Economics Working Paper 58

A Model of Financial Markets with Default and The Role of "Ex-ante" Redundant Assets*

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January 1994

Abstract

We consider trading on asset as an anonymous contract. Sellers of assets may default, fail to meet the terms of the contract, as long as they are for some punishment. As a result of default, the asset yields are no longer exogenous, but endogenous. Each borrower (seller of an asset) individually chooses the specific amounts of the payments he will make in the future. The effective payoffs of each type of assets is determined by the behaviour of *all* the sellers of that type of asset. The inmediate consequence of this behaviour is that the *spanning* of the asset returns becomes endogenous. In particular, by introducing a redundant asset to incomplete financial markets, we may Pareto-improve the welfare of the economy. This suggests the usefulness of a redundant asset, which cannot be explained in a standard (no default) incomplete market model. Indirectly, this result suggests that a great part of the incompleteness in financial markets arises because of a "wrong" design of securities rather than their numerical insufficiency.

I. Introduction

Default is an important issue both empirically and theoretically. The possibility of agents choosing to default has been present in any market oriented society. It is obvious that default is a main factor determining the performance of real markets. It is also theoretically meaningful, because it makes future asset returns endogenous. In fact, the determination of future returns has been a major issue among economic theorists. A clear exponent of this interest is the large literature developed in the last five years on financial innovation and security design. However, the problem of default *per se* has not been fully addressed in a general equilibrium setting except in Dubey, Geanakoplos and Shubik (1989).

Before commenting on their paper, let us briefly mention why the financial market model in a general equilibrium setting is an appropriate place to study default. In a multi-period model of financial markets, each household's future income depends partially on the future returns from the assets he traded in the previous period. Therefore, even if an agent correctly predicts the future price levels, the other agents' default behavior may reduce his future income and cause him to default -thus, generating a chain reaction of defaults. This cannot happen in a model with contingent commodities. In this sense, default is naturally analyzed in financial assets models.

Introducing default in the standard General Equilibrium model with financial assets is not an easy task. The first choice we have to make is how to treat default risk. This choice is tied to the choice of anonymity versus non-anonymity in the market. In our specification we preserve the competitive behaviour of agents as well as their anonymity in the market. Under our Rational Expectations specification (to be precisely defined below), the default that a borrower (or equivalently a seller, since all assets are in zero net supply in our economy) incurs is spread out proportionally to all owners (or, buyers) of that type of security. This means that each buyer does not bother about the default a particular seller may incur. He only cares about the aggregate default of all the sellers of

that type of security. Though this hypothesis is clearly questionable, we argue that it is a reasonable assumption on grounds of actual behaviour in financial markets. In general, the assumed sharing of losses is an appropriate specification for economies with a high level of financial intermediation.[1] However, this is not the only market for which our model seems to be a good approximation: there are many others. For instance, one of the most growing markets since the mid-eighties is the market for Mortgage Back Securities. To get an idea of the size and growth of this market, note that the ammount issued only by Federal agencies (GNMA, FNMA, FHLMC) went from $0.3 billions in 1970 to more than $1 trillon by the end of 1986.[2] In this market several baskets of mortgages are pooled and different securities are sold (usually by investment banks) offering a schedule of payoffs conditional on the effective payments of the original mortgages. The buyer of any of these securities has no idea of the identity or default characteristics of any particular mortgage holder. These securities are priced after an appropriate assessment of the likelihood of default of the whole pool of mortgages. These two examples, together with many others that we do not include here, point out that many traders in actual markets behave in a similar fashion to the behaviour imposed by our hypothesis. In our model, no trader has to bother about the identities of those he is trading with, but rather, about the behaviour of the whole market for each particular security.

Once default is allowed, the asset yields become endogenous as households choose which (and in what amount) payments to make and so does the spanning of the asset yields.[3] This has a negative effect (default induces corresponding punishments that we assume directly reduce one's utility), as a well as a positive one. The change on spanning can imply not only better risk sharing opportunities for the economy as a whole, but also

[1]It is a normal practice for commercial banks to first asses the fraction of each type of loan they offer that will be repaid and then to set the interest rate of each type of loan accordingly.

[2]Note that these figures do not include private issues and they correspond exclusively to what is known as "pass-through" securities. For information about "pay-through", CMO's and no-goverment issues, see Allen and Gale (1993) and the references in there.

[3]Asset Spanning can just be defined as the subspace of redistributions of wealth (or, "money") across states that can be obtained by arbitrary linear combinations of the existing assets.

for particular individuals. In particular, in our model it is possible that a particular individual ends up buying and selling the same security, which can greatly increase his "personalized" spanning. The reason for this is that, while a particular seller knows precisely how much he will default in a particular security he sells, the rest of the buyers of that security consider the aggregate default of all the sellers. It is easy to find situations in which a seller may find it worthwhile to simultaneously buy the same security. Dubey, Geanakoplos and Shubik (1989) stress these points and provide an example in which the existence of a small amount of punishment can activate financial markets and compensate the loss of utility through punishment.

Instead of this, we focus on the change of asset spanning in this paper. In particular, we consider the situation where the introduction of redundant assets changes the equilibrium allocation through the increase of the assets spanning. To accomplish this, we consider the incomplete financial market, although the existence proof in Section 3 does not depend on this fact.

"Ex ante" redundant assets can change the equilibrium allocation in the following way: They can be non-redundant at equilibrium if households default in each asset differently from the others. This asset specific default generates "ex post" non-redundant assets and effectively increases the dimension of the assets spanning. The situation is similar to that of sunspot equilibria in the sense that the households' expectations generate a new set of equilibria. The difference here is that the introduction of redundant assets can Pareto-improve the economy. A sunspot equilibrium cannot Pareto-improve a non-sunspot equilibrium. In some sense our model is a model of assets formation through market expectations.

The rest of the paper is organized as follows. In Section 2, we describe the model and state the precise meaning of default and its punishment. There we also define an (rational expectation) equilibrium where each household correctly predicts future asset returns along with future prices. The existence of equilibrium is established in Section 3,

following the argument by Dubey, Geanakoplos and Shubik (1989). In Section 4, we present two examples illustrating that the introduction of "ex-ante" redundant assets may improve efficiency in the economy. This point cannot be made in a model without default (fixed return matrix) and suggests the use of a model with default to explain the importance of "seemingly" redundant assets in the "real" world as well as the efficiency of the market *per se* as an innovator. In section 5 we comment on the role of redundant assets and in the possibility of Pareto Improvement in a general set-up. The final section, section 6, is dedicated to conclusions and topics for future research.

