Non-stationary Job Search When Jobs Are Not Forever: A Structural Estimation

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Abstract

This paper considers a job search model where the environment is not stationary along the unemployment spell and where jobs do not last forever. Under this circumstance, reservation wages can be lower than without separations, as in a stationary environment, but they can also be initially higher because of the non-stationarity of the model. Moreover, the time-dependence of reservation wages is stronger than with no separations.

The model is estimated structurally using Spanish data for the period 1985-1996. The main finding is that, although the decrease in reservation wages is the main determinant of the change in the exit rate from unemployment for the first four months, later on the only effect comes from the job offer arrival rate, given that acceptance probabilities are roughly equal to one.

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1 Introduction

In the recent past a large amount of research has been carried out about the job search behavior of unemployed workers. The analysis of unemployment duration has become an important tool for understanding better the issues behind the unemployment rate as an aggregate figure.

The classic labor supply model cannot explain important features of the typical problem faced by an unemployed worker searching for a job. Job search models, describing the behavior of unemployed individuals in a dynamic and uncertain world, characterize better their situation in the labor market.

These models study the problem of an unemployed worker searching for a new job. Their basic result is that the worker maximizes his expected wealth by using a stopping strategy based on accepting an offer when the offered wage is equal to or higher than a critical value called the *reservation wage*.

A traditional assumption in these models has been *stationarity:* parameters determining worker behavior were supposed to be constant over the spell of unemployment. But this assumption is often at variance with reality. Estimated reduced-form search models usually result in manifest negative duration dependence of the re-employment probability, even when unobserved heterogeneity is controlled for (see among others Meyer (1990) for US data, Narendranathan and Stewart (1993) for UK data, or Bover, Arellano and Bentolila (1997) for Spanish data). The natural way of taking into account this empirical fact is allowing for some time-dependence in one or more parameters of the model. Such time dependence is supported by various observed facts such as the lower number of offers arriving to long-term unemployed workers or the changes in the personal situation of or the environment faced by the unemployed worker.

The contribution of this paper is to introduce a new element not considered before in non-stationary models of search: an exogenous separation probability, which can represent both firing or quitting. Under this circumstance the unemployed worker knows that once employed, he can leave or can be compelled to leave the job and become unemployed again in the future. One of the most influential articles in the field of non-stationary job search is Van den Berg (1990). In this paper, nonstationarity is considered in a very general way but it is assumed that jobs are hold forever although it is recognized the importance of relaxing this assumption. The present paper is the first attempt to incorporate the separation probability to a discrete-time non-stationary search model. It is proved that this fact makes the reservation wage be more strongly time-dependent than otherwise. Moreover, in a stationary model, the effect of this separation probability on reservation wages is always negative. That is, reservation wages are lower when the separation probability is larger, basically because the future is discounted at a higher rate. However, when reservation wages change during the time the worker is unemployed this effect will be proved not to be unique: we can see lower reservation wages when the separation probability is considered but also higher ones, at least for the first part of the unemployment spell. This result means that, in some situations, the unemployed worker is choosier at accepting job offers because he knows that the probability of being separated from the job is different from zero. But this only happens at the beginning of the unemployment spell, when his situation as unemployed is not so bad. As time passes, the worker wants to be employed as soon as possible because not only he can access to better conditions once in a job, but also because he internalizes that, even after a possible separation, his situation as a new unemployed will be better than the present one.

There is another empirical objective in this paper: to estimate structurally the non-stationary model using Spanish data for the period 1985-1996. This estimation is carried out using data on unemployment duration and accepted wages. Moreover, it is reinforced by controlling for unobserved heterogeneity using a mixture technique inspired in Heckman and Singer (1984). Given that we have to estimate the model allowing for not much heterogeneity, the control for that not taken into account seems important. Some simulations about the identification of this model with unobserved heterogeneity and duration dependence have been carried out. Their results show that given the structure of the estimated model, the estimation procedure is able to distinguish between these two elements. Moreover, results from Elbers and Ridder (1982) ensure identification if we have more regressors apart form the duration and the unobserved heterogeneity considered. This is the case of the present estimation where we obtain marginally significative effects of unobserved heterogeneity. Two groups of workers are identified, one of them with much lower unemployment hazard rates.

The main results of the estimation are as follows. First, the predicted unemployment hazard rate is increasing up to the fourth month and decreasing thereafter. The structural estimation indicates that during the first four months, decreasing reservation wages are the main determinant of the hazard rate, but later on reservation wages are so low that acceptance probabilities are practically equal to one. Hence, the hazard is equal to the offer arrival rate, also estimated to be decreasing along the unemployment spell. The model predicts that those workers with access to unemployment benefits have a mean expected unemployment duration of more than four months whereas without such benefits this expected duration is less than three months.

The structure of the paper is the following. Section 2 presents the non-stationary job search model with the separation probability, jointly with some simulation exercises which help us understand better the results of the model. Section 3 describes the estimation procedure, the data used, and the main results, and Section 4 concludes.

2 The model

I consider a standard discrete-time search model (See, for example, Lippman and McCall, 1976 or Wolpin, 1987) where the parameters are not going to be stationary but will be allowed to vary with unemployment duration. Its main characteristics is based on the continuous-time model of Van den Berg (1990), but I modify it by introducing a probability of being separated from the job, once employed.[1]

Consider a discrete-time economy where agents either work receiving a constant wage, w, or are unemployed and searching for a job in each period t. The following conditions are assumed:

(**A1**) Wage offers at time t are random draws from a distribution function $F(w,t)$ where $w \in [0,\infty)$ and t is the amount of time the agent has been unemployed.[2]

[1] The other modification with respect to Van den Berg (1990) is introducing a discrete-time framework. The reason for departing from continuous time is not only the final objective of estimating the model using discrete data but also to understand better the effect of the separation rate. Nevertheless, the continuous version of the model is simply the limiting case of this model. Details on this continuous version can be found in García-Pérez (1998).

[2] Calendar time is assumed to start at the moment the individual becomes unemployed.

(**A2**) Job offers arrive at random intervals following a Poisson process with arrival rate $\alpha(t) \in [0, \infty)$ defined for each period t.

(**A3**) During the spell of unemployment, the agent has an income $b(t) \in [0, \infty)$, net of search costs. This income can be interpreted as the value of time for the unemployed worker, and includes, among other things, unemployment benefits and non-labor income.

(**A4**) When an offer is accepted, the agent works at the offered wage, w, but there is a constant separation probability $\delta \in [0, 1]$.[3]

(**A5**) $F(w, t), \alpha(t)$ and $b(t)$ are continuous functions of t.[4]

(**A6**) The individual has a constant subjective discount rate $r \in [0, \infty)$.

(**A7**) There exists some period T such that all the parameters depending on unemployment duration are constant on $[T, \infty)$.

These assumptions ensure the appropriate present values are well defined and, therefore, guarantee the existence of an optimal strategy.

The expected present value of future net income for an unemployed worker who is searching is defined as:

$$U(t) = b(t) + \frac{1}{1+r}\left[\alpha(t) E_{w,t+1} \max\left(W(w), U(t+1)\right) + \left(1 - \alpha(t)\right) U(t+1)\right] \quad (1)$$

Thus, $U(t)$ is the value of unemployment time, $b(t)$, received at the beginning of the period, plus the expected and discounted value of the optimal stopping decision at $t+1$. This expected value is, in the case an offer arrives in period t,[5] the maximum between the expected present value of accepting the offer, $W(w)$, and continuing to search one more period, $U(t+1)$. If no offer arrives, then the worker will have the value of being unemployed at period $t+1$, $U(t+1)$.

Thus, t refers both to calendar time and to the length of time over which the individual remains unemployed.

[3] That is, the job can be interrupted for whichever exogenous reason, for example, firing or quitting.

[4] These parameters can also be step functions of duration, as in Van den Berg (1990).

[5] I assume that this offer is received at the end of the period so that we have to apply the time discount factor to its expected value.

The expected present value of stopping search and beginning to work at wage w is:

$$W(w) = w + \frac{1}{1+r}\left[(1-\delta)W(w) + \delta U(0)\right] \tag{2}$$

That is, $W(w)$ is the value of the wage received in period t plus the expected present value of what can happen in period $t+1$: with probability $1-\delta$, the worker will continue employed and with the opposite probability the worker will leave the job and return to unemployment, where he will have a duration of zero periods, $U(0)$.

