On the Receiver Pays Principle[+]

Doh-Shin Jeon[*]

Jean-Jacques Laffont[**]

Jean Tirole[***]

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Abstract

This paper extends the theory of network competition between telecommunications operators by allowing receivers to derive a surplus from receiving calls (call externality) and to affect the volume of communications by hanging up (receiver sovereignty). We investigate the extent to which receiver charges can lead to an internalization of the calling externality. When the receiver charge and the termination (access) charge are both regulated, there exists an efficient equilibrium. Efficiency requires a termination discount.

When reception charges are market determined, it is optimal for each operator to set the prices for emission and reception at their off-net costs. For an appropriately chosen termination charge, the symmetric equilibrium is again efficient.

Lastly, we show that network-based price discrimination creates strong incentives for connectivity breakdowns, even between equal networks.

Keywords: Networks, Interconnection, Competition Policy.

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[*] University of Pompeu Fabra.
[**] IDEI, University of Toulouse I.
[***] IDEI, University of Toulouse I, CERAS, Paris, and MIT.

1 Introduction

1.1 Motivation

The deregulation of telecommunications, the most advanced among network industries, leads to new forms of competition. Operators compete for retail customers through sophisticated and discriminatory pricing while, at the wholesale level, their interconnection by and large remains regulated. The small literature on this new form of competitive environment[1] has neglected the facts that call receivers derive a utility from calls (call externality) and furthermore can affect volume by hanging up (receiver sovereignty). The purpose of this paper is to extend our understanding of network competition to environments with call externalities and receiver sovereignty in which firms can charge customers for receiving calls.

The operators' practice of charging their customers both for emission and for reception is usually referred to as the "receiver pays principle" (RPP). Reception charges play an increasingly important role in the case of mobile phones. The receiver pays principle is for example applied to mobile phone reception in the United States, Canada, and Hong Kong, as well as for international roaming on GSM mobile networks. Reception charges similarly play a key role in the new Internet economy, as both sides of the markets (e.g. dial-up customers and websites) are charged for the capacity and usage of their connection with Internet Service Providers or backbones.

Enriching the existing analysis to account for the existence of receiver surplus and sovereignty serves more than a descriptive purpose, though. On the positive side, reception charges alter the operators' competitive strategies. On the normative side, the joint determination of communications services by four parties (caller, receiver, and their operators) raises the question of whether proper incentives are in place for the maximization of joint surplus.

1.2 Overview of the analysis

To provide a roadmap for our analysis, it is convenient to introduce some notation. Two symmetrically differentiated networks compete in nonlinear prices for subscribers. A network i subscriber with outgoing volume q and incoming volume $\tilde{q}$ is charged (without loss of generality in our model)

$$T_i(q, \tilde{q}) = F_i + p_i q + r_i \tilde{q},$$

where F_i is the monthly subscriber charge, p_i is the per unit usage price and r_i the per unit reception charge. We let c_0 denote a network's marginal cost of terminating calls, a the reciprocal access charge paid by the originating network to the terminating network, and $c > c_0$ the total (origination plus termination) marginal cost of communications. Last, let α_1 and α_2 denote the two networks' market shares ($\alpha_1 + \alpha_2 = 1$).

[1] See Armstrong (1998, 2000), Carter-Wright (1999,2000), Cherdron (2000), Dessein (1999 a, b), Gans-King (1999), Hahn (2000) and Laffont-Rey-Tirole (1998 a, b).

Consumers derive a surplus both from calling and from being called. These surpluses in general differ and furthermore may be state contingent.

Part 1: regulated or contractually determined reception charges

Suppose that the reception charges (r_1, r_2) are regulated or else contractually agreed upon by the two operators before they wage competition in monthly fees and calling charges.

We first conduct the following thought experiment: Suppose the caller always determines the volume of communication. That is, either the receiver is not sovereign (i.e. is not allowed to hang up) or the reception charge is sufficiently low (relative to the caller's charge) that the receiver does not find it advantageous to hang up.

The inelastic demand for call reception implies that a network's reception charge has no incentive effect. And so, from the operator-subscriber pair's viewpoint, only the sum $\{F_i + r_i\tilde{q}\}$ matters, not its composition.

Our first insight is that, while a network's reception charge does not impact its profit given its rival's competitive offer, reception charges do matter. Indeed, we show that network i's equilibrium usage (calling) charge is equal to its "strategic marginal cost", namely:

$$p_i = [c + \alpha_j(a - c_0)] - \alpha_i r_j,$$

where, recall, c is the (industry's) marginal cost of a call, α_i is network i's market share, $(a - c_0)$ the access charge markup (the difference between the access charge and the termination cost) and r_j is network j's reception price. An average call originating on network i costs $[c + \alpha_j(a - c_0)]$ when the access markup (or discount) on the fraction α_j of calls that terminate off-net is accounted for. To obtain network i's perceived or strategic marginal cost, one subtracts the increase r_j in the monthly fee of the fraction α_i of consumers who subscribe to network i, that network i can afford implementing without losing market share.

To see this, consider an increase in the volume of calls from network i to network j. This increase has two opposite effects on the utility of network j's consumers. Their surplus increases because they receive more calls. However, they pay for receiving these extra calls. We call the first effect a *direct externality* and the second a *pecuniary externality*. Only pecuniary externalities matter. When network i lowers its price, the volume of calls received by consumers increases by the same amount regardless of their network affiliation, and therefore direct externalities are the same for all consumers. Pecuniary externalities result in a decrease in the perceived marginal cost $c + \alpha_j(a - c_0) - \alpha_i r_j$. That is, an increase in network j's reception charge makes it more desirable for network i to expand output.

Next, for a given symmetric reception charge r, we ask, does a symmetric equilibrium exist? We show that, if the reception charge is in the vicinity of the access charge discount $(c_0 - a)$, an equilibrium indeed exists. In contrast, when the reception charge substantially diverges from the access charge discount and networks are close substitutes, an equilibrium fails to exist; that is, competition is unstable. These results generalize the analysis in Laffont-Rey-Tirole (1998a), which ruled out reception charges $(r = 0)$ and showed that, with close substitutes, network competition is stable only if the access charge is in the vicinity of the termination cost.

Last, and on the normative side, we investigate the relationship between the competitive equilibrium and the second-best (Ramsey) optimum. Letting β denote the ratio of the marginal utilities of the caller and the receiver, second-best efficiency requires that the sum of the utilities be equal to the marginal cost c of a communication, or under caller-determined volume,

$$p = \frac{c}{1+\beta}.$$

Given that, in a symmetric equilibrium,

$$p = c + \frac{a - c_0 - r}{2}$$

from the above formula, the efficient outcome obtains when $r = c_0 - a$ (so an equilibrium exists) and

$$c_0 - a = \frac{\beta c}{1+\beta}.$$

That is, termination is priced at a discount, and this discount is steeper, the higher the receiver's marginal utility from receiving calls.

Last the fiction of caller-determined volume (our thought experiment), while open to the criticism that receivers are actually sovereign, turns out to be very useful, as we provide conditions under which it is still valid when receivers are allowed to hang up.

Part 2: market determined reception charges

Next, we assume that reception charges, just like monthly fees and calling charges, are set noncooperatively by the operators instead of being determined contractually or chosen by a regulator. In the absence of uncertainty about marginal utilities, a potential indeterminacy of equilibria arises: Over the range of parameters for which the volume is determined by the caller, only $\{F_i + r_i \tilde{q}\}$ matters, not its composition, as we have seen. And so, there might be a range of (nonequivalent) equilibria. We make the model more realistic and actually simplify it by letting the marginal utilities of communications be random (the utility of an extra minute of communication is state or time-of-the-day contingent). With wide enough supports for marginal utilities, both the calling and the reception charges have an incentive effect and therefore are determinate.

First, we show that, in a symmetric equilibrium, the usage and reception charges are set equal to the "off-net cost" of calls and call reception, respectively:

$$p = c + (a - c_0)$$

and

$$r = (c_0 - a).$$

That is, even though the networks have equal market shares in a symmetric equilibrium, each network sets prices for a subscriber's outgoing and incoming traffics at the marginal costs that it would incur if *all* other subscribers belonged to the rival network.

Second, when the randomness in the marginal utilities vanishes and provided that the access charge markup is larger than $-\beta c/(1+\beta)$, an equilibrium exists. The Ramsey optimum can then be approximated through the level of the access discount determined above. This latter result relies on a fixed ratio of marginal utilities (and thus in particular on the noise vanishing); more generally, one instrument (the access charge) cannot simultaneously adjust the incentives of both sides (caller, receiver) to internalize the other side's surplus.

Part 3: network-based price discrimination

Last, we allow networks to differentiate their emission and reception charges according to whether the communication is on- or off-net. In the presence of network-based discrimination, we need to separate the market for on-net calls and the market for off-net calls. In the former market, regardless of the introduction of reception charges, each network fully internalizes the externalities on callers and receivers. In contrast, in the latter market, the off-net caller and receiver charges affect the welfare of consumers on the rival network and are therefore subject to strategic manipulations.

Intuitively, a network has an incentive to charge a high off-net emission price if the receivers on the other network benefit almost as much from communications as the callers. Conversely, a network has an incentive to charge a high off-net receiver price if the receivers derive little utility from being called, since communications then benefit mainly the callers on the rival network. We provide sufficient conditions for these incentives to lead to a de facto connectivity breakdown between the networks. The logic of the connectivity breakdown here differs from the standard one emphasized in the network externalities literature[2] and stressing the incentive of a dominant player (characterized by a large installed base or a cost superiority) to reinforce dominance by reducing connectivity. Here, connectivity breakdowns occurs even with symmetric operators.

Finally, we show that, under an appropriate regulation of the reception charge, there exist equilibria in which both on-net and off-net charges are optimal. Furthermore, there exists an equilibrium that yields the monopoly profit and yet maximizes social welfare.

The paper is organized as follows. Section 2 describes the framework of analysis, introduces the relevant concepts and notation and derives the social optimum benchmark. Section 3 analyzes competition in nonlinear prices in the absence of network-based price discrimination and for regulated reception charges, and then Section 4 studies market determined reception charges. Section 5 performs similar analysis with network-based price discrimination. Section 6 concludes.

[2]See, e.g., Katz-Shapiro (1985) and Crémer et al. (2000).

1.3 Literature review

After investigating the British telephone industry, Oftel (1998) concluded that the price of calls from fixed networks to mobile phones was too high and envisioned price regulation. Indeed, with the caller pays principle (CPP), there might be little competitive pressure on the termination charge that a mobile phone operator can demand for terminating calls originated in the fixed network and the resulting high charges hurt the consumers of the fixed network. Opposing Oftel's proposed price regulation, Doyle and Smith (1998) offer instead to apply the receiver pays principle (RPP), by which mobile operators charge their own customers for receiving calls while callers pay a uniform per minute charge, e.g. the local call charge, regardless of where the calls terminate. They study a model with a monopoly fixed-link network and a duopoly of mobile operators. They first study CPP, assuming that each mobile network sets its own termination charge to be paid by the fixed network. This situation leads to a strong form of double marginalization on fixed-to-mobile calls (all the more that receivers are assumed not to derive surplus from being called). Turning to RPP, they show that mobile operators compete on reception charges to attract customers, which leads to a lower total charge of a fixed-to-mobile call as well as increased usage.

Kim and Lim (2000), as we do, address the question of how the RPP may help with the internalization of the call externalities when subscribers derive utility from receiving calls.[3] They consider two models without regulation of the access charge and the reception charge and with linear, unregulated pricing of calls. Their first model is a monopoly model. The introduction of a linear receiver charge decreases the perceived marginal cost of a call for the network and therefore leads to a lower price of a call. However, the effect on the total price (call price plus reception charge) and on welfare depends on how the price elasticity of demand varies with price. In the second model, they introduce call externalities in the Laffont, Rey and Tirole (1998a) model and assume that the access charge is set cooperatively by the networks before competing in linear prices. A network operator charges reception to all consumers (both his own subscribers and the subscribers of the other network) for calls initiated on his own network. In contrast, we assume that each operator sets reception charges for his own customers. Again, they show that the calling price decreases with the RPP but that the access charge is higher with the RPP, with ambiguous results on welfare.

DeGraba (2000) looks at a model with call externalities and network-based price discrimination and argues that the bill-and-keep policy (zero access charges) leads to efficient pricing in a symmetric case where the origination and termination costs are equal and the called party and the calling party benefit equally from a phone call.

[3] Littlechild (1977) suggests that these externalities can be internalized by cooperation between parties. However, this can only be true for subscribers who have repeated relationships.

2 Framework

2.1 The model

• *Demand Side*

We extend the analysis of network competition in Laffont, Rey and Tirole (LRT, 1998a, b) in two respects: Receivers obtain positive utility from receiving calls and firms can charge receivers for reception.

There are two operators (suppliers, networks), $i = 1, 2$, located at the two extremes of an Hotelling line of length one $(x_1 = 0, x_2 = 1)$. Consumers are differentiated along the Hotelling line. A consumer located at x and selecting network i incurs "transportation cost" $t|x - x_i|$.

