Interbank Market Integration under Asymmetric Information*

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Abstract

While domestic interbank markets are considered to work in an efficient way, cross-country bank lending appears to be subject to market imperfections leading to liquidity shortages and persistent interest rate differentials. Although these differences could be attributed to exchange rate risk, we argue that the main barrier to an integrated international interbank market is the existence of asymmetric information between different countries, which may prevail in spite of monetary integration or successful currency pegging. In order to address this issue, we consider a model where banks cope with liquidity shocks either by borrowing unsecured or through repo operations, or else by liquidating assets. We study the scope for international interbank market integration with unsecured lending when cross-country information is noisy. We find not only that an equilibrium with integrated markets need not always exist, but also that when it does, the integrated equilibrium may coexist with one characterized by interbank market segmentation. Therefore, market deregulation, per se, does not guarantee the emergence of an integrated interbank market. The effect of a repo market which, a priori, was supposed to improve efficiency happens to be more complex: it reduces interest rate spreads and improves upon the segmentation equilibrium, but it may destroy the unsecured integrated equilibrium. The introduction of other transnational institutional arrangements, such as multinational banking, correspondent banking and the existence of "too-big-to-fail" banks may reduce cross country interest spreads and provide more insurance against country wide liquidity shocks. Still, multinational banking, as the introduction of repos, may threaten the integrated interbank market equilibrium.

JEL Classifications: G15, G20, F36

Key Words: Banking theory, Asymmetric information, Financial integration, Interbank markets, Diamond-Dybvig

1 Introduction

The objective of this paper is to study the effects of cross-country asymmetric information on the structure of financial markets. Our main concern is the design of money markets and the role of repo and (unsecured) interbank markets in an international framework, but our results carry over to a more general framework of the analysis of cross-country direct investment, covering the cross-country market both for bonds and equity. The creation of an integrated interbank market is particularly relevant in order for banks to cope efficiently with liquidity shocks. Interbank markets are instrumental in allowing for a smooth working of the payment systems (so that a bank that is lacking liquidity in the payment system is able to borrow from another bank), and in channeling liquidity to the banks and countries that need it most. Both repo and unsecured interbank lending allow to cope with liquidity shocks. Still, because unsecured markets are based on peer monitoring, they introduce market discipline, thus playing the role unsecured deposits may play when depositors receive information (Calomiris and Kahn, 1991).

The collapse of a well functioning unsecured interbank market proved to be crucial in the context of Eastern Asia financial crises where interbank lending by foreign countries had played a key role in these countries' funding structure. It also sheds light on the construction of a European interbank market after the creation of the European Monetary Union. Finally, the issue has direct policy implications since, if the interbank market's provision of liquidity is inefficient, this calls for regulatory intervention.

The role of the interbank market to cope with bank specific liquidity shocks and avoid unnecessary liquidation of long term investments was first acknowledged in Bhattacharya and Gale (1987). Later contributions built upon this role while introducing either moral hazard (Rochet and Tirole), aggregated liquidity risk (Allen and Gale 2000) or else by introducing credit risk (Freixas, Parigi and Rochet 2000). These studies yield similar results pointing at the potential contagion provided by an interbank market as well as the effect the network of interbank lending may have on financial fragility. Furthermore, Bhattacharya and Fulghieri (1994) analyze the efficiency of an interbank market in a framework where banks face uncertain timing of liquidity returns, and Holmström and Tirole (1998) discuss the role of liquidity provision by the public sector.

Although our paper does not focus on financial crises, we also consider two different interbank lending networks, segmentation vs. integration, where the collapse of integration may be interpreted as a financial crisis.

As in Rochet and Tirole, we consider "peer monitoring" as a key factor in improving the efficiency of the interbank market; still we are concerned with asymmetric information and therefore about the quality of signals rather than about moral hazard and the way to discipline borrowers. From that perspective, our work is related to Broecker (1990) and to Flannery (1996) who consider models of asymmetric information and credit risk. An important difference to their work is that the value of a signal is given exogenously in their model, while in our model it is endogenous, as it results from the equilibrium behavior of borrowers.

Our model uses a Diamond and Dybvig type of framework, where consumers are uncertain about the timing of their consumption needs. This generates liquidity shocks, which we assume are present both at the individual and at the aggregate level. To be able to cope with these shocks, banks can invest in a storage technology. Because this technology has a lower return than alternative investment opportunities, it is efficient for banks to use the interbank market. We consider both an unsecured interbank market and a repo-market where government bills are traded. In order to introduce credit risk, the model assumes that banks have some risk of failure. As in Rochet and Tirole (1996), banks monitor each other in the interbank market, thus obtaining a signal on the solvency probability of each of their peers. The key assumption of our model is that cross border information about banks is less precise than home country information. Hence, when a bank tries to borrow from a foreign bank, it does so either because it belongs to a liquidity short country or else because it has generated a "bad" signal at a domestic level and is therefore unable to borrow in his home country. As it is intuitive, depending on the equilibrium probability distribution of the two types of motivations for borrowing abroad, an integrated interbank market may exist or not.

Our contribution is to show, using this framework, that having a single currency is no guarantee for having a single uninsured interbank market. Namely, a segmented interbank market is **always** an equilibrium, while the emergence of an integrated international market is only possible when the quality of cross-border information is sufficiently good. A further result is that the integration of

markets does not always yield a more efficient outcome.

On the other hand, the repo-market provides a perfect medium to channel liquidity between banks and across countries. However, the secured nature of the repo reduces banks' incentives for peer monitoring. As a result, banks with a low probability of solvency are able to obtain liquidity via the repo market and to avoid liquidation, although their liquidation value might exceed their expected continuation value. Surprisingly, we also establish that the combination of both types of markets need not yield a more efficient allocation, as it may lead to the collapse of the unsecured integrated market.

This paper is organized as follows. In section 2 we set up the basic model of interbank credit and the structure of signals. Section 3 analyzes the unsecured interbank market in a two country setting, while section 4 is devoted to the general and more complex case when the two markets -unsecured and repo- are coexistent. Section 5 extends the analysis by allowing for the introduction of correspondent banking, transnational banking and too-big-to-fail banks. Section 6 offers some concluding remarks on the policy implications of our results. All proofs are in the appendix.

2 The Model

We consider an economy with two countries.

Consumers

In each country, there is a continuum of consumers of a total measure of one, who possess one unit of endowment each at time 0. Consumers are risk neutral and face liquidity shocks as in a Diamond and Dybvig (1988) type of model: they need to consume either at time 1 or at time 2. At time 0, consumers deposit their endowments in a bank, and can withdraw funds at the time they need to consume. Deposits are fully insured by a deposit insurance, so no bank runs occur.[1] We assume that the demand-deposit contract promises them a consumption of 1 in either period, $C_1 = 1$ or $C_2 = 1$.

Banks' Investment

There is an infinite number of risk neutral banks. Each receives the endowments of a continuum of consumers at time 0, and invest them either in a risky

[1] The deposit insurance company is assumed to raise funds at unit cost. The deposit insurance is fair in that banks have to bear the expected cost of their failures.

technology or in reserves (storage technology). Furthermore, a bank can buy government T-Bills, which are issued at price B_0, and yield 1 in the second period with certainty. Denote the investment in the risky technology I, the one in T-bills $T_0 B_0$ and the one in reserves $s_0 \equiv 1 - I - T_0 B_0$. We simplify the analysis by assuming that the amount of banks' equity is negligeable but that banks are, nevertheless, residual claimants.

Each unit invested in the risky technology yields an uncertain payoff $\widetilde{R}$ at time 2, where $\widetilde{R} = R$ (solvency) with probability $p \geq \frac{1}{2}$, and $\widetilde{R} = 0$ (insolvency) with probability $1 - p$. Investment in the technology is assumed to be ex-ante efficient in that $pR > 1$. This risky asset can also be (partially) liquidated at time 1, with the following technology: liquidation of ΔI units gives a liquidation value of $l(\Delta I)$, where $l(\Delta I)$ is increasing and concave. For simplicity we use a logarithmic liquidation function $l(\Delta I) = \ln(\Delta I + 1)$.[2]

Because banks have limited liability they have incentives to forbear no matter how bad the prospects of the risky technology are. As a consequence, in our model bank closure will only occur if it is triggered by a liquidity shock.

