Economics Working Paper 27

Short Term Credit Versus Account Receivable Financing

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March 1993

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Abstract

Account receivables may be used as collateral for the loans a firm obtain. But if the markets price the credit risk correctly, this is equivalent to a loan to the firm that has bought on credit. We investigate the reasons why the two ways of funding the firms current operation differ in an asymetric information framework, and offer an explanation of some of the institutional aspects of account receivables financing that are currently observed.

Short term credit versus account receivable financing

by

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INTRODUCTION

Although trade credit operations account for some 20% to 25% of firms total debt, from a theoretical standpoint the very existence of this type of credit still remains partially unexplained and puzzling. Important contributions in this area have helped to understand the problem at hand, emphasizing the role of capital market imperfections (Schwartz(1974), Ferris(1981), Emery(1984))), market power (Schwartz and Whitcomb(1979), Brennan, Maksimovic and Zechner(1988)), taxes, and asymmetric information (J.K. Smith(1987)) as a possible origin of the trade credit operations, while the recent contribution of Mian and Smith(1992) offers an excellent perspective on the theoretical issues and on the empirical evidence for the U.S.

We will focus here on the way in which these operations are funded, and in particular in the use of the portfolio of account receivables as collateral for the loans obtained by the seller that provides the credit, that is account receivables discounting (A/Rd). The theoretical motivation for addressing this question is simply to try to understand why obtaining credit is easier for the seller than for the buyer, when interest rates correctly account for the default risk. Given that A/Rd implies a duplication of the administrative and monitoring costs associated with the credit decision, the A/Rd is a priori more costly. Therefore, our first motivation is

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simply to provide a rationale for A/Rd in a competitive world[2].

Our second motivation is based on cross country comparison of the institutional organization for A/Rd. Comparison of continental Europe and U.S. markets showed that there is a wide difference in the financial conditions the sellers obtain on A/Rd. Furthermore, depending on the extent and availability of this type of funding for the firms, A/Rd may signal an unsound financial position with a higher probability that the firm is exposed to financial distress[3]. According to Mian and Smith (1992) (table V, p.184) this is the case in the U.S. since the firms that use A/Rd have a lower credit rating. Still this is not the case in Europe. The question is then not only to understand why two different types of financial structure can be used, but also to determine whether one of them is superior to the other. This last question has important implications in banking policy: indeed, it may be profitable for the whole industry to make use of standardized procedures in order to decrease costs[4]. This has been an additional motivation for this paper, that is also justified by the questions concerning banking in Eastern Europe, as well as for building an integrated European credit market.

It seems natural to look for an explanation of the A/Rd operations assuming that the sellers and the financial intermediaries have different information. Indeed, the process of selling generates information on the buyer as a by product that may be easily used. Our model differs from the one of J.K.Smith(1987) and of Biais,

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[2]In a non competitive financial world, it is clear that the seller and the buyer may appropriate part of the rents of the financial institution by use of A/Rd, since the seller needs only to finance its own costs at the non competitive rate, while the buyer would have to finance the whole amount of the sale.

[3]For instance, Hempel and Simonson(1991, p. 224) define this situation as the one characteristic of a "weaker consumer which otherwise might not provide a viable loan credit"

[4] Thus in France, according to Burgard (1989), the use of standardized data processing in A/Rd has divided by five the cost of the procedure. Meanwhile, during the same period in Spain the cost has remained unchanged and A/Rd has been progressively substituted by direct short term lending.

Gollier and *Viala*(1992) because the information received by the banks is here explicitly modelled, and it is not necessarily inferior to the one received by the firms. Based on this assumption, we develop a simple model in order to obtain a well articulated explanation for the differences in the credit market that we observe across countries.

We find that there are two main types of sequential Bayesian equilibria that may emerge, whose characteristics could be grossly identified as those of European and U.S. markets.

An interesting feature of our model is that when the firms choose their level of information, it justifies the existence of a procedure for A/Rd where a) the financial intermediary rejects the invoices of the company that do not meet some standards b) the financial intermediary makes loans for less than 100 percent of the nominal amount of the invoices and c) the financial intermediary may require some additional amount of collateral (compensating balances)[5]. In addition, the two type of equilibria may coexist in the different industries explaining the empirical evidence of similar intraindustry levels and wide interindustry differences on trade credit.

We first examine the problem in a symmetric information setting in which we show that direct loans are equivalent to A/Rd. Therefore, in this context, A/Rd represents no advantage. In contrast, in section 3 we examine an asymmetric information setting and show that two different types of stable equilibria emerge, one with a low proportion of A/R discounted which is akin to the american financing scheme, while the other, with a high proportion of A/R discounting is similar to the one that prevails in continental Europe.

In section 4 we give a rationale for the existing procedures for pledging account receivables, based on the fact that the level of information of the seller is a non-observable endogenous variable.

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[5]A practice that is more common in Europe than in the U.S.

II A simple model

We build a simple model that is intended to capture the differences in the allocations determined by each financing scheme. We will first investigate which of these allocations correspond to an equilibrium, and then compare them. This implies that we investigate how the equilibrium prices and quantities of the market for the good that is the object of the transaction are affected by the financial conditions. This entails that we describe with some detail a general equilibrium model for this (apparently simple) buyer-seller transaction.

We distinguish three types of risk neutral agents: sellers or upstream firms (U), buyers or downstream firms (D), and financial intermediaries or banks (B).

We consider three goods. Good 1 and good 2 are consumption goods for period 1 and 2. The third good is the intermediate good that is produced at period 1 and is used as an input in order to produce good 2. There is a risk free technology that produces $1 + r_f$ units of period 2 consumption good out of 1 unit of period 1 consumption good.

