Endogenous Information Structures*

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April 1999

Abstract

Many models in the economics literature deal with strategic situations with privately informed agents. In those models the information structure is assumed to be exogenous and common knowledge. We consider whether such models, and the results they produce, are robust with respect the endogenization of the information structure. The results depend on whether information acquisition is secret or private, and on whether the strategic situation involves simultaneous or sequential moves. In particular we find that only when information is secretly acquired and moves are simultaneous, the results are fully robust. When information is acquired secretly but moves are sequential additional equilibria may appear. Instead, private information acquisition may make the equilibrium set smaller.

Journal of Economic Literature Classification Numbers: C72, D82, D83.

Keywords: Information acquisition, private information, equilibrium.

* Hurkens gratefully acknowledges financial support from CIRIT, Generalitat de Catalunya, grant 1997SGR 00138 and DGES grant PB96-0302. Vulkan gratefully acknowledges financial support from the EPSRC, grant no. M07052 and from the ESRC grant no. R000222642. We thank Esther Hauk and Antonio Cabrales for useful comments on an earlier version of this paper.

1 Introduction

Modern economic theory emphasizes the importance of private information in industrial organizations, principal-agent relationships, auctions and financial markets. In those models the private information of players is an *exogenous* part of the model. Such models explain what will happen *given* a certain information structure, but do not explain where that information structure *comes from*.[1] Private information of some player i in an extensive form game can be modelled as a random move of Nature, the realization of which is only disclosed to player i. One also says that Nature determines the *type* of player i. Players $j \neq i$ are not informed about the type of player i but since the structure of the game is common knowledge, they do know the probability distribution of the random move and have, therefore, exogenous (*ex-ante*) beliefs about the type of player i. In many instances we do not really think of a player as being born as some type with certain pieces of private information. Instead, players know they may profit from having private information and have, therefore, incentives to search for that information, even if it is costly to do so.

In order to get a full understanding of whether and how much effort agents are willing to exert in their search for information one needs to incorporate the decision to gather information into the strategic interaction and analyze the super-game, including an information gathering stage, as a whole. A static approach that tries to define the value of each piece of information for some player i may not be suitable in strategic interactions since the value of information, *i.e.* the amount by which player i can improve his payoff by obtaining the piece of information, may very well depend on how the other players in the game react, which in turn may also depends on how much information those other players gather. This is most easily seen in a model in which i's information gathering activity is observed by some player j. In this case the strategy of player j will, in general, be contingent on the level of this activity, that is, it will be a reaction function. However, even in situations where information gathering is secret and thus cannot be observed, players may react in an indirect way. Namely, just the fact that player i has the opportunity to gather information (which is known to other players), could possibly lead player j to behave differently.

[1]Notable exceptions are Vives (1988), Li *et al.* (1987), Hwang (1993, 1995), Hauk and Hurkens (1997), and Ponssard (1979) that consider information acquisition in oligopolies, Lee (1982), Milgrom (1981), Matthews (1984), and Persico (1997) who consider the incentives for information acquisition in auctions and Cremer and Khalil (1992, 1994), Cremer *et al.* (1998), and Kessler (1998) that consider principal agent relationships where the agent is initially uninformed but can acquire information, before or after the principal has offered him a contract. Grossman and Stiglitz (1980) initiated research on the incentives to acquire costly information on the value of a stock in financial markets. Hurkens and Vulkan (1996, 1997) considered information acquisition by potential entrants.

Analyzing the super-game that incorporates information gathering serves two purposes. First, it allows us to endogenously determine the private information structure. This means that we can check whether the private information structure exogenously given in the standard models of imperfect information is actually the one that would endogenously arise. If so, we would have some partial justification for the use of those standard models with exogenous information structure. Second, it allows us to examine the robustness of the predictions of models with exogenous private information structure. Namely, if the predictions of actions in the super-game coincide with the predictions of actions in the model with an exogenous private information structure, where this information structure is exactly the one predicted to arise endogenously in the super-game, then the results are robust and we would have a full justification for the use of models with an exogenous information structures.

