Job Dynamics, Correlated Shocks and Wage Profiles*

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Abstract

We generalize the Mortensen-Pissarides (1994) model of the labor market with a more realistic structure for the stochastic process of the shocks to the worker-firm match. In this way we can acommodate the empirical observation that hazard rates of job termination decrease and average wages increase with job tenure. Besides being able to fit better some observables of the model, the changes we introduce are nontrivial for the analysis of policies as well.

Keywords: Layoff costs; search; layoff hazard rates; wage-tenure relationship.

JEL: J60; J30.

1 Introduction

The last decade has witnessed a large body of empirical work which has substantively increased our knowledge about the facts concerning the performance of labor markets. One of the main features that arises from this literature is the large extent of labor reallocation that takes place in most developed economies, most of which is unrelated to sectoral, geographical or business cycle type shocks. This labor reallocation reflects the fact that opportunities do not arise always in the same places, so labor must be reallocated from less valuable matches to those that have become more valuable.

Several theoretical models have been used to study labor reallocation. Bentolila and Bertola (1990) study the hiring and firing decisions of firms which face idiosyncratic and aggregate uncertainty to the returns to labor. Hopenhayn and Rogerson (1993) incorporate this structure in a general equilibrium model, calibrating the stochastic process for firms' productivity to match US evidence on job creation and destruction. Mortensen and Pissarides (1994) use a labor matching model with idiosyncratic shocks to the worker firm pair and efficient bargaining to study the same issues. Millard and Mortensen (1995) calibrate a version of the Mortensen-Pissarides (1994) model. These models have also been used to study the effects of labor market policies.

The model of Mortensen and Pissarides (1994) is particularly interesting because it helps to understand the interplay between bargaining (and therefore wage formation) and the process of job creation and destruction. In particular, a model with exogenous wages (like the first two papers cited) is bound to produce unrealistic conclusions about the effects of obstacles to labor mobility, since the welfare efects of these obstacles are likely to be at least partially shared by the agents through the wages.

On the other hand, Mortensen and Pissarides (1994) and Millard and Mortensen (1995) make a number of simplifying assumptions for reasons of analytical tractability which have counterfactual implications for the analysis. In particular, the assumptions they make on the stochastic process for the shocks implies that hazard rates for job termination and average wages are independent of job tenure. In contrast, empirical work indicates quite robustly that hazard rates decrease and average wages increase with job tenure. The theory of job matching developed by Jovanovic (1982) was precisely motivated by this set of observations.

We extend the Mortensen-Pissarides model along these lines. The probability structure introduced is almost identical to the one used in Hopenhayn (1992) to explain patterns of exit and growth of firms which are qualitatively identical to the ones corresponding to wage growth and job termination.[1] Besides being able to fit better some observables of the model, the changes we introduce are nontrivial for the analysis of policies as well. The calibrated version of the model predicts effects of layoff costs that differ from those

[1]This connection between labor and firm reallocation was recognized in Jovanovic (1982), that applies a version of the job matching model to study firm dynamics.

obtained by Millard and Mortensen (1995), even though the model and parameters used are the same except in the probability structure.

The paper is organized as follows. Section 2 describes the model. A stationary equilibrium is defined in section 3. Section 4 provides a calibration of the extended Mortensen-Pissarides model and gives some numerical results.

2 The model

The model we develop is a modified version of the one in Mortensen and Pissarides (1994) (subsequently MP). We study an equilibrium process of job creation and job destruction. Job creation results from a costly process of two-sided matching, where workers and employers engage in search activities. The jobs that are created after the search process is completed have a level of productivity that is random and match specific. Once established, the productivity of a match evolves according to an exogenous Markov process. As analyzed below, when a match reaches a low level of productivity, it is destroyed. Jobs are also continually created as displaced workers look for new jobs and entrepreneurs engage in new search to create positions.

The basic unit of the model is the worker-employer match, which we will also call job or position. This match produces a flow of output $q(x)$, which is strictly increasing in the productivity state of the match $x \in X = [\underline{x}, \overline{x}]$, where $\underline{x}$ and $\overline{x}$ are real numbers.[2]. This state is match specific so if the match is dissolved the position becomes effectively destroyed.