II. The Model

We consider a pure exchange economy that lasts for two periods and has S states of nature in the second period. The symbol s = 0,1,...,S denotes, for s=0, the first period and, for s > 0, one of the S possible states in the second period.

For every s ≥ 0, there is a spot market for C physical commodities, labeled by the superscript c = 1, . . ., C. For s =0, there is also a market for I financial instruments (bonds), i = 1, . . .,I. Their overall yields, in units of account are represented by the matrix Y,

$$Y = \begin{bmatrix} \vdots \\ y^s \\ \vdots \end{bmatrix} = \begin{bmatrix} & \vdots & \\ \cdots & y^{si} & \cdots \\ & \vdots & \end{bmatrix}.$$

Since we will consider the possibility of default, it is understood that the matrix Y represents the *ex ante* yield. For example, the realized yield may differ from it because some borrower may not fulfill the promised payment.

There are also H household (denoted by the subscript h = 1, . . . , H) having utility functions $w_h : \Re_{++}^{(S+1)C} \to \Re$. We assume that w_h has the following structure: $w_h(\cdot)$ =

$u_h(x_h)$ + "punishment term", where $x_h = (x_h^0, x_h^1, \ldots, x_h^s) \in \Re_{++}^{(S+1)C}$ is the consumption vector and the punishment term will be specified later. The endowment vector is denoted by $e_h = (e_h^0, e_h^1, \ldots, e_h^s) \in \Re_{++}^{(S+1)C}$.

Throughout the paper we assume the following:

A1. $u_h(x_h)$ is C^2, differentiably strictly increasing and differentiably strictly concave, and satisfies the boundary condition.

A2. $Y >> 0$.

II. A. Household Maximization Problem

The price vector for the C commodities traded in spot s is denoted by $p^s = (p^{s1}, \ldots, p^{sC}) \in \Re_{++}^C$ and the whole price vector is written as $p = (p^0, p^1, \ldots, p^s) \in \Re_{++}^{(S+1)C}$. The bond prices are represented by $q \in (q^1, \ldots, q^I) \in \Re_+^I$. Since borrowers can default in the second period (and lenders know this fact), their expected future yield matrix may be different from the *ex ante* yield matrix Y. Denote this expected yield matrix by

$$Z = \begin{bmatrix} \vdots \\ z^s \\ \vdots \end{bmatrix} = \begin{bmatrix} & \vdots & \\ \cdots & z^{si} & \cdots \\ & \vdots & \end{bmatrix} \in \Re_+^{SI}.$$

This matrix is determined at equilibrium.

Let $\alpha_h \in \Re_+^I$ and $\beta_h \in \Re_+^I$ be the quantity of bond *selling* and of bond *buying* by household h, respectively. Also define the actual per unit return by household h at the state s for the bond i by $\tilde{Z}_h = (\tilde{z}_h^{si}) \in \Re_+^{SI}$. It may be called the *personal yield matrix*. Naturally, if he fails to pay back the promised amount, he will incur a punishment. Here we assume that the punishment is linear in the unfulfilled payment. Thus we write the punishment agent h incurs as:

$$(\pi_h^1, \ldots, \pi_h^S)\left(Y - \tilde{Z}_h\right)\alpha_h = \sum_{s=1}^S \pi_h^s \left(\sum_{i=1}^I (y^{si}\alpha_h^i - \tilde{z}_h^{si}\alpha_h^i)\right),$$

where $(\pi_h^1, \ldots, \pi_h^S) \in \Re_{++}^S$ is the punishment parameter. Note that under this specification we allow the punishment parameter to be parameter specific but not asset specific.

Now, after this preparation, we can describe the household's behavior as follows. Each agent h solves:

Given p, q, and Z,

$$\begin{aligned}
&\text{maximize} \quad u_h(x_h) - \textstyle\sum_{s=1}^{S} \pi_h^s \left(\sum_{i=1}^{I} \left(y^{si}\, \alpha_h^i - \tilde{z}_h^{si}\, \alpha_h^i\right)\right) \\
&x_h, \alpha_h, \beta_h, \tilde{Z}_h \\
&\qquad \text{subject to} \quad p^0 (x_h^0 - e_h^0) \le -q(\beta_h - \alpha_h), \\
&\qquad\qquad\qquad\quad p^s (x_h^s - e_h^s) \le Z^s \beta_h - \tilde{Z}_h^s \alpha_h, \quad s \ge 1, \\
&\qquad\qquad\qquad\quad 0 \le \tilde{z}_h^{si} \le y^{si}, s \ge 1 \text{ and all i}, \\
&\qquad\qquad\qquad\quad \alpha_h, \beta_h \ge 0, \text{ and} \\
&\qquad\qquad\qquad\quad x_h >> 0
\end{aligned}$$

Note that, since Y>>0, α_h>0 always means to pay something and $\beta_h > 0$ to receive something in the future. Also note that $\tilde{z}_h^{si}$ is multiplied by α_h^i, reflecting the fact that only the seller (borrower) can default, and that we do not allow a borrower to pay back more than promised (and get a reward).

II. B. Definition of Equilibrium

At an equilibrium the variables p, q, and Z are determined so that the goods and bonds markets clear. Since the variables involve those of the second period, we need to assume some sort of expectation scheme for the households. In this paper, we adopt the rational expectation scheme.

Definition 1: $(p, q, Z, x, \alpha, \beta, \tilde{Z}) \in \Re_{++}^{(S+1)C} \times \Re_{+}^{I} \times \Re_{+}^{SI} \times \Re_{++}^{(S+1)CH} \times \Re_{+}^{IH} \times \Re_{+}^{IH} \times \Re_{+}^{SIH}$ is called a **financial equilibrium with default** if it solves

(1) The household maximization problem

(2) $\sum_{h=1}^{H}(x_h - e_h) = 0$,

(3) $\sum_{h=1}^{H}(\beta_h - \alpha_h) = 0$, and

(4) $z^{si} \begin{cases} = \dfrac{\sum_{h=1}^{H} \tilde{z}_h^{si}\, \alpha_h^j}{\sum_{h=1}^{H} \alpha_h^j} & \text{if} \quad \sum_{h=1}^{H} \alpha_h^j > 0 \\ \in [0, y^{si}] & \text{otherwise} \end{cases}$

for all s ≥ 1 and i.

