The Adverse Effects of Environmental Policy in *Green Markets*

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November 9, 1998

[*] Thanks are given to Anette Boom and Louis Hotte for extensive comments. We have also benefited from conversations with J.P. Benoit, Emmanuel Petrakis and Praveen Kujal. Part of this research was carried out while the first author was at the Center for Industrial Economics, University of Copenhagen, whose financial support is gratefully acknowledged.

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Abstract

We model green markets in which purchasers, either firms or consumers, have higher willingness-to-pay for less polluting goods. The effectiveness of pollution reduction policies is examined in a duopoly setting. We show that duopolists' strategic behaviour may increase pollution levels. Maximum emission standards, commonly used in green markets, improve the environmental features of products. Nonetheless, overall pollution levels will rise because government regulation also affects market shares and boosts firms' sales. Consequently, social welfare may be reduced. We also explore the effects of technological subsidies and product charges, including differentiation of charges.

JEL Classification: L52, Q28.

Keywords: emission standards, subsidies, product charges, vertically differentiated duopoly.

1 Introduction

In this paper, we show that some typical environmental policy measures may increase pollution levels in markets where products differ in some environmental features *(green markets)*. Firms differentiate their goods when they are able to extract rents from purchasers' higher willingness-to-pay for less polluting varieties. In the presence of this strategic behavior, government regulation does not only affect product environmental features but also consumers' allocation in the market. This later effect may cause pollution increases even in the case where the environmental quality of all products improve.

The literature has not analyzed the consequences of environmental policy *on aggregate pollution* in markets with imperfect competition when products are heterogeneous. In homogeneous product markets, environmental regulation generally induces an overinternalization of pollution as firms may react by reducing their output levels.[1] In differentiated markets, Motta and Thisse (1993) examine the effects of the introduction of a *minimum environmental standard* on firms' quality choices and their international trade strategies. Cremer and Thisse (1994) analyze the effects of ad-valorem taxation on the average environmental quality in a natural oligopoly.

To capture the main characteristic of a green market, we build a duopoly model of *vertical* product differentiation.[2] Goods only differ in their associated level of emissions, which is an endogenous variable. Purchasers, either buyers or firms, differ in their willingness-to-pay for the goods but all prefer less-polluting varieties. On the supply side, there are two ex-ante symmetric duopolists. They engage in a two-stage game, first choosing the variety to offer, and then their prices. Producers' investments in less-polluting technology, product design or, more specifically, abatement devices, to reduce the level of emissions per unit of product are costly. This fixed cost is higher the lower is the good's unit emissions level chosen.

In the unique subgame perfect equilibrium, two different varieties of the product emerge: these are referred to as the *cleaner* and the *dirtier* variety. Interestingly, the higher the average willingness-to-pay for the good, the lower are emissions per unit of product, but the higher is aggregate pollution.

In this (imperfect) competitive market, the externality associated to pollution may still be considered too high by governments. We study the effects of three common environmental measures, namely, the introduction of mandated unit emission standards, technology subsidies and product charges.[3] Throughout, we do not describe

[1]As a result, optimal policies such as standards and Pigouvian taxes fall short of marginal environmental damage to account for additional reductions of output levels indirectly induced by the policy. Buchanan (1969), Barnett (1980), Misiolek (1980) and Oates and Strassman (1984) study the effects of optimal environmental policies in the monopoly case; Ebert (1991), Requate (1993) and Damania (1996) in duopoly; and Levin (1985), Katsoulakos and Xepapadeas (1992, 1995) and Katsoulakos *et al.* (1995) in the oligopoly case.

[2]Green markets can also present some degree of horizontal differentiation. For simplicity, we assume that the rest of products' features are identical. In addition, according to OCDE (1997), the number of firms active in green markets is usually very small.

[3]Moral hazard and observability problems limit the use of effluent fees and ambient charges in many real-world cases. Unit emission standards, subsidies and product charges are amply employed in western European countries, such as Germany, Holland, Norway, Spain, Sweden, etc. (see

optimal policies. Rather, our concern is to illustrate the effects of these measures on firms' product choices, aggregate pollution and social welfare.

We first show that after imposing a unit emission standard close to the original (under no regulation) equilibrium, total pollution increases. The intuition is that even though both firms offer less-polluting varieties, sales increase sufficiently to overweight this effect. As a result of the policy, the dirtier firm meets the requirement while the cleaner one best-responses by improving its product as well. However, the presence of decreasing returns in the abatement technology induces a lower effort of the cleaner firm to differentiate its good from the rival's. Thus, equilibrium product differentiation lowers, bringing about a tougher price competition stage. Environmentally-friendlier varieties offered at lower prices soar sales sufficiently to overweight the reduction in unit emissions.

This counter-productive quantitative effect does not emerge when firms are offered, instead, a subsidy that lowers technology costs, and, consequently, equilibrium unit emissions. The reason is that product differentiation does not change. This is also the case when uniform product charges are implemented. However, both unit emissions and total contamination increase as a result of the taxation policy. Finally, we investigate the effects of differentiating product charges. We show that by slightly increasing the tax rate of the dirtier firm, both firms' unit effluents increase but aggregate pollution decreases. In contrast, by slightly lowering the tax rate of the cleaner firm, both firms' unit effluents and total contamination decrease. Again, firms' strategic behavior drives the undesirable results.

As to the welfare consequences, we find that subsidies increase and uniform taxes decrease social welfare. However, the overall effect of policies such as the introduction of a unit emission standard and non-uniform product charges on social welfare depends on the marginal valuation of the environmental damage.

The remainder of paper is organized as follows. Section 2 describes the model and characterizes the unregulated equilibrium. In Section 3 we explore the effects of unit emissions standards. In Section 4 we analyze the effects of investment subsidies while uniform and non-uniform product charges are studied in Section 5. Section 6 offers concluding remarks.

2 The model

We present a duopoly model of environmentally differentiated products.[4] We assume that two firms enter a market where different varieties of a product may be produced, each of them identified by its observable level of associated polluting emissions (environmental wastes or effluents).[5] Manufacturing varieties whose levels of pollutant

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examples below).

[4]See Gabszewicz and Thisse (1979), Mussa and Rosen (1978) or Shaked and Sutton (1982) for typical product differentiation models.

[5]We will model emissions associated to the goods as a continuous variable for computational reasons. However, many of the examples we consider would better fit into a discrete description. For instance, recycled paper and non recycled, ecological batteries and non ecological, unleaded gasoline and leaded, beverages offered in plastic bottles and in glass bottles, etc.

discharges are lower will be assumed to be more costly.