The aim of this paper is to analyze in which circumstances the results are robust and in which circumstances they are not. We ask whether endogenous information acquisition can provide a foundation for the models that assume an exogenous private information structure. Can those latter models be generated when players have to search actively for information? If so, do the results of the models with exogenous and endogenous information structures correspond to each other? The reader may believe that the answer to both questions is obviously yes: It seems obvious that by choosing the cost of information gathering appropriately we can generate any information structure we want. In particular, assuming that information can be gathered at (almost) no cost would seem to provide a foundation for models with perfect information. Moreover, when a certain information structure is obtained endogenously, clearly the results should correspond to those obtained with the exogenous information structure. It is perhaps because of this apparent triviality that the issue of endogenous information acquisition has received so little attention in the literature. In this paper we argue that, in general, neither of the questions can be answered with a definite yes.

Let us illustrate what may go wrong by means of two models that have been applied widely in the literature: the principal-agent model and financial markets with informed speculators. In the standard principal-agent model the agent has private information about its cost to carry out a project the principal wants him to undertake. However, in real world contracting settings the agent will be able to calculate his cost only after the principal has spelt out the details of the project (e.g. about what materials to use). Cremer and Khalil (1994), Cremer *et al.* (1998) and Kessler (1998) show that the agent may strategically choose to stay uninformed about his cost, even if the cost of becoming perfectly informed is not prohibitively high. The reason for this is that a principal that knows that the agent may be uninformed will optimally offer different contracts than when he is sure that the agent is informed. The agent may actually benefit from those better contracts. This shows that the common principal-agent model with exogenously informed agents lacks a sound foundation: since it is not always in the interest of the agent to be informed we should not assume that they are. It is worthwhile to remark that the result is driven by the fact that in the models mentioned the agent has a first-mover advantage. The agent can gather information *before* the contract is offered and can credibly commit to abstain from gathering information. (The information gathering activity of the agent is observed by the principal.) Interestingly, in Cremer and Khalil (1992) it is assumed that the principal has the first-mover advantage and the agent can only gather information after the contract is offered. It is shown that in this case the principal will offer such a contract that induces the agent not to gather any information. Hence, in that model it turns out the agent will not be privately informed.

As a second example, let us consider financial markets. In financial markets it is usually assumed that some investors have a private and imprecise signal about the value of a stock. Grossman and Stiglitz (1980) argue that investors have no incentive to engage in costly information acquisition if the price of the stock fully reveals all information available, as is often the case in rational expectations equilibria. But when no investor searches for information, how can the price reveal information that nobody has? This paradox has puzzled many for some time. However, when one considers the super-game that incorporates the decision of investors to become informed the paradox disappears. In the super-game it is not possible in equilibrium that investors spend money on information that is freely available (through the availing price). Either the information must be costless, or the equilibrium price will be only partially revealing. (Often there exist rational expectations equilibria with partially revealing prices.)

Let us now outline the results obtained in this paper. We will focus on the class of games where information about fundamentals (*i.e.* moves of Nature) is acquired before players enter into a strategic situation. Hence, there is an information gathering stage followed by a game playing phase.[2] Since the actions of a player are chosen after information acquisition decisions are taken, they cannot influence those decisions. We will show that even within this class of information acquisition models the equilibrium results of the game with an exogenous structure need not coincide with the ones where the same structure is endogenously generated. The relation between the equilibrium sets of the endogenous and exogenous model depend on two characteristics. First, it is important whether the information acquisition decisions are observed or not before the

[2]This excludes the case analyzed by Cremer and Khalil (1992) where the principal offers a contract before the agent may gather information. It also excludes the case where a player can get information about *unobserved actions* chosen by other players. See Perea y Monsuwé (1997) for a setting in which players buy information about actions chosen by other players.

game playing phase starts. Second, it matters whether the game playing phase involves simultaneous or sequential play.