Remark: Conditions (1) - (3) are standard. Condition (4) means that the household's expectations over the future bond yields are correct at the equilibrium (rational expectations). Here we are implicitly assuming that the bond returns are proportionally rationed to the lender in case of default.

II. C. Difficulties

The standard procedure is to prove the existence of equilibrium at this stage. The formulation here, however, does not generate a standard concave programming problem. In fact, it is easy to verify that the objective function is not concave and the choice set is not convex. Therefore, we are forced to reformulate the maximization program.

Now, if we check the problem (1) again, we find that the term $\tilde{z}_h^{si}\, \alpha_h^j$ appears many times and it is the origin of the ill-behavedness of the problem. Therefore, we replace those terms by a single variable γ_h^{si} and reformulate the problem as below. As we will see, the original model can be analyzed with this new formulation. Proposition 1 will stablish the equivalence of both formulations.

II. D. Transformation of Variables

The new households' problem is:

Given p, q and Z

maximize $u_h(x_h) - \sum_{s=1}^{S} \pi_h^s (\sum_{i=1}^{I} (y^{si} \alpha_h^i - \gamma_h^{si}))$

subject to $p^o (x_h^0 - e_h^0) \leq -q(\beta_h - \alpha_h)$,

$p^s (x_h^s - e_h^s) \leq Z^s \beta_h - \sum_{h=1}^{H} \gamma_h^{si}$, $\forall s \geq 1$,

$y^{si} \alpha_h^i - \gamma_h^{si} \geq 0 \ \forall s,i$,

$\gamma_h^{si} \geq 0 \ \forall s,i$,

$\alpha_h^i \geq 0 \ \ \forall i$,

$\beta_h^i \geq 0 \ \ \forall i$,

and $x_h >> 0$

Definition 2: (p, q, Z, x, α, β, γ) $\in \Re_{++}^{(S+1)C} \times \Re_{+}^{I} \times \Re_{+}^{SI} \times \Re_{++}^{(S+1)CH} \times \Re_{+}^{IH} \times \Re_{+}^{IH} \times \Re_{+}^{SIH}$ is called a **financial equilibrium with default** if it solves

(1') The household maximization problem

(2) $\sum_{h=1}^{H} (x_h - e_h) = 0$,

(3) $\sum_{h=1}^{H} (\beta_h - \alpha_h) = 0$, and

(4) $$z^{si} \begin{cases} = \dfrac{\sum_{h=1}^{H} \tilde{z}_h^{si} \alpha_h^i}{\sum_{h=1}^{H} \alpha_h^i} & \text{if } \sum_{h=1}^{H} \alpha_h^i > 0 \\ \in [0, y^{si}] & \text{otherwise} \end{cases}$$

for all s ≥ 1 and i.

Now, problem (1') is a well-behaved concave programming problem. The next proposition shows that we can work the whole model with this second definition.

Proposition 1: The two definitions above define the same equilibrium in the following sense:

(i) If $(p,q,Z,x,\alpha,\beta,\tilde{Z})$ is an equilibrium of **Definition 1**, then (p, q, Z, x, α, β, γ) is an equilibrium of **Definition 2**, with $\gamma_h^{si} = \tilde{z}_h^{si}\,\alpha_h^i$, $\forall s$, h and i.

(ii) If (p, q, Z, x, α, β, γ) is an equilibrium of **Definition 2**, then $(p,q,Z,x,\alpha,\beta,\tilde{Z})$ is an equilibrium of **definition 1,** with

$$\tilde{z}_h^{si} = \begin{cases} \dfrac{\gamma_h^{si}}{\alpha_h^i} & \text{if } \alpha_h^i \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Proof: Straightforward.

In the following sections, we will always analyze the equilibrium in terms of **Definition 2**.

III. Existence of Equilibrium

III. A. Trivial Equilibrium

If we carefully look at the definition of equilibrium, we notice that there is always an equilibrium that is trivial; namely, (p, 0, 0, x, 0, 0, 0), where (p, x) solves

(i) Maximize $u_h(x_h)$

subject to $p^s(x_h^s - e_h^s) \leq 0, \forall s \geq 0,$

and $x_h >> 0$, and

(ii) $\sum_{h=1}^{H}(x_h - e_h) = 0.$

In this sense, the existence proof is trivial.

On the other hand, if we try to establish the existence of a non-trivial equilibrium, then we need to impose a certain condition on the punishment parameters. For when the punishment is zero, the trivial equilibrium is the unique equilibrium.

The following theorem states that a non-trivial equilibrium exists when the punishment parameters are sufficiently high.

III. B. Existence Theorem

In this section, we impose the following assumption.

A3. When Y has linearly dependent columns, it is written as Y = [Y' : Y"], where rank Y = rank Y'. Furthermore, Y" = Y'S with A > 0.

Theorem 1: For sufficiently high punishment parameters $(\pi_h^1, \ldots, \pi_h^S) \in \Re_{++}^S$, there exists a non-trivial equilibrium.

(Proof)

We proceed to prove the theorem in three steps.

Step 1: To show "when rank Y = I, then for any $(\pi_h^1, \ldots, \pi_h^S) \in \Re_{++}^S$, there exists an equilibrium (p, q, Z, x, α, β, γ) satisfying the following condition (5) ."

Condition (5): If $\sum_{h=1}^H \alpha_h^j = 0$, then there exists $t_h^j \geq 0, \quad \forall h$, satisfying $\sum_{h=1}^H t_h^j = y^{si}$, such that $z^{si} \geq \sum_{h=1}^H t_h^j \delta_h^{si}$, where $\delta_h^{si} = \begin{cases} 1 & \text{if } \dfrac{\partial u_h}{\partial x^{sc}} \leq \pi_h^s \\ 0 & \text{if } \dfrac{\partial u_h}{\partial x^{sc}} > \pi_h^s \end{cases}$

Step 2: To show that the conclusion of Step 1 is true when rank Y<I.

Step 3: To prove the theorem itself.