The demand side of the market is constituted by a continuum of purchasers, either consumers or firms, that differ in their marginal valuation for product green features, θ. This buyer-specific matching value is assumed to be uniformly distributed on $[0, \overline{\theta}]$ with measure one. Purchasers buy either one unit of the product or nothing. If they acquire a variety of the good whose level of emissions is e at price p, they obtain a (indirect) utility $W(\theta, e) = V - \theta e - p$. No consumption is assumed to give zero surplus.[6]

According to this specification, a purchaser that acquires the variety e obtains a gross surplus $V - \theta e$. One possible interpretation is that demand comes from environmentalist consumers. Then, V would stand for the utility obtained from the good regardless of the level of emissions of the variety acquired. Environmentalists, in addition, derive a desutility θe from purchasing a good whose level of emissions is e.[7] Since θ varies across individuals, consumers differ in their environmental awareness. The parameter $\overline{\theta}$ would then measure the degree of consumers' *environmental consciousness.*

An alternative interpretation is to consider that purchasers are a set of small firms non-competing among them that employ the goods as inputs for their production processes. This input is not substitutable, i.e. if a firm wants to enter its respective market must acquire it. Emissions associated to the inputs are by-products of the small firms production activities. These effluents are taxed according to the linear function $t(e) = e$. The parameter θ would then represent the firm-specific financial opportunity cost, and V would stand for the net benefits each firm would obtain from entering its market. Under either of these interpretations, note that if purchases were given a free choice between any pair of different varieties, they would agree and choose the one with the lowest level of emissions associated. This characteristic implies that our model falls into the category of product differentiation models.[8]

Throughout the analysis, we will use the following assumption:

Assumption 1 V is sufficiently small so that not all purchasers acquire the good in equilibrium.

This assumption, which is formalized below, implies that the market is not covered, i.e. some consumers do not consume the good or some firms do not enter their respective markets.[9]

[6]It is implicitly assumed that utility is also function of total environmental damage (Φ), that can not be affected by consumers' individual decision, and that it is a separable function. Therefore, consumers, either buying or not buying in this market, suffer the environmental externality associated to the production or consumption of the product.

[7]One could think of batteries. A battery gives a utility V to a consumer whatever its level of emissions associated (for example, 5 hours of radio music). However, an environmentalist derives a lower desutility from purchasing a low-cadmium battery than from a regular one.

[8]Note that it is implicitly assumed that the intrinsic valuation of the product is identical across purchasers, V. However, our main results would not substantially change if we considered them to be different (see Moraga-González and Padrón-Fumero (1997)).

[9]If this assumption was not satisfied, the interest of our analysis would be limited because both firms' reaction functions would be independent on the corresponding rivals' choices in the second

The ex-ante symmetric duopolists have access to the same technology. We assume that to produce the variety e, they must incur the fixed cost $C(e)$. Further, it is assumed that once a firm has incurred the technological cost to ensure the provision of the variety e, production takes place at a unit marginal cost c that is independent of the level of effluents chosen by the firm. We normalize c to zero. We also assume that producing varieties with lower levels of emissions is more costly, i.e. $C'(e) < 0$. For computational reasons, $C(e)$ is assumed to be a homogenous function of degree $\alpha < 0$. This implies that the technology for the production of the good e exhibits decreasing returns.

Competition between the duopolists takes place in two stages: In the first, firms simultaneously decide the variety to produce. In the second stage, when technological costs have already been invested, firms compete in prices. This two-stage modelling is motivated by the fact that, often, firms can rapidly change their prices while a change in the production technology takes place in the long run. In our context, it is reasonable to assume that abatement technology decisions are long-run variables while prices are short-run ones. We will look for the subgame perfect equilibrium of this game and proceed by backwards induction.

We will consider the following social welfare function:[10]

$$SW = CS + \Pi_T - \Phi(E_T) \tag{1}$$

where CS and Π_T denote consumers surplus and aggregate profits respectively, and $\Phi(\cdot)$ is the social valuation of environmental damage caused by aggregate pollution, E_T.

Next we compute the equilibrium under no regulation. Throughout, the firm that chooses a higher (lower) level of effluents will be referred to as the *dirtier* (*cleaner*) firm. Without any loss of generality, we consider firm 1 (firm 2) as the dirtier (cleaner) firm, offering a product with associated level of emissions e_1 (e_2) at price p_1 (p_2). Reasonably, $e_1 > e_2$ and $p_1 < p_2$.[11]

To derive the demand function for each variety, we define the following critical parameters. There is one purchaser indifferent between acquiring either of the products. This buyer is characterized by the taste parameter $\widetilde{\theta}$ satisfying $V - \widetilde{\theta}e_2 - p_2 = V - \widetilde{\theta}e_1 - p_1$. Therefore, $\widetilde{\theta} = (p_2 - p_1)/(e_1 - e_2)$. Assumption 2 implies that there are two more critical parameters: the first corresponds to that purchaser indifferent between buying the cleaner good and nothing. This buyer is identified by the parameter $\widehat{\theta} = (V - p_2)/e_2$. The second one is the purchaser indifferent between acquiring the dirtier product and not buying at all, i.e. $\widehat{\widehat{\theta}} = (V - p_1)/e_1$. In what follows, we will concentrate on the case where parameters satisfy $0 < \widetilde{\theta} < \widehat{\widehat{\theta}} < \widehat{\theta} < \overline{\theta}$.[12] Thus,

stage of the game. However, dropping it and introducing emissions-dependent variable cost, *a la* Crampes and Hollander (1995), would restore the interest of the problem. Our main results do not seem to be affected qualitatively for our modeling choice.

[10] See note 6 above.

[11] We check that these inequalities are satisfied in equilibrium below.

[12] We do not look at uninteresting cases. These appear when the above critical parameters are such that either only one firm faces positive demand, or when both firms enjoy positive market sizes but the demands are not connected among them. Their lack of interest lies on the fact that firms

demand for the dirtier firm stems from the group of consumers whose parameter θ is such that $0 \leq \theta \leq \widetilde{\theta}$ while demand for the cleaner firm comes from those customers θ such that $\widetilde{\theta} \leq \theta \leq \widehat{\widehat{\theta}}$. The rest of purchasers buy nothing. Therefore, dirtier and cleaner firm's demands are respectively

$$q_1(\cdot) = \frac{p_2 - p_1}{\overline{\theta}(e_1 - e_2)},\ q_2(\cdot) = \frac{V - p_2}{\overline{\theta} e_2} - \frac{p_2 - p_1}{\overline{\theta}(e_1 - e_2)}. \tag{2}$$

In the second stage, firms simultaneously choose prices to maximize their profits $\Pi_i = p_i q_i - C(e_i)$, $i = 1, 2$. Note that given any two levels of emissions satisfying $e_1 > e_2$, the second stage profits functions are strictly concave with respect to the prices. Therefore, necessary conditions also suffice for profits maximization. From the first order conditions (f.o.c.), we obtain the corresponding reaction functions

$$p_1 = \frac{p_2}{2},\ p_2 = \frac{V(e_1 - e_2) + e_2 p_1}{2e_1}. \tag{3}$$

By solving the system of equations (3), we obtain both duopolists' equilibrium prices

$$p_1^*(e_1, e_2) = \frac{V(e_1 - e_2)}{4e_1 - e_2},\ p_2^*(e_1, e_2) = \frac{2V(e_1 - e_2)}{4e_1 - e_2}. \tag{4}$$

Note that, as expected, $p_2^*(e_1, e_2) > p_1^*(e_1, e_2)$, i.e. the more polluting variety is offered at a lower price. Observe also that inequalities $0 < \widetilde{\theta} < \widehat{\theta} < \widehat{\widehat{\theta}}$ are satisfied in equilibrium:

$$0 < \frac{V}{4e_1 - e_2} < \frac{3V}{4e_1 - e_2} < \frac{V(2e_1 + e_2)}{e_2(4e_1 - e_2)}. \tag{5}$$

Finally, we can rewrite assumption 2 precisely as $V/\overline{\theta} < e_2(4e_1 - e_2)/(2e_1 + e_2)$, which guarantees that $\widehat{\widehat{\theta}} < \overline{\theta}$.