When information acquisition is secret (i.e. information acquisition decisions are not observed), the information structure that will endogenously arise in the equilibrium of the super-game will not become common knowledge. Players will hold some beliefs about how much information the other players possess and *in equilibrium* (of the super-game) these beliefs must be correct. However, in case of deviations players may be wrong: If some player gathers more (resp. less) information in the *information gathering stage* than what he is supposed to do in equilibrium, the other players will underestimate (resp. overestimate) the informedness of this player, since the secrecy implies that players cannot revise their beliefs. On the other hand, if some player deviates in the *game playing stage*, the other players may revise their beliefs (downward or upward) about the informedness of this player. Of course, if the players in the game playing stage choose their actions simultaneously this revision of beliefs will have no consequences since all actions have already been fixed. This means that any equilibrium of the endogenous information acquisition game induces an equilibrium in the game with an exogenous information structure. (Thm. 1 (ii).) However, if the original game has sequential moves, belief revision becomes important. In this case the super-game may have sequential equilibria that induce strategies in the game playing phase that do not form an equilibrium in the game with an exogenously fixed information structure. (Thm. 1(iii).) Such equilibria can be supported by having players hold incorrect beliefs about the informedness of any player that deviates in the game playing phase. On the other hand, we show that any equilibrium of the game with an exogenous information structure (with either simultaneous or sequential moves) is induced by some sequential equilibrium of the super-game. (Thm. 1(i).) Such equilibria are sustained by having players hold correct beliefs about the information structure (and, hence, never revise those beliefs).

It is worthwhile to spend some time discussing Theorem 1(iii) since it is a surprising and important result. It says that the super-game may have sequential equilibrium inducing strategies in the game playing phase that do not even constitute a Nash equilibrium in the game with an exogenously fixed private information structure. Hence, behavior that seemed irrational in the standard model turns out to be fully rational when information acquisition is endogenous! We would like to stress that the equilibria supporting seemingly irrational behavior are not unreasonable at all. In the super-game where information is acquired secretly players must hold some belief about the informedness of the other players. On the equilibrium path those beliefs must be correct but after observing some deviation in the game playing phase it is not unreasonable that players revise their belief about the informedness of the deviating player. In particular, such sequential equilibria may satisfy any refinement based on out of equilibrium restrictions.[3] This phenomenon may generate some surprising results as illustrated in Hurkens and Vulkan (1997). They consider a model of entry where potential entrants are initially uncertain about the demand but have the opportunity to acquire information before deciding whether to enter. Hurkens and Vulkan (1997) show the existence of equilibria where too few firms enter even if information about demand is costless. The beliefs which support such results are "pessimistic": unexpected entry (that is, off the equilibrium path) is interpreted as good news about the demand which makes the other firms behave more aggressively.. On the equilibrium path firms are reluctant to enter even if conditions are favorable, because they fear the aggressive reaction of other firms. Hence, firms will not necessarily enter up to the point where profits are almost zero (even when information is almost free) as opposed to what is assumed in many models and explained in many textbooks on Industrial Organization. More generally, equilibria of the type described in Thm. 1(iii) may have significant consequences in situations where information acquisition is important, like auctions, financial markets, contracting, and bargaining.

When information acquisition decisions are observed (information acquisition is *private*), the information structure generated in the first stage becomes common knowledge. The game playing phase is then a proper subgame of the super-game. Hence, any sequential or subgame perfect equilibrium of the super-game induces an equilibrium in the subgame, which is a game with an exogenous and fixed information structure. (See Thm. 2(i).) However, the game with the exogenous information structure may have several equilibria and not all of them are necessarily subgame perfect equilibrium outcomes of the super-game. (Thm. 2(ii).) We will give an example in Section 4, but we can already give the intuition for this result.[4] Suppose that for some information structure two continuation equilibria exist, one yielding player 1 a negative payoff and one a positive payoff. Suppose furthermore that player 1 could change his information structure in such a way that a new subgame is reached, and in this subgame there is a unique continuation equilibrium that yields him zero payoff. Then it is clear that choosing the original information structure and then continue with the continuation equilibrium that yields player 1 a negative payoff is not part of any subgame perfect equilibrium of the super-game. The subgame reached after having acquired some information structure is not exactly the same as the game where this same information structure has been exogenously been fixed. The fact that players have chosen their information structure is significant in determining what happens afterwards. The role of endogenously choosing one's information structure is similar to that of a sunk cost in entry models or burning

[3]See Hurkens and Vulkan (1997) for an example.

[4]In fact, the model of financial markets discussed before already serves as an example.

money in Van Damme (1989). Despite the fact that costs are sunk, they indicate that only some continuation equilibria are sensible. Hence, private information acquisition tightens the equilibrium set.

