Monetary Unions and the Transaction Cost Savings of a Single Currency

Hugo Rodríguez Mendizábal*†
Universitat Pompeu Fabra

May 1, 1998

Abstract

This paper studies the transaction cost savings of moving from a multi-currency exchange system to a single currency one. The analysis concentrates exclusively on the transaction and precautionary demand for money and abstracts from any other motives to hold currency. A continuous-time, stochastic Baumol-like model similar to that in Frenkel and Jovanovic (1980) is generalized to include several currencies and calibrated to fit European data. The analysis implies an upper bound for the savings associated with reductions of transaction costs derived from the European Monetary Union of approximately 0.6% of Community GDP. Additionally, the magnitudes of the brokerage fee and the volatility of transactions, whose estimation has traditionally been difficult to address empirically, are approximated for Europe.

Key words: Monetary Union, Demand for Money, Single Currency, Multivariate Brownian Motion.

JEL Classification: E41, F33

---

*I would like to thank Robert E. Lucas Jr. for his continuous help. Also, I am indebted to comments from Eduardo Andrade, Krishna Kumar, Wenqing Li, Casey Mulligan, Natxo Palacios, Gabriel Pérez Quirós, Ashok Rai, Apurva Sanghi, Tano Santos, Giorgio Topa, and participants at workshops at the University of Chicago and the University of Illinois at Urbana-Champaigne as well as at the 1996 Macroeconomics Summer Conference at Northwestern University. Carlos Kenig contributed to the technical part of the paper. Financial support from the Bank of Spain is gratefully acknowledged. All remaining errors are my own.

†Department of Economics, Universitat Pompeu Fabra. Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. E-mail: hugo.rodriguez@econ.upf.es

1 Introduction

This paper studies the transaction cost savings of moving from a multi-currency exchange system to a single-currency one. The analysis concentrates exclusively on the transaction and precautionary demand for money and abstracts from any other motive to hold currency. A continuous-time, stochastic, Baumol-like model similar to that in Frenkel and Jovanovic [8] is generalized to include several currencies.[1] As in most of this literature, the cost of managing a cash portfolio is minimized. In particular, the objective function to be optimized is the expected discounted cost over an infinite planning horizon.

The Commission of the European Community prepared a study of the economic effects of the move to the Economic and Monetary Union in Europe (EMU). In the words of its former president, Jacques Delors, and vice-president, Henning Christophersen, the investigation was directed "... to build a bridge between the political negotiators on the one hand, and the community of academic economists on the other, and to stimulate a two-way process of motivation of economic research and supply of economic advise."[2] This paper could be catalogued as one possible answer to their report from the academic community. For the Commission, one of the main sources for direct efficiency gains from a monetary union is the elimination of the currency exchange transaction costs. They estimate the savings associated with the suppression of the transaction costs derived from converting one EC currency into another to be approximately 0.4% of Community GDP. These savings could differ greatly from country to country starting at 0.1% to 0.2% of national GDP for Member States whose currency is extensively used as a means of international payments and belongs to the Exchange Rate Mechanism (ERM), to as much as 1% of national GDP for the small open and less developed economies within the Community.

The report prepared by the Commission represents an extraordinary effort to directly measure the likely effects of the EMU and recognizes the importance of these empirical studies to address issues on the optimality of the union. Two caveats, however, deserve mention. First, as the Commission points out, "...the basic message of this investigation is that both because [of] the present state of the theory of monetary unification and because [of] the

[1] As Baumol and Tobin [3] pointed out, the essence of these types of models is contained in a previous paper by Maurice Allais. I thank Casey Mulligan for this reference.

[2] Commission of the European Community [6] pg. 5.

diversity of the effects involved, an overall quantitative evaluation of the gains from EMU would be both out of reach and inappropriate."[3] Although the report contains an extensive list of the benefits and costs, it would be unrealistic to think that all of them are included. A similar circumstance happens when we concentrate on the part of the costs associated with exchanging currency. An attempt to directly measure them should not be expected to give a comprehensive answer.[4] Second, the study of the Commission may be seen as a one-point estimation and, therefore, it is not possible to analyze the effects of what should otherwise be relevant variables like levels of interest rates or income. It is for these reasons that an indirect measurement based on a theoretical model may add valuable hints about the empirical question of quantifying the size of the transaction cost gains associated with the EMU and how they may vary with other economic variables.

From a theoretical point of view, this paper extends the Baumol-Tobin model of money demand to a multi-currency environment in the presence of uncertainty in the payment schedule. As in Baumol [2] or Tobin [21], an agent faces a stream of payments which has to be paid for in cash. Because of the lack of synchronization between income receipts and cash expenditures, money is obtained by withdrawals from assets, bearing a constant known interest rate r, at a fixed cost per withdrawal, or brokerage fee, of c. The difference in this paper is that the payments have to be made in different currencies. A decisive characteristic of the model presented here is the existence of economies of scale in exchanging currencies. That is, there is a fixed cost of going to the bank to replenish any of the money stocks. Once this cost is paid, the agent will find it optimal to set all other currencies to their initial levels. Hence, the decision faced by the agent is dominated by the problem of coordinating the different flows of money so that they are depleted at the same time. The paper shows that only when uncertainty is present the exchange system matters in the solution of the problem. Uncertainty is going to be modeled as a stochastic process for the payment stream. When this stream is known in advance, the agent can perfectly replicate the solution of the one-currency system with the multi-currency one and viceversa and will, therefore, be indifferent between the two regimes. In a world with random flows of expenditures, the coordination of the different money stocks is no

[3] Commission of the European Community [6] pg. 44.

[4] In fact, the study presents its results as underestimating the true savings on transaction costs.

longer possible. It is proved in the paper that ex-ante the agent will prefer the single-currency system to the multi-currency one. On the other hand, even with a simple stochastic structure, the model presented here cannot be solved analytically. Numerical approximations will be used to compute the solutions. Additionally, it is shown that money demand decreases after the monetary union.

From the empirical side, the objective is to estimate the efficiency gains derived from the reduction in transaction costs that can be associated with the EMU (no attempt is made to analyze other types of benefits and costs) and to see how sensitive these estimates are to changes in interest rate and velocity of circulation of money. The results included here are to be seen as upper bounds for these gains. The robustness of these estimations to changes in the variables mentioned above is also studied. As a by-product, this paper sheds some light on a measurability issue encountered by these types of models. It has been repeatedly stated by researchers the problems involving the estimation of the brokerage fee and the variability of transactions.[5] Chang [4] represents one attempt to compute the former for U.S. data while there are no numerical estimates of the latter to the best of my knowledge. The magnitudes of both parameters are addressed below and the predictions for the brokerage fee agree with the ones obtained in Chang [4].

This paper is organized as follows. Section 2 analyzes a non-stochastic version of the model. It is shown there that without uncertainty the currency system does not affect the costs of managing a cash portfolio. In section 3, uncertainty is introduced into the model. To make the analysis easier, I will look at a simplified case throughout this section. An application of the model to compute the efficiency gains derived from the monetary union of the EEC countries is included in section 4. There, the parameters of the model will be calibrated to fit European data. The values of these coefficients agree with the corresponding estimations for the US economy. Also the predictions about the efficiency gains are similar to those computed in the Commission report. Section 5 concludes.

[5] See, for example, Goldfeld [9].

2 The Certainty Case

2.1 The Multi-Currency System

This economy is characterized by the following assumptions:

1. An agent receives exogenously wealth in the form of a domestic bond that yields a fixed return of r and uses this wealth to spend in n different countries. It is assumed that transactions in country i have to be paid for in the currency used in that country. Therefore, he manages a portfolio consisting of $n + 1$ assets: a domestic bond, a domestic currency, and n - 1 foreign currencies.

2. Initial holdings of the i-th currency at time t will be denoted by $M_n^i(t)$ $(i = 1, ..., n)$ in the n-currency system with $M_n^1(t)$ being the domestic currency.[6] The agent has to meet payments in all currencies at the same time. Cumulative payments up to time t in the i-th country are determined as the i-th component of a vector $X(t)$ defined as

$$X(t) = \begin{bmatrix} \mu_n^1 t \\ \vdots \\ \mu_n^n t \end{bmatrix}, \qquad (1)$$

   so if M_n^i denotes the initial holdings of the i-th currency, the agents stock of the i-th money at time t can be computed as the initial withdrawal minus whatever has been paid up to time t, i.e., the laws of motion between withdrawals for the different money holdings in the n-currency system can be written as

$$M_n^i(t) = M_n^i - X_n^i(t) = M_n^i - \mu_n^i t \quad ; \quad (i = 1, \ldots, n). \qquad (2)$$

3. All financial activities are made at the bank. There is a fixed cost of going to the bank, or brokerage fee, of c units, independent of the amount transacted. Once there, the agent can carry out, at no cost, any transaction needed, like transferring funds from the bond or exchanging currency.

[6]Unless otherwise is stated, subindexes denote the currency system (either 1 or n) and superindexes represent the particular currency (from 1 to n). Since under the single-currency regime there is only one currency, no superscripts will be included in the study of that system.

4. The objective of the agent is to minimize the long run discounted cost of managing the portfolio.

5. There exists a fixed exchange rate of s^i units of domestic currency per unit of the i-th currency $(i = 1, ..., n)$. Of course, $s^1 = 1$.

Let τ_n^i be the moment the i-th money stock is depleted and τ_n the time the agent has to go to the bank for the first time. They are equal to

$$\tau_n^i \equiv \frac{M_n^i}{\mu_n^i} \quad ; \quad (i = 1, \ldots, n), \tag{3}$$

and

$$\tau_n \equiv \quad \min\left\{\tau_n^i\right\}. \tag{4}$$

The total cost of holding money has two components: transaction costs $(PVTC_n)$ and opportunity costs $(PVOC_n)$. The present value of all transaction or adjustment costs over the infinite horizon is equal to

$$PVTC_n\left(M_n^1, \ldots, M_n^n\right) = c + ce^{-r\tau_n} + ce^{-2r\tau_n} + \ldots = \frac{c}{1 - e^{-r\tau_n}}, \tag{5}$$

since the fixed cost is paid at times $t = j\tau_n$ $(j = 0, 1, 2, \ldots)$, when the agent goes to the bank. On the other hand, since the bond is domestic, its interest rate is lost on the domestic counterpart of the money holdings. Therefore, the present value of the foregone interest earnings between withdrawals is equal to

$$\begin{aligned} & OC_n\left(M_n^1, \ldots, M_n^n\right) \\ = & \int_0^{\tau_n} e^{-rt} r \left[\sum_{i=1}^n s^i M_n^i(t)\right] dt = \int_0^{\tau_n} e^{-rt} r \left[\sum_{i=1}^n s^i \left(M_n^i - \mu_n^i t\right)\right] dt = \\ = & \sum_{i=1}^n s^i M_n^i - \left[1 - e^{-r\tau_n}\right] \frac{\sum_{i=1}^n s^i \mu_n^i}{r}. \end{aligned} \tag{6}$$

Because there is a withdrawal every τ_n periods, the present value of all opportunity costs equals

$$\begin{aligned} PVOC_n\left(M_n^1, \ldots, M_n^n\right) &= \sum_{j=1}^\infty OC_n\left(M_n^1, \ldots, M_n^n\right) e^{-jr\tau_n} = \\ &= \frac{OC_n\left(M_n^1, \ldots, M_n^n\right)}{1 - e^{-r\tau_n}}. \end{aligned} \tag{7}$$

The total cost of holding money under a multi-currency system (C_n) is then the sum of both present values

$$C_n\left(M_n^1,\ldots,M_n^n\right)=\frac{\sum_{i=1}^n s^i M_n^i+c}{1-e^{-r\tau_n}}-\frac{\sum_{i=1}^n s^i\mu_n^i}{r}, \tag{8}$$

which is the function to be minimized with respect to each initial stock of currency. The following proposition reduces the number of first order conditions to be determined in the optimization problem.