Financial intermediation exists because banks receive signals on the firms probability of bankruptcy, that give them an advantage in making loans to firms. On the other hand, sellers also receives a signal on the activity of these firms, as their usual suppliers.

Buyers production functions:
To encompass these features we assume that the buyers can be of two types, $\underline{\theta}$ or $\bar{\theta}$. The probabilities $\Pi(\bar{\theta})$ and $\Pi(\underline{\theta})$ of a firm being of type $\bar{\theta}$ and $\underline{\theta}$ respectively is common knowledge.

Buyers are supposed to have no capital and borrow entirely the amount of capital they need. Those whose projects are financed buy a quantity of input, x, and obtain random cash flows, y, characterized by a density function $g(y \mid \theta, x)$, and a cumulative distribution function $F(y \mid \theta, x)$ defined over $[0, \infty [$. The expected marginal productivity is decreasing, and the cash flow is zero if the firms uses no capital, so that the average product is decreasing.

We assume that the cash flows distributions of the two different types can be ordered according to the first order stochastic dominance criterium, with $\bar{\theta}$ dominating $\underline{\theta}$, so that for any t and any input level x we have

$$\int_0^t g(y \mid \bar{\theta}, x)) \, dy \leq \int_0^t g(y \mid \underline{\theta}, x) \, dy \qquad \forall t$$

In the same way, for a given value of Θ the distribution of y for x_1 will first order dominate the distribution of y for x_2 if $x_1 > x_2$.

Information structure:

-       Banks and firms perceive signals that give information on the firm's type. In order to have the simplest scheme, we assume that the signals observed either by U or by B can only take two values: $\bar{S}_u$ or $\underline{S}_u$ for the seller U, and $\bar{S}_B$ or $\underline{S}_B$ for the bank B. The value of these signals are known by the buyers that produces them.

We denote by α^j and β^j the conditional probabilities $\Pi(\bar{S}_j \mid \bar{\theta})$ and $\Pi(\bar{S}_j \mid \underline{\theta})$, $j = U, B$. Conditional on θ the signals S_u and S_B are independent random variables.

We assume $1 \geq \alpha^j \geq \frac{1}{2} \geq \beta^j \geq 0$. These conditional probabilities define the

quality of the information the agent j receives. If $\alpha^j = \beta^j$ the agent has no information; on the other hand, if $\alpha^j = 1$ and $\beta^j = 0$ he has perfect information.

We will refer to $I^j = (\alpha^j, \beta^j)$, $j = U, B$, as an _information structure_. Comparison of information structures will be crucial to understand how the financing schemes themselves compare.

- For the sake of simplicity, we will assume that the production of the intermediate good by sellers takes no time, so that the sellers needs credit only in order to grant credit to the buyers.

Equilibrium in the intermediary good market:

We consider a competitive market both for the buyers and for the sellers. Depending on the financing scheme, and on its level of input, a buyers characterized by signals $k = (S_B, S_u)$ has expected profits given by:

$$E(y \mid \bar{\theta}, X_k) - \int_o^\infty Min(y, [1 + r_k] p X_k) g(y \mid \bar{\theta}, X_k) dy$$

where p is the price of the input.

The demand for input of a type $\bar{\theta}$ buyer is therefore given by the equality of the expected marginal product to the expected unit cost of input:

$$\frac{\partial E(y \mid \bar{\theta}, x, X_k)}{\partial X_k} = [1 + \rho_{Dk}(\bar{\theta})] \, p \qquad (1)$$

with

$$[1+\rho_{Dk}(\bar{\theta})]p = \int_0^\infty Min(y,[1+r_k]pX_k)\frac{\partial g}{\partial X_k}(y \mid \bar{\theta},X_k)dy + [1+r_k]p[1-F[(1+r_k)pX_k \mid \bar{\theta},X_k]]$$

where p is the price of the intermediate good and r_k the nominal interest rate on the loan or the interest implicit in credit terms. Buyers with characteristic θ have the same demand, since otherwise they would be giving information on their true type.

The competitive seller will have a supply of the intermediate good given by

$$[1+\rho_{Dk}]\, p = (1+\rho_u) \quad C'_u(q_u) \tag{2}$$

where ρ_u is its expected borrowing cost (whose value will depend in particular on the quality of the A/R portfolio and on the collateral offered by the seller) and p is the seller's marginal expected income. If the seller is not granting any credit it obtains its profit $p - C_u(q_u)$ during period 1 and (2) still holds for $\rho_{Dk} = \rho_u = r_f$.

The equilibrium in the intermediate good market is then given by the supply demand equality:

$$N_u q_u = N(\bar{S}_u,\bar{S}_B)\, X_{UB} + N(\bar{S}_u,\underline{S}_B)\, X_U + N(\underline{S}_u,\bar{S}_B)\, X_B \tag{3}$$

where N_U is the number of sellers, $N(S_u,S_B)$ is the number of buyers with signals (S_u,S_B) and X_k the corresponding input demand of the firms given by equation (1) for $k = UB$, U, B.

In deriving (3) we assume that firms with signals $(\underline{S}_u,\underline{S}_B)$ are never financed whatever the financing scheme. Depending on the equilibrium, firms characterized by $(\bar{S}_u,\underline{S}_B)$ or $(\underline{S}_u,\bar{S}_B)$ are able to obtain a loan or not. We will say that the equilibrium is with a single financing type of agent if firms $(\underline{S}_u,\bar{S}_B)$ are not financed

in the A/Rd equilibrium and if $(\bar{S}_u, \underline{S_B})$ are not financed in the DL one. This will be the case if the expected return is inferior to the riskless rate r_f.