By gathering private information (that is, by making the information gathering activities public, say by publicly commissioning a marketing study), the firm is sometimes able to capture a large(r) market share for itself. More formally, the firm can use its information gathering activities to communicate its choice of most preferred post-entry Nash equilibrium to its competitors.

The remainder of the paper is as follows. Section 2 introduces the formal models of secret and private information acquisition. Section 3 contains the results on secret information acquisition while section 4 contains those on private information acquisition. Section 5 concludes.

2 The Model

There are n players involved in a strategic interaction represented by a game form G. This game form describes the actions that players may take, and the order in which they take them. It includes information sets, indicating that players may not know which actions have been chosen before. It is not, however, a fully defined extensive form game because there are no payoffs associated to the end points of the tree. The payoff functions to evaluate outcomes depend on the realized value of the state of Nature. We denote by Ω the finite set of states of Nature and will write ω to denote a generic state. We denote by S_i the finite set of pure strategies of player i in G (which does not depend on the state of Nature) and write $S = \prod_i S_i$ as the product set of strategy profiles. We denote by $u_i^\omega(s)$ the payoff obtained by player i when strategy profile s is chosen and the state of Nature is ω.

The players initially hold a common prior about the likelihood of each state of Nature which is represented by a probability distribution ρ on Ω. However, players can either endogenously improve their information or will be exogenously endowed with better information. The information structure of a player is given by some partition P of Ω. Hence, $P = \{P_1, ..., P_k\}$ where $\cup_{i=1}^k P_i = \Omega$ and $P_i \cap P_j = \emptyset$ for all $i \neq j$. We write $P(\omega)$ for the element of P that contains ω. The interpretation of the partition is that with information structure P a player can distinguish between two states ω and ω' if and only if $P(\omega) \neq P(\omega')$. We say that partition P is finer (more informative) than partition P' if $P(\omega) \subset P'(\omega)$ for all $\omega \in \Omega$ (with strict inclusion for some ω).

Each player i will either be endogenously or exogenously endowed with some information structure P^i. Let $P = (P^1, ..., P^n)$. We will denote the game with the exogenous information structure P by G^P. This is a standard game with private information. We will compare the equilibria of such games with those of the super-game where information structures are endogenously determined. This super-game has two stages. In the first stage all players choose simultaneously one of the feasible information structures. Not all information structures need to be feasible but we assume that $P^{\text{no}} = \{\Omega\}$ (no information acquisition) and $P^{\text{full}} = \{\{\omega\} : \omega \in \Omega\}$ (full information) are. Information acquisition is costly. Player i needs to pay $c(i, P)$ to acquire partition P, where $c(i, P) \geq c(i, P')$ if P is more informative than P'. For convenience, we assume $c(i, P^{\text{no}}) = 0$.

Information acquisition can either be *secret* or *private*. In the super-game where information is acquired secretly, denoted by Γ^s, the choice of partition P^i by player i is not observed by players $j \neq i$. A pure super-game strategy for player i in this game is a pair (P^i, σ_i), where P^i is the partition chosen and $\sigma_i : \Omega \to S_i$ maps states of Nature into strategies of G. Of course, the mapping σ_i must be measurable with respect to P^i, i.e. $\sigma(\omega) = \sigma(\omega')$ if $P^i(\omega) = P^i(\omega')$. On the other hand, in the super-game where information acquisition is private, denoted by Γ^p, the choice of partition P^i by player i is observed by all players. Hence, a pure super-game strategy for player i is now a pair (P^i, τ_i), where P^i is again the partition chosen by i and where $\tau_i(\omega, P) \in S_i$ denotes the strategy chosen in G when the state of Nature is ω and the (total) information structure is P. Again, the mapping τ_i must be measurable with respect to P^i.