Proposition 1 *In the economy just described it is optimal to choose M_n^i so that*

$$\tau_n=\frac{M_n^1}{\mu_n^1}=\frac{M_n^i}{\mu_n^i}\qquad;\qquad(i=2,\ldots,n). \tag{9}$$

Proof. See Appendix A.

Proposition 1 means that the agent will coordinate the withdrawals of all the money stocks so they are depleted at the same time. Expression (9) implies that all initial currency holdings are proportional to the domestic one, that is,

$$M_n^i=\frac{M_n^1}{\mu_n^1}\mu_n^i. \tag{10}$$

Substituting (10) into (8) will give the present value of money holding costs as a function of one of the currency stocks. This numeraire is chosen to be the domestic currency (M_n^1). The other initial money stocks are computed from (10) once the optimal initial holdings of domestic currency are determined. The cost function takes the form

$$C_n\left(M_n^1,\ldots,M_n^n\right)=C_n\left(M_n^1\right)=\frac{\frac{M_n^1}{\mu_n^1}\sum\limits_{i=1}^n s^i\mu_n^i+c}{1-\exp\left[-rM_n^1/\mu_n^1\right]}-\frac{\sum\limits_{i=1}^n s^i\mu_n^i}{r}. \tag{11}$$

The first order necessary condition for minimization is

$$\left(\sum_{i=1}^n s^i\frac{\mu_n^i}{\mu_n^1}\right)\left[1-e^{-rM_n^1/\mu_n^1}\right]-\frac{r}{\mu_n^1}\left[\frac{M_n^1}{\mu_n^1}\left(\sum_{i=1}^n s^i\mu_n^i\right)+c\right]e^{-rM_n^1/\mu_n^1}=0, \tag{12}$$

and approximating the exponential with a second order Taylor expansion around zero we get

$$M_n^{1*}\simeq\sqrt{\frac{2c\mu_n^1}{r}\left(\frac{\mu_n^1}{\sum_{i=1}^n s^i\mu_n^i}\right)}. \tag{13}$$

Using (10), the corresponding expression for M_n^{j*} is

$$M_n^{j*} = \frac{M_n^{1*}}{\mu_n^1}\mu_n^i \simeq \sqrt{\frac{2c\mu_n^j}{r}\left(\frac{\mu_n^j}{\sum_{i=1}^n s^i \mu_n^i}\right)}. \tag{14}$$

The total amount of money demanded measured in domestic currency is equal to

$$M_n^* = \sum_{i=1}^n s^i M_n^{i*} = \sqrt{\frac{2c}{r}\left(\sum_{i=1}^n s^i \mu_n^i\right)}. \tag{15}$$

2.2 The Single-Currency System

The way a monetary union works in this economy is as follows. One day the central banks of the n countries meet and decide to issue one currency which, without loss of generality, is assumed to be the domestic one. The agent wakes up in the morning and finds out that he will only need one currency for transactions. Therefore, he will manage a portfolio with two assets: the bond and the domestic currency. It is assumed that the monetary union does not change the transactions of the agent in each country (i.e. the vector $X(t)$.) The only difference with respect to the multi-currency system appears in the currency they are accounted for. After the monetary union the agent will translate all transactions to equivalents in the domestic currency using the fixed exchange rates s^i $(i = 1, 2, ..., n)$. Let M_1 denote the initial money holdings under this system. Then money holdings between withdrawals evolve as

$$M_1(t) = M_1 - S'X(t) = M_1 - \sum_{i=1}^n s^i X_n^i(t) = M_1 - \mu_1 t, \tag{16}$$

with

$$\mu_1 = \sum_{i=1}^n s^i \mu_n^i \tag{17}$$

where $S = [s^1, ..., s^n]$. In this economy without uncertainty money is exhausted exactly at times $\tau = j\tau_1 \equiv jM_1/\mu_1$ $(j = 1, 2, ...)$. Then the agent goes to the bank, withdraws M_1, and the process starts all over again.

The total cost of managing the portfolio is still the sum of the transaction ($PVTC_1$) and opportunity ($PVOC_1$) costs. The present value of all transaction or adjustment costs over the infinite horizon is equal to

$$PVTC_1(M_1) = c + ce^{-rM_1/\mu_1} + ce^{-2rM_1/\mu_1} + \ldots = \frac{c}{1 - e^{-rM_1/\mu_1}}. \tag{18}$$

On the other hand, the present value of all opportunity costs is

$$PVOC_1(M_1) = \sum_{j=1}^{\infty} OC_1(M_1)\, e^{-jrM_1/\mu_1} = \frac{OC_1(M_1)}{1-e^{-rM_1/\mu_1}}, \tag{19}$$

with

$$\begin{aligned} OC_1(M_1) &= \int_0^{M_1/\mu_1} e^{-rt} r M_1(t) dt = \int_0^{M_1/\mu_1} e^{-rt} r\left(M_1-\mu_1 t\right) dt = \\ &= M_1 - \left[1-e^{-rM_1/\mu_1}\right]\frac{\mu_1}{r}, \end{aligned} \tag{20}$$

so that

$$PVOC_1(M_1) = \frac{OC_1(M_1)}{1-e^{-rM_1/\mu_1}} = \frac{M_1}{1-e^{-rM_1/\mu_1}} - \frac{\mu_1}{r}. \tag{21}$$

Therefore, by adding both sources of cost, the present value of holding costs is

$$C_1(M_1) = PVTC_1(M_1) + PVOC_1(M_1) = \frac{M_1+c}{1-e^{-rM_1/\mu_1}} - \frac{\mu_1}{r}, \tag{22}$$

which is the function to be minimized with respect to M_1. The first order condition is

$$1-e^{-rM_1/\mu_1} - (M_1+c)\frac{r}{\mu_1}e^{-rM_1/\mu_1} = 0. \tag{23}$$

Equation (23) defines implicitly the transaction demand for money as a function of the interest rate, the cost of adjusting the portfolio, and the depletion rate. Approximating the exponential as before we get the usual square root formula

$$M_1^* \simeq \sqrt{\frac{2c\mu_1}{r}}. \tag{24}$$

If this formula is compared to that under the multi-currency system we see both expressions are the same if we consider the value of μ_1. The reason for this result is that under certainty the problem faced by the agent is the same no matter what monetary system prevails in the economy. In the multi-currency system, the equivalent in domestic currency of the agents holdings of the i-th currency is equal to the fraction of the total amount of money used to meet the i-th liability in the single-currency setup. That is

$$s^j M_n^{j*} \simeq s^j \sqrt{\frac{2c\mu_n^j}{r}\left(\frac{\mu_n^j}{\sum_{i=1}^n s^j \mu_n^i}\right)} = \left(\frac{s^j \mu_n^j}{\sum_{i=1}^n s^j \mu_n^i}\right) M_1^*. \tag{25}$$

Therefore, costs are the same independently of the monetary system existing in this economy. Without uncertainty, the agent can replicate perfectly a single-currency system with a multi-currency one and viceversa. This will no longer be true once uncertainty plays a role in the economy and that is the reason why both systems differ in the holding costs that they imply.

3 The Stochastic Case

The last section concluded that with certainty, the currency system does not have any effect on the costs of managing the cash portfolio. This result is no longer true in a stochastic environment. With uncertainty it is not possible to coordinate the withdrawals in each currency so they are exhausted at the same time. In fact, with a continuous time index, this is an event of zero probability measure. Therefore, with a multi-currency system the probability of one money stock to be depleted before the others is one. This characteristic is what makes, for given payment processes, the monetary system to affect both, the overall money demand and the expected holding costs. In this section it will be proved that it is optimal to have a single-currency system if the expected costs of holding money are to be minimized. Also, an expression for these costs under different currency systems will be computed.

3.1 The Stochastic Structure

The stochastic structure of the problem is defined as follows. Let $\mathcal{X} = \{X(t), \mathcal{F}_t; t \geq 0\}$ be an n-dimensional Brownian motion with drift vector and variance-covariance matrix

$$\Delta = \begin{bmatrix} \mu_n^1 \\ \mu_n^2 \\ \vdots \\ \mu_n^n \end{bmatrix} \quad ; \quad \Sigma = \begin{bmatrix} \sigma_n^1 & 0 & \cdots & 0 \\ 0 & \sigma_n^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n^n \end{bmatrix}. \tag{26}$$

Therefore, the law of motion for this process can be written as

$$X(t) = \Delta t - \Sigma W(t), \tag{27}$$

with

$$X(t) = \begin{bmatrix} X_n^1(t) \\ X_n^2(t) \\ \vdots \\ X_n^n(t) \end{bmatrix} \quad ; \quad W(t) = \begin{bmatrix} W_n^1(t) \\ W_n^2(t) \\ \vdots \\ W_n^n(t) \end{bmatrix} \tag{28}$$

where $W_n^i(t)$ are n independent Wiener processes. This stochastic process takes values in R^n and is defined on some probability space $(\Omega,\ \mathcal{F},\ P_x)$. Throughout this section $\mathcal{F} = \{\mathcal{F}_t, t \geq 0\}$ will be the filtration generated by $\mathcal{X}$ and is the one implicitly referred to when defining stopping times and martingales. Also, let E be the expectation associated with P_x. That is,

$$E(Z) \equiv \int_\Omega Z(\omega) P_x(d\omega). \tag{29}$$

This is the concept of expectation that is going to be used below.

3.2 The Multi-Currency System

The economy under uncertainty is characterized by the following assumptions:

1. As before, an agent receives exogenously wealth in the form of a domestic bond that yields a fixed return of r and uses this wealth to spend in n different countries. It is assumed that transactions in country i have to be paid for in the currency used in that country. Therefore, he manages a portfolio consisting of $n + 1$ assets: a domestic bond, a domestic currency, and n - 1 foreign currencies.

2. The agent faces stochastic transactions in all currencies at the same time. It is assumed that cumulative payments in the i-th currency $(i = 1, ..., n)$ up to time t follow the same process as the i-th component of $X(t)$, i.e. they fluctuate as a Brownian motion with drift μ_n^i and standard deviation parameter σ_n^i. If initial holdings of the i-th currency are denoted by M_n^i, the law of motion for the stock of currency i at time t would be

   $$M_n^i(t) = M_n^i - X_n^i(t) = M_n^i - \mu_n^i t + \sigma_n^i W_n^i(t) \ \ ; \ \ (i = 1, ..., n). \tag{30}$$

   Of course, this expression does not define the overall stochastic structure of $M(t)$ since this process is subject to interventions.