We consider three different financing schemes.

We define direct lending (DL) as the financial scheme in which the banks lend to the buyer for which the signal $\bar{S}_B$ is observed the amount they need to purchase all its inputs from the seller[6]. Sellers may act then as residual lender, lending at higher interest rates to the buyers that have been turned down by the banks.

In an A/Rd scheme, the sellers lend to the firms for which a signal $\bar{S}_u$ is observed. Then, the sellers obtain credit from the banks using A/Rs as collateral.

Finally, a third financing scheme we call mixed strategies (MS) emerges as a combination of DL and A/Rd. This obtains only if the financial conditions offered by the sellers and the banks are the same so that the buyers use mixed strategies to determine how they will be financed.

For a given realization of signals (S_u, S_B) what differentiates a financing scheme from another is i) which are the terms on which a particular buyer characterized by (S_u, S_B) is financed and ii) the price of the intermediate good. These conditions, in turn, will determine the quantity of intermediate good to be produced and therefore the expected quantity of good 2 that is obtained.

Since firms know the signal they produce, they try to obtain credit on the best terms. On the other hand, since each type of creditor observes only one kind of signal, the beliefs on the signal that is not observed are determined at equilibrium. A firm with signal $(\bar{S}_u, \underline{S_B})$ will obtain credit via A/Rd since it knows that it will

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[6] Since we assume that the loan the buyer receives is allocated to acquire the goods from the seller, the bank could also be a financial institution specialzed in non-recourse factoring.

never be financed by a bank, while a firm with signals $(\underline{S}_u, \bar{S}_B)$ will obtain a credit from the bank. But the firms with $(\bar{S}_u, \bar{S}_B)$ will choose the cheapest source of funds, and if the rates quoted by the banks and the sellers are equal, will choose randomly between them. If we denote by r_u and r_B the rates of interest quoted by the sellers and the banks, we therefore have three cases[7]:

i) $r_u > r_B$, and all the $(\bar{S}_u, \bar{S}_B)$ firms are financed by banks, corresponding to DL

ii) $r_u < r_B$, and all the $(\bar{S}_u, \bar{S}_B)$ firms are financed by the sellers corresponding to A/Rd

iii) $r_u = r_B$, in which case there is some distribution of the buyers between sellers and banks corresponding to MS

This implies that we cannot have more than the three financing schemes we have pointed out. In the next section we prove that, in general, if the banking industry is competitive, each of these financing schemes is feasible, so that we have multiple equilibria.

In the comparison of these financing schemes we will be particularly concerned with the efficiency issues. In the particular setting we develop, efficiency depends only upon the total amount of period 1 and period 2 consumption goods available, because the intermediate good is not storable and has no utility. Since, in addition there is a way to convert one unit of good 1 into $1+r_f$ units of good 2, we are allowed to value the total quantity of goods in terms of good 2 by capitalizing the amount of good 1 available at the riskless interest rate. Therefore, the stylized model we use will allow us to compare all allocations in terms of their efficiency.

In order to do so, efficiency has to be defined given an information structure and

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[7] We exclude the communication of signals between U and B that would allow the $(\bar{S}_u, \bar{S}_B)$ firms to be financed under more favourable conditions.

a realization of signals, $N(S_u, S_B)$. For a quantity ω of good 1, and an allocation $x = (x_{UB}, x_U, x_B)$, the quantity of the period 2 consumption good will be

$$
\begin{aligned}
\varphi(X) = {} & N(\bar{S}_u, \bar{S}_B) E[y | \bar{S}_u, \bar{S}_B, X_{U,B}] \\
& + N(\bar{S}_u, \underline{S}_B) E[y | \bar{S}_u, \underline{S}_B, X_u] + N(\underline{S}_u, \bar{S}_B) E[y | \underline{S}_u, \bar{S}_B, X_B] \\
& + (1 + r_f)\left(\omega - N_U C_u \left(\frac{N(\bar{S}_u, \bar{S}_B) X_{UB} + N(\bar{S}_u, \underline{S}_B) X_U + N(\underline{S}_u, \bar{S}_B) X_B}{N_U} \right) \right)
\end{aligned}
\tag{4}
$$

Maximization of this expression simply implies the usual marginal conditions

$$
\frac{\partial E[y \mid k, X_k]}{\partial X_k} = (1 + r_f) \, C_u'(q_u) \qquad k = UB, U, B
\tag{5}
$$

To compare with the competitive solution, we combine (1) and (2), to obtain:

$$
\frac{\partial E[y \mid \bar{\theta}, k, X_k]}{\partial X_k} = \left[\frac{1 + \rho_{Dk}(\bar{\theta})}{1 + \rho_{Dk}} (1 + \rho_u) \right] C_u'(q_u)
\tag{6}
$$

so that there are two reasons why an allocation may deviate from the efficient level: asymmetric information and the fact that financial markets may be imperfect, so that the borrowing expected rate differs from the lending expected rate, e.g. due to the regulation or to imperfect competition in the banking industry.

A particular feature of our model is that the value of $\varphi(X)$ is equal to the sum of second period profits of different types of firms for any allocation plus a constant, $\omega(1 + r_f)$. This is proved simply by adding and subtracting the financial costs of the buyers inputs and by using conditions (3) and (4)

$$\varphi(X) = N(\bar{S}_u, \bar{S}_B) \; E[\pi^D \mid \bar{S}_u, \bar{S}_B)] + N(\bar{S}_u, \underline{S}_B) E[\pi^D \mid \bar{S}_u, \underline{S}_B] +$$
$$+ N(\underline{S}_u, \bar{S}_B) E[\pi^D \mid \underline{S}_u, \bar{S}_B] + N_U E[\pi^U] + \pi^B + (1 + r_f)\omega \tag{7}$$

where π^D and π^U are the profits of a buyer and a seller and π^B are the profits of the whole banking industry.