We will be interested in comparing the results of the endogenous information acquisition games Γ^s and Γ^p with those obtained in games with an exogenous information structure, G^P. Since the strategy spaces in those three types of games are different, we need to make precise how we will compare the equilibrium results. First, in order for the comparison to make any sense we must restrict attention to those equilibria of the endogenous information acquisition games that use pure strategies in the first stage, i.e. where players choose a unique information structure and do not randomize. Because of the sequential structure of the information acquisition games we will also restrict attention to sequential equilibria so that our results do not rely on incredible out of equilibrium threats. Recall that a sequential equilibrium is a pair (μ, σ) where strategy σ is optimal given the beliefs μ, and the beliefs are consistent, i.e., $(\mu, \sigma) = \lim_n(\mu_n, \sigma_n)$, where σ_n is a completely mixed strategy profile and μ_n are beliefs determined by Bayes' rule based on σ_n. Finally, we will compare the strategies induced in the second stage by the sequential equilibria of the information acquisition games with the (sequential) equilibria of the games with an exogenous information structure, where this information structure is the one that endogenously emerged in the equilibrium at hand.

In order to compare sequential equilibria of the three models, we explain how the information sets in those models correspond to each other. In an information set I_i^s for player i in , ^{s}, the player knows its own information structure P^i, but does not know the information structure of the other players, P^{-i}. Moreover, it can distinguish states ω and ω' if and only if $P^i(\omega) \neq P^i(\omega')$. Hence $I_i^s = \bigcup_{P^{-i}} I_i^{G^{P^i,P^{-i}}}$, that is, I_i^s is simply the union over all possible information structures P^j of players $j \neq i$ of the corresponding information sets in the exogenous games $G^{P^i,P^{-i}}$. On the other hand, in an information set of player i in , ^{p}, player i knows the information structures of players $j \neq i$. Hence, each information set in , ^{p} corresponds one to one to an information set in G^P for exactly one information structure P. This allows us to identify belief assessments in , ^{p} and G^P, and to imbed belief assessments in G^P into the belief assessments in , ^{s} as follows: For some belief assessment μ in G^P, let $\mu^s_{|P}$ denote the assessment in , ^{s} at the corresponding information set that assigns zero probability to being in some node in $I_i^{G^{P^i,P'^{-i}}}$. In words, player i believes that the information structure is P (with probability one) and his beliefs about which point in the information set has been reached is then given by μ.

3 Secret Information Acquisition

THEOREM 1.
(i) For any information structure P and for any sequential equilibrium (μ, σ) of G^P there exist a feasible set of information structures, an information cost function $c(\cdot, \cdot)$, and a sequential equilibrium of , ^{s}, $(\mu', (P', \sigma'))$, such that $P' = P$, $\sigma' = \sigma$, and $\mu' = \mu^s_{|P}$.
(ii) Let (P, σ) be a sequential equilibrium of , ^{s} and suppose that the game playing phase involves only simultaneous moves. Then σ is a sequential equilibrium of G^P.
(iii) Let (P, σ) be a sequential equilibrium of , ^{s} and suppose that the game playing phase involves sequential moves. Then σ need not be an equilibrium of G^P.

Proof.

(i) Let the feasible set of information structures for player i consist of P^{no}, P^i, and P^{full} and suppose the first two cost zero and the last costs a huge amount (if $P^i \neq P^{\text{full}}$). Then buying information structure P followed by the play of σ is a sequential equilibrium of , ^{s}: Let the players always believe, independent of what happens in the game playing phase, that the information structure is P. If G is a game with sequential moves, let players hold the same beliefs as in the sequential equilibrium of G^P. It is clear that no player can profit from buying less information (i.e. no information), since information is costless, and other players' actions cannot be influenced in any way by such a strategy. Buying more information is dominated by assumption of high full information cost. It remains to be shown that the beliefs specified above are consistent.

Let (μ_n, σ_n) be the sequence which converges to (μ, σ) in the sequential equilibrium.

Let $\epsilon_n > 0$ be such that any pure strategy is played with at least probability ϵ_n in σ_n. Let τ_i be the strategy which randomizes uniformly over all pure strategies of player i in , ^{s}. Define $\tau_{n,i} = (1-\epsilon_n^2)\cdot(P_i,\sigma_i)+\epsilon_n^2\cdot\tau_i$. Clearly, $\tau_{n,i}$ converges to (P_i,σ_i) and μ'_n converges to μ', where the beliefs μ'_n are calculated using Bayes' rule based on τ_n.

