Heterogeneity-Promoting Optimal Procurement[1]

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Abstract

When procurement takes place in the presence of horizontally differentiated contractors, the design of the object being procured affects the resulting degree of competition. This paper highlights the interaction between the optimal procurement mechanism and the design choice. Contrary to conventional wisdom, the sponsor's design choice, instead of homogenizing the market to generate competition, promotes heterogeneity.

Keywords: Procurement, project design, horizontal differentiation.

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1 Introduction

Most procurement processes take place within a competitive environment characterized by horizontally differentiated potential contractors. In such cases, depending on the design of the object being procured, the sponsor may emphasize characteristics which one of the competitors has comparative advantage over, thus discriminating in favor of her. Within this framework, we address the following question: Which is the optimal design for the procured project, given that its choice affects the market structure, and consequently the induced degree of competition?

Consider, for example, a government interested in designating a telecommunications company as a universal service provider.[1] Firms interested in providing such services range from wired telecommunications companies and TV cable operators, to wireless telephone companies. Depending on the minimum quality standards set by the government for the services included in the universal service requirements (i.e., completion rate of calls, portability ease, etc.), different type of competitors may enjoy a comparative cost advantage. Hence, the government's choice of the minimum service requirements will affect the outcome of the competitive bidding process that might be used.[2]

When analyzing the issue of project design, the literature has focused on the case of vertically differentiated designs (e.g., Che (1993)). There, projects of higher quality are universally more costly to produce. Moreover, the contractors' ranking in terms of efficiency is preserved across designs, i.e., the efficiency parameters of contractors are perfectly correlated across designs. The result is that the optimal mechanism discounts quality in order to "homogenize" the market.[3]

In contrast here, not only we assume that designs are horizontally differentiated, and hence as we move along the design spectrum some firms become more efficient and others less efficient (in a stochastic sense), but moreover, we assume that a contractor's actual efficiency parameters in various designs are independent draws from a family of distributions related under first-order stochastic dominance. This implies that if contractors are asked to bid for a variety of possible designs[4], each contractor would have to prepare a multitude of proposals (or in a direct revelation

[1] According to the US 1996 Telecommunications Act, the Federal Communications Commission (FCC) and the state commissions should designate carriers with the responsibility for meeting universal service requirements. (For a more detailed analysis of the Act, see Brennan (1996).) According to FCC (CC docket n 96-45, Nov. 8, 1996), the role of universal service is to "ensure quality telecommunications services at affordable rates to consumers, including low-income consumers, in all regions of the nation, including rural, insular, and high-cost areas."

[2] According to the Telecommunications Act, a Carrier of Last Resort (COLR), i.e., the carrier designated to ensure that the universal service requirements are met, should receive subsidies in order to be able to provide these services. A number of involved parties, among them the California Public Utility Commission, have argued that auctions should be used to determine the level of the subsidies given to COLRs.

[3] A necessary condition for this result is that designs of lower quality attenuate the advantage of the more efficient contractors. This is the standard Spence-Mirrlees condition.

[4] This is a necessary condition for the sponsor to choose simultaneously both contractor and design, as it is the case in the literature of vertically differentiated designs.

mechanism to report the values of many parameters). Nonetheless, submitting proposals is costly. Hence, it is unnatural to assume that contractors bid over many different designs. This creates the need for design management. The sponsor should first choose a design, and then engage in a competitive bidding process.

The paper highlights the interaction between the optimal mechanism and the design choice when these two decisions are taken sequentially. The optimal mechanism in an environment characterized by heterogeneous competitors, as it is already known in the literature (e.g., Myerson (1981), McAfee and McMillan (1989)), resembles a discriminatory auction. Nonetheless, the intensity of discrimination is a function of the degree of the comparative advantage one of the potential contractors has over her competitors. This comparative advantage is endogenized here, and it is determined by the project's design choice.

We find that the sponsor's design choice, instead of homogenizing the market to generate competition, promotes heterogeneity (i.e., it increases the comparative advantage one of the potential contractors has). This is the case because of the way the optimal mechanism interacts with the degree of heterogeneity induced by the design choice. The more tilted in favor of a competitor is the design, the more discriminatory against this competitor the optimal mechanism will be. Intuitively, had not the sponsor been able to discriminate, she would have liked to choose a design that homogenizes the market to generate fiercer competition. Her ability to discriminate via the mechanism levels the field for the contractors even if one of them has a comparative advantage. This reduces the cost of having a less "competitive market", and therefore, the sponsor chooses a design that increases the chances that the "naturally" advantaged firm will get a favorable cost realization.