3. The objective of the agent is to minimize the long run expected discounted cost of managing the portfolio subject to the constraint $M(t) \geq 0$.

4. All financial activities are made at the bank. Each time the agent goes to the bank, he incurs in a fixed cost of c units of domestic currency, which is independent of the amount transacted. Once there, the agent can carry out any transactions needed at no additional cost.

5. There exists a fixed exchange rate of s^i units of domestic currency per unit of currency i. Of course, $s^i = 1$.

Define

$$\tau_n^i \equiv \inf \left\{ t \geq 0 : M_n^i(t) = 0 \right\} \qquad ; \qquad (i = 1, ..., n), \tag{31}$$

and

$$\tau_n \equiv \min \left\{ \tau_n^i \right\}, \tag{32}$$

that is, τ_n^i is the random variable determining the first passage time of $M_n^i(t)$ through the origin while τ_n determines the first moment one of the currencies is exhausted so the agent needs to go to the bank to withdraw funds. Because of the fixed cost of going to the bank, it is obvious that once the time to go to the bank comes, it is optimal to set all other money stocks to their initial level.[7] With these considerations in mind, the expected present value of the holding costs of managing the cash portfolio can be written as

$$\begin{aligned} C_n\left(M_n^1, \ldots, M_n^n\right) &= c + E\left[\int_0^{\tau_n} e^{-rt} r \left(\sum_{i=1}^n s^i M_n^i(t)\right) dt \right. \\ &\quad \left. + e^{-r\tau_n} C_n\left(M_n^1, \ldots, M_n^n\right)\right] \\ &= c + E\left[\int_0^{\tau_n} e^{-rt} r \left(\sum_{i=1}^n s^i M_n^i(t)\right) dt\right] \\ &\quad + E\left[e^{-r\tau_n}\right] C_n\left(M_n^1, \ldots, M_n^n\right). \end{aligned} \tag{33}$$

That is, the expected present value of money holding costs may be decomposed in three components. First, there is the opening cost of accumulating

[7] If the fee c has to be paid, once at the bank, the agent can deposit any money left over at zero cost and the problem is exactly the same he solved at $t = 0$.

the initial money stock. The second ingredient in the cost function is the expected present value of foregone interest earnings until the first withdrawal is made. The last term corresponds to the expected present value of all costs from then on. Since once the time to go to the bank comes the agent faces the same situation as the one encountered at $t = 0$, this element is just the expected discounted value of $C_n(M_n^1, ..., M_n^n)$. Following Appendix D, the solution of this functional equation is

$$C_n\left(M_n^1, \ldots, M_n^n\right) = \frac{\sum_{i=1}^n s^i M_n^i - \sum_{i=1}^n s^i E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] + c}{1 - E\left[e^{-r\tau_n}\right]} - \frac{\sum_{i=1}^n s^i \mu_n^i}{r}. \quad (34)$$

In order to simplify the analysis as well as the empirical application included in the next section, from now on I will consider the symmetric case where

$$\mu_n^i = \mu_n \quad ; \quad \sigma_n^i = \sigma_n \quad ; \quad s^i = 1 \quad (i = 1, ..., n), \quad (35)$$

so (34) becomes

$$C_n\left(M_n^1, \ldots, M_n^n\right) = \frac{\sum_{i=1}^n M_n^i - \sum_{i=1}^n E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] + c}{1 - E\left[e^{-r\tau_n}\right]} - \frac{n\mu_n}{r}. \quad (36)$$

It is easy to see that under the assumption of symmetry it is optimal to set

$$M_n^1 = M_n^i \quad (i = 1, ..., n), \quad (37)$$

so that

$$E\left[e^{-r\tau_n} M_n^1(\tau_n)\right] = E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] \quad (i = 2, ..., n) \quad (38)$$

and

$$P\left[\tau_n = \tau_n^1\right] = P\left[\tau_n = \tau_n^i\right] = \frac{1}{n} \quad (i = 2, ..., n), \quad (39)$$

and calling

$$M_n(t) \equiv \sum_{i=1}^n M_n^i(t), \quad (40)$$

and

$$M_n \equiv \sum_{i=1}^n M_n^i(0) = n M_n^i, \quad (41)$$

we can write (36) as

$$C_n(M_n) = \frac{M_n - E\left[e^{-r\tau_n} M_n(\tau_n)\right] + c}{1 - E\left[e^{-r\tau_n}\right]} - \frac{n\mu_n}{r}. \tag{42}$$

All the elements of this expression have a corresponding one in formula (8) besides the term $E[exp(-r\tau_n)M_n(\tau_n)]$. This element represents the present value of the expected cash in hand when the agent goes to the bank. As it was argued before, exhausting all currencies at the same time is an event of zero probability measure. Thus, with probability one the agent will go to the bank with some unspent cash. The expectations in (42) do not have a simple analytical characterization so they will have to be approximated by numerical methods in the next section. Appendix D includes the information needed to perform these computations.

The density function for τ_n can be computed now (see Appendix D) as

$$h_{\tau_n}(t; M_n) = n\left[1 - G_{\tau_n^i}(t; M_n)\right]^{n-1} g_{\tau_n^i}(t; M_n), \tag{43}$$

where

$$g_{\tau_n^i}(t; M_n) = \frac{M_n}{\sigma_n\sqrt{2n\pi t^3}} \exp\left[-\frac{(M_n - \mu_n t)^2}{2n(\sigma_n)^2 t}\right] \tag{44}$$

is the density function of the first arrival of the i-th money stock (identical for all i) and

$$\begin{aligned} G_{\tau_n^i}(t; M_n) \;&\equiv\; P\left[\tau_n^i < t\right] = 1 - \Phi\left(\frac{M_n - \mu_n t}{\sigma_n\sqrt{nt}}\right) \\ &\quad + \exp\left[-\frac{2M_n\mu_n}{(\sigma_n)^2}\right]\Phi\left(\frac{-M_n - \mu_n t}{\sigma_n\sqrt{nt}}\right) \end{aligned} \tag{45}$$

is its distribution function. $\Phi(\cdot)$ represents the standard normal distribution function.

3.3 The Single-Currency System

Here, as in the certainty case, I will assume that all transactions are met with one currency which corresponds to the domestic one. Let M_1 be the initial money holdings and $M_1(t)$ denote the money stock at time t under this system. Between withdrawals, this variable follows the process

$$M_1(t) = M_1 - S'X(t) = M_1 - \mu_1 t + \sigma_1 W_1(t) \tag{46}$$

where $S = [s^1, ..., s^n]'$, $W_1(t)$ is some function of the n original Wiener processes and

$$\mu_1 = \sum_{i=1}^{n} s^i \mu_n^i \qquad ; \qquad (\sigma_1)^2 = \sum_{i=1}^{n} \left(s^i \sigma_n^i\right)^2. \tag{47}$$

Define

$$\tau_1 \equiv \inf \{t \geq 0 : M_1(t) = 0\}, \tag{48}$$

that is, τ_1 is the random variable determining the first passage time of $M_1(t)$ through the origin. It is at this moment when the agent has exhausted all the currency and needs to go to the bank to replenish it. The random variable τ_1 follows an inverse Gaussian distribution. Its density is

$$g_{\tau_1}(t; M_1) = \frac{M_1}{\sigma_1 \sqrt{2\pi t^3}} \exp\left[-\frac{(M_1 - \mu_1 t)^2}{2 (\sigma_1)^2 t}\right]. \tag{49}$$

The expected present value of the holding costs of managing the cash portfolio, $C_1(M_1)$, can be written as

$$\begin{aligned} C_1(M_1) &= c + E\left[\int_0^{\tau_1} e^{-rt} r M_1(t) dt + e^{-r\tau_1} C_1(M_1)\right] = \\ &= c + E\left[\int_0^{\tau_1} e^{-rt} r M_1(t) dt\right] + E\left[e^{-r\tau_1}\right] C_1(M_1) \end{aligned} \tag{50}$$

where each component has the same interpretation as the one given for the n-currency regime. As proved in Frenkel and Jovanovic [8], the solution to this functional equation is equal to

$$C_1(M_1) = \frac{M_1 + c}{1 - e^{-\rho_1 M_1/\mu_1}} - \frac{\mu_1}{r} \tag{51}$$

where ρ_1 is the "interest rate adjusted for uncertainty"

$$\rho_1 = \frac{1}{(\sigma_1/\mu_1)^2}\left[\left(1 + 2(\sigma_1/\mu_1)^2 r\right)^{1/2} - 1\right] < r \tag{52}$$

with

$$\lim_{\sigma_1 \to 0} \rho_1 = r \qquad ; \qquad \lim_{\sigma_1 \to \infty} \rho_1 = 0. \tag{53}$$

Notice the similarity between (51) and (22). The only difference is that r is substituted by ρ_1. The first order condition for a minimum is

$$1 - e^{-\rho_1 M_1/\mu_1} - (M_1 + c)\frac{\rho_1}{\mu_1} e^{-\rho_1 M_1/\mu_1} = 0, \tag{54}$$

and approximating the exponential as above we get the counterpart of (24) for the uncertainty case

$$M_1^* \simeq \sqrt{\frac{2c\mu_1}{\rho_1}}. \tag{55}$$

Given the property $\rho_1 < r$, it is easy to see that introducing uncertainty in the model increases money demand. This is what has been associated with the precautionary motive to hold currency. The following theorem states the obvious fact that in this setup the multi-currency regime will never be preferred, ex-ante, to the single-currency system.

Theorem 1 *Let M_1^* be the optimal value for the money stock in the one-currency system which is determined by expression 54 and M_n^{i*} be the corresponding optimal i-th money stock for the n-currency system. Then*

$$C_1\left(M_1^*\right) \leq C_n\left(M_n^{1*}, \ldots, M_n^{n*}\right)$$

for the functions $C_1\left(M_1\right)$ defined in (51) and $C_n\left(M_n^1, \ldots, M_n^n\right)$ defined in (34).

Proof. See Appendix C.

The intuition behind this theorem is rather simple (although its formal proof is much more cumbersome.) Because of the fixed withdrawal costs, a policy will be optimal only if it exhausts the contents of the stock before its replenishment. This necessary characteristic of optimality is met by the one-currency system but not by the n-currency system where with probability one the agent will go to the bank with some unspent cash.

Once we specialize in the symmetric case, (47) implies

$$n\mu_n = \mu_1 = \mu \qquad ; \qquad \sigma_n\sqrt{n} = \sigma_1 = \sigma, \tag{56}$$

so (42) and (51) become, respectively

$$C_n\left(M_n\right) = \frac{M_n - E\left[e^{-r\tau_n} M_n(\tau_n)\right] + c}{1 - E\left[e^{-r\tau_n}\right]} - \frac{\mu}{r} \tag{57}$$

and

$$C_1\left(M_1\right) = \frac{M_1 + c}{1 - e^{-rM_1/\mu}} - \frac{\mu}{r}. \tag{58}$$

One immediate result is contained in the following proposition.

