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More on the Time Inconsistency of Optimal Monetary Policy

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Abstract

We introduce costs of unexpected inflation in a general equilibrium cash-in- advance model, by simply changing the timing of the constraints faced by consumers. We show that in this case the optimal monetary policy is still time inconsistent, but the nature of the inconsistency is very different from the standard result found in the literature. In particular, we find that depending on parameter valules, the government may have incentives to choose an inflation rate lower than the one expected by the private sector. By making a brief review of the monetary literature, we argue that the model of this paper is more reasonable than the ones proposed before to study the time consistency of monetary policy.

1 INTRODUCTION

In this paper we reconsider the well known result of Calvo (1978) Lucas and Stokey (L&S)(1983) and Persson, Persson and Svensson (PP&S)(1987) regarding the time inconsistency of optimal monetary policy. The basic insight of their time inconsistency result is that in the absence of lump sum taxation, a welfare optimizing government may have incentives to surprise the private sector by increasing the money supply at a higher rate than expected. In their models, unexpected inflation acts as a lump-sum tax on real money balances and allows the government to collect revenue without distorting the economy. In fact, it is optimal for the government to always increase the current price level, in order to make private sector's money holdings value approach zero.

We argue that this result depends crucially on an implicit assumption regarding the way in which the cash-in-advance restriction binds the representative agent at the time the government increases the money supply. We show that if we use an alternative formulation the results dramatically change, even though the time inconsistency problem does not vanish in general. The main difference between their result and the one we obtain is that with the alternative formulation surprise inflation is not lump-sum, because at the time the money supply is increased, the cash-in-advance constraint binds[1]. Thus, surprise inflation is a tax on cash goods only. To put it differently, unexpected inflation is costly because agents cannot adjust their nominal money balances immediately at zero cost. The key assumption is related to the timing of events within each period. The timing required to generate the standard result is the one proposed by Lucas (1980) and used by Lucas and Stokey (1983) among others. The formulation we use is equivalent to the one proposed by Svensson (1985) and used by Hodrick, Kocherlacota and Lucas (1989) among others[2]. Thus, the timing we use in this paper is not new. However, to the best of our knowledge, this version of the model has

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[1]Calvo and PPS use money in the utility function models. The way in which this ditinction is reflected in muf models will be clarified latter on.

[2]A very clear comparison between the two models, very similar to the one we follow, is presented in Giovannini and Labadie (1991).

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not been used to study the time inconsistency problem of monetary policy.

We argue that the alternative formulation is much more appropriate than the one used before when studying the impact effects of unexpected changes in monetary policy. We do it in two steps. First, we discuss the motivation behind the cash-in-advance constraint as a device to generate monetary equilibria. Second, we study the time inconsistency issue in other existing monetary models, which are more explicit regarding the economic environment and show that to generate the traditional result we require a perfect matching between all agents decisions, included the government. We use the results of these models to argue that the perfect matching is a knife edge, not robust to sensible changes in the environment and very hard to justify both on theoretical and empirical grounds.

The cash-in-advance model with the Svensson timing generates costs of unexpected inflation in general equilibrium monetary models in a very simple way, which, we argue, is the most reasonable way to build up a simple model to study welfare effects of surprise monetary shocks. A very extensive literature, pioneered by Barro and Gordon(1983), has developed to analyze reputational equilibria in monetary models. All these models[3] relied on ad-hoc payoff functions for the government arguing that monetary theory has not come up with good reasons why unexpected inflation is costly. This paper shows that a simple change[4] in the timing of the constraints is enough to generate costs of unexpected inflation.

There exists some independent work related to this paper. PP&S (1989) suggested a way to introduce costs of unexpected inflation in Sidrausky-type models which is similar in spirit to the one we propose. However, they did not - it was not their goal- explore the theoretical implications of this assumption and failed to see the most striking result of this paper, namely, that the government may want to surprise the private sector by choosing an inflation rate lower than expected. Their idea has been exploited by Calvo and Guidotti (1993). Their aim was not to investigate the theoretical implications of this way of modeling but to see whether their results were robust to costs of unexpected inflation. However, they did not solve the model and only run some simulations for a two period version of it.

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[3]For a very interesting survey see Persson and Tabellini(1991).

[4]The change is relative to the model used before to study this problem. As we mentioned above, the model we use was first developed by Svensson to study a different problem.