The following section sets up the model. Section 3 solves and discusses the two-type case. Section 4 extents these results to a continuum of types. Finally, Section 5 concludes.

2 The Model

2.1 Preferences and Technology

A risk neutral sponsor, S, wants to undertake a single indivisible project. The project may be implemented according to many different specifications, called designs (d), represented along the interval $[0,1]$. The sponsor's preferences over designs are described by $W(d)$, where $W(d)$ is strictly concave and symmetric around its unique maximum, attained at d^*. Finally, $W(d)$ is assumed to be sufficiently large for all possible designs, so that the sponsor always wants to undertake the project. The objective of the sponsor is to maximize her net surplus, i.e., $W(d) - T$, where T denotes the transfers made to contractors.

There are two risk neutral profit maximizing firms, denoted by $i \in \{A, B\}$. They are located at the end-points of the design spectrum, i.e., at 0 and 1 respectively. Each firm has comparative advantage over designs closer to its location. The comparative advantage is expressed in terms of lower expected costs to undertake the project. Formally, given a design d, the degree of *induced comparative advantage* is defined as $x(d)$, where

$$x(d) = \begin{cases} 2d - 1 & \text{if } d > 1/2 \equiv d^E \\ 1 - 2d & \text{otherwise.} \end{cases}$$

Clearly, at d^E no firm has a comparative advantage over the other, i.e., $x(d^E) = 0$.

Firm i has private cost c_i of implementing the project. This cost, c_i, is distributed on $[\underline{c}, \overline{c}]$. The distribution function is parametrized by $x(d)$. It is denoted as $F(x, c)$ if firm i is the firm with the comparative advantage, and $F(-x, c)$ otherwise. We make the following assumptions:

Assumption 1 *If* $x > y$ *then* $F(x, c) \geq F(y, c), \quad \forall c \in [\underline{c}, \overline{c}]$, *and* $x, y \in [0, 1]$.

This assumption states that depending on the degree of the *induced comparative advantage* the cost distributions are ordered in a first order stochastic dominance sense.

Assumption 2 $\frac{\partial}{\partial x}\left[\frac{F(x,c)}{f(x,c)}\right] > 0 \quad \forall x \in [0, 1]$, *and* $c \in [\underline{c}, \overline{c}]$.

This is a hazard rate dominance assumption, and implies that informational rents are larger for the firm that has the comparative advantage.[5]

Assumption 3 $\frac{\partial f(x,c)}{\partial x} = -\frac{\partial f(-x,c)}{\partial x}, \quad \forall c \in [\underline{c}, \overline{c}]$.

This is a "*neutrality*" of the design with respect to the industry cost assumption. A beneficial for one firm change in the design hurts equally the other firm. And, finally,

Assumption 4 $\frac{\partial}{\partial c}\left[\frac{F(x,c)}{f(x,c)}\right] > 0, \quad \forall x \in [0, 1]$, *and* $c \in [\underline{c}, \overline{c}]$.

This is the standard monotone hazard rate assumption.[6]

2.2 Description of Procurement Mechanism

Procurement proceeds in two steps. First, the sponsor announces the project's design d^o. Then, each firm learns its cost of undertaking the project under the announced design and participates in the mechanism. The sequential character of procurement, as explained in the Introduction, is

[5] Assumptions 1 and 2 imply the stronger conditional stochastic ordering (see, e.g., Maskin and Riley (1998)).

[6] Appendix C provides an example of a distribution satisfying these four assumptions.

due to the fact that it is costly to the firms to discover their cost for a particular design.[7] Working backwards, the sponsor's problem is addressed in two steps:

1. Characterization the optimal mechanism given an arbitrary design. The problem we address is a special case of Myerson (1981). Due to the revelation principle, we can concentrate on direct revelation mechanisms. The mechanism comprises two vectors $\psi(d) = \{p(c_i, c_j), T(c_i, c_j)\}$, where and $p_i \in [0, 1]$ denotes the probability of awarding the project to firm i, and $T_i \in I\!R$ the payment made to that firm.[8] The sponsor's objective is to find the mechanism that maximizes her expected net surplus subject to the relevant incentive compatibility, individual rationality, and feasibility constraints. The solution to this problem is the optimal mechanism, $\psi^*(d) = \{p^*(c_i, c_j), T^*(c_i, c_j)\}$.