Proposition 2 *Under the symmetric assumption, τ_1 dominates τ_n in the sense of first degree stochastic dominance, i.e.*

$$H_{\tau_n}(t;M) \geq G_{\tau_1}(t;M) \qquad ; \qquad \forall\ t \in [0,\infty), M \in [0,\infty)$$

where H and G are the distribution functions of τ_n and τ_1, respectively. Therefore

$$E[\tau_n] \leq E[\tau_1] = \frac{M_1}{\mu}.$$

Proof. See Appendix B.

Proposition 2 implies that starting with the same initial money stock, it is more likely to go to the bank sooner in the multi-currency system than when the economy uses only one currency. This has the effect of a larger initial money stock in the n-currency regime. Although it is not proven here, this greater level of initial currency holdings will not compensate fully the higher number of trips to the bank. Therefore a multi-currency economy will, on average, show both a larger money demand and more trips to the bank.

4 Empirical results

Theorem 1 implies that the single currency system constitutes a first best if the expected costs of holding money are to be minimized. However, these gains may vary a lot depending on parameter values. In this section the model will be calibrated to fit data for the countries in the EEC 15[8] and then used to compute the efficiency gains derived from the EMU. I will calculate the savings associated with what the model calls transaction costs, i.e., the costs related to trips to the bank. These costs represent all the resources diverted from productive uses due to the management of the cash portfolio.[9]

[8]This corresponds to the European Economic Community integrated by Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, and United Kingdom.

[9]Although this model does not include production, it implies a transaction technology that involves money and time. This technology may be used in a general equilibrium model that distinguishes between time dedicated to produce and time dedicated to transactions (see, among others, Lucas [17] and Guidotti [11].)

4.1 The Choice of Parameter Values

Five coefficients have to be specified in the model: the interest rate, r, the brokerage fee, c, the standard deviation, σ, the drift, μ and the number of currencies, n. Since Luxembourg does not use its own currency, n will take the value 14. Also, it is possible to normalize μ to be one so the whole model will be expressed relative to the drift parameter. In this case (57) and (58) take the form

$$\frac{C_n\left(M_n/\mu\right)}{\mu}=\frac{M_n/\mu-E\left[e^{-r\tau_n}M_n(\tau_n)/\mu\right]+c/\mu}{1-E\left[e^{-r\tau_n}\right]}-\frac{1}{r} \tag{59}$$

$$\frac{C_1\left(M_1/\mu\right)}{\mu}=\frac{M_1/\mu+c/\mu}{1-e^{-rM_1/\mu}}-\frac{1}{r} \tag{60}$$

so that the only parameters to be determined are relative transaction costs (c/μ) and relative volatility (σ/μ). The objective is then to identify these coefficients from observations on velocity of circulation of money and interest rates. Two comments are in order here. The first one is related to the stochastic nature of both the model and the data. By money velocity I mean the ratio of a flow (transactions within a period) to a stock (initial money holdings in that period.) In the model this variable is random since the volume of transactions, $X(t)$, follows a stochastic process. In real life money velocity is considered random too and data is available as a time series. With respect to interest rates, they are assumed constant and equal among countries while in reality are random and different among Member States. The second observation concerns the observability of variables in the model and their mapping into data. We only observe the n-currency world. Thus, I will relate a value from the time series of velocity with its population mean in the model which equals (μ/M_n) for EEC as a whole (or μ_n^i/M_n^i for country i).[10] From data on interest rates I will select some values that may serve as r in the model. Finally, expression (55) will give average money velocity in the single-currency regime (denoted by μ/M_1). Summarizing, values on interest rates and average velocity of money will be used to compute (c/μ) and (σ/μ) from two conditions to be specified below. Then I will use those values and expression (55) to recover (μ/M_1) and compute efficiency gains as the difference of the implied transaction costs under the two regimes.

The description of the data used in the calibration exercise is contained in Appendix F. The variable *money stock* includes demand deposits other

[10]Notice that the symmetry assumption implies that all these velocities are the same.

than those of the central government and currency outside banks and tries to give a close and homogeneous estimate to narrow money or M1 across countries.[11] The chosen *interest rate* is the rate on long term government bonds used to compute one of the convergence criteria. The choice of an appropriate variable to measure the return on alternative assets is still an open issue.[12] However, as with velocity, the exercise to carry out here is not to run regressions of historical series on money, income and interest rates, but to determine what are "reasonable" values for these variables. In this sense, the yield on government bonds will provide a range of possible values for this interest rate that should cover the return on other assets. Finally, GDP is used as a measure of the *transactions* in goods and services completed with each currency independently of the citizenship of the factors of production. Clearly, GDP underestimates the volume of transactions since it does not include purchases of intermediate and existing goods and financial transactions which should contribute to the demand for money. Alternative variables are wealth and debits to demand deposit accounts.[13] The choice of GDP has been made on grounds of simplicity and, as long as it provides reasonable values for the velocity of money, the same comments used for interest rates apply.[14]

With these observations in mind, the next step is to determine what values of the observables should be considered to approximate the parameters (c/μ) and (σ/μ). In this respect, I am going to use values of M1-velocity and interest rates observed since the creation of the European Monetary System (EMS) in 1979.[15] It is during this period that capital and good markets initiated their integration. Therefore, the information contained in this time interval should be the most relevant for the equilibrium values for these

[11]Although it could be arguable that M1 represents the best proxy of money held for transaction and precautionary motives in Europe, results are reasonably robust to include broader definitions of money that change, moderately, the velocity of money.

[12]Just to point out two examples, Orr's [20] three-asset model shown that, depending on the form of the transaction cost, the relevant interest rate may be a 'long' or 'cost of capital' rate or the return on some alternative, 'short', liquid asset. On the other hand, Barro and Santomero [1] argue that the return on demand deposits (one of the components of narrow money) "... should be taken into account in determining the opportunity cost of holding money."

[13]See Lieberman [16].

[14]For more on these measurement issues see Goldfeld [9], Goodhart [10] and Laidler [15].

[15]By belonging to the EMS it is meant that the country's currency is included in the ECU and not necessarily that the currency belongs to the ERM.

variables.[16] Figure 1 measures the velocity of money for 12 countries of the EEC as a whole.[17] This series covers 1979 to 1996 and has been constructed by adding equivalents in European Currency Unit (ECU) of national GDP and M1 for all Member States.[18]

Figure 2 shows the average interest rate of the three countries with lowest inflation rate. This average represents one of the convergence criteria established by the European Community to determine which countries will form the EMU since only countries with interest rates not higher than 2 percentage points above it will enter the union. Then this series is a good indication of the interest rates we will see after the monetary union.

A problem in choosing particular values for money velocity and interest rates appears since these two variables are related in general equilibrium economies. This implies that the choice of r should affect the choice of (μ/M_n). On the other hand, the countries belonging to the European Union have been lowering nominal interest rates as a result of the need to meet the convergence criteria outlined in the Maastricht Treaty. For these reasons I will use the last observed value for these variables. These values are 3.9 for money velocity and 7.1% for interest rate. Later in the paper the whole sample will be used to choose alternative combinations of velocity and interest rates and to determine the robustness of the results.

To estimate the relative volatility I will compare the expected number of trips to the bank under the two regimes. Imagine an agent in a single-currency world with initial money holdings relative to average transactions of (M/μ). For each value of (σ/μ) it is possible to compute the expected number of trips to the bank from the density function (49) as

$$E\left[z_1\right] = \int_0^\infty \frac{1}{t} g_{\tau_1}(t; M) dt \tag{61}$$

with $z_1 = 1/\tau_1$. Equivalently, if the agent had started with the same initial money holdings in a multi-currency economy the expected trips to the bank could have been calculated from the density function (43) as

$$E\left[z_n\right] = \int_0^\infty \frac{1}{t} h_{\tau_n}(t; M) dt \tag{62}$$

[16]It should be noted, however, that Greece joined in 1984 while Portugal and Spain did it in 1989.

[17]Finland, Sweden and United Kingdom are not included in these calculations because of lack of adecuate money series.

[18]Figures are included in the working paper series and can be sent upon request.

with $z_n = 1/\tau_n$. By Proposition 2 we know that $E[z_n] > E[z_1]$ and in fact simulations show that, at least for the parameter values used in this paper, their difference is an increasing function of (σ/μ). On the other hand the model predicts some economies of scale in money holdings when increasing the number of currencies so if the economy goes from one currency to two, we will not expect the trips to the bank to go from z to $2z$ since every time the agent makes a withdrawal in one of the currencies he replenishes all other money stocks. Hence, n times expected number of replenishments under a single-currency regime represents an upper bound for the frequency of withdrawals in the multi-currency case. This condition limits the variability of transactions for each observed money velocity since (M/μ) and (σ/μ) are the only parameters entering (61) and (62). This calibration procedure then will give us an upper bound for the parameter (σ/μ) which is independent of interest rates. For an average money velocity of 3.9, the estimated value for relative volatility is 0.265.

The other parameter to be estimated is the relative brokerage fee (c/μ). This coefficient has traditionally been "...conspicuous by [its] absence from empirical work."[19] To the best of my knowledge, the only attempt to measure it is included in Chang [4]. Also in a Baumol-Tobin framework, Chang obtained upper bound estimates for the U.S. economy in a range between 0.01 and 0.75 percent of GDP depending on money velocity and interest rates. Here, to estimate (c/μ) I will use the first order condition (FOC) associated with the n-currency cash-holding problem. This FOC, contained in Appendix E, determines (c/μ) as a function of the triplets $[(\mu/M), r, (\sigma/\mu)]$ and implies a negative relation between (c/μ) and (σ/μ) for given values of (μ/M) and r. Since I am using an upper bound for (σ/μ), the solution for (c/μ) represents a lower bound for this parameter. When introducing the point [3.9, 0.071, 0.265] the model gives an estimation of the relative transaction cost equal to 0.016 percent of GDP (while Chang's upper bound reaches 0.22 for the same values of (μ/M) and r.)

Summarizing, the values used to compute the efficiency gains derived from the EMU are $(\mu/M) = 3.9$, $(\sigma/\mu) = 0.265$, $r = 0.071$ and $(c/\mu) = 0.016$. Still, another combinations of money velocity and interest rates are possible implying, therefore, different values for (σ/μ) and (c/μ). Theoretically, we can expect high values of velocity to appear with high values of interest rates and low levels of velocity with low levels of interest rates although in

[19] Goldfeld [9], pg. 135.

practice this relation is not that sharp. The way to recover it would be by regressing money velocity on interest rates and dummies for each country. However, I am reluctant to perform this exercise. First, as it was said before, the data employed here may include a lot of measurement error when used to approximate velocity and interest rates in the model. Second, this relation, although clear for some individual countries, is very fuzzy for others.[20] The strategy followed here will be to analyze the efficiency gains for all possible combinations of velocity and interest rates. From Figure 1 we see that money velocity moves between 3 and 5 while from Figure 2 interest rates have fluctuated between 6% and 11%. It is clear that some of those pairs will be very unlikely to appear after the unification in Europe but nevertheless they are computed to see the variability of the calculations. Tables 1 and 2 include, respectively, estimations for (c/μ) and (σ/μ) for different values of money velocity and interest rates.

