NEGATIVITY EFFECT

IN

MULTIPARTY ELECTORAL COMPETITION*

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ABSTRACT

We construct a dynamic voting model of multiparty competition in order to capture the following facts: voters base their decision on past economic performance of the parties, and parties and candidates have different objectives. This model may explain the emergence of parties' ideologies, and shows the compatibility of the different objectives of parties and candidates. Together, these results give rise to the formation of political parties, as infinitely-lived agents with a certain ideology, out of the competition of myopic candidates freely choosing policy positions. We also show that in multicandidate elections held under the plurality system, Hotelling's principle of minimum differentiation is no longer satisfied.

1. INTRODUCTION

Most of the literature on electoral competition focuses on the study of static models. Parties decide strategically which policy to advocate in order to win the election. Voters decide which party they like best and the model typically only attempts to predict the elections' outcome. While models of this type capture some of the important features of party competition, they ignore some others. In reality, elections are repeated over time. The dynamic nature of the process changes both the circumstances under which elections are held and the voters' preferences. Finally, it introduces a distinction between a candidate and her party as separate decision makers who may have different goals. This paper presents a simple model of three-party competition, focusing on these aspects. For simplicity and clarity of exposition we ignore many of the standard aspects of party competition that are widely studied in the literature.

Since voters delegate the choice of a policy to parties, the choices available to the voters are the parties. The decision of the voters should be based on their available information about parties. In a dynamic setting, we consider past performance of the parties while in office as the most reliable information that is readily available to the voters.

We model the fact that in reality, voters often express dissatisfaction with the parties. Numerous studies of impression formation have found that negative information is weighted more heavily than positive information as impressions of others are formed. This tendency is known as the *negativity effect*. There are two different explanations of the negativity effect in political behavior. The perceptual "figure-ground" theory proposes that the negativity effect occurs because negative actions are contrasted against the generally positive expectations of presidential candidates' behavior. The "cost orientation" explanation argues that people are risk oriented and motivated to avoid loss. Several empirical studies have shown the importance of the negativity effect in political behavior (Kernell (1977), Lau (1982 and 1985), Klein (1991). For a survey, see Aragones (1997)).

As in Aragones (1997), our model assumes that it is dissatisfaction, rather than satisfaction that drives voters' choices; that is, voters vote *against* rather than *for* parties. Hence the record that voters keep of the parties reflects how much they disliked them rather than how much they liked them, and this record is updated every time that a new party takes office. Further, voters' dissatisfaction accumulates over time. Formally, elections take place at periods $t = 0,1,2,...$

Before each election voters update their evaluations of the parties by including the results of last election, given by the last party in office (p^t) and the policy it chose to implement (x^t). We can write the decision rule of voter i as maximizing:

$$U_i^T(p) = \begin{cases} u_i(x^{t-1}) + U_i^{T-1}(p) & \text{if } p \text{ was in office at } t-1 \\ U_i^{T-1}(p) & \text{if } p \text{ wasnot in office at } t-1 \end{cases}$$

Through this rule, the memory of past performance of the parties directly affects the nature and "utility" of present experiences. Therefore, even if voters know how much they like or dislike a policy implemented by one of the parties at a certain time, $u_i(x)$, this instantaneous utility function does not summarize all the relevant information. The function $u_i()$ should be thought of as some derivative of the "real" utility, $U_i()$, which in turn, is the aggregate of $u_i()$ values. This interpretation reflects the fact that the evaluation of parties by the voters is constantly changing over time as new parties are taking office. Since, as we have argued, voters get more tired of parties the longer they are in power, we define $u_i()$ as the negative of the distance between the voter's ideal point and the policy chosen by the party.

Since elections take place over time, we have to deal with the fact that candidates and parties have different "life" horizon. While candidates typically cannot be reelected forever, parties can. Thus it is natural to make a distinction between candidates and parties with respect to their objective functions. Since candidates cannot be reelected over and over again, they behave myopically. It is assumed that the objective of the candidates while in office is to maximize the proportion of votes that their party will get in next election[1]. Parties, on the other hand, care about the future and they try to maximize the number of times they will be in office over time. Candidates may be viewed as players in a one period game, whereas parties – as players in an infinite period one. We will see that the different objectives of parties and candidates are compatible. Furthermore, the choices that candidates make while in office result in the formation of different ideologies for different parties.

For simplicity we assume that the choices of parties and candidates are given by the set $\{0,1\}$. It may be interpreted as follows: given a position of the economic variables, the performance of a party in office for one period can decide

1 Since in this model voters base their decision on parties' past performance, the only relevant decision of parties and candidates is the policy they implement after they have been elected. Platforms do not have any effect on the voters' choice and therefore are omitted.

the direction in which the economy is going to move but cannot position the economy in a point of the state space far away from the original one (status quo). Grofman (1985) analyzes the relationship of this policy space with the standard one. As an example, policies of type 0 can be thought of as "to cut taxes" or "protectionist policies" or "decrease public expenditure" and policies of type 1 as "to raise taxes" or "nonprotectionist policies" or "increase public expenditure" respectively. The model can also apply to any other kind of policy, for instance, to increase or decrease aid to fledgling democracies.

The properties that characterize the electoral system in the present model are the following: each voter has a single vote to cast and there is a single-winner elected under plurality rule. Given the decision rule of the voters we have sincere voting and, because our voters do not consider the possibility of abstention, all votes are to be cast. There is a continuum of voters with "ideal points" uniformly distributed on the interval $[0,1]$. (An "ideal point" reflects a voter's preferences fro the frequency with which policy 1 is implemented.) The number of parties is exogenous and the main results are given for a three party competition. Parties do not have ideal points to begin with. These are endogenously determined by maximizing their objective functions. Each party presents one candidate for each election. Therefore there is no need for platforms: voters make their decision based on past performance alone. The winner of the election has to choose a policy to implement in the set $\{0,1\}$. Since candidates cannot be reelected forever, they maximize the proportion of votes stagewise. Parties, on the other hand, care about the future and maximize the limit frequencies of the number of times they win.

Aragones (1997) studies a similar model with two party competition. It shows that parties that only try to maximize the number of votes at each election, behave as if they had ideal points in the policy space (ideologies). In the present paper we focus on three party competition[2] for three reasons: first, to test the robustness of these results; second, to study the relationship between the policy space and the number of different "ideologies" that may emerge; and lastly, to study the effect of a party splitting into two formally distinct parties.

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[2] Although most of the literature on party competition focuses on the two-party case, in modern democracies there are many examples of multiparty competition. In Canada, two major parties together typically receive more than three quarters of the votes and a third party receives most of the remainder. India has a multiparty system with a dominant party. Even in the United States, the presence of a third party often interrupts the usual competition between Democrats and Republicans.

Equilibrium analysis for three-party competition yields the following predictions: the status quo (aggregate of policies implemented over time) converges to the median voter's ideal point and the policies chosen by the candidates characterize the ideology of the party to which they belong. The results for the two-party case should become clear from this derivation. In the concluding remarks we also suggest some conjectures about the results of competition among a larger number of parties.

First we present a solution of the candidates competition that relies on the assumption that candidates are loyal in the following sense: when they are indifferent between policies of type 0 and 1 they will choose the policy that the party has chosen in the past (this will turn out to be well defined). In this case the solution shows that no party mixes policies of different types, i.e., if a party starts choosing a policy of type 1 it will continue choosing this type of policy for ever (Theorem 1). Then we drop the assumption of loyalty and in the long run we have a similar result (Theorem 2), i.e., in the first periods one of the parties may switch between the two types of policies but at some point it chooses one of them and continues with the chosen one for ever. In both cases, it is in the interest of each party to have no other party choosing the same type of policy it has decided to implement. The two solutions described above yield the same results for the long run: one of the parties chooses one type of policy and wins one half of the time and the other two parties choose policies of the other type and each wins one fourth of the time. Since we assume that parties have no ideal point to begin with, this result can be interpreted as suggesting that ideologies may emerge from the actions of the candidates when maximizing the popularity of the party in the short run.

This result is corroborated by evidence. Empirical studies lend support to the claim that Democrats and Republicans have different effects on the economy while in office. Hibbs (1977), Beck (1982), and Chappel and Keech (1986) show the different effect on the unemployment rate. Alesina and Sachs (1988) and Tabellini and La Via (1989) show that republican administrations have been associated with tighter monetary policies. Frey and Schneider (1978) found that conservative presidents tend to restrict expenditures. The point we would like to emphasize is that these ideologies need not be assumed as primitive; they may simply result from the parties' attempt to differentiate themselves from others.