When the firm and the worker meet they form an impression of what the match productivity would be if they were to write a contract. They will use this information to decide whether to write the contract or not. This is an important consideration because the contracts that we are going to study have a mandated termination cost, and the level of match productivity that is demanded to start a relationship (when termination costs do not have to be paid) may be different from the productivity necessary to continue with an ongoing relationship. More precisely, once a worker and a employer meet, and prior to entering a contract, they receive a signal $y \in Y = [\underline{y}, \overline{y}]$ about the initial productivity of the match. This signal is drawn from a probability distribution $G_s(y)$. The stochastic process for productivity x evolves as follows. The initial productivity of the match is drawn from some probability distribution, $G_I(x|y)$, which depends on the signal received by employer and worker. We assume that $G_I(x|y')$ first order stochastically dominates $G_I(x|y'')$ when $y' > y''$ (that is, $G_I(x|y)$ is a decreasing function of y). This productivity is not known until the vacancy is filled. In particular, it could be that $x = y$ with probability 1, or it could be that x is independent of y. The evolution of productivity from that point on follows a Poisson jump process. New shocks arrive at a rate λ, and when

[2]An infinite state space can be easily handled if $q(x)$ is a bounded function

they arrive the new value of the state is drawn from a probability distribution $F(x|x')$ where x' is the previous value of productivity.[3] We also assume that $F(x|x')$ first order stochastically dominates $F(x|x'')$ when $x' > x''$. Aside from the endogenous termination rule which will be discussed below, separations occur exogenously at rate δ.

Matches provide a flow of benefits to the worker and the firms, contingent on the productivity of the match x. We will denote by $w(x)$ the gross wage received by the worker, which is determined below. The net income flow to the worker, denoted by $\pi^w(x, w(x))$, where π^w,is a continuously differentiable function, increasing in its first and second arguments. MP have $\pi^w(x, w(x)) = w(x)$, and Millard and Mortensen (1995) have $\pi^w(x, w(x)) = (1 - t_w)w(x)$, where t_w is a wage tax paid by the worker. The profit flow obtained from this match by the firm is $\pi^f(x, w(x))$, where π^f,is a continuously differentiable function, increasing in its first argument and decreasing in the second. MP have $\pi^f(x, w(x)) = x - w(x)$, and Millard and Mortensen (1995) have $\pi^w(x, w(x)) = x - (1 - t_f)w(x)$, where t_f is a payroll tax paid by the firm. We assume that both the owners of the firm and workers are risk neutral and discount future flows at a common interest rate r.

The rate at which the stock of posted vacancies v and the stock of unemployed workers u meet is given by a function $m_a(v, u, e)$, where e is the effort made by the worker in the search process. The function m_a is increasing in the three arguments and homogeneous of degree one in v and u. The rate at which workers meet vacancies is $\frac{m_a(v,u,e)}{u}$. By the assumption of constant returns in matching, this can be written $m_a(\frac{v}{u}, 1, e)$. Letting $\theta = v/u$, the matching technology can thus be described $m_a(\frac{v}{u}, 1, e) = m(\theta, e)$. Letting y^* be the minimum level of the signal above which the initial match is formed and x^* the minimum productivity level that makes continuing the match desirable [4], $\eta(\theta) = m(\theta, e)(\int_{y>y^*}(1 - G_I(x^*|y))dG_s(y))$ is the rate at which workers escape from unemployment and $1/\eta(\theta)$ the average duration of the unemployment spell. By the constant returns to scale assumption, and since all workers choose the same effort level in equilibrium and thus e is both the individual and average effort level, $\eta(\theta)/\theta$ is the rate at which vacancies are filled.

We assume that all separations between firms and workers are permanent. The termination value for the firm (which may depend continuously on x^*, y^* and θ) is denoted by T. The termination value for the worker (which may also depend continuously on x^*, y^* and θ) is denoted by B. The value of search for an unemployed worker is denoted by U.

Let $V(x)$ denote the value to a firm of having a match with productivity x and x^* be the level of productivity below which a match is terminated.