We see relative volatility to be estimated between 0.2 and 0.3 percent of GDP while the relative transaction cost per trip appears to be in the interval of 0.008 to 0.042 percent of GDP. These values still are consistent with the corresponding estimations in Chang [4] for the U.S. economy.

4.2 Efficiency Gains From Transaction Costs Savings

Once the values for the parameters of the model have been approximated, it is possible to compute the efficiency gains derived from savings in the transaction costs of holding the n currencies. As it was pointed out in the Introduction, these resources are distorted from productive uses and dedicated to managing the cash portfolio. We should not expect these costs to go to zero as the economy moves from n currencies to one since with one currency some resources still have to be used in economizing on liquidity. However, they may be decreased a lot when forming a monetary union. In reality people do not perform these activities themselves. They just hire resources from the banking sector. Therefore, one way to estimate these savings would be to compute the resources used by the banking sector and firms in exchange-related activities. This is the route taken by the Commission of the European Communities [6]. They separated the transaction costs in two parts. First, direct transaction costs to households and firms in the form

[20] Combinations of interest rates and money velocity for individual countries are not shown her for space considerations.

of foreign exchange commissions and differences between buying and selling rates. These are called external or financial costs. Second, there are costs borne inside companies and households from allocating man-hours and other resources to foreign exchange management. These costs are called internal or in-house costs. Their main findings are:

1. A single currency eliminates the present cost associated with converting one EC currency into another. The resulting savings can be estimated around ECU 13 to 19 billion per annum, or about 0.3% to 0.4% of 1990 Community GDP. The largest part of these gains are 'financial', consisting of the disappearance of the exchange margin and commission fees paid to banks. The other gains take the form of reductions in costs and inefficiencies inside the firms. The relative size of these costs are shown in Table 3.[21]

2. Transaction cost savings differ greatly from country to country. The gains for the larger Member States whose currency is extensively used as a means of international payments and belongs to the ERM may be on the order of between 0.1% and 0.2% of national GDP. In contrast, the small, open and less developed economies of the Community may stand to gain around 1% of their GDP.

In the model above, this corresponds to the costs of the brokerage fee times the expected number of trips to the bank. It is clear that once the brokerage fee is calculated, the difference between the two systems will appear from disparities in the frequency of trips to the bank. For values of money velocity and interest rate of 3.9 an 7.1% respectively, the efficiency gains appear to be 0.64% of Community GDP which was approximately ECU 43 billion or US$ 55 billion in 1996. This value is taken to be an upper bound for the true savings. This is for two reasons. The first one is the role that the symmetry assumption plays in the computations. This assumption overweights the magnitude of the foreign sector and, therefore, the importance of exchanging currency in the economy. The second reason comes from using an upper bound for volatility. It turns out that, for the values of the parameters used here, increasing the relative volatility implies both, decreasing the relative transaction costs and overestimating the efficiency gains. As with

[21] Cross-border payments are financial costs too and represent expenses and delays associated with cross-border bank payments.

estimates of the brokerage fee, it is possible to compute these savings, again, for several values of velocity and interest rates. The results are presented in Table 4.

The estimations range from 0.42% to 1.29% of Community GDP. These numbers may seem too high when compared to the predictions of the Commission. If we keep in mind that the Commission results are viewed as lower bounds for the efficiency gains derived from the reduction in transaction costs, and the estimates included in Table 4 represent upper bounds, these numbers seem adequate.

As it was said before, the assumption of symmetry in the payment processes is important in the empirical application of the model since it simplifies calculations a lot. However it adds two distortions in the computations. First, it overweights the magnitude of the foreign sector and, second, it overestimates the effective number of currencies used in international trade. To analyze how robust the estimations are with respect to these effects, Figure 3 shows the calculations of the efficiency gains as a function of the number of currencies used in the model. These computations are done by repeating the same process indicated above for values of money velocity and interest rate of 3.9 and 7.1%, respectively. The transaction cost savings appear as a convex function of n, the number of currencies. It should be pointed out that these savings do not change much for values of n larger than 5 while most of the increase arises when moving from two currencies to one. Figures 4 and 5 show the corresponding values for the relative volatility and the brokerage fee. They are convex functions of n, and have most of the decrease for the lower values of this parameter.

Additionally, one would like to look at the second conclusion of the Commission report, that is, the different impact of the monetary union on the efficiency gains in each country. To do this each of the countries is going to be characterized by a money velocity/interest rate pair. The argument here is that the development of a financial market should show up in the values of velocity and interest rates for that country. Therefore, financially developed countries will have combinations of velocity and interest rates that imply low values of both the relative volatility and the brokerage fee so that savings in transaction costs will be lower there. Again, the last observed values for money velocity and interest rates are used. Then, values for (c/μ), (σ/μ) and efficiency gains are computed for each country as indicated above.[22] These values are included in Table 5. Countries are ordered by savings.[23]

The results contained in Table 5 deserve several comments. First, the Southern European countries (typical examples of less financially developed countries) appear at the bottom together with Denmark that could be consider as a small open economy. Then, the two most important countries, France and Germany, appear together with the Benelux (Belgium, Netherlands and Luxembourg). This seems reasonable since these countries have a very close economic relationship. A little bit surprising is the position of Germany. One would expect it to be at the top of the table since it represents a country with well developed financial markets and with an internationally used currency. This could be the result of the German unification of 1990 and the effects that it could have had on the efficiency of the financial market for the unified Germany as compared with West Germany. To see this effect the same table is constructed for 1989. Germany jumps to third place and reduces the savings from 0.56% to 0.48% which signals a more efficient economy. The rest of the countries do not move significantly. On the other hand, the gains for the less developed countries appear to be around 1% which is the value advanced by the Commission, while the savings for the most efficient countries appear around 0.4% of their GDP.

To test the robustness of the results all the computations are repeated for 1992 and compared with the ones for 1996. The crisis of the EMS started this year and, thus, it should imply a complete different scenario for individual countries in terms of their values for money velocity and interest rates. However, since that shock was aggregate for the whole Union we should not expect their relative positions to change. Table 6 shows these results. We see that they are very similar to the ones in Table 5 with only two changes. Luxembourg and Belgium switch positions and France moves down below Netherlands. Therefore, the predictions of the model seems robust, at least in terms of the order of countries with respect to their savings. At the same time, three characteristics of the numbers in Table 6 indicate that 1992 was, in fact, a recession year. First, savings were generally larger in 1992 than in

[22] This exercise may push the model too much since symmetry is assumed throughout the theoretical exposition. The exercise performed here is similar to looking at the efficiency gains in the EEC when the average country is like one of the 15 member countries and to expect that this average is a reasonable approximation of the efficiency gains for individual countries. Of course, more research is needed to study the asymmetric case.

[23] Again, Finland, Sweden and United Kingdom are not included because of lack of data.

1996. This means lack of integration in the economies, which, at the end, was the main origin of the crisis. Second, for individual countries, the estimated 'brokerage fee' is larger in 1992 than in 1996. This means that the financial sector was using more resources in unproductive uses. Third, the estimated relative volatilities are lower in 1992 than in 1996 which could be implied by lower levels of activity.

Finally, another way of testing the model is to compare the estimated relative transaction costs with some measure of efficiency in the banking sector. One possibility is the ratio of operating expenses to non-bank deposits.[24] This gives an indication of the cost aspect of intermediation. The last available data is for 1991 so this ratio is compared to the estimated (c/μ) of Table 6. The results are included in Table 7.[25] Countries are ordered by efficiency measured by the operating costs. Again, the model performs surprisingly well. Although the model misses completely the relative position Portugal and Austria, the two lists agree for the rest of the countries. It is worthwhile to point out that the model succeeds in consistently estimating low levels of the transaction cost for Greece and high levels for the rest of Mediterranean countries and Denmark. In particular, the model always predicts the highest levels of transaction costs for Italy, clear example of an inefficient banking sector.[26]

5 Conclusions

This paper presents evidence on the savings in transaction costs associated with a monetary unification in Europe. For the Commission of the European Communities this element together with the elimination of the exchange rate uncertainty are "...the two main sources for direct efficiency gains from monetary union..."[27] The importance of these factors is not shared by all economists, though. For example, Viñals [22] believes that the economic benefits from the monetary union in Europe may be largest in further promoting

[24] Operating expenses include expenses related to the ordinary and regular business, in particular, salaries and other employee benefits. Taxes other than income or corporate taxes are also included. It does not include, though, some fee expenses related to borrowing operations and commissions paid in connection with payment services.

[25] Data is taken from OECD [19]. Ireland is not computed because of lack of data.

[26] These results are very robust to changes in the year as well as other measures of costs like staff expenses.

[27] Commission of the European Communities [6].

market integration. He cites official estimates from Commission of the European Communities [5] that quantify these effects to be an additional 2% of the Community's GDP. The total amount of savings derived from the EMU as well as the relative weights to be placed on these and other of its components are still open issues. Thus, more research should be done to answer this question both on theoretical as well as on empirical grounds especially if political decisions about the timing and extent of the integration have to be made in the future. The goal of this paper was to lessen this gap both, by giving new estimates of the gains of the EMU derived from savings in transaction costs and by introducing a theoretical framework suitable to be used in analyzing the effects that variables such as interest rates and incomes have on the estimations.

The model presented here performs surprisingly well considering all the simplifying assumptions made during its empirical application. The upper bound estimation for the EMU transaction cost savings is about 0.6% of Community GDP although it could be as large as 1.29% of Community GDP. Nevertheless, these estimates could be refined by substituting those assumptions by more realistic ones. In particular, the assumption of symmetry of the stochastic processes that represent transactions in each of the countries could be dropped. An alternative less complicated to analyze than the case with n different transaction processes is to assume that a fraction of the transactions made by each country is done with foreigners. Then this fraction can be approximated with a measure of openness of each economy.

Are the results of the paper large or small? Although any evaluation of these numbers has an important subjective component, they should not be seen as insignificant especially if we consider that the size of the European economy is US$ 8.6 trillion. In any case, this is just one component in the stream of benefits and costs derived from the monetary union and its estimation represents one step towards the computation of the net gains associated with the EMU. In this sense, that exercise will only be meaningful within the framework of a general equilibrium model. It is in that environment where all the sources of gains can be integrated and their relative importance assessed.

