The Daily Market for Funds in Europe: Has Something Changed with the EMU?*

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Abstract

This paper presents evidence that the existence of deposit and lending facilities combined with an averaging provision for the reserve requirement are powerful tools to stabilize the overnight rate. We reach this conclusion by comparing the behavior of this rate in Germany before and after the start of Stage III of the EMU. The analysis of the German experience is useful because it allows us to isolate the effects on the overnight rate of these particular instruments of monetary policy. To show that this outcome is a general conclusion and not a particular result of the German market, we develop a theoretical model of reserve management which is able to reproduce our empirical findings.

Keywords: Overnight Rates; Reserve Demand; Martingale Hypothesis;

JEL codes: E44, E52;

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1 Introduction

The daily market for funds is the generic denomination for the market where financial institutions trade overnight unsecured loans of their deposits at the central bank. The interest rate set in this market (henceforth called, indistinctly, the overnight rate, or the daily rate) plays a key role for the conduct of monetary policy. This is because the operating procedures of central banks are designed to affect the supply and demand of reserves among financial institutions.

Which are the determinants of the overnight rate? Central banks try to control it by using the instruments in their hands, namely, open market operations, reserve requirements and standing facilities. Control means an attempt to keep the daily rate around an "official rate" which in some countries is a "target rate" and in others is just the rate of the open market operations. One consequence of this control is that daily rates closely follow the rates determined by central banks. However, since this control is not perfect, the spread between market rates and official rates is usually different than zero. This difference gives an indication of the part of the daily rate which is driven by market forces. Figure 1 presents an example of such a series. It shows the spread between the overnight rate and the rate of the main refinancing operations in Germany for the period covering from September 1, 1996 until May 23, 2000. The dashed vertical lines represent the end of reserve maintenance periods.

The most remarkable feature of this series is the outstanding differences in its behavior before and after January 1999. Before this date, the last days of the reserve maintenance periods were characterized by significant peaks in the spread, which disappeared once the EMU was in place. Additionally, although it is less evident from the graph, the volatility before the start of Stage III was larger than after the beginning of the EMU.

This paper deals with characterizing and explaining such a dramatic change on the behavior of the overnight rate in Germany. We believe that this discussion goes beyond analyzing a particular historical episode in a particular country. On the contrary, we argue that it helps us understand the role of fundamental forces determining the time series properties of this rate in any economy. What makes the German experience with the EMU a singular one is that it represents the closest we can get to a controlled experiment in Macroeconomics. We show that this experiment allows us to trace the effect on the daily rate of changes in the operating procedures of central banks. In particular, it provides us with a way of evaluating the likely impact that the beginning of Stage III has had on the behavior of this rate in the Euro-area.

We develop a model to reproduce our empirical findings. This means explaining not only the properties of the overnight rate in the pre- and post-EMU periods but also the sudden change in its behavior. The explanation is based on modeling the degree of substitutability of funds within the reserve maintenance period. It is commonly said that if banks are risk neutral and there are no market frictions, funds should be perfect substitutes among days of the same reserve maintenance period. This would imply that banks would arbitrage away any expected differences between the current and future cost of funds. In other words, overnight rates should follow a martingale.

The main contribution of the paper is to demonstrate that this conclusion is not true even in an environment where agents are risk neutral and there are no impediments to trade. We reach this outcome by formalizing the instrumentation of monetary policy and by showing how it has different effects on the opportunity cost of funds for different days in the same reserve maintenance period. The corollary of this result is that banks do not see funds on different days as perfect substitutes. In addition, it allows us to rationalize the changes in the behavior of the overnight rate by comparing the implementation of monetary policy before and after the EMU.

The line of the argument is developed as follows. Section 2 characterizes the time series properties of the spread between the daily rate and the rate of the main refinancing operations in Germany. We show that there is a structural break in this series associated with the EMU. In particular, before January 1999, we find a significant increase in both the conditional mean and variance of the daily rate at the end of the reserve maintenance period. This effect is lost after 1999. Section 3 discusses why we concentrate in the German case and provides possible explanations for our empirical findings. It turns out that existing theories of the determination of the overnight rate within the reserve maintenance period have difficulties in explaining this episode. Section 4 associates these results with the changes in the implementation of monetary policy observed in Germany since the beginning of 1999. We develop a model of competitive, risk-neutral banks which is able to reproduce the features we find in the data. In this sense, the model generates a process for the overnight rate with increased volatility and peaks in the mean at the end of the maintenance period. We show that these features of the daily rate crucially depend on the rates of the central bank's standing facilities. Finally, section 5 concludes.

2 The Empirical Analysis

2.1 Data and Descriptive Statistics

The sample consists of daily observations covering the period from September 1, 1996 until May 23rd, 2000. For the period before January 1999, we use the spread between the overnight rate determined in the German money market and the rate of the main refinancing operations of the Bundesbank. The sample starts on September 1, 1996. We have 581 observations for this period. After January 1, 1999, the series studied is the difference between the Eonia and the rate of the main refinancing operations of the ESCB. The Eonia is a volume weighted average of all overnight unsecured lending transactions initiated within the euro area by a particular panel of banks. The contributors to Eonia are the banks with the highest volume of business in the euro zone money markets.[1] This series is indistinguishable from the corresponding interest rate

[1] In particular, these are 47 banks from EMU countries; 4 banks from non-EMU European countries; and 6 large international banks from non-EU countries but with important euro

in the German money market for that period. We have 357 observations of this variable from January 1, 1999 to May 23rd, 2000. Our linked series, therefore, includes a total of 939 observations and describes the part of the overnight rate determined by market forces. It is plotted in Figure 1.

A lot of useful information can be found by looking at some descriptive statistics of this series. Before January 1999, the overnight rate tended to show a peak at the end of the reserve maintenance periods. These peaks do not appear after the start of Stage III of the EMU. Also, as it was said in the Introduction, the volatility of this series is different for each subsample. Two distinctive properties define these differences. First, the variance of the spread before January 1999 is 0.043 and it is only 0.035 after that date. Second, if we eliminate from the samples the last and the first days of the reserve maintenance periods, the variance drops to 0.008 in the first subsample and to 0.024 in the second subsample. This means that, before the start of Stage III, the variance associated with settlement days was responsible for explaining over 81% of the total volatility of the series. With the beginning of the EMU, this percentage has been reduced to 31%.

Another important piece of information can be found by computing the average spread for each day of the reserve maintenance period. These computations are shown in Figures 2 and 3 for the first and second subsample, respectively. The dotted lines represent two standard error bands. As shown in the figures, before the EMU there is a clear increase in the unconditional mean and variance of the series on the last 2 days of the period. However, after the EMU, both the unconditional mean and variance are distributed more uniformly within the period.

The next subsection specifies a univariate model to describe the time series properties of the series as well as to perform statistical tests on the changes in its behavior. Nevertheless, this model does not pretend to be a full characterization of the overnight rates. In order to do that we would need to include the supply of funds and to construct a general equilibrium model as in Bindseil [2]. The purpose of this estimation is just to show some empirical regularities found in the data.

2.2 The Econometric Model

Linear analysis is usually inappropriate in financial econometrics. An extensive literature has shown the non-linearities that characterize most of the financial variables. Our series is not an exception. Just by looking at Figure 1, we can see that the days close to the end of the maintenance period, depicted in the graph by vertical lines, usually present higher uncertainty. Most of the "atypical" observations (in a linear sense) are associated with these days. For example, if we construct a two standard error band around i_t, we can observe that most of the times in which this variable is out of that interval, happen during the last

zone operations. For more information on this rate, see the European Banking Federation's internet page for the Euribor at www.euribor.org.

two days of the maintenance periods. In particular, before January 1999, i_t is 26 times outside this band, 25 of which are on the last two days of the period. After the EMU, this ratio is just 22 to 13.

In addition, an important deviation from the linear model is related to conditional heteroskedasticity problems. We encompass all these facts by proposing the following econometric specification:

$$i_t = i_{t-1} + \beta' X_t + h_t \epsilon_t \tag{1}$$

with

$$\begin{aligned} \ln(h_t) \quad &= \quad \lambda' \mathbf{V}_t + \sum_{j=1}^{4} \left[\delta_{j1} \left(\ln(h_{t-j}) - \lambda' \mathbf{V}_{t-j} \right) + \delta_{j2} \frac{\epsilon_{t-j}}{\sqrt{h_{t-j}}} \right. \\ & \left. + \delta_{j3} \left(\frac{|\epsilon_{t-j}|}{\sqrt{h_{t-j}}} - \sqrt{\frac{2}{\pi}} \right) \right] \end{aligned} \tag{2}$$

where i_t is the overnight rate, X_t collects a set of explanatory dummies that potentially could affect the mean while and V_t includes the ones that affect the variance.[2] The functional form of the variance comes from the specification proposed by Nelson [9] and used in Hamilton [8]. This specification captures an EGARCH type of persistence but allowing for different effects of positive and negative shocks. It also takes out the effect of changing the unconditional variance from the transmission of the conditional variance in day $t - j$ to day t [represented by the term $\ln(h_{t-j}) - \lambda' \mathbf{V}_{t-j}$] For the distribution of the error term, we use the mixture of normals proposed in Hamilton [8], because the fat tails and the excess of kurtosis made inappropriate the use of the normal distribution. Therefore, the density function of ϵ_t is:

$$f(\epsilon_t) = p \left(2\pi\sigma_1^2 \right)^{-1/2} \exp\left(\frac{-\epsilon_t^2}{2\sigma_1^2} \right) + (1-p) \left(2\pi\sigma_2^2 \right)^{-1/2} \exp\left(\frac{-\epsilon_t^2}{2\sigma_2^2} \right). \tag{3}$$

As in Hamilton [8], we use the normalization, $\sigma_1^2 = 1$ which implies that $E(\epsilon_t^2)$ instead of being 1 is $E(\epsilon_t^2) = p\sigma_1^2 + (1-p)\sigma_2^2$.

