts to the one shot B-N equilibrium \bar{R} from then on.

Suppose now that in the current period B observes \bar{R} , should he cheat or not? Weighting present gains and future losses, the following inequality summarizes the terms of B's choice

$$(\bar{R} - \bar{p}) + \left(\frac{\delta}{2(1 - \delta)} \right) ((\bar{R} - \bar{p}) + (\underline{R} - \underline{p})) \geq (\bar{R} - \underline{p}) \quad (4)$$

in words, cheating today gives a payoff $(\bar{R} - \underline{p})$ but from tomorrow on, given the reversion to the B-N equilibrium price, will grant zero surplus; while sticking to a *fair* behavior ensures B a constant average positive payoff over the time.

The last thing I need to check is that once S has obtained the information he needs about R_t , he does not abuse of it. In other words if S would get to know $\tilde{R}_t = \bar{R}$, nothing ensures B that he won't set $\bar{p} = \bar{R}$ and extract all the surplus from him. To avoid this possibility the buyer can threaten S to revert to the B-N equilibrium where he always reveals \underline{R} (or equivalently where he never reveals any signal), formally B's trigger strategy would be

$$\tilde{R}_t = \begin{cases} R_t & \text{if } \bar{p}_{t-1} = \bar{p} \text{ and } \underline{p}_{t-1} = \underline{p} \\ \underline{R} & \text{otherwise} \end{cases} \quad (5)$$

Given (5) the incentive compatibility constraint for S is

$$\bar{p} + \frac{\delta(\bar{p} + \underline{p})}{2(1 - \delta)} \geq \bar{R} + \frac{\delta\bar{R}}{2(1 - \delta)} \quad (6)$$

Summarizing, the problem for S is to find a couple of prices (\bar{p}, \underline{p}) that satisfy the participation constraints (1), (2) and the incentive compatibility constraints (4) and (6). Setting $\bar{R} = 5$ and $\underline{R} = 2$ the problem is represented graphically in Figure 1 where the rectangle represents the intersection of B's participation constraints (2), (a), (c) and (b) represent respectively, the incentive compatibility constraint of the seller (6), the participation constraint (1) and the incentive compatibility constraint of the buyer³ (4). The set of solutions is given by the triangle defined by the intersection of S's and B's incentive compatibility constraints. For instance, the shaded area shows the

³An issue that should be considered is the one of the *optimality* of the punishment.

set of solutions for $\delta = 5/6$. As one can see, the lower is δ the more difficult is to find prices that support an equilibrium: intuitively a *too low* buyer's probability of coming back, would require the seller to set prices giving him a profit not high *enough* in order for the buyer to reveal correctly the observed willingness to pay.

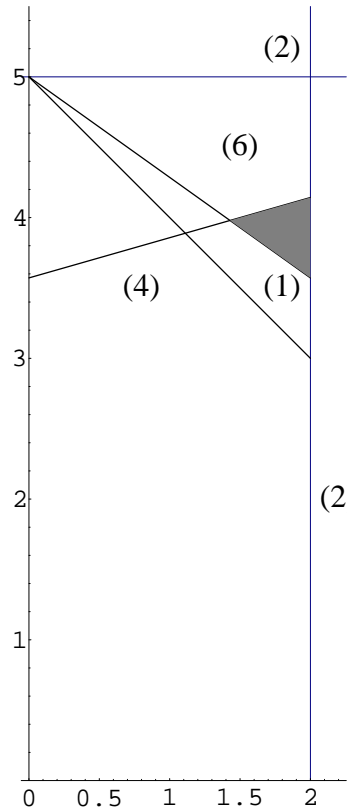


Figure 1: The Set of Solutions

The following claim shows that S can always find a solution to his problem, provided B is *sufficiently* locked-in the relationship

Claim 1 *There is a $\delta^* \in (0, 1)$ such that, for any $\delta \geq \delta^*$ the trigger strategies (3), (5) support a flexible-price subgame perfect equilibrium, where in each period the buyer announces his true reservation price and the seller sets the price according to this signal.*