2. Characterization of the optimal design, d^o. Given that the optimal mechanism, the sponsor solves the following problem: $\max_d E[\sum_{i=1}^{2}(W(d)p_i^* - T_i^*)]$.[9]

3 The Two-Type Case

In this section we assume that there are only two possible costs $c_i \in \{\underline{c}, \overline{c}\}$, $\forall\ i \in \{A, B\}$. Let $\nu_i(d)$ be the probability that firm i has $\overline{c}$. We moreover assume that $\nu_A(d) = \nu_B(1-d)$, and that $\nu'_A(d) = -\nu'_B(d) > 0$, $\forall d$.[10] It is straightforward to see that this cost structure satisfies the assumptions of the general model.

3.1 Symmetric Information Benchmark

Given an arbitrary design, d, the sponsor asks the firms to compute their costs, and awards the project to the firm with the lowest cost.[11] Hence, when deciding on the design, she can compute each design's expected cost, which is:

$$C(d) = [1 - \nu_i(d)\nu_j(d)]\underline{c} + \nu_i(d)\nu_j(d)\overline{c}.$$

[7] We make the extreme, but simplifying, assumption that it is costless for the firm to find its cost for one design, but infinitely costly for the subsequent ones.

[8] In order to simplify the notation we have omitted that both instruments depend on the design choice d.

[9] We would like to mention that the sponsor could follow an alternative route. She could adopt an "open rule", i.e., allow the firms to choose the design for which they will bid, and then by employing a scoring rule she could decide which proposal to accept. In such case, the procured design would arise endogenously as a result of a Hotelling-type location game between the two firms. We conjecture that in equilibrium the firms would choose designs close to their respective end-points. If this results to excessive differentiation, the sponsor could impose restrictions on the range of acceptable proposals.

[10] In this case, d^E is defined as the design, such that both probabilities are equal, i.e., $\nu_A(d^E) = \nu_B(d^E)$.

[11] When indifferent she randomizes between the two firms, say with equal probability.

The design that maximizes her net surplus, $W(d) - C(d)$, is given by the first order condition:

$$W'(d^s) = [\nu_i'(d^s)\nu_j(d^s) + \nu_i(d^s)\nu_j'(d^s)](\bar{c} - \underline{c}).$$

The next proposition summarizes the main feature of the optimal design.

Proposition 1 *The optimal design, d^s, is more extreme than the gross surplus maximizing design, d^*. This means that if $d^* < d^E$, $d^s \in (0, d^*)$, while if $d^* > d^E$, $d^s \in (d^*, 1)$.*

Proof: If $d^* < d^E$, the first order condition becomes $W'(d^s) = \nu_A'(d^s)(\nu_B - \nu_A)(\bar{c} - \underline{c})$. Since the RHS is positive, $W'(d^s)$ has to be positive, which implies that $d^s < d^*$. When $d^* > d^E$, by using the same argument, we can show that $d^s > d^*$. Q.E.D.

Note that the sponsor does not procure the gross surplus maximizing design. The intuition is the following: Moving away from d^* and towards the closest end-point reduces the expected cost of the advantaged firm, at the cost of an equal increase in the expected cost of the disadvantaged firm. Nonetheless, since the probability that the project is awarded to the advantaged firm is larger, the first effect dominates. The optimal design is then derived by trading off the cost enhancement effect vs the loss in gross surplus due to the procurement of a design other than d^*.

3.2 Asymmetric Information Case

Given an arbitrary design, the sponsor's problem is to design a mechanism to maximize her surplus net of the expected "*virtual cost*" (i.e., the sum of the expected production cost and of the expected informational rents offered to contractors).[12] The next lemma characterizes the optimal mechanism.

Lemma 1 *Assume w.l.o.g. that $d > d^E$, i.e., $\nu_i(d) > \nu_j(d)$. Then the optimal mechanism is:*

$$\psi^* = \begin{cases} p_i(c_i, c_j) = 1, \;\; p_j(c_j, c_i) = 0, & T_i(c_i, c_j) = \bar{c}, \;\; T_j(c_j, c_i) = 0 & \text{if } (c_i, c_j) = (\bar{c}, \bar{c}) \\ p_i(c_i, c_j) = 1, \;\; p_j(c_j, c_i) = 0, & T_i(c_i, c_j) = \bar{c}, \;\; T_j(c_j, c_i) = 0, & \text{if } (c_i, c_j) = (\underline{c}, \bar{c}) \\ p_i(c_i, c_j) = 0, \;\; p_j(c_j, c_i) = 1, & T_i(c_i, c_j) = 0, \;\; T_j(c_j, c_i) = \underline{c} & \text{if } (c_i, c_j) = (\bar{c}, \underline{c}) \\ p_i(c_i, c_j) + p_j(c_j, c_i) = 1, & T_k(c_i, c_j) = \underline{c} p_k(c_i, c_j), \quad k = i, j, & \text{if } (c_i, c_j) = (\underline{c}, \underline{c}) \end{cases}$$

Proof: See Appendix B.