Proof of step 1[4]:

Define:

- $w_h^\varepsilon(x_h, \alpha_h, \beta_h, \gamma_h) = u_h(x_h) - \sum_{s=1}^S \pi_h^s(\sum_{j=1}^J (y^{si}\alpha_h^j - \gamma_h^{si})) - \varepsilon\sum_{j=1}^J((\alpha_h^j)^2 + (\beta_h^j)^2)$.
- $\Pi_\varepsilon = \left\{(p,q) \in \Re_{++}^{(s+1)c} \times \Re_+^J \mid p^{sc} = 1 \text{ for all s} \geq 0, \text{ and } 0 \leq p^{sc}.q^j \leq \dfrac{1}{\varepsilon}\right\}$.

[4]We follow Dubey, Geanakoplos and Shubik (1989)

- $B_h^\varepsilon(p,q,Z) = \Big\{(x,\alpha,\beta,\gamma) \in \Re_{++}^{(s+1)c} \times \Re_+^I \times \Re_+^I \times \Re_+^{SI} \mid$

$$p^0(x_h^0 - e_h^0) \le -q(\beta_h - \alpha_h),$$

$$p^s(x_h^s - e_h^s) \le Z^s\beta_h - \textstyle\sum_{h=1}^H \gamma_h^{si}, \forall s \ge 1,$$

$$y^{si}\alpha_h^i - \gamma_h^{si} \ge 0, \quad \forall s,i$$

$$\text{and} \quad (x,\alpha,\beta,\gamma)_{\infty} \le \frac{1}{\varepsilon}\Big\},$$

- $E_h^\varepsilon(p,q,Z) = \arg\max_{x,\alpha,\beta,\gamma} \big\{w_h^\varepsilon(x,\alpha,\beta,\gamma) \mid (x,\alpha,\beta,\gamma) \in B_h^\varepsilon(p,q,Z)\big\}$,:

- $C_\varepsilon = \Big\{(\zeta,\xi) \in \Re^{(S+1)C} \times \Re^I \mid (\zeta,\xi)_{\infty} \le \frac{H}{\varepsilon}\Big\}$,

- $\Psi_\varepsilon : \Pi_\varepsilon \times C_\varepsilon \times \Pi_{s,i}[0,y^{si}] \to \Pi_\varepsilon \times C_\varepsilon \times \Pi_{s,i}[0,y^{si}]$

such that

$$(p,q,\zeta,\xi,Z) \to \Big(\{(p,q) \in \arg\max_{(p',q') \in \Pi_\varepsilon}(p',q')(\zeta,\xi)\},$$

$$\Big\{(\textstyle\sum_{h=1}^H (x_h - e_h), \sum_{h=1}^H (\beta_h - \alpha_h), Z) \mid z^{si} = \frac{\sum_{h=1}^H \gamma_h^{si} + \varepsilon y^{si}}{\sum_{h=1}^H \alpha_h^i + \varepsilon}$$

$$\text{and } (x_h,\alpha_h,\beta_h,\gamma_h) \in E_h^\varepsilon(p,q,Z) \text{ for all h}\Big\}\Big)$$

Then, following Dubey, Geanakoplos and Shubik (1989), it is easy to check that, for $\varepsilon > 0$, the set $\Pi_\varepsilon \times C_\varepsilon \times \Pi_{s,i}[0,y^{si}]$ is compact, Ψ_ε is well defined and convex valued and has closed graph. So, applying Kakutani's Theorem, we obtain a fixed point $(p^\varepsilon, q^\varepsilon, \zeta^\varepsilon, \xi^\varepsilon, Z^\varepsilon)$ of Ψ_ε.

Now consider a sequence $\varepsilon_\nu \to 0$. Then, it turns out that at least for some subsequence of ε_ν, $(p^{\varepsilon_\nu}, q^{\varepsilon_\nu}, Z^{\varepsilon_\nu}, x^{\varepsilon_\nu}, \alpha^{\varepsilon_\nu}, \beta^{\varepsilon_\nu}, \gamma^{\varepsilon_\nu})$ is bounded. Therefore, passing to a convergent subsequence, we obtain the limit $(\bar{p},\bar{q},\bar{Z},\bar{x},\bar{\alpha},\bar{\beta},\bar{\gamma})$. It is standard to check that $(\bar{p},\bar{q},\bar{Z},\bar{x},\bar{\alpha},\bar{\beta},\bar{\gamma})$ is an equilibrium and satisfies the condition (5)[5]. ∎

[5]For the details, see Dubey, Geanakoplos and Shubik (1989).

Proof of step 2:

From our assumption (A3), we know that Y= [Y':Y"], rank Y = rank Y' , Y" = Y'S and A>0. Using this matrix, we can define the prices, demand and supply of the remaining bonds so that $(p',(q',q''),(Z',Z''),x',(\alpha',\alpha''),(\beta',\beta''),(\gamma',\gamma''))$ is an equilibrium of the original model. ∎

Proof of step 3:

In the proof of step 1, if the limit $\sum_{h=1}^{H}\overline{\alpha}_h^j$ is strictly positive, then it is clear that the equilibrium obtained as a limit is not a trivial one. Even if $\sum_{h=1}^{H}\overline{\alpha}_h^j = 0$, for sufficiently large $(\pi_h^1, \ldots, \pi_h^S)$, h ∈H, the limiting $\overline{z}^{sl}$ satisfies $\overline{z}^{sl} = y^{sl}$ (see Dubey, Geanakoplos and Shubik (1989)). So, the equilibrium we get is non-trivial. ∎

Remark: We have shown the existence of a financial equilibrium with the prices $p^{0C} = p^{1C} = \cdots\cdots = p^{SC} = 1$. Since this normalization is not necessary, it suggests that there may be a real indeterminacy of equilibria, analogous to the case of the standard (non-default) incomplete market model.

IV. Examples

Once we know that an equilibrium exists and that, for large enough punishment parameters, it is non-trivial, we move on to analyze the role of ex-ante redundant assets, that is to say, assets whose future payoffs , are some linear combination of the payoffs of the other assets in the economy (defined by the matrix Y). In the standard FGE model (the one without default) these assets are "useless". They are priced by arbitrage and they do not expand the extent of risk sharing opportunities in the economy. But, before we explicitly deal with this issue, let's first consider a few examples. In these examples we will find that, while the introduction of a redundant asset enlarges the set of equilibria, its effect is different in each of them. In the first example, the real allocation does not change

and thus the equilibrium utility remains the same after the introduction of a redundant bond. In the second example, however, the real allocation *does* change and the resulting equilibrium allocation is Pareto-superior to the original one.