Proof

Rewrite (4) and (6) in terms of \bar{p} to obtain

$$\bar{p} \leq \frac{\delta \bar{R} + 2(1 - \delta)\underline{p}}{2 - \delta} \equiv \phi(\underline{p})$$

$$\bar{p} \geq \bar{R} - \left(\frac{\delta}{2 - \delta} \right) \underline{p} \equiv \varphi(\underline{p})$$

looking at the figure, two things turn out: first a solution will exist as long as $\phi(\underline{p})$ will intersect $\varphi(\underline{p})$ since the incentive compatibility constraint of the seller will be always above his participation constraint; second since the slope of $\phi(\underline{p})$ is always higher than the slope of $\varphi(\underline{p})$, the solution for S will be to set $\underline{p} = \underline{R}$ and $\bar{p} = \phi(\underline{R})$. Finally solving for $\phi(\underline{R}) \geq \varphi(\underline{R})$ one get the lower bound for δ , i.e.

$$\delta \geq \delta^* \equiv \frac{2(\bar{R} - \underline{R})}{2\bar{R} - \underline{R}} \tag{7}$$

QED

Remark 2.2.1 The intuition for this result is as follows: if S has to switch to a price regime more favorable to B, it must be the case that he will be able to *compensate* the loss he makes *ex-post* whenever $R_t = \bar{R}$ and $p(\bar{R}_t) = \phi(\underline{R})$, with the gains arising from the repetition of purchases over time. For this to be possible, it must be the case that the buyer comes back to shop with *sufficiently* high probability. Thus only *loyal* or *fair* buyers will obtain the improved price menu.

Remark 2.2.2 Changing the assumptions on the bargaining power, gives rise to the issue of multiplicity of equilibria. For instance if the buyer would make a *take-it-or-leave-it* offer, then the solution would be given by the intersection of his incentive compatibility constraint and the one of the seller. Looking at Figure 1 this would imply $\underline{p} < \underline{R}$ and $\bar{p} < \phi(\underline{R})$.

Remark 2.2.3 There are cases when the requirement for the existence of a solution is more *stringent*. Consider for instance the situation where $\bar{R} > 3\underline{R}$,

then a solution will exist iff

$$\delta \geq \frac{4}{5}$$

this makes sense, since according to the pricing policy of S, the buyer can earn a surplus only when $R_t = \bar{R}$ and increasing the distance $(\bar{R} - \underline{R})$, increases the gains from deviation, thus requiring a higher buyer's involvement in the relationship for the equilibrium to exist.

Remark 2.2.4 Intepreting δ as the extent to which B is locked in the relationship, it is natural to think that higher values of δ will give S more power in choosing \bar{p} , in other words

$$\frac{\partial \bar{p}}{\partial \delta} = \frac{2(\bar{R} - \underline{R})}{(2 - \delta)^2} > 0$$

So if $\delta \rightarrow 1$, the high price will converge to \bar{R} and S will earn all the surplus from the relationship.

Remark 2.2.5 The difference $\bar{R} - \bar{p}$ is the *informational rent* S has to pay to a high reservation price B in order for him not to cheat, again notice that the ability to extract surplus from the relationship is inversely related to the degree to which B is locked in the relationship: a buyer with no other supplier ($\delta \rightarrow 1$) cannot earn any positive gain from cheating and thus should not be paid any informational rent.

Remark 2.2.6 As in [1] the equilibrium strategy (3) is not *renegotiation proof*; in fact once S starts punishing B the buyer could propose him to go back to the old agreement that is more profitable for both. The seller has still the opportunity to threaten him with the reversion to B-N equilibrium price: in these respect subgame perfection is not compromised. But the threat becomes not credible since B, anticipating the possibility that S won't refuse his counteroffer, will always cheat and S therefore will always play the B-N equilibrium.

An example of the equilibrium price function is given in Figure 2, where the data are the same as in the previous example.

3 Two Types

Suppose now that it is common knowledge that S faces two types of potential buyers: on the one hand there are buyers to whom the threat of the reversion to the B-N equilibrium price is not *enough* to make them unwilling to cheat;