Thus, the three party model suggests the following conclusions. First, the emergence of ideologies is not an artifact of the two party model. Second, the number of "ideologies" seems to depend on the policy space, rather than on the number of parties; despite the parties incentive to differentiate themselves from each other, no party in our model chooses to appear as "moderate," and they all choose some "extreme." Finally, our results seem to explain Duverger Law: when the policy space consists of only two points, there is "no room" for a third party.

In the solutions described above we have considered candidates as the players of the game at each stage. If parties instead of candidates were to decide on policies, their objective would be to maximize the number of times that the party wins, i.e., they would take into account that they are going to participate in all elections and so they would prefer to sacrifice some of the votes in a given period in order to increase the total number of times in office. We show that if candidates are stagewise vote-maximizers and loyal, their choices constitute a Nash equilibrium path in the infinite-stage game. In the model we present we have that even in the case that candidates were completely responsible for the choice of policies, their choices would not conflict with their parties' long-run objectives. Furthermore, the resulting choice of policies over time characterizes the party by what we have called emergence of ideologies: in equilibrium parties are identified with a certain type of policy. Thus, the competition of myopic candidates that behave independently of the party label assigned to them gives as a result the characterization of political parties by an ideology.

The rest of the paper is organized as follows: section 2 relates the present model to the existing literature. Section 3 describes formally the model. In section 4 we present the results. Section 5 includes some concluding remarks.

2. RELATED LITERATURE.

There exist some theories of voting which suggest that voters base their decision on past performance of the parties. The Reward-Punishment theory proposed by Key (1966) is based on the assumption that voters only care about the effects of the policies that parties choose and they are looking at past performance when deciding how much confidence to give to each party. Downs (1957) proposed a theory according to which parties' past performance is the cheapest way for voters to predict future performance. In this model, voters care about the policy that a party implements on top of its effects. He assumes that political parties must be consistent over time in the policies they advocate and implement. Our interpretation of the voters' behavior is different from Downs' but we find that consistency over time in the policies implemented is a result of optimal choice of the parties. Fiorina (1981) builds a dynamic model for two parties that combines features of both theories and examines it at the empirical level. He assumes that voters base their decision not only on past performance of the parties but also on past promises and hypothetical choices of policies. He shows that most of the assumptions of his theory are supported by the data. Our model is much simpler than Fiorina's. Past promises or hypothetical choices are not considered by the voters in their evaluation of the parties. For simplicity we also assume that platforms have no effect on the evaluation of the parties.

Other variations of retrospective voting have been suggested in analyzing how voters ought to behave if they wish to get their representatives to pursue their interests. The solution is an optimal decision rule for the voters given that they know the objective function of the parties. Ferejohn (1986) and Austen-Smith and Banks (1989) are two examples. In our model, we assume that voters use very little information to make their decision. They do not know the objectives that define parties' behavior and they use a very simple rule to evaluate parties based on past performance. Even though the decision rule of the voters uses all the information in the voters' memory about past performance of the parties, at each point of time a voter only has to remember one number for each party, which represents the evaluation of the party by the voter. After he has seen the performance of a party while in office, the voter updates his evaluation of this party by adding a number to his previous evaluation.

Kramer (1977) presents a dynamic model of two-party competition whose main assumptions are the following: parties maximize votes myopically; the preferences of the voters are constant over time and are defined on the policy space; at any time the challenger can choose any policy in the policy space while the incumbent must defend the same policy. One of his results is that the challenger always has a strategy that defeats the incumbent, therefore parties alternate in office. Without restricting the actions of the party in office our model also shows that most of the time the challengers can defeat the party in office.

3. THE MODEL

We assume there is a continuum of voters. Each voter is characterized by his ideal point and they are distributed uniformly on the interval $[0,1]$. Their instantaneous utility functions are single peaked: $u_i(x) = -|x_i - x|$ where x_i represents the ideal point of voter i. It can be interpreted as follows: if voter i

were a dictator, he would choose policy 1 a proportion x_i of the times and policy 0 a proportion $(1-x_i)$ of the times.

At each time $t = 0,1,2,...$ an election takes place. At time t all voters have memory $\left(M^t\right)_i = M^t$ of past elections. An element of memory, which we call a case, is represented by $m^t = \left(t, p^t, x^t\right)$ where p^t represents the party that won the election at time t and x^t the type of policy it implemented. The memory of voters at time $t+1$ is $M^{t+1} = M^t \cup \left\{\left(t, p^t, x^t\right)\right\}$ and $M^0 = \emptyset$. For voter i at time T, each party p is ranked by aggregation of all cases in the memory M^T in which p won the election, i.e., $m^t = \left(t, p^t, x^t\right) \in M^T$ such that $p^t = p$ for $t < T$ and it is given by[3] :

$$U_i^T(p) = \sum_{\left(t,p,x^t\right) \in M^T} u_i\left(x^t\right)$$

When there are no cases in memory to calculate the utility, its default value is zero. Voters give their vote to the party that gives them the highest utility level (i.e., they do not vote strategically). If a voter is indifferent among different parties, he will choose each of them with equal probability. We will assume that the law of large numbers holds, i.e., that if a proportion μ of voters are indifferent among parties in a set P_o, each party in P_o gets a proportion $\frac{\mu}{|P_o|}$ of the votes[4] .

At each time t the party that obtains the largest proportion of votes wins the election and has to choose a policy to implement. Formally, let μ_p^t be the proportion of votes for party p at time t; party $p*$ wins election t only if $\mu_{p*}^t \geq \mu_p^t$ for all $p \in \{a,b,c\}$. Ties are broken by fair lotteries. The policies are $\{0,1\}$ to be interpreted as a small change in one of the two possible directions. Let $k_1^p(t)$ be the number of times that party p was in power and chose policy 1 up to time t. Similarly, we define $k_0^p(t)$ and $k^p(t) = k_0^p(t) + k_1^p(t)$. Thus, for every t,

$$t = k_0^a(t) + k_1^a(t) + k_0^b(t) + k_1^b(t) + k_0^c(t) + k_1^c(t).$$

It will simplify notation to define the infinite period game for the parties and embed the candidates' game in it. That is, instead of defining the candidates' game as a game with infinitely many stagewise vote-maximizer players we will model them as agents of the parties, and reflect their utility maximization by an appropriate choice of strategies for the parties.

3 Notice that this decision rule is equivalent to the one presented in the introduction. It has been suggested by Gilboa and Schmeidler (1993).

4 See Judd (1985) for the mathematical subtleties involved.

Let us formally define the parties game to be the following infinite-stage game. The set of players is $\{a,b,c\}$. For every $t \geq 0$ nature chooses a "winner" from $\{a,b,c\}$ as follows: if there exists a party p such that $\mu_p^t > \mu_{p'}^t$ for all $p' \neq p$ then p is the winner with probability one. Otherwise, nature chooses one of the maximizers of μ_p^t with equal probability. Then, for the same $t \geq 0$, the winner chooses an element of $\{0,1\}$, $U_i^t(p)$ is updated and the game proceeds to stage $t+1$. (The voters' choices are incorporated into the rules of the game as defining μ_p^t.)

Let H^t be the set of histories of the game at time t and let h^t denote an element of this set. That is, h^t is a sequence of t elements of the set

$$H = \{(p,x): p \in \{a,b,c\} \text{ and } x \in \{0,1\}\}.$$

We use $\circ$ for concatenation. Thus, $h^{t+1} = h^t \circ (p^t, x^t)$ where p^t is the party that wins election t and x^t is the policy it implements (h^0 is the empty sequence). The proportion of votes for party p at time t is a function of the history of the game at time t, formally $\mu_p^t: H^t \to [0,1]$.