A Proof of Proposition 1

To prove the proposition, assume first that there exists a M_n^k such that

$$\tau_n = \frac{M_n^k}{\mu_n^k} < \frac{M_n^i}{\mu_n^i} \quad ; \quad (i \neq k). \tag{63}$$

Then the objective function becomes

$$C_n\left(M_n^1, \ldots, M_n^n\right) = \frac{\sum_{i=1}^n s^i M_n^i + c}{1 - \exp\left[-rM_n^k/\mu_n^k\right]} - \frac{\sum_{i=1}^n s^i \mu_n^i}{r}, \tag{64}$$

which is a linear function in all M_n^i $(i \neq k)$. Therefore, to minimize the holding costs of the cash portfolio it is optimal to set $M_n^i = 0$ $(i \neq k)$. Since the solution for $M_n^k > 0$, that contradicts the initial assumption (63) of one money stock being depleted before the others. **Q.E.D.**

B Proof of Proposition 2

The distribution function of τ_n satisfies[28]

$$1 - H_{\tau_n}(t; M_n) = \left[1 - G_{\tau_1}(t; M_n)\right]^n \tag{65}$$

Since

$$0 \leq G_{\tau_1}(t; M_n) \leq 1 \qquad \forall\ t \in [0, \infty) \tag{66}$$

the result in the proposition is immediate. **Q.E.D.**

C Proof of Theorem 1

This problem is a particular case of Harrison et al. [13] where only two types of costs are considered. In order to increase the content of the stock the agent must pay a fixed charge c, and inventory holding costs are continuously incurred at rate $rM(t)$. Also, it must be assumed that the fixed cost of decreasing the stock is so large that this action is never taken. Then, Harrison et

[28] See Mood, Graybill and Boes [18] pg. 184.

al. [13] proved that if the objective is to minimize the expected present value of holding and control costs, the policy associated with the single-currency system is the optimal one among the class of all feasible policies including the ones that appear from the multi-currency system. **Q.E.D.**

D Computation of Expression [34]

In this Appendix, the expected present value of all costs of holding money will be calculated for the general multi-currency system. Without control, the money holdings follow the processes

$$\begin{array}{c} dM_n^1(t) = -\mu_n^1 dt + \sigma_n^1 dW_n^1(t) \\ dM_n^2(t) = -\mu_n^2 dt + \sigma_n^2 dW_n^2(t) \\ \vdots \\ dM_n^n(t) = -\mu_n^n dt + \sigma_n^n dW_n^1(t) \end{array} , \tag{67}$$

with $M_n^i(0) = M_n^i$, or, in more compact notation

$$M(t) = M(0) - \Delta t + \Sigma W(t) \tag{68}$$

where

$$M(t) = \begin{bmatrix} M_n^1(t) \\ M_n^2(t) \\ \vdots \\ M_n^n(t) \end{bmatrix} \quad ; \quad W(t) = \begin{bmatrix} W_n^1(t) \\ W_n^2(t) \\ \vdots \\ W_n^n(t) \end{bmatrix} \tag{69}$$

$$\Delta = \begin{bmatrix} \mu_n^1 \\ \mu_n^2 \\ \vdots \\ \mu_n^n \end{bmatrix} \quad ; \quad \Sigma = \begin{bmatrix} \sigma_n^1 & 0 & \cdots & 0 \\ 0 & \sigma_n^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n^n \end{bmatrix} . \tag{70}$$

First, it is necessary to prove that $E[\tau_n^i] < \infty$. Define the process

$$\widetilde{M}_n^i(t) \equiv M_n^i(t) + \mu_n^i t \qquad (i = 1, \ldots, n). \tag{71}$$

Since this process is a martingale, by the Martingale Stopping Theorem (MST),[29]

$$E\left[\widetilde{M}_n^i(\tau_n^i \wedge t)\right] = E\left[\widetilde{M}_n^i(0)\right] \Longrightarrow E\left[M_n^i(\tau_n^i \wedge t)\right] + \mu_n^i E\left[\tau_n^i \wedge t\right] = M_n^i. \tag{72}$$

[29] See Harrison [12] pg. 130.

Of course, $M_n^i(\tau_n^i \wedge t) \geq 0$, for all $t > 0$, so

$$M_n^i = E\left[M_n^i(\tau_n^i \wedge t)\right] + \mu_n^i E\left[\tau_n^i \wedge t\right] \geq \mu_n^i E\left[\tau_n^i \wedge t\right]. \tag{73}$$

Since this holds for all $t > 0$, we have that

$$E[\tau_n^i] \leq \frac{M_n^i}{\mu_n^i} < \infty. \tag{74}$$

This, in turn, implies $P\{\tau_n^i < \infty\} = 1$, for $i = 1, \ldots, n$.

The cost function is

$$\begin{aligned} C_n\left(M_n^1, \ldots, M_n^n\right) &= c + E\left[\int_0^{\tau_n} e^{-rt} r \left(\sum_{i=1}^n s^i M_n^i(t)\right) dt\right] \\ &\quad + E\left[e^{-r\tau_n}\right] C_n\left(M_n^1, \ldots, M_n^n\right) \end{aligned} \tag{75}$$

First,

$$\begin{aligned} E\left[\int_0^{\tau_n} e^{-rt} r \left(\sum_{i=1}^n s^i M_n^i(t)\right) dt\right] &= E\left[\int_0^{\infty} e^{-rt} r \left(\sum_{i=1}^n s^i M_n^i(t)\right) dt\right] \\ &\quad - E\left[\int_{\tau_n}^{\infty} e^{-rt} r \left(\sum_{i=1}^n s^i M_n^i(t)\right) dt\right] \end{aligned} \tag{76}$$

Then,

$$\begin{aligned} E\left[\int_0^{\infty} e^{-rt} r \left(\sum_{i=1}^n s^i M_n^i(t)\right) dt\right] &= \int_0^{\infty} e^{-rt} r E\left(\sum_{i=1}^n s^i M_n^i(t)\right) dt \\ = \int_0^{\infty} e^{-rt} r \left[\sum_{i=1}^n s^i \left(M_n^i - \mu_n^i t\right)\right] dt &= \sum_{i=1}^n s^i M_n^i - \frac{\sum_{i=1}^n s^i \mu_n^i}{r} \end{aligned} \tag{77}$$

where the first equality follows from a version of Fubini's Theorem (FT).[30]

[30] See Harrison [12] pg. 131. To apply FT here, we need the condition

$$E\left[\int_0^{\infty} \left|\exp(-rt) r \sum_{i=1}^n s^i M_n^i(t)\right| dt\right] < \infty$$

which is satisfied from expression (3.3.1) in Harrison [12] pg. 44.

On the other hand,

$$
\begin{aligned}
& E\left[\int_{\tau_n}^{\infty} e^{-rt} r \left(\sum_{i=1}^{n} s^i M_n^i(t)\right) dt\right] \\
= \; & E\left[e^{-r\tau_n} \int_{\tau_n}^{\infty} e^{-r(k-\tau_n)} r \left(\sum_{i=1}^{n} s^i M_n^i(k)\right) dk\right] \\
= \; & E\left[e^{-r\tau_n} \int_{0}^{\infty} e^{-rt} r \left(\sum_{i=1}^{n} s^i M_n^i(t+\tau_n)\right) dt\right] \\
= \; & E\left[e^{-r\tau_n} \int_{0}^{\infty} e^{-rt} r \left(\sum_{i=1}^{n} s^i M_n^i(\tau_n)\right) dt\right] \\
& + E\left\{e^{-r\tau_n} \int_{0}^{\infty} e^{-rt} r \left[\sum_{i=1}^{n} s^i \left(M_n^i(t+\tau_n) - M_n^i(\tau_n)\right)\right] dt\right\} \qquad (78)
\end{aligned}
$$

The second equality is just a change of variable ($t = k - \tau_n$). Using FT again,

$$
\begin{aligned}
& E\left[e^{-r\tau_n} \int_{0}^{\infty} e^{-rt} r \left(\sum_{i=1}^{n} s^i M_n^i(\tau_n)\right) dt\right] \\
= \; & \sum_{i=1}^{n} s^i E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] \int_{0}^{\infty} e^{-rt} r dt = \sum_{i=1}^{n} s^i E\left[e^{-r\tau_n} M_n^i(\tau_n)\right], \qquad (79)
\end{aligned}
$$

and

$$
\begin{aligned}
E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] \; &= \; \sum_{j=1}^{n} E\left[e^{-r\tau_n^j} M_n^i(\tau_n^j) \,\middle|\, \tau_n = \tau_n^j\right] P\left(\tau_n = \tau_n^j\right) = \\
&= \; \sum_{j \neq i} E\left[e^{-r\tau_n^j} M_n^i(\tau_n^j) \,\middle|\, \tau_n = \tau_n^j\right] P\left(\tau_n = \tau_n^j\right) \qquad (80)
\end{aligned}
$$

The first equality uses the Law of Total Probability (LTP)[31] and the second one is true since $M_n^i(\tau_n^i) = 0$.

With respect to the other term in (78),

$$
\begin{aligned}
& E\left\{e^{-r\tau_n} \int_{0}^{\infty} e^{-rt} r \left[\sum_{i=1}^{n} s^i \left(M_n^i(t+\tau_n) - M_n^i(\tau_n)\right)\right] dt\right\} \\
= \; & E\left[e^{-r\tau_n}\right] \int_{0}^{\infty} e^{-rt} r \left[\sum_{i=1}^{n} s^i E\left(M_n^i(t+\tau_n) - M_n^i(\tau_n)\right)\right] dt \\
= \; & -E\left[e^{-r\tau_n}\right] \sum_{i=1}^{n} \left(s^i \mu_n^i\right) \int_{0}^{\infty} e^{-rt} r dt = -\frac{\sum_{i=1}^{n} s^i \mu_n^i}{r} E\left[e^{-r\tau_n}\right]. \qquad (81)
\end{aligned}
$$

[31] See Karlin and Taylor [14] pg. 8.

In the first equality I am using both the FT and the fact that the random variable τ_n is independent of $M_n^i(t+\tau_n) - M_n^i(\tau_n)$. Then, rearranging,

$$E\left[\int_{\tau_n}^{\infty} e^{-rt} r \left(\sum_{i=1}^{n} s^i M_n^i(t)\right) dt\right] = \sum_{i=1}^{n} s^i E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] - \frac{\sum_{i=1}^{n} s^i \mu_n^i}{r} E\left[e^{-r\tau_n}\right], \qquad (82)$$

and

$$E\left[\int_{0}^{\tau_n} e^{-rt} r \left(\sum_{i=1}^{n} s^i M_n^i(t)\right) dt\right] = \sum_{i=1}^{n} s^i M_n^i - \sum_{i=1}^{n} s^i E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] - \frac{\sum_{i=1}^{n} s^i \mu_n^i}{r}\left[1 - E\left(e^{-r\tau_n}\right)\right]. (83)$$

Substituting into the cost function yields

$$C_n\left(M_n^1, \ldots, M_n^n\right) = \frac{\sum_{i=1}^{n} s^i M_n^i - \sum_{i=1}^{n} s^i E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] + c}{1 - E\left(e^{-r\tau_n}\right)} - \frac{\sum_{i=1}^{n} s^i \mu_n^i}{r}, \qquad (84)$$

which is expression (34) in the paper. The next step is to compute the expectations $E\left[\exp\left(-r\tau_n\right)\right]$ and $E\left[\exp\left(-r\tau_n\right) M_n^i(\tau_n)\right]$.