Liquidity Shocks

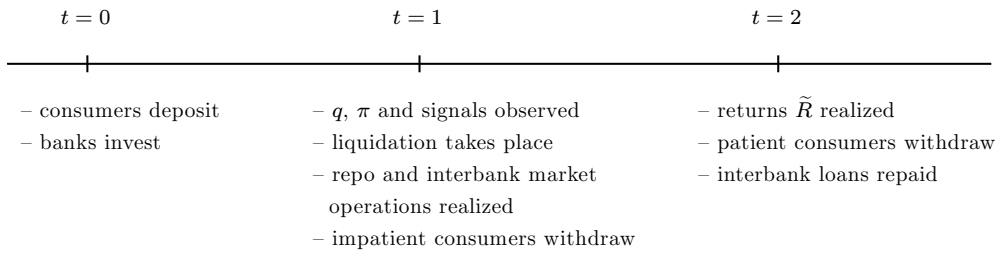
Our model combines bank specific liquidity shocks with country-wide ones. Banks are uncertain about the liquidity demand they face at time 1. For a fraction q of all banks, a high fraction of consumers π_H is impatient and wishes to withdraw at time 1. A fraction $1 - q$, on the other hand, faces a low liquidity demand π_L, $\pi_L < \pi_H$. The remaining consumers are impatient and withdraw at time 2.

The variable q reflects the country wide aggregate demand for liquidity and is uncertain as well. We assume liquidity shocks to occur with probability 1/2 and restrict our comparative statics analysis to changes in the probability of solvency. Thus with probability 1/2, $q = q_B$, in which case a country is in a state of high aggregate liquidity demand, because many banks face high time-1 withdrawals (π_H). On the other hand, with probability 1/2 the country faces a low liquidity demand with $q = q_A < q_B$, so that fewer banks face a high amount of withdrawals. For the sake of simplicity we assume $q_B + q_A = \overline{Q}$. The probability of solvency and liquidity are uncorrelated.

[2] We assume that liquidation of an asset is equivalent to selling the asset to someone outisde the banking sector, e.g. to institutional investors who are not participating in the interbank market.

Banks can manage their liquidity needs at time 1 by borrowing or lending in the interbank market, by buying or selling T-Bills on the repo market, by liquidating assets, and by storing reserves until the next period. Throughout the base model, we assume that a bank cannot split its loan demand among several banks. If illiquid banks fail to meet their creditors' demand, they are forced into bankruptcy, liquidating all assets. In this case, the proceeds are absorbed by the deposit insurance company.

The timing of the model is the following:



Information

At time 0, the ex-ante probability of being solvent, p, is common knowledge. At time 1, all banks in a given country receive a non-verifiable signal s_D about the solvency of their domestic counterparts. We assume that the same signal is observed about each bank from all its peers in a given country. The signal can either be good ($\overline{s}$) or bad ($\underline{s}$), it is defined as

$$prob(s_D = \overline{s}\,|\tilde{R} = R) = prob(\,s_D = \underline{s}\,|\tilde{R} = 0) = \alpha.$$

Our setting thus reflects the existence of "soft" domestic information regarding the different banks' strategies, their risk-taking behavior, and their accounting strategy (loan-loss provisions, window dressing and so on). This information is not directly observable, even at a cost by other banks that are not members of the place.

Denote $\theta \equiv p\alpha + (1-p)(1-\alpha)$ the ex-ante probability that the good signal is received about a bank.[3] We assume that the signal is informative, i.e., $\alpha \in (\frac{1}{2}, 1]$.

The above expressions allow us to compute the probability $\overline{p}$ of success condition on the bank having produced the signal $s_D = \overline{s}$. It is given by $\overline{p} \equiv prob(\tilde{R} = R\,|s_D = \overline{s}) = \frac{p\alpha}{\theta}$ and we denote, symmetrically, $\underline{p} \equiv prob(\tilde{R} = R|\underline{s}) = \frac{p(1-\alpha)}{1-\theta}$.

[3]Because we are dealing with a continuum of banks, the ex-ante probability is equal to the ex-post fraction of banks of this type.

The signals received in the foreign country can only be observed with some noise. We make the following assumption:

Assumption 1 (Noisy cross-country information)

$$prob(s_F = \overline{s}|s_D = \overline{s}) = prob(s_F = \underline{s}|s_D = \underline{s}) = 1 - \beta$$

This holds regardless of the bank's solvency, where $\beta \in (0, \frac{1}{2})$ denotes the probability that the domestic signal is received wrongly in the foreign country.

Thus, the lower β, the better is the information flow between countries. Each bank is then characterized by a pair (s_D, s_F), denoting the signals that have been received by domestic and foreign banks about this particular bank. Assumption 1 implies that s_D is a sufficient statistics for (s_D, s_F).[4]

Note that a bank cannot observe its own solvency, but only the signal. Therefore, it has no informational advantage over the other market participants regarding its own solvency. This assumption is required in order to leave aside the additional issues of moral hazard.

The second assumption we make regarding the signal structure is that it is not profitable to lend to a low-signal bank (in either country). Because $\underline{s}$-banks have a higher probability of failure than $\overline{s}$-banks, lenders need to demand a higher interest rate from those banks in order to break even. We assume here that this rate would need to be so high that it is impossible for a bank to meet its interest rate payments at time 2.

Assumption 2 (No lending to $\underline{s}$-banks) The expected net present value of a loan to a bank is positive if $s_F = \overline{s}$ and negative if $s_F = \underline{s}$.

Assumption 2 is stated for the foreign signal, however, because the domestic signal is more precise than the foreign one, this implies that the same result holds for $s = s_{D.}$.

Finally, we assume that when a bad signal is observed, it is efficient to close down the bank. That is, we assume

Assumption 3 (Efficient closure of bad-signal-banks)

$$prob(\widetilde{R} = R|s_F = \underline{s})R \leq l'(I)$$

[4] This can easily be checked that for any signal combination, for example for $(\overline{s}, \overline{s})$, we have $p(\tilde{R} = R|(\overline{s}, \overline{s})) = \frac{p\alpha(1-\beta)}{p\alpha(1-\beta)+(1-p)(1-\alpha)(1-\beta)} = p(\tilde{R} = R|s_D = \overline{s})$

Because of concavity of $l(\cdot)$, this assumption implies that it is efficient to liquidate the entire risky technology of a bank when the bad signal has been received in the foreign country. Assumption 3 obviously implies that it is also efficient to close down this bank with better quality of information, e.g. after observing the domestic bad signal $\big(\ \underline{p}R \leq l'(I)\big)$.

On the other hand, the assumption $pR > 1$ implies that $\bar{p}R > 1$, so that (partial) liquidation of a high-signal bank is never efficient.

3 Cross Country unsecured interbank market integration

In this and all subsequent sections, we focus on the banks' time 1 problem of managing their liquidity needs.[5] We regard two countries with different aggregate liquidity demands at time 1. W.l.o.g., we assume that for $q = q_A$, there is excess liquidity, while for $q = q_B$, the liquidity shortage is so high that both lenders and borrowers liquidate. Notice that while there are country-wide aggregate shocks, the assumption $q_B + q_A = \overline{Q}$ implies that there is no aggregate liquidity shock when both countries are taken together. Let us denote A the country with excess liquidity, and B the one with a liquidity deficit. We assume that there are no legal or infrastructural barriers to the emergence of an international money market.

3.1 Structure of equilibrium

At time 1, when the liquidity shock occurs, banks with liquidity needs can find themselves with one of the four pairs of signals: $(s_D, s_F) = (\overline{s}, \overline{s}), (\overline{s}, \underline{s}), (\underline{s}, \overline{s})$, or $(\underline{s}, \underline{s})$. Because of Assumption 2, only banks with the signal pair $(\overline{s}, \overline{s})$ are able to choose in which country to borrow. The other banks are constrained to using either the domestic market if they are $(\overline{s}, \underline{s})$, or the foreign market if they are $(\underline{s}, \overline{s})$. Banks of type $(\underline{s}, \underline{s})$ are not able to borrow at all. This is illustrated in figure 1. Moreover, in equilibrium, lenders will be able to update their information on foreign borrowers' solvency on the basis of their strategic behavior and on equilibrium interest rates. Because of the limited access of $(\overline{s}, \underline{s})$ banks to the foreign interbank market and of $(\underline{s}, \overline{s})$ to the local interbank market,

[5] The time 0 investment problem is not considered.

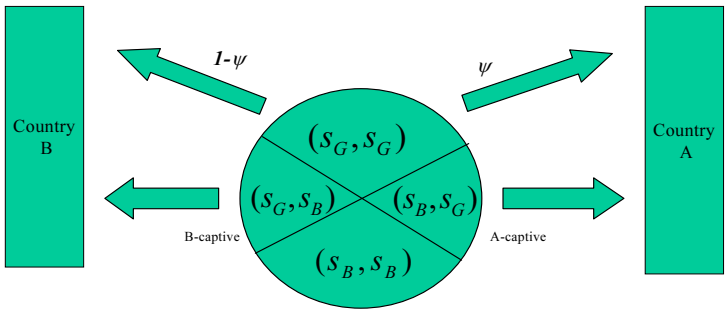


Figure 1: *Borrowing choices for country-B banks: banks can only obtain a loan in the country where the good signal* $\overline{s}$ *has been obtained about them. In equilibrium, a fraction* $1-\psi$ *of* $(\overline{s},\overline{s})$*-banks borrows domestically while* ψ *borrow abroad.*

the equilibrium is not a fully integrated market, and insurance against liquidity shocks is only partial.