A strategy for party p is defined by $x^p = (x_0, x_1, x_2, ..., x_t, ...)$ where $x_t: H^t \to \{0,1\}$ is a function that assigns to each history a policy $x \in \{0,1\}$. We consider the following two sets of strategies: the set of vote maximizing strategies

$$X^p = \left\{(x_0, x_1, x_2, ..., x_t, ...): \mu_p^{t+1}\left(h^t \circ \left(p, x_t\left(h^t\right)\right)\right) \geq \mu_p^{t+1}\left(h^t \circ (p,x)\right) \text{ for all } x \in \{0,1\} \text{ and all } t \geq 0\right\}$$

and its subset of loyal strategies

$$XL^p = \left\{ (x_0, x_1, x_2, ..., x_t, ...) \in X^p : \begin{array}{l} x_t\left(h^t\right) = x \text{ if } \mu_p^{t+1}\left(h^t \circ (p,0)\right) = \mu_p^{t+1}\left(h^t \circ (p,1)\right) \\ \text{and } x_j\left(h^j\right) = x, \text{ for every } j < t \text{ such that } p^j = p \end{array} \right\}$$

4. RESULTS.

First, we analyze the behavior of the candidates as autonomous agents in the game. At each election t we have a different candidate for each party. The candidate of the winning party has to choose a policy. We assume that at each time t, the candidate that wins the election, implicitly assumed to know the decision rule of the voters, chooses a policy in the set $\{0,1\}$ to maximize the proportion of votes that she can obtain in the next election (stagewise vote-maximizer). In addition, we have an assumption of loyalty: if a candidate is indifferent between the two policies (in terms of vote maximization) and if all previous candidates of her party happened to have chosen the same policy, so will she. (Notice that this assumption does not restrict a candidate choice if she is the first to win on her party's behalf or if past winners happened to choose different policies.) These agents' preferences are equivalent to assuming that the party chooses a strategy x^p in the set XL^p.

<u>Theorem 1: Competition of loyal candidates</u>.

If for all $p \quad P, x^p \quad XL^p$ we have the following results up to any permutation of parties and / or of policies:

$$
\begin{aligned}
&I.\ For\ all\ t = 4k,\ k = 1,2,...\\
&(i)\ k_0^a(t) = k^a(t) = k,\ k_0^b(t) = k^b(t) = k\ and\ k_1^c(t) = k^c(t) = 2k\\
&(ii)\ For\ all\ i,\ U_i^t(a) = -kx_i,\ U_i^t(b) = -kx_i\ and\ U_i^t(c) = -2k(1 - x_i)\\
&II. \lim_{t\to\infty} \frac{k^a(t)}{t} = \lim_{t\to\infty} \frac{k^b(t)}{t} = \frac{1}{4}\ and\ \lim_{t\to\infty} \frac{k^c(t)}{t} = \frac{1}{2}
\end{aligned}
$$

(All proofs are relegated to an appendix.)

Part I of the theorem states that, under the above assumptions, parties will always choose the same type of policy. In this case, we have that for all $t = 4k$ parties a and b, which have been choosing policies of type 0, win one fourth of the time and party c, which has been choosing policies of type 1, wins one half of the time.

Part II concludes that in the long run the limit frequencies of the time in office for the parties are as follows: party c, who has been choosing only policies of type 1, wins one half of the time and parties a and b, who have chosen only policies of type 0, win one fourth of the time each.

Next we drop the loyalty assumption. We still assume that at each time t, the candidate that wins the election, implicitly assumed to know the decision rule of the voters, chooses a policy in the set $\{0,1\}$ to maximize the proportion of votes that she can obtain in the next election (stagewise vote-maximizer). Formally, the party chooses a strategy $x^p \quad X^p$.

Theorem 2: Competition of candidates.

If for all $p \in P, x^p \in X^p$ then we have the following results up to any permutation of parties and / or of policies:

I. For all $t = 4k$, $k = 1,2,\ldots$ *there exist* k_1 *and* k_2 *with* $k = k_1 + k_2$ *and* $\min\{k_1, k_2\} \leq 3$ *such that*:

$$(i)\ k_0^a(t) = k_1, k_1^a(t) = k_2, k_0^b(t) = k^b(t) = k_1 + 2k_2 \text{ and } k_1^c(t) = k^c(t) = 2k_1 + k_2$$

$$(ii) \text{ For all } i, U_i^t(a) = -k_1 x_i - k_2(1 - x_i),$$

$$U_i^t(b) = -(k_1 + 2k_2)x_i \text{ and}$$

$$U_i^t(a) = -(2k_1 + k_2)(1 - x_i)$$

$$II.\ \lim_{t \to \infty} \frac{k^a(t)}{t} = \lim_{t \to \infty} \frac{k^b(t)}{t} = \frac{1}{4} \text{ and } \lim_{t \to \infty} \frac{k^c(t)}{t} = \frac{1}{2}$$

Furthermore, for all $p \in \{a,b,c\}$, *if* $k_0^p(t) > 0$ *and* $k_1^p(t) > 0$ *for some t, then* $\lim_{t \to \infty} \frac{k^p(t)}{t} = \frac{1}{4}$.

Theorem 2 states that two of the parties will always choose the same type of policy, regardless of how the third party chooses to mix the policies. That is, two of the parties are behaving as if they were loyal, while the third one is "almost" loyal: it will choose the same policy whenever in power, except for at most three times.

In the long run the limit frequencies of the periods in which the parties are in office are as follows: If party a, who started mixing, ends up choosing a policy of type 0, then party c, who has been choosing only policies of type 1 will win one half of the time and parties a and b, who have chosen mostly policies of type 0 will win one fourth of the time each. If party a ends up choosing a policy of type 1 then party b, who has been choosing only policies of type 0 will win one half of the time and parties a and c, who have chosen mostly policies of type 1, will win one fourth of the time each. At any rate, the long-run frequencies are as specified in Theorem 1; however, in case one party mixes the two types of policies, it cannot be the one that wins one half of the times.

Finally, we consider the case of competition of parties. As explained in the introduction, stagewise maximization characterizes the objective of the candidates. Since candidates cannot be reelected, they only care about their popularity one period ahead. In contrast, parties participate in elections at all periods. Therefore the objective of the parties can be characterized by the maximization of the limit frequency of the number of times that the party is in power. If candidates are stagewise vote-maximizers and loyal, as assumed in the first solution, we also have that their choices constitute a Nash equilibrium path in the parties game. The payoff function of party p is assumed to be $\Pi^p = \limsup_{t\to\infty} \frac{k^p(t)}{t}$ where $k^p(t)$ is the number of times that party p has been in office up to time t.

Theorem 3: Competition of parties.

For every x^a, x^b and x^c such that $x^p \quad XL^p$, (x^a, x^b, x^c) is a Nash equilibrium of the parties game.

Remark:

If x^a, x^b and x^c are such that $x^p \quad X^p$, (x^a, x^b, x^c) need not be a Nash equilibrium of the parties game. (A counterexample is provided in the appendix.)

5. CONCLUDING REMARKS

1. We start by assuming that all parties are identical, that they do not have preferences over policies; rather, they are vote maximizers. In equilibrium parties and candidates behave as if they had ideal points, i.e., each party chooses always the same policy. If, instead, we assume that parties have non-identical preferences over policies and their objective is not only to win elections but also to implement their most preferred policies, one can show that the result will not change. Therefore, the fact that parties always choose the same policy is compatible with (at least) two theories: (i) the parties are only interested in vote maximization, and ideologies "emerge" from strategic considerations; and (ii) parties do have ideologies to begin with, and these determine their initial choices, since vote maximization leaves them indifferent between the two policies; however, in later stages vote maximization and ideological considerations coincide.

2. The two solutions described in the paper for the "candidates' game" show the same results for the long run: one of the parties chooses one type of policy and wins one half of the time and the other two parties choose policies of the other type and each wins one fourth of the time. By comparison, consider the

two-party case in our model. It is easy to see that as a result of two-party competition none of the parties can win more than one half of the times in the limit. Hence our results may also be interpreted as if there were "no room" for a third party. That is, if we start with two parties, none of them seems to gain anything by dividing into two different parties. On the other hand, if we start with three parties, the two of them that end up choosing the same type of policy have no incentive to form a coalition, since by appearing as a single party they can only win one half of the times which is exactly what they do in aggregate. Notice that, as in Palfrey (1984), the presence of a third party is what drives the uniqueness of the strategies chosen by the parties. We conjecture that for competition of a larger number of parties, similar results should hold, that is, candidates whose sole objective is to win the election will not have any incentive to deviate from the ideology that characterizes the party they belong to.