This value satisfies the following dynamic programming equation:

$$rV(x) = \pi^f(x, w(x)) + \lambda\left[\int_{x'>x^*} V(x')dF(x'|x) - F(x^*|x)T - V(x)\right] - \delta V(x) \quad (1)$$

[3] MP assume $F(x|x') = F(x)$. They also assume that $F(x)$ is a uniform distribution and $G(x)$ has unit mass at the upper end of the support of $F(x)$.

[4] In the steady state -as is analyzed in the paper- these exit points are time independent.

The flow value $rV(x)$ for this firm consists of: *i)* the net profit flow; *ii)* the capital gain (or loss) associated to a change in productivity if the new productivity x' exceeds the cutoff level x^*; *iii)* the termination value T if the new shock is below x^* and *iv)* the loss of all value if the match is exogenously terminated.

Notice that this definition implicitly assumes that the value function is increasing in x, since the termination rule for the firms is assumed to be monotone. This is valid since for the wage rate $w(x)$ that will be specified later, $\pi^w(x, w(x))$ is increasing in x and because of the first order stochastic dominance relation satisfied by $F(x'|x)$ (see theorem 9.11, Stokey and Lucas, 1989).

In turn the value for the worker $W(x)$ satisfies the following dynamic programming equation:

$$rW(x) = \pi^w(x, w(x)) + \lambda\left[\int_{x'>x^*} W(x')dF(x'|x) + F(x^*|x)(U + B) - W(x)\right] + \delta(U - W(x)). \tag{2}$$

The interpretation of this equation is analogous to the one given for the firm's value. Notice that the value of an endogenous termination for the worker is $U + B$, consisting of the option value of unemployment plus the displacement transfers B. These transfers are assumed to be zero for an exogenous displacement.

The expected initial value for a worker of a match with signal y, $W_s(y)$ and the asset value of search for an unemployed worker $U(e)$ for a given level of search effort e are given by:

$$W_s(y) = \int_{x>x^*} W(x)dG_I(x|y) + G_I(x^*|y)(U + B) \tag{3}$$

$$rU(e) = b - e + m(\theta, e)\left[\int_{y>y^*} W_s(y)dG_s(y) + G_s(y^*)U - U(e)\right] \tag{4}$$

Then we define $U = \max_e U(e)$ and $e^* = \arg\max U(e)$.

The expected initial value for an employer of a match with signal y, $V_s(y)$ and the value V_e of posting a vacancy for a firm are given by

$$V_s(y) = \int_{x>x^*} V(x)dG_I(x|y) - G_I(x^*|y)T + V_e G_I(x^*|y) \tag{5}$$

$$rV_e = -c + \frac{m(\theta, e^*)}{\theta}\left[\left[\int_{y>y^*} V_s(y)dG_s(y)\right] - V_e(1 - G_s(y^*))\right] \tag{6}$$

These definition assume that the point at which a firm has to incur the layoff costs if it wants to fire a worker occurs after the signal y is revealed.

2.1 Bargaining process

The match-specific nature of productivity creates at each instant a bargaining situation between entrepreneur and worker. The approach followed is that the wage rate must be such that total surplus will be divided in proportions that depend on the (exogenously given) bargaining power of the partners. This results from Nash's (1953) bargaining solution which maximizes the product of the utilities minus the values of termination of the relationship for the players. The proportion that goes to the worker is denoted β.

We assume that all separations are interpreted as firings, so that the disagreement value of the worker is $B + U$ and the disagreement value of the firm is $-T$. This gives more bargaining power to the workers and a smaller expected value for firms. This leads to less job creation which is the channel through which layoff costs will lead to higher unemployment. Burda (1992) makes the opposite assumption, all separations that are not exogenous are interpreted as quits, and layoff costs (which in his model are only obtained after an exogenous separation) have no effects on unemployment since the layoff cost is considered by the bargaining pair as part of the workers' value and the wage is reduced accordingly. In our model the assumption that separations are interpreted as firings is more sensible that in Burda's model because in our model, unlike in his, we have idiosyncratic match productivity, which is not verifiable. A worker that wants to quit and obtain the layoff compensation can lower the productivity level of the match below x^* and make the match undesirable enough that even paying the layoff compensation is justified.