(ii) Suppose σ is not an equilibrium. Then some player can profitably deviate. But then the same player could deviate in the same way in the game playing phase in , ^{s}, which contradicts the presumption that (P,σ) is an equilibrium of , ^{s}. Since in G^P moves are simultaneous, all equilibria are sequential.

(iii) This will be shown by means of Example 1 below. The intuition is that some strategy profile s may not form a sequential equilibrium of G^P because some player has an incentive to deviate, whereas in the endogenous information acquisition game such deviations can be credibly punished by having players revise their beliefs about the informedness of the deviating player. □

Before we give the example to demonstrate (iii) note that the proof of (i) depends on the fact that some information can be acquired at zero cost. If we insist that any information costs a strictly positive amount, the statement becomes false. For example, consider the standard Stackelberg game where inverse demand is either High, $(P = 20 - Q)$, or Low $(P = 12 - Q)$. Assume that both firms are perfectly informed about demand. Then in the SPE the leader chooses quantities 10 and 6 in the High and Low states, respectively. However, if both firms can gather information at a strictly positive cost, then this is no longer a sequential equilibrium outcome of the endogenous information gathering game: The follower can free ride on the leader's information as long as the leader chooses different actions (quantities) in the different states. However, if the follower is not informed, the leader has an incentive to deceive the follower in the High state by choosing the quantity 6 (because $10\cdot(20-10-5) = 50 < 66 = 6\cdot(20-6-3)$). In fact, all sequential equilibria of , ^{s} have only the leader being informed, producing 6 when the state is Low, and quantity $q_L^{High} < 14 - 4\cdot\sqrt{6}$. These equilibria correspond to the separating equilibria of the game where the leader is exogenously informed and the follower is exogenously uninformed. The latter game with the exogenous structure allows also for pooling equilibria (i.e. equilibria in which the leader chooses the same quantity when demand is high as when it is low), but such equilibria can not be supported in the game with endogenous information acquisition if information is costly: the follower would also acquire information (if it is not too costly) and the leader would not be willing to pay any positive amount for information he will not use. This example illustrates the general result that in any model with privately informed agents that has a pooling equilibrium this agent would never pay to get information he will not use.

Hence, pooling equilibria are difficult to justify in situations where agents endogenously, secretly, and costly acquire information.

Now we come to the example that demonstrates point (iii).

Example 1. Consider the game from Fig. 1. There are two agents, 1 and 2. Agent 1 is an expert who is able to learn whether the true state of Nature is either A or B. (Ex ante the two states are equally likely.) The expert can capitalize on his information by investing in projects a or b. He should invest in a (b) if the true state is A (B). If the expert is not informed this investment is risky and would yield an expected loss of one. The expert can instead go to agent 2, and propose to him to collaborate. If agent 2 rejects the offer, both receive a payoff of zero (the first investment opportunity is gone). However, if agent 2 accepts the offer, agent 1 has to take a final decision. If the expert is informed it is optimal for him to choose a' (b') if the true state is A (B). Agent 2 will also make a profit when the expert chooses the right option. However, an uninformed expert will find it optimal to select an option c'. This option is however very bad for agent 2.

If it is common knowledge that the expert is informed, the game has a unique subgame perfect equilibrium: the expert gives the move to player 2, who will accept. Finally, the expert will choose the appropriate option and both will obtain a payoff of 3.

However, consider the case where agent 1 has the choice to become informed (at small or zero cost) or not and agent 2 does not observe that choice. Then the following strategies and beliefs constitute a sequential equilibrium: Agent 1 becomes informed and then invests in a (b) if the true state is A (B). Player 2 believes that when he gets to move that the expert is uninformed (with probability of at least 6/7) and rejects the offer. The uninformed expert chooses c' at his second information set while the informed expert chooses the optimal option (a' or b').