After an extensive search, we conclude that the sets X_t and V_t are composed of the variables included in Tables 1 and 2.[3]

[2] In the econometric specification we use the overnight rate instead of the spread. This is because we construct a model in first differences and, therefore, the level of the rate is not an issue. The use of the spread would generate an unnecessary noise in the estimation since markets participants usually discount changes in the official rates at the beginning of the reserve maintenance period while they usually occur within the period.

[3] Hamilton [8] considers a more sophisticated model for the first day of the new reserve maintenance period. In contrast, our specification is simpler because our sample includes a small number of observations for these days due to the longer reserve maintenance period.

Table 1
Variables in the Mean Equation (set X_t)

Variable	Meaning
X_{1t}	t occurs before January 1st, 1999
X_{2t}	t occurs after January 1st, 1999
X_{3t}	t occurs before January 1st, 1999 and is one of the last 2 days of the reserve maintenance period
X_{4t}	t occurs after January 1st, 1999 and is one of the last 4 days of the reserve maintenance period
X_{5t}	t occurs before January 1st, 1999 and is the first day of the reserve maintenance period
X_{6t}	t occurs after January 1st, 1999 and is the first day of the reserve maintenance period

Table 2
Variables in the Variance Equation (set V_t)

Variable	Meaning
V_{1t}	t occurs before January 1st, 1999
V_{2t}	t occurs after January 1st, 1999
V_{3t}	t occurs before January 1st, 1999 and is one of the last 3 days or the first day of the reserve maintenance period
V_{4t}	t occurs after January 1st, 1999 and is one of the last 4 days or the first day of the reserve maintenance period

2.3 Results

Tables 3 and 4 show the coefficient estimates.

Table 3
Estimations in the Mean Equation

Parameter	Estimate	Std. deviation
β_1	-0.003	0.001
β_2	0.000	0.002
β_3	0.371	0.041
β_4	-0.019	0.008
β_5	-0.627	0.065
β_6	0.238	0.052

Table 4
Estimations in the Variance Equation

Parameter	Estimate	Std. deviation
λ_1	-9.830	0.228
λ_2	-8.740	0.259
λ_3	5.986	0.215
λ_4	4.337	0.202
δ_{11}	0.066	0.071
δ_{21}	0.292	0.065
δ_{31}	0.121	0.062
δ_{41}	-0.148	0.070
δ_{12}	0.013	0.014
δ_{13}	0.190	0.025
p	0.051	0.010
σ_1	1.000	-
σ_2	7.802	0.621

As Table 3 shows, the behavior of the overnight rate is completely different before and after January 1999. Before the EMU, the positive and significant value of β_3 implies that there is an increase of rates at the end of the maintenance period, with an associated negative variation at the beginning as measured by the parameter β_5, the increase in the rate on the first day of the reserve maintenance period. Interestingly, the total increase of the spread on the last two days of the maintenance period equals 74 basis points which is very close to the value for β_5. We cannot reject the null hypothesis that $2 \times \beta_3 = \beta_5$ (the p-value of the test is 0.84). This means that the first day of the period washed out any changes occurred around the previous settlement day. After the EMU, there is a significant decrease of rates the last four days of the maintenance period, ($\beta_4 < 0$) also compensated with a significant increase at the beginning of the first day of the next period. In this case, we accept the null hypothesis of $4 \times \beta_4 + 0.05 = \beta_6$ (with a p-value of 0.07). The "over-compensation" of the first day of the reserve maintenance period comes from the expectation of changes in the main refinancing operations rate, which has suffered during these 16 periods an increase of 75 basis points. These 75 basis points imply an average increase of around 5 basis points per maintenance period. Additionally, contrary to the findings for the US case (Hamilton [8]), there is no significant effect of variables such as holidays or end-of-the week. This is probably due to the longer maintenance period in Europe.

With respect to the variance, the positive values of λ_3 and λ_4 indicate that volatility tends to be larger at the end of the reserve maintenance period both before and after the EMU. However, we cannot reject the hypothesis that $\lambda_3 > \lambda_4$ which means that volatility has increased more before than after the EMU. The p-value of the hypothesis $\lambda_3 = \lambda_4$ is 0.002. On the other hand, the significant positive value of the difference between λ_2 and λ_1 implies that the volatility of the overnight rate for the rest of the days within the reserve maintenance period has been higher in the Third Stage of the EMU as compared with the one observed before 1999. The p-value of the hypothesis $\lambda_2 = \lambda_1$ is 0.01.

Finally, a technical point deserve special attention. We impose in the estimation the null that the overnight rate during the reserve maintenance period follows a martingale or, equivalently, that banks consider funds as perfect substitutes during the maintenance period. We pretend to describe in the next sections of the paper the reasons that explains deviations of this null hypothesis.

3 Discussion of our Empirical Results

In this section we explain why we concentrate in the German experience with the EMU and discuss possible explanations for our empirical findings. First of all, the EMU seems a natural experiment to analyze. At least in principle, it is possible to identify all the institutional changes that this historical episode generated in the money markets of the Euro area. One of the main modifications in these markets has to do with the conduct of monetary policy. In this respect, most of the countries that initially entered the monetary union changed the instruments of their central banks so much that it is difficult to use their experience to learn anything about the determination of the overnight rate. Germany is the country with the most similar institutional framework before and after the EMU.[4] This means that we have to find a justification for our empirical results by searching within a limited set of possible answers. It is in this sense that Germany provides an interesting case to study and, as it will be clearer in the next section, by doing this exercise we learn some important lessons that should be applicable to any country.

What has been different in German money markets since January 1999? The first thing that comes to our minds is the increase in the number of participants. With the EMU, potentially any bank in the Euro area can have access to the German market to obtain liquidity. Can this fact explain our observations on the daily rate? The only way more agents could have any effect on the behavior of the rate is if they were different as compared with the existing ones. In particular, the only way the price of that market could have a lower volatility is if the liquidity shocks of newcomers were very negatively correlated with the liquidity shocks of the banks already installed. However, it is surprising that this negative correlation of shocks only affects the last day of the maintenance period. Additionally, the main source of volatility in the Euro-area comes from the Treasury deposits.[5] The standard deviation of the daily changes of government deposits since the start of Stage III of the EMU is Euro 5,507 million, whereas it is Euro 963 million for banknotes and Euro 506 million for net foreign assets. Germany does not have any volatility coming from Treasury deposits because the Bundesbank had an agreement with the German Treasury to deposit the

[4] See Escrivá and Fagan [5] for a description of the operating procedures of central banks in the Euro area before the EMU.

[5] European Central Bank [7], p. 16.

Treasury balances at the end of the day in private banks. On the other hand, it is Italy, Spain, and to a lesser extent, France, Ireland and Portugal, the countries where the Treasury balances represent a source of volatility for the money markets. Only if the Treasury shocks for these countries were very negatively correlated with banknote shocks in Germany we could have some smoothing effect on the German market. To think that something like that could happen is pretty adventurous.

With respect to the supply of reserves, it may be possible that differences in the behavior of the Bundesbank and the ESCB could explain the observed pattern of the overnight rate. This would explain the data only if the Bundesbank had restricted the supply of reserves at the end of the maintenance periods and the ESCB had not done so. It has been recognized that the Bundesbank tended to be harder on the market on the last day of maintenance periods. The reason was that, by maintaining the market shorter they avoided the possibility of rates dropping to zero on those days. On the other hand, it has been also recognized that the ESCB has provided a lot of liquidity, especially during the first months after the beginning of Stage III. The fear in this case was that liquidity was not circulating efficiently while agents were learning the new system. However, several problems arise when using these interpretations at face value. On the one hand, it is not clear how these policies can explain the differences in volatility. On the other hand, it does not provide an explanation as of why financial institutions did not take advantage of this situation. If German banks knew that it was systematically harder to get reserves at the end of the reserve maintenance periods, why did they keep demanding reserves on those days?