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In section I, we present the model and show the role of the assumption on the time inconsistency problem. We then show the solution of the problem and explain its implications. In section II we argue about the reasonability of the assumption and make reference to some existing monetary models. We close the paper with a discussion of the implications of this results and with some conclusions.

2   THE CASH-IN-ADVANCE MODEL

There is a representative consumer with preferences over goods and leisure of the form

$$\sum_{t=1}^{\infty} \beta^{t-1} \left[U(x_t) - \alpha.n_t \right] \tag{1}$$

where U is strictly increasing and concave and its derivative approaches infinity as the argument approaches zero, x_t is consumption and n_t is labor effort to be dedicated to production activities[5]. Goods are produced using labor as the only input according to the linear technology

$$x_t + g = n_t \tag{2}$$

where g > 0 is government expenditure which, for simplicity, is assumed constant over time. The government collects revenue only through the inflation tax[6]. The government also issues bonds, B_t which pay the nominal return R_t. Thus, the budget constraint for the consumer is

$$p_t.x_t + M_{t+1} + B_{t+1} = p_t.n_t + M_t + B_t(1 + R_t) \tag{3}$$

for t = 1,2,3,..... . The initial money holdings M_1 are given and B_1 is assumed equal to zero. This last assumption is made to sharpen the comparison of the results of the model under the two alternative assumptions. The implications of non zero initial holdings of nominal debt will be discussed later on.

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[5] Assuming that effort enters linearly in the utility function simplifies the conditions for the Ramsey solution. No relevant result hinges on this assumption.

[6] This simplifies the analysis. Distorting consumption or income taxes can be incorporated as well as credit goods and none of the results change.

Money is introduced into the model through a cash-in-advance constraint. The motivation behind this type of constraint is that every time period consists on two sub-periods, the securities trading period and the goods trading periods. In the first one, there is a centralized securities market, where agents buy and sell bonds and money. In the second, the representative agent splits into two agents, the buyer that is bound by the currency it carries and buys the goods, and the seller that decides the labor effort to produce the goods and sells them. Thus, the securities market is centralized and the goods market is decentralized.

L&S assume that monetary injections are done at the first sub period , when the cash-in-advance constraint is not binding[7]. Thus, as agents are at the securities market, they can instantaneously adjust their portfolios at zero cost. This is the reason why monetary surprises are lump-sum. However, if we assume that monetary increases are done at the second sub period, the time inconsistency results change substantially. In this case, a surprise inflation is a tax on cash goods only, so it is not lump-sum. The way in which this assumption enters into the formal structure of the model is through the time index of the nominal money stock in the cash-in-advance constraint . We assume, as in Svensson(1985), that there is a cash-in-advance constraint of the form

$$p_t.x_t \leq M_t \qquad (4)$$

The assumption of L&S would be equivalent[8] to a constraint of the form

$$p_t.x_t \leq M_{t+1}$$

The analysis of the time inconsistency problem would be the one in LS or Calvo if we use this equation instead of 4. We show now that the nature of the result is very different if we use 4. In the next section we interpret the two assumptions and discuss their reasonability.

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[7]The corresponding assumption in money in the utility function models is that end of period rather than beginnig of period money balances enter the utility function.

[8]They do not write the equation that way. However, they assume that at the begining of the period the consumer can exchange contingent claims for currency, which is equivalent to write the restrictions as above. With this restriction, we can exactly reproduce consumers first order conditions of LS.

To obtain a competitive equilibrium we must solve the problem of the representative consumer, that is to maximize 1 subject to 3 and 4 for given prices, taxes and interest rates. If we let λ_t and δ_t be the respective lagrange multipliers, the first order conditions of that problem are

$$\beta^{t-1}.U'(x_t) = (\lambda_t + \delta_t).p_t \tag{5}$$

$$\beta^{t-1} \cdot \alpha = \lambda_t.p_t \tag{6}$$

$$\lambda_t = \lambda_{t+1} + \delta_{t+1} \tag{7}$$

$$\lambda_t = \lambda_{t+1}.(1 + R_{t+1}) \tag{8}$$

for t = 1,2,3,......

Note that δ_1 appears only in equation 5 for t=1. This reflects the fact that if the cash-in-advance constraint is binding, x_1 is determined by it, and equation 5 for t=1 is only used to know the value of δ_1, the shadow price of the constraint. This is what makes the difference with respect to the existing time inconsistency literature. If we had used the Lucas formulation instead of 4, equation 7 would read

$$\lambda_t = \lambda_{t+1} + \delta_t \tag{9}$$

and δ_1 would be in equation 9 for t=1 also.