In this context, like in all job search models, every time an offer arrives the decision has to be made whether to accept or to reject it and search further. The individual will be indifferent between working and searching one more period for a wage called the *reservation wage,* $w_R(t)$. This wage equates $U(t)$ and $W(w)$ and, hence, verifies:

$$U(t) = \frac{(1+r)w_R(t)}{r} + \left(U(0) - \frac{(1+r)w_R(t)}{r}\right)\frac{\delta}{\delta + r} \tag{3}$$

Taking into account that $U(0) = \frac{(1+r)w_R(0)}{r}$, *i.e.* the value of $U(t)$ when $t = 0$, and substituting (3) in (1) we obtain the following difference equation for the reservation wage[6] :

$$w_R(t) = b(t) + \frac{\delta}{r}\left(b(t) - w_R(0)\right) + \frac{\alpha(t)}{r}\int_{w_R(t+1)}^{\infty}\left(w - w_R(t+1)\right)dF(w,t) + \frac{\Delta w_R(t)}{r} \tag{4}$$

where $\Delta w_R(t) = w_R(t+1) - w_R(t)$. It is straightforward to show that $w_R(0)$ satisfies:

$$w_R(0) = b(0) + \frac{\alpha(0)}{\delta + r}\int_{w_R(1)}^{\infty}\left(w - w_R(1)\right)dF(w,0) + \frac{\Delta w_R(0)}{\delta + r} \tag{5}$$

From (4) we can distinguish four terms in the reservation wage: (*i*) the value of time for the unemployed worker, $b(t)$; (*ii*) the value associated with a future job separation, given by the difference between income in period t and the value

[6] If we take the separation probability to be equal to zero, this equation is the same as in Van den Berg (1990) but in discrete time.

of being again at period 0 of the following unemployment spell; (*iii*) the expected discounted benefit associated with the arrival of a new offer; and, (*iv*) the appreciation or depreciation of the option represented by the reservation wage.

Given the expression of the reservation wage, equation (4), we can obtain the probability of exiting unemployment in t, conditional on not having exited before, the *hazard rate,* $\phi(t)$, which is defined as:

$$\phi(t) = \alpha(t)\,[1 - F(w_R(t+1), t)] \tag{6}$$

that is, the rate at which offers arrive times the probability that a given offer is acceptable. Note that, given (1), where the value of accepting a job offer, arriving at the end of period t, is compared with the expected present value of being unemployed at time $t+1$, the acceptance probability in period t is computed taking into account the reservation wage at time $t + 1$.[7]

2.1 Nonstationarity of the reservation wage

The nonstationarity of the reservation wage is derived from the nonstationarity of the parameters of the model, which is established by the following assumptions:[8]

(K1) $b(t) > b(t + 1)$, $\forall t \in [0, T)$.

(K2) $\alpha(t) > \alpha(t + 1)$, $\forall t \in [0, T)$.

(K3) $F(w, t)$ first order stochastically dominates $F(w, t + 1)$, $\forall t \in [0, T)$, which implies that $1 - F(w, t) > 1 - F(w, t + 1)$, $\forall w \in [0, \infty)$.

(K4) $F(w, t)$ is a mean preserving spread of $F(w, t + 1), \forall t \in [0, T)$, that is, $E(w, t) = E(w, t + 1)$, and $\forall x \in [0, \infty)$,

$$\int_0^x F(w, t)dw > \int_0^x F(w, t + 1)dw.$$

The economic meaning of these assumptions is simple. The value of time for an unemployed worker decreases with unemployment duration because his income

[7] This is a consequence of discrete time. In continuous time, see García-Pérez (1998), this acceptance probability would just be $1 - F(w_R(t), t)$.

[8] The derivation of the nonstationarity of the reservation wage is similar to Van den Berg (1990) but in discrete time.

and unemployment benefits decline over time. The offer arrival rate and the wage offered become smaller as time proceeds, as a result of the stigma effect that long-term unemployed workers may suffer (see Viswanath, 1989 or Berkovitch, 1990). The distribution of offers can be more concentrated around its mean for the long-term unemployed, because they may know more about this distribution (see Burdett and Viswanath, 1988). An important assumption is that people know how the parameters are related to the duration of unemployment.

The time dependence exhibited by the reservation wage is obtained in the following theorem, where it is helpful to use what I call a *stationary reservation wage,* $w_R^0(t)$. This wage is the optimal reservation wage at time t, for all $t \geq 0$, if the environment remains stationary after t, that is:

$$w_R^0(t) = b(t) + \frac{\delta}{r}\Big(b(t) - w_R^0(0)\Big) + \frac{\alpha(t)}{r}\int_{w_R^0(t)}^{\infty}\Big(w - w_R^0(t)\Big)\,dF(w,t) \qquad (7)$$

Theorem 1 *Let assumptions (A1) to (A7) be satisfied. Let one or more parameters satisfy assumptions (K1)-(K4) with strict inequality, while the remaining ones are constant over the time interval* $[0, \infty)$. *Then:*

(i) $w_R(t) < w_R^0(t)$,

(ii) $\Delta w_R(t) < 0$.

Proof : See Appendix A.

The meaning of this result is simple: any future decrease in the parameters of the model makes the value of search in the present be smaller than it would be, if the parameters were constant. So the unemployed worker, anticipating these future changes, sets a smaller reservation wage as his spell of unemployment lengthens.

2.2 The effect of the separation probability

In stationary search models (see, for example, Devine and Kiefer, 1991) the effect of the separation rate on reservation wages is negative. Given that the future is more risky, future opportunities are discounted at a higher rate and, thus, the reservation wage is lower. This is because the value of being employed is lower when jobs do not last forever. Given this, the value of being unemployed is also lower and the result is that the minimum acceptable wage for these workers is smaller.

However, in the present model, the nonstationarity of the search process introduces a new element at play: being separated from a job is not the same thing when considered by an unemployed worker at the beginning of the unemployment spell as after, for example, one year of unemployment. This fact can be confirmed by analyzing equation (4). The effect of the separation probability is not only direct, via the presence of δ in the expression for $w_R(t)$, but also indirect because of its effect on $w_R(0)$. Hence, in order to obtain the effect of the separation probability we need a general expression for $w_R(t)$ as a function only of exogenous parameters.

This can be obtained by taking into account that equation (4) determines a system of $T+1$ equations on reservation wages from period 0 to period T. If we work backwards in this system, we can firstly obtain an expression for $w_R(0)$, and after substituting in $w_R(t)$, obtain the following general expression for the reservation wage:

$$w_R(t) = (r+\delta)\mathrm{PV(b(t))} + \mathrm{PV(E(w,t))} - \frac{\delta \mathrm{D(t)}}{1+\delta \mathrm{D(0)}} \left[(r+\delta)\mathrm{PV(b(0))} + \mathrm{PV(E(w,0))}\right] \tag{8}$$

where,

$$\mathrm{PV(b(t))} = \sum_{i=t}^{T-1} \frac{b(i)}{1+r} \left(\prod_{j=t}^{i-1} \frac{1-\phi(j)}{1+r} \right) + \frac{b(T)}{r+\phi(T)} \left(\prod_{j=t}^{T-1} \frac{1-\phi(j)}{1+r} \right)$$

$$\mathrm{PV(E(w,t))} = \sum_{i=t}^{T-1} \frac{\alpha(i) \int_{w_R(i+1)}^{\infty} w dF(w,i)}{1+r} \left(\prod_{j=t}^{i-1} \frac{1-\phi(j)}{1+r} \right) + \frac{\alpha(T) \int_{w_R(T)}^{\infty} w dF(w,T)}{r+\phi(T)} \left(\prod_{j=t}^{T-1} \frac{1-\phi(j)}{1+r} \right)$$

$$\mathrm{D(t)} = \sum_{i=t}^{T-1} \frac{1}{1+r} \left(\prod_{j=t}^{i-1} \frac{1-\phi(j)}{1+r} \right) + \frac{1}{r+\phi(T)} \left(\prod_{j=t}^{T-1} \frac{1-\phi(j)}{1+r} \right)$$

That is, the reservation wage at time t is the present discounted value, $PV(\cdot)$, of $(r+\delta)b(t) + \alpha(t) \int_{w_R(t+1)}^{\infty} w dF(w,t)$ from period t to T minus a fraction of this present discounted value but from period 0 to T. In these actual discounted values, the discount factor involves all the parameters of the model via the unemployment hazard rate, $\phi(t)$. Hence, this expression takes into account both a time discount, r, and a probability discount, via the hazard rate. The latter considers whether the worker will be unemployed or not in each period considered.

Before discussing the sign of the derivative of $w_R(t)$ with respect to δ, we can realize of the following useful result.

Lemma 2 *If $b(t)$ is decreasing, the derivative of $w_R(t)$ with respect to δ is also decreasing in t.*

Proof : The derivative of $w_R(t)$ with respect to δ, dividing it by $D(t)$, is:

$$\frac{dw_R(t)}{d\delta} = \frac{PV(b(t))}{D(t)} - \frac{\delta D(0)}{1+\delta D(0)}\frac{PV(b(0))}{D(0)} - \frac{D(0)}{(1+\delta D(0))^2}\left[(r+\delta)\frac{PV(b(0))}{D(0)} + \frac{PV(E(w,0))}{D(0)}\right]$$

As $\frac{PV(b(t))}{D(t)}$ is a weighted average of the values of $b(t)$ from period t to T and $b(t)$ is decreasing in t, $\frac{PV(b(t))}{D(t)}$ is decreasing in t. But this mean that $\frac{dw_R(t)}{d\delta}$ will be also decreasing in t because the other terms in this derivative are constant in t. Q.E.D.