3. If we define the economic configuration or status quo at time t, e^t, as the ratio between the number of times that type 1 policies have been chosen in the past and the total number of elections we find that, in both cases, the economic configuration tends to stabilize at one half, that is, in the limit both policies have been chosen the same number of times. This result reflects the fact that parties tend to satisfy the median voter in some sense. To be precise, on the aggregate level there is some support to Hotelling's result. On the other hand, if we consider the aggregate economic variable when each party is in power, it tends to one half for all parties. However, this is a result of aggregation over time, taking into account the cumulative effect of past elections. By contrast, when we focus on each party's decision variable, i.e., whether to lead the economy to the "left" or "right" end of the state space we find the opposite of Hotelling's result of minimum differentiation, i.e., that parties are pushed to one of the extreme policies ("always right" or "always left"). Notice that the economic configuration is only a measure of the aggregate effects of the policies implemented and has no strategic implications.

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APPENDIX

To simplify notation we will write $\mu_p^t(x)$ and μ_p^t instead of $\mu_p^t\left(h^{t-1} \circ (p,x)\right)$ and $\mu_p^t\left(h^t\right)$ repectively.

Proof of Theorem 1:

Part I: first we prove (ii) by induction on k:

At $t = 0$, we have $U_i^0(a) = U_i^0(b) = U_i^0(c) = 0$, therefore $\mu_p^0 = \frac{1}{3}$ for all p which implies a tie among the three parties. If party a wins the first election $\mu_a^1(0) = \mu_a^1(1) = 0$, i.e., party a is indifferent between policies 0 and 1.

At $t = 1$, if $\left(p^0, x^0\right) = (a,0)$ then $U_i^1(a) = -x_i$ and $U_i^1(b) = U_i^1(c) = 0$. Therefore $\mu_a^1 = 0$ and $\mu_b^1 = \mu_c^1 = \frac{1}{2}$ implies a tie between parties b and c. If party b wins the second election $\mu_b^2(0) = \mu_b^2(1) = 0$, i.e., party b is indifferent between policies 0 and 1.

At $t = 2$, if $\left(p^0, x^0\right) = (a,0)$ and $\left(p^1, x^1\right) = (b,0)$ then $U_i^2(a) = U_i^2(b) = -x_i$ and $U_i^2(c) = 0$. Therefore $\mu_a^2 = \mu_b^2 = 0$ and $\mu_c^2 = 1$ implies that party c wins the third election and $\mu_c^3(0) = \frac{1}{3} < \frac{1}{2} = \mu_c^3(1)$, i.e., party c chooses policy 1.

At $t = 3$, given that $\left(p^0, x^0\right) = (a,0), \left(p^1, x^1\right) = (b,0)$ and $\left(p^2, x^2\right) = (c,1)$, we have $U_i^3(a) = U_i^3(b) = -x_i$ and $U_i^3(c) = -(1 - x_i)$. Therefore $\mu_a^3 = \mu_b^3 = \frac{1}{4}$ and $\mu_c^3 = \frac{1}{2}$ implies that party c wins the fourth election and $\mu_c^4(0) = 0 < \frac{1}{3} = \mu_c^4(1)$, i.e., party c chooses policy 1.

In addition to $\left[(a,0),(b,0),(c,1),(c,1)\right]$, other possible results for this period are: $\left[(a,0),(b,1),(c,0),(b,1)\right]$ and $\left[(a,0),(b,1),(c,1),(a,0)\right]$ up to any permutation of parties or policies. At $t = 4, i.e.\ k = 1$, for all possible results we have $U_i^4(a) = U_i^4(b) = -x_i$ and $U_i^4(c) = -2(1 - x_i)$ up to any permutation of parties or policies.

Now, suppose that the result is true for $k > 1$. Then, at $t = 4k$ we have $U_i^{4k}(a) = U_i^{4k}(b) = -kx_i$ and $U_i^{4k}(c) = -2k(1 - x_i)$. This implies that $\mu_p^{4k} = \frac{1}{3}$ for all p, i.e., a tie among the three parties. Here we have two cases depending on who breaks the tie.

Case 1.- If party a (or b) wins the election at time $t = 4k$ we have $\mu_a^{4k+1}(0) = \mu_a^{4k+1}(1) = 0$. Thus, party a will choose policy 0, because it is the policy it has already chosen in the past.

At $t = 4k + 1$ we have:

$$U_i^{4k+1}(a) = -(k+1)x_i, U_i^{4k+1}(b) = -kx_i \text{ and } U_i^{4k+1}(c) = -2k(1-x_i).$$

Therefore $\mu_a^{4k+1} = 0, \mu_b^{4k+1} = \frac{2}{3}$ and $\mu_c^{4k+1} = \frac{1}{3}$ which implies that party b wins this election and $\mu_b^{4k+2}(0) = \frac{k}{3k+1} > \frac{k-1}{6k-2} = \mu_b^{4k+2}(1)$, i.e., party b chooses policy 0.

At $t = 4k+2$ we have:

$$U_i^{4k+2}(a) = U_i^{4k+2}(b) = -(k+1)x_i \text{ and } U_i^{4k+2}(c) = -2k(1-x_i).$$

Therefore $\mu_a^{4k+2} = \mu_b^{4k+2} = \frac{k}{3k+1} < \frac{k+1}{3k+1} = \mu_c^{4k+2}$ which implies that party c wins this election and $\mu_c^{4k+3}(0) = \frac{1}{3} < \frac{k+1}{3k+2} = \mu_c^{4k+3}(1)$, i.e., party c chooses policy 1.

At $t = 4k+3$ we have:

$$U_i^{4k+3}(a) = U_i^{4k+3}(b) = -(k+1)x_i \text{ and } U_i^{4k+3}(c) = -(2k+1)(1-x_i).$$

Therefore $\mu_a^{4k+3} = \mu_b^{4k+3} = \frac{2k+1}{6k+4} < \frac{k+1}{3k+2} = \mu_c^{4k+3}$ which implies that party c wins this election and $\mu_c^{4k+4}(0) = \frac{k}{3k+1} < \frac{1}{3} = \mu_c^{4k+4}(1)$, i.e., party c chooses policy 1.

At $t = 4(k+1)$ we have: $U_i^{4(k+1)}(a) = U_i^{4(k+1)}(b) = -(k+1)x_i$ and $U_i^{4(k+1)}(c) = -2(k+1)(1-x_i)$.

Case 2.- If party c wins the election at time $t = 4k$ we have:

$$\mu_c^{4k+1}(0) = \frac{k-1}{3k-1} < \frac{k}{3k+1} = \mu_c^{4k+1}(1),$$

i.e., party c will choose policy 1.

At $t = 4k+1$ we have:

$$U_i^{4k+1}(a) = U_i^{4k+1}(b) = -kx_i \text{ and } U_i^{4k+1}(c) = -(2k+1)(1-x_i).$$

Therefore $\mu_a^{4k+1} = \mu_b^{4k+1} = \frac{2k+1}{6k+2}$ and $\mu_c^{4k+1} = \frac{k}{3k+1}$ which implies that party a (or b) wins this election and $\mu_a^{4k+2}(0) = \mu_a^{4k+2}(1) = 0$. Thus, party a chooses policy 0 because it is the policy it has already chosen in the past.

At $t = 4k+2$ we have:

$$U_i^{4k+2}(a) = -(k+1)x_i, U_i^{4k+2}(b) = -kx_i \text{ and } U_i^{4k+2}(c) = -(2k+1)(1-x_i).$$

Therefore $\mu_a^{4k+2}=0, \mu_b^{4k+2}=\frac{2k+1}{3k+1}>\frac{k}{3k+1}=\mu_c^{4k+2}$ which implies that party b wins this election and $\mu_b^{4k+3}(0)=\frac{2k+1}{6k+4}>\frac{1}{6}=\mu_b^{4k+3}(1)$, i.e., party b chooses policy 0.

At $t=4k+3$ we have:

$$U_i^{4k+3}(a)=U_i^{4k+3}(b)=-(k+1)x_i \text{ and } U_i^{4k+3}(c)=-(2k+1)(1-x_i).$$

Therefore $\mu_a^{4k+3}=\mu_b^{4k+3}=\frac{2k+1}{6k+4}<\frac{k+1}{3k+2}=\mu_c^{4k+3}$ which implies that party c wins this election and $\mu_c^{4k+4}(0)=\frac{k}{3k+1}<\frac{1}{3}=\mu_c^{4k+4}(1)$, i.e., party c chooses policy 1.