The Nash bargaining solution prescribes that the wage proposed in a given date and for a given productivity maximizes

$$\left(W\left(x\right)-\left(B+U\right)\right)^{\beta}\left(V\left(x\right)+T\right)^{1-\beta} \tag{7}$$

This implies that

$$(1-\beta)\pi_2^f\left(x, w\left(x\right)\right)\left(W(x)-(B+U)\right)=\beta\pi_2^w\left(x, w\left(x\right)\right)\left(V(x)+T\right) \tag{8}$$

Equations (1) and (2) imply

$$(r+\lambda+\delta)(V(x)+T)=\pi^f\left(x, w\left(x\right)\right)+(r+\delta)T+\lambda\left[\int_{x'>x^*}(V(x')+T)dF(x'|x)\right] \tag{9}$$

$$\begin{aligned}(r+\lambda+\delta)(W(x)-(B+U)) &= \pi_2^w(x, w(x))-(r+\delta)(B+U)+\delta U \\ &\quad +\lambda\left[\int_{x'>x^*}(W(x')-(B+U))dF(x'|x)\right]\end{aligned}$$

If we premultiply equation (9) by $\beta\pi_2^w(x, w(x))$ premultiply (10) by $(1-\beta)\pi_2^f(x, w(x))$and then subtract one from the other we obtain:

$$
\begin{aligned}
J(x, w(x)) = \beta\pi_2^w(x, w(x))\left(\pi^f(x, w(x)) + (r+\delta)T\right) \\
+(1-\beta)\pi_2^f(x, w(x))\left(\pi^w(x, w(x)) - (r+\delta)(U+B) + \delta U\right) = 0
\end{aligned}
\tag{10}
$$

In the particular case where $\pi^f(x, w(x)) = x - w(x)$ and $\pi^w(x, w(x)) = w(x)$ we have that $J(x, w(x)) = 0$ implies

$$w(x) = \beta(x + (r+\delta)T) + (1-\beta)(rU + (r+\delta)B)$$

.

$J(x, w(x)) = 0$ defines implicitly the wage schedule. We will make certain assumptions on J to guarantee that the wage schedule satisfies some desirable properties.

W1 For any fixed values U, B, T there is some w_m with $J(x, w_m) > 0, J(x, \infty) < 0, J_2(x, w) < 0$, for all x, w.

W2 $-J_1(x, w)/J_2(x, w) > 0$ for all x, w.

W3 $\pi_1^f(x, w(x)) - \pi_2^f(x, w(x))\, J_1(x, w)/J_2(x, w) > 0$ for all x, w.

If W1 is satisfied 10 defines uniquely the wage for each x. If W2 is satisfied then the wage rate is increasing in x, which is necessary for the definition of $W(x)$. If W3 is satisfied then the profit flow of the firm is increasing in x, which is necessary for the definition of $V(x)$. It is easy to check that W1, W2 and W3 are satisfied for the specifications of $\pi^f(x, w(x))$ and $\pi^w(x, w(x))$ used by MP and Millard and Mortensen (1995).

3 Stationary Equilibrium

The assumption of Nash bargaining implies that termination of the match is efficient for the firm/worker match. Matches are terminated when the state is such that total surplus is zero, The separation state x^* must then satisfy:

$$V(x^*) + T + W(x^*) - (U+B) = 0 \tag{11}$$

or given that (8) has to be satisfied

$$V(x^*) + T = 0. \tag{12}$$

If $V(x) < -T$ for all x then $x^* = \overline{x}$, and all matches are immediately destroyed. If $V(x) > -T$ for all x then $x^* = \underline{x}$, and matches are only destroyed at the exogenous rate δ.

Notice that since V and W are strictly increasing functions, x^* will be uniquely determined, for a given θ.