The utility of a consumer with income y located at x and joining network i is given by

$$y + v_0 - t|x - x_i| + u(q) + \tilde{u}(\tilde{q})$$

where $u(q)$ is the utility from calls placed by the consumer and $\tilde{u}(\tilde{q})$ represents the utility from received calls.[4] We assume that these utility functions $u(\cdot)$ and $\tilde{u}(\cdot)$ are twice continuously differentiable, with $u' > 0, u'' < 0$; $\widetilde{u}' > 0, \widetilde{u}'' < 0$, which implies that demand functions are differentiable. $u(q)$ and $\tilde{u}(q)$ can be thought of as the caller's and receiver's surpluses attached to a representative call (lasting q minutes). We assume that the receiver's marginal surplus from receiving a call is nonnegative.

We consider four different cases depending on whether network-based (on-net / off-net) price discrimination is allowed, and on whether the volume of calls is determined only by callers or jointly by callers and receivers.

a) *Price discrimination:* In the absence of network-based price discrimination (Sections 3 and 4), network i offers a three-part tariff $\{F_i, p_i, r_i\}$. F_i is the monthly subscriber charge, p_i is the (caller's) usage price, and r_i represents the per-unit price that network i's consumers pay for the calls received.

   Under network-based discrimination (Section 5), network i offers a five-part tariff, $\{F_i, p_i, \hat{p}_i, r_i, \hat{r}_i\}$. Here, $\hat{r}_i$ represents the price that a network i's consumer pays for receiving calls originating on network $j \neq i$, while r_i is the reception charge for on-net calls. Similarly, p_i and $\hat{p}_i$ refer to per-unit charges for calls that terminate on- and off-net, respectively.

b) *Demand function:* We first make the standard assumption that callers determine the volume. Letting $q(.)$ denote the caller's demand function, given by $u'(q(p)) = p$, the volume of calls placed by a customer of network i is given by $q(p_i)$ in the absence

[4] The constant v_0 in the utility function for a consumer who has joined a network ensures that all consumers will always choose to join one of the two networks if v_0 is high enough.

of discrimination, and by $q(p_i)$ and $q(\hat{p}_i)$ in the case of discrimination. Let $v(p)$ be the indirect utility function, i.e.,

$$v(p) = \max_q \{u(q) - pq)\}$$

We then consider the case in which receivers are sovereign, i.e., callers and receivers jointly determine the volume. Consider a representative caller-receiver pair. In the absence of discrimination, the volume of calls from (the caller's) network i to (the receiver's) network $j(= 1, 2)$ is given by $\min\{q(p_i), \tilde{q}(r_j)\}$ where the receiver's demand function is given by $\tilde{u}'(\tilde{q}(r)) = r$. Under discrimination, the volume of calls, if both belong to network i is given by $\min\{q(p_i), \tilde{q}(r_i)\}$ and the volume of calls, if the caller belongs to network i and the receiver to network $j \neq i$, is given by $\min\{q(\hat{p}_i), \tilde{q}(\hat{r}_j)\}$.

- *Supply side*

The local loop cost is decomposed into a traffic sensitive marginal cost, c_0 per unit of volume, and a traffic insensitive component. The traffic insensitive part is composed of a per-consumer connection component f, plus possibly some cost that is joint and common to all consumers. For notational simplicity, we will ignore the latter (introducing a joint and common cost would just require an overall upward adjustment of price levels keeping the price structure as given, if firm viability is an issue). The long distance (trunk) marginal cost is equal to c_1. So, the total marginal cost of a minute of a call involving two local loops and a long distance traffic is

$$c \equiv 2c_0 + c_1.$$

We let a denote the access charge or termination charge. The marginal cost of an off-net call is therefore $c + (a - c_0)$ for the caller's network and $(c_0 - a)$ for the receiver's network.

2.2 Ramsey benchmark

For future reference, we derive the social optimum. Consider an idealized situation in which a benevolent regulator would choose the market shares and the volume of calls. In our symmetric set-up, equal market division ($\alpha = \frac{1}{2}$) minimizes the average consumer's disutility from not being able to consume his preferred service. The benevolent regulator would choose the volume of calls so as to maximize

$$u(q) + \tilde{u}(q) - cq.$$

The optimal volume q^* is determined by

$$u'(q^*) + \tilde{u}'(q^*) = c.$$

To implement the optimal outcome, the benevolent regulator can use symmetric tariffs so as to implement equal market division ($\alpha = \frac{1}{2}$). When the volume is determined by callers, the optimal volume is obtained by choosing $p_1 = p_2 = p^*$, where

$$p^* = c - \tilde{u}'(q^*).$$

The regulator selects the fixed fee F (or r) in order to satisfy the industry's break-even constraint.

When the volume is jointly determined by callers and receivers, the regulator can still achieve the efficient outcome by choosing ($p = p^*, r = 0$). The regulator then uses F to satisfy the industry break-even constraint. More generally, any $\{F, r\}$ combination yielding the same level of $F + rq^*$ and such that $r \le \tilde{u}'(q^*)$ achieves the Ramsey outcome.

3 Regulated or contractually determined reception charges

This section studies competition in two-part tariffs without discrimination (F_i, p_i) (when the reception charges are exogenously regulated at some levels $\{r_i\}_{i=1,2}$). It first assumes that callers determine the volume of calls, and then finds sufficient conditions for this to be the case.

Ignoring the "transportation cost", the net surplus of a network i consumer is

$$w_i = v(p_i) + \alpha_i \tilde{u}(q(p_i)) + \alpha_j \tilde{u}(q(p_j)) - r_i[\alpha_i q(p_i) + \alpha_j q(p_j)] - F_i. \tag{1}$$

Network i's market share is given by

$$\alpha_i = \frac{1}{2} + \sigma(w_i - w_j), \tag{2}$$

where $\sigma = 1/2t$ measures network substitutability, and thus the intensity of competition. Equivalently,

$$\alpha_i = \frac{1}{2} + \sigma[v(p_i) - v(p_j) - (F_i - F_j) - (r_i - r_j)(\alpha_i q(p_i) + \alpha_j q(p_j))]. \tag{3}$$

Network i's profit is given by

$$\pi_i = \alpha_i\{[p_i - c - \alpha_j(a - c_0) + \alpha_i r_i]q(p_i) + (r_i + a - c_0)\alpha_j q(p_j) + F_i - f\}.$$

We will perform our analysis in two steps. First, we will maximize π_i with respect to p_i given α_i, yielding price $p_i^{**}(\alpha_i)$. This will allow us to define $\Pi_i(\alpha_i) \equiv \pi_i(\alpha_i, p_i^{**}(\alpha_i))$. Second, we will maximize $\Pi_i(\alpha_i)$ with respect to α_i.

3.1 Tariff structure: maximization keeping market share constant

We study the program of maximizing π_i given α_i. Let

$$\tilde{F}_i = F_i + r_i(\alpha_i q(p_i) + \alpha_j q(p_j)).$$

Intuitively, $F_i + r_i(\alpha_i q(p_i) + \alpha_j q(p_j))$ is a generalized fixed fee. Network i's consumers care only about this sum, not about its composition.

Market shares are determined by the net surplus differential :

$$w_i - w_j = v(p_i) - v(p_j) - \tilde{F}_i + \tilde{F}_j. \tag{4}$$

We have

$$\pi_i \equiv \alpha_i\{(p_i - c)q(p_i) - (a - c_0)(1 - \alpha_i)(q(p_i) - q(p_j)) + \tilde{F}_i - f\}.$$

Using (2) and (4) we have :

$$\tilde{F}_i = \tilde{F}_j + v(p_i) - v(p_j) + \frac{1}{\sigma}(\frac{1}{2} - \alpha_i).$$

After substitution of $\tilde{F}_i$ into the profit function, we have

$$\begin{aligned}\pi_i(p_i, \alpha_i) \ &\equiv \alpha_i\{(p_i - c)q(p_i) - (a - c_0)(1 - \alpha_i)(q(p_i) - q(p_j)) \\ &+v(p_i) - v(p_j) + \tilde{F}_j + \frac{1}{2\sigma} - \frac{\alpha_i}{\sigma} - f\}\end{aligned}$$

where $\tilde{F}_j$ is a function of p_i. Indeed,

$$\frac{\partial \tilde{F}_j}{\partial p_i} = r_j \alpha_i \frac{dq}{dp_i}.$$

The first-order derivative of network i's profit with respect to p_i keeping α_i constant is given by

$$\left.\frac{d\pi_i}{dp_i}\right|_{\alpha_i=\text{constant}} = \alpha_i\left[p_i - c - \alpha_j(a - c_0) + \alpha_i r_j\right]\frac{dq}{dp_i}, \quad \text{for } p_i > 0,$$

For a given α_i, the profit maximizing price p_i is therefore given by $p_i^{**}(\alpha_i)$:[5]

$$p_i^{**}(\alpha_i) = c + \alpha_j(a - c_0) - \alpha_i r_j. \tag{5}$$

[5]In fact, we have, for $p_i^{**}(\alpha_i) > 0$:

$$\begin{aligned}\left.\tfrac{d\pi_i}{dp_i}\right|_{\alpha_i=\text{constant}} &> 0, \quad \text{for all } 0 < p_i < p_i^{**}(\alpha_i),\\ \left.\tfrac{d\pi_i}{dp_i}\right|_{\alpha_i=\text{constant}} &= 0, \quad \text{for } p_i = p_i^{**}(\alpha_i),\\ \left.\tfrac{d\pi_i}{dp_i}\right|_{\alpha_i=\text{constant}} &< 0, \quad \text{for all } p_i > p_i^{**}(\alpha_i).\end{aligned}$$

If $r_j = 0$, the usage price is equal to the average marginal cost faced by network i as in LRT (1998a). However, in the presence of r_j, an increase in $q(p_i)$ imposes *pecuniary externalities* on network j consumers by making them pay more money for the calls received from network i. Hence, to maintain market share α_i constant, network i can charge more money to the consumers of its own network by increasing F_i. In other words, network j's charging for reception results in a decrease in the marginal cost perceived by network i. For example, if the access charge is near or above termination cost, charging receivers is socially desirable since this induces firms to lower emission charges.

If $c > r_j$ holds for $a - c_0 + r_j \geq 0$ or if $c + a - c_0 > 0$ holds for $a - c_0 + r_j < 0$, we have $p_i^{**}(\alpha_i) > 0$. Then, the profit maximizing price p_i is uniquely given by $p_i^{**}(\alpha_i)$ and we can define $\Pi_i(\alpha_i)$ by

$$\Pi_i(\alpha_i) \equiv \pi_i(p_i^{**}(\alpha_i), \alpha_i)$$

$$= \alpha_i \Big\{ (p_i^{**} - c)q(p_i^{**}) - (a - c_0)(1 - \alpha_i)(q(p_i^{**}) - q(p_j))$$

$$+ v(p_i^{**}) - v(p_j) + \tilde{F}_j(p_i^{**}, \alpha_i) + \frac{1}{2\sigma} - \frac{\alpha_i}{\sigma} - f \Big\}.$$

Proposition 1 ***(strategic marginal cost)***

(a) When the volume is determined solely by the caller, only the sum of the monthly subscriber charge and the subscriber's total reception charge matters given the rival network's competitive offering.

(b) In the absence of network-based price discrimination, a network's marginal cost and emission charge decrease with the other network's reception charge:

$$p_i^{**}(\alpha_i) = c + \alpha_j(a - c_0) - \alpha_i r_j.$$

As usual with competition in two-part tariffs, the marginal price is set at the *perceived* marginal cost. It is composed of the true marginal cost, c plus a mark up due to the access charge incurred in off-net calls (the number of which is proportional to the competitor's market share α_j), hence $\alpha_j(a - c_0)$, minus the pecuniary-effect correction $-\alpha_i r_j$.

Socially, and as explained in Section 2, the marginal price should be

$$c - \tilde{u}'(q^*),$$

which could be obtained by pricing access at its marginal cost $a = c_0$, and by subsidizing calls at a rate

$$\tau = \tilde{u}'(q^*)$$

in the absence of reception charges.

In the absence of subsidization, two instruments, the access charge and the reception charge, can be used to induce the optimal marginal price. Because network i internalizes only the positive externality on his own consumers, the reception charge would have to be (in a symmetric equilibrium) twice the marginal externality if the termination charge were set equal to termination cost ($a = c_0$). But this would induce receivers to hang up. *The fact that the reception charge cannot exceed the marginal externality calls for an access charge below the termination cost.*

Note that this simple first-order-condition approach is incomplete because proving the existence of equilibrium requires some constraints on these instruments (see Section 3.3).

3.2 Tariff level: Maximization with respect to market share

We now maximize $\Pi_i(\alpha_i)$ with respect to α_i. Since we will focus on symmetric equilibria, we will assume $r_j = r$ and study the maximization of $\Pi_i(\alpha_i)$ when network j uses the optimal $p_j = c + \frac{1}{2}(a - c_0 - r)$.