At $t=4(k+1)$ we have:

$$U_i^{4(k+1)}(a)=U_i^{4(k+1)}(b)=-(k+1)x_i \text{ and } U_i^{4(k+1)}(c)=-2(k+1)(1-x_i).$$

(i) follows directly from (ii). Part II follows from part I.

Proof of Theorem 2:

Part I: first we prove (ii) by induction on k. Assume that at $t=4k$ we have $k_1=k$ and $k_2=0$. Then $U_i^{4k}(a)=U_i^{4k}(b)=-kx_i$ and $U_i^{4k}(c)=-2k(1-x_i)$. Furthermore, suppose that party a wins the election at $t=4k$ and chooses policy 1. (If party a chooses policy 0, at $t=4(k+1)$ we have $k_1=k+1$ and $k_2=0$, from the previous proof.)

At $t=4k+1$ we have:

$$U_i^{4k+1}(a)=-kx_i-(1-x_i), U_i^{4k+1}(b)=-kx_i \text{ and } U_i^{4k+1}(c)=-2k(1-x_i).$$

Therefore $\mu_a^{4k+1}=0, \mu_b^{4k+1}=\frac{2}{3}$ and $\mu_c^{4k+1}=\frac{1}{3}$ which implies that party b wins this election and $\mu_b^{4k+2}(0)=\frac{1}{2}>\frac{2k-1}{6k-2}=\mu_b^{4k+2}(1)$, i.e., party b chooses policy 0.

At $t=4k+2$ we have:

$$U_i^{4k+2}(a)=-kx_i-(1-x_i), U_i^{4k+2}(b)=-(k+1)x_i \text{ and } U_i^{4k+2}(c)=-2k(1-x_i).$$

Therefore $\mu_a^{4k+2}=\frac{k-1}{6k-2}, \mu_b^{4k+2}=\frac{1}{2}$ and $\mu_c^{4k+2}=\frac{k}{3k-1}$ which implies that party b wins this election and $\mu_b^{4k+3}(0)=\frac{1}{3}>0=\mu_b^{4k+3}(1)$, i.e., party b chooses policy 0.

At $t=4k+3$ we have:

$$U_i^{4k+3}(a) = -kx_i - (1-x_i), U_i^{4k+3}(b) = -(k+2)x_i \text{ and } U_i^{4k+3}(c) = -2k(1-x_i).$$

Therefore $\mu_a^{4k+3} = \frac{3k-2}{9k-3}, \mu_b^{4k+3} = \frac{1}{3}$ and $\mu_c^{4k+3} = \frac{k}{3k-1}$ which implies that party c wins this election and $\mu_c^{4k+4}(0) = \frac{k-1}{3k-1} < \frac{1}{3} = \mu_c^{4k+4}(1)$, i.e., party c chooses policy 1.

At $t = 4(k+1)$ we have:

$$U_i^{4(k+1)}(a) = -kx_i - (1-x_i), U_i^{4(k+1)}(b) = -(k+2)x_i \text{ and } U_i^{4k+4}(c) = -(2k+1)(1-x_i)$$

Now suppose that at $t = 4k$ party c has won the election. From the previous proof we know that it chooses policy 1.

At $t = 4k+1$ we have:

$$U_i^{4k+1}(a) = U_i^{4k+1}(b) = -kx_i \text{ and } U_i^{4k+1}(c) = -(2k+1)(1-x_i).$$

Therefore $\mu_a^{4k+1} = \mu_b^{4k+1} = \frac{2k+1}{6k+2}$ and $\mu_c^{4k+1} = \frac{k}{3k+1}$ which implies that party a (or b) wins this election and $\mu_a^{4k+2}(0) = \mu_a^{4k+2}(1) = 0$. Suppose party a chooses policy 1. (The case in which party a chooses policy 0 is analized in the previous proof.)

At $t = 4k+2$ we have:

$$U_i^{4k+2}(a) = -kx_i - (1-x_i), U_i^{4k+2}(b) = -kx_i \text{ and } U_i^{4k+2}(c) = -(2k+1)(1-x_i).$$

Therefore $\mu_a^{4k+2} = 0, \mu_b^{4k+2} = \frac{2k+1}{3k+1} > \frac{k}{3k+1} = \mu_c^{4k+2}$ which implies that party b wins this election and $\mu_b^{4k+3}(0) = \frac{1}{2} > \frac{1}{3} = \mu_b^{4k+3}(1)$, i.e., party b chooses policy 0.

At $t = 4k+3$ we have:

$$U_i^{4k+3}(a) = -kx_i - (1-x_i), U_i^{4k+3}(b) = -(k+1)x_i \text{ and } U_i^{4k+3}(c) = -(2k+1)(1-x_i).$$

Therefore $\mu_a^{4k+3} = \frac{1}{6}, \mu_b^{4k+3} = \frac{1}{2}$ and $\mu_c^{4k+3} = \frac{1}{3}$ which implies that party b wins this election and $\mu_b^{4k+4}(0) = \frac{1}{3} > 0 = \mu_b^{4k+4}(1)$, i.e., party b chooses policy 0.

At $t = 4(k+1)$ we have:

$$U_i^{4(k+1)}(a) = -kx_i - (1-x_i), U_i^{4(k+1)}(b) = -(k+2)x_i \text{ and } U_i^{4k+4}(c) = -(2k+1)(1-x_i)$$

which proves the result for $k_1 = k$ and $k_2 = 1$. Similarly it can be proven for $k_1 = 1$ and $k_2 = k$.

Now suppose that it is true for $k_1 > 1$ and $k_2 > 1$ and $k_1 + k_2 = k$.

At $t = 4k$ we have:

$$U_i^{4k}(a) = -k_1 x_i - k_2(1-x_i), U_i^{4k}(b) = -(k_1 + 2k_2)x_i \text{ and } U_i^{4k}(c) = -(2k_1 + k_2)(1-x_i).$$

Case 1.- Suppose that party a wins the election at $t = 4k$. Then

$$\mu_a^{4k+1}(0) = \frac{3k_1k_2 - 2k_1 - k_2}{(3k_1+1)(3k_2-1)}, \mu_a^{4k+1}(1) = \frac{3k_1k_2 - k_1 - 2k_2}{(3k_1-1)(3k_2+1)},$$

$$\mu_a^{4k+1}(0) > \mu_a^{4k+1}(1) \text{ iff } \left\{ k_1 > k_2 \text{ and } k_1 > \frac{9k_2+1}{9k_2-9} \right\} \text{ or } \left\{ k_1 < k_2 \text{ and } k_1 < \frac{9k_2+1}{9k_2-9} \right\}$$

$$\mu_a^{4k+1}(0) < \mu_a^{4k+1}(1) \text{ iff } \left\{ k_1 > k_2 \text{ and } k_1 < \frac{9k_2+1}{9k_2-9} \right\} \text{ or } \left\{ k_1 < k_2 \text{ and } k_1 > \frac{9k_2+1}{9k_2-9} \right\}$$

$$\mu_a^{4k+1}(0) = \mu_a^{4k+1}(1) \text{ iff } k_1 = k_2$$

Case 1.1.- If at $t = 4k$ party a chooses policy 0. Then at $t = 4k+1$ we have:

$$U_i^{4k+1}(a) = -(k_1+1)x_i - k_2(1-x_i), U_i^{4k+1}(b) = -(k_1 + 2k_2)x_i \text{ and}$$
$$U_i^{4k+1}(c) = -(2k_1 + k_2)(1-x_i).$$

Therefore $\mu_a^{4k+1} = \frac{3k_1k_2 - 2k_1 - k_2}{(3k_1+1)(3k_2-1)}, \mu_b^{4k+1} = \frac{k_2}{3k_2-1}, \mu_c^{4k+1} = \frac{k_1+1}{3k_1+1}$ and

$$\mu_a^{4k+1} < \mu_c^{4k+1} \leq \mu_b^{4k+1} \text{ iff } k_2 \leq \frac{k_1+1}{2}.$$

Case 1.1.1.- If at $t = 4k+1$ party b wins then $\mu_b^{4k+2}(0) = \frac{1}{3} > \frac{k_2-1}{3k_2-2} = \mu_b^{4k+2}(1)$, i.e., party b chooses policy 0.