This discussion brings us to the crux of the matter. One of the central findings of the previous section is that we can reject the martingale hypothesis for the overnight rate for both subsamples; that is, we can reject the hypothesis that

$$i_t = E\left[i_{t+1}|\Phi_t\right],$$

where Φ_t represents the information set at time t. The idea behind this hypothesis is that risk-neutral banks together with an averaging provision for the reserve requirement will make financial institutions arbitrage away any misalignment between the current rate and its expected future value. An implication of accepting this proposition is that banks should be looking at funds at different days as perfect substitutes within the same maintenance period. Another set of explanations for our findings, then, arises from analyzing the reasons the literature has given to rationalize observed deviations of the daily rate from the martingale behavior.

The two obvious candidates to justify lack of substitutability of funds are risk aversion and impediments to trade. So it is of no surprise that the papers covering this issue assume one of them or both. For example, Hamilton [8] develops a model in which risk aversion together with reserve accounting conventions, transaction costs and credit line limits can reproduce the observed decrease in the level of the Fed funds rate on Fridays in particular, and over the reserve maintenance period in general. In Bartolini et al. [1], risk aversion and transaction costs are responsible for explaining why the level of the overnight rate as well as holdings of reserves tend to increase on settlement day. Another possibility has been provided by Campbell [3]. He uses risk aversion, transaction costs and information problems among banks about the level of aggregate reserve demand to generate more volatility of the funds rate towards the end of the reserve maintenance period. Spindt and Hoffmeister [11] are able to explain this increase in volatility with a model where a market maker dealer adjusts bid and ask rates to maximize profits subject to satisfying a reserve requirement. In such a model, reserve accounting conventions also play a central role.

Although these are valid reasons to rationalize deviations from the martingale behavior, they will hardly account for the fact that these deviations were so different before and after January 1999. In general, they would mean that the EMU has had a significant effect on bank's attitudes toward risk, transaction costs or available information, conjectures which are difficult to sustain. Additionally, when we try to use these models, there is some feature of the data that is left unexplained. For example, Hamilton's model is not designed to explain peaks at the end of the reserve maintenance period and is silent as to its implications for the variance of the overnight rate, a feature shared with Bartolini et al. [1]. Campbell's analysis is local around the full information solution and the implications for the level of the rate depend on parameter values. Finally, Spindt and Hoffmeister's results depend on the degree of market power by dealers, which, presumably, has decreased after the unification.

Still, the fact that the martingale hypothesis is rejected is an indication that funds on different days within the same reserve maintenance period are not perfect substitutes. In this paper, instead of generating this lack of substitutability by assuming risk averse agents or impediments to trade, we do so by modeling the role of the operating procedures of central banks in the determination of the overnight rate. In particular, we will show that these instruments of monetary policy make the cost structure of agents demanding reserves in money markets to be non-linear. It is this non-linearity what makes risk neutral agents to behave as if they were risk averse and it is this type of behavior what reduces the substitutability of funds across days.

We then use this results to explain the properties of the overnight rate in Germany by analyzing the changes in the operating procedures of the ESCB as compared with the ones of the Bundesbank.[6] These changes are:

- With respect to reserve requirements, both the Bundesbank and the ESCB have imposed a reserve maintenance period of one month. This period covered a calendar month in Germany whereas after 1999 it usually starts on the 24th of one month and ends on the 23rd of the following month. Reserves were not remunerated in Germany. On the contrary, the ESCB remunerates required reserves at the average rate of its main refinancing operations.

[6] For more details about the operating procedures of the Bundesbank and the ESCB see Deustche Bundesbank [4] and European Central Bank [6].

- The conduct of open market operations has been almost identical. In both cases they are the main source of liquidity for the system. Although divided in several categories, main refinancing operations have been the most important of them in both periods. These operations were conducted weekly, under a fixed-rate system, and had similar maturities.

- Finally, both the Bundesbank and the ESCB have maintained a marginal lending facility (called Lombard loans in the case of Germany). Under normal circumstances, access to this facility was not limited by the central banks. This lending rate have provided an upper limit for the overnight rate. In this respect, the ESCB has introduced a marginal deposit facility that was not in place in Germany before 1999. Financial institutions use this facility to make overnight deposits with national central banks. The rate of this facility provides now a floor for the overnight rate.

Could any of these differences explain the observed changes in the behavior of the daily rate in Germany? First, the remuneration of required reserves does not seem to have had any effect. This remuneration is paid after settlement day and, therefore, acts as a constant in the management problem of banks not affecting their decisions on reserve holdings. Secondly, the change in the ending day of the reserve maintenance period, if something, should have increased volatility. This is because main payment activities to the Treasury in Italy take place on the 23rd of each month. This may increase the volatility of the German market if Italian banks use it to obtain additional liquidity. Finally, the only possibility left is to see whether the introduction of the deposit facility by the ESCB can rationalize our empirical findings. This is the topic of the next section.

4 A Theoretical Model of the Overnight Rate

4.1 Overview of the Model

We construct a model where the interest rates of the central bank's two standing facilities will play a crucial role in determining the behavior of the daily rate over the reserve maintenance period. The model consists of identical, risk-neutral banks that exchange reserves in a competitive fashion. These agents demand reserves because they have to satisfy a reserve requirement imposed by a central bank. Furthermore, funds can be transferred between banks at no cost and there are no credit limits on their borrowing activities. Finally, there are no problems of private information in this economy. All variables are publicly known.

Our model tries to explain the data from the demand side. We want to show that an active monetary policy is not necessary to reproduce the observed behavior of the interest rate. For this reason we assume that the central bank does not intervene to modify the total liquidity of the system. The overall supply of reserves only changes unexpectedly from autonomous sources described by an exogenous, aggregate shock to the level of reserves of each bank. This shock is modeled as an i.i.d., zero mean random variable whose realization is known after the market is closed every day.

The last piece of the model is the specification of the marginal facilities the central bank provides to financial institutions. First, assume that there is only a lending facility where commercial banks can get liquidity after the shock is realized. Of course, the interest of these loans should be above the one in the market. When banks determine the demand for reserves they will balance several costs and gains. On the one hand, banks weight *static*, intraday costs. For that, they compare the opportunity cost of holding one additional unit of reserves (measured by the lost daily rate) with the marginal gain of not borrowing from the central bank (i.e., the lending rate). On the other hand, there are also *dynamic* costs. They have to do with the probability of having too much reserves at the end of the reserve maintenance period. This probability increases as banks accumulate reserves.

In this setup, how would the demand schedule for reserves behave over the reserve maintenance period? In other words, for each interest rate, would be optimal to have a constant demand for reserves? With a constant demand for reserves the static costs are constant over time. However, the dynamic costs are larger as time passes. Banks anticipate this effect by decreasing their demand for reserves at the beginning of the period and increasing it towards the end of the period. This behavior puts upward pressure on the daily rate as we get closer to settlement day. At the same time, the market rate should get more volatile since possible histories for the state are more diverse as shocks keep hitting the system.

With this model, it is also possible to conclude that the introduction of a deposit facility should reduce the effects of time on both the level and the variability of the market rate. First, by remunerating excess reserves, the facility reduces the costs of having more reserves than what is required by the central bank. This stabilizes the demand for reserves throughout the reserve maintenance period.[7] Second, by reducing the interval where the market rate can fluctuate, it also decreases its volatility. The following subsections reproduce these ideas in a formal model.

4.2 The Setup

This section develops a model of the overnight rate. It builds on the reserve management problem of a price-taking, representative bank. Implicitly, it is assumed that there exists a continuum of identical banks with measure one, each solving the same problem described here. The only perturbations hitting the system are aggregate shocks. Thus, there are no idiosyncratic risks and all aggregate variables coincide with their individual counterparts.

[7] Throughout the paper, the term "excess reserves" is used in a broad sense to indicate all reserves that are not required. They include reserves deposited at the deposit facility of the central bank. It is, therefore, a more general concept than the one used in the banking industry which refers to idle reserves, that is, reserves that are neither required nor deposited in the deposit facility.

Assume the central bank requires financial institutions to maintain a total of reserves of R monetary units over a reserve maintenance period of T days. Denote by A_t the accumulated reserves at the beginning of day t by the representative bank. The initial wealth of this bank is divided into reserves voluntarily deposited at the central bank (M_t) and reserves loaned to other banks in the money market (B_t), that is,

$$A_t = M_t + B_t \tag{4}$$

with

$$M_t \geq 0.$$

Reserves are exchanged in the market at the interest rate i_t. Assume that after banks have gone to the market they receive an aggregate liquidity shock ϵ_t. This shock is i.i.d. with zero mean and probability distribution function $F(\epsilon)$. It takes the same value for all banks.

The representative bank ends up the day with a balance in the central bank of $M_t + \epsilon_t$. It is assumed that any financial institution has unrestricted access to the central bank's marginal lending facility at the interest rate i^l. This means that if the end of day's balance is negative, the bank has to borrow from the central bank the funds needed to set it back to zero. If the bank ends up with a positive balance, those reserves work towards satisfying the reserve requirement. Denote by R_t the increase in reserves accounted for the requirement,

$$R_t = \max\{0, M_t + \epsilon_t\}$$

and by L_t, the accumulated reserves accounted for the requirement up to time t,

$$L_t = \sum_{\tau=1}^{t} R_\tau.$$

The reserve requirement is fulfilled if

$$L_T \geq R. \tag{5}$$

Define by e_t the reserves needed in t to fulfill the reserve requirement for the whole maintenance period, that is,

$$e_t \equiv \max\{0, R - L_{t-1}\}.$$

Another way of writing (5) is

$$e_{T+1} = 0.$$

Once the reserve requirement is fulfilled, financial institutions can deposit excess reserves at the central bank. These deposits are remunerated at the interest rate

i^d. It is assumed that $i^l > i^d$. In this model, required reserves do not earn any interest.

