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Economics Working Paper 40

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Goods of Heterogenous Quality**

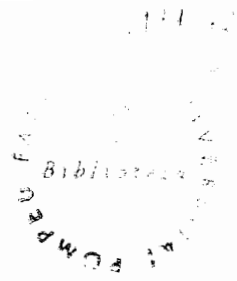
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Goods of Heterogenous Quality**

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Abstract

The aim of this paper is to demonstrate that uncertainty about the quality of a particular commodity does not necessarily exclude it from emerging as commodity money. This is shown in the context of the model of a simple economy with specialized agents and decentralized trade described in Kiyotaki and Wright (1989). In order to derive this result, considerations about marketability of the different goods are taken into account.

1. INTRODUCTION

The aim of this paper is to demonstrate that uncertainty of economic agents about the true quality of a particular commodity may not exclude it from being used as commodity money (i.e. a commodity that is accepted in trade not to be consumed or used in production, but to be used to facilitate further trade¹). This would happen even when commodities whose quality is perfectly known to everybody are available. We use a model of a simple decentralized economy with three different goods and three different types of agents who are specialized in production and consumption. The model is very close to the one described by Kiyotaki and Wright (1989) but, in contrast with their model, uncertainty about the actual quality of the goods is introduced. Specifically, the production functions in this model contain a random variable that will determine whether the good is high or low quality with some probability which is known by all agents in the economy. However, agents are unable to recognize the true quality of any particular good before accepting it for consumption. It will be shown, within this context, that there are equilibria in which goods of unknown quality play the role of commodity money.

One of the recurrent questions of the literature about money in economics is what circumstances determine which objects will come to be accepted as general media of exchange. Several authors like Brunner and Meltzer (1971), Alchian (1977) and, more recently, King and Plosser (1986) emphasize the role of the cost of verifying the quality of different goods ("recognizability" or "recognition costs") as the factor determining the selection of the medium of exchange. In their view, the cost of obtaining information about the attributes of the goods available for exchange is the determining

element in the emergence of a general medium of exchange. Their basic argument goes as follows: money exists because it serves to reduce the informational requirements of exchange. In a decentralized economy, individuals must spend resources acquiring information about the commodities they want to buy and the opportunities of exchange. When information about the qualities of goods varies, transactors are not indifferent about the commodities they accept for use in subsequent exchanges and they can reduce the amount of resources they use to acquire information about exchange opportunities by using goods with low recognition costs. In such a context, goods of heterogeneous quality would never become media of exchange when goods of homogeneous quality could also be used for such a purpose².

Other economists have emphasized the role of market characteristics of the goods in the determination of which commodity will appear as commodity money. Thus, Jones (1976) and Oh (1989) stress the fact that goods which are believed to be more commonly accepted in trade become medium of exchange, because agents accept them with the conviction that they will be able to exchange them more easily in the future for the goods they need (see also Iwai (1988)). In this framework, money is also a way of saving on exchange costs, or in other words, of minimising the individual search costs induced by the process of exchange. This market characteristic has been identified by Jones (1976) as commonness, although Kiyotaki and Wright (1989) refer to it as marketability, and more recently (Kiyotaki and Wright (1992)) as acceptability.

The results presented in this paper depend crucially on the interaction between these two different factors: goods of unknown quality can appear as commodity money because they are more marketable than other goods of homogeneous quality. This

combination of different factors is something that was not explicitly analyzed in the earlier works on money and recognizability of the qualities of goods by Brunner and Meltzer (1971) and Alchian (1977), and more recently, King and Plosser (1986). The particular structure of the model developed by Kiyotaki and Wright (1989), which we follow here very closely, allows the examination of these two factors together. In this sense, a paper related to this is Williamson and Wright (1992), which also analyzes exchange in economies with qualitative uncertainty in the context of a search theoretical model. However, this model focuses on the role of fiat money in overcoming problems of moral hazard derived from the existence of private information about the quality of the goods available for exchange in the economy. The decision of producing high or low quality goods is determined endogenously as an equilibrium outcome. The role of fiat money is to make it possible for agents to adopt strategies which increase the probability of acquiring a high quality good (without entering into the details of the model, it can be said that the use of money gives strategic incentives to the production of high quality commodities, something which would not be possible in a nonmonetary economy), and in this sense, there is a welfare improving role for a universally recognizable fiat money. Obviously, the scope of our research is different. We show that exogenous uncertainty about the quality of a particular good does not exclude it from emerging as a universally accepted medium of exchange. This is due to the fact that intrinsic characteristics of goods are not the only relevant variable which determine which objects are adopted as the medium of exchange. Considerations about the endogenously derived acceptability of those goods should also be taken into account.