The signal y^* was defined as the minimum signal that makes both agents interested in entering a contract. To make the initiation of the relationship individually rational y^* has to satisfy

$$\min\{W_s(y^*) - U, V_s(y^*)\} = 0. \tag{13}$$

If $\min\{W_s(y^*) - U, V_s(y^*)\} < 0$ for all y then $y^* = \overline{y}$, and no matches are created. If $\min\{W_s(y^*) - U, V_s(y^*)\} > 0$ for all y then $y^* = \underline{y}$, and for all initial productivities the match is transformed in a contrast. The reason why we need to express the condition as the minimum of two values is that, unlike in the definition of x^* we do not have a guarantee that both values are equal or, in other words, we do no have a guarantee that initiation of the match is efficient. This is so because there is no way to arrange transfers between the agents without writing a contract, but that is precisely what they are deciding.

Another assumption which we will use in the determination of an equilibrium is that there is free entry (except for the vacancy posting costs), new vacancies will be posted until $V_e = 0$ or

$$V_s(y) = \int_{x>x^*} V(x)dG_I(x|y) - G_I(x^*|y)T \tag{14}$$

$$0 = -c + \frac{m(\theta, e^*)}{\theta}\left[\left[\int_{y>y^*} V_s(y)dG_s(y)\right] - G_s(y^*)T\right] \tag{15}$$

If $V_e < 0$ for all values of x^* and θ then no vacancies are created and $\theta = 0$. Since productivity is bounded we will see that $V(\overline{x})$ is bounded, so it cannot be the case that V_e is always positive.

Since $m(\theta, e^*)$ is an increasing function of θ, given values $V(x)$ and the exit rule, this definition uniquely determines θ.

In a stationary equilibrium the rate of match separation must equal the rate of match creation so that the total stock of matches remains constant. Exit can thus be thought of as a renewal process which switches positions that become unproductive or that are exogenously destroyed into new ones, sampled from the distribution G_I, conditional on the set $\{x > x^*\}$ and on the set of signals $y > y^*$ sampled from the distribution G_s. For any fixed separation rule x^*, and assuming the expectation of the stopping time associated to x^* is finite[5], this process has a unique invariant distribution over productivity

[5]This condition is discussed in detail in Hopenhayn (1992)

states $\mu(x^*)$. Letting $\tau(x^*)$ denote the expected duration of the match associated to this separation rule and u the rate of unemployment, the hazard rate for match termination at the invariant distribution is $1/\tau(x^*)$ and thus the rate of escape of employment can be shown to be equal to $(1-u)/\tau(x^*)$.[6] In turn, the rate of match creation is given by $\eta(\theta)u$. The requirement that creation flows balance out with match destruction flows implies that

$$(1-u)/\tau(x^*) = \eta(\theta)u \tag{16}$$

A *stationary equilibrium* is given by the functions $V(x)$, $W(x)$ and $w(x)$, together with scalars U, x^*, y^*, e^*, θ and u that satisfy equations (1), (2), (3), (4), (5), (6), (10), (16), and the definitions of x^*, y^*, U, e^*, and θ.

Proposition 1 Assume that $F(x'|x)$ has the Feller property, $m(\theta, e)$ is a continuous function, which is bounded in e for all θ and is such that for all e $\lim_{\theta\to\infty} \frac{m(\theta,e)}{\theta} = 0$, then there is a stationary equilibrium.[7]

Proof: See the Appendix.

3.1 Wage and employment dynamics

In this section we provide conditions under which average wages increase and hazard rates for job destruction decrease as a function of job tenure. Remember that by assumption W2 wages are strictly increasing in the state x. This means that if the distribution of matches that survive t periods stochastically dominates the distribution of matches that survives $s < t$ periods, the average wage of workers with t years of tenure will be higher than the average wage of workers with $s < t$ years of tenure.

The following assumptions provide sufficient conditions for the distribution of matches of type x that have survived t years, $\mu_t(x)$ to dominate stochastically $\mu_s(x)$, when $s < t$.

A.1 $F(x|x')$ is decreasing in x' and jointly continuous.

A.2 G_I is continuous.

A.3 $\int F(x'|x)dG_I(x|y) \leq G_I(x')$ for all $x', y \in X$.

A.4 i) $\forall y \in X$ and $x' \geq z$, i) $[F(x'|x) - F(z|x)]/[1 - F(z|x)]$ is decreasing in x;
ii) $\int_{x'\geq z} F(x'|x)dG_I(x|y) \leq G_I(x'|y)[1 - F(z|x)]/[1 - G_I(z|y)]$.

