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A note on Measurement Error and Euler Equations: an Alternative to Log-Linear Approximations

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Abstract

Empirical research based on panel data has to pay special attention to measurement errors. Utility maximization often yields nonlinear decision rules in which measurement errors enter in a multiplicative way. The usual strategy to deal with them consists of taking log-linear approximations of the equations to estimate. The expression to be estimated then includes a new error component and the estimators could be biased and inconsistent. We describe one particular parameterization that avoids linearizing the equation we want to estimate.

1 Introduction

Most of the panel data tests based on the Euler equations associated with the life-cycle hypothesis , use a log-linear approximation of the equation they estimate and impose restrictions on the covariance structure of the forecast errors (See [Altonji and Siow 87], [MaCurdy 83], [Runkle 91] ,[Browning, Deaton and Irish 85] or [Zeldes 89]). Other authors do not need to resort to such approximations as they directly assume that markets are complete (See [Altug and Miller 89]) or impose a quadratic utility function which allows them to find a closed form solution for the dependent variable (See [Hall and Mishkin 82]).

To illustrate our point, consider the following example in wich the individual maximizes expected utility of consumption over a finite time horizon. At any time he/she must satisfy his/her budget constraint.

To make things as simple as possible, assume that individuals are free to lend and borrow as much as they want, consumption is separable from labor and consumption is intertemporally separable.

Individual i solves the following maximization problem

$$\max \quad \mathrm{E}_{i0} \textstyle\sum_{t=0}^{T} \beta^t u_i(c_i(t))$$

$$\text{s.t.} \quad \sum_{k=1}^{K} p_k(t) c_{ki}(t) + A_i(t) = (1 + r(t))A_i(t-1) + y_i(t)$$

$$t = 0, 1, ..., T \qquad \text{and} \qquad A_i(T+1) = 0$$

Here E_{i0} is the mathematical expectation conditional to the information set of individual i at time 0, T is the remaining life span, which we assume known with certainty, β is the time discount factor, u_i is the individual specific utility function, $c_i(t)$ is the consumption vector of individual i at time t, $y_i(t)$ is the labor income of individual i in period t, $c_{ki}(t)$ is commodity k consumption of individual i at time t, $p_k(t)$ is the price of commodity k at time t, $A_i(t)$ is total nominal wealth of individual i at the end of period t, $r(t)$ is the nominal net interest rate (could also be individual specific) and $A_i(T+1) = 0$ is a simplifying terminal condition.

In what follows we drop the i subscript to ease the notation.

The first order conditions for this problem can be combined to get the following equation

$$\mathrm{E}_t \left(\beta \frac{u'_k(c(t+1))}{u'_k(c(t))} R_k(t+1) - 1 \right) = 0 \qquad (1)$$

where $R_k(t+1) = (1 + r(t+1)\frac{p_k(t)}{p_k(t+1)}$ and $u'_k(c(t))$ is the partial derivative of the utility function with respect to $c_k(t)$.

In order to proceed to estimate the parameters of the model, we write

$$\beta \frac{u_k'(c(t+1))}{u_k'(c(t))} R_k(t+1) - 1 = \epsilon(t+1) \tag{2}$$

By construction, $\epsilon(t+1)$ is orthogonal to any variable belonging to the individual's information set at time t.

For the sake of simplicity assume[1] that $u_k'(c(t)) = \theta c_k(t)^\alpha$ with $\alpha < 0$.

Then

$$\beta \left(\frac{c_k(t+1)}{c_k(t)} \right)^\alpha R_k(t+1) - 1 = \epsilon(t+1) \tag{3}$$

If consumption is measured with error, that is if

$$c_k(t) = \tilde{c}_k(t)\phi(t)$$

where $\tilde{c}_k(t)$ is measured consumption and $\phi(t)$ is the measurement error, we can write

$$\beta \left(\frac{\tilde{c}_k(t+1)}{\tilde{c}_k(t)} \right)^\alpha R_k(t+1) = (1 + \epsilon(t+1)) \left(\frac{\phi(t)}{\phi(t+1)} \right)^\alpha \tag{4}$$

The right hand side of this equation includes a measurement error component. The model implies that $\epsilon(t+1)$ is orthogonal to the individual's information set at time t. Therefore its conditional expectation is 0. But the model does not give us the value of the composite error's expectation, so we can not derive orthogonality conditions that can be used with measured consumption.

The usual approach in the existing literature consist of taking natural logarithms and a Taylor expansion of $\log(1+\epsilon(t+1))$ around $\epsilon(t+1) = 0$. Therefore

$$\Delta \ln \tilde{c}_k(t) = -\frac{1}{\alpha} \ln \beta - \frac{1}{\alpha} \ln R_k(t+1) - \Delta \ln \phi(t) + \frac{1}{\alpha}(\epsilon(t+1) + \nu(t+1))$$

It is assumed that the composite error $-\Delta \ln \phi(t) + \frac{1}{\alpha}(\epsilon(t+1) + \nu(t+1))$ is orthogonal to information dated t-1 and earlier[2]. But here, $\nu(t+1)$ is an approximation error whose conditional expectation does not need to be 0 and therefore these estimates are, in fact, inconsistent.

Some authors try to minimize the error by taking a second degree approximation to $\ln(1+\epsilon(t+1))$ and postulate that $E(\epsilon^2(t+1)) = \sigma^2$ (See [Runkle 91]). But that

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[1] This specification has been used by [Runkle 91] and [Zeldes 89] among others because their data only provides information about food consumption. Consequently, they have to assume some sort of separability between the utility derived from food consumption and the utility derived from other cathegories of consumption.