The objective of this bank is to decide on a sequence for $\{M_t\}_{t=1}^T$ to maximize the expected profits derived from managing its reserves within the maintenance period, that is,

$$\max_{\{M_t\}_{t=1}^T} E_1 (A_{T+1})$$

with

$$A_{t+1} = (1 + i_t) A_t + \epsilon_t - c_t$$

for $t = 1, 2, \ldots, T$, where c_t represents the net costs the bank incurs in managing its reserves. This term includes the opportunity cost of holding reserves $(i_t M_t)$ but also comprises the interest paid on borrowing from the central bank net of the interest received from maintaining reserves there. The only information the bank needs to make its decision on M_t every day t, apart from interest rates, is its level of reserves A_t and its reserve deficiency e_t. The way to solve this problem is by backward induction. With this method, we first solve the problem at date T, and then work backwards towards the beginning of the maintenance period.

4.3 Solution of the Model

4.3.1 Problem at T

The problem of an individual bank is to maximize with respect to M_T

$$E_T (A_{T+1})$$

with

$$A_{T+1} = (1 + i_T) A_T + \epsilon_T - c_T,$$

given its initial accumulated reserves, A_T, its reserve deficiency, e_T, and the market's interest rate, i_T. The important point is to compute the variable c_T.

For any bank, total reserves at the end of the day are $M_T + \epsilon_T$. Given the reserves voluntarily accumulated at the beginning of the day (M_T) and given the reserve deficiency (e_T), the reserve requirement will be fulfilled depending on the value of the liquidity shock that day. For small shocks, that is, for shocks satisfying

$$\epsilon_T \leq e_T - M_T,$$

the reserve requirement will not be satisfied. This situation will imply a cost for the bank of

$$i^l (e_T - M_T - \epsilon_T) \quad \text{for all} \quad \epsilon_T \leq e_T - M_T.$$

On the other hand, for large shocks, that is, for shocks satisfying

$$\epsilon_T \geq e_T - M_T,$$

the requirement will be satisfied. Since excess reserves can be deposited at the deposit facility, the gain (negative cost) in this case is

$$i^d \left(M_T + \epsilon_T - e_T\right) \quad \text{for all} \quad \epsilon_T \geq e_T - M_T.$$

This makes the net cost of managing reserves equal to

$$\begin{aligned} c_T &= i_T M_T + i^l \left(e_T - M_T - \epsilon_T\right) I \left\{\epsilon_T \leq e_T - M_T\right\} \\ &\quad + i^d \left(e_T - M_T - \epsilon_T\right) I \left\{\epsilon_T \geq e_T - M_T\right\} \end{aligned}$$

where $I\{X\}$ is an indicator function taking value 1 if event X occurs.

It could also happen that $e_T = 0$. In this case, the bank has already satisfied the reserve requirement with the reserves accumulated up to time $T-1$. We denote this situation by saying that the bank is "locked-in". The only possible costs and gains are the ones associated with finishing the day with a negative or positive balance at the central bank. This makes the variable c_T equal to

$$c_T = i_T M_T - i^l \left(M_T + \epsilon_T\right) I \left\{\epsilon_T \leq -M_T\right\} - i^d \left(M_T + \epsilon_T\right) I \left\{\epsilon_T \geq -M_T\right\}.$$

The problem faced by this bank is summarized by the function

$$V\left(A_T, e_T, i_T\right) = \max_{M_T} E_T\left(A_{T+1}\right).$$

This expectation takes the value

$$\begin{aligned} E_T\left(A_{T+1}\right) &= \left(1 + i_T\right) A_T - i_T M_T - i^d \int_{e_T - M_T}^{\infty} \left(e_T - M_T - \epsilon_T\right) f\left(\epsilon_T\right) d\epsilon_T \\ &\quad - i^l \int_{-\infty}^{e_T - M_T} \left(e_T - M_T - \epsilon_T\right) f\left(\epsilon_T\right) d\epsilon_T. \end{aligned} \tag{6}$$

It is important to notice that the presence of the standing facilities makes this expression non-linear with respect to the choice variable M_T.

The first order condition for a maximum is

$$i_T = i^l F\left(e_T - M_T\right) + i^d \left[1 - F\left(e_T - M_T\right)\right]. \tag{7}$$

Using (4), the supply of funds in the market is

$$B_T = A_T - e_T + F^{-1}\left(\frac{i_T - i^d}{i^l - i^d}\right).$$

It is easy to show that the supply of funds is a positive function of the market rate (i_T), and the initial level of reserves (A_T), and a negative function of the reserve deficiency (e_T), the lending rate (i^l) and the deposit rate (i^d).

Given that the shock is aggregate, all banks are identical so individual decisions coincide with aggregate variables. In equilibrium it has to be the case that $B_T = 0$, and $M_T = A_T$. This means that the equilibrium interest rate if the economy is not locked-in ($e_T > 0$) is

$$i_T\left(e_T > 0\right) = i^d + \left(i^l - i^d\right) F\left(e_T - A_T\right). \tag{8}$$

This result is intuitive. The market interest rate differential with respect to the deposit rate is equal to its expected value if the market were to be open after the shock. That differential would be $i^l - i^d$ if the system as a whole does not have enough liquidity to satisfy the reserve requirement and needs to borrow from the central bank, and zero if there is excess liquidity in the system. The equilibrium interest rate if the system as a whole is locked-in ($e_T = 0$) is

$$i_T\left(e_T = 0\right) = i^d + \left(i^l - i^d\right) F\left(-A_T\right). \tag{9}$$

This expression has the same intuition as above. It is immediate to show that

$$i^d \leq i_T\left(e_T = 0\right) \leq i_T\left(e_T > 0\right) \leq i^l.$$

The value function is equal to

$$\begin{aligned} V\left(A_T, e_T, i_T\right) &= A_T - i^l \int_{-\infty}^{e_T - A_T} \left(e_T - A_T - \epsilon_T\right) f\left(\epsilon_T\right) d\epsilon_T \\ &\quad - i^d \int_{e_T - A_T}^{\infty} \left(e_T - A_T - \epsilon_T\right) f\left(\epsilon_T\right) d\epsilon_T. \end{aligned} \tag{10}$$

Although included for the sake of completeness, this expression no longer depends on i_T.

4.3.2 Problem at $T - 1$

The problem is to maximize with respect to M_{T-1}

$$E_{T-1}\left(A_{T+1}\right)$$

when the expectation is evaluated at the equilibrium level computed before. This problem can be expressed as

$$\begin{aligned} V\left(A_{T-1}, e_{T-1}, i_{T-1}\right) &= \max_{M_{T-1}} E_{T-1}\left[E_T\left(A_{T+1}\right)\right] \\ &= \max_{M_{T-1}} E_{T-1}\left[V\left(A_T, e_T, i_T\right)\right] \end{aligned} \tag{11}$$

with

$$A_T = \left(1 + i_{T-1}\right) A_{T-1} + \epsilon_{T-1} - c_{T-1} \tag{12}$$

and

$$\begin{aligned} e_T &= \max\{0, R - L_{T-1}\} = \max\{0, R - L_{T-2} - R_{T-1}\} \\ &= \max\{0, e_{T-1} - \max[0, M_{T-1} + \epsilon_{T-1}]\}. \end{aligned} \quad (13)$$

Substituting (10) in (11), the function to maximize is

$$\begin{aligned} E_{T-1}[V(A_T, e_T, i_T)] &= E_{T-1}(A_T) \\ &\quad -E_{T-1}\left[i^l \int_{-\infty}^{e_T - A_T} (e_T - A_T - \epsilon_T) f(\epsilon_T) d\epsilon_T \right. \\ &\quad \left. + i^d \int_{e_T - A_T}^{\infty} (e_T - A_T - \epsilon_T) f(\epsilon_T) d\epsilon_T \right]. \end{aligned}$$

When the agent changes M_{T-1} he will affect, in a non-linear fashion, the amount of reserves available to him on the following period, A_T. The agent values this change for two reasons. First, the bank likes to have more reserves on average. Second, the new level of reserves will affect the probabilities of going to the deposit or lending facility as well as the amounts deposited to or borrowed from the central bank. But, as it is explained in Pérez and Rodríguez [10] the derivative of the second term with respect to M_{T-1} is very close to zero. This means that we can approximate the solution to this problem as[8]

$$\arg\max E_{T-1}[V(A_T, e_T, i_T)] \simeq \arg\max E_{T-1}(A_T).$$

Evaluating the first order condition at the equilibrium produces an interest rate equal to

$$i_{T-1} = i^d + \left(i^l - i^d\right) F(-A_{T-1}) + \int_{-A_{T-1}}^{e_{T-1} - A_{T-1}} \left(i_T - i^d\right) f(\epsilon_{T-1}) d\epsilon_{T-1}$$

The intuition is the same as before. The equilibrium interest rate is an average of the possible interest rates that could happen should the market be open after the shock. If the system as a whole is not locked-in ($e_{T-1} > 0$), there are three possibilities, depending on the size of the shock:

- For shocks $\epsilon_{T-1} < -A_{T-1}$, all banks have to end up borrowing from the central bank. In that case, the interest differential would be $i^l - i^d$. This happens with probability $F(-A_{T-1})$.
- On the opposite side, for shocks satisfying $\epsilon_{T-1} > e_{T-1} - A_{T-1}$, banks accumulate so much reserves that all of them are locked-in. In such cases, the interest differential should be zero.