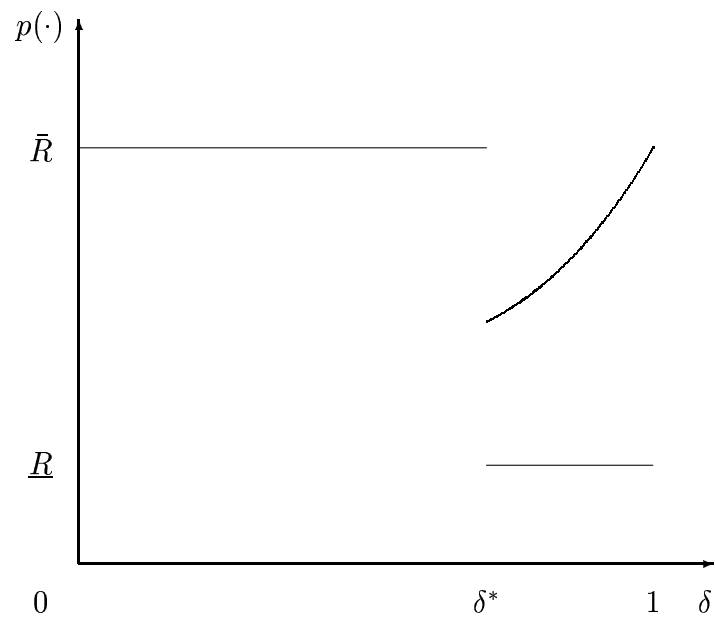


Figure 2: The equilibrium price function

on the other hand there are *fair* types that are supposed to come back to shop with sufficiently high probability that the threat will work correctly.

To fix ideas, I identify buyers of the first type with a probability of coming back $\delta_1 \in (0, \delta^*)$ and those of the second type with a probability of coming back $\delta_2 \in [\delta^*, 1)$.

Suppose S faces a new buyer who is entering his shop, what should he do? Should he ask him for \tilde{R}_t or should he just offer him a fixed price contract? The decision depends on two aspects:

- on the one hand S should have an opinion about B's type; accordingly, if he thinks that B is likely to be an *unfair* buyer, he should not run the risk of asking him \tilde{R}_t since then he will make a loss;
- on the other hand S has to compare the gains from a *risky* flexible price strategy with those from a *sure* fixed price one.

Therefore S needs two things, namely a rule to decide about B's type each moment he sees him entering the shop and a rule to decide whether or not prices should fluctuate. Although these two elements of S's decision are related, I will treat them separately for expositional simplicity.

3.1 Seller's beliefs

First assume that the only way S can decide about B's type is through the observation of his behavior, namely the situation he faces is similar to the following one:

S knows that all the buyers decide whether to come back or not flipping a biased coin. Some buyers have coins that with high probability give the result "go back", some others have a coin that deliver this result with low probability. Therefore when he sees a buyer entering again the shop, S infers that he has thrown the coin and that the coin gave the result "go back".

This story leads S to update his beliefs about B using Bayes' rule. Let $q_t < 1, \forall t \geq 0$ be the probability that S attaches to B being a δ_1 type at time

t. Given that B already bought once from S, upon seeing B entering his shop once again, S will think that he is type δ_1 with the following probability

$$\begin{aligned}
q_1 &\equiv P(\delta = \delta_1 | \text{he came back}) & (8) \\
&= \frac{q_0 P(\text{he came back} | \delta = \delta_1)}{q_0 P(\text{he came back} | \delta = \delta_1) + (1 - q_0) P(\text{he came back} | \delta = \delta_2)} \\
&= \frac{q_0 \delta_1}{q_0 \delta_1 + (1 - q_0) \delta_2}
\end{aligned}$$

Therefore if B comes back n times then the probability that he belongs to the *fair* buyers is likely to be very high, indeed using iteratively (8) this is given by the complement to one of

$$q_n = P(\delta = \delta_1 | \text{he came back } n \text{ times}) = \frac{q_0 \delta_1^n}{q_0 \delta_1^n + (1 - q_0) \delta_2^n} \quad (9)$$

Notice that since

$$\frac{\partial q_n}{\partial n} < 0 \quad (10)$$

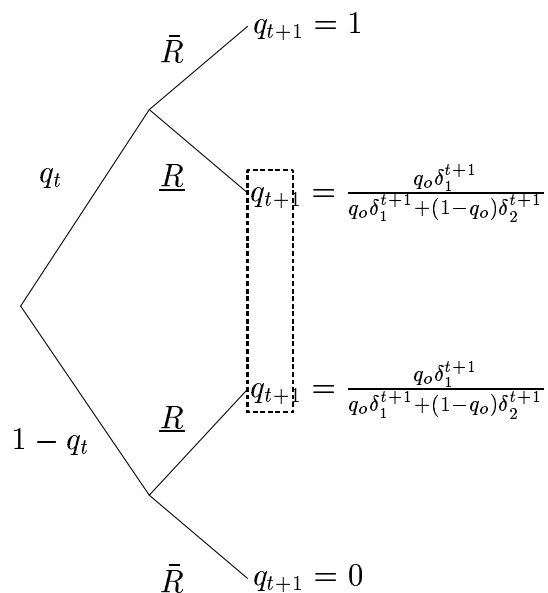
the probability that a buyer who comes back to shop belongs to the unfair types converges monotonically to zero, as n increases without bound.