[2] Note that the composite error is serially correlated due to the term $\Delta \ln \phi(t)$.

does not need to be true in general. In any case the resulting equation includes a new error component which would result in biased and, still, inconstistent estimators.

2 Alternative solution

Here we develop an alternative strategy which requires accepting a distributional assumption on the measurement error in exchange for not taking a log-linear approximation of the Euler equation. But at least, we have two alternatives whose relative performance can be compared and even tested with the aid of simulation techniques. Furthermore the solution proposed allows us to partly quantify the importance of measurement error in the data.

Let $\eta(t+1) = \frac{\phi(t+1)}{\phi(t)}$. We assume that $\eta(t)$ is serially independent and that $\ln(\eta(t)) \sim N(0, \sigma_\eta^2)$. We also require that $\eta(t+1)$ be independent from $\epsilon(t+1)$.

Let $\mathrm{E}_{t^*}(\cdot)$ denote the expectation conditional to Ω_{t^*}. This conditioning set is spanned by the same variables in the information set of the individual at time t plus past values of the measurement error.

Then

$$\mathrm{E}_{t^*}(1+\epsilon(t+1))(\eta(t+1))^{-\alpha}) = \mathrm{E}_{t^*}(1+\epsilon(t+1))\mathrm{E}_{t^*}(\eta(t+1))^{-\alpha} =$$

$$= \mathrm{E}_{t^*}(1+\epsilon(t+1))\mathrm{e}^{\alpha^2\frac{\sigma_\eta^2}{2}} = \mathrm{e}^{\alpha^2\frac{\sigma_\eta^2}{2}}$$

Therefore,

$$\mathrm{E}_{t^*}\left(\beta \mathrm{e}^{-\alpha^2\frac{\sigma_\eta^2}{2}}\left(\frac{\tilde{c}_k(t+1)}{\tilde{c}_k(t)}\right)^{\alpha} R_k(t+1) - 1\right) = 0$$

For any $z(t)$ belonging to Ω_{t^*},

$$\mathrm{E}\left[\left(\beta \mathrm{e}^{-\alpha^2\frac{\sigma_\eta^2}{2}}\left(\frac{\tilde{c}_k(t+1)}{\tilde{c}_k(t)}\right)^{\alpha} R_k(t+1) - 1\right) z(t)\right] = 0$$

Therefore we could use a generalized method of moments to estimate the relevant parameteres of this equation (see [Ventura 86] or [Ventura 89]), as long as the set of instruments belongs to Ω_{t^*}. This means, in particular, that the instruments list should not include measured consumption of period t because its error component $\phi(t)$ does not belong to the enlarged information set. But it can include measured consumption dated $t-1$ and earlier.

In this example , the parameters β and σ_η^2 are not identified. One way to identify these parameters would be to impose equilibrium conditions relating β to the the interest rate process and estimate β from these conditions.

3 Final remarks

We have proposed a simple alternative to deal with the presence of measurement errors in the Euler equation. This procedure does not rely on linear approximations to a non-linear function and, unlike the traditional approach it provides consistent and efficient estimators (in the generalized method of moments sense). The advantage of using this strategy depends on how strongly one feels against the approximation error introduced by the linearization of the equation, and how confortable one is with the assumed measurement errors distribution.

Although we have developed the strategy in a very simple context, the procedure can be readily extended to more complicated set-ups. The utility function may include a taste shifter component, the discount factor can vary over time and consumption does not need to be separable from labor or other types of consumption. The distribution for $\eta(t)$ does not need to be log-normal. In these cases, it may not be possible to obtain an analytic expression for the conditional expectation of the composite error. But it is still possible to numerically evaluate this expectation, which will be a function of the parameters of the model. The generalized method of moments technique could then be applied.

References

[Altonji and Siow 87] Altonji, J.G. and A. Siow *1987* : " Testing the response of consumption to Income changes with (Noisy) Panel Data", *The Quarterly Journal of Economics*, May 1987.

[Altug and Miller 89] Altug S. and R. A. Miller *1989*: "Household Choices in Equilibrium", *Econometrica 58*, pgs. 543-570.

[Browning, Deaton and Irish 85] Browning M. , A. Deaton and M. Irish *1985*:"A Profitable Approach to Labor Supply and Commodity Demands over the Life Cycle", *Econometrica 53.*

[Hall and Mishkin 82] Hall E. R. and F. S. Mishkin *1982*:"The sensitivity of consumption to transitory income: estimates from panel data on households." *Econometrica , Vol 50, no. 2*

[MaCurdy 83] MaCurdy, T.*1983*:"A Simple Scheme for Estimating an Intertemporal Model of Labor Supply and Consumption in the Presence of Taxes and Uncertainty", *International Economic Review 24.*

[Runkle 91] Runkle, D. E.*1991*:"Liquidity Constraints and the Permanent Income Hypothesis", *Journal of Monetary Economics 27*, pgs. 73-98.

[Ventura 86] Ventura, E. *1986*: "Heterogeneity and Goodness of Fit in an Asset Pricing Model", unpublished manuscript.

[Ventura 89] Ventura, E. *1989*: "Heterogeneous Consumers and Goodness of Fit in a Stock Pricing Model: A Test Using Panel Data", Ph. D. thesis, University of Minnesota.

[Zeldes 89] Zeldes, S. P. *1989*: "Consumption and Liquidity Constraints: An Empirical Investigation", *Journal of Political Economy, Vol 97, no 21*, pgs. 305-346.

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