[8] In particular, Pérez and Rodríguez [10] shows that the error derived from the approximation belongs to the interval $\left[-\left(i^l\right)^2, \left(i^l\right)^2\right]$.

- Finally, for intermediate cases, $-A_{T-1} < \epsilon_{T-1} < e_{T-1} - A_{T-1}$, banks accumulate reserves but still have a reserve deficiency that needs to be resolved in day T. In this case, the equilibrium interest differential should be equal to the interest rate differential in T, $i_T - i^d$.

Of course, if we are already locked-in at $T-1$ ($e_{T-1} = 0$), there are two possibilities depending on whether we end-up the day with a negative or positive balance:

- For shocks $\epsilon_{T-1} < -A_{T-1}$, all banks have to end up borrowing from the central bank. In that case, the interest differential would be $i^l - i^d$. This happens with probability $F(-A_{T-1})$.
- On the opposite side, for shocks satisfying $\epsilon_{T-1} > -A_{T-1}$, banks end up with excess reserves that are deposited in the central bank. In such cases, the interest differential should be zero.

4.3.3 Problem at t

The problem is to maximize $E_t(A_{T+1})$ with respect to M_t, when the expectation is evaluated at the equilibrium level computed before. This problem can be expressed as

$$\begin{aligned} V(A_t, e_t, i_t) &= \max_{M_t} E_t\left[E_T(A_{T+1})\right] \\ &= \max_{M_t} E_t\left[V(A_{t+1}, e_{t+1}, i_{t+1})\right] \end{aligned} \tag{14}$$

with

$$A_{t+1} = (1+i_t)A_t + \epsilon_t - c_t \tag{15}$$

and

$$e_{t+1} = \max\{0, e_t - \max[0, M_t + \epsilon_t]\}. \tag{16}$$

Applying the same reasoning as before, the solution of this problem can be approximated by

$$\arg\max E_t\left[V(A_{t+1}, e_{t+1}, i_{t+1})\right] \simeq \arg\max E_t(A_{t+1}).$$

Evaluating the first order condition at the equilibrium produces an interest rate equal to

$$i_t = i^d + \left(i^l - i^d\right)F(-A_t) + \int_{-A_t}^{e_t - A_t} \left(i_{t+1} - i^d\right) f(\epsilon_t)\, d\epsilon_t. \tag{17}$$

4.4 The Martingale Hypothesis

Is it true that $i_t = E_t(i_{t+1})$ in this model? From the previous discussion, it is clear that the state of the system, especially whether the financial sector is locked-in or not, should be important to answer this question. For example, if the economy is not locked-in at t, (17) expresses the interest rate on that day as an average over three possible values whose weights depend upon the measures associated with the three sets

$$\Phi_{1t} \equiv \{\epsilon : \epsilon_t < -A_t\}, \tag{18}$$

$$\Phi_{2t} \equiv \{\epsilon : -A_t < \epsilon_t < e_t - A_t\}, \tag{19}$$

and

$$\Phi_{3t} \equiv \{\epsilon : \epsilon_t > e_t - A_t\}. \tag{20}$$

From this expression we see that the validation of the martingale hypothesis will depend upon two elements. The first one is the measure of the set Φ_{2t}, since it is conditional on this set that the martingale hypothesis holds. The other factor is how different the expected value of i_{t+1} is from i^l and i^d on the sets Φ_{1t} and Φ_{3t}, respectively. On average, as t approaches T, daily reserves A_t tend to vary little while the reserve deficiency e_T tends to decrease. This means that the set of possible values for the shock where the martingale hypothesis is validated shrinks as we get closer to the end of the reserve maintenance period. Also, it is interesting to note that as $i^d \to i^l$, $i_t \to E_t(i_{t+1}) \to i^l$.

On the other hand, it is not possible to give a general assessment about the size and sign of these deviations from the martingale behavior. We answer this question in two ways. First, we will evaluate its sign for the particular case of a reserve maintenance period of two days. Second, we will present a simulation where the behavior of the interest rate over longer periods can be computed.

4.4.1 A 2-day reserve maintenance period

In general, using the possible values of i_T for each possible shock at $T-1$, $E_{T-1}(i_T)$ equals

$$\begin{aligned} E_{T-1}(i_T) &= i^d + \\ &+ \left(i^l - i^d\right) \int_{-\infty}^{-A_{T-1}} F\left[e_{T-1} - \left(1+i^l\right)\left(A_{T-1}+\epsilon_{T-1}\right)\right] f\left(\epsilon_{T-1}\right) d\epsilon_{T-1} \\ &+ \left(i^l - i^d\right) \int_{-A_{T-1}}^{e_{T-1}-A_{T-1}} F\left[e_{T-1} - 2\left(A_{T-1}+\epsilon_{T-1}\right)\right] f\left(\epsilon_{T-1}\right) d\epsilon_{T-1} \\ &+ \left(i^l - i^d\right) \int_{e_{T-1}-A_{T-1}}^{\infty} F\left[i^d e_{T-1} - \left(1+i^d\right)\left(A_{T-1}+\epsilon_{T-1}\right)\right] f\left(\epsilon_{T-1}\right) d\epsilon_{T-1}. \end{aligned} \tag{21}$$

Assume the system starts the reserve maintenance period with reserves $A_1 = r \equiv R/2$, so the economy has enough liquidity to satisfy the reserve requirement. In this case, e_1 would be $2r > 0$. Specializing (**??**) for this case we will have

$$i_1(e_1 > 0) = i^d + \left(i^l - i^d\right) F(-r) + \left(i^l - i^d\right) \int_{-r}^{r} F[-2\epsilon_1] f(\epsilon_1) d\epsilon_1. \tag{22}$$

On the other hand, using (21) $E_1(i_2)$ equals

$$\begin{aligned} E_1(i_2) &= i^d + \left(i^l - i^d\right) \int_{-\infty}^{-r} F\left[2r - \left(1 + i^l\right)(r + \epsilon_1)\right] f(\epsilon_1) d\epsilon_1 \\ &\quad + \left(i^l - i^d\right) \int_{-r}^{r} F[-2\epsilon_1] f(\epsilon_1) d\epsilon_1 \\ &\quad + \left(i^l - i^d\right) \int_{r}^{\infty} F\left[i^d 2r - \left(1 + i^d\right)(r + \epsilon_1)\right] f(\epsilon_1) d\epsilon_1. \end{aligned} \tag{23}$$

Then, comparing this expression with (22), i_1 would be larger, smaller or equal than $E_1(i_2)$ depending on whether the term

$$\int_{-\infty}^{-r} f(\epsilon_1) d\epsilon_1$$

is larger, smaller or equal than the term

$$\int_{-\infty}^{-r} F\left[2r - \left(1 + i^l\right)(r + \epsilon_1)\right] f(\epsilon_1) d\epsilon_1 + \int_{r}^{\infty} F\left[i^d 2r - \left(1 + i^d\right)(r + \epsilon_1)\right] f(\epsilon_1) d\epsilon_1.$$

Assuming a symmetric density function, the last term becomes

$$\begin{aligned} &\int_{-\infty}^{-r} f(\epsilon_1) d\epsilon_1 - \int_{-\infty}^{-r} F\left[\left(1 + i^l\right)(r + \epsilon_1) - 2r\right] f(\epsilon_1) d\epsilon_1 \\ &+ \int_{-\infty}^{-r} F\left[i^d 2r + \left(1 + i^d\right)(\epsilon_1 - r)\right] f(\epsilon_1) d\epsilon_1. \end{aligned}$$

This means that i_1 would be larger, equal or smaller that $E_1(i_2)$ depending on whether the term

$$NT = \int_{-\infty}^{-r} F\left[\epsilon_1 - r + i^l(\epsilon_1 + r)\right] f(\epsilon_1) d\epsilon_1 \tag{24}$$

is larger, equal or smaller than the term

$$PT = \int_{-\infty}^{-r} F\left[\epsilon_1 - r + i^d(\epsilon_1 + r)\right] f(\epsilon_1)\, d\epsilon_1. \quad (25)$$

Since it is assumed that $i^l > i^d$ and we are integrating over values of ϵ_1 satisfying $\epsilon_1 < -r$, it turns out that $NT < PT$ and $i_1 < E_1(i_2)$.