Equations 5 to 8 can be rearranged to obtain

$$\frac{p_{t+1}}{p_t} = \beta \cdot (1 + R_{t+1}) \tag{10}$$

$$\beta \cdot U'(x_t) = \alpha.\frac{p_{t+1}}{p_t} \tag{11}$$

$$U'(x_1) = (\delta_1.p_1 + \alpha) \tag{12}$$

A competitive equilibrium is sequences of quantities$\{x_t, y_t, n_t\}_{t=1}^{\infty}$ and prices $\{R_t, p_t\}_{t=1}^{\infty}$ such that equations 2 to 4 and 10 to 12 hold.

Alternative combinations of inflation rates lead to different equilibrium allocations. The Ramsey problem is to choose the sequence of inflation rates

that maximize the utility level of the representative consumer subject to the constraint that the resulting allocation must be a competitive equilibrium. Formally, that is to maximize 1 subject to equations 2 to 4 and 10 to 12. We can follow L&S and further reduce the dimension of the problem by eliminating all prices using consumers first order conditions.

First, we write the sequence of budget constraints 3 as a single lifetime budget constraint. We obtain

$$p_1.[x_1 - n_1] - M_1 + \sum_{t=2}^{\infty} Q_t.p_t.[x_t - n_t + \frac{M_t}{p_t}.R_t] \leq 0 \tag{13}$$

where Q_t is the intertemporal discount factor between period 1 and t. Then, we use equations 10 to 12 and 4 to eliminate prices, taxes, interest rates and money demands to obtain

$$- \alpha.n_1 + \sum_{t=2}^{\infty} \beta^{t-1}[U'(x_t).x_t - \alpha.n_t] \leq 0 \tag{14}$$

Note that x_1 does not show up in this implementability constraint, because it exactly cancels out with initial money holdings, and because of the way the problem is set up, there is no way to substitute cash goods at the first period for any other good at any other period. There is an irreversible decision the consumer has taken before the present time (initial money balances) and he is bound by that decision, and not by the total wealth.

If as in L&S we had assumed that money injections are done at the securities market, then we would be assuming that they are not bound by past decisions and can readjust their portfolios immediately at zero cost. In that case, x_1 would show up in the budget constraint because the consumer could substitute consumption goods at the first period with goods any other period and is only constrained by total wealth. In this case, the budget constraint would be

$$\sum_{t=1}^{\infty} \beta^{t-1}[U'(x_t).x_t - \alpha.n_t] - \alpha.(M_0/p_1) \leq 0$$

where M_0 is the amount of money the consumer holds at the beginning of the securities market. If the Ramsey problem is not trivial, the multiplier associated to that constraint is positive, which means that the derivative of the lagrangean with respect to the price level is always positive, reflecting

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the incentives the government has to generate an initial hyperinflation. This initial price level is the "additional degree of freedom " Calvo mentions in his paper. With our specification, there is no additional degree of freedom because we do not allow consumers to readjust their portfolios immediately at zero cost.

The optimal problem is then to maximize 1 subject to 2 and 14. If we let ϵ_t and ω be the lagrange multipliers of constraints 2 and 14 respectively, and we assume that the solution exists and it is interior, the following conditions must hold

$$\alpha.(1+\omega) = \frac{\epsilon_t}{\beta^{t-1}} \tag{15}$$

$$U'(x_t) + \omega.U'(x_t).[1 - \sigma(x_t)] = \frac{\epsilon_t}{\beta^{t-1}} \tag{16}$$

for t $= 2,3,4,....,$ and

$$U'(x_1) = \epsilon_1 \tag{17}$$

where $\sigma(z) = $ - $z.U''(z)/U'(z)$ (i.e., the relative risk aversion coefficient)

Equations 15 to 17 show that the optimum problem is to set a constant value over time for the consumption good except for the first period, in which it will be greater or smaller depending on $\sigma(.)$ being larger or smaller than one. Note that the problem is stationary, so if the government reoptimizes at t=2, we shall obtain a condition for x_2 similar to 17, which results in a value for x_2 different, in general, from the one we obtain in equation 16 . Thus, there is time inconsistency. However, the nature of the time inconsistency is quite different from the standard result in the literature. As we shall see, it is possible that the government may want to surprise by choosing an inflation rate lower than expected. From 16 and 17, and noting that 15 implies $\epsilon_t = \epsilon$ for all t, we can write

$$U'(x_1) = U'(x_2) + \omega \cdot U'(x_2) \cdot (1 - \sigma(x_2)) \tag{18}$$

Note first that the multiplier ω is positive, as it is the shadow price of constraint 14 . This can be formally proven by multiplying equations 15 to 17 by n_t, x_t, and x_1 respectively and adding up for all periods[9]. Then, x_1 will

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[9] See LS for details.