To show the existence of a symmetric equilibrium, let us restrict the analysis to meaningful values of the access charge and of the reception charge such that $\infty > a > c_0 - c$ and $c > r > -\infty$. Then, we have:

Lemma 1 $\Pi_i(\alpha_i) \equiv \pi_i(p_i^{**}(\alpha_i), \alpha_i)$ *is well defined and continuous. Furthermore, if* σ *is small enough or* $|a - c_0 + r|$ *is small enough, it is concave.*

Proof. See Appendix 1. ■

When $\Pi_i(\alpha_i)$ is concave, the unique solution is given by the first-order condition

$$(p_i - c)q_i - (a - c_0)(q_i - q_j)(1 - 2\alpha_i) + v_i - v_j - f + \frac{1}{2\sigma} - \frac{2\alpha_i}{\sigma}$$

$$+F_j + r(2\alpha_i q_i + (1 - 2\alpha_i)q_j) = 0. \tag{6}$$

The case of a small substitutability σ is relevant only if the networks are specialized in the sense of fitting geographic or technological niches.[6] The condition that the reception subsidy r not be too remote from the termination discount $(c_0 - a)$ underlines the potential instability that may exist in network competition when reception charges are regulated.

3.3 Symmetric equilibria

For a given reception charge r, equilibria (p_i, F_i, α_i) are characterized by (3), (5) and (6). The next proposition characterizes symmetric equilibria: $(p, F, \alpha = \frac{1}{2})$ for a given value of r.

Proposition 2 ***(existence)***: *(a) If* $\Pi_i(\alpha_i)$ *is concave, a symmetric equilibrium* $(p, F, \alpha = \frac{1}{2})$ *exists.*

[6] It may also be relevant to study the competition between one fixed and one mobile networks.

(b) No cornered-market equilibrium exists.

(c) For any $\varepsilon > 0$, if $|a - c_0 + r| > \varepsilon$, no equilibrium exists for σ large enough.

Proof. See Appendix 2. ■

So, if a isn't close to $c_0 - r$ and substitutability is high, there exists no pure strategy equilibrium. Indeed, the only candidate for equilibrium is subject to undercutting by one network.[7]

Specializing to symmetric equilibria (5),

$$p + \frac{r}{2} = c + \frac{1}{2}(a - c_0); \tag{7}$$

and (6),

$$F = f + \frac{1}{2\sigma} - (p + r - c)q(p). \tag{8}$$

After some computations, we obtain

$$\pi = \frac{1}{4\sigma}, \text{ when } (F, p) \text{ satisfy (7) and (8).}$$

Thus, the profit is always equal to the Hotelling profit with unit demand, as in LRT (1998a). Provided that symmetric equilibria exist, the access charge and the reception charge have no impact on profit.

Summarizing we have:

Proposition 3 ***(characterization)****: The symmetric equilibrium is characterized by:*

(a) $p + \frac{r}{2} = c + \frac{1}{2}(a - c_0), F = f + \frac{1}{2\sigma} - (p + r - c)q(p).$

(b) Each firm's profit is equal to $\frac{1}{4\sigma}$.

3.4 Social welfare maximizing equilibrium

What matters for social welfare is p only. From (7), p is equal to p^*, if and only if

$$r = a - c_0 + 2(c - p^*). \tag{9}$$

We have the following proposition.

Proposition 4 ***(efficiency)****: Suppose that $\tilde{u}(q) = \beta u(q)$ holds with $\beta \geq 0$. If the access charge a satisfies $-c < a - c_0 < \frac{1-\beta}{1+\beta}c$ for σ small enough or $a - c_0 + r \approx 0$ for σ large enough, the social welfare maximizing price p^* can be implemented as an equilibrium of network competition by choosing $r = (a - c_0) + \frac{2\beta}{1+\beta}c$.*

[7]The logic is here similar to that in LRT (1998a).

Proof. When $\tilde{u}(q) = \beta u(q)$, efficiency requires $a - c_0 = r - \frac{2\beta}{1+\beta}c$. For σ small enough, this condition, together with $a - c_0 + c > 0$ and $c > r$, gives $-c < a - c_0 < \frac{1-\beta}{1+\beta}c$. The existence comes from Proposition 2. For σ large enough, if $a - c_0 + r \approx 0$, we have $p_i^{**}(\alpha_i) > 0$ for all α_i. ■

3.5 Receiver sovereignty

The above analysis has not taken into account the fact that receivers may want to hang up. However, the above existence and optimality results are readily extended to the case of receiver sovereignty as long as the regulated reception charge does not exceed the marginal utility of reception; $r \leq \tilde{u}'(q(p))$.

Proposition 5 *: A symmetric equilibrium $(p^e, F^e, \alpha = 1/2)$ in the absence of receiver sovereignty (where p^e and F^e are given in Proposition 3) is still an equilibrium under receiver sovereignty as long as*

$$r \leq \tilde{u}'(q(p^e)).$$

Proof. Consider an equilibrium in the absence of receiver sovereignty. In particular, for each network i, p^e maximizes profit (given market share) over the domain $P = \{p_i : r \leq \tilde{u}'(q(p_i))\}$. Because under receiver sovereignty network i's profit becomes insensitive to p_i for p_i such that $r > \tilde{u}'(q(p_i))$ (since demand is then determined by r), p^e still maximizes profit in the extended domain $P' = \{p_i : p_i \geq 0\}$. ■

Let us summarize the main result of the section.

For low network substitutability, existence is always guaranteed. For any termination charge a, there exists a value of the reception charge (given by (9)) which induces the optimal marginal calling charge $p^* = c - \tilde{u}'(q^*)$. Guaranteeing that receivers do not want to hang up sets an upper bound on the termination charge which equals $c_0 - \frac{\beta}{1+\beta}c$ when $\tilde{u}(q) = \beta u(q)$, such that the termination charge must be below the termination cost.

For the more interesting case of high network substitutability, again there is for each termination charge a value of the reception charge that yields efficiency, but the existence of equilibrium calls for $a \approx c_0 - r$. In the limit of perfect network substitutability, and for $\tilde{u}(q) = \beta u(q)$, these two equations impose $r = \frac{\beta c}{1+\beta}$ and $a = c_0 - \frac{\beta c}{1+\beta}$. For this value of r, the caller and the receiver hang up together. A single value of the access charge is compatible with efficiency; it is smaller than the termination cost.

Altogether, these results allow us to conclude that a proper regulation of the reception charge enables the regulator to achieve the internalization of call externalities.

4 Market determined reception charges

As we discussed in the introduction, the absence of uncertainty about marginal utilities make operators locally indifferent as to the level of their reception charge in the range of parameters in which the caller's net marginal utility strictly exceeds the receiver's net marginal utility. This both is unrealistic and complicates the analysis. In reality, the receiver's utility, for example, may be subject to noise: for example, one is less eager to stay long on the phone when a visitor is in one's office or when watching over young children. When reception charges are market determined, it turns out to be convenient to allow for such state-contingent marginal utilities, since then both the calling and reception charges have incentive effects.

Suppose that the marginal utility that a receiver derives from receiving a call is subject to a noise ϵ.[8] The receiver's utility is:

$$\widetilde{u}(q) + \epsilon q.$$

We assume that ϵ follows the distribution function $F(\cdot)$, with wide enough support $[\underline{\epsilon}, \overline{\epsilon}]$, zero mean and density $f(\cdot)$, which is strictly positive for all ϵ in $[\underline{\epsilon}, \overline{\epsilon}]$; and that the noise ϵ is identically and independently distributed for each caller-receiver pair.

For simplicity, we further assume in this section that:

$$\widetilde{u}(q) = \beta u(q) \text{ with } \beta > 0.$$

We first study how the volume is determined given (p_i, r_j) and a realized value ϵ of the random variable. Unless the caller interrupts the conversation first, the receiver with noise ϵ will equate his marginal utility $\widetilde{u}' + \epsilon$ to the reception charge r_j. Hence, the volume of call is given by $q(\max(p_i, \frac{r_j - \epsilon}{\beta}))$. Therefore, the volume of calls from network i to network j is given by:

$$\alpha_i \alpha_j D(p_i, r_j),$$

$$\text{with } D(p_i, r_j) \equiv [1 - F(r_j - \beta p_i)]\, q(p_i) + \int_{\underline{\epsilon}}^{r_j - \beta p_i} q(\frac{r_j - \epsilon}{\beta}) f(\epsilon) d\epsilon.$$

Similarly, the utility that a network i consumer derives by making calls to network j consumers is given by:

$$\alpha_j U(p_i, r_j),$$

$$\text{with } U(p_i, r_j) \equiv [1 - F(r_j - \beta p_i)]\, u(q(p_i)) + \int_{\underline{\epsilon}}^{r_j - \beta p_i} u(q(\frac{r_j - \epsilon}{\beta})) f(\epsilon) d\epsilon.$$

The utility that a network j consumer derives from receiving calls from network i consumers is given by:

$$\alpha_i \widetilde{U}(p_i, r_j),$$

[8] The caller's marginal utility could also be subject to a noise without any change in the results. The important feature of the following analysis is that both the caller and the receiver have positive probability of hanging up first.

with $\widetilde{U}(p_i.r_j) \equiv \int_{r_j-\beta p_i}^{\overline{\epsilon}} \left[\widetilde{u}(q(p_i)) + \epsilon q(p_i)\right] f(\epsilon)d\epsilon + \int_{\underline{\epsilon}}^{r_j-\beta p_i} \left[\widetilde{u}(q(\frac{r_j-\epsilon}{\beta})) + \epsilon q(\frac{r_j-\epsilon}{\beta})\right] f(\epsilon)d\epsilon.$

Therefore, the net surplus of a network i consumer is given by:

$$w_i = \alpha_i U(p_i, r_i) + \alpha_j U(p_i, r_j) + \alpha_i \widetilde{U}(p_i, r_i) + \alpha_j \widetilde{U}(p_j, r_i)$$

$$-p_i \left[\alpha_i D(p_i, r_i) + \alpha_j D(p_i, r_j)\right] - r_i \left[\alpha_i D(p_i, r_i) + \alpha_j D(p_j, r_i)\right] - F_i.$$

And the profit of network i is given by:

$$\pi_i \equiv \alpha_i \left\{\alpha_i (p_i - c) D(p_i, r_i) + \alpha_j \left[p_i - c - (a - c_0)\right] D(p_i, r_j)\right.$$

$$\left. +\alpha_j (a - c_0) D(p_j, r_i) + r_i \left[\alpha_i D(p_i, r_i) + \alpha_j D(p_j, r_i)\right] + F_i - f\right\}.$$

We now analyze the first-order conditions. Given market share α_i, the first-order derivative of π_i with respect to p_i is given by:

$$\alpha_i \left[1 - F(r_i - \beta p_i)\right] \alpha_i \left[u' + \widetilde{u}' + E(\epsilon \mid \epsilon \geq r_i - \beta p_i) - c\right] q'$$

$$+\alpha_i \left[1 - F(r_j - \beta p_i)\right] \left[\alpha_j (u' - c - a + c_0) + \alpha_i \left(r_j - \widetilde{u}' - E(\epsilon \mid \epsilon \geq r_j - \beta p_i)\right)\right] q'.$$

Consider a small decrease in p_i. This increases the volume of on-net calls by $\left[1 - F(r_i - \beta p_i)\right] q'$ and the volume of off-net calls by $\left[1 - F(r_j - \beta p_i)\right] q'$. In the market for on-net calls, network i consumers' utility increases by $u' + \widetilde{u}' + E(\epsilon \mid \cdot)$, which the network can extract by increasing the fixed tariff F_i. In the market for off-net calls, network i consumers' utility increases by u' and network j consumers' utility increases by $\widetilde{u}' + E(\epsilon \mid \cdot) - r_j$. As before, $\widetilde{u}' + E(\epsilon \mid \cdot)$ represents the direct externalities, r_j represents the pecuniary externalities and an increase in network j consumers' utility requires a decrease in F_i in order to keep α_i constant.[9]

When $r = r_i = r_j$, the first-order derivative simplifies to:

$$\alpha_i \left[1 - F(r - \beta p_i)\right] \left[p_i - c - \alpha_j (a - c_0) + \alpha_i r\right] q',$$

which gives the following first-order condition:

$$p_i = c + \alpha_j (a - c_0) - \alpha_i r.$$

We note that this condition is the one we found in Section 3 in the absence of noise.

The first-order derivative of π_i with respect to r_i is given by:

$$\alpha_i F(r_i - \beta p_i) \alpha_i E\left[(u' + \widetilde{u}' + \epsilon - c) q' \mid \epsilon \leq r_i - \beta p_i\right] \frac{1}{\beta}$$

[9]The reader will check that the strategic-marginal-cost pricing formula of Proposition 1 holds as the noise vanishes and the caller determines volume with probability (close to) one.

$$+\alpha_i F(r_i - \beta p_j) E\left[\alpha_j(\widetilde{u}' + \epsilon + a - c_0)q' + \alpha_i(p_j - u')q' \mid \epsilon \leq r_i - \beta p_j\right] \frac{1}{\beta}.$$

Consider a small decrease in r_i. This will increase the volume of on-net calls by $F(r_i - \beta p_i)\ E(q' \mid \cdot)/\beta$ and the volume of the off-net calls received from network j by $F(r_i - \beta p_j) E(q' \mid \cdot)/\beta$. In the market for on-net calls, network i consumers' utility increases by $E(u' + \widetilde{u}' + \epsilon \mid \cdot)$. In the market for off-net calls, network i consumers' utility increases by $E(\widetilde{u}' + \epsilon \mid \cdot)$ and network j consumers' utility increases by $E(u' - p_j \mid \cdot)$. u' represents the direct externalities and p_j represents the pecuniary externalities.