4.4.2 A Numerical Example

In order to get a view for the behavior of the equilibrium interest rate in this model the following exercise is conducted. We define a grid for the shock with probabilities associated with each point. Given the distribution of the shock, the interest rates of the central bank, i^l and i^d, the total reserve requirement R, and the initial reserve holdings A_1, it is possible to compute the distribution for A_t and e_t, $t = 1, \ldots, T+1$. This implies a distribution for i_t, $t = 1, \ldots, T$. The only problem in working out this simulation is that the number of possibilities increases exponentially with t. This constraints the size of the grid we can use if we want to make calculations for a sizable length of the reserve maintenance period (RMP).

Table 5 includes the values of the parameters used in the simulation.

Table 5

Value of Parameters in Simulation

Parameter	Value	Meaning
T	11	Length of the RMP
R	1100	Total reserve requirement
A_1	100	Initial reserve holdings
i^l	0.05	Lending rate of central bank

We leave i^d as a free parameter and do all computations for different levels of this coefficient. It will take values between 0 and i^l.

Table 6 represents the distribution of the shock.

Table 6

Distribution of the Shock

Value	Probability
-50	0.2
0	0.6
50	0.2

With his distribution, a bank that starts with reserves A_1 and does nothing to change its reserve holdings will face a probability of borrowing from the central bank before reaching settlement day of 14 percent.

Figure 4 shows the change of the overnight rate with respect to the value on the first day for each day in the reserve maintenance period, that is,

$$\Delta_t = i_t - i_1; \quad t = 1, \ldots, T.$$

Computations are done for two cases only, when $i^d = 0$ and $i^d = 0.03$. The first thing we notice is its similarity with Figure 2. The rate is almost constant until the very last days of the period. Then, it spikes. For the case of no deposit facility ($i^d = 0$) the average interest rate on the last day of the period is about 32 basis points above what it was at the beginning of the period. With a deposit rate of 3 percent (which produces a band for the overnight rate of 2 percent, equal to the one in the euro area), this spike is about 13 basis points.

Figure 5 presents the standard deviation of the unconditional distribution for the interest rate. We see the familiar pattern of increase in volatility as the period progresses. From the figure it is also clear the negative effect that increases in the deposit rate has on the volatility of this process.

4.5 Intuition and Policy Implications

To interpret these results think of a bank on the first day of the reserve maintenance period. We ask the following question: What should the expected interest rate on a future day t be so as to make this bank willing to substitute a unit of reserves between those days? When banks decide their demand for reserves they have to weight the different costs and benefits of increasing their deposits at the central bank. The cost of not having enough reserves is the lending rate which is above the daily rate. The cost of having too much reserves is the possibility of being locked-in earlier in the period and to receive the deposit rate which is smaller than the overnight rate.

The likelihood of these two outcomes is measured by the probability of having a shock belonging to the sets Φ_{1t} and Φ_{3t}, respectively. Define these probabilities as

$$\pi_{jt} = prob\left\{\epsilon_t \in \Phi_{jt}\right\}, \quad j = 1, 2, 3.$$

Since the shock is i.i.d. with zero mean, each bank expects its level of reserves, A_{ht}, to be constant. This means it takes the probability of reaching the set Φ_{1t}, π_{1t}, to be constant throughout the reserve maintenance period. For the same reason, the bank expects the reserve deficiency, e_{ht}, to be decreasing over time. This implies that, as we get closer to T, π_{2t} should be decreasing while π_{3t} should be increasing. The key to understanding the properties of the interest rate is to look at the evolution of the probability of reaching the set Φ_{3t}. At the beginning of the period, π_{3t} is close to zero. On those days, then, it is more likely to have a high interest rate, i^l, than a low one, i^d. Banks compensate this expectation by asking for a low rate, i_t, on those days. This is why, on average, the overnight rate tends to decrease throughout the reserve maintenance period. However, as we get close to T, the probability of being locked-in increases a lot. In that case, the probability of ending up with a low rate is very large. Banks try to compensate this expectation by bidding up the daily rate and this is why the overnight rate increases at the end of the reserve maintenance period. It is easy to see that this effect is smaller the larger the deposit rate is.

This example shows that the spread between the lending and the deposit rate plays a crucial role in the determination of the statistical properties of the overnight rate. This is for several reasons. First, these two rates define an interval inside which the daily rate has to fluctuate. Thus, they limit the volatility of this series for the *whole* maintenance period. Second, the rates of the two standing facilities affect the costs of being locked-in and of borrowing from the central bank. We show that the relative importance of these costs changes as we move toward settlement day. In this sense, the deposit and lending rate also affect the relative variation in the overnight rate *within* the maintenance period.

This model can be used to make sense of the empirical findings that motivated the paper. As our numerical example shows, the reduction in the opportunity costs associated with the creation of a deposit facility in the ESCB may be behind the differences in the level and volatility of the overnight rate experienced after the start of Stage III of the EMU. To have a complete explanation, though, we have to address one more issue. It has to do with the observation that the Eonia tends to go down at the end of the maintenance period. As shown in Figure 4, we would have predicted a small upward peak. This observation can be reconcile with our model by noticing that the ESCB has been introducing an excess of liquidity in the system, as it was pointed out above. An indication of this statement is the fact that the use of the deposit facility has been much larger than the use of the lending facility.[9] In our model, this means that the central bank is shifting the distribution function of the shock so that the probability of being locked-in increases. Thus, it is more likely that rates will drop at the end of the reserve maintenance period.

We believe these results have important implications for policy. Central banks could achieve a stable profile for the overnight rate in two ways. On the one hand, they could *actively* try to reduce the volatility of that rate by intervening in the market at the end of each maintenance period. Alternatively, they could *passively* obtain that goal by setting a window for the daily rate with the two standing facilities. The resources needed to follow the first option seem much larger than what is required to design the second one. This discussion suggests that the introduction of two standing facilities appears as a preferable system to stabilize the overnight rate.

5 Conclusions

This paper presents evidence about the time series properties of the overnight rate in Germany. It shows that the process governing this rate has become closer to a martingale after the start of the EMU. Additionally, the paper also documents a reduction in its volatility after January 1999, which is a result of the smaller variance on the last days of the reserve maintenance period.

[9] The use of the deposit facility, from February 24th 1999 to May 23rd 2000, has been almost twice as much as the use of the lending facility. We exclude the first maintenance period after the EMU because the use of the lending facility was atypically high (around 12 times the average of the rest of the periods) probably due to the accommodation to the new system.

We develop a model of reserve management by banks that reproduces our empirical findings. An important theoretical implication is that, with an averaging provision for the reserve requirement, banks do not necessarily see funds on different days of the same reserve maintenance period as perfect substitutes even if they expect rates to be constant in the future. In fact, this type of behavior implies a process for the interest rate that tends to be higher on average as we approach settlement day. At the same time, the accumulation of shocks makes the volatility of the overnight rate to increase over time too. The paper also shows that these deviations from the martingale hypothesis are reduced as the spread between the central bank's lending and deposit rate decreases. We obtain these results neither by invoking market frictions nor by imposing noncompetitive behavior. It is just a consequence of paying particular attention in modeling the opportunity costs faced by banks and how these costs change as we move along the reserve maintenance period.

Summarizing, it seems that it is the institutional framework of the new system what is producing a smoother patter for the market rate in Germany. In particular, we can trace the origin of this change to the introduction of a deposit facility by the ESCB that it was not in place before the EMU.