Assume now S has decided that letting prices fluctuate is a good deal. Therefore he asks B his reservation price; B's answer can be, depending on his type and on the shock he faces, either \bar{R} or \underline{R} ; notice that if B reports $\tilde{R}_t = \bar{R}$ then S can set $q_t = 0$ since only a δ_2 type would report sincerely a high willingness to pay, in other words \bar{R} is a *type-revealing* answer; however B may reveal \underline{R} , then waiting one period S could be able to decide about his type or not depending on the *true* value of the shock he observes. Summarizing, S's belief is defined $\forall t \geq 1$

$$q_t = \begin{cases} 1 & \text{if } \tilde{R}_{t-1} = \underline{R} \text{ and } R_{t-1} = \bar{R} \\ 0 & \text{if } \tilde{R}_t = \bar{R} \\ \frac{\delta_1^t q_0}{\delta_1^t q_0 + \delta_2^t (1 - q_0)} & \text{if } \tilde{R}_{t-1} = \underline{R} \text{ and } R_{t-1} = \underline{R} \end{cases} \quad (11)$$

The updating rule S follows is summarized in Figure 3. A δ_1 type faced with any type of shock will always reveal $\tilde{R}_t = \underline{R}$ thus S will identify him with probability (1/2) (the upper part of the tree); equivalently a δ_2 type will always reveal truthfully $\tilde{R}_t = \bar{R}$, again S will identify him with probability (1/2) (the lower part of the tree); receiving $\tilde{R}_t = \underline{R}$ and observing with one

Figure 3: S's updating rule



period delay \underline{R} does not allow S to distinguish from whom the signal has come (the middle part of the tree).

Remark 3.1.1 Notice that as long as S does not let prices fluctuate, the only way he has to update his beliefs, is through observation of B's behavior. In other words, S can choose between two options: *slow* learning, obtained through the observation of B's behavior and *fast* learning, obtained asking B his reservation price R_t . The decision between which of the two learning strategies is better to adopt will depend on the expected payoff S can gain with each one of them. Therefore I now turn to the analysis of S's decision rule.

3.2 Seller's decision rule

As far as the decision rule that weights the gains from adopting the two alternative price strategies, it will depend on the expected payoff S will get adopting one or the other option. On the one hand if a fixed price contract has to be offered, then S will offer the equilibrium price of the one-shot game $p(R_t) = \bar{R}$ since this is the price that among the fixed price contracts maximizes his expected profits.

On the other hand choosing to let prices fluctuate, opens the problem of what kind of prices S should propose to B. Formally, S should choose \bar{p} and \underline{p} according to the solution proposed in the first part only if he thinks that B belongs with probability 1 to the *fair* buyers. For what I've shown before as long as S keeps prices rigid, he has no way to infer B's type with certainty in finite time. Eventually he will learn it only when the duration of the relationship is infinite. Deciding to switch to a flexible price regime in finite time, casts a doubt over the best pricing strategy he should choose. Intuitively one may think that the prices S should propose to B in the short run should be different than those he should propose in the long run. However since it is common knowledge that only two types of buyers live in this economy, S has basically not too much freedom in making his choice.

Lemma 1 *If S decides to let prices fluctuate in finite time, he will choose $\bar{p} = \phi(\underline{R})$ and $\underline{p} = \underline{R}$.*

Proof

First of all notice that S cannot provide any incentive to separate the two buyers' type: a type δ_2 does not gain anything from misrepresenting himself as a δ_1 (S would not allow a high price tailored to δ_1 since this would not be worth to him); the type who would gain something from cheating is δ_1 since he could get a better price regime in the high state; however since \bar{p} is computed for $\delta = \delta_2$, it is not incentive compatible for him, hence S should not expect an attempt to cheat from any of the two buyers for what concerns their true type.

Next, if B reveals $R_t = \bar{R}$, then the best choice S can do is to set $\phi(\underline{R})$ since this price is incentive compatible, given δ_2 , and maximizes his profit. Equivalently no type δ_1 would ever reveal \bar{R} , therefore since the only reason why S sets $\bar{p} < \bar{R}$ is to make the price incentive compatible for a *high* type, there is no reason (in fact, according to the first section, no possibility) to find an incentive compatible price for a low type.