In order to characterize the equilibrium, notice, first that the interbank interest rates, $r_i, i = A, B$ are crucial in determining how much of the long run technology the banks will liquidate. In equilibrium, in each country i the demand for liquidity Ω_i equals the supply of liquidity Λ_i. These are given by the following expressions:

$$\Omega_i \equiv \sigma_L^i \pi_L + \sum_{k=D,F} \sigma_k^i \pi_H \tag{1}$$

$$\Lambda_i \equiv \sum_{k=L,D,F} \sigma_k^i \left[s_0 + l(\Delta I_k^i) \right] \tag{2}$$

where the superindex $i = A, B$ denotes the country, and subindex k refers to the different agents active in the local interbank market, $k = L, D, F$. Here, L denotes lenders, D refers to domestic borrowers, and F to foreign borrowers, and σ_k^i is the (equilibrium) measure of each type of bank in country i. Using the bank's first order decisions, we can establish the following Lemma:

Lemma 1 *The supply of liquidity in country* i*,* Λ_i*, is non-decreasing in the domestic interest rate* r_i*, where* $i = A, B$*.*

Proof. See appendix. ■

For low levels of interbank interest rates, only reserves are used and the credit market does not develop. For larger levels of interest rate, a market begins to

develop where lending banks do not liquidate, but lend out their reserves and have a zero expected yield. When we consider higher interest rates, both borrowing and lending banks liquidate their long term assets and the expected yield for the lending banks is strictly positive.

Unsurprisingly, the interbank interest rate is an increasing function of the aggregate demand for liquidity. However, markets do not provide for a perfect smoothing of liquidity shocks: banks with a high amount of early withdrawals π_H liquidate less than banks facing π_L. The reason is that the expected cost of borrowing is higher than the expected return from lending. This can be seen by computing the expected return on lending and the expected cost of borrowing conditional on the banks' survival: lenders obtain an expected return of $\bar{p}(1+r)$ on loans, while borrowers pay $(1+r)$ on each unit borrowed.

Equilibrium in both countries requires

$$F_A \equiv \Omega_A - \Lambda_A = 0 \tag{3}$$

$$F_B \equiv \Omega_B - \Lambda_B = 0 \tag{4}$$

It is quite intuitive that the equilibrium will be characterized by a unilateral flow of borrowers from the country with high liquidity needs and high interest rates to the country with excess liquidity and low interest rates. The following lemma establishes this point. Denoting by ψ_i the fraction of $(\overline{s}, \overline{s})$-banks in country i that choose to borrow abroad, we prove that $\psi_A = 0$.

Lemma 2 *Borrowers from the excess liquidity country A borrow only in country A ($\psi_A = 0$). In addition, in the case of segmented markets ($\psi_A = \psi_B = 0$), interest rates in country B are higher than in country A ($r_A < r_B$).*

Proof. See appendix. ■

This will simplify our notations, since it implies having only one foreign market, the one for borrowers of country B that wish to borrow in country A. Thus, denote $\psi \equiv \psi_B$, and r_A, r_B, and r_F the domestic rates of countries A and B and the rate for foreign borrowers, respectively. A perfect Bayesian equilibrium in the inter-bank market is a quintuple $(\psi^*, r_A^*, r_B^*, r_F^*, p_F^*)$, specifying the fraction ψ^* of $(\overline{s}, \overline{s})$-banks from country B borrowing abroad as well as the interest rates demanded and the corresponding rationally updated probability p_F^*. *Integration* will be defined as the case where in equilibrium $\psi^* > 0$, while *segmentation* is defined as the opposite case when $\psi^* = 0$.

In an integrated equilibrium, with $0 < \psi^* < 1$, two conditions need to be satisfied: on the one hand, lenders in country A should be indifferent between lending to either country:

$$\overline{p}\,(1 + r_A) = p_F\,(1 + r_F)\,. \tag{5}$$

On the other hand, borrowers able to borrow at home or abroad should be indifferent where to borrow, thus

$$r_B = r_F. \tag{6}$$

In the segmented equilibrium, $\psi^* = 0$, $p_F = prob(\tilde{R} = R|\underline{s}) = \underline{p}$, and r_F is indeterminate. The equilibrium results simply from the two market clearing conditions (3) and (4), one for each country.

Proposition 3 *Under integration we have* $r_B(\psi^*) \geq r_F(\psi^*) > r_A(\psi^*)$, $p_F(\psi)(1+r_F^*) = \bar{p}(1+r_A^*)$. *Under segmentation we have* $r_F(0) > r_B(0)$, $p_F = prob(\tilde{R} = R|\underline{s}) = \underline{p}$.

Proof. See appendix. ■

Notice that in our terminology, integration includes the case where $\psi = 1$, and $r_B(1) > r_F(1)$: the interest rate charged abroad is still lower than the domestic one, and all banks who are able to, will borrow abroad. Only "captive" $(\overline{s}, \underline{s})$ banks will be forced to borrow domestically at the higher rate $r_B(1)$.

When integration occurs, we have the following implications:

- Lenders in the B country draw an informational rent from the liquidity shortage.
- Lending banks in the A country finance a heterogenous population of foreign banks, consisting of all the "bad risks"$(\underline{s}, \overline{s})$, and a fraction ψ of the "good risks", $(\overline{s}, \overline{s})$.

3.2 Integration versus Segmentation

We will now proceed to establish under what conditions segmentation and integration occur, that is to determine the equilibrium values of ψ. A crucial element in the analysis is the information updating of foreign lenders. Observing the

measure of banks trying to borrow abroad, lenders can infer ψ and therefore the fraction of $(\overline{s},\overline{s})$ and $(\underline{s},\overline{s})$ banks among the foreign borrowers. As it is intuitive, a higher fraction of $(\overline{s},\overline{s})$ banks implies a higher updated probability of solvency p_F. Moreover, from (5) it follows that the premium on foreign loans, $r_F - r_A$ is decreasing in ψ.

Lemma 4 *A foreign bank's updated probability of solvency $p_F(\psi)$ is increasing in ψ. For $\psi > 0$, the premium charged to foreign borrowers $r_F - r_A$ is decreasing in ψ.*

Proof. See appendix. ■

In order to obtain ψ^*, we first derive the demand and supply of liquidity in the domestic and foreign markets for given values of ψ. Market clearing then allows us to characterize interest rates in both markets as functions of ψ. The results of this analysis are presented in the following Lemma:

Lemma 5 *The domestic interest rate in country B, r_B, is decreasing in ψ.*

Proof. See appendix. ■

Unsurprisingly, a higher level of cross-country borrowing eases the liquidity shortage in country B and leads to a lower interest rate r_B.

For $\psi = 0$, interest rates in country B are lower than the foreign interest rates, $r_B(0) < r_F(0)$. This, together with the results from Lemma 5, has two implications: first, it might be the case that $r_F(\psi) > r_B(\psi)$ for all values of ψ. In this case, borrowing abroad is never an equilibrium strategy. Second, if $r_F(\psi) \leq r_B(\psi)$ for some ψ, an equilibrium with integration exists. However, nothing precludes the segmentation equilibrium to be obtained as well.

Figure 2 illustrates the interest rates $1 + r_B$ and $1 + r_F$ as functions of ψ for different parameter constellations. In all of them, a segmented equilibrium with separated inter-bank markets (point A) is possible. In case (i), it is the only equilibrium because for all values of ψ, the rate charged abroad is strictly higher than the one in country B.