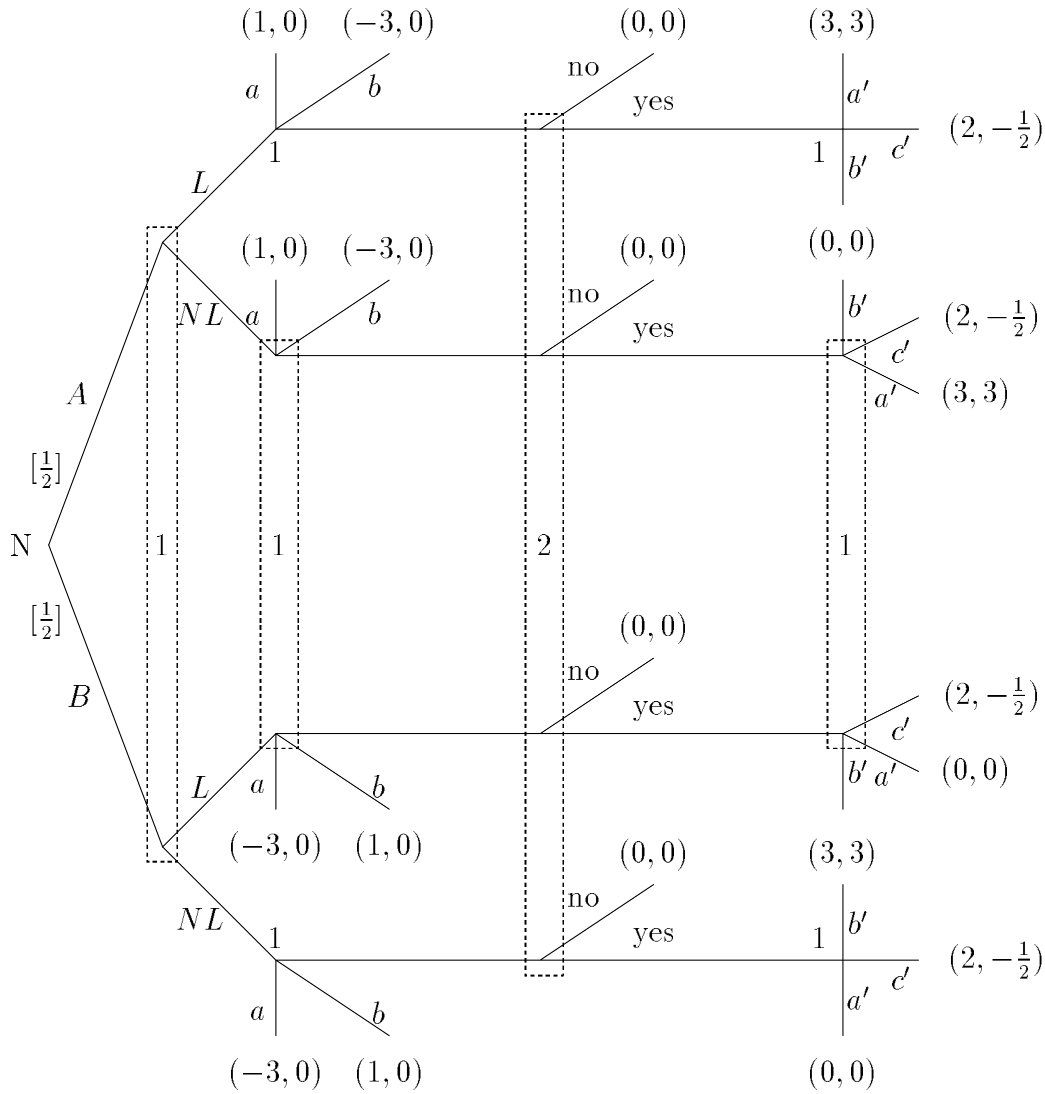

Fig. 1.

4 Private Information Acquisition

In the previous section we saw that the game with secret endogenous information acquisition may have an equilibrium (P^*, s^*) where s^* is not an equilibrium of G^{P^*}. Such additional equilibria are sustained by beliefs that do not assign all weight to the information structure P^* when some out of equilibrium action is observed in the game playing stage. Clearly, when information acquisition is private, the information structure is always common knowledge and players cannot hold incorrect beliefs about the information structure. By the very definition of a subgame perfect (or sequential) equilibrium, for any sequential equilibrium (P^*, s^*) of , ^{p} it is necessarily true that s^* be an equilibrium of G^{P^*}. On the other hand, it is not necessarily true that for every equilibrium s of G^{P^*}

there exists a sequential equilibrium of $,^{p}$ in which information structure P^* is chosen and s is induced in the game playing stage.