When $p = p_i = p_j$, the first-order derivative simplifies to:

$$\alpha_i F(r_i - \beta p) E\left[(r_i - \alpha_i c + \alpha_j(a - c_0) + \alpha_i p)q' \mid \epsilon \leq r_i - \beta p_j\right] \frac{1}{\beta},$$

which gives the following first-order condition:

$$r_i = \alpha_i c - \alpha_j(a - c_0) - \alpha_i p$$

We note that *this condition is the one we would find in the absence of noise if the volume were determined by receivers.*

Consider now a symmetric equilibrium with $\alpha_i = \frac{1}{2}$. From the two first-order conditions, we have:

$$p = c + \frac{a - c_0 - r}{2},$$

$$r = \frac{c - (a - c_0) - p}{2}.$$

These two conditions yield $p = c + (a - c_0)$ and $r = c_0 - a$. Furthermore, we show below that, as the noise vanishes (the distribution F converges to a spike at value 0 while keeping a wide enough support to confer an incentive role upon the reception charges), the equilibrium in which the volume is determined by callers exists if the access charge markup is larger than $-\frac{\beta}{1+\beta}c$. This analysis is summarized in:

Proposition 6 ***(off-net-cost pricing)*** *Suppose that the reception charges are non-cooperatively set by the networks and that the marginal utility of call reception is random.*

(a) There exists a unique symmetric candidate equilibrium. For this candidate, the reception charge is equal to the access charge discount:

$$r = c_0 - a.$$

And the emission charge is

$$p = c + (a - c_0).$$

That is, the networks price calls and call receptions at their off-net cost.

(b) Furthermore, when $a - c_0 \geq -\frac{\beta}{1+\beta}c$ holds, as the noise converges to zero, the candidate equilibrium is an equilibrium and in the equilibrium the volume is determined by callers.

(c) As the noise converges to zero, the optimum can be approximated by choosing an access charge a such that $a - c_0 = -r^* \equiv -\beta c/(1+\beta)$.

Proof: See Appendix 3. ■

Note the remarkable property that the privately optimal reception charge (equal to the off-net reception cost) ensures the existence of equilibrium for any value of the access charge. Furthermore, a proper choice of the access charge yields efficiency.

However, efficiency is of course impossible to achieve with a single instrument in the presence of noise. For example, in the caller-determined-volume region ($u'(q) = p$ and $\tilde{u}'(q) + \epsilon > r$), the sum of the marginal utilities always exceeds c: There is always underprovision of communications.

5 Network-based discrimination

Network i's tariff is characterized now by a five-uple $\{F_i, p_i, \widehat{p}_i, r_i, \widehat{r}_i\}$, where hats refer to off-net communications. Again, we can distinguish the case in which the reception charges $(r_i, \hat{r}_i)$ are regulated from the case in which they are not.

We show below that network-based discrimination is a mixed blessing. While network-based discrimination induces network i to choose the on-net price p_i to fully internalize the externalities on its receivers, it also allows networks to implement *selective connectivity breakdown* by charging very high or even infinite prices ($\widehat{p}_i$ or $\widehat{r}_i$) for off-net calls, which results in a global lack of connectivity.

For simplicity, we will assume again in this section that $\widetilde{u}(q) = \beta u(q)$ with $\beta \geq 0$.

We identify two different reasons why network competition results in connectivity breakdown:

- In the absence of reception charges, each network's equilibrium off-net caller charge tends to infinity as the receiver's utility converges toward the caller's (that is, β converges to 1). The intuition for this result is that a receiver on the rival network (who, recall, does not pay any reception charge) fully enjoys her surplus from the call. In contrast, the caller-network pair perceives only the net surplus (caller surplus minus calling cost). And so, off-net calls make the rival network relatively more attractive for β large, which leads to a connectivity breakdown.

- The introduction of reception charges should a priori reduce this incentive for connectivity breakdown. Reception charges, however, provide a second instrument for implementing selective connectivity breakdown: Instead of inducing the caller to hang up (or not to call off-net) as above, each network can induce the receiver to hang up off-net calls. We provide sufficient conditions for the equilibrium off-net emission or reception charges to be infinite. This suggests looking at the regulation of the reception charge for off-net calls $\widehat{r}_i$. Hence, we study network competition under the regulation of off-net reception charges and show that the efficient allocation can be achieved with an appropriate regulation of $\widehat{r}_i$.

5.1 Connectivity breakdown in the absence of reception charge

We show that, in the absence of reception charge, the equilibrium price for off-net calls $\widehat{p}$ goes to infinity as β goes to one. Thus, we have a *de facto* connectivity breakdown. Furthermore, for $\beta \geq 1$, we show that network competition always results in connectivity breakdown ($\widehat{p} = \infty$).

In the absence of reception charge, the volume is automatically determined by callers. The net surplus of a network i consumer is given by:

$$w_i = \alpha_i v(p_i) + \alpha_j v(\widehat{p}_i) + \alpha_i \widetilde{u}(q(p_i)) + \alpha_j \widetilde{u}(q(\widehat{p}_j)) - F_i. \tag{10}$$

Network i's market share is given by

$$\alpha_i = \frac{1}{2} + \sigma(w_i - w_j).$$

Equivalently,

$$\alpha_i = \frac{1}{2} + \sigma\left[\alpha_i v(p_i) + \alpha_j v(\widehat{p}_i) - \alpha_j v(p_j) - \alpha_i v(\widehat{p}_j)\right.$$

$$\left. + \alpha_i \widetilde{u}(q(p_i)) + \alpha_j \widetilde{u}(q(\widehat{p}_j)) - \alpha_j \widetilde{u}(q(p_j)) - \alpha_i \widetilde{u}(q(\widehat{p}_i)) - F_i + F_j\right].$$

Network i's profit is given by

$$\pi_i = \alpha_i \left\{(p_i - c)\alpha_i q(p_i) + (\widehat{p}_i - c - (a - c_0))\alpha_j q(\widehat{p}_i)\right.$$

$$\left. + (a - c_0)\alpha_j q(\widehat{p}_j) + F_i - f\right\}.$$

As in the no-discrimination case, we will perform our analysis in two steps. First, we maximize π_i with respect to p_i and $\widehat{p}_i$ keeping market share α_i constant. Second, we perform the maximization with respect to the market share.

For all α_i (> 0), the profit maximizing price p_i is equal to the social welfare maximizing price p^*.

$$p_i^{**}(\alpha_i) = p^* = c - \widetilde{u}'(q(p^*)). \tag{11}$$

For on-net calls, network i fully internalizes the externalities on receivers. Since network i is a monopoly in the market for on-net calls, under a two-part tariff, it maximizes the pie. Hence, both networks choose the same price p^* regardless of the market shares.

The first-order derivative of profit with respect to $\widehat{p}_i$ is given by:

$$\alpha_i \left\{ \alpha_j \left[\widehat{p}_i - c - (a - c_0) \right] - \alpha_i \widetilde{u}'(q(\widehat{p}_i)) \right\} \frac{dq}{d\widehat{p}_i}, \text{ for } \widehat{p}_i > 0.$$

The first term represents the direct impact on the profit while the second term represents the indirect impact arising from the condition of keeping market share constant. The latter represents the direct externalities on the consumers of network j. Consider an incremental increase in the volume of calls from network i to network j, $q(\widehat{p}_i) \equiv \widehat{q}_i$. Then, network j's consumers' utility from reception increases by $\widetilde{u}'(q(\widehat{p}_i))$. When the market share α_i is kept constant, an increase in the utility obtained by network j's consumers implies an increase in the marginal cost perceived by network i. Since $\widetilde{u}'(q(\widehat{p}_i)) = \beta\widehat{p}_i$, the optimal price $\widehat{p}_i^{**}$ depends upon the market share as follows:

$$\widehat{p}_i^{**}(\alpha_i) = \begin{cases} \frac{(1-\alpha_i)(c+a-c_0)}{1-(1+\beta)\alpha_i} & \text{if } \alpha_i < \frac{1}{1+\beta}, \\ \infty, & \text{otherwise,} \end{cases}$$

where we assume that $c + a - c_0 > 0$.

Therefore, social and network i's private incentives regarding the choice of $\widehat{p}_i$ are in conflict. From the social welfare point of view, the positive externalities on the consumers of network j should be internalized by a decrease in $\widehat{p}_i$. In contrast, from network i's point of view, these externalities are costly in that it must increase the utility of its own consumers in order to make the marginal consumer indifferent between two networks. This results in an increase in its perceived marginal cost and, as a consequence, an increase in $\widehat{p}_i$. The conflict becomes larger as β increases or α_i increases. This is because the utility that consumers of network j derive from receiving calls originating in network i becomes larger as β increases or α_i increases. To illustrate the point, consider the case $a = c_0$ with $\alpha_i < \frac{1}{1+\beta}$. If $\beta = 0$, there is no conflict between social and private incentives and the price is optimal: $\widehat{p}_i^{**}(\alpha_i) = p^* = c$. However, as β increases, $\widehat{p}_i$ increases. Furthermore, for any $\beta > 0$, if $\alpha_i > \frac{1}{1+\beta}$, $\widehat{p}_i^{**}(\alpha_i) = \infty$. We note that

$$\frac{d\widehat{p}_i^{**}}{d\alpha_i} = \frac{\beta(c + a - c_0)}{\left[1 - (1 + \beta)\alpha_i\right]^2} > 0, \text{ for } \alpha_i < \frac{1}{1 + \beta}.$$

Remark: The fact that network i takes into account the direct externalities on network j consumers when it chooses the price for off-net calls offers a new explanation of why the price of calling mobile phones from fixed networks is high. Doyle and Smith (1998), who do not consider the possibility that consumers can obtain utility from reception argue that there exists no competitive force that can drive mobile companies to charge low termination prices for off-net calls from fixed networks and that the resulting exorbitant termination charges substantially inflate the price of calling mobile phones from fixed networks.[10] Our model offers an alternative explanation for this phenomenon based on the incentive of fixed networks to choose high off-net call prices. Usually, the fixed-phone service is offered by a dominant firm with large market share compared to those of mobile

[10] Doyle and Smith (1998)'s proposal of RPP does not really work in their model because receivers would hang up.

companies. Therefore, the direct externalities on the consumers of mobile networks are large, which induces the fixed network to charge a high price for calling mobile phones. Furthermore, contrary to Doyle and Smith's argument, when consumers derive utility from reception, there exist competitive pressures which may induce mobile companies to choose moderate termination charges.

Proposition 7 *: In the absence of reception charge, and if a symmetric equilibrium with network-based price discrimination exists:*

(a) The price for on-net calls is socially optimal: $p = p^*$.

*(b) (**connectivity breakdown**) (i) For* $0 \leq \beta < 1$, *as* β *tends to one, network competition results in a de facto connectivity breakdown in that the price for off-net calls goes to infinity:*

$$\widehat{p} = \frac{c + a - c_0}{1 - \beta}.$$

(ii) For $\beta \geq 1$, *any symmetric equilibrium exhibits connectivity breakdown:* $\widehat{p} = \infty$.

Appendix 4 studies the existence of equilibrium for a constant-elasticity demand function. The second-order derivative for the program of maximizing the profit Π_i with respect to α_i is negative if σ is small enough and $a \simeq c_0$. These are sufficient conditions for a symmetric equilibrium to exist.

5.2 Connectivity breakdown with reception charge

In this section, we examine how the introduction of reception charges affects connectivity. In this case the volume is determined by both callers and receivers.

Let $q_{ij} = \min\{q(\widehat{p}_i), \widetilde{q}(\widehat{r}_j)\}$ for $i \neq j$ and $q_{ii} = \min\{q(p_i), \widetilde{q}(r_i)\}$.

We have

$$w_i = \alpha_i u(q_{ii}) + \alpha_j u(q_{ij}) + \alpha_i \widetilde{u}(q_{ii})) + \alpha_j \widetilde{u}(q_{ji})$$

$$-[p_i \alpha_i q_{ii} + \widehat{p}_i \alpha_j q_{ij}] - [r_i \alpha_i q_{ii} + \widehat{r}_i \alpha_j q_{ji}] - F_i.$$

Network i's profit is given by

$$\pi_i = \alpha_i \left[(p_i - c + r_i) \alpha_i q_{ii} + (\widehat{p}_i - c - (a - c_0)) \alpha_j q_{ij} \right.$$

$$\left. + (\widehat{r}_i + a - c_0) \alpha_j q_{ji} + F_i - f \right].$$

As in the absence of reception charge, it is optimal for network i to maximize the pie in the market for on-net calls. Therefore, we have:

$$(p_i = p^*, r_i \leq r^*) \text{ or } (p_i \leq p^*, r_i = r^*).$$

Once the volume is determined by p^* (respectively, r^*), r_i (respectively, p_i) does not affect the profit as long as $r_i \leq r^*$ (respectively, $p_i \leq p^*$).