In cases (ii) and (iii), the curves $1+r_B(\psi)$ and $1+r_F(\psi)$ cross for some $\psi > 0$. In case (ii), three equilibria coexist, two of them with an active international market (at the crossing points B and C): borrowers face the same interest rates

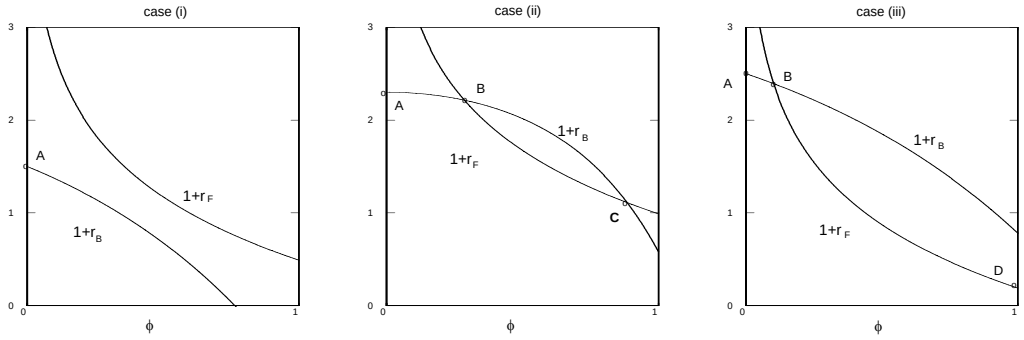


Figure 2: *Interest rates for country-B borrowers as functions of ψ: Liquidity short banks in country B can either borrow domestically at rate r_B, or in the foreign market at rate r_F. Both rates depend on the fraction of $(\overline{s}, \overline{s})$-banks borrowing abroad, ψ. The existence of an integrated equilibrium depends on the relative location of both rates*

in both countries, and they are indifferent as to where to demand a loan. Still, C implies a higher level of integration than B. Finally, in case (iii), there is an equilibrium with integration at point D, where all banks prefer to borrow abroad ($\psi = 1$) and the only banks in country B to borrow domestically are the $(\overline{s}, \underline{s})$ ones.

Under which circumstances can an equilibrium with an integrated market be obtained? Since the foreign interest rate is increasing in the informational premium, integration becomes impossible for very high β. Another important parameter is the difference in liquidity needs across countries, $\Delta q \equiv q_B - q_A$: Only when Δq is sufficiently large, integration is possible.

The following proposition outlines the results of this market imperfection.

Proposition 6 *(Multiple equilibria) If the difference in aggregate liquidity demand in the two countries is large enough, both the integrated and segmented market equilibrium exist. Otherwise, only the segmented market equilibrium exists.*

Proof. See appendix. ■

Proposition 6 implies first, that there is a threshold $\overline{\Delta q}$ (which depends on the exogenous parameters α, β, and $\overline{p}$) such that only for $\Delta q \geq \overline{\Delta q}$ an integrated equilibrium exists. Second, and more important for its policy implications, it

implies that whenever an integrated market exists the segmented market also exists. So even if integration is possible it is not necessarily reached, and even if integration is reached, the possibility to revert to the segmentation equilibrium is always present. This possible collapse of the integrated equilibrium is somewhat reminiscent of Flannery's (1996) argument for the collapse of a domestic interbank market. In both cases it is the existence of excessively noisy information that drives the collapse of the market.

In this model, segmentation is quite a "robust" equilibrium because there exist self-fulfilling beliefs that support it, independently of the existence of an integrated interbank market equilibrium. This is of concern because usually, but not necessarily, integration will Pareto dominate segmentation, a point the following proposition establishes.

Proposition 7 *The integrated equilibrium does not always dominate the segmented equilibrium, but it does so for β sufficiently small.*

Proof. See appendix. ■

As is intuitive, for most parameter constellations integration leads to a higher expected welfare than segmentation because of higher cross-country interest rate smoothing. However, if the signals in the foreign country are sufficiently imprecise, foreign lenders make more 'mistakes' in granting loans to insolvent banks, which decreases welfare. Still, parameter values for which this effect dominates are hard to find since the integrated equilibrium exists only for small β.

The analysis shows that a high level of cross-border information (i.e. low β) is essential for an integrated inter-bank market to exist. However, even when the difference in information across borders is sufficiently low, there is no guarantee that private market forces reach the most efficient equilibrium.

Furthermore, in all possible equilibria, an inefficiency remains that is due to the informational asymmetry between countries. Because of the concavity of the liquidation technology, the most efficient outcome would involve all banks with the domestic good signal to liquidate the same amount. However, since borrowers from country B pay a premium that reflects the asymmetry in information, they liquidate more than banks in country A in either one of the equilibria.

4 Coexistence of unsecured and repo markets in an international setting

Finally, we discuss the general case where both types of markets, repo and unsecured, coexist. We focus on the more interesting case where banks hold few T-Bills so that borrowers have to rely on both markets, and borrowing banks without access to unsecured markets have to close down (The opposite case where banks are able to cope with liquidity shocks by selling T-Bills is trivial). In this way we are able to examine the combined effect of integration through the repo market and market discipline by peer monitoring.

In this context, as it is intuitive, it is possible to show that the excess liquidity banks of the liquidity short country will never hold T-Bills until time 2. This is in line with the fact that these lenders have an informational rent from lending to the interbank market because of more accurate information, so that they can lend the liquidity they obtain through the sale of T-Bills. Therefore, equilibrium is now obtained with a transfer of liquidity equal to Δs_0 through the sale of T-Bills (notice that T-Bills are not redeemable and therefore do not produce liquidity before time $t = 2$). That is,

$$\Omega_B = \Lambda_B + \Delta s_0 \tag{7}$$

$$\Omega_A = \Lambda_A - \Delta s_0 \tag{8}$$

The following Proposition extends Proposition 3 and characterizes the integrated equilibrium

Proposition 8 *The equilibrium in the interbank market is characterized by* $r_B^* \geq r_F^* > r_A^*$ *and either*

1. *Integration if* $0 < \psi^* \leq 1$, $r_B(\psi^*) \geq r_F(\psi^*)$, $p_F(1+r_F^*) = \bar{p}(1+r_A^*) = \frac{1}{B_1} < \overline{p}(1+r_B)$, *and country-B banks will sell all their T-Bills to the A country ones.*

2. *Integration with unsecured markets segmentation if* $r_B^* = r_A^*$ *and* $\psi^* = 0$ *in which case* $p_F = prob(\tilde{R} = R|\underline{s})$. *This will occur when the aggregate liquidity shocks are small with regard to the T-Bill market.*

3. *Segmentation if* $\psi^* = 0$ *and* $r_B^* > r_A^*$, *in which case* $p_F = prob(\tilde{R} = R|\underline{s})$, *and the B country banks will sell all their T-Bills.*

Proof. See appendix. ■

Proposition 8 considers a situation where banks with excess liquidity are willing to both buy repos and lend in the unsecured market. Therefore, we assume that the price for liquidity on both markets is the same one for lenders in country A, i.e. $\overline{p}(1 + r_A) = \frac{1}{B_1}$. This implies that for borrowers in both countries, the cost of obtaining liquidity through the repo market, $\frac{1}{B_1}$, is smaller than the cost it faces in the unsecured market, which is $1 + r_s$, $s = A, B$. The borrowing banks will resort to the unsecured market only after having sold all their T-Bills, i.e. $T_H^{1s} = 0$. Thus, in an international framework, the repo-facility will be used to transfer liquidity from the excess liquidity country to the liquidity short one, while the unsecured market will be used to provide liquidity to the $\bar{s}$ borrowers.

However, because the repo market results in a lower interest rate spread of domestic rates, the potential gain from integration of unsecured markets is now lower. At the same time, the cost that is associated with integration remains unchanged, namely the fact that credit is extended to insolvent foreign banks. Indeed, recalling Proposition 6, we saw that only if the difference in liquidity needs was sufficiently high could an equilibrium with $\psi > 0$ be obtained. But, the existence of an international repo market reduces Δq, and hence makes the integration of unsecured markets *less likely*. This point is established in the following Proposition.

Proposition 9 *The introduction of a cross-border repo market implies*

- *for a segmented equilibrium, ($\psi^* = 0$), $r_B - r_A$ is reduced.*
- *the integrated equilibrium ($\psi^* > 0$) collapses for Δs_0 sufficiently high.*

Proof. See appendix. ■

The possible equilibria in the unsecured market are illustrated in figure 3. The left graph illustrates the case where Δs_0 is relatively low. Here, equilibria with both separated and integrated unsecured markets exist. In the right graph, on the other hand, Δs_0 is so high that integration in the unsecured market is not possible any longer.