Notice that the optimal mechanism discriminates against the advantaged firm. When both firms report high costs the sponsor awards the project to the disadvantaged one. By doing so, she drives to zero the informational rents she has to offer to the advantaged firm. When it reports $\bar{c}$, the project is awarded to the disadvantaged firm regardless of its cost report. Hence, the advantaged firm can not benefit by overstating its cost.

[12] The method employed to construct the sponsor's maximization problem is a widely known procedure (see, e.g., Laffont and Tirole (1993)) and is relegated to Appendix A.

To solve for the optimal design, we first compute the expected cost of implementing the project:

$$C(d) = (1-\nu_j(d))\underline{c} + \nu_j(d)\overline{c}$$

The project's expected cost depends only on the expected cost of the advantaged firm. Hence, the sponsor has stronger incentives to choose a design that increases the probability that the advantaged firm gets a favorable cost realization. The optimal design is given by the first order condition:

$$W'(d^o) = \nu_j'(d^o)(\overline{c} - \underline{c})$$

Proposition 2 *The optimal design, d^o, is more extreme than the one being procured under symmetric information, d^s. This means that if $d^* < d^E$, $d^o \in (0, d^s)$, while if $d^* > d^E$, $d^o \in (d^s, 1)$.*

Proof: If $d^* < d^E$, the first order condition becomes $W'(d^o) = \nu_A'(d^o)(\overline{c} - \underline{c})$. Clearly, comparing the RHS of this first order condition with the RHS of the one derived under symmetric information, we get that $d^o < d^s$. When $d^* > d^E$, by the same argument, we can show that $d^o > d^s$. Q.E.D.

The sponsor's optimal design makes the firms more heterogeneous than under symmetric information. The intuition is the following: The optimal mechanism allows the sponsor to eliminate the informational rents of the advantaged firm. On the other hand, the informational rents given to the disadvantaged firm are large (it always gets $\overline{c}$). Therefore, the sponsor is even more willing to distort the design in a way that increases the chances that the advantaged firm is awarded the project, i.e., it gets a favorable cost realization. The optimal design choice expresses the trade off between rent extraction (the reason for which the sponsor procures a more extreme design) and allocative inefficiency (the cost of moving away from d^*).

Remark. One should not interpret this result as showing that when discriminatory mechanisms are employed competition is not important. It is straightforward to show that net surplus increases as more firms locate at each end-point (a proxy for more competition). Not only expected cost decreases, but also the procured design converges fast towards d^*.

4 The Continuum-of-Types Case

4.1 Asymmetric Information Case when Discrimination Is Allowed

The continuum-of-types case is an application of Myerson's (1981) optimal auction design.

Lemma 2 *Assume w.l.o.g. that design d gives firm j a comparative advantage. Then the optimal procurement mechanism is:*

$$\psi^*(d) = \begin{cases} p_i(c_i, c_j) = 1, p_j(c_j, c_i) = 0 & \textit{if } c_i + \frac{F(-x, c_i)}{f(-x, c_i)} < c_j + \frac{F(x, c_j)}{f(x, c_j)} \\ p_i(c_i, c_j) = 0, p_j(c_j, c_i) = 1 & \textit{otherwise.} \end{cases}$$

Given the optimal mechanism, the sponsor's expected cost of implementing the project is:

$$C(x) = E\left[\min\left\{c_i + \frac{F(-x, c_i)}{f(-x, c_i)}, c_j + \frac{F(x, c_j)}{f(x, c_j)}\right\}\right].$$

Lemma 3 *$C(x)$ is decreasing in the degree of the induced comparative advantage, $x(d)$.*

Proof: See Appendix B.

The intuition behind this result is the same as that in the two-type case. Since the optimal mechanism reduces the informational rents given to the more efficient firm, the sponsor's expected cost decreases in the probability that the more efficient firm is awarded the project. As a result, the optimal design is similar to the one in the two-type case.