Under the symmetry assumption, the stopping times τ_n^i $(i = 1, \ldots, n)$ are independent and identically distributed with common density and distribution functions[32]

$$f_{\tau_n^i}(t) = \frac{M_n^i}{\sigma_n^i \sqrt{2nt^3}} \exp\left[\frac{\left(M_n^i - \mu_n^i t\right)^2}{2\left(\sigma_n^i\right)^2 t}\right] \qquad (85)$$

and

$$F_{\tau_n^i}(t) \equiv P\left[\tau_n^i < t\right] = 1 - \Phi\left(\frac{M_n^i - \mu_n^i t}{\sigma_n^i \sqrt{t}}\right) + \exp\left[\frac{2M_n^i \mu_n^i}{\left(\sigma_n^i\right)^2}\right] \Phi\left(\frac{-M_n^i - \mu_n^i t}{\sigma_n^i \sqrt{t}}\right), \qquad (86)$$

[32] See Harrison [12] pg. 14.

where $\Phi(\cdot)$ is the $N(0,1)$ distribution function. Now it is easy to get the density function of τ_n as[33]

$$f_{\tau_n}(t) = n\left[1 - F_{\tau_n^i}(t)\right]^{n-1} f_{\tau_n^i}(t). \tag{87}$$

The expectation $E[\exp(-r\tau_n)]$ is just the Laplace transform of this density. Although no closed form exists for this expression, it can be approximated by numerical methods. At the same time, because of the symmetry assumption

$$P\left[\tau_n = \tau_n^i\right] = \frac{1}{n} \tag{88}$$

so

$$E\left[\exp(-r\tau_n) M_n^i(\tau_n)\right] = \frac{n-1}{n} E\left[\exp\left(-r\tau_n^j\right) M_n^i(\tau_n^j) \middle| \tau_n = \tau_n^i\right]. \tag{89}$$

To compute the expectation on the right hand side of (89), first define the random variable

$$Y_n^{i,j} = M_n^i(\tau_n^j), \tag{90}$$

which gives the value of the process $M_n^i(t)$ at the epoch when the process $M_n^j(t)$ first reaches the origin. Then the expectation in (89) can be computed as the double integral

$$E\left[\exp\left(-r\tau_n^j\right) M_n^i(\tau_n^j) \middle| \tau_n = \tau_n^i\right] = \int_0^\infty \int_0^\infty e^{-rt} y p(t,y) dt dy, \tag{91}$$

where $p(t,y)$ is the joint distribution of τ_n^j and $Y_n^{i,j}$ conditional on $M_n^j(t)$ being the first process to reach the origin.

In order for the analysis to be similar to that in Harrison [12] where a process that starts at zero and reaches a positive boundary is studied, I will characterize the distribution $p(t,y)$ for the processes

$$Z_n^i(t) \equiv M_n^i - M_n^i(t) \tag{92}$$

and perform the corresponding transformations afterwards. With the preceding definition. the processes $Z_n^i(t)$ follow the stochastic differential equations

$$\begin{array}{ll} dZ_n^1(t) = \mu_n^1 dt - \sigma_n^1 dW_n^1(t) & Z_n^1(0) = 0 \\ dZ_n^2(t) = \mu_n^2 dt - \sigma_n^2 dW_n^2(t) & Z_n^2(0) = 0 \\ \vdots & \vdots \\ dZ_n^n(t) = \mu_n^n dt - \sigma_n^n dW_n^1(t) & Z_n^2(0) = 0 \end{array} \tag{93}$$

[33] See Mood, Graybil and Boes [18] section V.3.2.

and, as before, define the stopping times

$$\tau_n^i \equiv \left\{t \geq 0 : Z_n^i(t) = a\right\} \qquad (i = 1, \ldots, n), \tag{94}$$

and

$$\tau_n \equiv \min\left\{\tau_n^i\right\}. \tag{95}$$

Let $S_n^i(t) \equiv \sup\left\{Z_n^i(t), 0 \leq s \leq t\right\}$, then[34]

$$P\left[Z_n^i(t) \in dz, S_n^i(t) \leq s\right] = v(x, s; t)dx, \tag{96}$$

where

$$v(x, s; t) = \frac{1}{\sigma_n^i} \exp\left[\frac{\mu_n^i x}{\left(\sigma_n^i\right)^2} - \frac{\left(\mu_n^i\right)^2 t}{2\left(\sigma_n^i\right)^2}\right] u\left(\frac{x}{\sigma_n^i}, \frac{s}{\sigma_n^i}\right), \tag{97}$$

$$u(a, b) = \frac{\phi[at^{-1/2}] - \phi[(a-2b)t^{-1/2}]}{t^{1/2}} \tag{98}$$

and $\phi(x)$ is the $N(0,1)$ density function. Now

$$\begin{aligned}
& P\left[Z_n^i(\tau_n^j) \in dz \,\middle|\, \tau_n^j = t\right] \\
= \; & P\left[Z_n^i(t) \in dz \,\middle|\, S_n^i(t) \leq a\right] = \frac{P\left[Z_n^i(t) \in dz, S_n^i(t) \leq a\right]}{P\left[S_n^i(t) \leq a\right]} \\
= \; & \frac{v(x, a; t)}{\Phi\left(\frac{a - \mu_n^i t}{\sigma_n^i \sqrt{t}}\right) + \exp\left[\frac{2a\mu_n^i}{\left(\sigma_n^i\right)^2}\right] \Phi\left(\frac{-a - \mu_n^i t}{\sigma_n^i \sqrt{t}}\right)} dx \\
= \; & w(x; a, t)dx,
\end{aligned} \tag{99}$$

so that $w(x; a, t)$ is the density function of $Z_n^i(\tau_n^j)$ conditional on $\tau_n^j = t$ and $\tau_n = \tau_n^j$. Let $g_{\tau_n^j|\tau_n = \tau_n^j}(t; a)$ denote the density function of τ_n^j conditional on $\tau_n = \tau_n^j$. Then

$$q(x, t; a) = w(x; a, t) g_{\tau_n^j|\tau_n = \tau_n^j}(t; a) \tag{100}$$

where, as before, $q(x, t; a)$ is the joint distribution of τ_n^j and $Z_n^i(\tau_n^j)$ conditional on $Z_n^j(t)$ being the first process to reach a. But the τ_n^j are i.i.d. so

$$g_{\tau_n^j|\tau_n = \tau_n^j}(t; a) = g_{\tau_n^i|\tau_n = \tau_n^i}(t; a), \tag{101}$$

[34]See Harrison [12] pg. 11.

and since it is possible to write the density of τ_n as a mixture of these densities

$$\begin{aligned} h_{\tau_n}(t;a) &= \sum_{i=1}^{n} g_{\tau_n^i|\tau_n=\tau_n^i}(t;a) P\left[\tau_n = \tau_n^i\right] = \sum_{i=1}^{n} g_{\tau_n^i|\tau_n=\tau_n^i}(t;a)\frac{1}{n} \\ &= g_{\tau_n^i|\tau_n=\tau_n^i}(t;a), \end{aligned} \tag{102}$$

then

$$q(x,t;a) = h_{\tau_n}(t;a)w(x;a,t) = n\left[1 - G_{\tau_n^i}(t;a)\right]^{n-2} g_{\tau_n^i}(t;a)v(x,a;t), \tag{103}$$

where g and G are defined in (44) and (45) respectively. To transform these results into the variables $M_n^i(t)$ notice

$$p(m,t) = q(x,t;M_n^i) \qquad \text{with} \qquad x = M_n^i - m. \tag{104}$$

Therefore,

$$\begin{aligned} p(m,t) &= h_{\tau_n}(t;M_n^i)w(M_n^i - m;M_n^i,t) = \\ &= n\left[1 - G_{\tau_n^i}(t;M_n^i)\right]^{n-2} g_{\tau_n^i}(t;M_n^i)v(M_n^i - m, M_n^i;t). \end{aligned} \tag{105}$$

Finally,

$$\begin{aligned} & E\left[e^{-r\tau_n^j} M_n^i(\tau_n^j) \,\middle|\, \tau_n = \tau_n^i\right] \\ =& \int_0^\infty \int_0^\infty e^{-rt} m p(t,m) dm dt \qquad (106) \\ =& \int_0^\infty e^{-rt} h_{\tau_n}(t;M_n^i) \int_0^\infty m w(M_n^i - m; M_n^i, t) dm dt \\ =& \int_0^\infty e^{-rt} h_{\tau_n}(t;M_n^i) \int_0^\infty m f(m) dm dt \qquad (107) \end{aligned}$$

with

$$f(m) = \frac{\exp\left[\frac{2\mu_n^i(M_n^i - m) - \left(\mu_n^i\right)^2 t}{2(\sigma_n^i)^2}\right] u\left(\frac{M_n^i - m}{\sigma_n^i\sqrt{t}}, \frac{-M_n^i - m}{\sigma_n^i\sqrt{t}}\right)}{\sigma_n^i\sqrt{t}\left[\Phi\left(\frac{M_n^i - \mu_n^i t}{\sigma_n^i\sqrt{t}}\right) + \exp\left[\frac{2M_n^i\mu_n^i}{(\sigma_n^i)^2}\right]\Phi\left(\frac{-M_n^i - \mu_n^i t}{\sigma_n^i\sqrt{t}}\right)\right]}. \tag{108}$$

After some computations,

$$E\left[M_n^i(\tau_n^j)\,|\tau_n = t\right] = \int_0^\infty m f(m) dm = M_n^i - \mu_n^i t + 2\xi_n^i(t) \tag{109}$$

with

$$\xi_n^i(t) = \frac{\exp\left[\frac{2\mu_n^i M_n^i}{(\sigma_n^i)^2}\right]}{\Psi_n^i(t) - \exp\left[\frac{2M_n^i\mu_n^i}{(\sigma_n^i)^2}\right]} \tag{110}$$

and

$$\Psi_n^i(t) = \frac{\Phi\left(\frac{M_n^i - \mu_n^i t}{\sigma_n^i\sqrt{t}}\right)}{\Phi\left(\frac{-M_n^i - \mu_n^i t}{\sigma_n^i\sqrt{t}}\right)}. \tag{111}$$

Then,

$$\begin{aligned} E\left[e^{-r\tau_n^j} M_n^i(\tau_n^j)\,\middle|\,\tau_n = \tau_n^i\right] &= M_n^i E\left[e^{-r\tau_n}\right] - \mu_n^i E\left[\tau_n e^{-r\tau_n}\right] \\ &\quad + 2M_n^i E\left[e^{-r\tau_n}\xi_n^i(t)\right] \end{aligned} \tag{112}$$

and, from (89),

$$\begin{aligned} &E\left[e^{-r\tau_n} M_n^i(\tau_n)\right] \\ &= \left(\frac{n-1}{n}\right)\left[M_n^i E\left(e^{-r\tau_n}\right) - \mu_n^i E\left(\tau_n e^{-r\tau_n}\right) + 2M_n^i E\left(e^{-r\tau_n}\xi_n^i(t)\right)\right], \end{aligned} \tag{113}$$

where all the expectations are with respect to the density of τ_n and can be computed, again, numerically.