Finally if B reveals \underline{R} then S may be tempted not to believe him, since this answer could come either from a low type (independently from the *true* shock he has observed) or from a high type that received a low shock. Thus S may want to set a price $\underline{p} > \underline{R}$ in order to reduce the eventual losses he would make if B was cheating. Any price $\underline{p} < \bar{R}$ would not be optimal, since if B was not cheating a $\underline{p} > \underline{R}$ would in any case trigger B's reversion to the B-N equilibrium. Therefore S should set $\underline{p} = \bar{R}$. This price would make sense only if S is sure that B belongs to the *unfair* buyers since otherwise, say at time τ , with probability $0.5q_\tau + (1 - q_\tau)$ S would loose the buyer and pay the consequences of the B-N reversion, while only with probability q_τ he would get the buyer cheating. QED

Lemma 2 *S's expected profit from price rigidity at any time $\tau \geq 0$ is given by*

$$\begin{aligned} E[\Pi_\tau(\delta, \bar{R})] &= \left(\frac{\bar{R}}{2}\right) \left(\sum_{i=0}^{\infty} \delta_1^i q_{\tau+i} + \sum_{i=0}^{\infty} \delta_2^i (1 - q_{\tau+i})\right) \end{aligned} \quad (12)$$

While the expected profit from price flexibility is given by

$$\begin{aligned} E[\Pi_\tau(\delta, p(\tilde{R}_t))] &= \left(\frac{1}{2}\right) \left((1 - q_\tau) \left(\bar{p} + \frac{\delta_2(\bar{p} + \underline{R})}{2(1 - \delta_2)} \right) + q_\tau \left(\underline{R} + \frac{\delta_1 \bar{R}}{2(1 - \delta_1)} \right) \right) \\ &+ \left(\frac{1}{2}\right) \left\{ \sum_{i=0}^{\infty} \left(\frac{1}{2}\right)^i \underline{R} (\delta_2^i (1 - q_{\tau+i}) + \delta_1^i q_{\tau+i}) \right. \\ &\left. + \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^i \left(\delta_2^i (1 - q_{\tau+i}) \left(\bar{p} + \frac{\delta_2(\bar{p} + \underline{R})}{2(1 - \delta_2)} \right) + \delta_1^i q_{\tau+i} \left(\underline{R} + \frac{\delta_1 \bar{R}}{2(1 - \delta_1)} \right) \right) \right\} \end{aligned} \quad (13)$$

Proof

To understand (12) consider that as long as S does not decide to ask \tilde{R}_t , he won't be able to learn immediately B's type, thus the only way he can get to know him is through the *slow* learning process which is induced by seeing him purchasing repeatedly. As a consequence the expected payoff he will make in each period is the weighted average of the one-shot B-N equilibrium payoff and the belief he holds about B. For instance, starting at time zero

this will be

$$\begin{aligned}
E[\Pi_0(\delta, \bar{R})] &= \left(\frac{\bar{R}}{2}\right) \left(\sum_{i=0}^{\infty} \delta_1^i q_i + \sum_{i=0}^{\infty} \delta_2^i (1 - q_i)\right)
\end{aligned}$$

Turning to the flexible price regime, consider Figure 4 and assume that S decides to let prices fluctuate at time zero (i.e. he asks \tilde{R}_t). Given the structure of the random process governing the evolution of B's willingness to pay, S can face four situations, namely B can have high or low willingness to pay and he can be a type δ_2 or a type δ_1 . The situation is summarized in Table 2: In a sense S should *hope* that when he decides to introduce flexible price, B

Table 2: The possible situations S can face at time zero.

	δ_2	δ_1
\bar{R}	$((1 - q_0)/2)$	$(q_0/2)$
\underline{R}	$((1 - q_0)/2)$	$(q_0/2)$

is facing a high shock, since only in this way he can be sure that he will learn immediately his type.

In statistical terms the problem S is facing is equivalent to the choice between two distinct sampling procedures whose risk functions are given by (12) and (13), the first one is sure but less remunerative, while the second one is risky but more promising. Ideally there should exist a τ^* after which the expected loss S would make facing an *unfair* buyer is overcompensated by the expected gains he would net facing a *fair* one.