IV. A. First Example

Let S=2, C=1, and H=2, and take a quadratic utility function: $u_h(x_h) = (k\, x_h^0 - \frac{1}{2}(x_h^0)^2) + \sigma^1(k\, x_h^1 - \frac{1}{2}(x_h^1)^2) + \sigma^2(k\, x_h^2 - \frac{1}{2}(x_h^2)^2)$ for $h = 1$ and 2. We will use the following numerical values:

$$e = (e_1, e_2) = \begin{pmatrix} 99 & 1 \\ 50 & 50 \\ 1 & 99 \end{pmatrix}$$

$k = 150$

$\sigma^1 = 0.01$, and $\sigma^2 = 0.99$.

First, consider the case where I = 1 and Y = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. By choosing different punishment parameters, we obtain different equilibria.

(i) If $(\pi_1^1, \pi_1^2, \pi_2^1, \pi_2^2) = (0,0,0,0)$, then nobody has an incentive to repay debt in the second period. Given our rational expectation assumption, the only possible equilibrium is the one with no asset trading. Therefore, every equilibrium has the form,
$((p^0, p^1, p^2), q, (z^1, z^2)) = ((p^0, p^1, p^2), 0, (0,0))$ with corresponding allocations
$((x_1^0, x_1^1, x_1^2), (x_2^0, x_2^1, x_2^2)) = ((e_1^0, e_1^1, e_1^2), (e_2^0, e_2^1, e_2^2))$.

With higher punishment parameters, households have greater incentive to repay the debt. We are especially interested in the situation where the realized yields (z^1, z^2) are both strictly between zero and one. The next case is one of those examples.

(ii) If $(\pi_1^1, \pi_1^2, \pi_2^1, \pi_2^2)$ = (200, 200, 1.3, 85), then we can find an equilibrium $((p^0, p^1, p^2), q, (z^1, z^2)) = ((1, 1, 1), 0.8096, (0.5728, 0.6656))$ with corresponding allocations $((x_1^0, x_1^1, x_1^2), (x_2^0, x_2^1, x_2^2)$ = ((56.6, 80.0, 35.9), (43.4, 20.0, 64.1)).[6] At this equilibrium, household 1 is buying the bond, and household 2 is selling it. As one might expect, the indeterminacy result in the standard incomplete market model is also true here, and, for each $(p^0, p^1, p^2) = (1, 1, p^2)$ such that p^2 is sufficiently close to 1, we get different equilibrium allocations.[7] In the following analysis, we will focus on the case $p^2 = 1$, to simplify the presentation.

Now introduce a second asset that is exactly the same as the first asset, i.e., $Y = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, and use the punishment parameters of (ii) above. Then the equilibrium is characterized by the 9-tuple $((p^0,p^1,p^2), (q^1,q^2), (z^{11},z^{12},z^{21},z^{22}))$, and one of them is given by ((1, 1, 1), (0.8096, 0.8096), (0.5728, 0.5728, 0.6656, 0.6656)). We will only consider the equilibria around this one.

To characterize these equilibria, we use the first-order conditions of each household, which is obtained from the maximization problem:

$$\underset{x_h,\, \alpha_h, \beta_h, \gamma_h}{\text{maximize}} \; u_h(x_h) - \pi_h^1((\alpha_h^1 - \gamma_h^{11}) + (\alpha_h^2 - \gamma_h^{12})) - \pi_h^2((\alpha_h^1 - \gamma_h^{21}) + (\alpha_h^2 - \gamma_h^{22}))$$

$$\begin{aligned}
\text{subject to} \quad & -(x_h^0 - e_h^0) + q^1((\alpha_h^1 - \beta_h^1) + q^2(\alpha_h^2 - \beta_h^2) \geq 0, \\
& -(x_h^1 - e_h^1) + z^{11}\beta_h^1 + z^{12}\beta_h^2 - \gamma_h^{11} - \gamma_h^{12} \geq 0, \\
& -(x_h^2 - e_h^2) + z^{21}\beta_h^1 + z^{22}\beta_h^2 - \gamma_h^{21} - \gamma_h^{22} \geq 0, \\
& \alpha_h^i - \gamma_h^{si} \geq 0 \quad \forall i, \\
& \gamma_h^{si} \geq 0, \alpha_h^i \geq 0, \; \beta_h^i \geq 0, \quad \forall i \\
& \text{and} \quad x_h >> 0.
\end{aligned}$$

---

[6] These numbers are calculated numerically, i.e., approximations. In the following analysis, we will see "=" even when the equality holds approximately.

[7] $p^0 = p^1 = 1$ is a standard normalization.

We also need the following conditions.

- Rational expectation conditions:

$$z^{11}(\alpha_1^1+\alpha_2^1)-(\gamma_1^{11}+\gamma_2^{11})=0,$$
$$z^{12}(\alpha_1^2+\alpha_2^2)-(\gamma_1^{12}+\gamma_2^{12})=0,$$
$$z^{21}(\alpha_1^1+\alpha_2^1)-(\gamma_1^{21}+\gamma_2^{21})=0,$$
$$z^{22}(\alpha_1^2+\alpha_2^2)-(\gamma_1^{22}+\gamma_2^{22})=0,$$

- Market clearing conditions:

$$(x_1^0-e_1^0)+(x_2^0-e_2^0)=0,$$
$$(x_1^1-e_1^1)+(x_2^1-e_2^1)=0,$$
$$(x_1^2-e_1^2)+(x_2^2-e_2^2)=0,$$
$$(\beta_1^1-\alpha_1^1)+(\beta_2^1-\alpha_2^1)=0,$$
$$(\beta_1^2-\alpha_1^2)+(\beta_2^2-\alpha_2^2)=0.$$

Consider the equilibrium ((1, 1, 1), (0.8096, 0.8096), (0.5728, 0.5728, (0.6656, 0.6656)). Calculating the first-order conditions, and after eliminating unbinding or redundant constraints, we obtain the following equations locally characterizing the equilibrium.