Consider the off-net calls from network i to network j. The volume of calls q_{ij} is determined by $\max(\widehat{p}_i, \frac{\widehat{r}_j}{\beta})$. For expositional convenience, we introduce the following notation:

$$\pi_i^{\widehat{p}}(\widehat{p}_i : \alpha_i, \widehat{r}_j) \equiv \alpha_i \left\{ \alpha_j \left[u(q_{ij}) - (c + a - c_0) q_{ij} \right] + \alpha_i \left[\widehat{r}_j q_{ij} - \widetilde{u}(q_{ij}) \right] \right\},$$

$$\pi_j^{\widehat{r}}(\widehat{r}_j : \alpha_i, \widehat{p}_i) \equiv \alpha_j \left\{ \alpha_i \left[\widetilde{u}(q_{ij}) + (a - c_0) q_{ij} \right] + \alpha_j \left[\widehat{p}_i q_{ij} - u(q_{ij}) \right] \right\}.$$

$\pi_i^{\widehat{p}}(\widehat{p}_i)$ (respectively, $\pi_j^{\widehat{r}}(\widehat{r}_j)$) represents the share of π_i (respectively, π_j) that can be affected by $\widehat{p}_i$ (respectively, $\widehat{r}_j$) when α_i is kept constant. We note that $\pi_i^{\widehat{p}}(\infty) = \pi_j^{\widehat{r}}(\infty) = 0$. Therefore, each network can have at least zero profit in the market for off-net calls from network i to j by implementing selective connectivity breakdown. This defines an "individual rationality constraint". Since, in the absence of reception charge, $\pi_j^{\widehat{r}} < 0$ is possible at equilibrium, the introduction of reception charges adds a new individual rationality constraint: $\pi_i^{\widehat{r}} \geq 0$.

When $\beta\widehat{p}_i \geq \widehat{r}_j$, the first-order derivative of $\pi_i^{\widehat{p}}$ with respect to $\widehat{p}_i$ is given by:

$$\alpha_i \left\{ \alpha_j \left[\widehat{p}_i - c - (a - c_0) \right] + \alpha_i \left[(\widehat{r}_j - \widetilde{u}'(q(\widehat{p}_i))) \right] \right\} \frac{dq}{d\widehat{p}_i}, \text{ for } \widehat{p}_i > 0.$$

As it was the case in the absence of reception charge, the first term represents the direct impact on the profit while the second term represents the indirect impact which arises from the condition of keeping the market share constant. The second term has two components: pecuniary externalities and direct externalities on the consumers of network j. The pecuniary externalities occur since the consumers of network j have to pay for reception. The optimal price $\widehat{p}_i$ depends upon the market share in a complex way and can be infinite for a certain range of market shares. When $\beta\widehat{p}_i < \widehat{r}_j$, the first-order derivative with respect to $\widehat{p}_i$ is zero.

When $\widehat{r}_j \geq \beta\widehat{p}_i$, the first-order derivative of $\pi_j^{\widehat{r}}$ with respect to $\widehat{r}_j$ is given by:

$$\alpha_j \left\{ \alpha_i \left[\widehat{r}_j + a - c_0 \right] + \alpha_j \left[(\widehat{p}_i - u'(q(\frac{\widehat{r}_j}{\beta}))) \right] \right\} \frac{dq}{d\widehat{r}_j}, \text{ for } \widehat{r}_j > 0.$$

The interpretation is similar to the one given for the first-order derivative with respect to $\widehat{p}_i$: the first term represents the direct impact on the profit while the second term represents the indirect impact, which is composed of pecuniary externalities and direct externalities. The direct externalities represent the utilities that consumers of network i derive from making calls to consumers of network j.

The following proposition focuses on symmetric equilibria. We ignore equilibria based on "weakly dominated strategies" (an equilibrium with total connectivity breakdown always exists since $\widehat{p}_i = \infty$ is a best response to $\widehat{r}_j = \infty$ and vice versa).

Proposition 8 *(a) (**connectivity breakdown**): (i) For β small enough and $a < c_0$, any symmetric equilibrium exhibits connectivity breakdown: $\widehat{r} = \infty$.*

(ii) For β large enough and $c + a - c_0 > 0$, any symmetric equilibrium exhibits connectivity breakdown: $\widehat{p} = \infty$.

*(b) (**inefficiency**): No efficient symmetric equilibrium exists for $\beta \neq 1$.*

Proof See Appendix 5. ■

Reception charges are a mixed blessing in the context of network-based price discrimination. On the one hand, a positive $\widehat{r}_j$, through the pecuniary externalities, reduces the marginal cost perceived by network i, which helps network i to internalize the externalities on receivers. On the other hand, $\widehat{r}_j$ is set strategically by network j, whose private incentive is in conflict with social welfare maximization: the gain that consumers of network i derive from placing calls to consumers of network j increases the marginal cost perceived by network j.

Connectivity breakdowns occur for β small enough if the access charge markup is negative ($a - c_0 < 0$). When β is small, callers on network i derive some utility from interconnection of the two networks while receivers of network j derive almost no utility from interconnection. This makes network j's profit associated with $\widehat{r}_j$ ($\pi_j^{\widehat{r}}(\widehat{r}_j)$) negative whenever the access revenue from interconnection is negative and, consequently, network j is better off setting $\widehat{r}_j = \infty$ when $\widehat{p}_i < \infty$. Since, in the absence of reception charge, connectivity breakdown is not an issue for β small, this result shows that reception charges can make it even harder to internalize call externalities. By symmetry, when β is large enough, receivers on network j derive a large utility from interconnection while callers on network i derive a relatively small utility. Hence, network i is better off setting $\widehat{p}_i = \infty$ when $\widehat{r}_j < \infty$. Finally, there exists no efficient symmetric equilibrium. In Appendix 5, we show that efficiency requires $\widehat{p} = p^*$, $\widehat{r} = r^*$, and $a - c_0 + r^* = 0$. However, in this case, $\pi_j^{\widehat{r}}$ is strictly negative for β smaller than one and $\pi_i^{\widehat{p}}$ is strictly negative for β larger than one. Therefore, one of the two networks has the incentive to break down connectivity.

5.3 Regulation of reception charges

We just saw that network-based price discrimination allows each network to implement *selective connectivity breakdown*, that is breakdown of connectivity in one direction. This selective connectivity breakdown results in a two-way lack of connectivity. This observation calls for some form of regulation (broadly defined), in the same way termination charges cannot just be left to the discretion of the terminating networks. This "regulation" can take the form of a cooperatively determined off-net reception charge. Alternatively, the off-net reception charge may be set by a regulatory agency. In this section, we consider a specific form of regulation. Namely, we will assume that the regulator sets $\widehat{r}_j$ such that

$$\widehat{r}_j = \begin{cases} g(\underline{p}), & \text{if } \widehat{p}_i < \underline{p}, \\ g(\widehat{p}_i), & \text{if } \widehat{p}_i \geq \underline{p}, \end{cases}$$

$$\text{with } g(\widehat{p}_i) \equiv \beta \frac{\eta}{\eta - 1} \widehat{p}_i - \varepsilon \widehat{p}_i^{\eta} \text{ and } 0 < \underline{p} < c + a - c_0,$$

where η is the elasticity of demand, which we will assume is constant, and ε is a positive constant such that $g(\underline{p}) = \beta\underline{p}$ and $g(\widehat{p}_i) \leq \beta\widehat{p}_i$ for all $\widehat{p}_i \geq \underline{p}$. Hence, $\widehat{r}_j$ is chosen indirectly by network i.

The regulation of $\widehat{r}_i$ has two main consequences. First, since network i cannot control $\widehat{r}_i$, the regulation eliminates its strategic behavior regarding the choice of $\widehat{r}_i$. In particular, network i's opportunity profit related to off-net calls from network j to network i can be negative (network i would prefer to set $\hat{r}_i = \infty$ if it could). Second, the regulation is designed so as to ensure that a change in network i 's off-net caller charge ($\widehat{p}_i$) does not impact the welfare of a receiver on network j. This is why we focus on this form of regulation. It may look peculiar but it obeys some logic. A change in $\widehat{p}_i$ induces a change $dq/d\widehat{p}_i$ in the volume of off-net calls of network i's customers for all $\widehat{p}_i \geq \underline{p}$. The total externality on a network j receiver is

$$\frac{d}{d\widehat{p}_i}\left[\widetilde{u}(q(\widehat{p}_i)) - \widehat{r}_j(\widehat{p}_i)q(\widehat{p}_i)\right] = 0,$$

if $\widehat{r}_i$ is set as above. We note that when $\widehat{p}_i < \underline{p}$, both $\widehat{r}_j$ and q_{ij} are independent of $\widehat{p}_i$.

We continue to assume joint volume determination. As earlier, we will perform our analysis in two steps. First, we will maximize π_i given α_i. This will allow us to define $\Pi_i(\alpha_i) \equiv \pi_i(\alpha_i, p_i^*(\alpha_i), \widehat{p}_i^*(\alpha_i))$. Second, we will maximize $\Pi_i(\alpha_i)$ with respect to α_i.

5.3.1 Tariff structure: Maximization keeping market share constant

Since the regulation of $\widehat{r}_j$ has no impact on the choice of p_i and r_i, we have as before:

$$(p_i = p^*, r_i \leq r^*) \text{ or } (p_i \leq p^*, r_i = r^*). \tag{12}$$

The first-order derivative of profit with respect to $\widehat{p}_i$ is given by:

$$\alpha_i \left\{ \alpha_j \left[\widehat{p}_i - c - (a - c_0)\right] \frac{dq}{d\widehat{p}_i} + \alpha_i \left[(\widehat{r}_j - \widetilde{u}'(q(\widehat{p}_i))) \frac{dq}{d\widehat{p}_i} + q(\widehat{p}_i) \frac{d\widehat{r}_j}{d\widehat{p}_i} \right] \right\}, \text{ for } \widehat{p}_i > 0.$$

Under the rule chosen for the regulation of $\widehat{r}_j$, the second terms disappears and the profit maximizing price $\widehat{p}_i$ is uniquely given by:

$$\widehat{p}_i^*(\alpha_i) = \widehat{p}^* = c + (a - c_0). \tag{13}$$

The off-net price is equal to the off-net marginal cost as in LRT (1998b). Both networks choose the same price $\widehat{p}^*$ regardless of market shares.

In what follows, we will focus on the equilibria in which $p_i = p^*, r_i = r \leq r^*$. We can define $\Pi_i(\alpha_i)$ by

$$\Pi_i(\alpha_i) \equiv \pi_i(p^*, \widehat{p}^*; \alpha_i)$$

$$= \alpha_i \Big\{ (p^* - c)\alpha_i q(p^*) + (\widehat{p}^* - c)\alpha_j q(\widehat{p}^*) - f + F_j + \frac{1}{2\sigma}(1 - 2\alpha_i)$$

$$+ r\alpha_j q(p^*) + \widehat{r}(\widehat{p}^*)\alpha_i q(\widehat{p}^*) + (\alpha_i - \alpha_j)\left[v(p^*) - v(\widehat{p}^*) + \widetilde{u}(q(p^*)) - \widetilde{u}(q(\widehat{p}^*))\right] \Big\}.$$

5.3.2 Tariff level: Maximization with respect to market share

We now study the program of maximizing $\Pi_i(\alpha_i)$ with respect to α_i.

Lemma 2 $\Pi_i(\alpha_i) \equiv \pi_i(p_i^*, \widehat{p}_i^*; \alpha_i)$ *is well defined and continuous. Suppose* $c+(a-c_0) > 0$*. Then, if* σ *is small enough or if* $|a - c_0 + r|$ *is small enough and* $r \geq \widehat{r}(\widehat{p}_i^*)$*, it is concave.*

Proof See Appendix 6.

When $\Pi_i(\alpha_i)$ is concave, the unique solution is given by the first-order condition:

$$\Big\{(p^* - c)\alpha_i q(p^*) + (\widehat{p}^* - c)\alpha_j q(\widehat{p}^*) - f + F_j + \frac{1}{2\sigma}(1 - 2\alpha_i)$$

$$+r\alpha_j q(p^*) + \widehat{r}(\widehat{p}^*)\alpha_i q(\widehat{p}^*) + (\alpha_i - \alpha_j)\left[v(p^*) - v(\widehat{p}^*) + \widetilde{u}(q(p^*)) - \widetilde{u}(q(\widehat{p}^*))\right]\Big\}$$

$$+\alpha_i\Big\{(p^* - c)q(p^*) - (\widehat{p}^* - c)q(\widehat{p}^*) - \frac{1}{\sigma}$$

$$+2\left[v(p^*) - v(\widehat{p}^*) + \widetilde{u}(p^*) - \widetilde{u}(\widehat{p}^*)\right] - rq(p^*) + \widehat{r}(\widehat{p}^*)q(\widehat{p}^*)\Big\} = 0. \qquad (14)$$

5.3.3 Symmetric equilibria

We already know that both networks choose the same pair of prices $p_i, \widehat{p}_i$ regardless of market shares. Equilibria $(p^*, \widehat{p}^*, r_i, F_i, \alpha_i)$ are characterized by (2), (12), (13) and (14). Since r_i does not affect π_i as long as $r_i \leq r^*$, there are multiple equilibria.