Given this result, we can easily prove a general result for the sign of the derivative of $w_R(t)$ with respect to δ. This is established in the following proposition:

Proposition 3 *The effect of the separation rate on the reservation wage at period t, $\frac{dw_R(t)}{d\delta}$, will be negative if and only if*

$$\frac{PV\,(b(0))}{D(0)} \leq \frac{PV\,(E(w,0))}{1-rD(0)}$$

In the opposite case, the effect will be positive until period t^* when it is verified that

$$\frac{PV(b(t^*))}{D(t^*)} = \frac{\delta D(0)}{1+\delta D(0)}\frac{PV(b(0))}{D(0)} + \frac{D(0)}{(1+\delta D(0))^2}\left[(r+\delta)\frac{PV(b(0))}{D(0)} + \frac{PV(E(w,0))}{D(0)}\right]$$

Proof : Given the result of the previous Lemma, in order to have that $\frac{dw_R(t)}{d\delta} \leq 0$, a sufficient condition is that this derivative at time 0 is negative. This sufficient condition is satisfied if and only if, evaluating $\frac{dw_R(t)}{d\delta}$ at time 0, we have that

$$\frac{PV(b(0))}{D(0)} - \frac{\delta D(0)}{1+\delta D(0)}\frac{PV(b(0))}{D(0)} - \frac{D(0)}{(1+\delta D(0))^2}\left[(r+\delta)\frac{PV(b(0))}{D(0)} + \frac{PV(E(w,0))}{D(0)}\right] \leq 0$$

Rearranging terms, we obtain that the condition to be satisfied is:

$$\frac{PV\,(b(0))}{D(0)} \leq \frac{PV\,(E(w,0))}{1-rD(0)}$$

In the opposite case, $\frac{dw_R(0)}{d\delta} \geq 0$ and it will continue being positive until period t^*, where the second expression in the proposition is verified. For all periods after t^*, the derivative will be negative. Q.E.D.

This proposition tells us that in a non-stationary environment, the effect of the separation probability on reservation wages is not always negative. If the weighted average of $b(t)$ from period 0 to T, $\frac{PV(b(0))}{D(0)}$, is high enough with respect to the present value of expected wages, we can find a positive initial effect which lasts for t^* periods. This is totally new and different to a stationary environment: when the parameters of the model change with the time the worker is unemployed, a higher separation wage can provoke the reservation wage to be higher instead of lower. That is, in presence of a positive separation probability, the worker may be choosier at the beginning of the unemployment spell and this is so when he enjoys a much better situation that it is expected to have in a possible job. However, as time passes, the worker realizes that his income or his chances of a new offer will be lower. But, he also knows that if he is hired, then in the case of a future separation, he will have access to greater values of all the parameters of the model. This fact provokes that the reservation wage decreases very fast as time passes.[9]

But, the separation probability affects also the time dependence of the reservation wage. The following proposition tells us that when the unemployed worker considers a future possibility of being unemployed, reservation wages will be even more negatively time dependent.

Proposition 4 *If $b(t)$ is decreasing, a higher separation probability will make a negative time dependence of reservation wages be even more negative.*

Proof : As $\Delta w_R(t) = w_R(t+1) - w_R(t)$, substituting each reservation wage by its own expression, we will have that

$$\frac{d\Delta w_R(t)}{d\delta} = \frac{\Delta b(t)}{1+r} + \frac{1}{1+r}\frac{d\Delta w_R(t+1)}{d\delta}.$$

[9] Of course, this effect comes from assuming that the situation at the beginning of the unemployment spell is the same whatever the duration of the previous job was. This is clearly at odds with the observed fact that, for example, unemployment benefits depend on the length of the previous job and on its associated wage. However, given the difficulty of controlling for these aspects, I omit them in the analysis.

Given that $\forall t \in [T, \infty)$ we have that $\Delta w_R(t) = 0$, then $\frac{d\Delta w_R(T)}{d\delta} = 0$ and thus, we have $\frac{d\Delta w_R(T-1)}{d\delta} = \frac{\Delta b(T-1)}{1+r} < 0$ because $b(t)$ is decreasing. Therefore, we have that a higher separation probability increases the negative time dependence of $w_R(t)$ provided:

$$\frac{d\Delta w_R(t)}{d\delta} < \frac{d\Delta w_R(t+1)}{d\delta} < 0 \qquad \forall t \in [0, T). \; Q.E.D.$$

Hence, the worker's requirements for accepting job offers will be even more decreasing given that he can be unemployed in the future. This fact will make that acceptance probabilities increase quickly along the unemployment spell. Moreover, they can be equal to one very soon in the unemployment spell for certain values of the model's parameters.

I have carried out some simulations with the model in order to determine when we observe positive or negative effects of the separation probability over the reservation wage. In these simulations I have combined five possible values for each of the parameters in the model and I have calculated reservation wages in each of these combinations. As we saw in Proposition 3, all the parameters of the model determine whether we obtain a positive or negative effect, but the main feature in order to obtain an initial positive effect is that the mean of the distribution of wages has to be low enough with respect to the value of unemployment time. This is confirmed with these exercises, whose results are in Figure 1. Given the values used in the simulations, the level of the parameter $b(t)$ at period 0 has to be at least 40% larger than the mean offered wage in order to obtain an initially positive effect of the separation rate. The coefficient of variation of offered wages is also very important. The larger is this coefficient, the higher has to be $b(0)$ with respect to the mean offered wage in order to observe a positive effect of the separation probability on reservation wages at time zero. The remaining parameters, the effects of $\alpha(t)$ and the time dependence of $b(t)$ are also shown in this figure, play a much smaller role in the determination of the sign of this derivative.

Hence, to conclude, I have studied the effect of the separation probability on non-stationary reservation wages and I have obtained that this analysis changes the results substantially with respect to the stationary case. Not only we can obtain a positive sign in the derivative of reservation wages with respect to the separation probability, but also I have found that the time dependence of the reservation wage will be even more negative when the probability of being separated from the job is larger.

3 Structural estimation

The estimation of the previous model is performed with Spanish data: the Spanish Continuous Family Expenditure Survey (*Encuesta Continua de Presupuestos Familiares (ECPF)*) for the period 1985-1996. The *ECPF* is a rotating panel which interviews about 3,200 households every quarter. One eighth of the sample is renewed quarterly and hence an individual can be followed for a maximum of two consecutive years. This source gives information on unemployed workers over their spells of unemployment and on their post-unemployment wages, in addition to information on consumption and other household characteristics.

The estimation sample is composed of unemployed household heads, who are the only group for which the educational level is reported. Also, I restrict the sample to married men to reduce heterogeneity, since, given the estimation procedure, I am not able to introduce many regressors in the estimation.

The individuals in the sample are all entrants to unemployment. The observed spells can be either *complete*, if the worker exits from unemployment or *censored*, if he does not. For the complete spells, the re-employment wage is computed for those who continue answering the survey two quarters after the unemployment spell ends, from the labor income of the second quarter of employment. The reason for doing this is to reduce measurement error about the amount, which is simply quarterly income (see Appendix B).

As we can see in Table 1, there are 869 completed spells of unemployment and 698 censored spells. Of the former, 446 have an observed re-employment wage. The shape of the Kaplan-Meier estimates of the hazard rate and the histogram of re-employment wages, which are expressed in real terms of December 1996, can be seen in Figures 2 and 3, respectively.

Although the *ECPF* is a quarterly survey, it is possible to calculate monthly values of the variables. Monthly data are preferred because they will reflect better the nonstationarity of the job search behavior. Indeed, with monthly data the changing patterns of the parameters are likely to be estimated better.[10] In order

[10] Hence, the time period in our discrete-time model is one month. This length could be quite

to obtain monthly data, a few transformation rules, explained in Appendix B, have been applied.

The model has been estimated structurally using the monthly data described before and the assumption made in Appendix C that wages are lognormal.[11] But the difficulties in the process of estimation make other simplifying assumptions necessary.

Estimation involves solving for the reservation wages of each worker at each evaluation of the likelihood function. However, it is computationally very time consuming to solve for each worker. The solution I have adopted is to restrict the heterogeneity of the sample and to build types of workers based on a few dichotomous variables.