(5) $k-x_1^0=\lambda_1^0,$

(6) $\sigma^1(k-x_1^1)=\lambda_1^1,$

(7) $\sigma^2(k-x_1^2)=\lambda_1^2,$

(8) $-\lambda_1^0 q^1+\lambda_1^1 z^{11}+\lambda_1^2 z^{21}=0,$

(9) $-\lambda_1^0 q^2+\lambda_1^1 z^{12}+\lambda_1^2 z^{22}=0,$

(10) $-(x_1^0-e_1^0)+q^1\beta_1^1+q^2\beta_1^2=0,$

(11) $-(x_1^1-e_1^1)+z^{11}\beta_1^1+z^{12}\beta_1^2=0,$

(12) $-(x_1^2-e_1^2)+z^{21}\beta_1^1+z^{22}\beta_1^2=0,$

(13) $k - x_2^0 = \lambda_2^0,$

(14) $\sigma^1(k - x_2^1) = \lambda_2^1,$

(15) $\sigma^2(k - x_2^2) = \lambda_2^2,$

(16) $-\pi_2^1 - \pi_2^2 + \lambda_2^0 q^1 = 0,$

(17) $-\pi_2^1 - \pi_2^2 + \lambda_2^0 q^2 = 0,$

(18) $\pi_2^1 - \lambda_2^1 = 0,$

(19) $\pi_2^2 - \lambda_2^2 = 0,$

(20) $-(x_2^0 - e_2^0) + q^1\beta_2^1 + q^2\beta_2^2 = 0,$

(21) $-(x_2^1 - e_2^1) + z^{11}\beta_2^1 + z^{12}\beta_2^2 = 0,$

(22) $-(x_2^2 - e_2^2) + z^{21}\beta_2^1 + z^{22}\beta_2^2 = 0,$

(23) $z^{11}\alpha_2^1 - \gamma_2^{11} = 0,$

(24) $z^{12}\alpha_2^2 - \gamma_2^{12} = 0,$

(25) $z^{21}\alpha_2^1 - \gamma_2^{21} = 0,$

(26) $z^{22}\alpha_2^2 - \gamma_2^{22} = 0,$

(27) $(x_1^1 - e_1^1) + (x_2^1 - e_2^1) = 0,$

(28) $(x_1^2 - e_1^2) + (x_2^2 - e_2^2) = 0$[8]

From (16) and (17), we get immediately $q^1 = q^2$. So we use q instead of q^1 or q^2 Furthermore, it turns out that the following transformation of variables is useful: $z^{12} = z^{11} + td$ and $z^{22} = z^{21} - d$. Since $q^1 = q^2$, the new equilibrium must satisfy the conditions Rank $\begin{bmatrix} z^{11} & z^{11} & +td \\ z^{21} & z^{21} & -d \end{bmatrix} = 2$ and $t > 0$. Now, the first step is to calculate the demand function for h=1, using equations (5) to (12). They are:

[8]By Walras' law, we only need to consider any two of the market clearing conditions.

$$x_1^1 = e_1^1 + \frac{\bar{z}\{-\sigma^2 q k_1^0 + \sigma^1\sigma^2 k_1^2 \bar{z}\} + (q)^2 t\{\sigma^1 t k_1^1 - \sigma^2 k_1^2\}}{(q)^2\{\sigma^1 (t)^2 + \sigma^2\} + \sigma^1\sigma^2(\bar{z})^2} \text{ and}$$

$$x_1^2 = e_1^2 + \frac{\bar{z}\{-\sigma^1 tq k_1^0 + \sigma^1\sigma^2 k_1^2 \bar{z}\} - (q)^2\{\sigma^1 t k_1^1 - \sigma^2 k_1^2\}}{(q)^2\{\sigma^1 (t)^2 + \sigma^2\} + \sigma^1\sigma^2(\bar{z})^2},$$

where $\bar{z} = z^{11} + t z^{21} > 0$ and $k_h^s = k - e_h^s$.

The next step is to get the demand function for h=2, from (15) and (19). They are calculated as:

$$x_2^1 = k - \frac{\pi_2^1}{\sigma^1} \text{ and } x_2^2 = k - \frac{\pi_2^2}{\sigma^2}$$

Furthermore, by eliminating the variables $x_2^0, x_2^1, x_2^2, \lambda_2^0, \lambda_2^1, \lambda_2^2, \alpha_2^1, \alpha_2^2, \gamma_2^{11}, \gamma_2^{12}, \gamma_2^{21}$ and γ_2^{22} from the equations (13)-(26), we get the following equation (29):

(29) $(q)^2(\sigma^1 t(\pi_2^2 - \sigma^2(k - e_2^2)) + \sigma^2(\pi_2^1 - \sigma^1(k - e_2^1))$
$- \sigma^1\sigma^2\bar{z}(q(k - e_2^0) - (\pi_2^1 + \pi_2^2)) = 0.$

Now, define the functions f and g by $f(t) = \sigma^1 t(\pi_2^2 - \sigma^2(k - e_2^2)) + \sigma^2(\pi_2^1 - \sigma^1(k - e_2^1))$ and $g(q) = q(k - e_2^0) - (\pi_2^1 + \pi_2^2)$. Then, from equation (29), we obtain $\bar{z} = \frac{(q)^2 f(t)}{\sigma^1\sigma^2 g(q)}$.[9]

By substituting this to the demand functions of household 1, we get the following equations.

$$x_1^1 = e_1^1 + \frac{a(q,t)}{c(q,t)} \text{ and } x_1^2 = e_1^2 + \frac{b(q,t)}{c(q,t)}, \text{ where}$$

$$a(q,t) = k_1^1(q)^2 f(t)^2 - \sigma^2 q k_1^0 f(t) g(q) + \sigma^1\sigma^2 t g(q)^2(\sigma^1 t k_1^1 - \sigma^2 k_1^2),$$

$$b(q,t) = k_1^2(q)^2 f(t)^2 - \sigma^1 tq k_1^0 f(t) g(q) - \sigma^1\sigma^2 g(q)^2(\sigma^1 t k_1^1 - \sigma^2 k_1^2),$$

$$\text{and } c(q,t) = \sigma^1\sigma^2(\sigma^1(t)^2 + \sigma^2) g(q)^2 + (q)^2 f(t)^2.$$

---

[9] At an equilibrium, g(q)>0 because $x_2^0 - e_2^0 = q(\alpha_2^1 + \alpha_2^2) > 0$ so that

$q(k - e_2^0) - (\pi_2^1 + \pi_2^2) > q(k - x_2^0) - (\pi_2^1 + \pi_2^2) = 0$

Note that we effectively eliminated z^{11}, z^{21} and d from the demand functions. So, we are left with two equations, (27) and (28), and two unknowns, q and t. Approximately, the solution is (q,t) = (0.8096, 161.4).