At $t = 4k+2$ we have:

$$U_i^{4k+2}(a) = -(k_1+1)x_i - k_2(1-x_i), U_i^{4k+2}(b) = -(k_1 + 2k_2 + 1)x_i \text{ and}$$
$$U_i^{4k+2}(c) = -(2k_1 + k_2)(1-x_i).$$

Therefore $\mu_a^{4k+2} = \frac{3k_1-1}{9k_1+3}, \mu_b^{4k+2} = \frac{1}{3}$ and $\mu_c^{4k+2} = \frac{k_1+1}{3k_1+1}$ which implies that party c wins this election and $\mu_c^{4k+3}(0) = \frac{1}{3} < \frac{k_1+1}{3k_1+2} = \mu_c^{4k+3}(1)$ i.e., party c chooses policy 1.

At $t = 4k+3$ we have:

$$U_i^{4k+3}(a) = -(k_1+1)x_i - k_2(1-x_i), U_i^{4k+3}(b) = -(k_1 + 2k_2 + 1)x_i \text{ and}$$
$$U_i^{4k+3}(c) = -(2k_1 + k_2 + 1)(1-x_i).$$

Therefore $\mu_a^{4k+3} = \dfrac{3k_1+1}{9k_1+6}, \mu_b^{4k+3} = \dfrac{1}{3}$ and $\mu_c^{4k+3} = \dfrac{k_1+1}{3k_1+2}$ which implies that party c wins this election and $\mu_c^{4k+4}(0) = \dfrac{k_1-2}{3k_1-1} < \dfrac{1}{3} = \mu_c^{4k+4}(1)$, i.e., party c chooses policy 1.

Case 1.1.2.- If at $t = 4k+1$ party c wins then $\mu_c^{4k+2}(0) = \dfrac{1}{3} < \dfrac{k_1+1}{3k_1+2} = \mu_c^{4k+2}(1)$, i.e., party c chooses policy 1.

At $t = 4k+2$ we have:

$$U_i^{4k+2}(a) = -(k_1+1)x_i - k_2(1-x_i), U_i^{4k+2}(b) = -(k_1+2k_2)x_i \text{ and}$$
$$U_i^{4k+2}(c) = -(2k_1+k_2+1)(1-x_i)$$

Therefore $\mu_a^{4k+2} = \dfrac{3k_1k_2-2k_1+k_2-1}{(3k_1+2)(3k_2-1)}, \mu_b^{4k+2} = \dfrac{k_2}{3k_2-1}$ and $\mu_c^{4k+2} = \dfrac{k_1+1}{3k_1+2}$ which implies:

$$\mu_a^{4k+2} < \mu_c^{4k+2} \le \mu_b^{4k+2} \text{ } iff \text{ } k_2 \le k_1+1.$$

Case 1.1.2.1.- If at $t = 4k+2$ party b wins then $\mu_b^{4k+3}(0) = \dfrac{1}{3} > \dfrac{k_2-1}{3k_2-2} = \mu_b^{4k+3}(1)$, i.e., party b chooses policy 0.

At $t = 4k+3$ we have:

$$U_i^{4k+3}(a) = -(k_1+1)x_i - k_2(1-x_i), U_i^{4k+3}(b) = -(k_1+2k_2+1)x_i \text{ and}$$
$$U_i^{4k+3}(c) = -(2k_1+k_2+1)(1-x_i).$$

Therefore $\mu_a^{4k+3} = \dfrac{3k_1+1}{9k_1+6}, \mu_b^{4k+3} = \dfrac{1}{3}$ and $\mu_c^{4k+3} = \dfrac{k_1+1}{3k_1+2}$ which implies that party c wins this election and $\mu_c^{4k+4}(0) = \dfrac{k_1}{3k_1+1} < \dfrac{1}{3} = \mu_c^{4k+4}(1)$, i.e., party c chooses policy 1.

Case 1.1.2.2.- If at $t = 4k+2$ party c wins then $\mu_c^{4k+3}(0) = \dfrac{k_1}{3k_1+1} < \dfrac{1}{3} = \mu_c^{4k+3}(1)$, i.e., party c chooses policy 1.

At $t = 4k+3$ we have:

$$U_i^{4k+3}(a) = -(k_1+1)x_i - k_2(1-x_i), U_i^{4k+3}(b) = -(k_1+2k_2)x_i, \text{ and}$$
$$U_i^{4k+3}(c) = -(2k_1+k_2+2)(1-x_i).$$

Therefore $\mu_a^{4k+3} = \dfrac{3k_2-2}{9k_2-3}, \mu_b^{4k+3} = \dfrac{k_2}{3k_2-1}$ and $\mu_c^{4k+3} = \dfrac{1}{3}$ which implies that party b wins this election and $\mu_b^{4k+4}(0) = \dfrac{1}{3} > \dfrac{k_2-1}{3k_2-2} = \mu_b^{4k+4}(1)$, i.e., party b chooses policy 0.

At $t = 4(k+1)$, for all possible results of Case 1.1. we have:

$$U_i^{4(k+1)}(a) = -(k_1+1)x_i - k_2(1-x_i),\ U_i^{4(k+1)}(b) = -((k_1+1)+2k_2)x_i \text{ and}$$

$$U_i^{4(k+1)}(c) = -(2(k_1+1)+k_2)(1-x_i).$$

Case 1.2.- If at $t = 4k$ party a chooses policy 1 then at $t = 4k+1$ we have:

$$U_i^{4k+1}(a) = -k_1x_i - (k_2+1)(1-x_i),\ U_i^{4k+1}(b) = -(k_1+2k_2)x_i \text{ and}$$
$$U_i^{4k+1}(c) = -(2k_1+k_2)(1-x_i).$$

Therefore $\mu_a^{4k+1} = \dfrac{3k_1k_2 - k_1 - 2k_2}{(3k_1-1)(3k_2+1)}$, $\mu_b^{4k+1} = \dfrac{k_2+1}{3k_2+1}$ and $\mu_c^{4k+1} = \dfrac{k_1}{3k_1-1}$. Hence:

$$\mu_a^{4k+1} < \mu_c^{4k+1} \le \mu_b^{4k+1} \ \mathit{iff}\ k_1 \ge \frac{k_2+1}{2}.$$

By a similar argument we can prove that at $t = 4(k+1)$ we have:

$$U_i^{4(k+1)}(a) = -k_1x_i - (k_2+1)(1-x_i),\ U_i^{4(k+1)}(b) = -(k_1+2(k_2+1))x_i \text{ and}$$

$$U_i^{4(k+1)}(c) = -(2k_1+(k_2+1))(1-x_i).$$

Case 2.- Suppose that party b wins the election. Then $\mu_b^{4k+1}(0) = \dfrac{k_2}{3k_2+1} > \dfrac{k_2-1}{3k_2-1} = \mu_b^{4k+1}(1)$, i.e., party b chooses policy 0.

At $t = 4k+1$ we have:

$$U_i^{4k+1}(a) = -k_1x_i - k_2(1-x_i),\ U_i^{4k+1}(b) = -(k_1+2k_2+1)x_i \text{ and } U_i^{4k+1}(c) = -(2k_1+k_2)(1-x_i).$$

Therefore $\mu_a^{4k+1} = \dfrac{3k_2+2}{9k_2+3}$, $\mu_b^{4k+1} = \dfrac{k_2}{3k_2+1}$ and $\mu_c^{4k+1} = \dfrac{1}{3}$ which implies that party a wins $\mu_a^{4k+2}(0) = \dfrac{3k_1-1}{9k_1+3}$, $\mu_a^{4k+2}(1) = \dfrac{3k_1k_2 + k_1 - 2k_2 - 1}{(3k_1-1)(3k_2+1)}$ and

$$\mu_a^{4k+2}(0) \ge \mu_a^{4k+2}(1)\ \mathit{iff}\ k_2 \le \frac{9(k_1)^2 - 6k_1 + 5}{9k_1 - 9}.$$

Case 2.1.- If at $t = 4k+1$ party a chooses policy 0 then at $t = 4k+2$ we have:

$$U_i^{4k+2}(a) = -(k_1+1)x_i - k_2(1-x_i),\ U_i^{4k+2}(b) = -(k_1+2k_2+1)x_i \text{ and}$$
$$U_i^{4k+2}(c) = -(2k_1+k_2)(1-x_i).$$

Therefore $\mu_a^{4k+2} = \dfrac{3k_1 - 1}{9k_1 + 3}$, $\mu_b^{4k+2} = \dfrac{1}{3}$ and $\mu_c^{4k+2} = \dfrac{k_1 + 1}{3k_1 + 1}$ which implies that party c wins and $\mu_c^{4k+3}(0) = \dfrac{1}{3} < \dfrac{k_1 + 1}{3k_1 + 2} = \mu_c^{4k+3}(1)$, i.e., party c chooses policy 1.