In the results I present, there are three explanatory variables which are used both for predicting their effect on hazard rates and also for identifying better the parameters of the model. *Skill,* which is measured by the level of education: a skilled worker is one with education equal to or above secondary. *Age,* divided in three groups (less than 30 years old, *Age1830;* between 30 and 45 years old, *Age3045;* and more than 45 years old, *Age4565*). Finally, I use a variable which indicates if the individual has access to unemployment benefits or not, *Benefits.*[12]

In the estimation, a monthly discount rate of 0.3% (i.e. a 3.66% annual rate) was imposed and not estimated and T was set to 24 months in calculating the final condition for the reservation wage. Different discount rates have been used and

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long for some economies, what could create problems in the estimation of the offer arrival rate. However, I think this is not a problem for Spain where the duration in unemployment is long enough to imply monthly offer arrival rates lower than one.

[11] It is well-known that not all wage offer distribution function satisfies the *recoverability condition* which is crucial for identifying the model (See Flinn and Heckman, 1982). One function which satisfies it is the lognormal and this is the main reason for choosing it. Moreover, this function works well also in Wolpin (1987) and fits the empirical distribution of accepted wages (See Figure 3).

[12] This variable requires further comment. It indicates not only whether the unemployed worker actually receives unemployment benefits or not, but also whether he has received them. The basic idea behind this distinction is that workers who have accumulated and used their entitlement to unemployment benefits have a different behavior in their search process than those without those rights. However, the empirical motivation for this distinction is different: to correctly estimate the effect of benefits on a structural estimation, we would need to know the complete sequence of benefit receipt over the spell of unemployment of each worker, both for workers with or without a complete spell. This requirement is clearly far from being satisfied with the data used. So, we have to follow an intermediate solution which leads to obtain a not fully structural effect of unemployment benefits.

the estimations results change only marginally. With respect to the separation probability, instead of estimating it which could be difficult to identify given no data on employment spells, I have used a previously estimated value, obtained from García-Pérez (1997), for each group of workers in the estimation procedure. The mean separation probability for the estimation sample is 4.99% per month, being higher for unskilled and young workers.

Now, the way the structural model is estimated and some issues of identification are explained. After this, the results are presented and discussed.

3.1 The likelihood function

There are not many papers estimating structurally dynamic programming models of individual behavior. In the search context some references are Lancaster and Chesher (1983), Miller (1984) or Narendranathan and Nickell (1985). But one of the most influential articles in this area, which is the basis for the maximum likelihood estimation in this paper, is Wolpin (1987). This paper develops a discrete-time model of search which is non-stationary because of a finite horizon of search. It is estimated by maximum likelihood using data on duration, accepted wages and a few individual characteristics.

The estimation presented here is clearly inspired in Wolpin's technique but it contains a new element: unobserved heterogeneity. I will first explain the likelihood function without unobserved heterogeneity, and, afterwards, we will see the one which controls for its presence.

In the sample of unemployed workers there exist three types of individuals: those with complete spells and an observed re-employment wage, those with complete spells but without an observed re-employment wage and finally, those with censored spells. Thus, the likelihood function will have three different components (for the whole derivation of this likelihood function see Appendix C):[13]

$$\ln \pounds \quad = \quad \sum_{i=1}^{N} c_i y_{it} \left[v_i \ln \left(\Pr(T_i = t, W_{o_i}) \right) + (1 - v_i) \ln \left(\Pr \left(T_i = t \right) \right) \right]$$

[13] v_i is an indicator variable which takes a value of 1 if the re-employment wage of worker i is observed and zero otherwise. c_i is an indicator of censoring: it takes a value of 1 if the individual i has a complete spell and zero otherwise. T_i represents worker i's unemployment spell duration and W_{o_i} is his observed re-employment wage. Finally, y_{it} is equal to one if the individual i has his last observation, d_i, at period t.

$$
\begin{aligned}
&+(1-c_i y_{it})\ln\left(\Pr(T_i > t)\right) \qquad (9)\\
= \; & \sum_{i=1}^{N} L_i = \sum_{i=1}^{N}\sum_{t=1}^{d_i} c_i y_{it}\left[v_i \ln\left(f_{W_o}(W_o|\, t)\phi_i(t)\right) + (1-v_i)\ln\left(\phi_i(t)\right)\right] \\
&+(1-c_i y_{it})\ln\left(1-\phi_i(t)\right)
\end{aligned}
$$

Given this likelihood function and taking into account the reservation wage, equation (4), we can estimate the parameters of the model, $\alpha(t)$, $b(t)$, δ, $\bar{W}$, σ_u and σ_ε provided they are all identified.

The general idea behind identification is the following: given data on accepted wages, along with data on unemployment duration, the parameters of the wage offer distribution, $\bar{W}, \sigma_u$ and σ_ε are clearly identified in the first component of the likelihood function. Further, given that, for some workers the acceptance probability is equal to one due to the effect of the separation probability, I can identify the offer arrival rate in both the second and the third components of the likelihood function. Finally, the separation probability, δ, and the value of time for unemployed workers, $b(t)$, are identified making use of the system of reservation wages from 0 to T. However, the distinction between these two parameters can be quite poor without data on previous employment spells' durations or data on the value of unemployment time. Therefore, we will estimate the model imposing some previously estimated values for the separation probability.

3.1.1 The likelihood function with unobserved heterogeneity

The requirement of restricting the heterogeneity in the sample implies that a lot of sample heterogeneity is not captured by the explanatory variables used. This problem together with the fact that unobserved heterogeneity generates spurious negative duration dependence in the estimation, motivates the consideration and estimation of unobserved heterogeneity in the hazard rate.

Although unobserved heterogeneity can not be controlled by a fixed effect approach, because we do not have multiple spells, I can apply a random effect technique as it has been used, for example, in Flinn and Heckman (1982). In order not to restrict more the estimation procedure, we are going to estimate nonparametrically the distribution of unobserved heterogeneity, with a technique inspired in that of Heckman and Singer (1984).

As we want to separate the effect of unemployment duration from that of unobserved heterogeneity, this element is introduced in the two parameters where duration dependence is allowed: the offer arrival rate and the value of unemployment time. It is the same unobserved heterogeneity distribution but with a possibly different effect on both parameters. Note that the effect of this heterogeneity is the same for each individual, who has so many likelihood contributions as the length of his unemployment spell. Hence, having this unobserved heterogeneity term in two different parameters facilitates its identification.

With unobserved heterogeneity, η, the log-likelihood function takes the form:

$$\ln \pounds_h = \sum_{i=1}^{N} \ln \int L_i(\eta) dF(\eta) \tag{10}$$

where $F(\eta)$ is the cumulative distribution function of η, which is a discrete function with two mass points, η_1 and η_2.[14] These mass points are selected in order to verify the assumption of $E(\eta) = 0$ which is necessary given the presence of a constant term in the two parameters where unobserved heterogeneity is introduced. Besides, it is estimated the probability p for the variable η to be equal to its value η_1.

The function $L_i(\eta)$ is the likelihood function described in the previous subsection, where its arguments are all functions of the unobserved heterogeneity variable, η.

The addition of unobserved heterogeneity as a two mass point distribution function adds a new dichotomous variable to the estimation procedure. Hence, we have twenty-four types of workers (twelve in the case without unobserved heterogeneity), so I have to compute reservation wages these times for each evaluation of the likelihood function.

To understand to what extent we can jointly identify the effect of unobserved heterogeneity and duration dependence in the unemployment hazard rate, I have carried out some simulations. I have generated forty random samples of 500 workers with some binary variables as those present in our data set[15] and I have

[14] I have used just two mass points in the distribution of the unobserved heterogeneity. It is known that increasing the number of mass points could be a way of improving the control for unobserved heterogeneity. Moreover, a promising avenue for improving our control for unobserved heterogeneity in structural models will be to study how the estimation results change as the number of mass points increases. This exercise could be complementary to that of Baker and Melino (2000) but it is left for future research.

[15] These variables are included, respectively, in the offer arrival rate, the mean of offered wages and in the value of unemployment time.

applied to them the estimation procedure previously described. For those exercises which deal with the presence of unobserved heterogeneity in the offer arrival rate and the value of unemployment time, one of the generated binary variables in these parameters is dropped and assumed to be the unobserved heterogeneity.

The first conclusion emerging from these simulations, see Table 2, is that almost all parameters are well identified given typical significance levels.

With respect to the introduction of unobserved heterogeneity, I have obtained that the estimation procedure identifies its presence and its differential effect with respect to duration dependence. That is, when the offer arrival rate is parameterized as $1 - \exp(-\exp(-1 - 0.14t))$ for half the population and $1 - \exp(-\exp(-0.6 - 0.14t))$ for the other half, the estimation procedure makes the duration dependence coefficient to be, on average, -0.144 with a mean standard error of 0.023. The same happens for the time dependence of the value of unemployment time: its true value is -0.1 and its mean estimated value is -0.129 with a mean standard error of 0.048. The probability of the unobserved heterogeneity distribution, which is equal to 0.5 when generating the data, is estimated to be, on average, 0.568 with a mean standard error of 0.259. However, the results with respect to its level and its differential effect over the value of unemployment time are poor: the mean estimated values are close to the real ones but the standard errors of both parameters are quite high.