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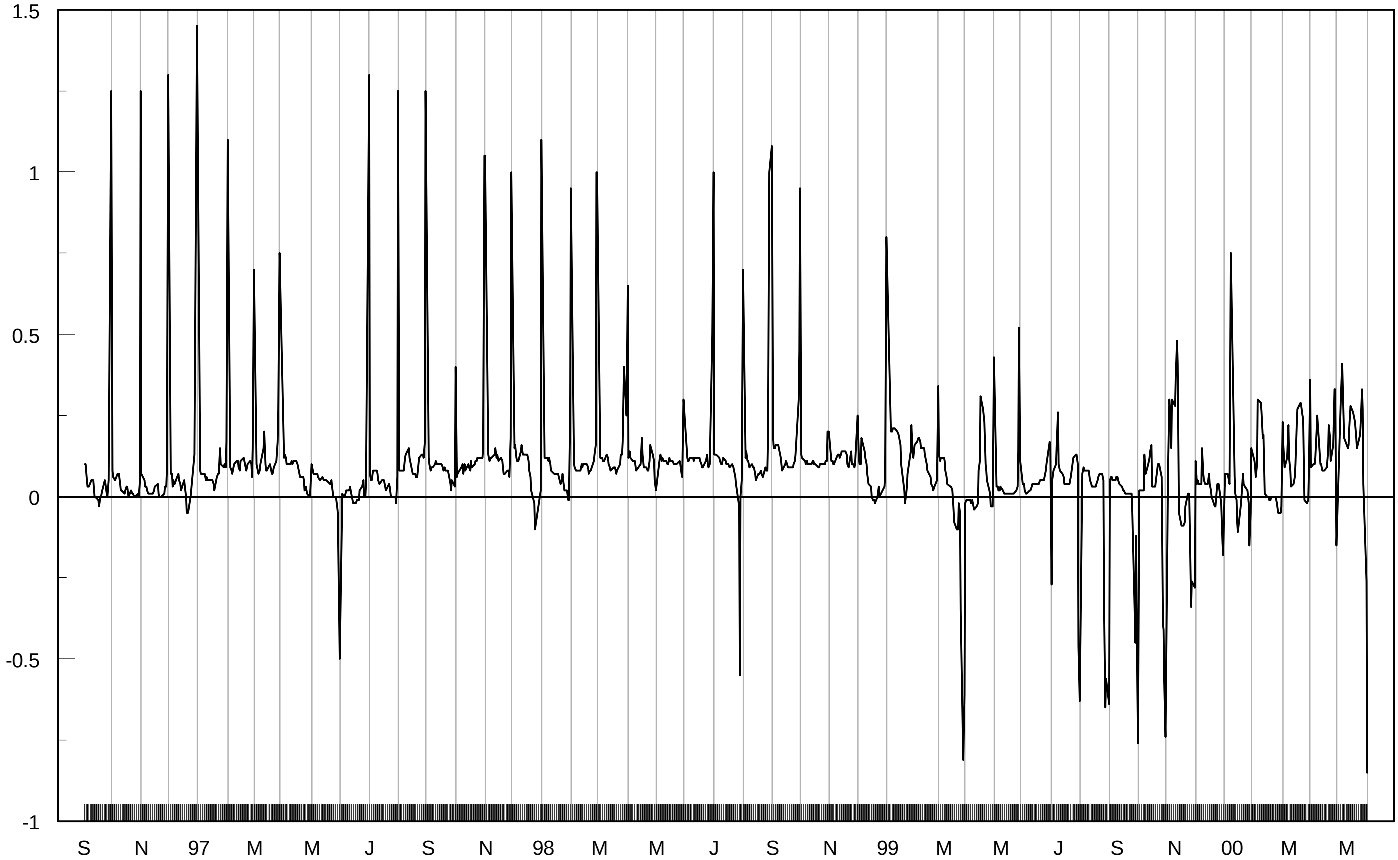
FIGURE 1

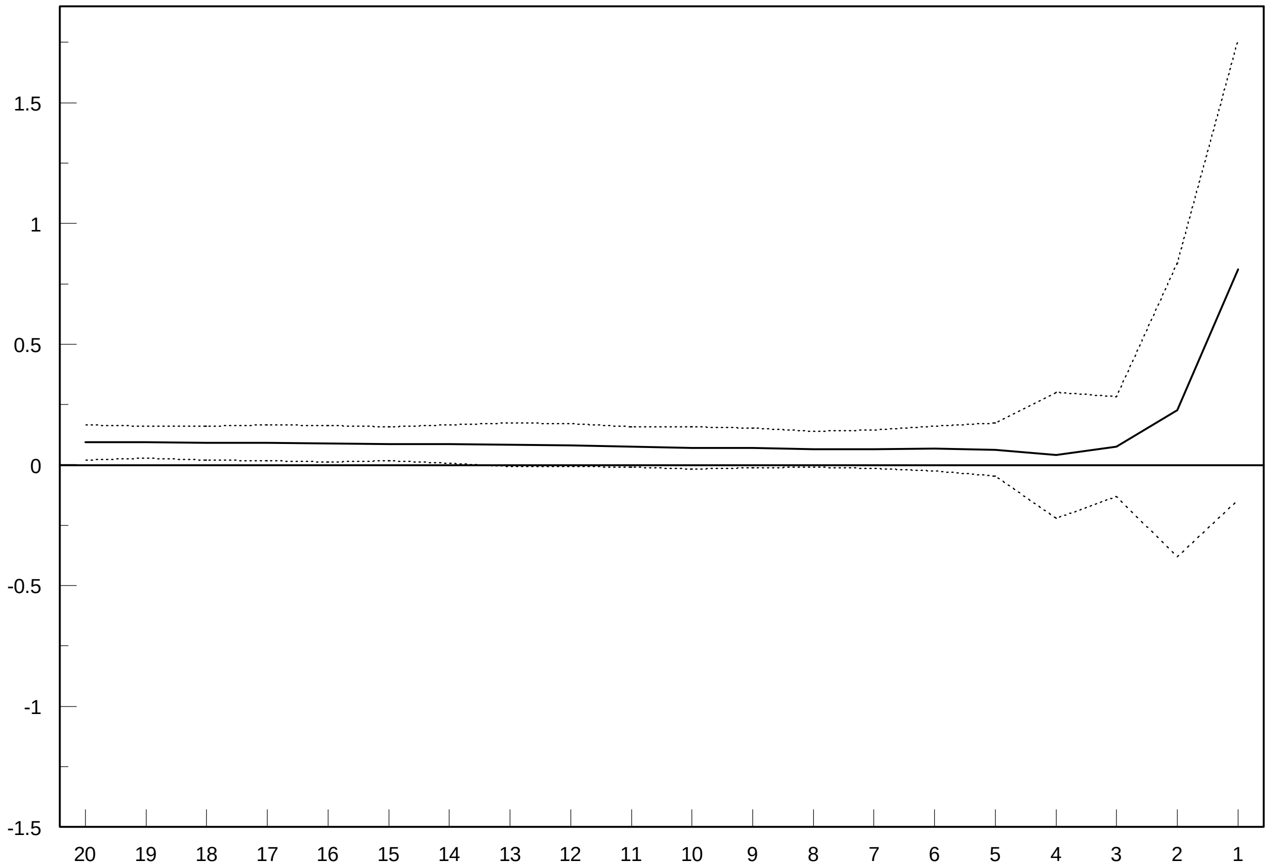
Overnight rate - MRO rate (Sep. 1st 1996: May 23rd 2000)



Note: The solid line plots the spread between the overnight rate and the main refinancing operations rate. The vertical lines represent end of the reserve maintenance periods.

FIGURE 2

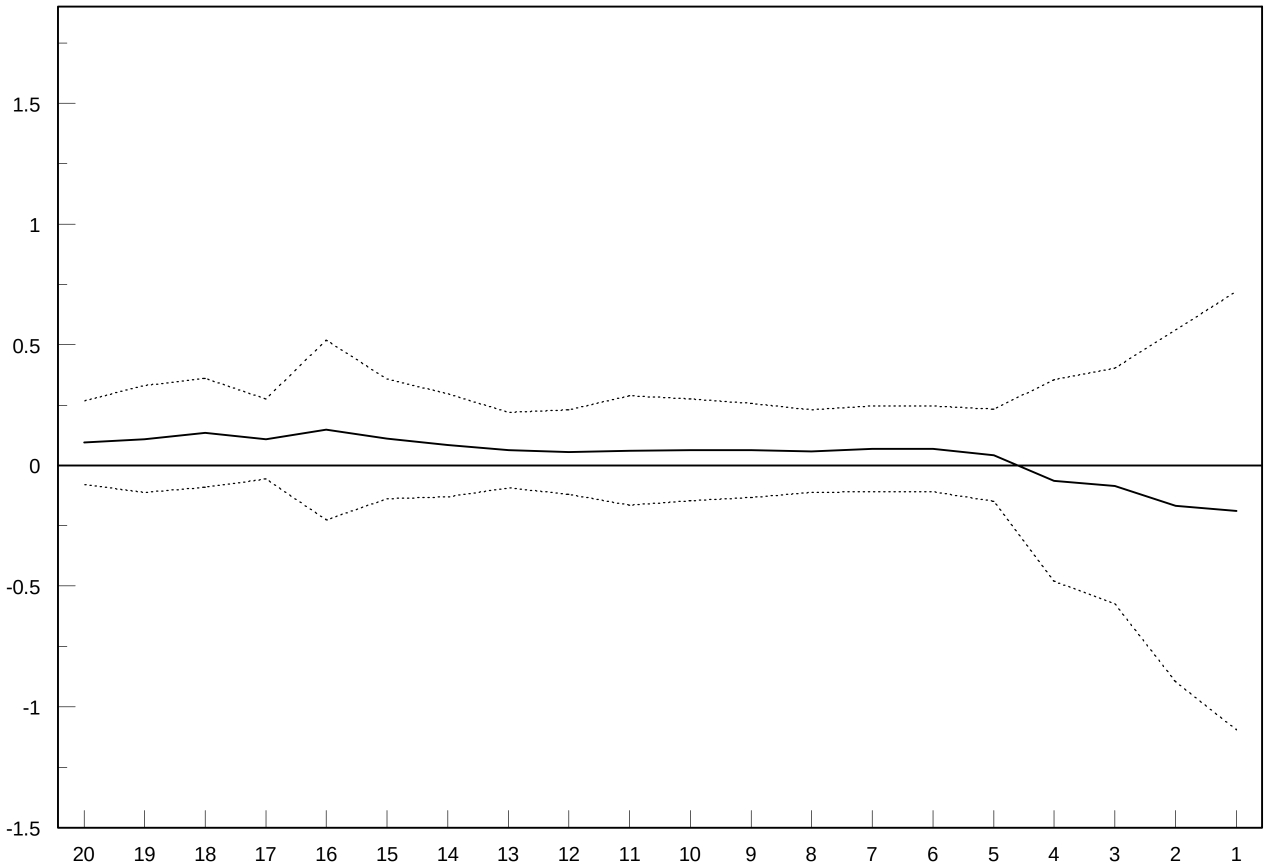
Average spreads in Germany: Before EMU



Note: The horizontal axis represents the number of days before the end of the maintenance period. The solid line plots the average rate in that day.The dotted line represents the two standard error band.