III Competitive Banking Structure

In this section we consider a competitive banking industry. Since, in addition, we assume that there are no intermediation cost, this implies that banks have zero expected profits, so that at equilibrium

$$\rho_u = \rho_{DB}^m = r_f \tag{8}$$

where ρ_{Dj}^m is the expected average return of a loan to a buyer granted by agent j. Now the sellers are indifferent between selling directly to the buyers or granting them credit, so that we also obtain:

$$\rho_{DU} = r_f \tag{9}$$

The following result holds:

Proposition 1:

i)   If $\pi(\bar{\theta} \mid \bar{S}_u, \underline{S}_B) < \pi(\bar{\theta} \mid \bar{S}_B)$ and $\pi(\bar{\theta} \mid \underline{S}_u, \bar{S}_B) < \pi(\bar{\theta} \mid \bar{S}_u)$ there exist three types of equilibria corresponding to DL, A/Rd and MS

ii)   If $\pi(\bar{\theta} \mid \bar{S}_u, \underline{S}_B) > \pi(\bar{\theta} \mid \bar{S}_B)$ only A/Rd exists

iii)   If $\pi(\bar{\theta} \mid \underline{S}_u, \bar{S}_B) > \pi(\bar{\theta} \mid \bar{S}_u)$ only DL exists

conditions i) ii) and iii) are mutually exclusive.

Inequality ii) covers the case in which the information structure S_u is so clearly superior to S_B that a favourable signal $\overline{S}_u$, that is controverted by the opposite signal, $\underline{S}_B$, yields a better information than the uncontroverted signal of the poorest information structure, $\overline{S}_B$. Inequality iii) covers the opposite case.

Under i) the three financing schemes are feasible. Still, the three types of equilibrium differ with regard to stability: DL and A/Rd are stable equilibria while this is not so for MS. Indeed, any departure from equilibrium will produce a difference between the rates r_u and r_B quoted by the firms, and all the $(\overline{S}_u, \overline{S}_B)$ firms will choose to be financed at the lower rate, so that the equilibrium becomes either DL or A/Rd.

We will use Blackwell's comparison of experiments in order to compare the different information structures.

<u>Definition</u>: Let
$$\begin{aligned} a &= \pi(\overline{\underline{S}} \mid \overline{\theta}) &;& \quad a' = \pi(\overline{\underline{X}} \mid \overline{\theta}) \\ \beta &= \pi(S \mid \underline{\theta}) &;& \quad \beta' = \pi(X \mid \underline{\theta}) \end{aligned}$$

where S and X are two random variables, (a,β) is more informative than (a',β') if there exists a stochastic matrix Σ

such that
$$\begin{bmatrix} a' & \beta' \\ 1-a' & 1-\beta' \end{bmatrix} = \Sigma \begin{bmatrix} a & \beta \\ 1-a & 1-\beta \end{bmatrix}$$

Under the conditions $a > \frac{1}{2} > \beta$, $a' > \frac{1}{2} > \beta'$, it is easy to check that the necessary and sufficient conditions for (a,β) to be more informative than (a',β') is that:

$\alpha - \alpha' - (\beta - \beta') \geq \alpha\beta' - \alpha'\beta \geq 0$

This can be rewritten as $\Delta\alpha = \alpha - \alpha'$; $\Delta\beta = \beta - \beta'$

$$\Delta\alpha - \Delta\beta \geq -\alpha\Delta\beta + \Delta\alpha\beta \geq 0 \qquad (10)$$

It will be helpful to have the information structure as a function of some index I, in which case dividing (10) by ΔI we will have in the limit:

$$\alpha'(I) - \beta'(I) \geq \alpha'(I)\,\beta(I) - \alpha(I)\,\beta'(I) \geq 0 \qquad (11)$$

that is, necessary and sufficient conditions are that $\alpha'(I) - \beta'(I) \geq 0$ and $\alpha(I) / \beta(I)$ increasing.

The following propositions establishes a quite intuitive result based on the idea that having more information will improve efficiency. In order to do so we have to introduce an additional definition:

given an information structure for banks and sellers we are able to define a coarser information structure, we will call the common information structure, by assuming that agents can only observe whether the signal (S_U, S_B) belongs to the set $\{(\bar{S}_u, \bar{S}_B), (\bar{S}_u, \underline{S}_B), (\underline{S}_u, \bar{S}_B)\}$ or not.

Proposition 2:

Under the competitive markets assumption,

i)if the information structures of U and B are the same, then DL and A/Rd generate the same aggregate allocation.

ii)the MS financing scheme generates the same allocation as the DL or the A/Rd one endowed with the common information structure.

Since in general the competitive equilibrium differs from the optimal allocation, the comparisons between the different financing schemes cannot be made without strong assumptions on the distribution of the cash flows. To avoid dealing with particular cases we impose the following assumption.

Assumption 1: $\dfrac{\partial E\,(y \mid \theta\,, \kappa\, X_k^*)}{\partial X_k^*} > [\,1 + \rho_{DK}(\,\theta\,)]\, p$

Assumption 1 implies that in equilibrium the $\underline{\theta}$ firms would gain from am expansion of their production. This is justified because they are borrowing at a rate which is subsidized by the existence of the $\overline{\theta}$ firms. Still, since the expected marginal productivity of x_K is inferior of $\underline{\theta}$, this effect also comes into play. Assumption 1 states that the benefits from the lower expected cost of borrowing is the dominating effect.