Proposition 3 *The optimal design, d^o, is more extreme than the gross surplus maximizing design, d^*. This means that if $d^* < d^E$, $d^o \in (0, d^*)$, while if $d^* > d^E$, $d^o \in (d^*, 1)$.*

Proof: The proof is done in two steps. First, note that if there are two designs that give the same degree of implied comparative advantage, i.e., $x(d^A) = x(d^B)$, the design that is closer to d^* is preferred. Both designs have the same expected cost, but the gross surplus generated by the design that is closer to d^* is larger. Hence, if d^* is closer to firm i, d^o lies between d^E and the location of firm i. The second step is to show that d^o lies between d^* and the location of firm i. This can be shown by using Lemma 3 and the definition of $x(d)$. Q.E.D.

4.2 Asymmetric Information Case when Discrimination Is Not Allowed

The sponsor is now not allowed to discriminate against any of the two firms, i.e., attention is restricted to anonymous mechanisms. Although we do not make any claim about its optimality among this restricted class of mechanisms, we consider a second-price auction; the project is awarded to the firm with the lowest cost announcement, and the winner receives a transfer equal to the cost announcement of the losing firm. The sponsor's expected cost of implementing the project is:

$$C^{SPA}(x) = E[\max\{c_i, c_j\}].$$

Lemma 4 *$C^{SPA}(x)$ is increasing in the degree of the induced comparative advantage, $x(d)$.*

Proof: See Appendix B.

The result is now the opposite. By making the two firms more homogeneous, the sponsor intensifies competition, and this results to a lower transfer. Not surprisingly, the result on the optimal design is now reversed.[13]

[13]The proof of the proposition is equivalent to the one of Proposition 3 and is omitted.

Proposition 4 *The optimal design, d^{SPA}, is less extreme than the gross surplus maximizing design, d^*. This means that if $d^* < d^E$, $d^{SPA} \in (d^*, d^E)$, while if $d^* > d^E$, $d^{SPA} \in (d^E, d^*)$.*

This result highlights the fact that the discriminatory nature of the optimal mechanism is crucial for the characterization of the optimal market structure. Only if discrimination is allowed the sponsor would actually want to make, by the appropriate design choice, the contractors more heterogeneous.

5 Conclusions

When procurement takes place within an environment characterized by horizontally differentiated potential contractors, the design of the project being procured becomes a strategic variable the sponsor can use to influence the degree of competition among the contractors. We find that when the sponsor is able to use discriminatory mechanisms the optimal design promotes heterogeneity among contractors.

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Appendix A

This appendix derives the sponsor's maximization problem in the two-type case.

The set of feasible mechanisms must satisfy the incentive compatibility and the individual rationality constraints both for high- and low-cost types. The high- and low-cost incentive compatibility constraints are respectively,

$$\underline{\pi_i} \equiv E_{c_j|d}\{\pi_i(\underline{c}, c_j)\} = E_{c_j|d}[T_i(\underline{c_i}, c_j) - p_i(\underline{c_i}, c_j)\underline{c_i}] \geq E_{c_j|d}[T_i(\overline{c_i}, c_j) - p_i(\overline{c_i}, c_j)\underline{c_i}], \quad \forall i \in \{A, B\},$$
$$\overline{\pi_i} \equiv E_{c_j|d}\{\pi_i(\overline{c}, c_j)\} = E_{c_j|d}[T_i(\overline{c_i}, c_j) - p_i(\overline{c_i}, c_j)\overline{c_i}] \geq E_{c_j|d}[T_i(\underline{c_i}, c_j) - p_i(\underline{c_i}, c_j)\overline{c_i}], \quad \forall i \in \{A, B\}.$$

The respective individual rationality constraints are $\underline{\pi_i} \geq 0$, and $\overline{\pi_i} \geq 0$. As it is usually the case, only the low-cost incentive compatibility and the high-cost individual rationality constraints will be binding at the optimal mechanism.

Using these constraints, we can compute the incentive compatible and individually rational expected profits for the two types,

$$\begin{aligned}
\underline{\pi_i} &= [\nu_j(d)p_i(\overline{c}, \overline{c}) + (1 - \nu_j(d))p_i(\overline{c}, \underline{c})]\,(\overline{c} - \underline{c}), \quad \forall\, i, j \in \{A, B\}, \\
\overline{\pi_i} &= 0, \quad \forall\, i, j \in \{A, B\}.
\end{aligned}$$