[6] See Hopenhayn (1992) for details.

[7] Notice that a special case of the proposition holds when the matching function m_a is independent of v, and the parameters $c = 0$, $\beta = 1$ and $T = 0$, which corresponds to a pure search model (by the worker), connected for example to the work of Lucas and Prescott (1974) and Diamond (1982).

Assumption A.1 says that higher values of x' leads to a conditional distribution that puts more weight on higher values of x. Assumption A.3 says that the initial realization of the shock is on average worse than the ones that come later. This assumption contrasts with the one in MP, who assume that the initial shock comes at the top of the distribution of subsequent shocks. Assumption A.4 i) is the analog of assumption A.1 for a distribution of shocks, conditional on an exit rule that eliminates shocks with values $z' < z$. Assumption A.4 ii) is the analog of A3 for a distribution of shocks, conditional on an exit rule that eliminates shocks with values $z' < z$.

Lemma 1 in Hopenhayn (1992) shows that under assumptions A.1 to A.4 the distribution of matches that survive from time s to time t conditional on any number of shocks j_{st}, $\mu_t(x|j_{st})$ stochastically dominates $\mu_s(x)$ for all j_{st}. Therefore $\mu_t(x)$ stochastically dominates $\mu_s(x)$. [8]

This stochastic dominance result implies that wages increase and hazard rates for match destruction decrease with the length of job tenure.

Other properties of the equilibrium that are of interest concern the average values of firms and workers. Those are useful because when the model is calibrated one would like to match some moments of observables, and since the values of firms that are typically accessible are the averages (not the expectations) those are the ones whose match will be attempted, and we would like to know their relationship with other equilibrium values.

Let α_t be the probability that a match has at least a duration of t. The average value of a firm of age t is $\overline{V}_t = \int_{x'>x^*} V(x')d\mu_t(x')$. The average value of firms in the industry is thus $\overline{V} = \int_0^\infty \alpha_t \overline{V}_t / \int_0^\infty \alpha_t$. If $\mu_t(x)$ dominates stochastically $G_I(x|y)$ we have that $\overline{V} > \int_{x'>x^*} V(x')dG_I(x|y)$.

4 Quantitative Analysis

In this section we calibrate the model presented in the previous sections to US data in order to assess the effects of changes in the policy parameters on the equilibrium values for employment, turnover and wages. Our approach will be to try to follow the parametrization of Millard and Mortensen (1995) as close as possible, so that our results can be compared to theirs. This will provide a measure of the sensitivity of the model to the specific assumptions about the stochastic process for job productivity.

The time unit in the model is a quarter and all flows will be reported in per quarter terms. The rate of interest is $r = 0.01$. The exogenous rate of attrition (quit to unemployment), $\delta = 0.015$. The matching function $m(\theta) = \theta^\eta$, where $\eta = 0.6$. We normalize all values of

[8] As shown in Hopenhayn (1992) conditions A.1 to A.4 are satisfied if $F(x'|x)$ follows an AR1 process $x' = \rho x + \epsilon$, where $0 < \rho < 1$ and ϵ is normally distributed with mean $\bar{\epsilon}$ and variance $\sigma_\epsilon^2 > 0$, and G is a normal distribution with mean $x_0 < \bar{\epsilon}/(1-\rho)$ and variance $\sigma_\epsilon^2/(1-\rho^2)$.

productivity to a numeraire which will be the long run value of output. The training cost and recruiting cost figures must be interpreted in terms of this numeraire. The vacancy cost $c = 0.33$ and the training cost $k = 0.275$. The bargaining power of the worker is $\beta = 0.3$. The specific values chose are justified in Millard and Mortensen (1995).

We now consider the policy parameters. The social security taxes are $t_f = 0.075$ and $t_w = 0.075$. The number of quarters during which unemployment insurance is received is 2. The average replacement ratio of unemployment insurance is 25%, the product of the mandated replacement ratio, which is 50% and the percentage of people who qualify for the benefits, around 50%. Severance pay in the US is zero, but there is a cost of layoff since employers pay an average of about 60 cents for each additional dollar that former employees receive in the form unemployment insurance.