Now, $\bar{z} = \frac{(q)^2 f(t)}{\sigma^1 \sigma^2 g(q)} = 108.0 = 0.5728 + 162.4 * 0.6656$. Therefore, if we take two numbers (z^{11}, z^{21}) that are close to (0.5728,0.6656) and that satisfy $z^{11} + 161.4 * z^{21} = 108.0$ and take (z^{11}, z^{21}) such that $z^{12} = z^{11} + 161.4 * d$ and $z^{22} = z^{21} - d$ for sufficiently small $d \neq 0$, then ((1, 1, 1), (0.8096, 0.8096), $(z^{11}, z^{12}, z^{21}, z^{22})$) is always an equilibrium.[10] Therefore, we have 2-dimensional *nominal indeterminacy even with fixed* p^2.

This nominal indeterminacy, however, does not translate into real indeterminacy. The reason is simple: the demand of household 2 (who is selling) is constant regardless of the values of the z's. By the market clearing conditions, household 1's demand is necessarily constant, although the financial market is now complete for him. In the next example, we introduce another household (who buys the bond at the equilibrium), and show that the equilibrium can be Pareto-superior when we introduce a redundant bond.

IV. B. Second Example

Let S=2, C=1, and H=3, and take the same quadratic utility function: $u_h(x_h) = (k\, x_h^0 - \frac{1}{2}(x_h^0)^2) + \sigma^1 (k\, x_h^1 - \frac{1}{2}(x_h^1)^2) + \sigma^2 (k\, x_h^2 - \frac{1}{2}(x_h^2)^2)$ for $h = 1, 2$ and 3. This time, we will use the following numerical values:

$$e = (e_1, e_2, e_3) = \begin{pmatrix} 99 & 99 & 2 \\ 50 & 51 & 99 \\ 26 & 25 & 149 \end{pmatrix}$$

K=250, $\sigma^1 = 0.01$, and $\sigma^2 = 0.99$.

[10] As long as the β's are strictly positive.

As in example 1, we start with the single asset case.

Consider the case where I=1 and $Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. By choosing the punishment parameters $(\pi_1^1, \pi_1^2, \pi_2^1, \pi_2^2, \pi_3^1, \pi_3^2) = (300, 300, 300, 300, 2, 170)$, we get an equilibrium $((p^0, p^1, p^2), q, (z^1, z^2)) = ((1, 1, 1), 0.9739, (0.6685, 0.9647))$ with corresponding allocations $((x_1^0, x_1^1, x_1^2), (x_2^0, x_2^1, x_2^2), (x_3^0, x_3^1, x_3^2))$=((63.553, 74.331, 61.115), (63.060, 75.669, 60.603), (73.387, 50.000, 78.283)) and the equilibrium utility levels (u_1, u_2, u_3)=(27303.94, 27118.44, 31620,03). At this equilibrium, household 1 and 2 are buying the bond and household 3 is selling it. We focus on the case p^2=1.

Now we introduce a second asset that is the same as the first asset, i.e., $Y = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, and use the same punishment parameters. The equilibrium is characterized by the 9-tuple $((p^0,p^1,p^2), (q^1,q^2), (z^{11},z^{12},z^{21},z^{22}))$. One of them is given by ((1, 1, 1), (0.9739, 0.9739), (0.6685, 0.6685, 0.9647, 0.9647)). As before, we consider the equilibria around this equilibrium.

Using the same notation, we can calculate the demand function of each household as follows:

- $x_1^1 = e_1^1 + \dfrac{a_1(q,t)}{c(q,t)}$ and $x_1^2 = e_1^2 + \dfrac{b_1(q,t)}{c(q,t)}$,
- $x_2^1 = e_2^1 + \dfrac{a_2(q,t)}{c(q,t)}$ and $x_2^2 = e_2^2 + \dfrac{b_2(q,t)}{c(q,t)}$, where

  $a_h(q,t) = \left(k_h^1(q)^2 f(t)^2 - \sigma^2 q k_h^0 f(t) g(q) + \sigma^1 \sigma^2 t g(q)^2 (\sigma^1 t k_h^1 - \sigma^2 k_h^2)\right)$,

  $b_h(q,t) = \left(k_h^2(q)^2 f(t)^2 - \sigma^1 t q k_h^0 f(t) g(q) - \sigma^1 \sigma^2 g(q)^2 (\sigma^1 t k_h^1 - \sigma^2 k_h^2)\right)$,

  h = 1,2 and $c(q,t) = \sigma^1 \sigma^2 (\sigma^1 (t)^2 + \sigma^2) g(q)^2 + (q)^2 f(t)^2$.
- $x_3^1 = k - \dfrac{\pi_3^1}{\sigma^1}$ and $x_3^2 = k - \dfrac{\pi_3^2}{\sigma^2}$.

This time, the market clearing conditions are

$(x_1^1 - e_1^1) + (x_2^1 - e_2^1) + (x_3^1 - e_3^1) = 0$, and

$(x_1^2 - e_1^2) + (x_2^2 - e_2^2) + (x_3^2 - e_3^2) = 0$.

Thus, we have again two equations and two unknowns. The solution is approximately (q, t) = (0.9739, 107.0). The situation is the same as in example 1. Since

$\bar{z} = \dfrac{(q)^2 f(t)}{\sigma^1 \sigma^2 g(q)} = 103.9 = 0.6685 + 107.0\ (0.9647)$, if we take (z^{11}, z^{21}) close to (0.6685, 0.9647) and satisfying $z^{11} + 107.0 * z^{21} = 103.9$, and $z^{12} = z^{11} + 107.0 * d$ and $z^{22} = z^{21} - d$, for sufficiently small $d \neq 0$, then ((1, 1, 1), (0.9739, 0,9739), $(z^{11}, z^{12}, z^{21}, z^{22})$) is always an equilibrium.[11] So, again, we have 2-dimensional nominal indeterminacy even with fixed p^2.

The prime importance of this example lies in the fact that the equilibrium allocation is different. Although the demand of household 3 is constant, the demand of household 1 and 2 are now $((x_1^0, x_1^1, x_1^2), (x_2^0, x_2^1, x_2^2))$ = ((63.552, 75.230, 61.107), (63.061, 74.770, 60.060)) and their corresponding utilities are higher than in the previous equilibrium.