Here, we are interested in symmetric equilibria: $p^*, \widehat{p}^*, r, F, \alpha = \frac{1}{2}$. When $\Pi_i(\alpha_i)$ is concave, there exist symmetric equilibria:

Proposition 9 ***(existence)****: Suppose that* $c + (a - c_0) > 0$ *holds and that* $\Pi_i(\alpha_i)$ *is concave. Then multiple symmetric equilibria* $(p^*, \widehat{p}^*, r, F)$ *exist.*

Proof Under the condition, (2), (12), (13) and (14) are satisfied for $\alpha_i = \frac{1}{2}$. ■

We note that r_i is indeterminate since it has no strategic impact on p_j or $\widehat{p}_j$.

In a symmetric equilibrium, we have from (14),

$$F = f + \frac{1}{2\sigma} - (p^* - c)q(p^*) - \widehat{r}(\widehat{p}^*)q(\widehat{p}^*) - \left[v(p^*) - v(\widehat{p}^*) + \widetilde{u}\left(q(p^*)\right) - \widetilde{u}(q(\widehat{p}^*))\right]. \quad (15)$$

In the case of no-discrimination, the fixed tariff is given by $F = f + \frac{1}{2\sigma} - (p - c + r)q(p)$.

After some computations, we have

$$\pi = \frac{1}{4\sigma} + \frac{1}{2}\left[-(p^* - c - r)\frac{q(p^*)}{2} + (\widehat{p}^* - c - \widehat{r}(\widehat{p}^*))\frac{q(\widehat{p}^*)}{2} - (v - \widehat{v} + \widetilde{u} - \widehat{\widetilde{u}})\right].$$

Thus, the profit can differ from $\frac{1}{4\sigma}$. In particular, the profit is increasing in r as long as customers remain connected.

What matters for social welfare is $p, \widehat{p}, \alpha$. Since $p = p^*, \alpha = \frac{1}{2}$, we need to have $\widehat{p} = p^*$ to maximize social welfare. This can be achieved provided the access price satisfies $a - c_0 = p^* - c = -r^*$. We noted that in the no-discrimination case as well the access price had to be equal to $c_0 - r^*$ in the efficient equilibrium.

Suppose now that the firms negotiate the access price a. Then, we can show that there exists an equilibrium where the firms maximize social welfare and make the monopoly profit under the constraint of serving all consumers.

Suppose that $c + a - c_0 = p^*$. This implies that $p = \widehat{p} = p^*$, $\widehat{r} = g(p^*)$ and $q = \widehat{q} = q^*$. Then, we have

$$\pi = \frac{1}{4\sigma} + \frac{1}{4}\left[r - \widehat{r}\right] q^*.$$

Hence, the profit is increasing in the difference between r and $\widehat{r}$.

Consider r given by

$$r = \widehat{r} + \frac{2}{q^*}\left[v_0 - f - \frac{3}{4\sigma} + u(q^*) + \widetilde{u}(q^*)\right] - 2c.$$

Then, the sum of the profits is given by,

$$\sum_{i=1,2} \pi_i = v_0 + u(q^*) + \widetilde{u}(q^*) - cq^* - f - \frac{1}{4\sigma},$$

which is the monopoly profit.

The idea is quite simple. The firms maximize the pie by choosing the social welfare maximizing price and use r to satisfy the individual rationality constraint of the consumer located at the middle point.[11]

We have the following proposition:

Proposition 10 ***(characterization)****: (a). The symmetric equilibria are characterized by:*

(i) $p = p^*, \widehat{p} = c + (a - c_0)$.

(ii) Each firm's profit is increasing in r.

(b) If the access price is given by $a - c_0 + r^* = 0$, *the symmetric equilibrium maximizes the social welfare.*

(c) There exists an equilibrium in which the firms maximize social welfare and obtain the monopoly profit.

[11] So, we assume implicitely that the firms serve the whole market.

Proof. See Appendix 7. ■

The above proposition shows that the access charge plays a crucial role in determining social welfare. Regardless of whether there is network-based discrimination or not, and regardless of whether there exists regulation of reception charges or not, the optimal outcome requires $a - c_0 + r^* = 0$.

We give below the intuition for the result that the profit is increasing in the difference between r and $\widehat{r}$ in the efficient equilibrium. Consider network i's deviation in terms of α_i (equivalently, in terms of F) from a symmetric equilibrium with $p = \widehat{p} = p^*$ and $\beta p^* \geq r > \widehat{r}$. Precisely, suppose that α_i increases by $\Delta\alpha_i > 0$ after the deviation. Although this deviation does not change the total volume of calls placed or received by a consumer, it affects the composition of the volume between on-net and off-net calls. In particular, after the deviation, a network i consumer receives more on-net calls than off-net calls while a network j consumer receives more off-net calls than on net-calls. This implies that, when $r > \widehat{r}$ holds, after the deviation, a network i consumer will pay more reception charge while a network j consumer will pay less. Since stealing a fraction of consumers from the other network makes the consumers of its own network unhappier and the remaining consumers of the other network happier in terms of reception charge, competition becomes less intense when $r - \widehat{r}$ is larger, which results in a higher fixed fee F and a higher equilibrium profit.

6 Conclusion

We provided a comprehensive overview of the main insights in the introduction, and so there is no need to reproduce it fully here. Suffices it to reiterate the key lines of the analysis:

- From a normative viewpoint, when receivers value receiving calls, calling charges must lie below the communications' marginal cost. This "calling subsidy" in principle could be obtained by setting the termination charge below the marginal cost of termination.
- By lowering each network's "strategic marginal cost", reception charges also contribute to an internalization of the externality on receivers. The termination charge and the reception charge can be regulated in such a way that a symmetric equilibrium exists and is efficient.
- When both emission and reception demands are elastic and reception charges are market determined, it is optimal for each operator to equate the prices for emission and reception with their off-net costs. Consequently, the equilibrium reception charges decrease with the termination charge, which reinforces the encouragement provided by termination discounts to set low caller charges. For an appropriately chosen termination charge, the symmetric equilibrium is again efficient.
- Last, network-based price discrimination creates strong incentives for connectivity breakdowns, even among equal networks.

Because the issues studied here are central to the development of network industries exhibiting externalities between the various sides of the market, we hope that this paper will stimulate further research extending the analysis in several important directions, including competition among an arbitrary number of networks, asymmetric networks (installed bases, cost structures,...), and alternative descriptions of call externalities.

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Appendix 1

That $\Pi_i(\alpha_i)$ is continuous is obvious. Let us show that it is concave. The first-order derivative with regard to α_i is given by

$$\frac{d\Pi_i}{d\alpha_i} = \frac{\partial \pi_i}{\partial \alpha_i} + \frac{\partial \pi_i}{\partial p_i^{**}} \frac{dp_i^{**}}{d\alpha_i} = \frac{\partial \pi_i}{\partial \alpha_i}$$

$$= (p_i - c)\, q_i - (a - c_0)(q_i - q_j)(1 - 2\alpha_i) + v_i - v_j - f + \frac{1}{2\sigma} - \frac{2\alpha_i}{\sigma}$$

$$+F_j + r(2\alpha_i q_i + (1 - 2\alpha_i) q_j).$$

The second-order derivative with respect to α_i is given by:

$$f(\alpha_i, p_i^{**}(\alpha_i)) \equiv -\frac{2}{\sigma} + (a - c_0 + r) \left\{ 2 \left[q_i(p_i^{**}(\alpha_i)) - q_j \right] - \alpha_i \left[a - c_0 + r \right] q_i'(p_i^{**}(\alpha_i)) \right\}.$$

The existence is trivial if $a - c_0 + r = 0$.

We study below the case $a - c_0 + r \neq 0$, given $a < \infty$, $r > -\infty$.

If $a - c_0 + r > 0$, let us write

$$p_i^{**}(\alpha_i) = c - r + \alpha_j(a - c_0 + r) \geq c - r.$$

Therefore, $p_i^{**}(\alpha_i)$ is bounded below uniformly in α_i by a positive number.

If $a - c_0 + r < 0$, let us write

$$p_i^{**}(\alpha_i) = c + a - c_0 - \alpha_i(a - c_0 + r) \geq c + a - c_0.$$

Therefore, $p_i^{**}(\alpha_i)$ is bounded below uniformly in α_i by a positive number.

Since $p_i^{**}(\alpha_i)$ is also bounded above uniformly in α_i, the continuous functions $q_i(p_i)$ and $q_i'(p_i)$ are also bounded uniformly.

Therefore, $f(\alpha_i, p_i^{**}(\alpha_i))$ is negative uniformly in α_i if either σ is small enough or $|a - c_0 + r|$ is small enough.

Appendix 2

(a) The candidate symmetric equilibria satisfy (3), (5) and (6) with $\alpha_i = \frac{1}{2}$. Substituting the first-order condition in α_i and p_i in the objective function of the firm, the profit is independent of r.

(b) Suppose that network 1, say, corners the market. We have $\pi_1 = F_1 - f + (p_1 + r - c)q(p_1) \geq 0$ and $\pi_2 = 0$. But network 2 can charge $p_2 = p_1$, and $F_2 = F_1 + \varepsilon$ and attract a share $\frac{1}{2} - \varepsilon\sigma$ of the market which is strictly positive for ε small enough. Its profit $\widetilde{\pi}_2 = (\frac{1}{2} - \varepsilon\sigma)(\pi_1 + \varepsilon)$ is then strictly positive for ε small enough.

(c) We study the conditions for non-existence of equilibrium. We have

$$\frac{d^2\Pi_1}{d\alpha_1^2} + \frac{d^2\Pi_2}{d\alpha_2^2} = -\frac{4}{\sigma} - (a - c_0 + r)^2 \left[\alpha_1 q_1' + \alpha_2 q_2'\right]$$

$$\geq -\frac{4}{\sigma} + (a - c_0 + r)^2 \min\left[-q_1', -q_2'\right].$$

Since $|a - c_0 + r| > \varepsilon$ and prices are bounded above and below, $(a - c_0 + r)^2 \min\left[-q_1', -q_2'\right]$ is bounded below by a strictly positive number. Therefore, for σ large enough, at least one of the second-order derivatives, $\frac{d^2\Pi_1}{d\alpha_1^2}$ and $\frac{d^2\Pi_2}{d\alpha_2^2}$, is strictly positive. Hence, no equilibrium exists.

Appendix 3

We show that a symmetric equilibrium exists in the absence of noise. By continuity, the candidate symmetric equilibrium is an equilibrium for a small enough noise.

Let (p^{**}, r^{**}) denote a symmetric equilibrium under joint determination in the absence of noise. We first show that $\max\left(p^{**}, \frac{r^{**}}{\beta}\right) \geq p^*$.

Lemma 3 $\max\left(p^{**}, \frac{r^{**}}{\beta}\right) \geq p^*$.

Proof. Consider the case in which the volume is determined by callers: $\beta p^{**} \geq r^{**}$. We study one-dimensional deviations. Since the first-order derivative of π_i with respect to p_i for $p_i \geq p^{**}$ (keeping market share α_i equal to $\frac{1}{2}$) must not be strictly positive at $p_i = p^{**}$, we have

$$p^{**} - c - \frac{a - c_0}{2} + \frac{r^{**}}{2} \geq 0. \tag{16}$$

Since the first-order derivative of π_i with respect to r_i for $r_i \geq \beta p^{**}$ (keeping market share α_i equal to $\frac{1}{2}$) must not be strictly positive at $r_i = \beta p^{**}$[12], we have

$$\beta p^{**} + \frac{1}{2}(p^{**} - c) + \frac{a - c_0}{2} \geq 0. \tag{17}$$

[12]We note that the first-order derivative of π_i with respect to r_i is zero for $r_i < \beta p^{**}$.

After summing (16) and (17), we have

$$0 \le \beta p^{**} + \frac{r^{**}}{2} + \frac{3}{2}(p^{**} - c) \le \frac{3}{2}((1+\beta)p^{**} - c)$$

Hence, we have

$$p^{**} \ge \frac{c}{1+\beta} = p^*.$$

When the volume is determined by receivers ($\beta p^{**} < r^{**}$), after applying a similar logic, we obtain that $\frac{r^{**}}{\beta} \ge p^*$ holds. ■

We now show the existence of the following equilibrium focusing on the caller-determined volume case: $p^{**} = c + \frac{1}{2}(a - c_0 - r^{**})$, $r^{**} = c_0 - a$. In the equilibrium candidate, the call volume is determined by the caller as long as the following inequality holds

$$p^{**} \ge \frac{r^{**}}{\beta} \quad \Leftrightarrow \quad a - c_0 \ge -\frac{\beta c}{1+\beta}.$$

Lemma 4 *If $a - c_0 \ge -\frac{\beta c}{1+\beta}$ holds, there exists a symmetric equilibrium (p^{**}, r^{**}) satisfying $p^{**} = c + \frac{1}{2}(a - c_0 - r^{**}) \ge \frac{r^{**}}{\beta}$ and $r^{**} = c_0 - a$.*

Proof. Since the equilibrium candidate satisfies $p^{**} + r^{**} = c$, $p^{**} \ge \frac{r^{**}}{\beta}$ implies $p^{**} \ge p^*$, $r^{**} \le r^*$. We first examine one-dimensional deviations and then joint deviations.