At this point it is convenient to define the variable $\lambda = e_1/e_2$, $\lambda > 1$, and rewrite both prices and quantities. This new variable measures the relative *degree of product differentiation*.[13] Suppose for the moment that an equilibrium exists (below, we analyze its existence, see Proposition 3). First, equilibrium prices can be rewritten as

$$p_1^*(\lambda) = \frac{V(\lambda - 1)}{4\lambda - 1},\ p_2^*(\lambda) = \frac{2V(\lambda - 1)}{4\lambda - 1}. \tag{6}$$

Note that as the relative product differentiation diminishes (λ), firms face a tougher price competition stage and equilibrium prices consequently fall down ($\partial p_i/\partial\lambda > 0$, $i = 1, 2$).

Each duopolist's aggregate pollution is given by $E_i = q_i e_i$, $i = 1, 2$. Using (6), equilibrium market shares can be rewritten as:

$$q_1(\lambda, e_2) = \frac{V}{\overline{\theta}(4\lambda - 1)e_2},\ q_2(\lambda, e_2) = \frac{2\lambda V}{\overline{\theta}(4\lambda - 1)e_2}. \tag{7}$$

do not compete for the consumers. In appendix A we show that when critical parameters are such that $0 < \widetilde{\theta} < \widehat{\theta} < \widehat{\widehat{\theta}} < \overline{\theta}$ an equilibrium does not exist.

[13] This variable simplifies computations. Moreover, it facilitates the interpretation of our findings (see Ronnen (1991) and Motta (1993)).

Therefore, dirtier and cleaner firm's aggregate pollution is

$$E_1 = \frac{\lambda V}{\overline{\theta}(4\lambda - 1)},\ E_2 = \frac{2\lambda V}{\overline{\theta}(4\lambda - 1)}, \tag{8}$$

respectively. Interestingly, aggregate pollution crucially depends on the degree of product differentiation. By differentiating, it is obtained that $\partial E_i/\partial\lambda < 0$, $i = 1, 2$. This means that total pollution increases as the degree of product differentiation decreases. The intuition is that if firms differentiate their products to a lower extent, they face a stronger price competition stage, that consequently lower equilibrium prices (see (6)). As a result, both firms' market sizes increase. Since total contamination depends on sales, it increases. To summarize:

Lemma 1 *Both firms' equilibrium aggregate pollution (E_i, $i = 1, 2$) increases as the equilibrium degree of product differentiation decreases (λ).*

As explained below, the equilibrium degree of product differentiation does not depend on V and $\overline{\theta}$ (see equation (15)). This enable us to already extract some comparative statics. First, note that both firms' aggregate pollution (see (8)) increase as the parameter V raises. This means that the higher the purchasers' valuation for the goods, the higher is the equilibrium industrial total contamination. This is simply due to the fact that the size of the active market is larger because buyers' willingness to pay for the goods is also higher. Second, observe that aggregate pollution decreases as parameter $\overline{\theta}$ increases. Therefore, contamination is a decreasing function of the degree of population's environmental awareness. This is also reasonable because the higher the average consumers' environmental consciousness, the lower is their average willingness-to-pay for the products. As a result, equilibrium sales are lower and, consequently, aggregate pollution is also lower. We summarize next:

Proposition 1 *Equilibrium aggregate pollution (a) increases as the valuation of the product (V) increases and (b) decreases as the degree of purchasers' environmental awareness ($\overline{\theta}$) increases.*

Next, we analyze firms' first stage decisions, i.e. environmental variety choices. Anticipating that second stage equilibrium prices will be given by equations (4), the dirtier firm chooses e_1 to maximize

$$\Pi_1(e_1, e_2) = \frac{V^2(e_1 - e_2)}{\overline{\theta}(4e_1 - e_2)^2} - C(e_1), \tag{9}$$

and the cleaner one selects e_2 to maximize

$$\Pi_2(e_1, e_2) = \frac{4V^2 e_1(e_1 - e_2)}{\overline{\theta}(4e_1 - e_2)^2 e_2} - C(e_2). \tag{10}$$

Note that both firms would obtain zero revenues in case they offered identical varieties ($e_1 = e_2$). To increase revenues (and hence profits), duopolists have an incentive to relax price competition by differentiating their goods. This is what actually happens in equilibrium. The f.o.c. for both firms' decision problems are:

$$\frac{V^2(4e_1 - 7e_2)}{\overline{\theta}(-4e_1 + e_2)^3} - C'(e_1) = 0 \tag{11}$$

$$\frac{4V^2 e_1(4e_1^2 - 3e_1e_2 + 2e_2^2)}{\overline{\theta}(-4e_1 + e_2)^3 e_2^2} - C'(e_2) = 0 \tag{12}$$

Using λ, we can rewrite conditions (11) and (12) as:

$$\frac{V^2\lambda^2(4\lambda - 7)}{\overline{\theta}(4\lambda - 1)^3} = -e_1^2 C'(e_1) \tag{13}$$

$$\frac{4V^2\lambda(4\lambda^2 - 3\lambda + 2)}{\overline{\theta}(4\lambda - 1)^3} = -e_2^2 C'(e_2) \tag{14}$$

Since $C(\cdot)$ is a homogeneous function of degree α, we can divide equations (13) and (14) to obtain:

$$4\lambda - 7 - 4\lambda^{\alpha}(4\lambda^2 - 3\lambda + 2) = 0 \tag{15}$$

Equation (15) describes those pairs $(\alpha, \lambda(\alpha))$ that may constitute an equilibrium. By graphically representing the zero-level contour curve defined by the left-hand side of equation (15), it is easily checked that this equation has a unique real solution satisfying $\lambda > 1$ for all $\alpha \leq -2$. Moreover, the solution verifies that $\lambda'(\alpha) > 0$, i.e. the lower the degree of homogeneity of the cost function, the lower is the equilibrium product differentiation. This finding allow us to state that:

Proposition 2 *The lower the degree of homogeneity (α) of the (fixed) cost function $C(\cdot)$, the higher is each firm equilibrium aggregate pollution (E_i, $i = 1, 2$).*

Note that α measures the degree of convexity of the cost function. The lower parameter α, the more convex is $C(\cdot)$. An increase in α implies that firms must incur higher fixed costs in order to differentiate their products. As a consequence, duopolists differentiate their goods to a lower extent in equilibrium. This fact together with lemma 1 proves the result.