Proposition 3

If the markets are competitive whenever condition i) of proposition 1 holds with a single financing type, both for DL and for A/Rd, then if assumption 1 holds,

i) DL dominates A/Rd if and only if the banks have a better information than the sellers.

ii) MS is dominated both by DL and by A/Rd.

Thus, proposition 3 establishes that welfare improves if the financing scheme allocates the right to grant credit to the agent endowed with a better information.

III Costly Information

We now extend our model so as to take into account the fact that information is costly. We will focus on the decisions of the seller concerning the level of information structure under A/Rd, assuming that there is variable a cost of information. We assume that the seller is able to acquire a level of information I, where I represents a level of information. If the information level acquired by the seller was known to the banks, then it should be treated as an additional input and this would not affect the analysis of the contracts between the banks and the sellers. We will focus instead on the case in which the level of information chosen by the sellers is not observable by the banks. This will lead to a moral hazard problem whose effect can be mitigated by the use of collateral and by the banks observation of A/R credit quality.

When the level of information the seller chooses is not observable by the banks, this choice generates an externality. Indeed, the benefits of improving the information level goes to the average buyer that is financed and to the firm´s lenders. Since a fraction of the profits are appropriated by the other agents while the seller is paying for all the costs, there is no reason why the equilibrium level for

information is the efficient one. On the other hand, the benefits of investing in information will also depend on the financial contract that the sellers has in order to finance its A/Rd. Use of collateral and of banks information produces an allocation that will dominate the one obtained with unsecured loans. We show that, in general, the equilibrium that obtains is one in which the collateral level is strictly positive and in which the bank will choose to lend only if the pledged A/R is considered of good quality.

In this context, the banks compete by offering contracts. Since banks behave competitively and obtain zero expected profits the equilibrium contracts will be the ones that maximize the sellers profits[8].

Since the banks observe a signal S_B they are able to offer different contracts ($\overline{r,c}$) and ($\underline{r,c}$), where r denote interest rates and c collateral, depending on the observed value of $S_B,(\overline{S}_B$ or $\underline{S}_B$ respectively).

If the seller's level of information was observable by the banks, all the contracts that lead to a zero expected profit it makes for the banks would be equivalent. Since it is not, the problem at hand is therefore to obtain the contracts that minimize the moral hazard distortion. The following result is then quite intuitive.

Proposition 4:

A pair of contracts $(\overline{c,r},\underline{c,r})$ in which the bank bears no risk is an equilibrium contract under a competitive structure.

Proof: see appendix

Proposition 4 is of interest because it is related to the commonly observed

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[8] This increase in the profit may be in fact channelled through lower prices to the D firms.

characteristics of the A/R discounting process: first, the bank accepts or rejects the invoices that the sellers wants to discount; next, if the invoice is accepted, the loan is made for less than its nominal amount, and a compensating balance may be required so that the risk of default is negligible.

It is interesting to see that the effect of collateral is here due to the value it has to the borrower, that makes him act so as to diminish the probability of bankruptcy. The Proposition 3 extends to the case in which collateral has little or no value to the lender, for instance the case in which there is a non monetary cost of bankruptcy (as in Diamond (1984))[9]

Therefore, in this setting, the fact that the banks check the quality of each invoice instead of adjusting the interest rate to the average risk of default and the fact that they use strong collateral for loans, is a way to deal with the moral hazard issue. If the equilibrium loans are safe then, the rates of interest equal the risk free rate and the loans only differ in the required collateral.Consequently there is no cross-subsidiation between the good and the bad signal loans.

Conversely, it is possible to prove that if there is a risk of default, or if the collateral is costly, cross-subsidation may be used to implement the optimal level of information.

CONCLUSION

To summarize, our paper states that if we consider that the information concerning a firm that is received by the banks and by its suppliers is different, then two types of equilibrium may prevail in the market for A/R discounting, one characterized by a low level of A/Rd, high risk and corresponding high interest

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[9] There are other institucional mechanisms that will induce the U firm to acquire more information. Their examination is out of the scope of this paper.

rates as in the U.S. market and another characterized by a high level of A/Rd, with low default risk and low interest rates, as in the continental Europe market. The more efficient one will be, in general, the one in which the agent endowed with better information is the one which provides more credit to the firms. Still, nothing prevents the dominated equilibrium from taking place. In addition, the extension of this setting to the case where the level of information of the seller is endogenous shows that the procedures that are used in A/Rd can be perfectly justified as a way to limit the underproduction of information due to the unobservability of the sellers level of information.

APPENDIX

Proof of proposition 1

a) *To begin with, we consider the MS equilibrium. We first prove that there exists a probability μ_j for the $(\bar{S}_u, \bar{S}_B)$ firms to be financed by j, j = U, B for which we have*

$$\int_0^\infty Min(y, (1+r)px) \; g(y, X, \bar{S}_B(\mu_B)) \, dy = \int_0^\infty Min(y, (1+r)px) \; g(y, X, \bar{S}_u(\mu_u)) \, dy$$

A.1

where px is the amount lend to each buyer, $r = r_U = r_B$ and $X = X_{UB} = X_U = X_B$; $g(y, X, \bar{S}_j)$ is the density function of y when $\bar{S}_j$ is observed <u>in this equilibrium</u>, and in addition, the banks zero profit condition holds:

$$\int_0^\infty Min(y, (1+r)L_D) \; g(y, X, \bar{S}_B(\mu_B)) \; dy = (1+r_f)L_D$$