The structure of the paper is as follows: in section 2 a model of a simple economy with decentralized trade is described in detail. In section 3 several equilibrium results which

imply the emergence of goods of heterogeneous quality as media of exchange are presented. Section 4 outlines some concluding comments about the main results derived in the paper.

2. THE ECONOMY

In this section a model of a simple economy with decentralized trade and goods of heterogeneous quality is specified. The model is as the one studied by Kiyotaki and Wright (1989). The crucial modification of our model with respect to the original is the assumption that goods may be of high or low quality. Production processes are characterized such that the quality of output depends on a stochastic variable and agents cannot know the true quality of a particular good before accepting it for consumption. The details of this model are provided in the following subsection.

2.1. General environment

Time is discrete. There are three different indivisible goods: good 1, good 2, and good 3. Goods may be of high quality or low quality (lemons). High quality goods can be consumed and lemons can also be consumed, although their consumption yields a lower utility to consumers. There is a continuum of infinitely lived agents who are, in equal proportion, of type I, type II, or type III. Agents of type i ($i=I,II,III$) consume only goods of type i ($i=1,2,3$) and they are also specialized with the following alternative patterns: either agent of type i produces goods $i+1$ (Model A) or he produces good $i-1$ (Model B).

Consuming a high quality good of type i adds U_i units of utility to agent of type i , while consuming a lemon of type i yields a level of utility $U_i' = U_i - d_i$ ($d_i > 0$). Thus, the cost of consuming a lemon is d_i , and this is expressed in terms of a lower level of utility. Agents can store any good, but only one at a time. Storing goods is a costly activity and c_{ij} denotes the cost in terms of disutility incurred by agent i in order to store good j during one period. The following storage costs structure is assumed: $c_{i3} > c_{i2} > c_{i1}$. It should be noticed that the patterns of specialization of Model A and Model B, together with the assumed storage costs structure, constitute two different economies and no mere relabellings of the same economy. After consuming a unit of good i , agent of type i produces a unit of good $i+1$ ($i-1$ in the case of Model B) with production costs in terms of disutility being denoted by D_i . The production function of agents of type i includes a stochastic factor leading to the following situation: good $i+1$ can be of high quality with probability α_{i+1} ($1 \geq \alpha_{i+1} \geq 0$) and of low quality with probability $1-\alpha_{i+1}$ (analogously for Model B). These parameters α_i are known by all agents in the population.

This economy is such that no agent produces the good he consumes and, as a consequence, the existence of trade is a sine qua non condition for consumption. No centralized market exists and agents only meet through an anonymous random matching process. Every period of time, agents meet randomly in pairs and make decisions about whether to exchange their goods or not (according to some trading strategies; see below for details), and also about consumption and production. Trade only takes place when both paired agents agree that it should occur and always on a quid pro quo basis³.

Let $p_{ij}(t)$ be defined as the proportion of type i agents who are holding good j in inventory ($\forall i,j$) at the time t , $0 \leq p_{ij}(t) \leq 1$ and $\sum_j p_{ij}(t) = 1$. Since there is an equal proportion of individuals of all types in the population, the probability of meeting an individual of type i is $1/3$ for all agents. Consequently, for any agent the probability of being paired with another agent of type i holding good j at time t can be simply characterized by the vector $p(t) = (\dots p_{ij}(t) \dots)$. Let $p(t)$ be called the distribution of inventories.

In this economy, agents cannot tell in general the true quality of a particular good. However, an agent of type i can tell the difference between a high quality good and a lemon of type i (his consumption good), although he can only do this only after inspecting that particular good. We will assume that the quality of a good which is accepted by some agent to be consumed immediately is determined before trade takes place. If the good is determined to be a lemon (something that is going to happen with probability $(1-\alpha_i)$), resources for an amount d_i , which is the difference between consuming a lemon or a high quality good, will have to be invested to make the lemon as good as a high quality commodity. Let γ_i be the proportion of this cost d_i which will be paid by the agent who holds the lemon and, consequently, $(1-\gamma_i)$ the proportion of the cost which will be borne by the agent who is going to consume the lemon himself. We will assume exogenous determination of γ_i . As a consequence of all this, agents who are holding goods of type i with positive probability of being lemons (i.e., goods for which $\alpha_i < 1$), are accepting the risk of having to pay $\gamma_i d_i$ with probability $(1-\alpha_i)$ whenever they trade with an agent who accepts good i in order to consume it.