In Appendix B, we demonstrate that $\alpha \leq -2$ is a necessary condition for a solution to equation (15) to be an equilibrium. In addition, it is checked that a solution to (13) and (14) satisfies the second order conditions and that both firms make positive profits. Unfortunately, it is not possible to prove in general that neither of the firms has an incentive to leapfrog its rival's choice. However, for the function $C(e) = \frac{k}{e^2}$, this is easily shown. The following proposition states the existence of the equilibrium:

Proposition 3 *For a subgame perfect equilibrium to exist, it must be the case that $\alpha \leq -2$. If an equilibrium exists, it is unique (up to a permutation of firms). The equilibrium product differentiation is given by equation (15) while the dirtier and the cleaner firm's equilibrium unit emissions are given by equations (13) and (14), respectively. The set of costs functions $C(e)$ for which an equilibrium exists is not empty.*

From equations (13) and (14), we can obtain additional comparative statics results. Using Euler's theorem and the fact that $\alpha \leq -2$, it is easily seen that the right-hand side of equation (13) is decreasing in e_1. Analogously, the right-hand side of expression (14) also decreases with e_2. These two facts enable us to state that:

Proposition 4 *Both firms' equilibrium unit emissions (e_i, $i = 1, 2$) (a) decrease as the valuation of the product (V) increases, and (b) increase as purchasers' environmental awareness ($\overline{\theta}$) increases.*

In equilibrium, consumers' surplus, aggregate profits, and total pollution can be written as:

$$CS = \frac{V^2\lambda(4\lambda+5)}{2\overline{\theta}(4\lambda-1)^2 e_2}, \tag{16}$$

$$\Pi_T = \frac{V^2(4\lambda+1)(\lambda-1)}{\overline{\theta}(4\lambda-1)^2 e_2} - C(e_1) - C(e_2), \tag{17}$$

$$E_T = \frac{3V\lambda}{4\lambda-1}. \tag{18}$$

These expressions will be used to evaluate policy effects on social welfare as explained above.

3 Unit Emission Standards

Consider now that the regulator imposes a *unit (maximum) emission standard.* Indeed, standards in the form of input content or emissions per unit of output is the most used form of environmental regulation. Let $\overline{e}$ denote this mandated standard and consider the unregulated market equilibrium analyzed previously (equations (13), (14) and (15)). Note first that an emission standard set above the level of emissions of the dirtier product (e_1) would not have effects at all on the previous unregulated equilibrium. Observe also that a very stringent standard might cause non-existence of a duopoly equilibrium. To avoid such troubles we here restrict the analysis to standards set arbitrarily close to and below the level of emissions chosen by the dirtier firm in the original (unregulated) equilibrium.[14]

Consider that $\overline{e}$ is set slightly below the dirtier firm's unit emissions in the unregulated equilibrium (e_1). In the new (regulated) equilibrium, the dirtier firm meets the requirement and the cleaner one chooses its emissions by best-responding its rival's choice (the standard). Thus,

$$e_1 = \overline{e} \tag{19}$$

and e_2 satisfies[15]

$$\frac{-4V^2\overline{e}(4\overline{e}^2 - 3\overline{e}e_2 + 2e_2^2)}{\overline{\theta}(4\overline{e} - e_2)^3 e_2^2} - C'(e_2) = 0 \tag{20}$$

[14] See Moraga-González (1997) for emission standards causing exit of the dirtier firm.

[15] Equation (20) is simply the f.o.c. for the cleaner firm's maximization problem.

It is useful to analyze the sign of the derivative

$$\frac{de_2}{d\overline{e}} = -\frac{\frac{8V^2(5\overline{e}+e_2)}{\overline{\theta}(4\overline{e}-e_2)^4}}{\frac{8V^2\overline{e}(16\overline{e}^3-16\overline{e}^2e_2+6\overline{e}e_2^2-3e_2^3)}{\overline{\theta}e_2^3(4\overline{e}-e_2)^4} - C''(e_2)}. \tag{21}$$

Note that the denominator of this expression is nothing else than the s.o.c. of firm 2's maximization problem, whose sign is negative in a neighborhood of the original equilibrium (see equation (47) in Appendix B). Since the numerator is clearly positive, we conclude that:

Lemma 2 *After imposing a unit emission standard arbitrarily close to and below the emissions of the variety chosen by the dirtier firm in the unregulated equilibrium, the cleaner firm best-responses by reducing its unit effluents as well.*

The direct implication of this lemma is that the establishment of a unit emission standard seems a proper policy to reduce goods' unit effluents.[16] The intuition stems from the fact that firms differentiate their products to avoid a tougher price competition stage (see (6)). Thus, anticipating that the dirtier firm will meet the standard, the best strategy for the cleaner one is to reduce the emissions of its variety too.

By using variable λ, we can describe the (new) equilibrium by equations:

$$e_2 = \frac{\overline{e}}{\lambda} \tag{22}$$

$$\frac{-4V^2\lambda^3(4\lambda^2-3\lambda+2)}{\overline{\theta}(4\lambda-1)^3\overline{e}^2} - \frac{C'(\overline{e})}{\lambda^{\alpha-1}} = 0. \tag{23}$$

Equation (23) gives the implicit relation between the equilibrium product differentiation and the effluent standard imposed. By employing both Euler's and the implicit function theorems, we find that

$$\frac{d\lambda}{d\overline{e}} = -\frac{\overline{e}(\alpha+1)C'(\overline{e})\overline{\theta}(4\lambda-1)^4}{4V^2\lambda^{\alpha+1}(\alpha(16\lambda^3-16\lambda^2+11\lambda-2)+16\lambda^3-16\lambda^2+\lambda-4)}. \tag{24}$$

We can evaluate the sign of this derivative in a neighborhood of the original equilibrium, i.e. where α and λ satisfy 15. Since $\alpha \leq -2$, the numerator of this expression is positive. Therefore $d\lambda/d\overline{e}$ has positive sign whenever $\alpha(16\lambda^3-16\lambda^2+11\lambda-2)+16\lambda^3-16\lambda^2+\lambda-4<0$. In other words, whenever

$$\frac{\ln\frac{4\lambda-7}{4(4\lambda^2-3\lambda+2)}}{\ln\lambda}(16\lambda^3-16\lambda^2+11\lambda-2)+16\lambda^3-16\lambda^2+\lambda-4<0. \tag{25}$$

It is easily checked that this expression is negative. Therefore

Lemma 3 *Equilibrium product differentiation reduces after imposing a maximum emission standard arbitrarily close to and below the emissions of the variety chosen by the dirtier firm in the unregulated equilibrium.*

[16] This result is in line with Ronnen (1991), Motta and Thisse (1993) and Crampes and Hollander (1995).

The existence of decreasing returns in the abatement technology is crucial for this result. As a result of the mandated standard, the dirtier firm diminishes its unit effluents, which reduces product differentiation and fosters price competition. To alleviate the effects of the tougher interaction, the cleaner firm decreases its unit emissions as well. However, due to the fact that abatement technology exhibits decreasing returns, the cleaner firm's effort to reduce effluents is lower than the dirtier's one. As a result, after-policy equilibrium product differentiation reduces.

Taking into consideration the effects of the measure on product differentiation, and hence on prices, firms' sales, and purchasers' allocation in the market, we can state that:

Proposition 5 *After imposing a maximum emission standard arbitrarily close to and below the emissions of the variety chosen by the dirtier firm in the unregulated equilibrium, both firms' aggregate pollution increases.*

The proof follows immediately from lemmas (1) and (3). The key issue here is that the decrease in the product differentiation fosters price competition and, therefore, firms' equilibrium prices fall. Both effects, the decrease in prices and the decrease in products' unit effluents, soar firms' sales. This negative effect with respect to aggregate pollution compensates for the positive effects of the policy on the equilibrium unit emissions of the varieties, thus leading to an increase in total contamination.