THEOREM 2.
(i) Let (P, τ) be a sequential equilibrium of $,^{p}$. Then s defined by $s(\omega) = \tau(\omega, P)$ is an equilibrium of G^P.
(ii) Suppose that $s' \neq s$ is another equilibrium of G^P, where P and s are as in (i). Then it is *not* necessarily true that there exists a sequential equilibrium (P, τ') of $,^{p}$ where $\tau'(\omega, P) = s'(\omega)$.

Proof. (i) By definition of sequential equilibrium.
(ii) By example: Suppose that Nature determines which of the two bimatrix games of Fig. 2 is going to be played, I or II. Game I is picked with probability 11/18.

	L	R
T	4,8	0,0
B	0,0	5,1

I

	L	R
T	0,0	6,10
B	5,2	0,0

II

Fig. 2.

Suppose that only player 2 has the possibility to learn the state of Nature and that his decision to learn or not is observed by player 1. If player 2 does not learn the players will play the average game which is dominance solvable, yielding them to play (T,L) with expected payoff vector (44/18,88/18). If player 2 knows which game is going to be played, and player 1 knows that, then there are 3 equilibria (i.e. in the game with the exogenous structure and in the subgame of the game with endogenous information acquisition): (1) Player 1 plays T and player 2 plays L (resp. R) in game I (resp. II); (2) Player 1 plays B and player 2 plays R (resp. L) in game I (resp. II); (3) Player plays T with probability 1/9 and player 2 plays L in game II and in game I he plays L with probability 10/11. Only equilibrium (1) gives player 2 a payoff higher than 88/18. So the endogenous information acquisition game has two subgame perfect equilibria outcomes: either player 2 learns and equilibrium (1) is played, or player 2 does not learn and the average game is played. Of course, to support this as an equilibrium outcome players would continue with equilibrium (2) or (3) in case player 2 deviated and decided to learn. So equilibria (2) and (3) are equilibria in the game with an exogenously informed player 2, but when information acquisition is endogenous, player 2 may become informed but play will not continue with equilibria (2) or (3). Hence, the equilibrium set has shrunk. □

5 Discussion

In this paper we investigated the robustness of equilibrium results of game models with respect to the endogenization of the information structure of the players. It turned out that only when information acquisition is secret and players choose actions simultaneously in the original game, the results are fully robust. When information acquisition is private, the endogenization process eliminates some of the equilibria of the game with an exogenous information structure. On the other hand, when players move sequentially (and information acquisition is secret), the endogenization may generate additional equilibria. In the latter case endogenous information acquisition may explain, what seemed to be irrational behavior in the game with an exogenous information structure, as rational equilibrium outcomes. The fact that we need *secret* information acquisition to obtain robustness is somewhat surprising, since in that case the information structure will not become common knowledge, while in the case of private information acquisition and games with an exogenous information structure the latter is always common knowledge.

Our results also demonstrate the important difference between secret and private information acquisition.[5] When information acquisition is secret, information is acquired for informational purposes only. That is, it is acquired because it allows the person who possesses it to make better decisions. When information acquisition is observed, however, it may be acquired (or not) for strategic reasons: By committing to have (or not have) some piece of information the actions of other players can be influenced in a way that is favorable to the first player. Since the difference between secret and private information acquisition is so important, the choice between the two should be determined by which resembles reality best, and not so much by analytical convenience.

We have considered information acquisition as refining one's partition of the state space. In the literature private information is sometimes modelled by players receiving an imprecise signal about the true state. Also in this case one can endogenize the information structure by having players decide on the precision of information they want. We conjecture that our results also hold in this case: When information acquisition is secret and play is simultaneous, there will be a one-to-one correspondence between the results of the exogenous and endogenous information models. With sequential moves (and secret information) additional equilibria will appear, while in the case of private

[5] This point was already made, in the context of Cournot competition, by Hauk and Hurkens (1997) and Hurkens and Vulkan (1997) who focussed on the incentives to gather information.

information acquisition some equilibria may disappear. We can even use the same examples to prove the latter statements. Just interpret no learning as information with precision zero, and full learning as information of precision 1 (or infinite). For the case where precisions can be chosen from a continuum, one would have to look further to come up with some examples, but we conjecture that there is no fundamental difference and that such examples can be constructed.

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