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be larger (smaller) than x_2 when $\sigma(x_2)$ is larger (smaller) than one. Thus, if $\sigma(x_2)$ is larger than one, the government will have incentives to chose an inflation rate lower than expected, if $\sigma(x_2)$ is smaller than one, it will have incentives to choose an inflation rate higher than expected, but in general will not have incentives to let the initial price level go to infinity. Finally, if $\sigma(x_2)$ is equal to one (the log-utility case), the Ramsey problem is time consistent.

The reason why it is important the value of $\sigma(x_2)$ relative to one is at the heart of the Ramsey problem. Note that from 10 and 11 we can write the demand function of x_2 as

$$U'(x_2) = \alpha.(1 + R_2) \tag{19}$$

Thus, $\sigma(x_2)$ as defined above is the inverse of the price elasticity of x_2. However, the x_1 demand function is determined by the cash-in-advance constraint, and it is thus a unit price elasticity good. The Ramsey solution is to tax more heavily the goods which are less elastic. Thus, if $\sigma(x_2)$ is larger than one, it means that the demand function of x_2 is less elastic than the demand function of x_1. Thus, you must tax more heavily x_2 and that is why the initial inflation must be lower than the following ones.

The reason why time inconsistency arises in this context is because over time consumers make irreversible decisions, such that the elasticities of the demand functions change. As the optimal tax depends on the elasticity, the optimal tax also changes. There is a sense in which there is a "short run" and "long run" demand for money balances, and the time inconsistency problem depends on the relative sizes of the elasticities of these two demand curves.

3   OTHER RELATED MODELS

In this section we want to discuss the reasonability of the assumption required to generate the traditional time inconsistency result.

The interpretation of a single time period as consisting of two sub periods is made for tractability. It is a simple way to have a centralized- decentralized model in which it is possible to have a representative consumer and still think about a shopper and a worker. Thus, the possibility of a surprise inflation at a point in time such that all agents are at the bank and can readjust their portfolios for free comes from a very particular assumption of the model

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which has no counterpart with any observable economy. In Lucas (1980), the cash-in-advance constraint is motivated as "...Individual behavior resembles that captured in inventory theoretic models of money demand, as studied by Baumol (1952) and Tobin (1956)...". However, in those models, the individual time profile of money demand has a very particular feature (resembling very much actual individual patterns). To generate aggregate behavior consistent with observations one must assume that different consumers attend the securities market at different times, such that average money holdings resemble aggregate money holdings. But then, there is no hope that the government can increase the money supply at a point in time in which every single consumer can readjust his portfolio for free. The cash-in-advance constraint is a shorthand for the frictions that exist in markets and that make the exchange of liquid assets (money) and nonliquid assets (bonds) a costly process. The same frictions that justify the existence of a dominated asset cast doubt on the assumption that it is possible to increase the money supply at a fully centralized securities market where every agent can exchange money for bonds at zero cost.

In this type of models, goods do not buy goods. It matters in which units the government is charging the inflation tax because agents cannot exchange some goods for other goods at the market price unless they have enough liquid assets. The inflation tax is charged in cash, and the distortion generated by that tax is given by the marginal value of liquidity. That is basically what liquidity is all about, and it does not seem very reasonable to assume (for the purpose of the welfare effects of money surprises) that liquidity services have value at one sub period but do not have value in the other sub period for the economy as a whole.

In equilibrium cash is demanded, in spite of being return dominated, because of the liquidity services it provides. For unexpected inflation to be lump-sum, we require that there exists a sub-period in which agents hold a return dominated asset -cash- which provides, during the sub period, no liquidity services.

The discussion above suggests that in order to study the welfare effects of unexpected monetary changes, the Svensson version of the cash-in-advance model is more reasonable than the Lucas version. However, it has at least two shortcomings. The first, is that in most economies monetary policy is typically executed by means of open market operations. Thus, it makes much more sense to assume that money injections are done at the securities market.

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Second, the way in which we think about liquidity is as an excess of cash, rather than a squeeze of cash, as in the case we just analyzed.