Fix the realization of the shock, say $R_0 = \bar{R}$, then with probability $(1 - q_0)$ S will face a *fair* buyer and with probability q_0 an *unfair* one; in the first case asking him \tilde{R}_t will lead to immediate learning of his type and therefore to a payoff from then on equal to the expected discounted stream of payoffs S can earn trading with a fair buyer through the implementation of the subgame perfect equilibrium derived in section 2. In case he faces an *unfair* buyer he will gain \underline{R} immediately and learn with one period delay his type, thus netting from the next period on the payoff delivered by the implementation of the punishment (the static B-N equilibrium payoff).

Hence with probability $(1/2)$ S can learn immediately B's type asking him his

reservation value. However things may turn out to be more difficult, namely B can experience a *low* shock and thus S cannot screen his type since both δ_2 and δ_1 will truthfully reveal \underline{R} . In this case too the continuation payoff S can earn is a function of the buyer's type and of S's beliefs but now the learning process becomes slower, at least for one period, since starting from the next period the same situation S was facing is there again; in other words S has once again the possibility of learning immediately B's type with probability (1/2).

In this way the following series can be obtained

$$\begin{aligned} & \left(\frac{1}{2}\right) \left((1 - q_0) \left(\bar{p} + \frac{\delta_2(\bar{p} + \underline{R})}{2(1 - \delta_2)} \right) + q_0 \left(\underline{R} + \frac{\delta_1 \bar{R}}{2(1 - \delta_1)} \right) \right) \\ & + \left(\frac{1}{2}\right) \left\{ (1 - q_0) \underline{R} + q_0 \underline{R} + \left(\frac{1}{2}\right) \left[\delta_2(1 - q_1) \left(\bar{p} + \frac{\delta_2(\bar{p} + \underline{R})}{2(1 - \delta_2)} \right) \right. \right. \\ & \left. \left. + \delta_1 q_1 \left(\underline{R} + \frac{\delta_1 \bar{R}}{2(1 - \delta_1)} \right) \right] + \left(\frac{1}{2}\right) [\delta_2(1 - q_1) \underline{R} + \delta_1 q_1 \underline{R} + \dots \right. \end{aligned} \quad (14)$$

The expression in the claim is just the compact form of (14) evaluated at an arbitrary $\tau \geq 0$. QED

Therefore S will decide to switch to flexible prices at time $\tau \geq 0$ iff

$$E[\Pi_\tau(\delta, p(\tilde{R}_t))] \geq E[\Pi_\tau(\delta, \bar{R})] \quad (15)$$

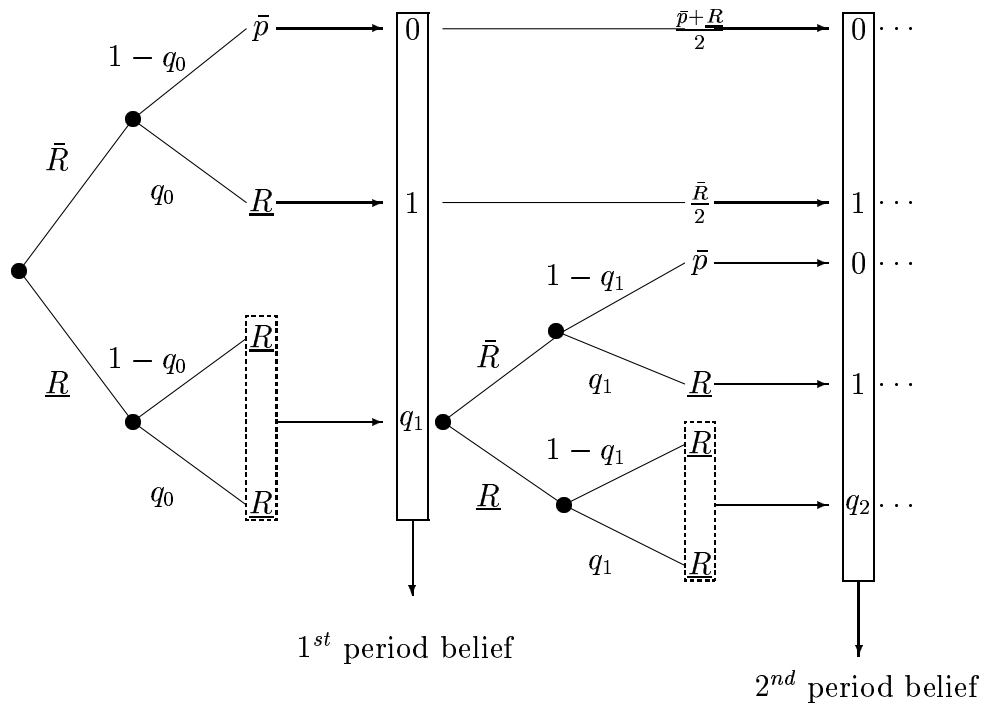
Remark 3.2.1 Notice that if (15) is not satisfied at time zero, B has no way to cheat since S simply does not ask him any signal, therefore S can *learn* B's type at *no cost* and let price fluctuate only when he is *sufficiently* sure that he is facing a δ_2 type.