Summing up, a sketch of the temporal sequence of events is as follows. Every period of time, agents, who always hold a good in inventory, are randomly matched in pairs. Then they make decisions about exchange, according to trading strategies which will depend on the good they have in inventory and also on the good held by the agent with whom they are matched. Details about these strategies will be given in the next section. If an agent gets his consumption good and he consumes it, he will immediately produce a new good which he will hold in inventory for the next period. Before an agent gets a good that he is going to consume, he will inspect it and determine its quality. If the good turns out to be a lemon, resources will have to be invested (by the holder, the acceptant, or both, depending on the value of the parameter γ_i) to make the lemon as good as a high quality good. If the agent gets something different from his consumption good, he will hold it in inventory until next period. Then, a new period of time starts.

2.2. Trading strategies and equilibrium

The behaviour of agents in this model will be characterized by their chosen trading strategies. A trading strategy is a rule defining the conditions under which an agent of type i is intending to trade. Specifically, this will depend on the good held by the agent himself and the good being held by the agent with whom he has been matched. As in Kiyotaki and Wright (1989), the following notation is introduced. Let $\tau_i(j,k) = 1$ if agent of type i wants to trade good j for k , and $\tau_i(j,k) = 0$ otherwise. It follows from this that when type i with good j meets type h with good k , they only trade if $\tau_i(j,k) \cdot \tau_h(k,j) = 1$. A trading strategy will be a rule that specifies the actions of the agent (trade denoted by 1, no trade denoted by 0) in all possible states. States are characterized by the goods being held in inventory by the agent and his partner. Formally, a trading

strategy for an agent of type i is a vector $s_i \in \mathbb{R}^9$ composed of elements 0 and 1 as follows,

$$s_i = (\tau_i(1,1), \tau_i(1,2), \dots, \tau_i(3,3))$$

At this point, we will assume that agents do not randomise between strategies and do not change them over time. Consequently, we are only looking at pure and steady-state strategies⁴. It should also be noted that, since we will only consider symmetric equilibria, we can summarize the strategies of the continuum of agents by simply stating a strategy for each type of agent.

Given an initial distribution, the strategies of the different agents will determine the resulting distribution of inventories at any time t (i.e. $p(t) = p(t, s_1, s_2, s_3)$). Given a strategy vector (s_1, s_2, s_3) , we can define a steady-state distribution of inventories $p(s_1, s_2, s_3)$, as an inventory distribution that satisfies the following condition⁵:

$$p(t, s_1, s_2, s_3) = p(t+1, s_1, s_2, s_3).$$

Finally, let an equilibrium be a vector of strategies (s_1^*, s_2^*, s_3^*) and a steady-state distribution of inventories, p^* , such that for each agent of type i

- 1) s_i^* ($\forall i=1,2, \text{ or } 3$) maximizes individual expected discounted lifetime utility of agent i given the strategies of the other agents and the steady-state distribution of inventories,
- and

$$2) p(s_1^*, s_2^*, s_3^*) = p^*$$

Condition 1 is the usual Nash equilibrium optimisation condition; and condition 2 is a consistency condition that states that given the vector of strategies (s_1^*, s_2^*, s_3^*) , p^* is a resulting steady-state distribution.

3. EQUILIBRIUM RESULTS

The main aim of this section is to present several results that prove the existence of equilibria in which a good of heterogeneous quality is used as commodity money. In order to do this, we will first present some general results.

3.1. Some general results.

Again, following Kiyotaki and Wright (1989), we will first introduce some notation and that will help us to characterize the equilibrium strategies of the agents⁶. Let V_{ij} be the value function of storing good j for agent of type i . In other words, the expected discounted utility of agent of type i when he exits a trade matching holding good j , and given that he follows a maximizing strategy and the strategies of the other agents. If $i = j$, and the agent consumes the good he has and produces a new good, V_{ii} will be equivalent to

$$V_{ii} = u_i + V_{ii+1},$$

where $u_i = U_i - D_i - (1-\alpha_i)(1-\gamma_i)d_i$. Thus, this expression is equivalent to the expected net utility of consuming good i plus the expected utility of holding the good he produced, $i+1$. When $i \neq j$, V_{ij} is equivalent to

$$V_{ij} = -c_{ij} + \max \beta E (V_{ij'} | j),$$

where β is the discount rate for all agents and $0 < \beta < 1$. This latter expression is a standard Bellman's equation of dynamic programming, where $E (V_{ij'} | j)$ is the expected indirect utility of agent i at next period random state j' , conditional on j , and the maximization is over strategies.