One-dimensional deviations

Consider network i's deviation in terms of p_i for $\beta p_i \ge r^{**}$ (keeping α_i and $r_i = r^{**}$ constant). The first-order derivative is given by

$$\alpha_i \left[p_i - c - (1-\alpha_i)(a - c_0) + \alpha_i r^{**}\right] \frac{dq}{dp_i}.$$

Thus, it is optimal to have

$$p_i^{**}(\alpha_i) = p^{**} = c - r^{**} \text{ for all } \alpha_i \in [0,1].$$

Consider now network i's deviation in terms of r_i for $r_i \ge \beta p^{**}$ (keeping α_i and $p_i = p^{**}$ constant). The first-order derivative is given by

$$\alpha_i \left[r_i + (1-\alpha_i)(a - c_0) + \alpha_i(p^{**} - c)\right] \frac{dq}{dp_i}, \tag{18}$$

which is negative since we have

$$r_i + (1-\alpha_i)(a - c_0) + \alpha_i(p^{**} - c) = r_i + (1-\alpha_i)(a - c_0) - \alpha_i r^{**}$$

$$= r_i - r^{**} \ge 0.$$

Hence, the above deviation is not profitable for all α_i.

Joint deviations

We now study joint deviations. We will perform our analysis in three steps. First, given p_i, we study the best choice of r_i keeping market share α_i constant. Second, we study the joint deviation in terms of (p_i, r_i) keeping market share constant. Last, we study the deviation in terms of market share.

Step 1. Choice of r_i given (p_i, α_i)

Case 1: $p_i \leq \frac{r^{**}}{\beta}$

If $r_i \leq \beta p_i$, r_i does not affect π_i.

If $\beta p_i \leq r_i \leq \beta p^{**}$, r_i affects only the volume of on-net calls. Then, $r_i = r^*$ is optimal.

If $\beta p^{**} \leq r_i$, r_i affects both the volume of on-net calls and that of off-net calls from network j. We know, from the first-order condition (18), that $r_i = \beta p^{**}$ is optimal.

Therefore, when $p_i \leq \frac{r^{**}}{\beta}$, $r_i = r^*$ is optimal.

Case 2: $\frac{r^{**}}{\beta} < p_i \leq p^{**}$

If $r_i \leq \beta p_i$, r_i does not affect π_i.

If $\beta p_i \leq r_i \leq \beta p^{**}$, r_i affects only the volume of on-net calls. Then, $r_i = \max[\beta p_i, r^*]$ is optimal.

If $\beta p^{**} \leq r_i$, r_i affects both the volume of on-net calls and that of off-net calls from network j. We know, from the first-order condition (18), that $r_i = \beta p^{**}$ is optimal.

Therefore, when $\frac{r^{**}}{\beta} \leq p_i \leq p^{**}$, $r_i = r^*$ is optimal for $\frac{r^{**}}{\beta} < p_i < p^*$ and $r_i \leq \beta p_i$ is optimal for $p^* \leq p_i \leq p^{**}$.

Case 3: $p_i > p^{**}$

If $r_i \leq \beta p^{**}$, r_i does not affect π_i.

If $\beta p_i \leq r_i$, r_i affects both the volume of on-net calls and that of off-net calls from network j. We know, from the first-order condition (18), that $r_i = \beta p_i$ is optimal.

In the case in which $\beta p^{**} \leq r_i \leq \beta p_i$ holds, the analysis is a little bit long. In what follows, we briefly sketch the proof. In this case, r_i affects only the volume of off-net calls from network j. Then, the first-order derivative of π_i with respect to r_i is given by:

$$f(r_i; \alpha_i) \equiv \alpha_i \left[\alpha_j (a - c_0 + \widetilde{u}'(q(\frac{r_i}{\beta}))) - \alpha_i (u'(q(\frac{r_i}{\beta})) - p^{**}) \right] q'(\frac{r_i}{\beta}) \frac{1}{\beta}.$$

Let $A(r_i; \alpha_i) \equiv \alpha_j (a - c_0) + \alpha_i p^{**} + [(1 - \alpha_i)\beta - \alpha_i] \frac{r_i}{\beta}$.

We first note that when $r_i = \beta p^{**}$, $f(\beta p_i^{**}; \alpha_i) \leq 0$ since $A(\beta p^{**}; \alpha_i) = \alpha_j (a - c_0 + \beta p^{**}) \geq 0$. Let $\alpha^0 \equiv \frac{\beta}{1+\beta}$. If $\alpha_i \leq \alpha^0$, $A(\cdot)$ is increasing in r_i. Hence, $r_i = \beta p^{**}$ is optimal. If $\alpha_i > \alpha^0$, $A(\cdot)$ is strictly decreasing in r_i. Hence, either $r_i = \beta p^{**}$ or $r_i = \beta p_i$ is optimal.

Consider now the joint deviation with $r_i = \beta p_i$ for $p_i \geq p^{**}$. Then, the first-order derivative of π_i with respect to p_i (keeping α_i constant) is given by:

$$\alpha_i \left[\alpha_j(p_i + \beta p_i - c) + \alpha_i(p^{**} + r^{**} - c)\right] \frac{dq}{dp},$$

which is negative since we have $p^{**} + r^{**} - c = 0$. Hence, the profit with $(p^{**}, \beta p^{**})$ is larger than the profit with $(p_i, r_i = \beta p_i)$ for $p_i > p^{**}$. Therefore, without loss of generality, we can say that $r_i = \beta p^{**}$ is optimal for $\beta p^{**} \leq r_i \leq \beta p_i$.

Hence, we can conclude that when $p_i > p^{**}$, $r_i \leq \beta p^{**}$ is optimal.

Step 2. Choice of (p_i, r_i) given α_i

Case 1: $p_i \leq p^*$

Without loss of generality, we can assume $r_i = r^*$.

If $p_i \leq \frac{r^{**}}{\beta}$, p_i does not affect π_i.

If $\frac{r^{**}}{\beta} \leq p_i \leq p^*$, p_i affects only the volume of off-net calls toward network j. The first-order derivative with respect to p_i is given by:

$$\alpha_i \left[(1-\alpha_i)(p_i - c - a - c_0) + \alpha_i(r^{**} - \beta p_i)\right] \frac{dq}{dp_i},$$

which is positive since we have

$$p_i - c - a - c_0 \leq p^* - c + r^{**} \leq p^* - c + r^* = 0.$$

Hence, $p_i = p^*$ is optimal.

Case 2: $p^* \leq p_i$

From the previous study of the choice of r_i given p_i, we have:

$$r_i = \left\langle \begin{array}{ll} \leq \beta p_i & \text{for } p^* \leq p_i \leq p^{**}, \\ \leq \beta p^{**} & \text{for } p_i > p^{**}. \end{array} \right.$$

Hence, without loss of generality, we can still assume $r_i = r^*$.

Then, p_i affects the volume of on-net and off-net calls. From the first-order condition, $p_i = p_i^{**}(\alpha_i) = p^{**}$ is optimal.

Finally, when we consider both case 1 and case 2, given r^*, choosing p^{**} is better than choosing p^* for every α_i. Furthermore, given that $p_i^{**}(\alpha_i) = p^{**}$, choosing $r_i = r^{**}$ instead of r^* does not affect π_i since $p^{**} \geq p^*$. Therefore, for all α_i, network i's optimal choice of prices is given by: $p_i = p^{**}, r_i = r^{**}$.

Step 3. Choice of α_i

Since the optimal choice of prices is given by $p_i = p^{**}, r_i = r^{**}$, we are back to the previous case in which the volume is determined by callers. From lemma 1, we know that the symmetric equilibrium with $\alpha_i = \frac{1}{2}$ exists if $a - c_0 + r^{**}$ is small enough. ■

Appendix 4

In the program of maximizing network i's profit with respect to its prices given market share α_i, we can easily see by checking the sign of the first-order derivatives that $\{p_i^{**}(\alpha_i), \widehat{p}_i^{**}(\alpha_i)\}$ is the unique global maximizer if $c + a - c_0 > 0$ holds.

Define $\Pi_i(\alpha_i)$ by

$$\Pi_i(\alpha_i) \equiv \pi_i(p_i^{**}(\alpha_i), \widehat{p}_i^{**}(\alpha_i), \alpha_i).$$

We now study the concavity of the program of maximizing network i's profit Π_i with respect to market share α_i when network j charges $p_j = p^*, \widehat{p}_j = \widehat{p}_j^{**}(\frac{1}{2}) = \frac{c+a-c_0}{1-\beta}$. We have

$$\frac{d^2\Pi_i}{d\alpha_i^2} = \frac{\partial^2\Pi_i}{\partial\alpha_i^2} + \frac{\partial^2\Pi_i}{\partial\alpha_i\partial\widehat{p}_i}\frac{d\widehat{p}_i}{d\alpha_i},$$

$$\frac{\partial^2\Pi_i}{\partial\alpha_i^2} = 2(p^* - c)q^* - 2(\widehat{p}_i - c)\widehat{q}(\widehat{p}_i) - 2(a - c_0)(\widehat{q}(\widehat{p}_j) - \widehat{q}(\widehat{p}_i))$$

$$-\frac{2}{\delta} + 4\left[v(p^*) + \widetilde{u}(q^*)\right] - 2\left[v(\widehat{p}_i) + \widetilde{u}(q(\widehat{p}_i)) + v(\widehat{p}_j) + \widetilde{u}(q(\widehat{p}_j))\right];$$

$$\frac{\partial^2\Pi_i}{\partial\alpha_i\partial\widehat{p}_i}\frac{d\widehat{p}_i}{d\alpha_i} = \left[(2\alpha_i - 1)c + (1 - (1+\beta)2\alpha_i)\widehat{p}_i\right]\frac{d\widehat{q}}{d\widehat{p}_i}\frac{d\widehat{p}_i}{d\alpha_i} \text{ for } \alpha_i < \frac{1}{1+\beta},$$

$$\frac{\partial^2\Pi_i}{\partial\alpha_i\partial\widehat{p}_i}\frac{d\widehat{p}_i}{d\alpha_i} = 0, \text{ for } \alpha_i \geq \frac{1}{1+\beta}.$$

If $a \simeq c_0$ holds, we have

$$\frac{\partial^2\Pi_i}{\partial\alpha_i^2} \leq 2(p^* - c)q^* - \frac{2}{\delta} + 4\left[v(p^*) + \widetilde{u}(q^*)\right];$$

$$\frac{\partial^2\Pi_i}{\partial\alpha_i\partial\widehat{p}_i}\frac{d\widehat{p}_i}{d\alpha_i} \simeq \frac{\beta^2\eta}{c}\frac{\alpha_i}{(1-\alpha_i)^2}\widehat{p}_i^{-(\eta-1)} \leq \eta(1+\beta)c^{-\eta}, \text{ for } \alpha_i < \frac{1}{1+\beta}.$$

Hence, the program is concave if the following inequality holds,

$$-2\beta p^* q^* + 4\left[v(p^*) + \widetilde{u}(q^*)\right] + \eta(1+\beta)c^{-\eta} < \frac{2}{\delta}.$$

This inequality holds for σ low enough. Therefore, the symmetric equilibrium with $p_i = p^*, \widehat{p}_i = \frac{c+a-c_0}{1-\beta}$ for $i = 1, 2$ exists if σ is low enough and if $a \simeq c_0$ holds.

Appendix 5

We first analyze the connectivity breakdown with $\widehat{p} = \widehat{r} = \infty$. In the program of maximizing profit given market share α_i, for any value of α_i, $\widehat{p}_i = \infty$ is a best response of network i to $\widehat{r}_j = \infty$ and vice versa. We also know that prices are efficiently chosen in the market for on-net calls regardless of the value of α_i. Therefore, for any value of α_i, $\{p_i = p^*, r_i \leq r^*, \widehat{p}_i = \widehat{r}_i = \infty\}$ is a best response of network i when network j charges $\widehat{p}_j = \widehat{r}_j = \infty$.

We now examine the program of maximizing profit with respect to market share when both networks charge $p = p^*$, $r \leq r^*$, $\widehat{p} = \widehat{r} = \infty$. This program is concave if the following inequality holds:

$$2\left[v(p^*) + \widetilde{u}(q^*)\right] - (r^* + r)q^* \leq \frac{1}{\sigma}.$$

This inequality holds for σ small enough. Therefore, for σ small enough, a symmetric equilibrium with connectivity breakdown exists.

(a) (i) Suppose that a symmetric equilibrium $(p, r, \widehat{p}, \widehat{r}, \alpha = \frac{1}{2})$ exists. We first show that for $1 > \beta > 0$, if $\beta\widehat{p} < \widehat{r}$ holds, then $\widehat{r} = \infty$.