It would be tempting to jump to the conclusion that the collapse of the integrated equilibrium decreases welfare. Still, in the light of Proposition 7, we know

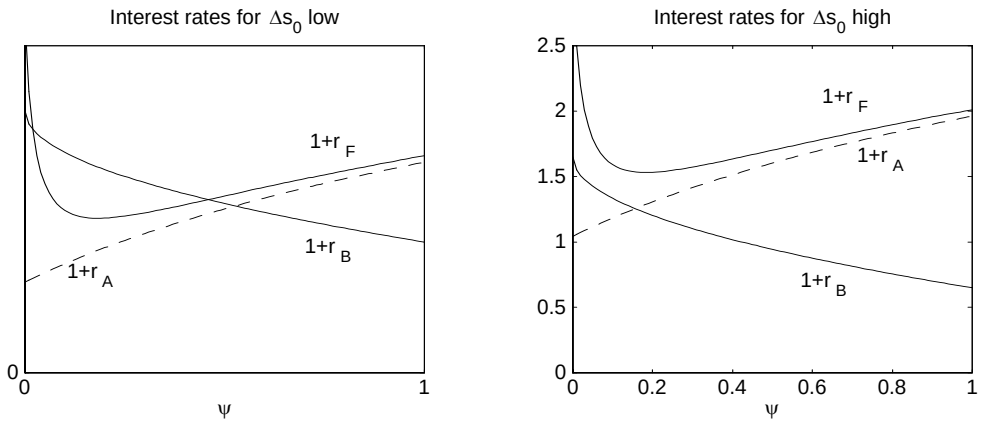


Figure 3: *Equilibria with different levels of T-Bill holdings.*

that the analysis is more envolved. Indeed, the following Proposition establishes that the collapse of an integrated equilibrium need not be welfare decreasing.

Proposition 10 *The break-down of the international unsecured interbank market due to a higher Δs_0 can lead to an increase in welfare only for high β.*

Proof. See appendix. ■

5 Extensions

Up to now we have restricted our focus to a simplified world where banks were either domestic or foreign, correspondent banking services were excluded and the population of banks was homogeneous in regard of their credit risk. In the following three sections we consider these extensions, starting with correspondent banking, then turning to transnational banks and concluding with the introduction of safe banks.

5.1 Correspondent Banking

Correspondent banking will develop when some banks are able to borrow from the liquid country and lend to the illiquid one. These are liquidity long banks from the B-country with a $\overline{s}$ foreign signal that will borrow at the rate r_F, and lend at the domestic rate r_B. As correspondent banking develops, some aggregated amount of liquidity Z is channeled into the illiquid country and this will result in new equilibrium interest rates $r_B(Z)$ and $r_F(Z)$.

For correspondent banking to be profitable, the cost of borrowing in country A, $1 + r_F(Z)$, cannot exceed the average return from lending to borrowers in country B, $\overline{p}(1 + r_{CB}(Z))$, where $r_{CB}(Z)$ represents the interest rate charged by correspondent banks. Therefore, it is required that

$$1 + r_F(Z) \leq \overline{p}(1 + r_B(Z)). \tag{9}$$

Condition (9) implies that $r_F(Z) < r_B(Z)$, requiring that the equilibrium is of the $\psi = 1$ type, which corresponds to the case where all $(\overline{s}, \overline{s})-$banks of country B borrow abroad. In addition, corresponding banking is unable to reduce the spread $r_B - r_F$ to zero as the following proposition establishes

Proposition 11 *Correspondent banking will develop only in the $\psi = 1$ equilibrium, provided that the interest rate wedge $r_B(0) - r_F(0)$ is sufficiently large, and in equilibrium the wedge $r_B(Z^*) - r_F(Z^*)$ is strictly positive.*

Proof. See appendix. ■

This interest rate differential will trigger the arbitrage operated by correspondent banking: those banks able to borrow abroad would do so, even if they are liquidity long, and then use the liquidity obtained to lend it to those banks in their home country.

Since operating in the interbank market does not deteriorate the credit rating of the bank, there is no limit in the amount correspondent banks face in order to borrow abroad. Therefore, the interbank market equilibrium will be such that competition among correspondent banks will lead to the limit point Z^* where there are no more gains from correspondent banking:

$$1 + r_F(Z^*) = \overline{p}(1 + r_{CB}(Z^*)) = \overline{p}(1 + r_B(Z^*)). \tag{10}$$

From this discussion it is clear that correspondent banking has a positive effect on welfare, since it helps channeling liquidity to where it is most needed: it is efficient that banks with the signals $(\overline{s}, \underline{s})$ liquidate as little as possible. This fact together with a concave liquidation technology implies that having correspondent banking is welfare-improving. The difference with respect to the introduction of a repo market is that there is no possible switch to another equilibrium, as correspondent banking requires $\psi = 1$. The inefficiency due to the asymmetry of information across countries is not completely removed since an interest rate differential continues to remain.

5.2 Transnational Banks

We will define a transnational bank as one which is part of the financial systems in the two countries and thus issues the same domestic signal in both countries. Therefore, there is no cross-country information asymmetry for transnational banks, so that they are able to operate in both markets when the signal they receive is good, and in none when it is bad. This implies that transnational banks borrow from the country with lower interest rates and lend in the country with the higher ones. Contrasting with the corresponding banking case that emerged only for $\psi = 1$, a transnational bank will operate whenever there is an interest rate differential, including the case of $\psi = 0$.

Because in our model there is no limit to the amount a bank can borrow other than its credit appraisal, transnational banks will borrow from country A and lend to country B, as correspondent banks do.

The difference with respect to correspondent banking is that in the correspondent banking case, when arbitraging interest rate differentials is not profitable because condition (9) is not met, transnational banks will still choose to borrow from the cheapest source, in country A, if they are liquidity-short (π_H), and to lend at the best rates, in country B, if they are liquidity long (π_L). Consequently, we will have a "variable size market", as transnational banks will choose the market they enter. If we assume there is a measure μ of transnational banks in each country, the effect we will have is an increase in the supply of loans in the country with a liquidity shortage, and an increase in the demand for loans in the excess liquidity country. Equilibrium will occur with

$$\widehat{\Omega}_B = \Lambda_B \tag{11}$$

$$\widehat{\Omega}_A = \Lambda_A \tag{12}$$

where the expression for $\widehat{\Omega}_B$ and $\widehat{\Omega}_A$ are given by:

$$\begin{aligned} \widehat{\Omega}_B &= (1-\mu)\Omega_B(\psi) + \mu(\sigma_L^A + \sigma_L^B)\pi_L \\ \widehat{\Omega}_A &= (1-\mu)\Omega_A(\psi) + \mu\left(\sigma_D^A + \sigma_F^A + \sigma_D^B\right)\pi_H \end{aligned}$$

As a consequence, the effect of transnational banks is, even in the absence of cross-country arbitrage, to diminish the liquidity shocks by allowing a fraction of banks to choose the best rates without facing any information asymmetries.

It is also worth noticing that whenever transnational banks are present, correspondent banking is not profitable. This means that obviously, all transnational banks are able to act as correspondent banks. Still, if the amount of transnational banks is sufficiently large, banks with a good foreign signal will be indifferent between borrowing at home or borrowing abroad, as an integrated market with $r_B = r_F$ can be achieved. This is in contrast to the result we obtain with correspondent banking (Proposition 11). If the amount of transnational banks is even larger, then the spread between $r_B = r_F$ and r_A diminishes and the informational rents in the illiquid country begin to be eroded. This is so as a transnational bank is able to obtain liquidity in the country with excess liquidity at interest rate r_A (while a correspondent bank would borrow at r_F).

Notice that the presence of transnational banks is similar to an economy with a cross-border repo market: in both cases, the transfer of liquidity is facilitated and this is efficient provided there is no switch to an inferior equilibrium, as established in proposition 9.

5.3 Heterogeneous default risk levels

Assume now that banks are not homogeneous anymore, and that some banks in each country have a lower probability of failure which is common knowledge. For the sake of simplicity we take the extreme case where some banks are perfectly safe while others are risky. We consider this case to be particularly meaningful, as the too-big-to-fail argument implies that some banks have the unlimited support of the regulatory authority, while others face default with a non-zero probability.

The assumption $\overline{p} = 1$ immediately implies that such a bank is able to borrow freely from any market, as a transnational bank. But in addition, equation (10) implies a complete integration of the markets. The existence of noisy information for foreign banks will imply that lending unsecured to banks abroad is not profitable, as there is no spread to compensate for the risk. In equilibrium, liquidity is transferred from the excess liquidity country to the one experiencing the liquidity shortage by the safe banks at no cost. This equilibrium is efficient, (independently of the existence of an implicit transfer from the government to the too-big-to-fail bank which is out of the focus of our analysis).

This type of equilibrium seems to describe rather well the current situation in the European Monetary Union. While interest rates in the different member

states have converged rapidly (see ECB 2000), the interbank market seems to be characterized by a two-tier structure: In a recent study, Ciampolini and Rhode (2000) report the results of a survey conducted among European Banks according to which only a few large banks are actice in the international market while smaller ones are confined to domestic sources for liquidity.

The implications of this finding are far reaching. It implies that a country that lacks the resources (or the credibility) to back up its major banks in case of distress will be at a disadvantage in obtaining liquidity in the international interbank market. For developed countries that are able to bail-out their banks, it may also imply that there is an interest on behalf of the country to build strong "too-big-to-fail" banks in order to compete in the international arena. If this was so, governments might have an interest in promoting national mergers by creating national banks rather than allowing for the creation of transnational banks that they would not be able to support.