Where L_D is the amount of the loan equal to px

In equilibrium we have

$$g(y \mid \bar{S}_j(\mu_j), X) = \pi^*(\bar{\theta} \mid \bar{S}_j(\mu_j)) \, g(y \mid \bar{\theta}, X) + [1 - \pi^*(\bar{\theta} \mid \bar{S}_j(\mu_j))] \, g(y \mid \underline{\theta}, X)$$

where
$$\pi^*(\bar{\theta} \mid \bar{S}_u, \mu_u) = \frac{[\mu_u \pi((\bar{S}_u, \bar{S}_B) \mid \bar{\theta}) + \pi((\bar{S}_u, \underline{S}_B) \mid \bar{\theta})] \, \pi(\bar{\theta})}{\mu_u \pi((\bar{S}_u, \bar{S}_B) \mid \bar{\theta}) \, \pi(\bar{\theta}) + \mu_u \pi((\bar{S}_u, \bar{S}_B) \mid \underline{\theta}) \pi(\bar{\theta}) + \pi((\bar{S}_u, \underline{S}_B)}$$

and $\pi^*(\bar{\theta} \mid S_B, \mu_B)$ *is defined in an analogous way, with* $\mu_u + \mu_B = 1$.

Since the distribution for $\bar{\theta}$ *first order dominates the one with* $\underline{\theta}$, *a necessary and sufficient condition for* ***A.1*** *is that:*

$$\pi^*(\bar{\theta} \mid \bar{S}_u, \mu_u) = \pi^*(\bar{\theta} \mid \bar{S}_B, 1 - \mu_u)$$

Let

$$A = \pi(\bar{S}_u, \bar{S}_b, \bar{\theta}) \quad , \quad B = \pi(\bar{S}_u, \underline{S}_b, \bar{\theta}) \quad , \quad C = \pi(\underline{S}_u, \bar{S}_b, \bar{\theta})$$
$$A' = \pi(\bar{S}_u, \bar{S}_b) \quad , \quad B' = \pi(\bar{S}_u, \underline{S}_b) \quad , \quad C' = \pi(\underline{S}_u, \bar{S}_b)$$

where $\pi(k)$ *denotes the probability of an event k*

Since $\alpha_i \geq 1 - \alpha_i$ *and* $\beta_j \leq 1 - \beta_j$, *it is easy to check that*

$$\frac{A}{A'} \geq \frac{B}{B'} \text{ and } \frac{A}{A'} \geq \frac{C}{C'}$$

We want to show that there exists a $\mu_u \in (0,1)$ for which (A.4) holds, that is:

$$\frac{\mu_u A + B}{\mu_u A' + B'} = \frac{(1-\mu_u) A + C}{(1-\mu_u) A' + B'}$$

A.5

Now, the LHS of (A.5) is increasing with μ_u while its RHS is decreasing with μ_u; in addition for $\mu_u = 0$ we have $\dfrac{B}{B'} < \dfrac{A+C}{A'+C'}$, because $\dfrac{B}{B'} = \pi(\bar{\theta} \mid \bar{S}_u, \underline{S}_b)$ and $\dfrac{A+C}{A'+C'} = \pi(\bar{\theta} \mid \bar{S}_u)$ and for $\mu_u = 1$ $\dfrac{A+B}{A'+B'} > \dfrac{C}{C'}$ holds because $\dfrac{A+B}{A'+B'} = \pi(\bar{\theta} \mid \bar{S}_u)$ and $\dfrac{C}{C'} = \pi(\bar{\theta} \mid \underline{S}_u, \bar{S}_b)$. Therefore there exists $\mu_u \in (0,1)$ for which (A.5) holds. So that there exists an equilibrium mixed strategy independent of the equilibrium prices or quantities, and the continuity of demand and supply functions suffices to prove the existence of equilibrium

ß) Consider now case ii). Since $\pi(\bar{\theta} \mid \bar{S}_u, \underline{S}_B) > \pi(\bar{\theta} \mid \bar{S}_B)$ we have $\dfrac{C}{C'} > \dfrac{A+B}{A'+B'}$. Therefore, for $L_D^B = L_D^U$ and $r^B = r^U$ we obtain:

$$\rho(\frac{L_D^B}{P}) = L_D^{\frac{1}{B}} \int_0^\infty Min(y, (1+r_B)L_D^B) \, g(y \mid \bar{S}_B, L_D^B/p) \, dy < L_D^{\frac{1}{U}} \int_0^\infty Min(y, (1+r_u)L_D^U) \, g(y \mid \bar{S}_u, L_D^U) dy$$

A.6

so that MS equilibrium fails to exist.

Now, DL also fails to exist, because under DL we have $r_B < r_U$ and $L_D^B > L_D^U$, and it is possible to show that the average return on a loan is a decreasing function of the amount of the loan, that is $\rho'(X) \leq 0$. In order to do that, notice first that the concavity of $E(y \mid X)$ implies that for $\Delta > 0$,

$$\int_0^\infty \left[\frac{y}{x} g(y \mid x) - \frac{y}{x+\Delta} g(y \mid x+\Delta) \right] dy \leq 0$$

which can be rewritten as

$$\int_0^\infty \left[\frac{y}{x} g(y \mid x+\Delta) - g(y \mid x) - \left(\frac{y}{x} - \frac{y}{x+\Delta} \right) g(y \mid x+\Delta) \right] dy \leq 0$$