For this example, we know that the introduction of an-ex ante redundant asset may Pareto improve the equilibrium allocation, when defaulting is allowed. We can interpret this result as an explanation, with the help of default, of the abundance of "seemingly" redundant assets in the real world. Though ex-ante they seem redundant, the fulfillment of expectations at equilibrium may create new risk sharing opportunities.

V. The Role of Ex-ante Redundant Assets.

In a General Equilibrium framework, the introduction of an asset has two effects. First, it may change the spanning of the return (payoff) matrix. In the case the spanning is increassed, more risk sharing opportunities are open for the agents in the economy. Second, prices may change. This change in prices may affect differently to different

[11] As long as the β's are strictly positive.

agents and therefore, even when the spanning is increased, the new equilibrium may not Pareto-dominate the previous one (without the new asset). In our model, both effects are present.[12]

Our existence proofs tell us that an equilibrium exists for any Y matrix. There are two ways in which we can analyze the role of redundant assets in our model. The first, and obvious, approach would be to compare utility levels in the equilibrium obtained with a particular Y with the equilibrium obtained in an economy in which Y is increased to include a redundant asset (let's denote it by asset i). Since nothing guarantees here that both equilibria are going to be close by, utility comparisons are going to be difficult. This loss of continuity is of great difficulty to get any result on Pareto Improvement. Our second example illustrates the possibility of such a Pareto Improvement but, in general, we know that some agents may be worse off in the new equilibrium.

To get some general result we need to restore continuity. One possibility would be to "construct" the result rather than to "check" it. From our existence proof we know that there always exists an equilibrium in which ex-ante redundant assets remain redundant (see the proof of step 2). At this equilibrium we could perturb some (or, all) of the agents' punishment parameters for that asset (π_h^j) and try to find the existence of perturbations that would create a Pareto Improvement (i.e., $Du_h(x_h^*) \geq 0$ for all h and with strict inequality for some h). The question is, what would a positive answer to this question imply? After all, that perturbation would imply that some agents' consider the punishment associated to that asset differently to the one of the asset (or, combination of assets) to which the new asset is redundant. There are many arguments to assume the punishment parameter being asset specific. For instance, instead of thinking on the punishment term as days in jail, deportation, etc., if we think of it as the utility loss when some assets are expropriated after defaulting, it is easy to understand the parameter being asset specific.

[12]In our two examples we obtained an equilibrium, after the introduction of the redundant asset, in which assets spanning increases ($t \neq 0$ and d can take any value). However, we only get a Pareto improvement in the second example.

An individual may get two different mortgages for two of his homes. He may like living in one of them more than in the other which explains the different π's for each asset.[13] Another argument to sustain the possibility of different parameters for different assets would be reputation effects. The loss of reputation when defaulting can be very different for different assets. Keeping this in mind, a positive answer to the previous exercise would imply that, provided there is the appropriate diversity on "default tastes" about assets, an equilibrium will arise in which everybody is better off. Unfortunately, we still cannot offer a result in this direction.

Our model is a model of asset formation (or, on the origin of assets). Starting from any asset structure, the individual decisions of the agents in the economy give birth to new assets. We believe this the most genuine approach to financial innovation. The final asset configuration depends explicitly on the primitives (preferences, endowments, default attitudes and set of contracts (Y)) of the whole economy rather than on the characteristics of some privileged traders. The existence of such a Pareto-improvement in our set-up would tell us that the "market" is able to design a "better" security than the naive one we arbitrarily introduce.

Finally, just to mention that in our model we explicitly assume the agents' punishment parameter to be agent and state specific but not asset specific. If we assume this last property, the obtention of a non-redundant asset from a redundant one would be an easy exercise (at least in a "generic" sense). However, the Pareto-improvement result is not straightforward.

VI. Conclusions and topics for future research.

In a similar model to ours, Dubey, Geanakoplos and Shubik (89) show by mean of an example that by choosing the appropriated punishment parameter we can improve the

[13]Strictly speaking, the different services obtained by using each home should be included in the agent's utility function. However, the specification of different π's for different assets captures this types of preferences.

allocation in an economy where agents are allowed to fail to honor their commitments. Our analysis goes one step beyond in this direction. With the help of a set of examples we illustrate how fulfilled agents expectations may create new risk sharing opportunities for the agents in the economy. In particular, we show that assets spanning can be larger at equilibrium than before the opening of the markets.

In a pure exchange economy, assets are just financial arrangements between agents. There is not a production technology determining the extent of risk sharing opportunities in the economy. Our result suggests that one of the reasons of incompleteness in this type of economies is the inefficient design of contracts rather than the numerical insufficiency of such contracts. In our model, the market is sometimes able to "define" better securities (contracts) than those agents previously agreed on trading among each other. A second implication of our analysis is that in the design of securities it is important to take into account the possible default the sellers of such securities may incur. For instance, there is an incentive for designers to create securities in which future sellers will not default. The reason being that this future seller will pay a premium to buy such security since he knows that if he sells it in the future he will not default and consequently, he will not incur in the "punishment" cost we defined in our model.

There is still a long way to go to get a good understanding of the General Equilibrium effects of default and, most importantly, to get a good framework in which to price default risk. In addition to pricing default risk, there are other lines of research that can be pursued following the present framework. First, our model sets the basis for a financial intermediary or ownership of a new exchange to arise. If at equilibrium we obtain a financial structure that Pareto-improves the original one, there is an incentive for an agent to invest resources in learning that, more efficient, financial structure and to offer to the rest of market participants to trade those securities on his exchange as opposed to trading the original ones among each other (therefore incurring in big losses of utility due to the punishment term). Second, it seems quite interesting to introduce a monetary

punishment (say, a trader who defaults can be excluded from trading on that market from the moment he defaults on) as opposed to our non-monetary punishment. Finally, other possible extension of the model would consist on relaxing (never eliminating) the anonymity in the market. For instance, we could introduce some specific traders, completely identified by the rest of the market, with known propensities to default. Explicit examples could be banks, as a clear example of a low propensity to default, and issuers of "junk" bonds, as a clear example of a high propensity to default. In this framework, it would be interesting to analyze how the demand for (and price of) the assets these agents offer changes when the default risk in the other markets changes. In future work we will try to address some of these issues.

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