At $t = 4k + 3$ we have:

$$U_i^{4k+3}(a) = -(k_1 + 1)x_i - k_2(1 - x_i),\ U_i^{4k+3}(b) = -(k_1 + 2k_2 + 1)x_i \text{ and}$$
$$U_i^{4k+3}(c) = -(2k_1 + k_2 + 1)(1 - x_i).$$

Therefore $\mu_a^{4k+3} = \dfrac{3k_1 + 1}{9k_1 + 6}$, $\mu_b^{4k+3} = \dfrac{1}{3}$ and $\mu_c^{4k+3} = \dfrac{k_1 + 1}{3k_1 + 2}$ which implies that party c wins and $\mu_c^{4k+4}(0) = \dfrac{k_1}{3k_1 + 1} < \dfrac{1}{3} = \mu_c^{4k+4}(1)$, i.e., party c chooses policy 1.

At $t = 4(k + 1)$ we have:

$$U_i^{4(k+1)}(a) = -(k_1 + 1)x_i - k_2(1 - x_i),\ U_i^{4(k+1)}(b) = -((k_1 + 1) + 2k_2)x_i \text{ and}$$

$$U_i^{4(k+1)}(c) = -(2(k_1 + 1) + k_2)(1 - x_i).$$

Case 2.2.- If at $t = 4k + 1$ party a chooses policy 1 then at $t = 4k + 2$ we have:

$$U_i^{4k+2}(a) = -k_1x_i - k_2(k_2 + 1)(1 - x_i),\ U_i^{4k+2}(b) = -(k_1 + 2k_2 + 1)x_i \text{ and}$$
$$U_i^{4k+2}(c) = -(2k_1 + k_2)(1 - x_i).$$

Therefore $\mu_a^{4k+2} = \dfrac{3k_1k_2 + k_1 - 2k_2 - 1}{(3k_1 - 1)(3k_2 + 2)}$, $\mu_b^{4k+2} = \dfrac{k_2 + 1}{3k_2 + 2}$ and $\mu_c^{4k+2} = \dfrac{k_1}{3k_1 - 1}$ which implies that party c wins and $\mu_c^{4k+3}(0) = \dfrac{k_1 + 1}{3k_1 - 2} < \dfrac{1}{3} = \mu_c^{4k+3}(1)$, i.e., party c chooses policy 1.

At $t = 4k + 3$ we have:

$$U_i^{4k+3}(a) = -k_1x_i - k_2(k_2 + 1)(1 - x_i),\ U_i^{4k+3}(b) = -(k_1 + 2k_2 + 1)x_i \text{ and}$$
$$U_i^{4k+3}(c) = -(2k_1 + k_2 + 1)(1 - x_i).$$

Therefore $\mu_a^{4k+3} = \dfrac{3k_1 + 1}{9k_1 + 6}$, $\mu_b^{4k+3} = \dfrac{k_2 + 1}{3k_2 + 2}$ and $\mu_c^{4k+3} = \dfrac{1}{3}$ which implies that party b wins and $\mu_b^{4k+4}(0) = \dfrac{1}{3} > \dfrac{k_2}{3k_2 + 1} = \mu_b^{4k+4}(1)$, i.e., party b chooses policy 0.

At $t = 4(k + 1)$ we have:

$$U_i^{4(k+1)}(a) = -k_1x_i - (k_2 + 1)(1 - x_i),\ U_i^{4(k+1)}(b) = -(k_1 + 2(k_2 + 1))x_i \text{ and}$$

$$U_i^{4(k+1)}(c) = -\left(2k_1 + \left(k_2 + 1\right)\right)\left(1 - x_i\right).$$

<u>Case 3</u>.- Suppose that party c wins the election. Then, using an argument symmetric to case 2 we obtain the same results.

By combining the different cases we conclude that if a party mixes the two policies (because, given that candidates are stagewise vote-maximizers, they are indifferent between 0 and 1), eventually it will decide for one of them (candidates are no longer indifferent) and will continue choosing this same policy forever. This result is represented in Figure 1. For k_1 and k_2 with values in region I, party a maximizes the proportion of votes for next election by choosing policy 1. For k_1 and k_2 with values in region II, party a maximizes the proportion of votes for next election by choosing policy 0. Party a is indifferent between policies 0 and 1 if $k_1 = k_2$ or $k_j = 0\, or\, 1\ for\ j = 1\, or\, 2$. Finally, for k_1 and k_2 with values outside of these regions, party a maximizes the proportion of votes for next election by choosing policy 0 or 1 depending on the results of previous elections. In all regions parties b and c maximize the proportion of votes for next election by choosing always the same policy, 0 and 1 respectively. Figure 1 also shows that, in any possible path, a party will not switch between the two policies more than three times.

(i) follows directly from (ii). Part II follows from part I.

<u>Lemma 1</u>: (i) If $k_0^a(t) < k_0^b(t)$ then $k_1^a(t) \geq k_1^b(t)$.

(ii) If $k_0^a(t) = k_0^b(t)$ then $\left|k_1^a(t) - k_1^b(t)\right| \leq 1$.

<u>Proof</u>: First we prove part (i). Suppose it is not true, i.e., at time t.we have $k_0^a(t) < k_0^b(t)$ and $k_1^a(t) < k_1^b(t)$. Consider $t_b < t$, the last time that party b won an election. Since $t_b < t$ we must have $k_0^a(t_b) \leq k_0^a(t)$ and $k_1^a(t_b) \leq k_1^a(t)$. If at t_b party b chose policy 0 then $k_0^b(t_b) = k_0^b(t) - 1$ and $k_1^b(t_b) = k_1^b(t)$. Therefore $k_0^a(t_b) \leq k_0^b(t_b)$ and $k_1^a(t_b) < k_1^b(t_b)$ which implies that at t_b party b could not win. If at t_b party b chose policy 1 then $k_0^b(t_b) = k_0^b(t)$ and $k_1^b(t_b) = k_1^b(t) - 1$. Therefore $k_0^a(t_b) < k_0^b(t_b)$ and $k_1^a(t_b) \leq k_1^b(t_b)$ which implies that at t_b party b could not win. Part (ii) follows from a similar argument.

<u>Lemma 2</u>: $\dfrac{k^p(t)}{t} \leq \dfrac{1}{2} + \dfrac{1}{t}$ for all p $\{a,b,c\}$ and all $t > 0$.

<u>Proof</u>: Define $X^t(p,q) = \left\{x_i \quad [0,1]: U_i^t(p) = U_i^t(q)\right\}$ and let $x^t(p,q)$ be an element of $X^t(p,q)$. To prove the Lemma we consider different cases depending on the values of $k_0^p(t)$ and $k_1^p(t)$ for all p (to simplify notation we will drop the time index). By the previous Lemma, it suffices to study the following cases:

$$(i)\ k_0^b > k_0^a > k_0^c\ and\ k_1^b < k_1^a < k_1^c$$

$$(ii)\ k_0^b > k_0^a > k_0^c\ and\ k_1^b = k_1^a < k_1^c$$

$$(iii)\ k_0^b > k_0^a > k_0^c\ and\ k_1^b < k_1^a = k_1^c$$

$$(iv)\ k_0^b = k_0^a > k_0^c\ and\ k_1^b = k_1^a < k_1^c$$

$$(v)\, k_0^b = k_0^a > k_0^c \text{ and } k_1^b < k_1^a = k_1^c$$

$$(vi)\, k_0^b = k_0^a = k_0^c \text{ and } k_1^b = k_1^a = k_1^c$$

$$(vii)\, k_0^b = k_0^a = k_0^c \text{ and } k_1^b = k_1^a < k_1^c$$

$$(viii)\, k_0^b > k_0^a > k_0^c \text{ and } k_1^b < k_1^a = k_1^c$$

Suppose the contrary, i.e., $\frac{k^p(t)}{t} > \frac{1}{2} + \frac{1}{t}$ for some p. Next we find a contradiction for each party in each case:

(i) Suppose that parties' performance has been such that $k_0^a + k_1^a > k_0^b + k_1^b + k_0^c + k_1^c + 1$. Since $k_0^a < k_0^b$ and $k_1^a < k_1^c$ we have a contradiction.