To ensure that agents in this economy will not want to drop out of it, it is sufficient to assume that U_i is large enough⁷. In order to characterize optimal strategies for agents of type i , we will first corroborate that whenever an agent of type i is matched with another agent who holds good i , the agent of type i will be willing to trade (i.e., $\tau_i(\cdot, i) = 1$), consume good i , and produce a new good $i+1$. The following lemma is equivalent to Lemma 1 in Kiyotaki and Wright (1989), although its contents have been slightly modified to adapt it to the particular model we are examining.

Lemma $\max_j V_{ij} = V_{ii} = u_i + V_{ii+1} = U_i - D_i - (1-\alpha_i)(1-\gamma_i)d_i + V_{ii+1}$. That is, each agent of type i will accept good i , consume it and produce a new good whenever he has the opportunity.

This lemma need not be proved explicitly. It is obvious that a high enough value of u_i , as it has been assumed above, will be sufficient for the result to hold.

In order to characterize the equilibrium strategies of agents, we must first recapitulate two things we have already seen: agents of type i will always accept good i whenever it is offered to them; and also, agents of type i will never carry good i when they meet other agents because it is optimal for them to consume it immediately. Moreover, when agent of type i meets another agent carrying the same good that he is holding, he will never be willing to exchange (i.e., $\tau_i(j,j) = 0, \forall i,j = 1,2,3$). This is because we assume, as in Kiyotaki and Wright (1989), that trade does not take place when one of the agents is indifferent between holding his good and the good held by his trading partner.

Consequently, the set of possible equilibrium strategies of an agent of type i can be characterized in the following way: he will be willing to "exchange good $i+1$ for good $i+2$ " (i.e., $\tau_i(i+1,i+2) = 1$ and $\tau_i(i+2,i+1) = 0$) iff $V_{i+2} > V_{i+1}$; vice versa, he will be willing to "exchange good $i+2$ for good $i+1$ " (i.e., $\tau_i(i+2,i+1) = 1$ and $\tau_i(i+1,i+2) = 0$) iff $V_{i+1} \geq V_{i+2}$. Note the asymmetry between the two equilibrium conditions for the two strategies. In the second case, it is necessary and sufficient for good $i+1$ to be held that it is at least as good as the alternative possibility. Instead, in the first case, we have a strict inequality since to exchange good $i+1$ (good produced by agent i) for good $i+2$ requires that the latter is strictly preferred to the former. Otherwise, there would be a transaction when the individual is indifferent among holding both goods, which goes against the maintained assumption. The following notation will be used to denote a vector of strategies (one for each type of agents): $s = (s_1, s_2, s_3)$, where

$$s_i = 1 \text{ iff agent of type } i \text{ plays strategy "exchanges good } i+1 \text{ for } i+2"$$

$$= 0 \text{ otherwise}$$

To find the equilibria of this model, we shall adopt exactly the same procedure as in Kiyotaki and Wright (1989): first, a strategy vector is conjectured; secondly, the distribution p determined by it is computed; and finally, it has to be checked that the conjectured strategies satisfy the equilibrium conditions for each type of agent, given the strategies of the other agents and the resulting distribution p . Since there is a finite number (eight) of possible strategy vectors, we can find all the equilibria of the model simply by repeating this procedure for all cases.

3.2. Equilibria with goods of heterogeneous quality as commodity money in a model with zero storage costs

In this section, we shall examine an economy with no storage costs and only one good of heterogeneous quality. In this context, Model A and Model B are only relabellings of the same economy, because there is perfect symmetry among all goods. It will be assumed that $c_{ij} = 0$ ($\forall i, j = 1, 2, 3$) and $\alpha_1 = \alpha_2 = 1$ and $\alpha_3 < 1$. Moreover, we will simplify the exposition by assuming that $\gamma = 1$, which can be done without loss of generality, and denoting $(1 - \alpha_i)d_i = K_i$. Note that in the model without storage costs (or equivalently, when all goods have identical storage costs), all goods are symmetric and the analysis of the equilibria under the assumption that there is a good of heterogeneous quality is equivalent independently of which one is that good. The following proposition states that in such an economy there is an equilibrium in which the only good of heterogeneous quality is generally accepted as commodity money by agents who do not consume it and use it only to improve their trade success. This is shown in the context of Model A, but it would be identical for Model B in this version of the model without storage costs.