We next evaluate the social welfare effects of the introduction of a unit emission standard. We analyze its effects on consumers surplus, aggregate firms' profits and total pollution separately. From (16), we can compute

$$\frac{dCS}{d\overline{e}} = -\frac{V^2(28\lambda+5)}{2\overline{\theta}e_2(4\lambda-1)^3}\frac{d\lambda}{d\overline{e}} - \frac{V^2\lambda(4\lambda+5)}{2\overline{\theta}e_2^2(4\lambda-1)^2}\frac{de_2}{d\overline{e}}.$$

Since $\frac{d\lambda}{d\overline{e}} > 0$ and $\frac{de_2}{d\overline{e}} > 0$ (see lemmas 2 and 3), then $\frac{dCS}{d\overline{e}} < 0$. Therefore, consumer surplus increases after imposing the unit effluent standard. Consumers clearly beneficiate from the policy not only because it lowers goods' unit emissions but also because it brings about a price war strong enough to lower prices substantially.

Aggregate profits (see (17)) can be written as

$$\Pi_T = \frac{V^2(4\lambda^2-3\lambda-1)\lambda}{\overline{\theta}(4\lambda-1)^2\overline{e}} - C(\overline{e}) - C(\frac{\overline{e}}{\lambda}).$$

By taking derivatives,

$$\begin{aligned}\frac{d\Pi_T}{d\overline{e}} &= \left(\frac{V^2(16\lambda^3-12\lambda^2+10\lambda+1)}{\overline{\theta}\overline{e}(4\lambda-1)^3} + C'\left(\frac{\overline{e}}{\lambda}\right)\frac{\overline{e}}{\lambda^2}\right)\frac{d\lambda}{d\overline{e}} \\ &\quad -\frac{V^2\lambda(4\lambda^2-3\lambda-1)}{\overline{\theta}\overline{e}^2(4\lambda-1)^2} - C'\left(\frac{\overline{e}}{\lambda}\right)\frac{1}{\lambda} - C'(\overline{e}).\end{aligned}$$

In a neighborhood of the original equilibrium, $C'\left(\frac{\overline{e}}{\lambda}\right)$ satisfies equation (14). Employing this relationship, and that $\frac{d\lambda}{d\overline{e}} > 0$, it is easily seen that $\frac{d\Pi_T}{d\overline{e}} > 0$. Therefore, equilibrium industry profits decrease after the introduction of the standard.

Putting together consumers' and producers surplus and evaluating its sign in a neighborhood of the unregulated equilibrium (i.e. where α and λ satisfy equation (15)), we obtain that the net effect of the standard is an increase in market surplus.

Finally, by lemma (1), we know that aggregate pollution increases. This allows us to state that:

Proposition 6 *After imposing a unit emission standard arbitrarily close to and below the emissions of the dirtier firm in the unregulated equilibrium, social welfare increases (decreases) if and only if*

$$\left.\frac{\partial\Phi(\cdot)}{\partial\overline{e}}\right|_{\overline{e}=e_1^*} > (<) \left.\frac{\partial(CS+\Pi_T)}{\partial\overline{e}}\right|_{\overline{e}=e_1^*}.$$

This proposition shows that social welfare might decrease after introducing a maximum emissions standard that lower both products' unit emissions. This is due to the adverse effect that an increase in sales causes on aggregate pollution.

4 Technology Subsidization

Besides unit standards, direct subsidy on technological costs are widely used to impel firms' investment in abatement or to mitigate the economic impact of compliance with regulation. The subsidization of technological acquisition results in lower capital costs through, generally, cheaper loans, tax allowances or grants. For instance, France offer loans to control water pollution; Italy favors industries that commit themselves to introduce production processes that recuperate and recycle solid waste; The Netherlands offers financial assistance to promote compliance with regulation, promote technology research and the introduction of pollution control; Sweden employ subsidies to diminish pesticide sprays (see Hanley *et al.* (1997)). This type of subsidies does not generally depend on emissions' reduction but on firms' investment. Indeed, most of government aids to environmental investment are not subject to pollution abatement levels. That is the case for example in the PITMA program of the Spanish Ministry of Industry and Trade since 1991 or the US subsidies to construct water treatment plants and soil conservation efforts of farmers.[17]

Consider that the industry we have described benefits from a subsidization policy that reduces abatement costs. Let the subsidized cost function be

$$C_i = C(e_i)(1-s)\ i=1,2. \tag{26}$$

As above, to avoid non-existence problems of a duopoly equilibrium, we concentrate on the analysis of small subsidies. F.o.c. would be in this case:

$$\frac{V^2\lambda^2(4\lambda-7)}{\overline{\theta}\left(-1+4\lambda\right)^3(1-s)} = -e_1^2C'(e_1) \tag{27}$$

[17] These type of aids tend to avoid conflicts with the 'polluters-pay-principle' which would be violated using subsidies defined over the emission reduction.

$$\frac{4V^2\lambda(4\lambda^2-3\lambda+2)}{\overline{\theta}\left(-1+4\lambda\right)^3(1-s)} = -e_2^2C^{'}(e_2). \tag{28}$$

Note first that a subsidy s per dollar of abatement cost does not affect equilibrium product differentiation. Indeed, by dividing equations (27) and (28), it is obtained that λ must satisfy equation (15). However, both goods' unit emissions fall. In fact, by applying both the Euler's and implicit function theorems to equations (27) and (28), we obtain that

$$\frac{de_i}{ds} = \frac{e_i}{(1-s)(1+\alpha)} > 0, \ i = 1, 2. \tag{29}$$

Observe also that this effect is smaller the greater the curvature of firms' cost function, i.e. $\frac{de_i^2}{d\alpha ds} = e_i/\left(s-1\right)\left(1+\alpha\right)^2 < 0$, $i = 1, 2$.

Since λ does not change in the new equilibrium, neither equilibrium prices (see (6)) nor aggregate emissions (see (8)) varies with s. Even though both firms lower unit emissions, sales increase such that the reduction of unit effluents is compensated by the increase in production.

We next evaluate the effects of the introduction of the subsidy on social welfare. First, consumers surplus increases as s increases (from (16) and (29)). Second, aggregate profits can be written in this case as

$$\Pi_T = \frac{V^2(4\lambda^2-3\lambda-1)}{\overline{\theta}e_2(4\lambda-1)^2} - (1-s)(1+\lambda^\alpha)C(e_2). \tag{30}$$

By properly deriving and employing Euler's theorem, we obtain

$$\begin{aligned}\frac{\partial\left(\Pi_T\right)}{\partial s} &= \frac{-V^2}{\overline{\theta}(1-s)e_2(4\lambda-1)^3(1+\alpha)} \\ &\quad \left[(4\lambda^2-3\lambda-1)(4\lambda-1) + \frac{4\lambda(1+\lambda^\alpha)(4\lambda^2-3\lambda+2)}{\alpha}\right].\end{aligned}$$

Evaluating this expression in a neighborhood of the unregulated equilibrium, i.e. employing equation (15), it is easily seen that $\frac{\partial\Pi_T}{\partial s} > 0$. Finally, since λ does not vary with s, aggregate pollution remains unaltered. We summarize our findings next:

Proposition 7 *A direct subsidy on abatement investment (a) reduces both firms unit emissions and (b) has no impact on the equilibrium aggregate pollution in the market. As to the welfare consequences, it increases consumers surplus and firms' profits. Therefore, social welfare increases.*