By substituting these expected profits in the sponsor's expected net surplus expression, we can eliminate the transfers from her maximization program, which can then be represented as the following linear program: $\max_{p_i(\cdot), p_j(\cdot)}$

$$\begin{aligned}
(1 - \nu_i(d))(1 - \nu_j(d)) \quad &\times \quad \Big[p_i(\underline{c_i}, \underline{c_j})(W(d) - \underline{c_i}) + p_j(\underline{c_i}, \underline{c_j})(W(d) - \underline{c_j})\Big] \\
+(1 - \nu_i(d))\nu_j(d) \quad &\times \quad \left[p_i(\underline{c_i}, \overline{c_j})(W(d) - \underline{c_i}) + p_j(\underline{c_i}, \overline{c_j})\left(W(d) - \overline{c_j} - \frac{1 - \nu_j(d)}{\nu_j(d)}(\overline{c_j} - \underline{c_j})\right)\right] \\
+\nu_i(d)(1 - \nu_j(d)) \quad &\times \quad \left[p_i(\overline{c_i}, \underline{c_j})\left(W(d) - \overline{c_i} - \frac{1 - \nu_i(d)}{\nu_i(d)}(\overline{c_i} - \underline{c_i})\right) + p_j(\overline{c_i}, \underline{c_j})(W(d) - \underline{c_j})\right] \\
+\nu_i(d)\nu_j(d) \quad &\times \\
&\quad + p_j(\overline{c_i}, \overline{c_j})\left(W(d) - \overline{c_j} - \frac{1 - \nu_j(d)}{\nu_j(d)}(\overline{c_j} - \underline{c_j})\right)\Bigg].
\end{aligned}$$

Appendix B

Proof of Lemma 1: The solution to the sponsor's program simply sets to zero the probability of awarding the project to the firm with the higher virtual cost. Hence, $p_i(\overline{c_i}, \underline{c_j}) = p_j(\underline{c_i}, \overline{c_j}) = 0$, i.e., $p_i(\underline{c_i}, \overline{c_j}) = p_j(\overline{c_i}, \underline{c_j}) = 1$. To determine which allocation minimizes the cost when both firms report $\overline{c}$, it is sufficient to check that $d \gtrless d^E \Longrightarrow \nu_A(d) \gtrless \nu_B(d) \Longleftrightarrow \frac{1-\nu_A(d)}{\nu_A(d)} \lessgtr \frac{1-\nu_B(d)}{\nu_B(d)}$. Therefore, since

d is such that j is the advantaged firm, it is optimal to set $p_i(\overline{c_i}, \overline{c_j}) = 1$ and $p_j(\overline{c_i}, \overline{c_j}) = 0$. When both firms report $\underline{c}$, the sponsor is indifferent, and the optimal mechanism is indeterminate. The associated transfers are uncovered by plugging the optimal allocation probabilities to the incentive compatible profits, and solving for the transfers. Q.E.D.

Proof of Lemma 3: Let j be the advantaged firm. $C^A(x)$ is rewritten as

$$C^A(x) = \int_{\underline{c}}^{\overline{c}} \left[\int_{\underline{c}}^{\widehat{c(c_i)}} (c_j + \frac{F(x,c_j)}{f(x,c_j)}) dF(x,c_j) + \int_{\widehat{c(c_i)}}^{\overline{c}} (c_i + \frac{F(-x,c_i)}{f(-x,c_i)}) dF(x,c_j) \right] dF(-x,c_i), \quad (1)$$

where $\widehat{c(c_i)}$ is implicitly defined by the following equation:

$$c_i + \frac{F(-x,c_i)}{f(-x,c_i)} = \widehat{c(c_i)} + \frac{F(x,\widehat{c(c_i)})}{f(x,\widehat{c(c_i)})}. \quad (2)$$

In order to simplify (1), we rewrite the first integral in the square brackets:

$$\begin{aligned} \int_{\underline{c}}^{\widehat{c(c_i)}} (c_j + \frac{F(x,c_j)}{f(x,c_j)}) dF(x,c_j) &= \int_{\underline{c}}^{\widehat{c(c_i)}} c_j dF(x,c_j) + \int_{\underline{c}}^{\widehat{c(c_i)}} \frac{F(x,c_j)}{f(x,c_j)} dF(x,c_j) \\ &= [c_j F(x,c_j)]_{\underline{c}}^{\widehat{c(c_i)}} - \int_{\underline{c}}^{\widehat{c(c_i)}} F(x,c_j) dc_j + \int_{\underline{c}}^{\widehat{c(c_i)}} F(x,c_j) dc_j \\ &= \widehat{c(c_i)} F(x,\widehat{c(c_i)}). \quad (3) \end{aligned}$$