Hence, we conclude from these simulation results that the structure of the model, basically the different regressors we have in each parameter of the model, helps to identify the model. This is also what is obtained in Elbers and Ridder (1982) but for a proportional hazard model. In order to differentiate duration dependence from the effect of unobserved heterogeneity we need other regressors in the parameter where they are included. This is our identifying strategy in the following estimation.

The selected functional forms for the parameters of the model are shown in Table 3. The offer arrival rate, $\alpha(t)$, is parameterized using the extreme value distribution function. The idea is to use a proportional assumption for the underlying continuous offer arrival rate. It is well known, see Meyer (1990), that in discrete time, a continuous proportional hazard rate follows this distribution. I want to use a proportional form for this arrival rate in order to identify separately the effect of unobserved heterogeneity from that of the duration of unemployment (see Elbers and Ridder's (1982)). The other parameters are assumed to be exponential because of the assumption of lognormal wages, in order to reduce their scale or to restrict them to be non-negative.

With respect to the parameterization of the offer arrival rate, I distinguish between people with and without access to unemployment benefits. It may be argued that the access to unemployment benefits should not only make the value of time for the unemployed worker to be different, but also his search effort, reflected in the offer arrival rate, although not modelled here, should differ from that of workers without benefits. We have to remember that this variable is not unemployment benefits. It is just an indicator of wether the worker has access to them or not. Estimates without this indicator in the offer arrival rate are much poorer in terms of likelihood values and significance of the rest of parameters.

3.2 Results

The main results of the structural estimation can be seen in Tables 4 and 5. Table 4 shows the estimated coefficients of the model both when unobserved heterogeneity is and is not controlled for. Table 5 reports the predicted values of the main elements in the model estimated, for the sample mean values of the regressors and both for skilled and unskilled workers and for workers with and without access to unemployment benefits. It also presents the main predictions for the two estimated groups with respect to unobserved heterogeneity.

The first result shown in Table 4 is that the presence of unobserved heterogeneity in the data cannot be rejected (the likelihood ratio of a test of no unobserved heterogeneity has a value of 11.451 with a p-value lower than 0.005). Hence, there is unobserved heterogeneity in the data but we confirm from this table that its control does not affect to the duration dependence of the two parameters where it is considered. As the model which controls for unobserved heterogeneity is more general, we discuss thereafter its results.

Duration dependence is estimated in the offer arrival rate, $\alpha(t)$, and in the value of time for unemployed workers, $b(t)$. We can observe that there is a strong negative duration dependence in both parameters: a 2.69% monthly decrease in $b(t)$ and a 13.87% mean monthly decrease in $\alpha(t)$ which is a much higher rate than the 2.5% found in Wolpin (1987) for the offer arrival rate with US data. Furthermore, both parameters are highly significant despite unobserved heterogeneity being controlled for.

With respect to the skill variable, we can see that it is marginally significant in both the offer arrival rate and in mean offered wages. There are more offers for skilled unemployed workers, as in Van den Berg (1990), and the offered wages are quite higher for these workers. The same result is obtained in Wolpin (1987).

The effect of having access to unemployment benefits is very strong. Not only is the value of time for the unemployed worker higher for those with such benefits, but also the offer arrival rate is much lower for these workers (see Table 4). These results might be revealing a lower search effort of this type of workers, reflected in a lower offer arrival rate but also in a higher valuation of time. What is important is that the known stylized fact of lower hazard rates for workers with unemployment benefits can be interpreted much better within this structural estimation.[16]

The estimated values of $\alpha(t)$ and $E(w)$ are quite reasonable: the offer arrival rate at the sample mean values of both the regressors and the effect of unobserved heterogeneity begins at 41.08% in the first month of unemployment and has a value of only 5.13% fourteen months later. This parameter is higher for skilled unemployed workers: 48.11% in the first month and 6.34% in the fourteenth. The estimated mean of offered wages, $E(w)$, at the sample mean of the regressors, is 117,423 pesetas, around 899 dollars per month (at the December 1996 exchange rate), which is only 7.75% lower than the mean monthly accepted wage in the sample (see Table 1). Finally, the value of the parameter $b(t)$ is estimated to be quite high, although it has a rapid decrease over the spell of unemployment. However, it is still more than 50% larger than both the mean offered wage and the reservation wage along the studied fourteen months of the unemployment spell. In Table 5 we have the effect of unobserved heterogeneity in these two parameters. The estimated distribution of unobserved heterogeneity reveals the existence of two groups (See Table 4): with 34.7% probability, the workers have both a lower offer arrival rate and a higher value of unemployment time. These two effects of unobserved heterogeneity make this group of workers to have larger reservation wages and hence, lower hazard rates. The other 65.3% of the sample has larger offer arrival rates and lower values of unemployment time for all unemployment

[16] We should not forget, however, that the estimation is not totally structural with respect to this variable.

durations. Hence, their reservation wages are lower and their hazard rate larger than for the first group of workers.

There exists a problem with the estimation of the variance of the offered wage and of the measurement error. In fact, real variation of offered wages is estimated to be too low: the estimated fraction of the wage variance accounted for by real variation of wages is of only 12.63%. Although measurement error is present in our data, due to the construction procedure, this unexpected result may be reflecting problems of identification which have also appeared in other structural estimations as, for example, Eckstein and Wolpin (1990). Nevertheless, the total variation of wages, $\sigma_u + \sigma_\varepsilon$, is estimated quite well: the coefficient of variation of accepted wages is 39.45% and the estimated coefficient of variation of observed wages is 34.91%.

Estimated reservation wages and hazard rates can be obtained given these estimated parameters. Reservation wages are decreasing with unemployment duration (See Table 5), as the theoretical model predicts, and higher for skilled unemployed workers (141,506 pesetas for skilled workers, *i.e.* 1,083 dollars, and 135,443 pesetas, 1,037 dollars, for the unskilled ones in the first month of unemployment). But the main characteristic of reservation wages is that they begin being quite high although their decreasing pattern is very important along the fourteen months analyzed: for sample mean values of the regressors, the reservation wage in the first month of unemployment is 15.74% higher than the mean of the distribution of wages but, after 14 months of unemployment, the reservation wage is 56.81% lower than it was in the first month. The effect of the separation probability on reservation wages is present and very important. In fact, the estimation results show that we are obtaining an initial positive effect of the separation probability over reservation wages. If we evaluate these results with a 10% higher separation probability, reservation wages are higher for the first two periods in unemployment and lower afterwards.

The estimated low reservation wages lead, as in Van den Berg (1990), to high *acceptance probabilities.* It seems that in Spain they are even larger: after 5 months, they are practically equal to one (see Figure 4 where we distinguish between those with and those without access to unemployment benefits). However, the acceptance probability begins at a low level, 7.62% at the beginning of the spell (opposed to a mean value of 77% in Van den Berg, 1990), basically because the offer arrival rates are quite high in these first months. But it grows rapidly, reaching the value of one in 5 months for the larger group with respect to unobserved heterogeneity and in 8 months for the other group of workers.

The final result of this estimation is the hazard rate. This rate is the product of the offer arrival rate and the acceptance probability. As shown in Figure 5, the hazard rate increases until 4 months and then it decreases, becoming equal to the offer arrival rate, as the acceptance probability approaches one. The initial increase in the hazard is due to the large increase in the acceptance probabilities.

The hazard rates for workers with and without access to unemployment benefits are, as shown in Figure 5, very different. The known stylized fact estimated in some reduced-form estimations, see García-Perez (1997) or Bover *et al.* (1997), is obtained also here: a worker without unemployment benefits has higher probabilities of exiting unemployment in all of the fourteen months of the spell which are studied here. But with the structural estimation carried out, we can interpret this result and conclude that in the early stage of the spell, the main element at work is the acceptance probability, which is much larger for those without unemployment benefits, but, once this probability is estimated to be equal to one, the difference between the two groups of workers remains because the offer arrival rate is still quite higher for those without benefits.

Figure 6 shows the effect of unobserved heterogeneity on the hazard rates. As stated above, the estimation identifies two groups of workers: one with higher offer arrival rates and lower values for $b(t)$, and thus with much higher hazard rates and another group with much lower hazard rates because they have smaller offer arrival rates and higher values for $b(t)$.

If we compute the predicted mean unemployment durations, those with access to unemployment benefits have a larger mean and a much higher probability of becoming a long-term unemployed, that is, surviving as unemployed for more than 12 months: their predicted mean is 4.14 months and their predicted survival probability at 12 months is 25.93% whereas these figures are, respectively, 2.96 and 7.34% for those workers without access to such benefits. I have simulated the effect of a different duration of the reception of unemployment benefits over these outcomes of the model. Lengthening unemployment benefits entitlement from 3 to 12 months makes the expected mean duration of unemployment to be 9.17% larger and almost doubles the probability of being long-term unemployed.

Nevertheless, we have to take with precaution these predictions, given that we have not used the different durations of unemployment benefit reception in the sample to estimate the model.