6 Conclusion

In this paper, we analyzed a model of interbank markets in an international context. We focused on the respective roles played by an unsecured money market and a repo-market on the domestic and international levels, and developed their welfare properties.

In an economy with unsecured markets, lending takes place on the basis of peer monitoring. This is shown to be efficient, as funds are channeled to the most efficient projects. In repo-markets, on the other hand, monitoring plays no role because all loans are collateralized. Therefore, markets are unable to achieve efficient liquidation of unprofitable projects, and insolvent banks forebear. Still, contrary to unsecured markets, a repo market is able to achieve liquidity smoothing across solvent market participants.

In an international context, interbank markets seem to work less efficiently, leading to market imperfections such as liquidity shortages or interest rate differentials. Although these differences could be attributed to exchange rate risk, we argue that the main barrier to an integrated international market is the existence of asymmetric information between different countries.

We have shown that as long as peer monitoring across borders is less effi-

cient than on a domestic level, the integration of unsecured markets can never be perfect. In particular, cross-border lending involves the payment of interest rate premia which reflect the adverse selection of borrowers in the international market. This implies that a perfect liquidity smoothing across borders cannot take place. As a consequence, we show, first, that an equilibrium with an integrated interbank market does not always exist. Second, even if it does exist, at the same time market segmentation is always an equilibrium. Therefore, even with monetary integration or currency pegging, market integration is not necessarily achieved. Interestingly, we also found that in an integrated equilibrium, welfare is not necessarily higher than in the segmented one.

A repo-market, on the other hand, is always able to function on an international basis, since it overcomes the problem of asymmetric information. Thus, it is able to achieve liquidity smoothing at least to some degree. Still, the welfare effects from a repo facility remain ambiguous: our analysis shows that a repo market reduces the benefits from peer monitoring and might even impede the integration of markets. Furthermore, even the combination of both types of markets is not necessarily beneficial since integration comes at the cost of a higher degree of inefficient forbearance.

Finally, the effects of correspondent banks, transnational banks with varying degrees of riskiness are analyzed. We show that these institutions can play, in varying degrees, a crucial role in the cross-border liquidity transfer.

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Appendix: Proof of Lemmas and Propositions

Proof of Lemma 1

W.l.o.g., assume that $T_0 = 0$, as the repo market is inactive and T-Bills play no role. For a bank of type k, $k = L, D, F$, denote ΔI_k the amount liquidated, L_k^S (L_k^D) the loan given to (demanded by) a bank of type k, and s_k^1 the storage of reserves. The time-1 optimization problem for a bank facing withdrawals π_L is:

$$\max_{\{\Delta I_L, s_L^1, L_D^S, L_F^S\}} p\left\{R(I - \Delta I_L) + \overline{p}(1 + r_D)L_D^S + p_F(1 + r_F)L_F^S + s_L^1 - (1 - \pi_L)\right\}$$

$$\text{s.t.} \quad s_0 + l(\Delta I_L) \geq \pi_L + s_L^1 + L_D^S + L_F^S$$

as well as the appropriate non-negativity constraints. Here, r_D and r_F denote the domestic and foreign interbank market rates, and p_F refers to the expected probability of solvency of foreign banks. Notice that we have assumed that investment in the risky technology I is so large that the bank fails whenever the risky project is not successful. Similarly, a bank of type $k = D, F$ with withdrawals π_H faces the problem

$$\max_{\{\Delta I_k, s_k^1, L_k^D\}} p\left\{R(I - \Delta I_k) - (1 + r_k)L_k^D + s_k^1 - (1 - \pi_H)\right\}$$

$$\text{s.t.} \quad s_0 + L_k^D + l(\Delta I_k) \geq \pi_H + s_k^1.$$

The first-order conditions for the Lagrangian for an L-bank, with the multiplier $p\lambda$ for the liquidity constraint are the following:

$$\begin{aligned} \overline{p}(1 + r_D) - \lambda &\leq 0 \\ p_F(1 + r_F) - \lambda &\leq 0 \\ -R + \lambda l'(\Delta I_L) &\leq 0 \\ 1 - \lambda &\leq 0 \end{aligned} \tag{13}$$

while in a state of nature $k = D, F$ (and multiplier $p\mu$) they are characterized by:

$$\begin{aligned} -(1 + r_k) + \mu &\leq 0 \\ -R + \mu l'(\Delta I_k) &\leq 0 \\ 1 - \mu &\leq 0 \end{aligned} \tag{14}$$

In order for L-banks to lend to liquidity short banks from both countries, we need $\overline{p}(1+r_D) = p_F(1+r_F)$. If $\overline{p}(1+r_D) \leq 1$, only reserves are used, $\Delta I_k = 0$ for every k, and the interbank markets are inactive. Banks with a liquidity shortage liquidate positive amounts only for $1 + r_k > 1/R$, while excess liquidity banks offer loans for $\overline{p}\,(1 + r_D) > 1$, and liquidate for $\overline{p}\,(1 + r_D) > 1/R$. The optimal liquidation decisions are

$$\begin{aligned} l(\Delta I_L) &= \max\left\{0, \ln\left(\frac{\overline{p}\,(1+r_D)}{R}\right)\right\} \\ l(\Delta I_D) &= \max\left\{0, \ln\left(\frac{1+r_D}{R}\right)\right\} \\ l(\Delta I_F) &= \max\left\{0, \ln\left(\frac{1+r_F}{R}\right)\right\} = \max\left\{0, \ln\left(\frac{\overline{p}\,(1+r_D)}{p_F R}\right)\right\}. \end{aligned} \tag{15}$$

From (2), it follows that Λ_i is non-decreasing in the local interest rate. ■

Proof of Lemma 2

We will prove the lemma in two steps. First we prove that only one ψ_i can be non-zero, the one corresponding to the country with high interest rates. Second, we prove that if $\psi_L > 0$, the interest rates differentials between the two countries would imply a contradiction.

1) Assume by way of contradiction that $r_i \geq r_j$, and $\psi_j > 0$. Denote by r_{iF} the rate offered to foreign banks that want to borrow in country i. Because country i banks have access to coarser information on foreign borrowers, we have $r_i < r_{iF}$ for $i = L, H$.

On the other hand, a necessary condition for $\psi_j > 0$ is that $r_{iF} \leq r_j$ (borrowing in country i is attractive for $(\overline{s}, \overline{s})$ borrowers in country j) but this would imply $r_j > r_i$, a contradiction, so that $r_i \geq r_j$, implies $\psi_j = 0$.

2) To prove this second point, consider first the interest rates when $\psi_L = \psi_H = 0$. Equilibrium demands $\Omega_i = \Lambda_i(r_j)$ for $i = L, H$. Since the liquidation technology is the same in the two countries, so is the supply function, and as we have $\Omega_L < \Omega_H$ and the supply is increasing in r_j, we obtain $r_L < r_H$.

Consider now the case $\psi_L > 0$ and $\psi_H = 0$. In this case, the demand Ω'_H in country H is larger than in the $\psi_L = \psi_H = 0$ case, while the demand for Ω'_L is lower than before. This would imply therefore that the market clearing interest rates satisfy again $r_L < r_H$. But then , using the argument in 1), yields a contradiction, since we have proved in 1) that the in order to have $\psi_L > 0$ we need $r_L > r_H$. ■

Proof of Proposition 3

A necessary condition for $\psi > 0$ is that borrowing abroad is no more expensive than borrowing domestically, so that $r_H \geq r_F$, with $\psi = 1$ in case of strict inequality. On the other hand, (5) implies $r_F \geq r_L$.

If $\psi = 0$, there is segmentation, since any potential borrower is identified as a $(\underline{s}, \overline{s})$-type and therefore, $p_F = \underline{p}$. But, if $r_F < r_H$, we would have $\psi > 0$, a contradiction. ■

Proof of Lemma 4

The updated probability of solvency of a foreign borrower is

$$\begin{aligned} p_F(\psi) &= \frac{\psi prob(\tilde{R} = R \text{ and } (\overline{s}, \overline{s})) + prob(\tilde{R} = R \text{ and } (\underline{s}, \overline{s}))}{\psi prob(\overline{s}, \overline{s}) + prob(\underline{s}, \overline{s})} \\ &= \frac{\psi p\alpha(1-\beta) + p(1-\alpha)\beta}{\psi(1-\beta)\theta + (1-\theta)\beta}. \end{aligned} \tag{16}$$

Taking into account that $\theta = \alpha p + (1-\alpha)(1-p)$, it is easy to see that $p'_F(\psi) > 0$.