Suppose that parties' performance has been such that $k_0^b + k_1^b > k_0^a + k_1^a + k_0^c + k_1^c + 1$. It is necessary that, given the performances of parties a and c, party b wins an election with either $k_0^b - 1$ or $k_1^b - 1$. If party b chose policy 1 and case (i) applies, we have $\mu_b = \min\{x(a,b), x(b,c)\}$. For party b to win we need $x(a,b) \geq \frac{1}{3}$ for $\left(k_0^b, k_1^b - 1\right)$. This condition implies $2k_1^a + k_0^a - k_1^b + 2 \geq k_0^b + k_1^b$ and contradicts the initial assumption. If party b chose policy 0 and we have $k_0^b - 1 > k_0^a$ then case (i) applies and we have $\mu_b = \min\{x(a,b), x(b,c)\}$. For party b to win we need $x(a,b) \geq \frac{1}{3}$ for $\left(k_0^b - 1, k_1^b\right)$. This condition $2k_1^a + k_0^a - k_1^b + 1 \geq k_0^b + k_1^b$ contradicts the initial assumption. If party b chose policy 0 and we have $k_0^b - 1 = k_0^a$ then case (iii) applies and we have $\mu_b = x(b,c)$. For party b to win we need $x(b,c) \geq \frac{1}{2}$ for $\left(k_0^b - 1, k_1^b\right)$. This condition $k_0^c + k_1^c + 1 \geq k_0^b + k_1^b$ contradicts the initial assumption.

Suppose that parties' performance has been such that $k_0^c + k_1^c > k_0^a + k_1^a + k_0^b + k_1^b + 1$. Since in this case party c is symmetric to party b the last argument applies here.

(ii) For parties a and b the contradiction follows directly.

Suppose that parties' performance has been such that $k_0^c + k_1^c > k_0^a + k_1^a + k_0^b + k_1^b + 1$. It is necessary that, given the performances of parties a and b, party c wins an election with either $k_0^c - 1$ or $k_1^c - 1$. If party c chose policy 0 then case (ii) applies and we have $\mu_c = 1 - x(a,c)$. For party c to win we must have $k_0^a = k_0^b$ and $1 - x(a,c) \geq \frac{1}{3}$ for $\left(k_0^c - 1, k_1^c\right)$. This condition $2k_0^a + k_1^a - k_0^c + 2 \geq k_0^c + k_1^c$ contradicts the initial assumption. If party c chose policy 1 and we have $k_1^c - 1 > k_1^a$ then case (ii) applies and we have $\mu_c = 1 - x(a,c)$. For party c to win we must have $k_0^a = k_0^b$ and $1 - x(a,c) \geq \frac{1}{3}$ for $\left(k_0^c, k_1^c - 1\right)$. This condition $2k_0^a + k_1^a - k_0^c + 1 \geq k_0^c + k_1^c$

contradicts the initial assumption. If party c chose policy 1 and we have $k_1^c - 1 = k_1^a$ by previous lemmata this case is not possible.

(iii) For parties a and c the contradiction follows directly.

Suppose that parties' performance has been such that $k_0^b + k_1^b > k_0^a + k_1^a + k_0^c + k_1^c + 1$. It is necessary that, given the performances of parties a and c, party b wins an election with either $k_0^b - 1$ or $k_1^b - 1$.. If party b chose policy 1 then case (iii) applies and we have $\mu_b = x(b,c)$. For party b to win we need $x(b,c) \geq \frac{1}{3}$ for $(k_0^b, k_1^b - 1)$.. This condition $2k_1^c + k_0^c - k_1^b + 2 \geq k_0^b + k_1^b$ contradicts the initial assumption. If party b chose policy 0 and we have $k_0^b - 1 > k_0^a$ then case (iii) applies and we have $\mu_b = x(b,c)$. For party b to win we need $x(b,c) \geq \frac{1}{3}$ for $(k_0^b - 1, k_1^b)$. This condition $2k_1^c + k_0^c - k_1^b + 1 \geq k_0^b + k_1^b$ contradicts the initial assumption. If party b chose policy 0 and we have $k_0^b - 1 = k_0^a$ then case (v) applies and we have $\mu_b = x(b,c)$. For party b to win we need $x(b,c) \geq \frac{1}{2}$ for $(k_0^b - 1, k_1^b)$. This condition $k_1^c + k_0^c + 1 \geq k_0^b + k_1^b$ contradicts the initial assumption.

(iv) For parties a and b the contradiction follows directly.

Suppose that parties' performance has been such that $k_0^c + k_1^c > k_0^a + k_1^a + k_0^b + k_1^b + 1$. It is necessary that, given the performances of parties a and b, party c wins an election with either $k_0^c - 1$ or $k_1^c - 1$. If party c chose policy 0 then case (iv) applies and we have $\mu_c = 1 - x(a,c)$. For party c to win we need $1 - x(a,c) \geq \frac{1}{3}$ for $(k_0^c - 1, k_1^c)$. This condition $2k_0^a + k_1^a - k_0^c + 2 \geq k_0^c + k_1^c$ contradicts the initial assumption. If party c chose policy 1 and we have $k_1^c - 1 > k_1^a$ then case (iv) applies and we have $\mu_c = 1 - x(a,c)$. For party c to win we need $1 - x(a,c) \geq \frac{1}{3}$ for $(k_0^c, k_1^c - 1)$. This condition $2k_0^a + k_1^a - k_0^c + 1 \geq k_0^c + k_1^c$ contradicts the initial assumption. If party c chose policy 1 and we have $k_1^c - 1 = k_1^a$ then case $(viii)$ applies and we have $k_0^c = k_0^a - 1$. The contradiction follows directly.

For cases (v) to $(viii)$ the contradiction follows directly.

<u>Proof of Theorem 3</u>:

An implication of Theorem 1 is that the given strategies yield the following payoffs: $\Pi^a = \Pi^b = \frac{1}{4}$ and $\Pi^c = \frac{1}{2}$ up to any permutation of parties. If party a (or b) deviates we will have a situation as the one described in Theorem 2: one party mixes between policies 0 and 1 and the other two parties always choose the same policy. In this case we have already seen that the limit frequency of times in office of the party that mixes, in this case party a, is one fourth. Therefore, parties a and b have no incentive to deviate. Now consider a deviation of party c. Lemma 2 in the appendix proves that at each $t > 0$ an for all $p \in \{a,b,c\}$, $\frac{k^p(t)}{t} \leq \frac{1}{2} + \frac{1}{t}$.

Therefore, $\limsup_{t\to\infty} \dfrac{k^p(t)}{t}$ for any party is bounded by $\dfrac{1}{2}$, for every strategies of its opponents. Since in this case the limit frequency of party c is already one half, it does not have any incentive to deviate.■

Proof of Remark:

Suppose that the winners of the first three elections are a,b and c respectively. Assume that parties a and b are stagewise vote-maximizers. Suppose that parties a and b have chosen policy 0 the first time they took office (it is stagewise maximizing). Further, suppose that they choose the following (stagewise maximizing) strategy for $t > 2$: if it is indifferent between the two policies in terms of stagewise maximization then it will choose the policy that party c chose at $t = 2$ (the third election). In this case we have that $x^a \quad X^a$ and $x^b \quad X^b$. Given the strategies of parties a and b, it pays to party c not to maximize next election votes the first time it takes office (by choosing policy 1) in order to make sure that its limit frequency of times in office will be one half.

If at $t = 2$ and $t = 3$ party c uses a stagewise maximizing strategy it will choose policy 1, since parties a and b have chosen policy 0. At $t = 4$ there is a tie among the three parties. If parties a (or b) wins this election it will be indifferent between policies 0 and 1 in terms of vote maximization and it will choose policy 1 and from then on we will have the following result: parties a and c choose always policy 1 and win one fourth of the times each and party b chooses always policy 0 and wins one half of the times. If at $t = 4$ party c wins the election then at $t = 5$ party a (or b) will win and we will have the situation described above.

If at $t = 2$ party c chooses policy 0, instead of a vote maximizing policy, at $t = 3$ there is a tie among the three parties. If party a (or b) wins the election it will be indifferent and will choose policy 0, imitating party c at $t = 2$. If the next time that party c wins an election it chooses policy 1 then the result will be as follows: parties a and b choose always policy 0 and win one fourth of the times each and party c chooses always policy 1 and wins one half of the times.■