In the case in which $\beta\widehat{p} < \widehat{r}$ holds, $\widehat{r}$ determines the volume of off-net calls. The first-order derivative of $\pi_j^{\widehat{r}}$ with respect to $\widehat{r}_j$ keeping $\alpha_j = \frac{1}{2}$ is given by:

$$f(\widehat{r}_j) \equiv \frac{1}{4}\left\{[\widehat{r}_j + a - c_0] + \left[(\widehat{p} - u'(q(\frac{\widehat{r}_j}{\beta})))\right]\right\}\frac{dq}{d\widehat{r}_j}\frac{1}{\beta}.$$

$f(\widehat{r}_j)$ is strictly increasing in $\widehat{r}_j$. If $f(\beta\widehat{p}) \geq 0$ holds, we have $f(\widehat{r}_j) > f(\beta\widehat{p}) \geq 0$ for all $\widehat{r}_j > \beta\widehat{p}$. Hence, $\widehat{r}_j = \infty$ is optimal. If $f(\beta\widehat{p}) < 0$ holds, define $\widehat{r}_j^0$ by $f(\widehat{r}_j^0) \equiv 0$. Then, $\pi_j^{\widehat{r}}(\beta\widehat{p}) \geq \pi_j^{\widehat{r}}(\widehat{r}_j)$ for all $\widehat{r}_j \in \left[\beta\widehat{p}, \widehat{r}_j^0\right]$ and $\pi_j^{\widehat{r}}(\infty) \geq \pi_j^{\widehat{r}}(\widehat{r}_j)$ for all $\widehat{r}_j \in \left[\widehat{r}_j^0, \infty\right]$. Hence, it is optimal to have either $\widehat{r}_j = \infty$ or $\widehat{r}_j = \beta\widehat{p}$. However, the optimal $\widehat{r}_j$ has to be strictly larger than $\beta\widehat{p}$. Thus, $\widehat{r}_j = \infty$ is optimal. Therefore, $\widehat{r}$ should be equal to ∞ regardless of the value of $f(\beta\widehat{p})$.

Consider now the case in which $\infty > \beta\widehat{p} \geq \widehat{r}$ holds. Suppose that β is close to zero and $a < c_0$. In a symmetric equilibrium in which the quantity q_{ij} is determined by $\widehat{p}_i = \widehat{p} < \infty$, we have

$$\pi_j^{\widehat{r}}(\widehat{r}_j) \equiv \frac{1}{4}\left\{[\widetilde{u}(q(\widehat{p})) + (a - c_0)q(\widehat{p})] + [\widehat{p}q(\widehat{p}) - u(q(\widehat{p}))]\right\} \text{ for } \widehat{r}_j \leq \beta\widehat{p}.$$

Since $\widehat{p} = u'(q(\widehat{p}))$, we have for β small enough

$$\pi_j^{\widehat{r}}(\widehat{r}_j) = \frac{1}{4}\left\{[\beta u(q(\widehat{p})) + (a - c_0)q(\widehat{p})] + [u'(q(\widehat{p}))q(\widehat{p}) - u(q(\widehat{p}))]\right\} \text{ for } \widehat{r}_j \leq \beta\widehat{p}.$$

$$\approx \frac{1}{4}\left\{(a - c_0)q(\widehat{p}) + [u'(q(\widehat{p}))q(\widehat{p}) - u(q(\widehat{p}))]\right\} < 0 \text{ for } \widehat{r}_j \leq \beta\widehat{p}.$$

Hence, network j has the incentive to choose $\widehat{r}_j = \infty$ to have $\pi_j^{\widehat{r}}(\widehat{r}_j) = 0$, which is a contradiction. Therefore, we have to have $\widehat{r} = \infty$ when β is close to zero and $a < c_0$ holds.

(ii) Suppose $\beta > 1$. What happens in this case is symmetric to the previous case in which $0 < \beta < 1$ holds. Suppose first $\beta\widehat{p} > \widehat{r}$. The first-order derivative of $\pi_i^{\widehat{p}}$ with respect to $\widehat{p}_i$ keeping $\alpha_i = \frac{1}{2}$ is given by:

$$g(\widehat{p}_i) \equiv \frac{1}{4}\left\{[\widehat{p}_i - c - (a - c_0)] + [(\widehat{r} - \widetilde{u}'(q(\widehat{p}_i)))]\right\}\frac{dq}{d\widehat{p}_i}.$$

$g(\widehat{p}_i)$ is strictly increasing in $\widehat{p}_i$. $\pi_i^{\widehat{p}}$ is maximized when $\widehat{p}_i = \infty$ or $\widehat{p}_i = \widehat{r}$. Since $\widehat{p}_i = \widehat{r}$ is contradictory, $\widehat{p}_i = \infty$ must hold.

Consider now the case in which $\beta\widehat{p} \leq \widehat{r} < \infty$ holds. Suppose that β is large enough. We have:

$$\pi_i^{\widehat{p}}(\widehat{p}_i) \equiv \frac{1}{4}\left\{\left[u(q(\frac{\widehat{r}}{\beta})) - (c + a - c_0)q(\frac{\widehat{r}}{\beta})\right] + \left[\widehat{r}q(\frac{\widehat{r}}{\beta}) - \widetilde{u}(q(\frac{\widehat{r}}{\beta}))\right]\right\} \text{ for } \widehat{r} \geq \beta\widehat{p}_i$$

$$= \frac{1}{4}\beta\left\{\frac{1}{\beta}\left[u(q(\frac{\widehat{r}}{\beta})) - (c + a - c_0)q(\frac{\widehat{r}}{\beta})\right] + \left[u'(q(\frac{\widehat{r}}{\beta}))q(\frac{\widehat{r}}{\beta}) - u(q(\frac{\widehat{r}}{\beta}))\right]\right\}$$

$$\approx \frac{1}{4}\beta\left\{-\frac{c + a - c_0}{\beta}q(\frac{\widehat{r}}{\beta}) + \left[u'(q(\frac{\widehat{r}}{\beta}))q(\frac{\widehat{r}}{\beta}) - u(q(\frac{\widehat{r}}{\beta}))\right]\right\} < 0.$$

Hence, network i has the incentive to choose $\widehat{p}_i = \infty$ to have $\pi_i^{\widehat{p}}(\widehat{p}_i) = 0$. Therefore, we have to have $\widehat{p} = \infty$ when β is large enough.

(b) Suppose that a symmetric equilibrium $(p, r, \widehat{p}, \widehat{r}, \alpha = \frac{1}{2})$ exists. We have

$$\pi_i^{\widehat{p}}(\widehat{p}_i = \widehat{p} : \frac{1}{2}, \widehat{r}) + \pi_j^{\widehat{r}}(\widehat{r}_j = \widehat{r} : \frac{1}{2}, \widehat{p}) = \frac{1}{4}(\widehat{p} + \widehat{r} - c)q_{ij}.$$

From the network i's individual rationality constraint with respect to $\widehat{p}_i$ and the network j's individual rationality constraint with respect to $\widehat{r}_j$, each term of the left hand side of the above equality should be non-negative. Therefore, at any symmetric equilibrium, $\widehat{p} + \widehat{r} \geq c$ should hold. This implies that the only possible efficient symmetric equilibrium candidate is $\widehat{p} = p^*$ and $\widehat{r} = r^* = \beta p^*$. We show below that this candidate violates one of the two networks' individual rationality constraint.

In order to make network i's deviation to $\widehat{p}_i > p^*$ not profitable, the first-order derivative of $\pi_i^{\widehat{p}}(\cdot)$ with respect to $\widehat{p}_i$ keeping $\alpha_i = \frac{1}{2}$ must not be strictly positive at the point $\widehat{p}_i = p^*$ and $\widehat{r}_j = r^*$, which implies

$$[(p^* - (c + a - c_0)) + r^* - \widetilde{u}'(q(p^*))] \geq 0. \quad (19)$$

In order to make network j's deviation to $\widehat{r}_j > r^*$ not profitable, the first-order derivative of $\pi_j^{\widehat{r}}(\cdot)$ with respect to $\widehat{r}_j$ keeping $\alpha_j = \frac{1}{2}$ must not be strictly positive at the point $\widehat{p}_i = p^*$ and $\widehat{r}_j = r^*$, which implies

$$\left[(r^* + a - c_0) + p^* - u'(q(\frac{r^*}{\beta}))\right] \geq 0. \quad (20)$$

After summing (19) and (20), we have

$$p^* + r^* - c \geq 0.$$

However, we know that $p^* + r^* - c = 0$. Therefore, both (19) and (20) must hold with equality, implying $a - c_0 + r^* = 0$. Then, we have:

$$\pi_j^{\widehat{r}}(r^* : \frac{1}{2}, p^*) = \frac{1}{4}\left[(p^* + a - c_0)q^* - (1-\beta)u(q^*)\right] = \frac{1}{4}(1-\beta)(p^*q^* - u(q^*)),$$

$$\pi_i^{\widehat{p}}(p^* : \frac{1}{2}, r^*) = \frac{1}{4}\left[(r^* - c - a + c_0)q^* + (1-\beta)u(q^*)\right] = \frac{1}{4}(\beta - 1)(p^*q^* - u(q^*)).$$

Hence, for β smaller than one, $\pi_j^{\widehat{r}}$ is negative and network j has the incentive to choose $\widehat{r}_j = \infty$. For β larger than one, $\pi_i^{\widehat{p}}$ is negative and network i has the incentive to choose $\widehat{p}_i = \infty$.

Appendix 6

Since it is obvious that $\Pi_i(\alpha_i)$ is continuous, we show below that it is concave.

The first-order derivative with respect to α_i is given by:

$$\frac{d\Pi_i}{d\alpha_i} = \Big\{(p^* - c)\alpha_i q(p^*) + (\widehat{p}^* - c)\alpha_j q(\widehat{p}^*) - f + F_j + \frac{1}{2\sigma}(1 - 2\alpha_i)$$

$$+ r\alpha_j q(p^*) + \widehat{r}(\widehat{p}^*)\alpha_i q(\widehat{p}^*) + (\alpha_i - \alpha_j)\left[v(p^*) - v(\widehat{p}^*) + \widetilde{u}(q(p^*)) - \widetilde{u}(q(\widehat{p}^*))\right]\Big\}$$

$$+ \alpha_i \Big\{(p^* - c)q(p^*) - (\widehat{p}^* - c)q(\widehat{p}^*) - \frac{1}{\sigma}$$

$$+ 2\left[v(p^*) - v(\widehat{p}^*) + \widetilde{u}(p^*) - \widetilde{u}(\widehat{p}^*)\right] - rq(p^*) + \widehat{r}(\widehat{p}^*)q(\widehat{p}^*)\Big\}.$$

The second-order derivative with respect to α_i is given by

$$\frac{d^2\Pi_i}{d\alpha_i^2} = 2\Big\{(p^* - c - r)q(p^*) - (\widehat{p}^* - c - \widehat{r}(\widehat{p}^*))q(\widehat{p}^*) - \frac{1}{\sigma}.$$

$$+ 2\left[v(p^*) - v(\widehat{p}^*) + \widetilde{u}(p^*) - \widetilde{u}(\widehat{p}^*)\right]\Big\}.$$

Thus, if σ is small enough, Π_i is concave.

When $a - c_0 + r^* \simeq 0$, we have

$$\frac{d^2\Pi_i}{d\alpha_i^2} \simeq -2(r - \widehat{r}(\widehat{p}^*))q^* - \frac{2}{\sigma}.$$

Hence, for σ large, Π_i is concave if $a - c_0 + r^* \simeq 0$ and $r \geq \widehat{r}(\widehat{p}^*)$.

Appendix 7

We only need to prove (c). For this, we will show that there exists $\varepsilon(>0)$ such that in the equilibrium associated with this ε, social welfare is maximized and the firms obtain the monopoly profit.

First, from the defined rules for the regulation of reception charge, ε must satisfy the following three conditions.

1) $g(\underline{p}) = \beta \underline{p}$.

This condition is satisfied if the following equality holds;

$$\underline{p} = \left[\frac{\beta}{(\eta - 1)\varepsilon} \right]^{\frac{1}{\eta - 1}}.$$

2) $g(\widehat{p}_i) \leq \beta \widehat{p}_i$ for all $\widehat{p}_i \geq \underline{p}$.

Since $g''(\cdot) < 0$ for $\widehat{p} > 0$ and $g'(\underline{p}) \leq \beta$, we have $g(\widehat{p}_i) \leq \beta \widehat{p}_i$ for all $\widehat{p}_i \geq \underline{p}$.

3) $\underline{p} \leq \widehat{p}^*(= c + a - c_0)$

This condition is satisfied if the following inequality holds.

$$\frac{\beta}{\eta - 1} \widehat{p}^{*-(\eta-1)} \leq \varepsilon.$$

Second, to obtain the monopoly profit, each network should choose r satisfying the following equality:

$$r = \widehat{r} + \frac{2}{q^*} \left[v_0 - f - \frac{3}{4\sigma} + u(q^*) + \widetilde{u}(q^*) - cq^* \right].$$

$r \leq \beta p^*$ holds if the following inequality holds:

$$\frac{\beta}{\eta - 1} p^{*-(\eta-1)} + \frac{2}{q^*} \left[v_0 - f - \frac{3}{4\sigma} + u(q^*) + \widetilde{u}(q^*) - cq^* \right] p^{*-\eta} \leq \varepsilon.$$

Third, the existence of the equilibrium is obtained when $a - c_0 + r^* = 0$ and $r \geq \widehat{r}$. Efficiency requires $a - c_0 + r^* = 0$ and $r \geq \widehat{r}$ is a necessary condition for each firm's profit to be larger than the Hotelling profit under unit demand $\frac{1}{4\sigma}$. Since there exists ε that satisfies all the above conditions, we can conclude that if $a - c_0 + r^* = 0$ holds, there exists an equilibrium in which social welfare is maximized and the firms obtain the monopoly profit.