Proposition 1

In Model A, and under the maintained assumptions, there exist only the following two pure strategy equilibria: a) for values of the parameters such that $K_3 < (\sqrt{2}-1)u_1$, there is an equilibrium in which goods 1 and 3 are used as commodity monies; and b) for values of the parameters such that $K_3 \geq 0.5u_1$, there is an equilibrium in which good 1 is used as commodity money.

These equilibria coincide respectively with the speculative and fundamental equilibria of Theorem 1 in Kiyotaki and Wright (1989). In particular, the equilibrium strategies for agents of type I, II and III are, respectively, $s=(1,1,0)$ for equilibrium a) and $s=(0,1,0)$ for equilibrium b).

Proof

Following the procedure outlined above, we will show the conditions for existence of the equilibria of this model by conjecturing all the possible strategy profiles, computing the steady-state distribution of inventories resulting from those profiles, and checking whether the conjectured strategies are best responses and constitute an equilibrium as defined above.

In order to illustrate this procedure, we will specify the details for the case of equilibrium a). To check that there is only one other equilibrium in that model is only a matter of repeating the procedure for all possible strategy profiles of agents. Consequently, we first conjecture the same strategy vector as in the speculative equilibrium in Kiyotaki and Wright (1989). This is $s = (1,1,0)$. Secondly, the

probability distribution corresponding to the steady state is computed. As in Kiyotaki and Wright (1989), this can be done easily and can be expressed by three numbers like $p = (p_{12}, p_{23}, p_{31})$, since it is known that $p_{ii}=0$ ($\forall i$) and $\sum_j p_{ij} = 1$. Since the strategies of agents are identical to the ones in the speculative equilibrium in Theorem 1 of Kiyotaki and Wright (1989), the resulting distribution should be the same, $p = (0.5\sqrt{2}, \sqrt{2}-1, 1)^8$. Thirdly, it has to be checked that the strategies conjectured above satisfy the equilibrium conditions. Thus, given the strategies of other agents, the strategy conjectured for an agent of type I would imply that

$$V_{12} = b \{ V_{12} + p_{21}[u_1 + V_{12} - K_2] + p_{23} [V_{13} - K_2] + p_{31}V_{12} + p_{32}V_{12} \} \quad (1) \quad \text{and}$$

$$V_{13} = b\{V_{13} + p_{21}V_{13} + p_{23}V_{13} + p_{31}[u_1 + V_{12} - K_3] + p_{32}V_{13} \} \quad (2)$$

where $b = \beta/3$ (remember that there is an identical number of players of type I, II and III in this population). To understand better how the model works, an explanation of how expression (1) is derived is in order. Agent of type I holding good 2 has a value function V_{12} equivalent to the sum of the following terms: 1) V_{12} if he meets another agent of type I, since then no exchange will take place; 2) with probability p_{21} , he meets an agent of type II holding good 1. They trade and agent I consumes good 1 and produces a new good 2. If the good 2 he was holding is a lemon (which happens with probability $(1-\alpha_2)$)⁹, agent I must also compensate agent II in order to induce him to accept a good which has been determined as a lemon (the expected amount of the compensation being $K_2 = (1-\alpha_2)d_2$); 3) with probability p_{23} agent I meets an agent II holding good 3. Trade takes place because the strategy of agent I tells him to trade 2 for 3. Consequently, agent I leaves such a trade opportunity with value V_{13} . Since agent

I has given away good 2 to an agent of type II (who is going to consume it), agent I has to bear the expected cost of good 2 being a lemon, K_2 ; 4) with probability p_{31} , agent I will meet an agent III holding good 1. Trade does not take place because agent III never accepts good 2; and 5) with probability p_{32} , agent I will meet an agent III who is holding good 2 (as agent I is) and, consequently, exchange will not take place.