To conclude, the estimation of the search model shows that Spanish unemployed workers do not differ so much from unemployed workers elsewhere: their acceptance probabilities are very high except for the first four to six months of unemployment (see Wolpin, 1987 for US data or Van den Berg, 1990 for the Netherlands). Thus, the main mechanism at play in the process of exit from unemployment is the arrival of offers from employers. The offer arrival rate, in spite of its initial high value, is very low for workers who are unemployed for more than 12 months, the so-called long-term unemployed. Thus, this group of unemployed workers, among with the unskilled ones, has serious problems in order to leave unemployment in Spain.

4 Conclusions

This paper presents a non-stationary job search model where jobs do not last for ever. When the unemployed worker is looking for a new job, he takes into consideration that once employed he can be unemployed again. This future risk makes him, normally, reduces his reservation wages because, if he loses his job in the future, he will be unemployed again. However, given the nonstationarity of the process, at the beginning of the unemployment spell, the worker can be in a situation quite good with respect to his expectations for the future. Therefore, we can also observe higher reservation wages in the first steps of the unemployment spell when the separation rate is larger.

I have implemented a structural estimation of this search model for the Spanish economy using data which are observed in discrete intervals of time. Furthermore, the estimation procedure controls for the presence of unobserved heterogeneity by using the Heckman and Singer (1984) mixture technique.

One of the basic results of the estimation of the search model is that the re-employment probability, the hazard rate, is increasing up to the fourth month of the unemployment spell, but then it becomes clearly decreasing. This result remains even when we control for the presence of unobserved heterogeneity. The interpretation of this result is that in the first months of unemployment, the main element at work is the rapid increase of the acceptance probability, given the highly decreasing pattern of reservation wages. But as soon as these first months pass, the only element present in the hazard rate is the offer arrival rate, because acceptance probabilities are, in fact, equal to one.

As to other results, we obtain that there are some differences between skilled and unskilled unemployed workers: the offer arrival rate and the mean of the distribution of offered wages are higher for the former. Furthermore, the worker who receives or has received unemployment benefits has a much lower probability of exiting unemployment. The reason differs between the early stages of unemployment and the latter ones: at first, the reason is that those with unemployment benefits have higher reservation wages and thus, lower acceptance probabilities. From the fourth month of unemployment onwards, the only difference is in the offer arrival rates, which are much higher for those without unemployment benefits, possibly because they have a higher search effort than those without such benefits.

Finally, we can assert that the long-term unemployed, those who are unemployed for more than a year, have very small probabilities of exiting unemployment: this is estimated to be around 5% per month for the fourteenth month in unemployment. This result is consistent with the fact that more than 50% of Spanish unemployed workers are long-term unemployed. Given that the acceptance probability is estimated to be equal to one in this stage of unemployment, we can conclude that long-term unemployed workers do receive almost no offers once they spend more than a year in unemployment.

Appendix A: Proof of Theorem 1

This proof consists of: firstly, proving the following Lemma which, basically, requires that $\Delta w_R^0(t) < 0$ for (i) and (ii) to hold, and, secondly, proving that $\Delta w_R^0(t) < 0$.

Lemma 5 *If assumptions (A1)-(A7) are satisfied and if, for every* $t \in [0, T)$, *we have that* $\Delta w_R^0(t) < 0$, *then:*

(i) $w_R(t) < w_R^0(t)$,

(ii) $\Delta w_R(t) < 0$.

Proof: Suppose that at some $t \in [0, T)$ $w_R(t) \geq w_R^0(t)$ holds. Then, because of the relationship between $w_R(t)$ and $w_R^0(t)$ we will have that $\Delta w_R(t) > 0$. However, given that $w_R(t)$ and $w_R^0(t)$ are continuous functions and, by the Lemma's assumptions, $\Delta w_R^0(t) < 0$, it cannot be true that $w_R(T) = w_R^0(T)$, which must be verified at time T given the assumptions of the model. Thus, the opposite must hold: $w_R(t) < w_R^0(t)$ and implied by this, that $\Delta w_R(t) < 0$. $Q.E.D.$

Now we have to prove that $w_R^0(t)$ is a decreasing function of t under all the assumptions (K1)-(K4). The proofs under each of them are quite similar so we will show only the proof under (K1), i.e. for $b(t)$:

Given (7) we will have that:

$$w_R^0(t) - w_R^0(t+1) - \frac{\alpha(t)}{\delta + r}\Big(G(w_R^0(t), t) - G(w_R^0(t+1), t)\Big) = b(t) - b(t+1)$$

where $G(w_R^0(t), t) = \int_{w_R^0(t)}^{\infty} \left(w - w_R^0(t)\right) dF(w, t)$.

If $b(t)$ is decreasing in t, the right-hand side of this expression will be positive and since the function $w_R^0(t) - \frac{\alpha(t)}{\rho} G(w_R^0(t), t)$ is a increasing function of $w_R^0(t)$, we will have that $w_R^0(t) > w_R^0(t+1)$, that is, $w_R^0(t)$ is decreasing in the time the worker is unemployed.

Appendix B

To obtain monthly wages, the labor income and the unemployment benefits declared in the correspondent quarter have been compared. If there are no unemployment benefits, the monthly wage is the declared labor income divided by three. If there are unemployment benefits, their amount is compared with the labor income: if the benefits are bigger than 80% of the labor income (70% for periods posterior to 1992:2), then the monthly wage is the total amount declared as labor income. On the contrary, the monthly wage is the labor income divided by two. This rule is based on the characteristics of the unemployment benefits system in Spain, which lowered the replacement rate from 80% of the previous wage to 70% in the second quarter of 1992.

Calculation of monthly duration data is more difficult. The numbers of months of unemployment in the spell can be computed once it is established how many months of unemployment there are in the first quarter of unemployment and, if the worker exits unemployment, how many months he has been employed in the first quarter of employment. The general rule applied to compute the number of unemployment months in these two quarters is based on comparing the labor income of each quarter, if it is positive, with the unemployment benefits received that quarter or with the labor income of the following quarter. If there is no labor income in the first quarter the individual answers he is unemployed, it is considered that he is unemployed during all the quarter. If the reported labor income is low enough a duration of two months is imputed in the correspondent quarter but if this income is sufficiently large, it is considered that the worker has been only one month in unemployment in that quarter.

Appendix C

The likelihood function in equation (9) is based on the relationship between the hazard rate and the distribution function of a random variable. In a sample of unemployed workers, those with censored spells or completed spells but without an observed re-employment wage will have a likelihood contribution which is only a function of the unemployment hazard rate $\phi_i(t)$ for the t periods of unemployment:

$$\Pr(T_i = t) = \phi_i(t) \prod_{j=0}^{t-1} (1 - \phi_i(j)) \tag{C1}$$

$$\Pr(T_i > t) = \prod_{j=0}^{t} (1 - \phi_i(j)) \tag{C2}$$

For those with completed spells and an observed re-employment wage the likelihood contribution is the following:

$$\Pr(T_i = t, W_{o_i}) = \Pr(T_i = t, W_{o_i} \mid T_i \geq t) \prod_{j=0}^{t-1} (1 - \phi_i(j)) \tag{C3}$$

where, in period t, this contribution is the joint probability of T_i being equal to t and of observing the wage W_{o_i}.

Here, an assumption about the wage offer distribution is needed. Like in other papers (Van den Berg, 1990 or Wolpin, 1987) it is assumed that wages have a lognormal distribution. In addition to this, as in Wolpin (1987) and justified by the construction of the wage data, the re-employment wages are assumed to be measured with error. Thus, the observed re-employment wage has the following expression:

$$\ln W_{o_i} = \ln \bar{W}_i + u_i + \varepsilon_i \tag{C4}$$

where u_i is normal with zero mean and variance σ_u^2, and ε_i, the measurement error, follows a normal distribution with zero mean and variance σ_ε^2. I assume that ε_i is distributed independently of u_i .

The joint distribution of W_o and $T \mid T \geq t$ is given by the following equations:

$$\Pr(T = t, W_o \mid T \geq t) = f_{W_o}(W_o \mid T = t) \Pr(T = t \mid T \geq t) = f_{W_o}(W_o \mid t) \times \phi(t) \tag{C5}$$

Note that the distribution of W_o conditional on t is the truncated distribution of the observed wages, with the reservation wage at $t + 1$ being the truncation point, thus, $f_{W_o}(W_o \mid t) = f_{W_o}(W_o \mid W \geq W_R(t + 1))$. For an expression for this density, see Wolpin (1987).

Finally, we can express the likelihood function in logarithms and in the usual way of expressing likelihood functions for discrete data (see Jenkins, 1995). This is exactly the second equation in (9).