To further discuss these issues, we study the time inconsistency problem in two simple versions of more stylized models, the Grossman and Weiss(1982) - Rotemberg(1983) (GWR) model and the Fuerst(1992) model. The analysis of the time inconsistency problem in these two models will show that the shortcomings in the interpretation of the Svensson model arise because of its very simple structure, rather than because of having costs of unexpected inflation and that all we know about the monetary transmission mechanism and liquidity effects suggest that it is more appropriate than the Lucas version when studying the welfare effects of unanticipated monetary shocks.

3.1 The GWR model

In this model, there are two representative agents, with identical preferences[10] over the single perishable consumption good and leisure as

$$\sum_{t=1}^{\infty} \beta^t \left[U(x_t^i) - \alpha.n_t^i \right] \tag{20}$$

for i = a,b. There is a linear technology that relates total available consumption to labor effort

$$c_t^a + c_t^b + g = n_t^a + n_t^b \tag{21}$$

where g is, as before, government consumption. We assume that cash must be used to purchase consumption goods, but agents can only go to the bank to exchange interest earning assets for money once every two periods. In fact, the only source of heterogeneity arises in the assumed restrictions agents face regarding trips to the bank. We assume that agent a visits the bank on even periods, and agent b on odd periods. Thus, the first period agent a is at the bank, and must withdraw enough money to finance consumption for the first and second period, because he will return to the bank only on the third period. On the contrary, agent b must finance first period consumption out of initial money balances and on the second period is at the bank where he can withdraw money to finance second and third period consumption.

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[10]To sharpen the comparison with the previous model, we assume the economic environment as close as possible to the original model.

Both agents receive credit in their bank accounts every period for the wage receipts. The restrictions that reflect these assumptions are

$$\sum_{t=1}^{\infty} Q_t.n_t^a \leq \sum_{t=1}^{\infty} Q_{2t-1}.M_{2t-1}^a \qquad (22)$$

$$c_{2t-1}^a.p_{2t-1} + c_{2t}^a.p_{2t} \leq M_{2t-1}^a \qquad (23)$$

for agent a, and

$$\sum_{t=1}^{\infty} Q_t.n_t^b + M_0^b - c_0^b.p_0 \leq \sum_{t=1}^{\infty} Q_{2t}.M_{2t}^b \qquad (24)$$

$$c_{2t}^b.p_{2t} + c_{2t+1}^b.p_{2t+1} \leq M_{2t}^b \qquad (25)$$

$$c_1^b.p_1 \leq M_0^b \qquad (26)$$

for agent b. It seems clear, given the discussion of last section regarding general equilibrium versions of Baumol-Tobin models, that an injection of currency in the first period taxes only first period consumption for agent b so that it cannot be lump-sum.

The equilibrium conditions of this model are

$$U'(c_1^b) = \alpha + \frac{\delta_0.p_1}{\beta} \qquad (27)$$

$$U'(c_{2t-1}^a) = \alpha = U'(c_{2t}^b) \qquad (28)$$

$$U'(c_{2t}^a) = \alpha.(1 + R_{2t}) \qquad (29)$$

$$\alpha.(1 + R_{2t+1}) = U'(c_{2t+1}^b) \qquad (30)$$

$$Q_t.p_t.\beta = Q_{t+1}.p_{t+1} \qquad (31)$$

for all t. It is possible to construct implementability constraints for each agent by replacing first order conditions in the budget constraints, the same way we did before[11]. If we do it, we obtain

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[11] As there are two representative agents in this model, we need two implementability constraints, because both budget constraints must hold.

$$\sum_{t=1}^{\infty} \beta^t . [\alpha.n_t^a - U'(c_t^a).c_t^a] \geq 0 \tag{32}$$

$$\alpha.n_1^b + \sum_{t=2}^{\infty} \beta^t . \left[\alpha.n_t^b - U'(c_t^b).c_t^b\right] \geq 0 \tag{33}$$

Note that as before, c_1^b does not show up in agent's b implementability constraint. This is so, because with positive nominal interest rates, there are no incentives to save some of the initial money balances for future periods and thus they exactly cancel out with first period consumption. Note also that equation 28 implies that consumption on odd periods for agent a and consumption in even periods for agent b are determined only by the marginal utility of leisure, and do not depend in any way in policy variables. The government can only manipulate agent a consumption on even periods and agent b consumption on odd periods.