Then, by integrating the second integral in the square brackets in (1), we get:

$$\int_{\widehat{c(c_i)}}^{\overline{c}} (c_i + \frac{F(-x,c_i)}{f(-x,c_i)}) dF(x,c_j) = [c_i + \frac{F(-x,c_i)}{f(-x,c_i)}][1 - F(x,\widehat{c(c_i)})]. \quad (4)$$

Now, by plugging (3) and (4) back into (1), and then factorizing, we get:

$$C^A(x) = \int_{\underline{c}}^{\overline{c}} \left[[\widehat{c(c_i)} - c_i - \frac{F(-x,c_i)}{f(-x,c_i)}] F(x,\widehat{c(c_i)}) + (c_i + \frac{F(-x,c_i)}{f(-x,c_i)}) \right] dF(-x,c_i).$$

By the definition of $\widehat{c(c_i)}$ in (2), the first term of the integrand becomes

$$[\widehat{c(c_i)} - c_i - \frac{F(-x,c_i)}{f(-x,c_i)}] F(x,\widehat{c(c_i)}) = -\frac{F(x,\widehat{c(c_i)})^2}{f(x,\widehat{c(c_i)})}.$$

Moreover, by integrating the second term of the integrand we get

$$\int_{\underline{c}}^{\overline{c}} \left[c_i + \frac{F(-x,c_i)}{f(-x,c_i)} \right] dF(-x,c_i) = \overline{c}.$$

Hence, $C^A(x)$ can be written as

$$C^A(x) = \overline{c} - \int_{\underline{c}}^{\overline{c}} \frac{F(x,\widehat{c(c_i)})^2}{f(x,\widehat{c(c_i)})} dF(-x,c_i).$$

Define $I(x,c_j) \equiv \frac{F(x,\widehat{c(c_i)})^2}{f(x,\widehat{c(c_i)})}$. To complete the proof is enough to show that $\int_{\underline{c}}^{\overline{c}} I(x,c_j)dF(-x,c_i)$ is increasing in x.

We can split the proof in two steps: 1) Proof that $\int_{\underline{c}}^{\overline{c}} I(x,c_i)\frac{\partial f(-x,c_i)}{\partial x}dc_i \geq 0$.

By Assumption 1, $\int_{\underline{c}}^{c} \frac{\partial f(-x,c_i)}{\partial x}dc_i = \frac{\partial F(-x,c_i)}{\partial x} < 0$. The boundary condition, $\int_{\underline{c}}^{\overline{c}} \frac{\partial f(-x,c_i)}{\partial x}dc_i = 0$, implies that $\int_{c}^{\overline{c}} \frac{\partial f(-x,c_i)}{\partial x}dc_i > 0$. This means that there must be $2N-1$ cutoff points, such that $\frac{\partial f(-x,c_i)}{\partial x}|_{c_i=c_\kappa} = 0$, where $\kappa = 1, \cdots, 2N-1$. By employing these cut off points, the support of c can be partitioned in $2N$ intervals

$$\int_{\underline{c}}^{\overline{c}} \frac{\partial f(-x,c_i)}{\partial x}dc_i = \int_{\underline{c}}^{c_1} \frac{\partial f(-x,c_i)}{\partial x}dc_i + \int_{c_1}^{c_2} \frac{\partial f(-x,c_i)}{\partial x}dc_i + \cdots + \int_{c_{2N-1}}^{\overline{c}} \frac{\partial f(-x,c_i)}{\partial x}dc_i = 0.$$

If we denote each integral by ϕ_k, the equation can be rewritten as $\phi_1 + \phi_2 + \cdots + \phi_{2N} = 0$. Hence, $0 > \sum_{k=1}^{z} \phi_k = -\sum_{k=z+1}^{2N} \phi_k, \quad \forall\ 1 < z < 2N-1$. Therefore, $\phi_{2k-1} < 0$ and $\phi_{2k} > 0$, $\forall\ k = 1, \cdots, N$. Now:

$$\begin{aligned}
&\int_{\underline{c}}^{\overline{c}} I(c_i)\frac{\partial f(-x,c_i)}{\partial x}dc_i \\
&\quad = \int_{\underline{c}}^{c_1} I(c_i)\frac{\partial f(-x,c_i)}{\partial x}dc_i + \int_{c_1}^{c_2} I(c_i)\frac{\partial f(-x,c_i)}{\partial x}dc_i + \cdots + \int_{c_{2N-1}}^{\overline{c}} I(c_i)\frac{\partial f(-x,c_i)}{\partial x}dc_i \\
&\quad > I(c_1)\phi_1 + I(c_1)\phi_2 + I(c_3)\phi_3 + I(c_3)\phi_4 + \cdots + I(c_{2N-1})\phi_{2N-1} + I(c_{2N-1})\phi_{2N} \\
&\quad \equiv [I(c_1) - I(c_3)]\sum_{k=1}^{2} \phi_k + [I(c_3) - I(c_5)]\sum_{k=1}^{4} \phi_k + \cdots + [I(c_{2N-3}) - I(c_{2N-1})]\sum_{k=1}^{2N-2} \phi_k \\
&\quad > 0.
\end{aligned}$$

The first inequality follows from the fact that $I(c)$ is an increasing function, and that $\phi_{2k-1} < 0$ and $\phi_{2k} > 0$. To see that the second line is equivalent to the third, add and subtract from the third line $I(c_{2N-1})(\phi_{2N-1} + \phi_{2N})$, use the fact that $I(c_{2N-1})\sum_{k=1}^{2N} \phi_k = 0$, and rearrange terms. Finally, the last inequality follows again from the fact that $I(c)$ is increasing, and from $\sum_{k=1}^{z} \phi_k < 0, \ \forall\ z$.

2) Proof that $\int_{\underline{c}}^{\overline{c}} \frac{\partial I(x,c_i)}{\partial x}dF(-x,c_i) \geq 0$. It is sufficient to prove that $\frac{\partial I(x,c_i)}{\partial x} \geq 0$. Assumption 2 ensures that this is the case. Note that due to the envelope theorem we do not have to consider the effect of x on $\widehat{c(c_i)}$. Q.E.D.

Proof of Lemma 4: Let j be the advantaged firm. We rewrite $C^{SPA}(x)$ as

$$C^{SPA}(x) = \int_{\underline{c}}^{\overline{c}} \left[\int_{\underline{c}}^{c_i} c_i dF(x,c_j) + \int_{c_i}^{\overline{c}} c_j dF(x,c_j)\right] dF(-x,c_i). \tag{5}$$

In order to simplify (5), we first integrate the two integrals in the square brackets:

$$\int_{\underline{c}}^{c_i} c_i dF(x,c_j) \quad = \quad c_i F(x,c_i), \tag{6}$$

$$\int_{c_i}^{\overline{c}} c_j dF(x,c_j) \quad = \quad \overline{c} - c_i F(x,c_i) - \int_{c_i}^{\overline{c}} F(x,c_j)dc_j. \tag{7}$$

Now, by plugging (6) and (7) back into (5), we get:

$$\begin{aligned} C^{SPA}(x) &= \overline{c} - \int_{\underline{c}}^{\overline{c}} \left[\int_{c_i}^{\overline{c}} F(x, c_j) dc_j \right] dF(-x, c_i) \\ &= \overline{c} - \left[[\int_{c_i}^{\overline{c}} F(x, c_j) dc_j] F(-x, c_i) \right]_{\underline{c}}^{\overline{c}} - \int_{\underline{c}}^{\overline{c}} F(x, c_i) F(-x, c_i) dc_i \\ &= \overline{c} - \int_{\underline{c}}^{\overline{c}} F(x, c_i) F(-x, c_i) dc_i. \end{aligned}$$

The result obtains by checking that, due to Assumptions 1 and 2, $F(x, c_i)F(-x, c_i)$ is decreasing in $x(d)$. Q.E.D.

Appendix C

Consider the following density function $f(x, c) = 1 + x - 2xc$ defined over $c \in [0, 1]$ and $x \in [-1, 1]$. Notice that when no firm has a comparative advantage (i.e., $x = 0$), the density function is reduced to a standard uniform. It is straightforward to show that it satisfies the assumptions of the model:

Assumption 1 $x > y \Longrightarrow F(x, c) = (1 + x)c - xc^2 > (1 + y)c - yc^2$.

Assumption 2 $\frac{\partial}{\partial x}\left[\frac{F(x,c)}{f(x,c)}\right] = \frac{c^2}{(1+x-2xc)^2} > 0$.

Assumption 3 $\frac{\partial f(x,c)}{\partial x} = 1 - 2c = -\frac{\partial f(-x,c)}{\partial x}$.

Assumption 4 $\frac{\partial}{\partial c}\left[\frac{F(x,c)}{f(x,c)}\right] = \frac{[1+(1-c)^2]+x^2c^2}{(1+x-2xc)^2} > 0$.