It follows from (1) and (2) that

$$V_{13} - V_{12} = b [(p_{31} - p_{21})u_1 - p_{31}K_3] / [1 - b(2 + p_{32} - p_{23})] > 0$$

$$\text{iff } (p_{31} - p_{21})u_1 > p_{31}K_3$$

$$\text{Substituting for the } p\text{'s, } V_{13} - V_{12} > 0 \quad \text{iff} \quad K_3 < (\sqrt{2}-1)u_1,$$

which confirms that the conjectured strategy for agents of type I is optimal in the stated parameter space and given the strategies of the other players. Similarly, it can be shown for agents of type II that

$$V_{21} - V_{23} = b K_3 / [1 - b(2 - p_{12})] > 0$$

And the same for agents of type III,

$$V_{31} - V_{32} = b p_{13}u_3 / (1 - 2b) > 0$$

In order to show that there is another equilibrium, we would have to repeat this same procedure for the same strategy vector as in the fundamental equilibrium in Kiyotaki

and Wright (1989). This is $s = (0,1,0)$. It is a matter of simple algebra (available from the author upon request) to check that the conjectured strategy of each type of agents is a best response given the strategies played by the other agents: $V_{12} - V_{13} \geq 0$ iff $K_3 \geq 0.5u_1$, and that $V_{21} - V_{23} > 0$ and $V_{31} - V_{32} = 0$ for all values of the parameters. In the same way, it can be shown that no other strategy vector constitutes an equilibrium in this model. In particular, we disregard the case when $s = (0,1,1)$ as an equilibrium because although these strategies imply that $V_{12} > V_{13}$, $V_{21} > V_{23}$, and $V_{31} = V_{32}$ and so they are best responses to themselves, they require agents of type 3 to trade when they are indifferent, which goes against the maintained assumption. Q.E.D.

The previous proposition shows that there is a perfect analogy between our results and Theorem 1 in Kiyotaki and Wright. In equilibrium, it is always optimal for agents of type II to adopt the strategy "exchange good 3 for good 1" and for agents of type III the strategy "exchange good 2 for good 1". This means that agents of type II always use good 1 as medium of exchange and agents of type III simply keep the good they produce until they can barter it for their consumption good. In the case of agents of type I, it will be optimal to adopt the strategy "exchange good 2 for good 3" when $p_{31}[u_1 - K_3] > p_{21}u_1$, that is, when the expected utility of holding good 3 is greater than the expected utility of holding good 2. The expected utility of holding good 3 is the level of utility obtained from consuming good 1 (u_1) multiplied by the probability of being matched with an agent of type III holding good 1 (p_{31}), (in which case exchange will take place) taking into account the additional costs derived from the possibility of good 3 being a lemon, K_3 . The expected utility of holding good 2, which is a good of homogeneous quality, is given by $p_{21}u_1$, i.e., the level of utility obtained from consuming good 1 multiplied by the probability of meeting an agent of type II who is

holding good 1. This inequality is reversed when K_3 is too big and in this case good 1 is the only good accepted as commodity money in equilibrium. As is obvious, the factors which determine the optimality of the different strategies are not only considerations about the intrinsic quality of the goods, but also about the marketability of them (i.e. the value of the p 's, which are derived by the strategies adopted by the agents).

Note that in this model, the discount rate does not play any role in the determination of which strategies are maximizing. This is something that does not happen in the model by Kiyotaki and Wright (1989). The reason for this is that until now storage costs have not been considered. A typical individual will produce a good he cannot consume. Depending on the strategies the other agents are using, he will find it more convenient to store (at no cost) the good he produced or to exchange it for another good that he will eventually exchange for his consumption good. The optimality of these actions is independent of the discount rate because the disutility caused by storing any good is identical (zero in our case). In other words, getting a more marketable good has no cost, because they are all freely storable. In Kiyotaki and Wright (1989), getting a more marketable good implies sometimes paying a higher storage costs. Obviously, in that case, the optimality of such action will depend on the discount rate of the individual.

The results from Proposition 1 state that there are not multiple equilibria for any of the values of the parameters, but there is a set of parameters for which no equilibrium is found (specifically, when $(\sqrt{2}-1)u_1 \leq K_3 < 0.5u_1$). However, a natural mixed strategy equilibrium which connects the two pure strategy equilibria found in Proposition 1 can

be constructed as in Kehoe et al. (1991). This is done in Proposition 2 below and this allows us to prove existence of a steady-state equilibrium for all parameter values of the model.

Proposition 2

In Model A, when mixed strategies are considered, there is a steady-state equilibrium for all values of the parameters.

This fills the gap in Proposition 1, where there was a region of the parameter space where no pure-strategy equilibrium was found and ensures that there is a steady-state equilibrium in which exchange occurs and one or more commodity monies are used for all values of the parameters.