Remark 3.2.2 The decision rule (15) evolves through time only in the prior S uses to evaluate B's type. At any time $\tau \geq 0$ if prices were kept rigid till then, q_τ will be the τ -th iteration of Bayes' rule on the prior q_0 .

Claim 2 *In the game where S faces two types of buyers $\delta_1 \in (0, \delta^*)$, $\delta_2 \in [\delta^*, 1)$, S's optimal price strategy is*

$$p(R_t) = \begin{cases} \bar{R} & \text{if } E[\Pi_\tau(\cdot, \bar{R})] > E[\Pi_\tau(\cdot, p(\tilde{R}_t))] \\ p(\tilde{R}_t) & \text{otherwise} \end{cases}$$

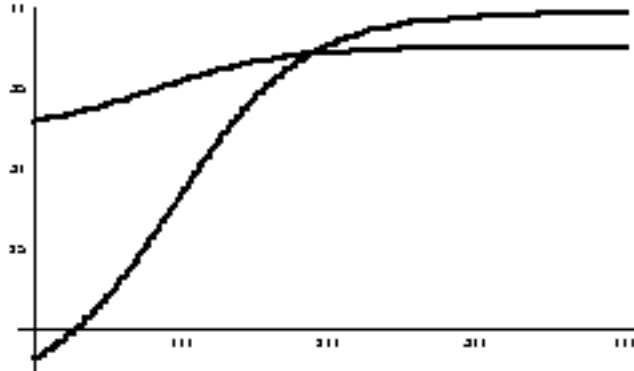
Figure 4: S's game



Proof

The point is to show whether S will ever find it worth to switch to flexible price. According to **Lemma 4** in the appendix, such a *switch point* exists. QED

Figure 5: Example $\delta_1 = 0.87$ and $\delta_2 = 0.88$



Consider Figure 5, there are plotted the two functions $E[\Pi_\tau(\cdot, \bar{R})]$ and $E[\Pi_\tau(\cdot, p(\tilde{R}_t))]$ when $\tau = 0, 1, \dots, 400$ and the sums approximate the limit of the respective series. The data are $\bar{R} = 9$, $\underline{R} = 2$, therefore $\delta^* = (7/8)$, choosing $\delta_1 = 0.87$, $\delta_2 = 0.88$ and $q_0 = 0.9$, prices are kept fixed for almost 200 periods, the corresponding belief for S is then roughly $q_{200} = .47$.

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Appendix

To simplify the notation, set

$$\Phi_1 = \left(\bar{p} + \frac{\delta_2(\bar{p} + \underline{R})}{2(1 - \delta_2)} \right)$$

$$\Phi_2 = \left(\underline{R} + \frac{\delta_1 \bar{R}}{2(1 - \delta_1)} \right)$$

To prove **Claim 2** I need the following two lemmas:

Lemma 3 (Monotonicity)

$$E[\Pi_{\tau+1}(\delta, p(\tilde{R}_t))] \geq E[\Pi_{\tau}(\delta, p(\tilde{R}_t))] \quad \forall \tau$$

Proof

Rewrite (13) as follows

$$\begin{aligned} & E[\Pi_{\tau}(\delta, p(\tilde{R}_t))] \\ &= \left(\frac{1}{2} \right) (\Phi_1 - q_{\tau}(\Phi_1 - \Phi_2)) \\ & \quad + \left(\frac{1}{2} \right) \left\{ \sum_{i=0}^{\infty} \left(\frac{1}{2} \right)^i \underline{R}(\delta_2^i - q_{\tau+i}(\delta_2^i - \delta_1^i)) \right. \\ & \quad \left. + \sum_{i=1}^{\infty} \left(\frac{1}{2} \right)^i (\delta_2^i \Phi_1 - q_{\tau+i}(\delta_2^i \Phi_1 - \delta_1^i \Phi_2)) \right\} \end{aligned} \tag{16}$$

then notice that shifting (3) one period ahead and comparing term to term the elements of the two sums

$$\Phi_1 - q_{\tau+1}(\Phi_1 - \Phi_2) > \Phi_1 - q_{\tau}(\Phi_1 - \Phi_2)$$

since q_{τ} is decreasing in τ , the same hold for the other terms of the sum, hence the lemma holds. QED

The same argument can be used to show that (12) is increasing in τ . Since both the sequences are bounded (set $q = 1$ and compute the sum of the series) it follows that they are convergent.