5 Product charges

Polluting product charges are widely used to influence firms' behavior to reduce the quantity or improve the quality of pollution. By putting a charge directly on the product or input that causes environmental damage, the regulation avoids the information problems associated with first-best schemes such as emission or ambient charges. This is the case for Sweden and Norway, where product charges are applied to batteries, fertilizers and pesticides. Tobacco, fossil-fuels and cars, are other examples of goods facing special charges. Moreover, current government agendas, e.g. Italy's one, include special taxes for energy production through an ecotax and product containers (e.g. Italy levies a tax on plastic bags).[18]

Consider that an ad-valorem charge t_i is imposed on firm i, $i = 1, 2$. Firm i's profit is then given by:

$$\pi_i = (1 - t_i)p_i q_i - C(e_i),\ i = 1, 2. \tag{31}$$

We define $\tau_i = 1/(1 - t_i)$ as the tax burden of the firm i. Note first that $\tau_i \geq 1$ defines a tax ($0 \leq t_i \leq 1$) while $\tau_i \leq 1$ defines a subsidy ($t_i \leq 0$). Besides, setting $\tau_1 = \tau_2 = 1$, we obtain the unregulated equilibrium analyzed above. As explained above, we concentrate the analysis on values of τ_i near 1. Multiplying (31) by τ_i, we obtain:

$$\tau_i \pi_i = p_i q_i - \tau_i C(e_i),\ i = 1, 2. \tag{32}$$

The following observations are useful in what follows: Both the firm i's optimal strategy facing the commodity tax t_i and the cost function $C(e_i)$, and the one facing the cost function $\tau_i C(e_i)$ and not being taxed at all, are exactly the same. Note also that, for any (e_1, e_2), equilibrium prices under emission charges are the same as those under no regulation, i.e. prices (6).

Given equilibrium prices, optimal emissions are given by the equations:

$$\frac{V^2(4\lambda - 7)}{\overline{\theta}\,(4\lambda - 1)^3\, e_2^2 \tau_1} = -C'(e_1) \tag{33}$$

$$\frac{4V^2\lambda(4\lambda^2 - 3\lambda + 2)}{\overline{\theta}\,(-1 + 4\lambda)^3\, e_2^2 \tau_2} = -C'(e_2). \tag{34}$$

By dividing these two equations, we obtain that

$$\gamma = (4\lambda - 7) / \left(4\lambda^2 - 3\lambda + 2\right) 4\lambda^{\alpha}, \tag{35}$$

where the parameter $\gamma = \tau_1/\tau_2$ measures firms' charge differentiation. Interestingly, if $\tau_1 = \tau_2$, then $\gamma = 1$, therefore, equations (15) and (35) coincide. This implies that the equilibrium product differentiation does not change whenever both firms charges are identical. By properly differentiating (35), we obtain

$$\frac{d\lambda}{d\gamma} = -\frac{-4\left(4\lambda^2 - 3\lambda + 2\right)\lambda^{\alpha}}{4(1 - \lambda^{\alpha}\gamma((8\lambda - 3) + \left(4\lambda^2 - 3\lambda + 2\right)\frac{\alpha}{\lambda})} \tag{36}$$

Evaluating this derivative in a neighborhood of the unregulated equilibrium, i.e. where $\gamma = 1$ and α and λ satisfy equation (15), it is obtained that $d\lambda/d\gamma > 0$. Therefore:

[18] See Hanley *et al.* (1997).

Lemma 4 *(a) If the dirtier firm's charge increases, equilibrium product differentiation increases. (b) If the cleaner firm's charge increases, equilibrium product differentiation decreases. If both firms' charges vary such that $\gamma = 1$, equilibrium product differentiation does not change.*

This lemma implies that the strategic effects of imposing a product charge to an individual firm are essentially different. This will have interesting implications on the policy effects as explained below.

5.1 Uniform commodity taxation

Consider first that both firms are charged equally, i.e. $\tau_1 = \tau_2 = \tau$. A uniform rate can be justified in terms of information requirements or due to possible legal constraints impeding charge rates differentiation among firms participating in the same industry. An illustrative example of this policy is the green point charge in Spain, where all types of market containers face a common charge on the product value.

Applying both the Euler's and the implicit function theorems to equations (33) and (34) we obtain that

$$\frac{de_i}{d\tau} = -\frac{e_i}{\tau(1+\alpha)} > 0; \; i = 1, 2. \tag{37}$$

Equation (37) and lemma (4) allow us to state that:

Proposition 8 *The introduction of a uniform ad-valorem product charge a) increases both firms' equilibrium unit emissions and b) does not affect aggregate pollution levels.*

The reason is that even though unit emissions increase, firms end up selling less quantity of their products.

Finally we evaluate social welfare effects. Consumer surplus is given by (16). Since equilibrium product differentiation does not change with τ, from (37) we conclude that consumer surplus decreases. Firms' profits, on the other hand, are given by

$$\Pi_T = \frac{V^2(4\lambda^2 - 3\lambda - 1)}{\tau e_2 (4\lambda - 1)^2 \overline{\theta}} - (1 + \lambda^\alpha) C(e_2). \tag{38}$$

By taking derivatives and using Euler's Theorem, we obtain:

$$\frac{\partial \Pi_T}{\partial \tau} = -\frac{V^2}{\tau e_2 \overline{\theta} (4\lambda - 1)^3} \left(\frac{\alpha}{\alpha + 1} (4\lambda + 1)(\lambda - 1) + 4\lambda\lambda^\alpha (4\lambda^2 - 3\lambda + 2) \right). \tag{39}$$

which is clearly negative. To summarize:

Proposition 9 *A small uniform product charge decreases consumers' surplus and firms' profits while aggregate contamination does not vary. As a result, social welfare decreases.*

5.2 Non-uniform product charges

Consider now the case where firms face different charge rates. Given that a uniform charge does not affect total emissions, we want to investigate the effects of tax differentiation in this market. This is the case, for example, of fuels in most European countries, where unleaded fuel faces lower tax rates.

By undertaking some algebra and evaluating the following derivatives in a neighborhood of the unregulated equilibrium (i.e. where $\tau_i = 1$, $i = 1, 2$ and α and λ satisfy (15)), it is checked that

$$\frac{de_i}{d\tau_i} = \frac{\partial e_i}{\partial \tau_i} + \frac{\partial e_i}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_i} > 0; i = 1, 2. \tag{40}$$

$$\frac{de_j}{d\tau_i} = \frac{\partial e_j}{\partial \lambda}\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_i} > 0;\ j \neq i;\ i = 1, 2. \tag{41}$$

By lemmas 1 and 4, and (40) and (41) we can state that:

Proposition 10 *Consider the duopoly game described above where both firms face a uniform ad-valorem product charge policy. Then,*
(1) by slightly raising (diminishing) the charge of the dirtier firm, (a) both firms' unit emissions increase (decrease) and (b) aggregate pollution decreases (increases).
(2) by slightly raising (diminishing) the charge of the cleaner firm, (a) both firms' unit emissions increase (decrease) and (b) aggregate pollution increases (decreases).

This result illustrates the existence of an equivalence between the effects on aggregate pollution of an increase (decrease) of the dirtier firm's charge and a decrease (increase) of the cleaner firm's charge. Both policies diminish (raise) total pollution.