(See Appendix for a proof of this proposition)

3.3. Equilibria with goods of heterogeneous quality as commodity money in a model with storage costs

An obvious extension of the previous results is the consideration of the same model with the addition of positive storage costs with the structure assumed above. The analysis of this new model may add some interesting insights to the conclusions derived in section 3.2. The equilibrium results of the model would now resemble more closely the results obtained by Kiyotaki and Wright (1989) in their analysis of Model A and Model B. However, the equilibrium set can be shown to be more complicated due to

the addition of the new parameters to the model. There is an important difference between a model with storage costs and a model with no storage costs: goods are no longer symmetric as in the model analyzed in section 3.2. Thus, storage costs make goods asymmetric in the following sense: goods have now different intrinsic properties that might make them more or less suitable to appear as commodity money. In this context, the equilibrium analysis of the model with only one good of heterogeneous quality is different and will depend on which of the goods of the economy is of heterogeneous quality. Interestingly, although the analysis is now more complicated and many more equilibria appear than in Kiyotaki and Wright (1989) or in the model analyzed above, the main conclusion of the paper still holds: it can be proved that, in a model with storage costs and one good of heterogeneous quality, there are always equilibria in which the good of heterogeneous quality appears as commodity money, independently of which is this good. This result holds for Model A and Model B, which are now models of different economies. Nevertheless, the equilibrium set obtained with this formulation of the model has become larger and more complicated. It seems a good strategy to restrict the number of equilibria of the model by assuming positive, but arbitrarily small storage costs. Interestingly enough, although now the number of equilibria of the model becomes smaller, the main result of this paper still holds and there are equilibria in which the only good of heterogeneous quality is used as commodity money, whatever this good is and for the two different specifications of the economy, namely Model A and Model B. This establishes our claim about the robustness of the main result of this paper to the introduction of storage costs. We will not provide the specific details of the proof of this last result because they follow exactly the same type of algebra described in the proof of Proposition 1, with the additional consideration of storage costs.

4. CONCLUDING COMMENTS

Money exists as a mechanism of saving in exchange costs in decentralized economies with specialized agents. Several authors, who emphasize the role of the recognition costs of goods in the determination of which goods will appear as commodity money, conclude that goods of heterogeneous quality cannot emerge as commodity money when other goods of homogeneous quality are available. These conclusions should be revised when the analysis is enlarged to take into account market characteristics of goods. It has been shown in this paper that goods of heterogeneous quality may appear as medium of exchange when the trading strategies of the agents in the economy make those goods marketable enough.

APPENDIX

Proof of Proposition 2

This proof is partially based on section 4 of Kehoe et al. (1991). Let r_i be the probability that agents of type i play the strategy $s_i=1$ ($0 \leq r_i \leq 1$) and let $r = (r_1, r_2, r_3)$. With mixed strategies, the assumption of no trade when agents are indifferent between holding their good or the good held by their trading partners will be modified: it will be assumed that agents may randomize between trade and no trade whenever they are indifferent between two goods. Then, best response mixed-strategies will be characterized as follows:

$$\begin{aligned}
& \{0\} \quad \text{if } V_{\bar{i}+1} > V_{\bar{i}+2} \\
r_i \in & [0,1] \quad \text{if } V_{\bar{i}+1} = V_{\bar{i}+2} \\
& \{1\} \quad \text{if } V_{\bar{i}+1} < V_{\bar{i}+2}
\end{aligned}$$

In order to construct a mixed-strategy equilibrium which connects naturally the two pure-strategy equilibria found in Proposition 1, the following strategies are conjectured: $r_1 \in [0,1]$, $r_2 = 1$, and $r_3 = 0$. Following Kehoe et al. (1991), we can compute the steady-state inventory distribution p as a function of r_1 . This is

$$p = \left[\frac{\sqrt{1+r_1}}{1+r_1}, \frac{\sqrt{1+r_1}-1}{r_1}, 1 \right]$$

(this holds for $r_1 > 0$; for $r_1 = 0$, take the limit).

The equivalent to expressions (1) and (2) in the proof of Proposition 1 are the following expressions (3) and (4):

$$V_{12} = b\{V_{12} + p_{21}(u_1 + V_{12}) + p_{23}[r_1 V_{13} + (1-r_1)V_{12}] + p_{13}V_{12} + p_{32}V_{12}\} \quad (3)$$

$$V_{13} = b\{V_{13} + p_{21}V_{13} + p_{23}V_{13} + p_{31}(u_1 + V_{12} - K_3) + p_{32}[r_1 V_{13} + (1-r_1)(V_{12} - K_3)]\} \quad (4)$$