From (5), the foreign premium is given by $r_F - r_L = (1 + r_L)\left(\frac{\overline{p}}{p_F(\psi)} - 1\right)$. Because $p'_F(\psi) > 0$, it is decreasing in ψ.■

Proof of Lemma 5

In country H, there are $\sigma_L^H = 1 - q_H$ lenders, $\sigma_D^H(\psi) = q_H\theta\left[1 - \psi(1-\beta)\right] \equiv q_H\xi^H$ domestic borrowers, and no foreign borrowers, $\sigma_F^H = 0$. Using the first order conditions (15), we can express liquidity demand and supply, (1) and (2), as functions of ψ,

$$\begin{aligned} \Omega_H(\psi) &= \sigma_L^H\pi_L + \sigma_D^H(\psi)\pi_H \\ \Lambda_H(\psi) &= \sigma_L^H\left[s_0 + \ln\left(\frac{\overline{p}(1+r_H)}{R}\right)\right] + \sigma_D^H(\psi)\left[s_0 + \ln\left(\frac{1+r_H}{R}\right)\right]. \end{aligned}$$

Since $\overline{p}R > 1$, it is never efficient to choose s_0 so that $\Omega_H(\psi) > s_0\left(\sigma_L + \sigma_D^H(\psi)\right)$. This implies that some banks (i.e. domestic borrowers) will always liquidiate positive amounts. The interest rate r_H adjusts so that

$$F_H \equiv \Lambda_H(\psi) - \Omega_H(\psi) = 0. \tag{17}$$

Then, $\frac{\partial F_H}{\partial r_H} = \frac{\sigma_L^H + \sigma_D^H}{1+r_H} > 0$ (resp. $\frac{\partial F_H}{\partial r_H} = \frac{\sigma_D^H}{1+r_H} > 0$ if lenders do not liquidate) and $\frac{\partial F_H}{\partial \sigma_D^H(\psi)} = l(\Delta I_D^H) - (\pi_H - s_0) \equiv X_D^H$ which is negative because π_H-banks borrow. Then, the Implicit Function Theorem implies $\frac{dr_H}{d\sigma_D^H(\psi)} > 0$, and together with $\frac{d\sigma_D^H(\psi)}{d\psi} < 0$, we obtain $\frac{dr_H}{d\psi} = \frac{dr_H}{d\sigma_D^H(\psi)}\frac{d\sigma_D^H(\psi)}{d\psi} < 0$. ■

Proof of Proposition 6

The segmented equilibrium exists since for $\psi = 0$, any $(\overline{s}, \overline{s})$ bank would prefer to borrow at rate r_H rather than not obtaining any credit. Therefore, $\psi = 0$ is consistent with their behavior, $F_L = 0$ and $F_H = 0$ determine r_L and r_H, respectively, while r_F is undetermined since there is no cross-border interbank lending.

An integrated equilibrium does not exist if and only if

$$\min_{\psi} r_F(\psi) - r_H(\psi) > 0 \tag{18}$$

Let $\psi^*(\Delta q)$ be the solution to (18) with equality. We will show that if Δq satisfies $\min_{\psi^*(\Delta q)} r_F(\psi^*(\Delta q)) - r_H(\psi^*(\Delta q)) = 0$, then for any $\Delta q'$ such that $\Delta q' \leq \Delta q$, condition (18) holds, i.e. the integrated equilibrium does not exist. We only have to prove that $\frac{d(r_F - r_H)}{d\Delta q} < 0$.

Since $q_H + q_L = \overline{Q}$, we have $dq_L = -dq_H$, so that $d\Delta q = 2dq_H$. Consider first $\frac{dr_H}{d\Delta q}$. In country H, r_H solves $\Lambda_H = \Omega_H$ (as in the proof of Lemma 5). From (17), denote $F_H \equiv \Lambda_H - \Omega_H = (1 - q_H) X_L^H + q_H \xi^H X_D^H = 0$ where $X_L^H \geq 0$and $X_D^H \leq 0$. From the proof of Lemma 5, we have $\frac{\partial F_H}{\partial r_H} = \frac{1}{1+r_H}\left(\sigma_L^H + \sigma_D^H\right)$. On the other hand,

$$\frac{\partial F_H}{\partial \Delta q} = 2\frac{\partial F_H}{\partial q_H} = 2\left\{-X_L^H + \xi^H X_D^H\right\} = -\frac{2}{q_H} X_L^H \leq 0$$

where the last equality follows from (17).

Next, consider $\frac{dr_F}{d\Delta q}$. Liquidity demand and supply in country L are

$$\begin{aligned}
\Omega_L &= \sigma_L^L \pi_L + \sigma_D^L(\psi)\pi_H + \sigma_F^L(\psi)\pi_H \\
\Lambda_L &= \sigma_L^L \left[s_0 + \ln \frac{p_F(1+r_F)}{R}\right] + \sigma_D^L \left[s_0 + \ln \frac{p_F(1+r_F)}{\overline{p}R}\right] + \sigma_F^L \left[s_0 + \ln \frac{1+r_F}{R}\right]
\end{aligned}$$

with $\sigma_L^L = 1 - q_L$, $\sigma_D^L = q_L\theta$, and $\sigma_F^L = q_H\left[\psi(1-\beta)\theta + \beta(1-\theta)\right] \equiv q_H \xi^L$, and where we have used (5). Denote

$$F_L = \Lambda_L - \Omega_L \equiv (1 - q_L) X_L^L + q_L \theta X_D^L + q_H \xi^L X_F^L = 0 \tag{19}$$

with $X_L^L \geq 0$, $X_D^L \leq 0$ and $X_F^L \leq 0$. Then, $\frac{\partial F_L}{\partial r_F} = \frac{\sigma_L^H + \sigma_D^L + \sigma_F^L}{1+r_F}$ and

$$\frac{\partial F_L}{\partial \Delta q} = 2\frac{\partial F_L}{\partial q_H} = 2\left\{\left[-X_L^L + \theta X_D^L\right]\frac{dq_L}{dq_H} + \xi^L X_F^L\right\} = \frac{2}{q_H}\left\{\left[X_L^L - \theta X_D^L\right](q_H + q_L) - X_L^L\right\}$$

where the last equality follows from (19).

Using the Implicit Function Theorem and that $r_F = r_H$ for Δq belonging to F, we find

$$\begin{aligned}\frac{d(r_F - r_H)}{d\Delta q} &= -\frac{\partial F_L/\partial \Delta q}{\partial F_L/\partial r_F} + \frac{\partial F_H/\partial \Delta q}{\partial F_H/\partial r_H} \\ &= -\frac{2\,(1+r_F)}{q_H}\left\{\frac{\left(X_L^L - \theta X_D^L\right)(q_H+q_L)}{\sigma_L^L+\sigma_D^L+\sigma_F^L} + \frac{X_L^H}{\sigma_L^H+\sigma_D^H} - \frac{X_L^L}{\sigma_L^L+\sigma_D^L+\sigma_F^L}\right\}\end{aligned}$$

The first term is positive. Furthermore, the second term exceeds the third term: first, $X_L^H > X_L^L$ because $r_H > r_L$ implies $l(\Delta I_L^H) > l(\Delta_L^L)$. Second, it is easy to show that $\sigma_L^H + \sigma_D^H < \sigma_L^H + \sigma_D^L + \sigma_F^L$, as it is equivalent to $(q_H - q_L)(1-\theta) + q_L\psi(1-\beta) + q_H\xi^L \geq 0$. Hence, $\frac{d(r_F-r_H)}{d\Delta q} < 0$. ■

Proof of Proposition 7

Suppose the banks hold a small amount of T-Bills at time 0, and that the integrated equilibrium exists. Denoting by index $i = L, H$ the country, and k the bank type. Expected welfare can be shown to be

$$W = I\,(pR-1) + T_0(1-B_0) - \frac{1}{2}\sum_{i=L,H}\sum_k \sigma_k^i\left[p_k R\Delta I_k^i - l(\Delta I_k^i)\right] \qquad (20)$$

where p_k denotes the expected probability of solvency for a given type k, i.e. $p_k \in \{p, \overline{p}, \underline{p}\}$.