Now, $\text{Min}\left[\frac{y}{x},(1+r)p \right]$ is a non decreasing function whose derivative when it

exists is always inferior or equal to the one of $\frac{y}{x}$. It is possible to prove integrating
by parts and using first order dominance that

$$\int_0^\infty \frac{y}{x} \left[(g(y \mid x+\Delta) - g(y \mid x)) \right] dy \geq$$

$$\geq \int_0^\infty \text{Min}\left[\frac{y}{x},(1+r)p \right] \left[g(y \mid x+\Delta) - g(y \mid x) \right] \ dy$$

Combining the two inequalities, we have:

$$\int_0^\infty \text{Min}\left[\frac{y}{x},(1+r)p \right] \left[g(y \mid x+\Delta) - g(y \mid x) \right] dy -$$

$$- \left(\frac{y}{x} - \frac{y}{x+\Delta} \right) g(y \mid x+\Delta) \leq 0$$

dividing by Δ and making Δ tend to zero, we obtain,

$$\int_0^\infty Min\left[\frac{y}{x},(1+r)p\right]\frac{\partial g}{\partial x}(y \mid x)\,dy - \int_0^\infty \frac{y}{x^2}\,g(y \mid x+\Delta)\,dy \le 0$$

that is $p'(x) \le 0$

To obtain the equality in (A.5) we need $r^B > r^U$ and $L_D^B < L_D^U$ conditions that characterize A/Rd. To show existence, we have to distinguish two cases depending on the behaviour of the LHS of (A.6) when L_D^B goes to zero. If the inequality reverses, by continuity there exists a A/Rd with banks financing the buyers who have obtained a signal $(\bar{S}_B, \underline{S}_u)$. On the other hand, if the inequality in (A.6) is preserved, then the firms with signal $(\bar{S}_B, \underline{S}_u)$ are not financed.

Proof of Proposition 2:

Since the two information structures are equivalent, the probability of observing a number N of good signals is the same under DL and under A/Rd. We therefore consider the allocation obtained in both financing schemes when $N[\bar{S}_u] = N[\bar{S}_B]$ and $N[\underline{S}_u, \bar{S}_B] = N[\underline{S}_B, \bar{S}_u]$.

Consider the equilibrium under DL, that is given by equations

$$\frac{\partial E(y \mid \bar{\theta}, \bar{S}_B, X)}{\partial X} = [1 + \rho_{D\bar{S}_a}(\bar{\theta})]\,p$$

$$\frac{\partial E(y \mid \bar{\theta}, \bar{S}_u, \underline{S}_B, X)}{\partial X} = [1 + \rho_{D(\bar{S}_u, \underline{S}_B)}(\bar{\theta})]\,p \qquad (1)$$

$$p = C'_u(q_u) \qquad (2)$$

$$\rho_D(\bar{S}_B) = r_f \qquad (8)$$

$$\rho_D(\underline{S}_B, \bar{S}_u) = r_f \qquad (9)$$

Equations (8) and (9) imply

$$(1+r_f)\, L_{DB} = \int_0^\infty Min(y,(1+r_B^D)\, L_{DB})\ g(y \mid \bar{S}_B, L_{DB}/p)\ \ dy$$

and either $L_{DU} = 0$ or else if $L_{DU} > 0$, we have:

$$(1+r_f)\, L_{DU} = \int_0^\infty Min(y,(1+r_D^U)\, L_{DU})\ g((y \mid \bar{S}_u, \underline{S}_B), L_{DU}/p)\ \ dy$$

where $L_{DB} = X_B\, p = X_{UB}\, p$ and $L_{DU} = X_U\, p$

Since $g(y \mid \bar{S}_B, X) = g(y \mid \bar{S}_u, X)$ and $g(y \mid \underline{S}_u, \bar{S}_B) = g(y \mid \bar{S}_u, \underline{S}_B)$, the equilibrium under A/Rd is obtained simply by exchanging the role of U and B.

ii)   An MS equilibrium is characterized by equations (1), (2), (3) and the zero profit conditions

$$(1+r_f)\, p\, X = \int_0^\infty Min(y,(1+r)\, p\, X)\ g(y \mid \bar{S}_B(1-\mu), X)\ \ dy$$

while a DL or a A/Rd financing scheme will satisfy (1), (2), (3) and

$$(1+r_f)\,p\,X = \int_0^\infty Min(y,(1+r)\,p\,X)\ g(y\ |\ \bar{S}_U\ \cup\ \bar{S}_B, X)\,dy$$

to show that these last two equations are equal recall that the equality of the d.f. that obtains under MS implies the equality

$$\pi^*(\bar{\theta}\ |\ \bar{S}_u,\mu_u) = \pi^*(\bar{\theta}\ |\ \bar{S}_B,(1-\mu))$$

with the definition of π^*, given by (A3). We add the numerators and the denominators of these expression μ_u vanishes and we obtain:

$$\pi^*(\bar{\theta}\ |\ \bar{S}_u,\mu_u) = \pi^*(\bar{\theta}\ |\ \bar{S}_B,(1-\mu)) = \pi(\bar{\theta}\ |\ (\bar{S}_U\ \cup\ \bar{S}_B, X))$$

Proof of proposition 3:

Assume that banks have a better information than sellers. Using lemma 1, we know that the allocation obtained under A/Rd is equivalent to the allocation obtained under DL with the banks endowed with the information structure of the sellers.

We only have to check that total expected profit increases when information increases. When there is only one financing type, expression (7) can be rewritten as:

$$\gamma((\,I\,))\,.\,N\,(\bar{S}_B) = H\,(\bar{S}_B) + N_U\,E\,(\Pi^U) + \Pi^B + (1+r_f)\,W$$

with

$$H(\bar{S}_B) = \pi(\bar{\theta} \mid \bar{S}_B) H(\bar{S}_B, \bar{\theta}) + \pi(\underline{\theta} \mid \bar{S}_B) H(\bar{S}_B, \underline{\theta})$$

where $H(\bar{S}_B, \theta)$ are the profits obtained by Θ .