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Table 1
Distribution of unemployment duration and other variables in the sample

	Completed Spells			Censored Spells	
	Number	Percentage	Mean Accepted Wage	Number	Percentage
Months					
0-1	88	10.13		95	13.61
1-2	109	12.54		87	12.46
2-3	157	18.07		110	15.76
3-4	162	18.64		58	8.31
4-5	105	12.08		37	5.30
5-6	59	6.79		57	8.17
6-7	39	4.49		18	2.58
7-8	50	5.75		9	1.29
8-9	27	3.11		31	4.44
9-10	21	2.42		24	3.44
10-11	21	2.42		13	1.86
11-12	12	1.38		32	4.58
12-13	11	1.27		17	2.44
13-14	5	0.58		16	2.29
14-15	3	0.35		94	13.47
Age1830	136	15.65	125,788	84	12.03
Age3045	418	48.10	131,546	269	38.54
Skill	63	7.25	153,888	67	9.60
With benefits	517	59.49	126,442	460	65.90
TOTAL	869		127,294	698	

Note : Mean accepted wages are in 1996 Spanish pesetas (exchange rate: 130.6 pesetas/dollar).

Table 2

Identification with the estimation procedure: some simulations results

Without unobserved heterogeneity														
Coef.	a_0	a_1	a_2	a_3	e_0	e_1	v_0	b_0	b_1	b_2	b_3	r_0	γ	p	η_1
True	-1	-0.14	0.5	0.4	11.5	0.1	-3.4	12.5	-0.1	0.5	-0.2	1	-	-	-
Estim.	-1.06	-0.137	0.55	0.42	11.48	0.10	-3.39	12.40	-0.12	0.69	-0.19	0.98			
St. Er.	0.16	0.02	0.14	0.14	0.024	0.03	0.23	0.215	0.04	0.23	0.15	0.32			
With unobserved heterogeneity															
True	-0.8	-0.14	0.5	-	11.5	0.1	-3.4	12.4	-0.1	0.5	-	1	-0.5	0.5	-0.2
Estim.	-0.70	-0.143	0.58		11.46	0.09	-3.20	12.26	-0.13	0.80		1.39	-0.28	0.57	-0.40
St. Er.	0.25	0.02	0.26		0.03	0.03	0.25	0.32	0.05	0.35		0.46	1.40	0.26	0.36

Notes : The parameters take the following form:

$$\alpha(t) = 1 - \exp(-\exp(a_0 + a_1 \times t + a_2 \times var2 + a_3 \times var1)),$$
$$E(w) = e^{e_0 + e_2 \times var2},\ b(t) = e^{b_0 + b_1 \times t + b_2 \times var3 + b_3 \times var1},$$
$$Var(w) = e^{v_0},\ \rho^2 = \frac{Var(w)}{Var(w) + Var(\varepsilon)} = \frac{e^{r_0}}{1 + e^{r_0}}.$$

The distribution of the unobserved heterogeneity term is a discrete one with two mass points. The probability of $\eta = \eta_1$ is p. In the estimation with unobserved heterogeneity, the effect of $var1$ is dropped and taken as the one of unobserved heterogeneity. The differential effect of this on $b(t)$ is measured by the parameter γ.

Table 3

Functional forms of the estimated parameters

Job offers arrival rate: $\alpha(t,\eta) = 1 - \exp(-\exp(\beta_1 + \beta_2 \times dur + \beta_3 \times skill + \beta_4 \times benefits + \eta))$
Distribution of wages: $W_o = \bar{W} e^u e^\varepsilon$ *with* $u \sim N(0, \sigma_u^2)$ $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ $\bar{W} = \exp(\beta_5 + \beta_6 \times skill + \beta_7 \times age1830 + \beta_8 \times age3045)$ $\sigma_u^2 = \exp(\beta_9)$ $\sigma_\varepsilon^2 = \exp(\beta_{10})$
Value of time for the unemployed worker: $b(t) = \exp(\beta_{11} + \beta_{12} \times dur + \beta_{13} \times benefits + \beta_{14} \times \eta)$

Table 4
Main results of the structural estimation

	without Unob. Het.		with Unob. Het.	
Parameter	Coef.	t-ratio	Coef.	t-ratio
$\boldsymbol{\alpha}_i(\mathbf{t}, \boldsymbol{\eta})$				
Constant	-0.754	-6.014	-0.442	-2.784
Duration	-0.139	-8.513	-0.165	-6.742
Skill	0.152	0.839	0.237	1.554
Benefits	-0.539	-4.340	-0.296	-1.934
$\bar{\mathbf{W}}_i$				
Constant	11.659	458.986	11.652	480.729
Skill	0.035	1.226	0.058	1.617
Age18-29	-0.014	-0.789	-0.001	-0.052
Age30-45	0.032	1.747	0.039	2.020
$\boldsymbol{\sigma}^2_{\mathbf{u}}$				
Constant	-5.996	-5.907	-6.302	-6.806
$\boldsymbol{\sigma}^2_{\varepsilon}$				
Constant	-2.178	-32.471	-2.179	-33.112
$\mathbf{b}_i(\mathbf{t})$				
Constant	12.064	85.451	12.083	83.947
Duration	-0.028	-2.582	-0.027	-3.085
Benefits	0.246	2.093	0.267	2.228
Unobs. Het.			-0.723	-1.690
Unobs. Het.				
η_1			-0.304	-1.525
p			0.347	5.361
Log-likelihood	-7,992.82		-7,987.09	
No. of observ.	8,520		8,520	