Now we proceed to evaluate the effects of charge differentiation on social welfare. First, changes in consumers surplus are given by (see (16))

$$\frac{\partial CS}{\partial \tau_i} = \left(\frac{\partial CS}{\partial e_2}\frac{\partial e_2}{\partial \lambda} + \frac{\partial CS}{\partial \lambda}\right)\frac{\partial \lambda}{\partial \gamma}\frac{\partial \gamma}{\partial \tau_i};\ i = 1, 2. \tag{42}$$

To evaluate the sign of this derivative, note that from (16), it follows that $\partial CS/\partial e_2 < 0$ and $\partial CS/\partial \lambda < 0$. From (34), it is seen that $\partial e_2/\partial \lambda > 0$. Lemma 4 implies that $\partial \lambda/\partial \gamma > 0$. Therefore, the sign of the derivative (42) depends on the sign of $\partial \gamma/\partial \tau_i$.

Indeed, consumers surplus increases as cleaner firm's charge raises, while it decreases as dirtier firm's tax increases. The intuition behind this difference is that, even though unit emissions increase in both cases (diminishing CS), the increase of τ_2 decreases product differentiation, thus fostering price competition (hence increasing CS). The later dominates the former.

Second, aggregate profits are given by

$$\Pi_T = \frac{V^2(\lambda - 1)(4\lambda + \frac{1}{\gamma})}{e_2\overline{\theta}(4\lambda - 1)^2\tau_2} - (1 + \lambda^a)C(e_2).$$

Then, the effects of changes of firms charge rates on aggregate profits are given by

$$\frac{\partial \Pi_T}{\partial \tau_i} = \left(\left(\frac{\partial \Pi_T}{\partial e_2}\frac{\partial e_2}{\partial \lambda} + \frac{\partial \Pi_T}{\partial \lambda}\right)\frac{\partial \lambda}{\partial \gamma} + \frac{\partial \Pi_T}{\partial \gamma}\right)\frac{\partial \gamma}{\partial \tau_i};\ i = 1, 2. \tag{43}$$

Note that $\partial\Pi_T/\partial e_2 < 0$, $\partial e_2/\partial\lambda > 0$ and $\partial\Pi_T/\partial\lambda > 0$. By evaluating this derivative in a neighborhood of the equilibrium ($\gamma = 1$ and α and λ satisfy (15)), we obtain that the sign of this derivative is negative.

Moreover, the net effect over market surplus (i.e. consumer surplus plus total profits) is negative in both cases. Taking into account lemmas 1 and 4, we can state that:

Proposition 11 *(a) By slightly increasing the product charge to the dirtier firm market surplus and aggregate emissions decrease. Therefore, social welfare increases if and only if:*

$$\left.\frac{\partial\Phi(\cdot)}{\partial\tau_1}\right|_{\tau_1=\tau_2,\gamma=1} > (<) \left.\frac{\partial(CS+\Pi_T)}{\partial\tau_1}\right|_{\tau_1=\tau_2,\gamma=1}.$$

(b) By slightly increasing the product charge to the cleaner firm market surplus decreases and aggregate emissions increase. Therefore, social welfare decreases.

Note that these results are particularly interesting in the case where total pollution levels are considered by the regulator in order to differentiate the tax rate.

6 Discussion and conclusions

In this paper we have examined the impact of some environmental policy instruments on aggregate pollution in a differentiated industry. Our results demonstrate that pollution might increase as a result of government regulation. More precisely, in our model contamination increases after setting a unit emission standard and by imposing a product charge to the cleaner firm. This counter-productive effect is due to the firms' strategic response to the regulation. Moreover, environmental policies might result in social welfare losses. Our findings suggest that environmental regulatory policy in differentiated markets must take into consideration not only its effects on the products' environmental features but also its implications on the consumers' allocations between firms. This issue has not been pointed out by the literature so far.

Throughout we have employed some assumptions that deserve a discussion. Our modeling choice included non-covered market situation and technological fixed costs. Alternatively, we could have used a covered market with emission-dependent variable costs (as in Crampes and Hollander (1995)). Some computations have turned out to show that the fact that pollution might increase as a result of government regulation is robust to these modeling changes. In fact, in this latter case, regulation affects also product differentiation, which leads to a re-division of market shares between the firms. As a result, pollution may also increase.

Goods' emissions do not necessarily enter linearly in the decision rule of the purchasers, as opposed to our assumption. For instance, we could have used the purchasers' decision rule $V - \theta t(e) - p \geq 0$. The main difference is that pollution would not always increase as a result of the reduction of product differentiation (as opposed to our lemma 1). However, it can be seen that the result that contamination might increase as a consequence of government policy would be reproduced under some parametric conditions.

The effects of other instruments – such as marketable permits – on aggregate pollution remain to be investigated. First-best schemes – such as Pigouvian taxes – introduce complex computations in this type of models. Even though the effects of Pigouvian taxes must be carefully studied, we conjecture that there might be parametric spaces for which they are welfare reducing.

7 Appendix

7.1 Appendix A

Here we show that if critical parameters are such that $0 < \widetilde{\theta} < \widehat{\theta} < \widehat{\widehat{\theta}} < \overline{\theta}$, an equilibrium does not exist. Demands would be given by

$$q_1(\cdot) = \frac{p_2 - p_1}{(e_1 - e_2)\overline{\theta}},\ q_2(\cdot) = \frac{V - p_2}{\overline{\theta} e_2} - \frac{p_2 - p_1}{\overline{\theta}(e_1 - e_2)} \tag{44}$$

By optimizing profits with respect to prices and solving the reaction functions we obtain prices

$$p_1 = \frac{V(\lambda - 1)}{4\lambda - 1},\ p_2 = \frac{2V(\lambda - 1)}{4\lambda - 1}. \tag{45}$$

However, since $\lambda > 1$, this equilibrium prices do not satisfy $\widehat{\theta} < \widehat{\widehat{\theta}}$. Therefore, this parameter configuration cannot be part of an equilibrium.

7.2 Appendix B

First note that equation (15) has a real solution satisfying $\lambda \geq 1$ only if $\alpha \leq -2$. Next, we prove that second order conditions (s.o.c.) are verified and that both firms make positive profits in equilibrium. Finally, we show that for the case $C(e) = \frac{k}{e^2}$, neither of the firms has an incentive to leapfrog its rival's choice. Therefore, the set of functions for which an equilibrium exists is not empty.

Both firms s.o.c. are

$$\frac{\partial^2 \Pi_1}{\partial e_1^2} = \frac{16V^2(2e_1 - 5e_2)}{\overline{\theta}(4e_1 - e_2)^4} - C''(e_1), \tag{46}$$

$$\frac{\partial^2 \Pi_2}{\partial e_2^2} = \frac{8V^2 e_1(16e_1^3 - 16e_1^2 e_2 + 6e_1 e_2^2 - 3e_2^3)}{\overline{\theta} e_2^3 (4e_1 - e_2)^4} - C''(e_2). \tag{47}$$

By using Euler's theorem and equation (11), equation (46) can be rewritten as

$$\frac{\partial^2 \Pi_1}{\partial e_1^2} = \frac{16V^2(2e_1 - 5e_2)}{\overline{\theta}(4e_1 - e_2)^4} + \frac{(\alpha - 1)V^2(4e_1 - 7e_2)}{\overline{\theta} e_1 (4e_1 - e_2)^3}. \tag{48}$$