Defining a Ramsey problem in a two representative consumer set up is not a trivial problem. If we assume that the government maximizes a linear combination of both agents utilities, the policy problem is to maximize

$$\sum_{t=1}^{\infty} \beta^t \left[U'(x_t^a) - \alpha.n_t^a\right] + \theta. \sum_{t=1}^{\infty} \beta^t \left[U'(x_t^b) - \alpha.n_t^b\right] \tag{34}$$

subject to equations 32, 33 and 21, where θ is the relative preference of the government for agent b[12]. We let λ^i be the lagrange multiplier on the implementability constraint of agent i, and ϵ_t the multiplier on the market clearing constraint at time t. If a solution exists and it is interior, the necessary conditions are

$$\left[\theta + \lambda^b\right] .\alpha = \frac{\epsilon_t}{\beta^{t-1}} \tag{35}$$

$$[1 + \lambda^a] .\alpha = \frac{\epsilon_t}{\beta^{t-1}} \tag{36}$$

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[12] The resulting allocation must be a competitive equilibrium, and the only policy instrument is the inflation tax, thus it is not clear that all the -restricted - utility frontier can be implemented. However, with positive government expenditures, there exists a nontrivial interval such that the problem is well defined for all relative preference parameters in that interval.

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$$U'(c_{2t}^a) + \lambda^a.U'(c_{2t}^a).\left[1 - \sigma\left(c_{2t}^a\right)\right] = \frac{\epsilon_{2t}}{\beta^{2t-1}} \qquad (37)$$

$$\theta.U'(c_{2t+1}^b) + \lambda^b.U'(c_{2t+1}^b).\left[1 - \sigma\left(c_{2t+1}^b\right)\right] = \frac{\epsilon_{2t+1}}{\beta^{2t}} \qquad (38)$$

$$\theta.U'(c_1^b) = \epsilon_1 \qquad (39)$$

for t=1,2,3,... . These conditions are very similar to the ones we obtained in the first section, except for the parameter θ, which rises distributional considerations into the model. From the last two equations, it is possible to obtain

$$U'(c_1^b) - U'(c_3^b) = \frac{\lambda^b}{\theta}U'(c_3^b)\left[1 - \sigma(c_3^b)\right] \qquad (40)$$

It is clear then that the incentives to deviate depend not only on the elasticities, but also on the distributional parameter θ . The intuition behind this solution is the same as before. Given the structure of the model, at any time period, there is always one consumer who cannot adjust money balances (go to the bank) till next period. A surprise injection of currency squeezes the purchasing power of money balances held by that consumer and creates distortions measured at the margin by the multiplier of the cash-in-advance constraint of that consumer. As it is clear from the expression above, the solution depends on how much the government cares for that consumer. However, the cualitative nature of the optimal deviation (i.e. if it is positive, negative or zero) depends only on the elasticities, as in the case before. Incidentally, note that if $\theta = 1$, so the planner gives the same weight to both consumers, the set of equations 35 to 39 is identical to equations 15 to 17 with $\lambda^a = \lambda^b = \omega$. Thus, the equal weight case exactly reproduces the set of conditions of the first model.

3.2   The Fuerst model

One of the shortcomings of the above model is that it is not possible to isolate the distributional issue from the liquidity issue. To separate the effects, Fuerst(1992) proposes a model in which there is a representative family with many members, that separate at the beginning of every time period. Each member must take decisions in separate markets, and then they get together

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at the end of the period to pull the leftovers of all transactions carried on during the period. In this way, any redistribution generated by policy among different members of the family, washes out at the end of the day. We will consider a simplified version of the model, without uncertainty and without capital. The effects we discuss hold in the general model as well.

The representative family preferences are given by

$$\sum_{t=1}^{\infty} \beta^t \left[U(x_t) - \alpha.n_t \right] \tag{41}$$

and the technology is

$$x_t + g = n_t \tag{42}$$

The family has three members. The shopper, who takes a fraction of money balances and travels to the goods market, the broker who takes the other fraction of initial money balances and travels to the bank to deposit it, and the worker who travels to the labor market. There is a bank who accepts deposits from the broker and lends them to the firm. The firm hires labor to produce, and also faces a cash-in-advance constraint. It must pay wages with cash. The asymmetry in the model arises because it is assumed that money injections are performed at the banking sector, and thus can be used by the firm to hire employees, but cannot be used by the shopper to buy goods. The constraints that reproduce this behavior are given by

$$p_t.x_t \le C_t = M_t - N_t \tag{43}$$

$$w_t.n_t \le N_t + H_t \tag{44}$$

where M_t, N_t, H_t, and C_t, are initial money balances, the amount of money to be deposited at the bank by the broker, the nominal increase of money by the government and the amount of money for goods consumption that the shopper takes to the goods market. As the firm and the bank are constant returns to scale industries, they generate no profits. Thus, the budget constraint of the family can be written as

$$B_{t+1} + C_{t+1} + N_{t+1} + p_t.c_t - (N_t + B_t).(1 + R_t) - w_t.n_t - C_t \le 0 \tag{45}$$