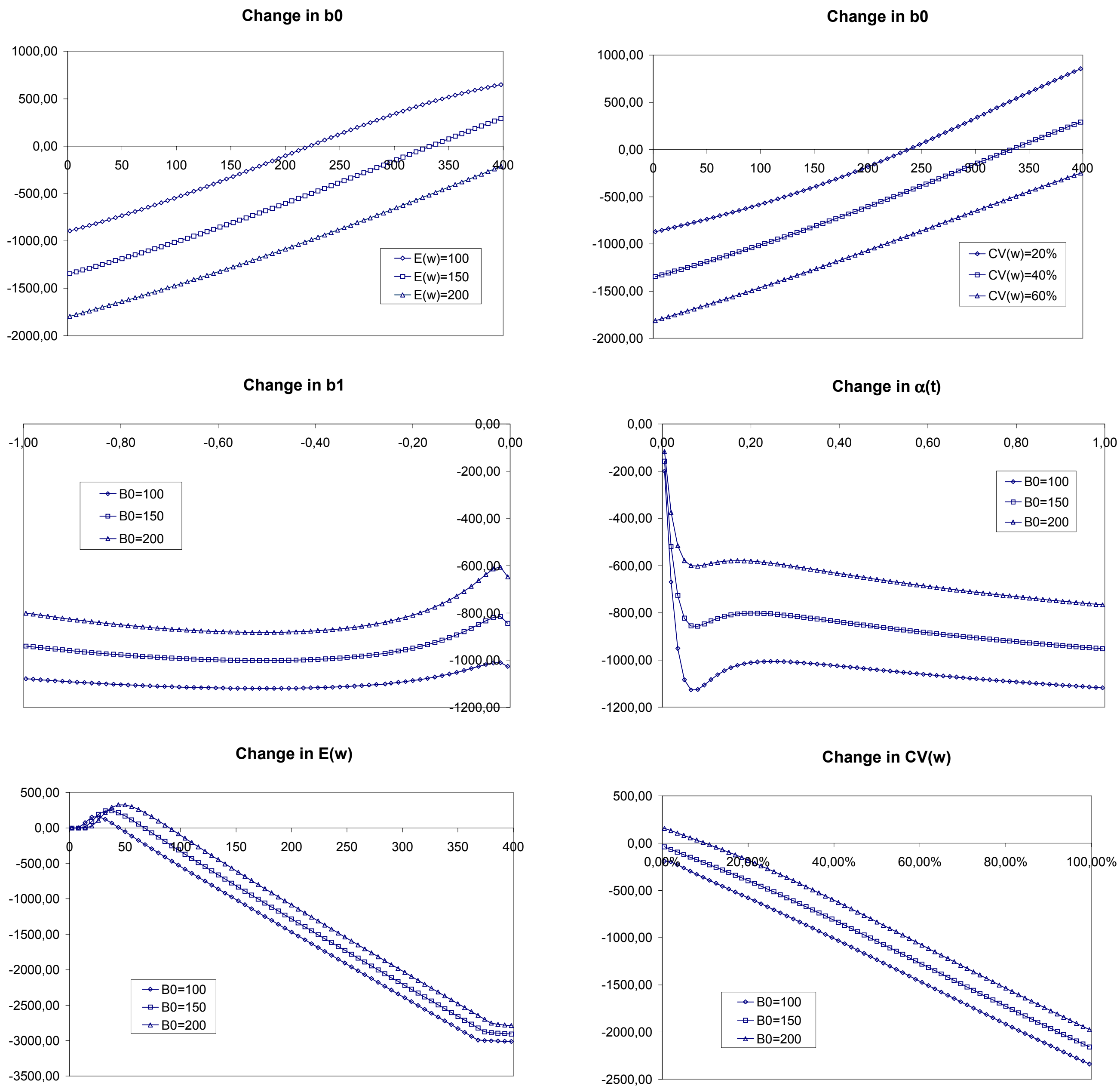
Table 5

Predicted values for the main elements of the model

t	$\alpha(\cdot)$	$\phi(\cdot)$	$\bar{F}(w_R(\cdot))$	$w_R(\cdot)$	$b(\cdot)$
Mean values for all variables:					
0	41.08	3.13	7.62	135,902	219,667
4	23.99	18.12	75.53	117,809	196,943
14	5.13	5.13	100.00	58,691	149,892
For the group with η_1 :					
0	32.00	0.00	0.00	151,919	270,079
4	18.04	1.66	9.20	129,070	242,140
14	3.79	3.79	100.00	84,351	184,291
For the group with η_1 :					
0	45.90	4.79	10.44	127,390	192,878
4	27.16	26.86	98.89	111,825	172,925
14	5.87	5.87	100.00	45,054	131,612
With access to Unempl. Benefits:					
0	38.94	1.27	3.26	140,489	234,107
4	22.57	16.11	71.38	121,249	209,889
14	4.79	4.79	100.00	65,929	159,745
Without access to Unempl. Benefits:					
0	48.36	11.89	24.59	125,874	179,213
4	29.06	28.48	98.00	111,019	160,674
14	6.38	6.38	100.00	38,772	122,288
Skilled workers:					
0	48.11	3.25	6.75	141,506	219,667
4	28.88	21.50	74.45	124,879	196,943
14	6.34	6.34	100.00	62,535	149,892
Unskilled workers:					
0	40.49	3.11	7.68	135,443	219,667
4	23.60	17.80	75.42	117,237	196,943
14	5.04	5.04	100.00	58,504	149,892

Notes : $\bar{F}(w_R(\cdot)) = 1 - F(w_R(\cdot))$. The first three columns are percentages and the other two are expressed in 1996 pesetas. The predictions are carried out using the model with unobserved heterogeneity.

Figure 1: Effect of the separation rate on reservation wages at period 0



Note: The baseline parameters in each graph are $b(t) = b0 * exp(b1 * t) = 100 * exp(-0.03t)$, $\alpha(t) = 0.3 * exp(-0.05t)$, $E(w) = 150$, $CV(w) = 40\%$, $\delta = 2\%$.

Figure 2: Kaplan-Meier estimates of the hazards

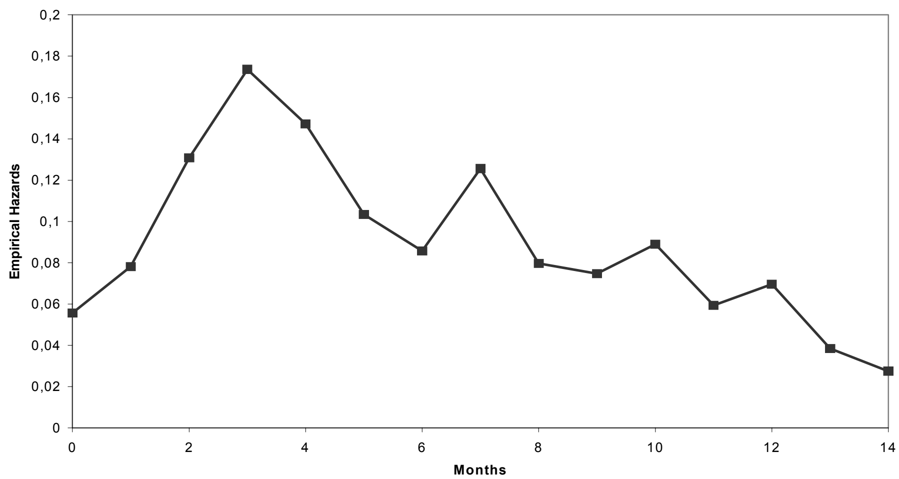


Figure 3: Histogram of the reemployment wages

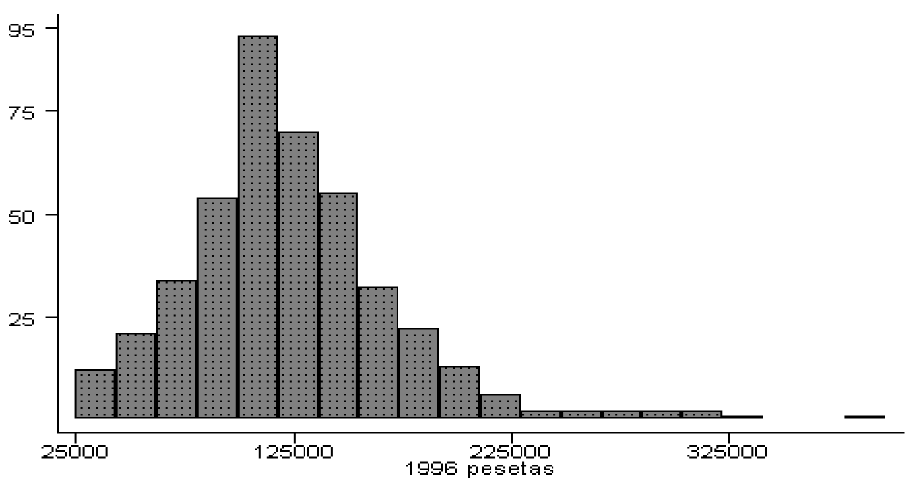


Figure 4: Estimated acceptance probabilities

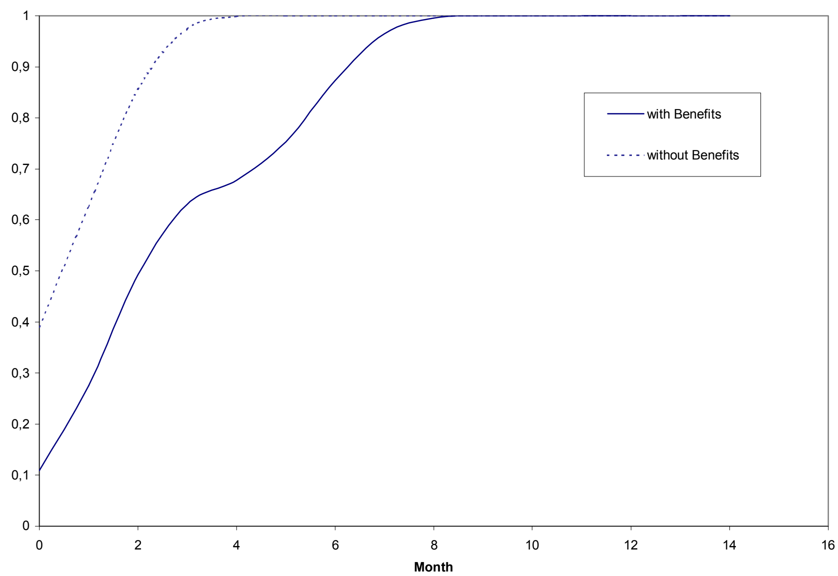


Figure 5: Estimated hazard rates: the effect of unemployment benefits

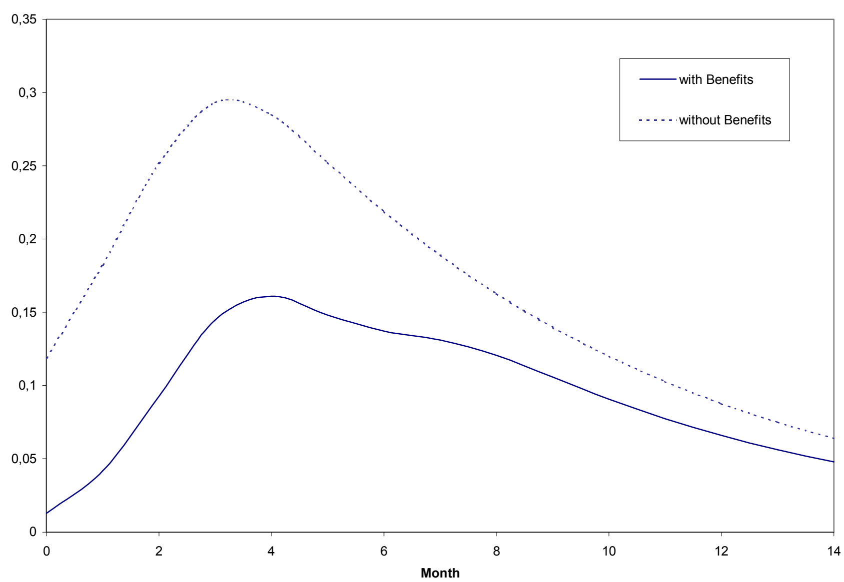


Figure 6: Estimated hazard rates: the effect of unobserved heterogeneity