E First Order Condition for the n-Currency System

The first order condition (FOC) for the n-currency system is

$$\begin{aligned} 0 &= \left[1 - E\left(e^{-r\tau_n}\right)\right]\left(\frac{n-1}{n}\right)\left[\frac{n}{n-1} - E\left(e^{-r\tau_n}\right) - 2E\left[e^{-r\tau_n}\xi_n^i(t)\right]\right] \\ &\quad - \left(\frac{n-1}{n}\right)\left[2M_n E\left(e^{-r\tau_n}\xi_n^i(t)\right) - \mu E\left(\tau_n e^{-r\tau_n}\right) - \frac{1}{n-1}M_n \right. \\ &\quad \left. - \frac{nc}{n-1}\right]\frac{\partial E\left(e^{-r\tau_n}\right)}{\partial M_n} + \mu\left(\frac{n-1}{n}\right)\left[1 - E\left(e^{-r\tau_n}\right)\right]\frac{\partial E\left(\tau_n e^{-r\tau_n}\right)}{\partial M_n} \\ &\quad - 2M_n\left(\frac{n-1}{n}\right)\left[1 - E\left(e^{-r\tau_n}\right)\right]\frac{\partial E\left(e^{-r\tau_n}\xi_n^i(t)\right)}{\partial M_n} \end{aligned} \tag{114}$$

where

$$\begin{aligned}\frac{\partial E\left[e^{-r\tau_n}\right]}{\partial M_n} &= 2(n-1)n\int_0^\infty e^{-rt}\left[1-G_{\tau_n^i}(t;a)\right]^{n-2}A(t;M_n)dt \\ &-\frac{M_n}{n\sigma^2}E\left[\frac{e^{-r\tau_n}}{\tau_n}\right]+\left[\frac{1}{M_n}+\frac{\mu}{n\sigma^2}\right]E\left(e^{-r\tau_n}\right) \\ &-(n-1)\frac{2\mu}{n\sigma^2}E\left[e^{-r\tau_n}\xi_n^i(t)\right] \end{aligned} \tag{115}$$

$$\begin{aligned}\frac{\partial E\left[\tau_n e^{-r\tau_n}\right]}{\partial M_n} &= 2(n-1)n\int_0^\infty te^{-rt}\left[1-G_{\tau_n^i}(t;a)\right]^{n-2}A(t;M_n)dt \\ &-\frac{M_n}{n\sigma^2}E\left[e^{-r\tau_n}\right]+\left[\frac{1}{M_n}+\frac{\mu}{n\sigma^2}\right]E\left(\tau_n e^{-r\tau_n}\right) \\ &-(n-1)\frac{2\mu}{n\sigma^2}E\left[\tau_n e^{-r\tau_n}\xi_n^i(t)\right] \end{aligned} \tag{116}$$

$$\begin{aligned}\frac{\partial E\left[e^{-r\tau_n}\xi_n^i(t)\right]}{\partial M_n} &= \left[\frac{1}{M_n}+\frac{3\mu}{n\sigma^2}\right]E\left[e^{-r\tau_n}\xi_n^i(t)\right]-\frac{M_n}{n\sigma^2}E\left[\frac{e^{-r\tau_n}}{\tau_n}\xi_n^i(t)^2\right] \\ &+2(n-2)n\exp\left(\frac{2\mu M_n}{n\sigma^2}\right)\times \\ &\int_0^\infty e^{-rt}\left[1-G_{\tau_n^i}(t;a)\right]^{n-3}\Phi\left(\frac{-M_n-\mu t}{\sigma\sqrt{t}}\right)A(t;M_n)dt \\ &-(n-2)\frac{2\mu}{n\sigma^2}E\left[e^{-r\tau_n}\xi_n^i(t)^2\right] \\ &-n\int_0^\infty e^{-rt}\left[1-G_{\tau_n^i}(t;a)\right]^{n-2}A(t;M_n)dt \end{aligned} \tag{117}$$

$$A(t;M_n)=\frac{M_n}{2n\sigma^2t^2\pi}\Phi\left(\frac{(M_n-\mu t)^2}{n\sigma^2 t}\right) \tag{118}$$

and $\xi_n^i(t)$ was defined in Appendix D.

F Data Sources

The sources for the data used in essay one were collected from the IMF, *International Financial Statistics* (IFS), Commission of the European Communities [7] (CEC), Boletín Estadístico del Banco de España (BEBE) and OECD [19] (OECD). The variables are:

- Money stock: Money (line 34, IFS)
- Interest rate: Government Bond Yield (line 61, IFS) and Rendimientos de la deuda pública a largo plazo utilizados para la medición de la convergencia de los países de la UE (table 26.23, BEBE)
- Transactions: Gross Domestic Product (table 4, CEC)
- Exchange rate: ECU exchange rates (table 54, CEC)
- Operating expenses (line 6, OECD)
- Non-bank deposits (line 25, OECD)

References

[1] Barro, R. J. and A. J. Santomero (1972): "Household Money Holdings and the Demand Deposit Rate" *Journal of Money, Credit and Banking* 4, May: 397-413.

[2] Baumol, W. J. (1952): "The Transaction Demand for Cash: An Inventory-Theoretic Approach" *Quarterly Journal of Economics* 66 (4): 545-556.

[3] Baumol, W. J. and J. Tobin (1989): "The Optimal Cash Balance Proposition: Maurice Allais' Priority" *Journal of Economic Literature* 27, September: 1160-1162.

[4] Chang, F. R. (1992): "Homogeneity and the Transactions Demand for Money" Unpublished paper, Indiana University.

[5] Commission of the European Communities (1988): "The Economics of 1992" *European Economy* 33, March.

[6] Commission of the European Communities (1990): "One Market, One Money: An Evaluation of the Potential Benefits and Costs of Forming and Economic and Monetary Union" *European Economy* 44, October.

[7] Commission of the European Communities (1996): "Statistical Annex" *European Economy* 62.

[8] Frenkel, J. and B. Jovanovic (1980): "On Transactions and Precautionary Demand for Money" Quarterly Journal of Economics 95(3): 27-43.

[9] Goldfeld, S. M. (1987): "Demand for Money: Empirical Studies" in *The New Palgrave: A Dictionary of Economics*, J. Eatwell, M. Milgate, and P. Newman (eds.), London: Macmillan, New York Press.

[10] Goodhart, C. A. E. (1989): *Money, Information and Uncertainty*, 2nd ed. The MIT Press.

[11] Guidotti, P. E. (1993): "Currency Substitution and Financial Innovation" *Journal of Money, Credit and Banking* 25 (1): 109-124.

[12] Harrison, J. M. (1985): *Brownian Motion and Stochastic Flow Systems.* John Wiley and Sons.

[13] Harrison, J. M., T. M. Selke and A. J. Taylor (1983): "Impulse Control of Brownian Motion" *Mathematics of Operations Research* 8 (3): 454-466.

[14] Karlin, S. and H. Taylor (1975): *A First Course in Stochastic Processes.* Academic Press, New York.

[15] Laidler, D. E. (1985): *The Demand for Money: Theories, Evidence, and Problems*, 3rd ed. Harper and Row Publishers.

[16] Lieberman, C. (1977): "The Transactions Demand for Money and Technological Change" *Review of Economics and Statistics* 59: 307-317.

[17] Lucas, R. E. Jr. (1993): "On the Welfare Cost of Inflation" mimeo, University of Chicago.

[18] Mood, A. M., F. A. Graybill and D. C. Boes (1965): *Introduction to the Theory of Statistics*, Mc Graw Hill.

[19] Organisation for Economic Co-operation and Development (1993): *Bank Profitability, Statistical Supplement. Financial Statements of Banks 1982-1991.*

[20] Orr, D. (1970): *Cash Management and the Demand for Money*, Praeger Publisher.

[21] Tobin, J. (1956): "The Interest Elasticity of the Transactions Demand for Cash" *Review of Economic and Statistics* 38 (3): 241-247.

[22] Viñals, J. (1994): "Building a Monetary Union in Europe: Is it Worthwhile, Where Do We Stand, and Where Are We Going?" *CEPR Occasional Paper* 15.

Table 1

Upper Bound Estimation of (σ/μ)

μ/M	3	**4**	5
σ/μ	0.300	**0.260**	0.235

Table 2

Lower Bound Estimation of $(c/\mu)^a$

μ/M r	3	**4**	5
0.06	0.023	0.013	0.008
0.07	0.027	**0.015**	0.009
0.08	0.030	0.017	0.011
0.09	0.034	0.019	0.012
0.10	0.038	0.021	0.013
0.11	0.042	0.024	0.015

[a] As a percentage of GDP

Table 3

Cost Savings on Intra-EC Settlements by Single Currency[a]

	Estimated range	
1. Financial transaction costs	**9.5**	**14.4**
Bank transfers	6.4	10.6
Bank notes, eurochecks, credit cards, etc.	1.8	2.5
Reduction of cross-border payment costs	1.3	1.3
2. In-house costs	**3.6**	**4.8**
TOTAL	**13.1**	**19.2**

[a] In billion ECU, 1990

Table 4
Upper Bound Estimation of Efficiency Gains[a]

μ/M r	3	**4**	5
0.06	0.704	0.527	0.421
0.07	0.821	**0.615**	0.491
0.08	0.939	0.703	0.561
0.09	1.056	0.792	0.631
0.10	1.174	0.880	0.701
0.11	1.291	0.968	0.772

[a] As a percentage of GDP

Table 5
Estimation of Savings for Individual Countries (1996)

Country	μ/M	r	(σ/μ)	$(c/\mu)^a$	Savings[a]
Ireland	7.2	7.3	0.195	0.005	0.36
Austria	5.8	6.3	0.215	0.006	0.38
Belgium	5.7	6.5	0.220	0.008	0.40
Luxembourg	4.9	6.3	0.235	0.009	0.45
France	4.3	6.3	0.250	0.012	0.52
Germany	3.9	6.2	0.265	0.014	0.56
Netherlands	3.4	6.1	0.280	0.018	0.63
Greece	6.8	13.5	0.200	0.010	0.70
Denmark	3.1	7.2	0.295	0.026	0.82
Spain	3.6	8.7	0.275	0.023	0.85
Portugal	3.3	8.6	0.285	0.027	0.92
Italy	3.1	9.4	0.295	0.034	1.07

[a] As a percentage of GDP

Table 6
Estimation of Savings for Individual Countries (1992)

Country	μ/M	r	(σ/μ)	$(c/\mu)^a$	Savings[a]
Ireland	9.4	9.3	0.170	0.004	0.35
Austria	7.2	8.2	0.195	0.005	0.40
Luxembourg	5.0	7.9	0.235	0.011	0.55
Belgium	5.2	8.6	0.230	0.011	0.58
Germany	4.8	7.9	0.235	0.012	0.58
Netherlands	4.2	8.1	0.255	0.016	0.68
France	4.4	8.6	0.250	0.015	0.69
Greece	7.9	19.9	0.185	0.011	0.89
Denmark	3.3	9.0	0.285	0.028	0.96
Spain	3.5	11.7	0.280	0.032	1.17
Portugal	3.7	15.4	0.270	0.039	1.47
Italy	2.9	13.3	0.305	0.054	1.62

[a] As a percentage of GDP

Table 7
Estimation of Relative Costs for
Individual Countries

Country	$(c/\mu)^a$	Operating Costs[b]
Luxembourg	0.011	1.14
Greece	0.011	2.81
Germany	0.012	3.18
Belgium	0.011	3.64
Netherlands	0.016	3.65
Portugal	0.039	3.79
Austria	0.005	3.89
France	0.015	4.40
Spain	0.032	4.53
Denmark	0.029	4.99
Italy	0.054	5.59

[a] As a percentage of GDP
[b] As a percentage of non-bank deposits