FIGURE 3

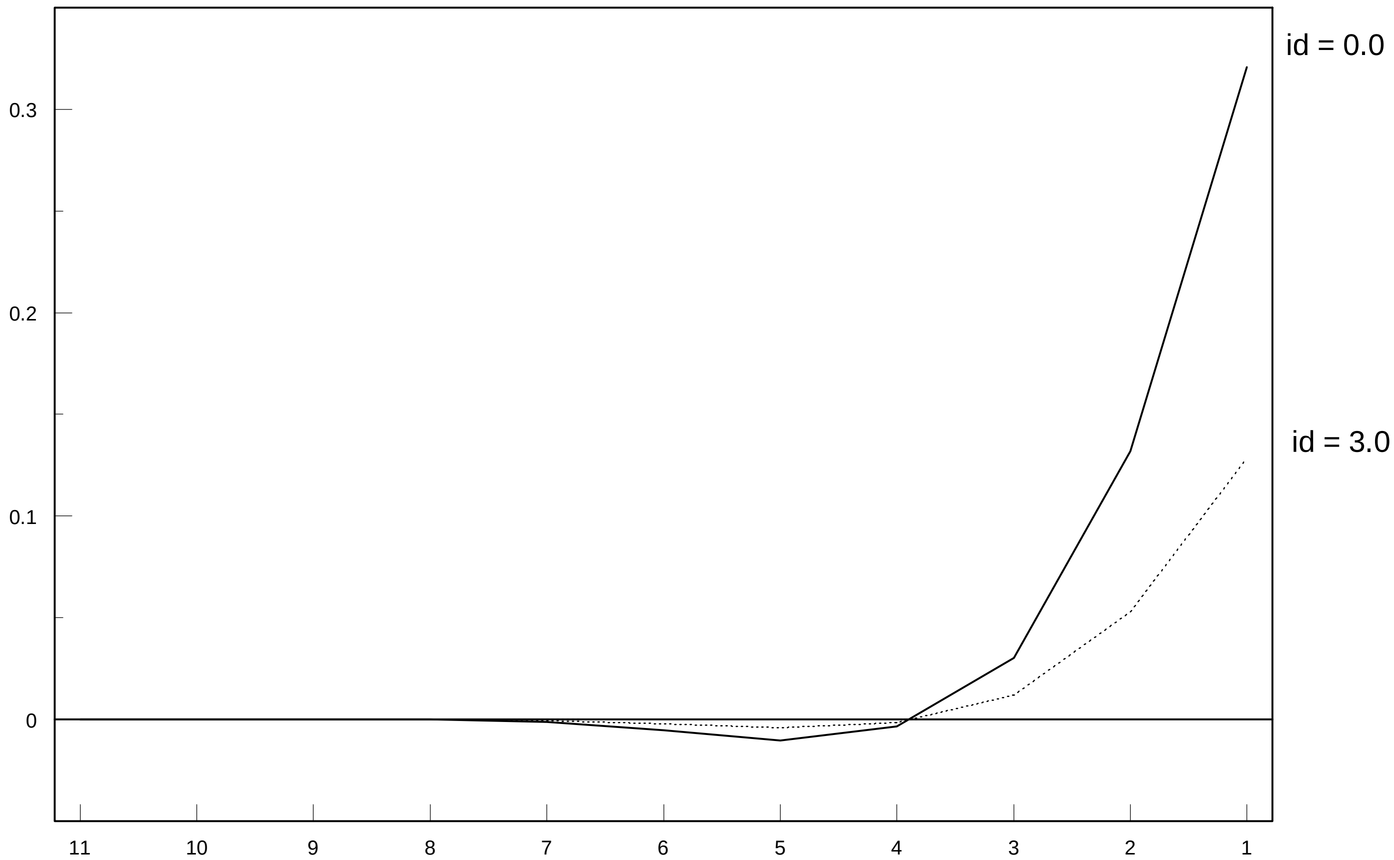
Average spreads in Germany: After EMU



Note: The horizontal axis represents the number of days before the end of the maintenance period. The solid line plots the average rate in that day.The dotted line represents the two standard error band.

FIGURE 4

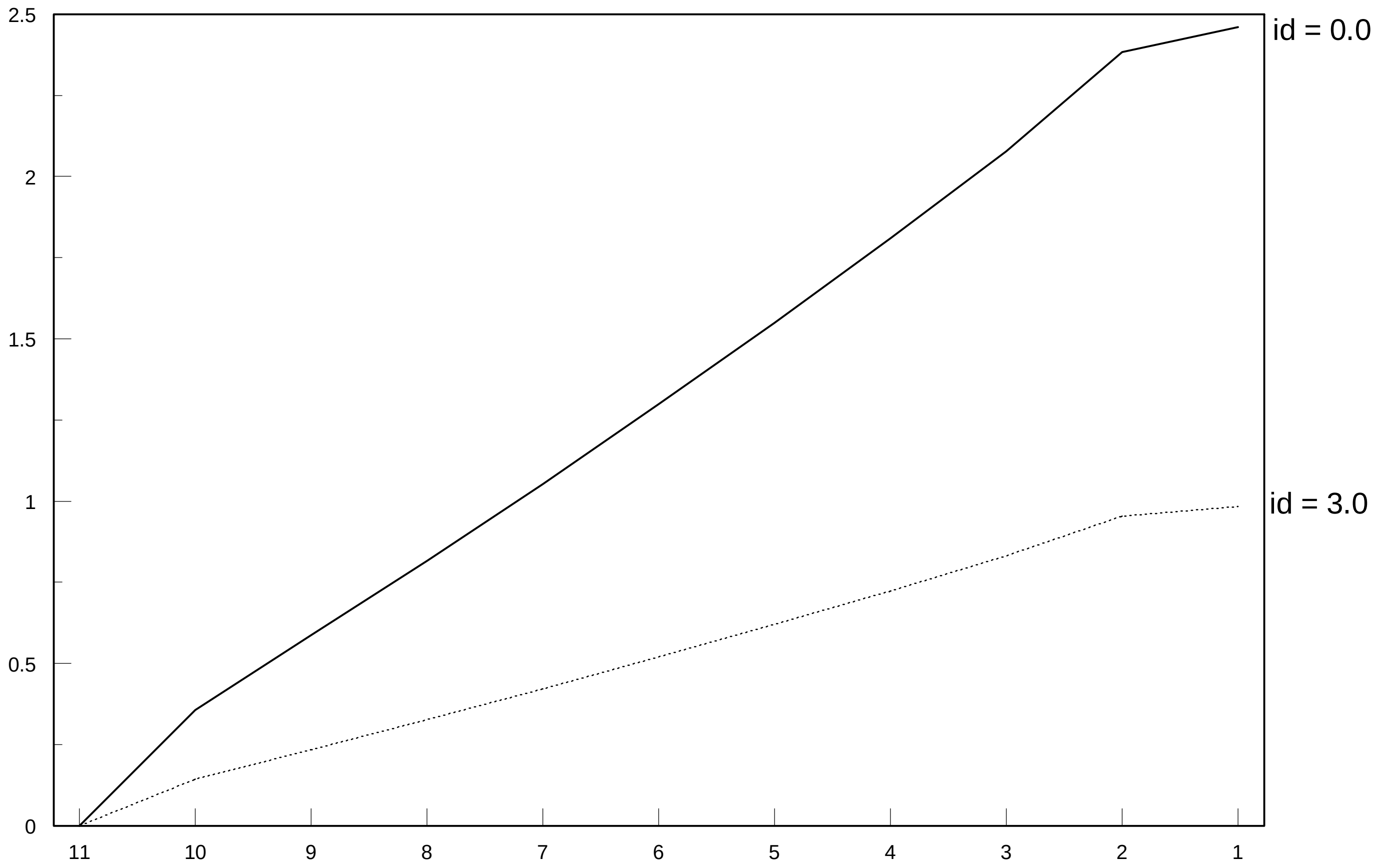
Change in unconditional mean of interest rate



Note: The horizontal axis represents the number of days before the end of the maintenance period. The solid line represents the expected unconditional mean when the deposit rate is 0%. The dotted line represents the expected unconditional mean when the deposit rate is 3%.

FIGURE 5

Unconditional variance of interest rate



Note: The horizontal axis represents the number of days before the end of the maintenance period. The solid line represents the expected unconditional variance when the deposit rate is 0%. The dotted line represents the expected unconditional variance when the deposit rate is 3%.