It follows from (3) and (4) that

$$V_{12} - V_{13} = b\{(p_{21}-p_{31})u_1 - [p_{31} + p_{32}(1-r_1)]K_3\} / \{1-b[2+r_1(p_{21}-p_{31})]\}$$

The strategy r_1 is a best response as long as $V_{12} = V_{13}$ is satisfied. Given the values of

the p 's, it can be shown that this equality holds for the following combinations of values of the parameters and r_1

$$(\sqrt{2}-1)u_1 \leq K_3 < 0.5u_1 \quad \text{and} \quad r_1 = [u_1(u_1-2K_3)] / K_3^2.$$

In the same way, it can be easily derived that for this same combination the following inequalities hold:

$$V_{21} - V_{23} = b\{p_{12}(1-r_1)u_2 + K_3\} / \{1-b(1+p_{13})\} > 0$$

$$V_{31} - V_{32} = (bp_{13}r_1u_3) / \{1-b[2-p_{13}(1-r_1)]\} \geq 0.$$

This means that both $s_2=1$ and $s_3=0$ are also best response strategies. This, together with the results from Proposition 1, completes the proof of the proposition. Q.E.D.

NOTES

1. Definition extracted from Kiyotaki and Wright (1989).

2. Thus, King and Plosser affirm that "in our framework, general acceptance of a specific commodity derives from its low verification costs (recognizability)" (King and Plosser (1986), p.113).

3. This is a consequence of the particular assumptions of the model: indivisibility of goods, anonymity of agents (which means that there cannot be credit), conditions of storage, etc. It means that trade always take place in the form of one-for-one swap of inventories. As a result, prices are left aside of this model (the price of every good is fixed and equal to one in terms of any other good in the economy).

4. This means that we are only looking for pure-strategy and steady-state equilibria (see Kehoe, Kiyotaki and Wright (1991) for an extension of the analysis of Kiyotaki and Wright (1989) which includes mixed-strategy equilibria and dynamic equilibria).

5. This definition is no more than the definition of stationary distribution of a Markov stochastic process.

6. The following lines only refer to the case of Model A. The case of Model B is exactly the same changing the subscripts appropriately.

7. This is also the role of Assumption A in Kiyotaki and Wright (1989). In our model the equivalent assumption would be like it follows: $u_i = U_i - D_i - (1-\alpha_i)(1-\gamma_i)d_i > (c_{i+1}-c_{ik})/(1-\beta)$, for all k . This is, the expected net utility of consuming plus producing is positive for all agents, even when they take into account the costs derived from the probability of consuming a good of poor quality and the difference of storage costs between a good k they hold before accepting and consuming good i , and good $i+1$ they will hold after consuming good i .

8. This result is computed as the steady-state probability distribution of the Markov process generated jointly by the different trade strategies of agents and the particular matching technology of our model.

9. Note that, since $\alpha_1 = \alpha_2 = 1$, in this particular case $(1-\alpha_1)d_1 = K_1 = (1-\alpha_2) = K_2 = 0$. The notation is maintained in expressions (1) and (2) in order to make clearer the way the equilibria in the model are found.

REFERENCES

- Alchian, A.A. "Why Money?" J. Mon. Cred. Bank., 9, 133-140 (1977).
- Brunner, K. and A.H. Meltzer "The Uses of Money: Money in the Theory of an Exchange Economy" Amer. Econ. Rev., 61, 784-805 (1971).
- Iwai, K. "The Evolution of Money: A Search-Theoretic Foundation of Monetary Economics" CARESS Working Paper, 88-03. University of Pennsylvania (1988).
- Jones, R. "The Origin and Development of Media of Exchange" J. Pol. Econ., 84, 757-775 (1976).
- Kehoe, T.J., N. Kiyotaki and R. Wright "More on Money as a Medium of Exchange" Econ. Theo., 3, 297-314 (1993).
- King, R.G. and C.I. Plosser "Money as the Mechanism of Exchange" J. Mon.Econ., 17, 93-115 (1986).
- Kiyotaki, N. and R. Wright "On Money as a Medium of Exchange" J. Pol.Econ., 97, 927-954 (1989).
- Kiyotaki, N. and R. Wright "Acceptability, Means of Payment and Media of Exchange" in TheNew Palgrave Dictionary of Money and Finance; London: Macmillan Press (1992).
- Oh, S "A Theory of a Generally Acceptable Medium of Exchange and Barter" J.Mon.Econ., 23, 101-119 (1989).
- Williamson, S. and R. Wright "Barter and Monetary Exchange Under Private Information" Amer. Econ. Rev. (forthcoming).

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