By rearranging and using the variable λ, this expression reduces to

$$\frac{\partial^2 \Pi_1}{\partial e_1^2} = \frac{V^2(16\lambda(2\lambda - 5) + (\alpha - 1)(4\lambda - 7)(4\lambda - 1))}{\overline{\theta}\lambda(4\lambda - 1)^4 e_2^3}, \tag{49}$$

which has negative sign whenever $16\lambda(2\lambda - 5) + (\alpha - 1)(4\lambda - 7)(4\lambda - 1) < 0$. By using the equilibrium equation (15) we can substitute the value of α to obtain the inequality

$$16\lambda(2\lambda - 5) + \left(\frac{\ln \frac{4\lambda - 7}{4(4\lambda^2 - 3\lambda + 2)}}{\ln \lambda} - 1 \right)(4\lambda - 7)(4\lambda - 1) < 0. \tag{50}$$

It is easy to check that the left-hand side of inequality (50) defines a negative continuously decreasing function in the relevant range (i.e. its derivative is a strictly negative concave function for all $\lambda > 1$; in fact, it reaches a maximum of $-82.4654 < 0$ at $\lambda = 2.68915$). Therefore, the s.o.c. for the dirtier firm is always satisfied.

By proceeding analogously, the cleaner firm's s.o.c. (47) can be rewritten as

$$\frac{\partial^2 \Pi_1}{\partial e_1^2} = \frac{4\lambda V^2(32\lambda^3 - 32\lambda^2 + 12\lambda - 6 + (\alpha - 1)(4\lambda - 1)(4\lambda^2 - 3\lambda + 2))}{\overline{\theta}(4\lambda - 1)^4 e_2^3}. \tag{51}$$

This expression is negative whenever $32\lambda^3 - 32\lambda^2 + 12\lambda - 6 + (\alpha - 1)(4\lambda - 1)(4\lambda^2 - 3\lambda + 2) < 0$. By again using (15), this inequality reduces to

$$32\lambda^3 - 32\lambda^2 + 12\lambda - 6 + \left(\frac{\ln \frac{4\lambda - 7}{4(4\lambda^2 - 3\lambda + 2)}}{\ln \lambda} - 1 \right)(4\lambda - 1)(4\lambda^2 - 3\lambda + 2) < 0. \tag{52}$$

This expression has indeed a negative sign everywhere (for $\lambda > 1$). In fact, it is a strictly concave expression reaching its maximum value at $\lambda = 2.03375$. At this point, the inequality reduces to $-406.415 < 0$. Therefore, the cleaner firm's s.o.c. is satisfied too.

We now check that both firms make positive profits in equilibrium. First, dirtier firm's profits are given by (9). By using Euler's theorem and properly rearranging, firm 1's benefits can be rewritten as

$$\Pi_1 = \frac{V^2(\alpha(4\lambda - 1)(\lambda - 1) + \lambda(4\lambda - 7))}{\overline{\theta}\alpha(4\lambda - 1)^3 e_2}. \tag{53}$$

This expression is positive as long as $\alpha(4\lambda - 1)(\lambda - 1) + \lambda(4\lambda - 7) < 0$, i.e. whenever (using (15))

$$\frac{\ln \frac{4\lambda - 7}{4(4\lambda^2 - 3\lambda + 2)}}{\ln \lambda}(4\lambda - 1)(\lambda - 1) + \lambda(4\lambda - 7) < 0. \tag{54}$$

The function defined by the left-hand side of inequality (54) is strictly concave and reaches a maximum value of $-36.9502 < 0$ at $\lambda = 2.05982$. Therefore, dirtier firm obtains positive profits in equilibrium.

Second, equation (10) gives cleaner firm's equilibrium profits. It can be rewritten as

$$\Pi_2 = \frac{4\lambda V^2(\alpha(4\lambda - 1)(\lambda - 1) + 4\lambda^2 - 3\lambda + 2)}{\overline{\theta}\alpha(4\lambda - 1)^3 e_2}. \tag{55}$$

This expression has a positive sign as long as $\alpha(4\lambda - 1)(\lambda - 1) + 4\lambda^2 - 3\lambda + 2) < 0$. By using (15) again, this inequality reduced to

$$\frac{\ln \frac{4\lambda - 7}{4(4\lambda^2 - 3\lambda + 2)}}{\ln \lambda}(4\lambda - 1)(\lambda - 1) + 4\lambda^2 - 3\lambda + 2 < 0. \tag{56}$$

The left hand-side of this equation is strictly concave, reaching a maximum of $-26.579 < 0$ at $\lambda = 2.13086$. Therefore, firm 2's profits are positive.

Finally, we show that for the cost function $C(e) = \frac{k}{e^2}$, neither of the firms has an incentive to leapfrog its rival's choice. Therefore, the set of cost functions for which an equilibrium exists is not empty. Suppose first that firm 2 chooses $e_2^*(\lambda^*)$ given by equations (12) and (15). Suppose that firm 1 deviates by choosing $e_1 < e_2^*$. Then, in the second stage, since environmental choices are observed before firms set their prices, firm 2 will optimally fix price p_1 in equation (4) while firm 1 will set price p_2 also en (4). Profits from such a deviation are given by

$$\Pi_1(e_1 < e_2^*, e_2^*) = \frac{4V^2 e_2^*(e_2^* - e_1)}{\overline{\theta}(4e_2^* - e_1)^2 e_1} - C(e_1). \tag{57}$$

For $C(e) = \frac{k}{e^2}$, we have

$$\Pi_1(e_1, e_2^*) = \frac{31.5818kV^2(7.89544k\overline{\theta} - e_1V^2)}{e_1(31.5818k\overline{\theta} + e_1V^2)^2} - \frac{k}{e_1^2}. \tag{58}$$

The unique solution satisfying $e_1 > e_2^*$ and the first and second order conditions is $\widehat{e}_1 = 5.95014k\overline{\theta}/V^2$ which gives profits $\widehat{\Pi}_1 = -0.00271284V^8/k\overline{\theta}^4 < 0$. Clearly, seller 1 has no incentives to leapfrog the cleaner seller's choice.

Suppose second that firm 1 chooses $e_1^*(\lambda^*)$ given by equations (11) and (15) and that firm 2 deviates by choosing $e_2 > e_1^*$. Then, as above, firm 2 will optimally fix price p_1^* in equation (4) while firm 1 will set price p_2^*. Profits from such a deviation are given by

$$\Pi_2(e_1^*, e_2 > e_1^*) = \frac{V^2(e_2 - e_1^*)}{\overline{\theta}(4e_2 - e_1^*)^2} - C(e_2). \tag{59}$$

For $\alpha = -2$, we have

$$\Pi_2(e_1^*, e_2) = \frac{0.0241192V^4(0.0241192e_2V^2 - k\overline{\theta})}{\overline{\theta}(k\overline{\theta} - 0.0964767e_2V^2)^2} - \frac{k}{e_2^2} \tag{60}$$

In this case, the only solution satisfying $e_2 > e_1^*$ and the first and second order conditions is $\widehat{e}_2 = 95.2009k\overline{\theta}/V^2$ which gives profits $\widehat{\Pi}_2 = 0.00356346V^4/k\overline{\theta}^2$. These profits are clearly lower than equilibrium profits for this case $\Pi_2^* = 0.0122193V^4/k\overline{\theta}^2$. Therefore, firm 2 will not leapfrog its rival's choice.

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