Lemma 4 (Crossing Property) *For all \bar{R} , \underline{R} , δ_1 , δ_2 , $\exists \tau^*$ such that*

$$E[\Pi_{\tau^*}(\delta, p(\tilde{R}_t))] \geq E[\Pi_{\tau^*}(\delta, \bar{R})]$$

Proof

For $q_0 = 0$, the statement is true since then the seller knows that no buyer will ever cheat hence the optimality of price flexibility in this case follows immediately.

For $q_0 > 0$, suppose the statement is false, then there is always the possibility to find \bar{R} , \underline{R} , δ_1 and δ_2 such that

$$E[\Pi_{\tau^*}(\delta, p(\tilde{R}_t))] < E[\Pi_{\tau^*}(\delta, \bar{R})]$$

for *any* τ . Now the following inequality holds

$$\sum_{i=0}^{\infty} \frac{\bar{R}}{2} \delta_2^i \geq E[\Pi_{\tau^*}(\delta, \bar{R})]$$

since $\delta_2 > \delta_1$, moreover

$$\begin{aligned} E[\Pi_{\tau^*}(\delta, p(\tilde{R}_t))] &\geq \\ &\left(\frac{1}{2}\right) ((1 - q_\tau)\Phi_1 + (1 - q_{\tau+1})\delta_2\Phi_1) \geq \\ &\left(\frac{1}{2}\right) ((1 - q_\tau)(1 + \delta_2)\Phi_1) \end{aligned}$$

since $q_{\tau+1} < q_\tau$. Then, putting together the last two inequalities the assumed condition implies

$$q_\tau > 1 - \frac{2\bar{R}}{(1 + \delta_2)((2 - \delta_2)\bar{p} + \delta_2\underline{R})} \quad (17)$$

Notice that the r.h.s. of (17) is always between zero and one since

$$2\bar{R} < (1 + \delta_2)(\delta_2(\bar{R} - \underline{R}) + 2\underline{R})$$

given that $\delta_2 \geq \delta^*$.

Given this, since the l.h.s. of (17) decreases monotonically with τ , there will exist a τ^* for which the assumed inequality is false, hence the assumption led to a contradiction, therefore the lemma holds. QED

To characterize completely the set of solutions to the bargaining problem one should solve the following program:

$$\max_{\underline{p}, \bar{p}} \gamma(\bar{p} + \underline{p}) + (1 - \gamma)((\bar{R} - \bar{p}) + (\underline{R} - \underline{p}))$$

under the constraints

$$\bar{p} + \underline{p} \geq \bar{R}$$

$$\bar{p} \leq \bar{R}$$

$$\underline{p} \leq \underline{R}$$

$$(\bar{R} - \bar{p}) + \frac{\delta}{2(1-\delta)}((\bar{R} - \bar{p}) + (\underline{R} - \underline{p})) \geq \bar{R} - \underline{p}$$

$$\bar{p} + \frac{\delta(\bar{p} + \underline{p})}{2(1-\delta)} \geq \bar{R} + \frac{\delta\bar{R}}{2(1-\delta)}$$

where $\gamma \in [0, 1]$ represents and index of the bargaining power the two parties have. The solution is as follows:

- for $\gamma > (1/2)$ (more bargaining power to the seller) and $\delta \geq \delta^*$, $\underline{p} = \underline{R}$ and

$$\bar{p} = \frac{\delta\bar{R} + 2\underline{R}(1-\delta)}{2-\delta}$$

therefore it coincides with what I find in the theorem;

- for $\gamma < (1/2)$ (more bargaining power to the buyer) and $\delta \geq \delta^*$

$$\underline{p} = \bar{R} - \frac{\delta\underline{R}}{2(1-\delta)}$$

$$\bar{p} = \frac{(2-\delta)(2-3\delta)\bar{R} + \delta^2(\underline{R} + \bar{R})}{2(1-\delta)(2-\delta)}$$

selecting the couple of prices that solve the intersection of the incentive compatibility constraint of the buyer and the one of the seller.