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and the equilibrium conditions are given by

$$U'(c_t) = \frac{p_t}{w_t}.\alpha.(1 + R_t) \tag{46}$$

$$p_t = w_t.(1 + R_t) \tag{47}$$

$$\frac{U'(c_t)}{\beta.U'(c_{t+1})} = (1 + R_t).\frac{p_t}{p_{t+1}} \tag{48}$$

where the first equation makes the marginal rate of substitution between consumption and leisure equal to its relative price, the second equates the real wage to its productivity, and the last one is the standard Fisher equation[13]

Following the same strategy, we can build up an implementability constraint as

$$- n_1.\alpha + \sum_{t=2}^{\infty} \beta^{t-1}[U'(x_t).x_t - n_t.\alpha] \le 0 \tag{49}$$

The Ramsey problem in this case, is to maximize 41 subject to 42 and to 49. Note that it is identical to the Ramsey problem in the cash-in-advance model with the Svensson timing which we solved in section 1. Thus, we exactly replicate the solution we found there.

3.3 Discussion of the models

At the beginning of this section we raised two shortcomings to the Svensson version of the cash-in-advance model. Then, we presented two alternative models that are much more explicit regarding the timing of agents decisions and do not share any of the shortcomings. In the two models presented in this section, monetary policy is executed through open market operations at the financial sector and the liquidity effects they generate are consistent with the empirical evidence[14]. It is worth pointing out that excess of liquidity is a relative concept, such that there must exist a market or sector with relative squeeze of liquidity, as in the model of section 1.

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[13]In this case, the liquidity effect of Fuerst is absent because there is no uncertainty.

[14]These models were developed to capture that pattern.

The time inconsistency analysis in these two models is equivalent to the one in the Svensson timing cash-in-advance model. The shortcomings we mentioned above are due to the very simple structure of the model, in particular, the separation of the time period in two sub periods[15]. This unrealistic assumption is innocuous to deal with certain very relevant questions, like the long run relationship between money and prices. However, to separate the financial market from the goods market as events occurring at different points in time rather than occurring at the same time in different places has important implications for the welfare analysis of surprise changes in the quantity of money. As the more sophisticated models suggest, the two shortcomings of the Svensson version are not related to the fact that there are costs of unanticipated inflation, but to the very simple economic environment assumed.

These two models make clear that to generate the standard time inconsistency result we require a perfect matching between all agents financial decisions and between these decisions and the policy actions of the government. They also suggest that this perfect matching is a knife edge situation. not robust to small changes in the environment.

There seems to be a good degree of consensus regarding the effect of the perfect matching assumption in monetary models: it generates too much neutrality of monetary shocks relative to the empirical evidence. In fact, the two models studied in this section, were developed to reproduce the non neutralities and short run departures from quantity theory exhibited by the data. In the original papers, they explicitly assume away the perfect matching, to be able to fit the data. The evidence on liquidity effects and short run no neutralities can be taken as evidence against the perfect matching assumption.

Before considering the effect of nonzero initial holdings of government debt, we briefly discuss the time inconsistency problem in other monetary models. Consider first the overlapping generations model of money (see Wallace (1980)). Given that in equilibrium money holdings are unevenly distributed across agents, the effects discussed in the analysis of the GWR model apply. In fact, if one defines the welfare function as a linear combi-

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[15]It seems that the story of separating one period in two sub-periods can be misleading in some cases. As we mentioned before, this interpretation requires the coexistence, during the subperiod, of three facts -positive interest rates, nonnegative cash holdings and zero value for liquidity services- inconsistent with equilibrium.

nation of all generations utilities - including the current old - the first order conditions of the policy problem for the first period consumption are the same as in the GWR model. Consider also the Townsend(1980) model with spatially separated agents. The redistribution problem arises also there, and to avoid the liquidity issue, it is required that the government injects the currency across markets with a symmetry that hardly resembles the way monetary policy is executed. Finally, consider the Kiyotaki-Wright (1989) model. Even though it is not obvious how one would model monetary policy in the model, it seems clear that the redistributional problem arises and to avoid the liquidity problem, a very strong symmetry on the way money injections reach the pairs of agents is required.