An increase in I will affect both the prices and the quantities of the equilibrium. The effect of the prices on total profits is null, because the effect on one type of firms compensates with the effect on another. The effect on U firms of an increase in the quantity produced can be shown to be zero since the marginal profit is zero. Since the banks have zero profits, we only have to examine the effect on the profits of the average D firm that is financed, $H(\bar{S}_u)$.

To examine the effect of an increase in information we want to compute:

$$\frac{\partial H(\bar{S}_B)}{\partial I} = \frac{\partial \pi(\bar{\theta} \mid \bar{S}_B)}{\partial I} \left[H(\bar{S}_B, \bar{\theta}) - H(\bar{S}_B, \underline{\theta}) \right] +$$

$$\cdot \left[\pi(\bar{\theta} \mid \bar{S}_B) \frac{\partial H(\bar{S}_B, \bar{\theta})}{\partial X_B} + \pi(\underline{\theta} \mid \bar{S}_B) \frac{\partial H(\bar{S}_B, \underline{\theta})}{\partial X_B} \right] \frac{dX_B}{dI}$$

The first term of this expression is positive because so is $H(\bar{S}_u, \bar{\theta}) - H(\bar{S}_u, \underline{\theta})$ given that the cash flow distribution for $\bar{\theta}$ first order dominates $\bar{\theta}$, and because in addition, when information on B improves $\pi(\bar{\theta} \mid \bar{S}_B)$ increases and $\pi(\underline{\theta} \mid \bar{S}_B)$

decreases, since

$$\pi(\bar{\theta} \mid \bar{S}_B) = \frac{\pi(\bar{\theta}) \cdot a_B(l)}{\pi(\bar{\theta}) \cdot a_B(l) + \pi(\underline{\theta}) \beta_B(l)}$$

and $a_B(l) / \beta_B(l)$ is increasing. (condition 11), so that $\dfrac{\partial \pi \ (\bar{\theta} \mid \bar{S})}{\partial I} > 0$.

The second term is also positive under assumption 1, since $\dfrac{\partial H}{\partial X_B} (\bar{S}_B, \underline{\theta}) > 0$, and given that $\dfrac{\partial H}{\partial X_B} (\bar{S}_B, \bar{\theta}) = 0$ is determined by the profit maximizing behaviour or $\bar{\theta}$.

The proof of ii) is exactly the same, since lemma 2 establishes that MS is endowed with a poorer information than either A/Rd or DL.

Proof of proposition 4:

An equilibrium contract will maximize the sum of the profits the seller and of the bank.

Let $C_i(l)$ be the cost of producing an information level l. Then the seller will demand a loan $L = C_i(C) + C_u(Qu)$ an will promise rate $\bar{r}$ on good A/R and $\underline{r}$ on the bad ones. If we denote $\bar{\alpha}$ the proportion of good A/R, then the promised repayment is $R = [\bar{\alpha} \ (1 + \bar{r}) + (1 - \bar{\alpha})(1 + \underline{r})]L$.

Let W be the random return the seller obtains on the loans granted to the buyer. For a given value of α, the sellers profit is

$$\pi^U(\alpha) = \int_0^{(R-C)/Q_\bullet} -Cg(W \mid I, \bar{S}_u) \ dw + \int_{(R-C)/Q_\bullet} (W Q_u - R) \ g(W \mid I, \bar{S}_u) \ dw$$

where $\qquad C = \bar{\alpha} \ \bar{C} + (1 - \bar{\alpha}) \ \underline{C}$

Denoting by $\gamma(\alpha \mid I)$ the probability density function for α, given information level I, the x firm will solve the following problem.

$$\underset{(Q_u,I)}{Max} \int_0^1 \Pi^U(\alpha) \; \gamma(\alpha \mid I) \; dx$$

The first order conditions will be:

$$\frac{\partial \; E(\pi^U(\alpha))}{\partial \; Q_u} = 0$$

and

$$\int_0^1 \frac{\partial \; \Pi^U \; (\alpha)}{\partial \; I} \gamma(\alpha \mid I) \; dx \; + \; \int_0^1 \pi^U \; \frac{\partial \gamma(\alpha \mid I)}{\partial \; I} \; dx = 0$$

On the other hand, we are able to determine the value of Q_u and I that maximize the sum of the profits of the sellers and the bank as:

$$\underset{(Q_u,I)}{Max} \int_0^\infty q_u g(W \mid I) dw - (1+rf)(C_I(I)+C_u(Q_u))$$

obtaining as first order conditions:

)i
$$\int_0^\infty W g(W \mid I) dw = (1+rf) C_u^{'}(Q_u)$$

and

)ii
$$\frac{\partial \Pi}{\partial I} \; (\overline{\Theta} \mid \overline{S}, I) \int_0^\infty w[(g(W \mid \overline{\theta}) - g(w \mid \theta)] \; dw = (1+rf) \; C_I^{'}(I)$$

Now, under the conditions of proposition 4, competition will lead to $\underline{r}=\overline{r}=rf$,
and $\Pi_U(\alpha)$ is independent of α. Maximization with respect to Qu will lead to i).
On the other hand, since $\Pi_U (\alpha) = \Pi^U$ and γ is a probability density function.
We have:

$$\int_0^1 \pi^U \frac{\partial \gamma(\alpha \mid I)}{\partial I} = 0$$

Differentiating Π^U with respect to I we obtain condition ii) for these value to.

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