Denote the integrated equilibrium I and the separated one S. In country L, suppose that even in the integrated equilibrium, $\overline{p}\,(1+r_L) = 1$. This implies that country L banks liquidate the same amount in both equilibria, so we can neglect them in the welfare comparison. For country H, denote interest rates in the segmented equilibrium $r_H(S)$ and those in the integrated one $r_H(I) = r_F(I)$, where $r_H(S) > r_H(I)$. A change from a separated to an integrated equilibrium then affects the following banks in country H: $\sigma_L^H \equiv 1 - q_H$ lenders and $\sigma_D^H \equiv q_H\theta$ borrowers with the good domestic signal will face higher interest rates $1 + r_H$. Thus, $\Delta I_k^H(S) > \Delta I_k^H(I)$, $k = L, D$. Furthermore, $\sigma_C^H \equiv q_H(1-\theta)\beta$ borrowers with the signal pair $(\underline{s}, \overline{s})$ will go bankrupt in the equilibrium with separation, having to liquidate $\Delta I_C^H(S) \geq \Delta I_D^H(S)$, but obtain a foreign loan with integration and liquidate $\Delta I_D^H(I)$.

For $p_L = p$, $p_D = \overline{p}$, and $p_C = \underline{p}$, we can manipulate the change in welfare

from (20) so that

$$\begin{aligned}\Delta W &\equiv W(S) - W(I) \qquad (21)\\ &= \sum_{k=L,D,C} \sigma_k^H \left\{ \left[p_k R \Delta I_k^H(S) - l(\Delta I_k^H(S)) \right] - \left[p_k R \Delta I_k^H(I) - l(\Delta I_k^H(I)) \right] \right\} \\ &= \sum_{k=L,D,C} \sigma_k^H \left\{ \int_{\Delta I_k^H(I)}^{\Delta I_k^H(S)} l'(\Delta I) d\Delta I - \int_{\Delta I_k^H(I)}^{\Delta I_k^H(S)} p_k R d\Delta I \right\} \\ &= \sum_{k=L,D,C} \sigma_k^H \left\{ \int_{\Delta I_k^H(I)}^{\Delta I_k^H(S)} \left[l'(\Delta I) - p_k R \right] d\Delta I \right\}.\end{aligned}$$

Our model assumptions imply $\overline{p}R \geq pR \geq l'(\Delta I)$ and hence that the terms corresponding to $k = L, D$ are negative, while Assumption 2 implies $\underline{p}R \leq l'(\Delta I)$ so that the term for $k = C$ is positive.

The sign of ΔW depends therefore on parameters. Suppose first that $\beta \to 0$, so that $\sigma_C^H \to 0$. Then, $\Delta W < 0$, and the the break-down of the integrated equilibrium decreases welfare.

On the other hand, consider the following set of parameters: $R = 2$, $p = 0.8$, $\alpha = 0.9$, $\beta = 0.45$, $I = 0.5$, $T_0 = 0$, $q_H = 0.55$, $q_L = 0.425$, $\pi_H = 0.8$, and $\pi_L = 0.3$. Calculations show that expected welfare (20) is $W^{seg} = 0.2968$ in the segmented equilibrium. Furthermore, an integrated equilibrium exists for $\psi \approx 0.77$, leading to expected welfare of $W^{int} = 0.2964 < W^{seg}$.■

Proof of Proposition 8

We know that $p_F(1 + r_F^*) = \bar{p}(1 + r_L^*) < \overline{p}(1 + r_H^*) < 1 + r_H^*$. Now, depending on the T-Bills yield $\frac{1}{B_1}$, banks will be willing to buy and sell T-Bills rather than making loans. Notice first that if $\frac{1}{B_1} > 1 + r_H^*$ there is an excess demand in the T-Bill market as no agent wants to sell T-Bills, and if $\frac{1}{B_1} < \bar{p}(1 + r_L^*)$ there is an excess supply as no one wants to buy them. As a consequence, for possible equilibria we have to focus on the following cases:

1. $p_F(1 + r_F^*) = \overline{p}(1 + r_L^*) = \frac{1}{B_1} < \overline{p}(1 + r_H^*)$
2. $\overline{p}(1 + r_L^*) = \frac{1}{B_1} < \overline{p}(1 + r_H^*)$
3. $\overline{p}(1 + r_L^*) < \frac{1}{B_1} = \overline{p}(1 + r_H^*)$
4. $\overline{p}(1 + r_L^*) = \frac{1}{B_1} = \overline{p}(1 + r_H^*)$

Case 1 corresponds to integration. Country H banks will all prefer to sell their T-Bills and use the liquidity thus obtained either to borrow less or to lend more in the interbank market. Case 2 corresponds to segmentation. For a higher T-Bill yield, we would reach case 3. This, however, cannot be sustained as an equilibrium as we have assumed that the T-Bill holdings are insufficient to guarantee survival by borrowing on the T-Bills market only. Consequently, liquidity short L-banks will bid higher interest rates rather than being liquidated. If this occurs, the interest rate hike either restores the case 1-2 type of equilibrium or else it involves only the domestic interbank market. In the latter case, we reach case 4 which corresponds to the case where unsecured interbank market segmentation occurs so that there is no strict inequality between r_L^* and r_H^*. In case 4, the T-Bill market leads to interest rate equalization without any cross-country unsecured borrowing. If the yield on interest rates is higher, then only liquidity short H-banks will sell T-Bills. Competition among buyers will then restore the equality of either 1,2 or 4. ■

Proof of Proposition 9

Suppose the cross country trading of T-Bills leads to a transfer of liquidity of Δs_0 as in (7) and (8). It is sufficient to show that $\frac{\partial(r_H - r_L)}{\partial \Delta s_0} < 0$. Consider country L. From (8), equilibrium with the market now requires a lower supply of liquidity Λ_L. Lemma 1 implies that $\frac{\partial r_L}{\partial \Delta s_0} \geq 0$. Consider now country H. Notice first that without the T-Bill market, there is a liquidity shortage in H so that in equilibrium, a positive measure of banks need to liquidate positive amounts. W.l.o.g. assume that $\Delta I_L^H = 0$. From (1), (7), and (15) it follows $1 + r_H = R \exp\left\{\frac{\Omega_H - \Delta s_0}{\sigma_D^H}\right\}$ so that $\frac{\partial r_H}{\partial \Delta s_0} < 0$. This proves the first part of the proposition.

To establish the second part, suppose that T_0 is so high that an equilibrium of the second type in Proposition 8 is obtained, where $r_L(\psi = 0) = r_H(\psi = 0)$. Since $r_F \geq r_L$, r_L is non-decreasing and r_H non-increasing in ψ (see Lemma 5), if follows that $r_F(\psi) > r_H(\psi)$ for all ψ. ■

Proof of Proposition 10

An integrated equilibrium does not exist if and only if (18) holds. Suppose that Δs_0 is such that

$$\min_{\psi(\Delta s_0)} r_F(\psi(\Delta s_0)) - r_H(\psi(\Delta s_0)) = 0. \tag{22}$$

We first show that if Δs_0 satisfies (22), then for any $\Delta s_0'$ such that $\Delta s_0' > \Delta s_0$, condition (18) holds, i.e. the integrated equilibrium does not exist. For this, we have to prove that $\frac{d(r_F - r_H)}{d\Delta s_0} > 0$. From (7) and (8), define $\widehat{F}_L \equiv F_L - \Delta s_0$ and $\widehat{F}_H \equiv F_H - \Delta s_0$ with F_H and F_L as defined in (17) and (19). From the proof of Lemma 5 and Proposition 6, we have $\frac{\partial F_H}{\partial r_H} > 0$ and $\frac{\partial F_L}{\partial r_L} < 0$. The Implicit Function Theorem then indeed yields $\frac{dr_H}{d\Delta s_0} = -\frac{\partial \widehat{F}_H / \partial \Delta s_0}{\partial \widehat{F}_H / \partial r_H} > 0$ and $\frac{dr_F}{d\Delta s_0} = -\frac{\partial \widehat{F}_L / \partial \Delta s_0}{\partial \widehat{F}_L / \partial r_F} < 0$.

As a second step, we consider Δs_0 so that (22) holds, and consider an infinitesimal increase in Δs_0, $\Delta s_0' = \Delta s_0 + \varepsilon$. We will show under which circumstances welfare is decreased.

Because the difference in welfare between the integrated and the segmented equilibrium is independent of Δs_0, we are entitled to take a sufficiently small change in Δs_0 whose effect on welfare will be negligeable in regard to the effect of the equilibrium switch.

Finally, suppose Δs_0 satisfies (22). The change in welfare is given by (21). From Proposition (7) follows that for β sufficiently small, the switch to a segmented equilibrium reduces welfare. ■

Proof of Proposition 11

Using the fact that $r_F'(Z) \geq 0$ and $r_H'(Z) \leq 0$, condition (9) for $Z = 0$ implies

$$\frac{1 - \bar{p}}{\bar{p}} (1 + r_F(0)) \leq r_H(0) - r_F(0).$$

On the other hand, the same inequality will hold in equilibrium replacing 0 by Z, so that the wedge $r_H(Z) - r_F(Z)$ has a lower bound. ■