3.3.1 Positive initial holdings of government debt

Note also that the results of last section would be different if there exists nominal debt. In the simple cash-in-advance model, the implementability constraint with nonzero initial holdings of government debt would be

$$-n_1.\alpha - \alpha \cdot x_1 \cdot \frac{B_1}{M_1} + \sum_{t=2}^{\infty} \beta^{t-1}[U'(x_t).x_t - n_t.\alpha] \leq 0$$

where B_1 is the total value of government debt (including interest payments) held by the private sector. Note that the ratio B_1/M_1 are givens for the policy problem. Note that now the first order condition with respect to consumption in the first period will be

$$U'(x_1) - \omega\alpha\frac{B_1}{M_1} = \epsilon_1$$

Combined with 16, this equation yields

$$U'(x_1) - U'(x_2) = \omega \cdot \left[U'(x_2)(1 - \sigma(x_2)) + \alpha\frac{B_1}{M_1}\right]$$

In this case, present inflation will not only depend on elasticities but also on the marginal gain derived from changing the value of the existing debt. Note that if $\sigma(x_2) = 1$, which means that the optimal policy is time consistent without nominal debt, then

$$U'(x_1) - U'(x_2) = \omega \cdot \alpha\frac{B_1}{M_1}$$

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Note that when B_1 is positive, consumption in the first period must be lower than consumption in all other periods reflecting the incentives to have a higher initial inflation and reduce the real value of the debt. However, if B_1 is negative, there exists incentives to have a lower initial inflation, because that increases the value of government credit and reduces the distortion. In the more general case, both elasticities and nominal debt are important in determining the optimal deviation. The reason why this new effect show up is because unexpected taxation of government debt acts as a lump-sum tax. This is the effect discussed in PP&S (1987) and (1989).

Several interesting issues arise which deserve further research. First, there is the possibility to use nominal debt to reduce the temptation to change the inflation rate as suggested by Persson, Persson and Svensson (1987). Second, the incentives to inflate depend on the size of government debt, a feature that can give rise to very interesting implications. Third, the interiority of the optimal deviation can have important implications regarding the relevance of the time structure of government debt. Finally, the interactions between the size of the government debt over time with the incentives to inflate the debt away open a new avenue to interpret the high inflation rates experienced in the last thirty years.

4 CONCLUSIONS

We considered the robustness of the traditional time inconsistency result of the literature (Calvo (1978), Lucas and Stokey (1983), Persson, Persson and Svensson (1987)). We showed that result to crucially depend on a particular assumption regarding the constraints faced by the private sector at the time money injections are done.

We argued against the reasonability of that assumption when studying the welfare effects of monetary shocks, by exploring the intuition behind the cash-in-advance constraint in the literature. In order to shed more light into the issue, we also solved the time inconsistency problem in two other existing models which were constructed to better fit the data after an unexpected change in monetary policy. In none of these two models it is possible to reproduce the traditional time inconsistency result.

The first objective of the paper is to argue that the economic environment required to generate the traditional result is a knife edge situation. It requires

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a perfect matching between all agents actions and the policy actions of the government which is hard to justify in real economies.

In all the models analyzed in this paper the optimal policy is not, in general, time consistent. However, the nature of the time inconsistency problem is quite different from the traditional result. First, the optimal deviation is finite in general. Second, the optimal deviation depends on the stock of nominal debt. Finally, and this is the most striking result, the optimal deviation can be negative. That is, the government can be tempted to chose an inflation rate lower than the one chosen in the planning period.

The models solved in this paper show one easy and reasonable way to introduce costs of unexpected inflation in simple general equilibrium monetary models. Contrary to the analysis of reputation in capital taxation (see Chari and Kehoe(1992), Persson and Tabellini(1990) or Stokey(1992)) and debt repudiation (see Chari and Kehoe (1993)) , the literature on monetary policy has relied on ad-hoc models (see Persson and Tabellini(1990)), and many of the results of the literature depend on the special features of the assumed payoff function of the government. In particular, the payoff function typically assumed rules out the possibility of optimal negative deviations. It remains an open question which of the established results of the literature hold in general equilibrium models with benevolent governments.

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