The values of the remaining parameters are chosen to obtain an adequate fit with values of unemployment duration, incidence and the existing estimates of the effects of tenure on wage profiles and hazard rates of termination of the employment.

In our calibration $\lambda = 0.15$. This implies the arrival of shocks in our model is more frequent than in Millard and Mortensen (1995) who use $\lambda = 0.1$. Notice that in our model, unlike in theirs, there are two measures of persistence of shocks, namely, λ the parameter that controls their rate of arrival and ρ the parameter that weights the last value of the shock in the autoregressive process that controls the value of the shock once it arrives. We also make the value of leisure, $b = 0.37$.

The stochastic process assumed for the shocks is an AR(1) with normal innovations. The persistence of the shocks we impose is $\rho = 0.80$. The mean of the innovation is 0.025. The variance of the innovation is 1.2. As for the initial value of productivity we assume a normal distribution with mean 0.11 and variance 0.05.

The benchmark values obtained for our simulations are shown in table A.1. A comparison with US data shows that the fit for the incidence of unemployment and duration are adequate. The qualitative picture of dynamics is as we predicted. A comparison of the evolution of wages with tenure and the rates of growth obtained from Topel (1991), reported on Table A.2 shows a remarkable approximation. The variance of wages grows with tenure (after the second year) which also squares with available evidence. The qualitative features of the hazard rates of separations, decreasing hazards and decreasing rate of change, are also similar to available estimates (see McLaughlin 1991). Inspection of the invariant distribution (Table A.1.a) shows that the distribution of wages is quite skewed.

Two policy experiments were performed and the results are reported in tables A.3. and A.4. Table A.3. reports the calibration for the economy where a severance payment of 0.28, around a month of mean wages. Consistently with the findings of Millard and Mortensen (1995) the effect on unemployment is rather sizable, but in our case the increase in unemployment is dramatic, almost 6 percentage points compared with about

2.2 percentage points in their case. The difference can be explained because in our case the increase in unemployment comes largely through an increase in its duration, since the effect on the rate of separations is negligible. In Mortensen and Millard's case there was also a reduction in the rate of turnover that mitigated the effect of a lower rate of employment creation. Since salary dispersion is relatively low for Mortensen and Millard's calibration, there is a high density at every point of the distribution of qualities. When the cutoff point for the destruction of employment falls, lots of workers that otherwise would have been laid off are kept employed. In our case however, the fall affects substantially fewer workers because the distribution is far more disperse around the cutoff point. This is an example of a seemingly harmless assumption with drastic implications in the policy analysis.

Besides the effect on unemployment, the layoff compensation raises wages as a reflection of the increased bargaining power of workers. The value of an employed worker, $W(x)$, shows that all employed workers benefit from the severance pay. The effect is negative for the firms, whose value decrease and for the unemployed.

The second policy experiment (Table A.4) performed is given by a suppression of the payroll tax. Our results show a relatively low decrease in the unemployment rate as the tax is eliminated. The turnover rate changes by a very small amount. In this case the productivity increases since less of it leaks to the government, and this induces large increases in value of around 15%. The qualitative dynamic figures are almost the same.

One conclusion from these experiments is that this type of models is rather sensitive to modeling details. The change in the value of leisure, and the specification of a different stochastic process for the shocks are not merely an instrument to match a few extra observables, like hazard rates of employment and wage profiles; they have implications on the effects of policies and therefore they should be considered carefully when specifying the model.

5 Conclusions

We have shown in this paper that it is easy to extend the model of Mortensen and Pissarides (1994) to account for several empirical observations in labor markets. The model is useful because it is manageable and it provides insights into the workings of the labor markets and the effects of policies, but the conclusions seem to be sensitive to the precise specification chosen. For this reason further research into these issues would be necessary to know the extent to which the conclusions here would need to be modified.

A point that should be mentioned is that since the quality of workers is only match dependent we cannot explain decreasing hazard rates for unemployment. It is probable that the reason why the long term unemployed are less likely to find employment is a sorting argument similar to the one we use to explain the hazard rates for employment.

People with longer unemployment tend to be less skilled. We would need to have a quality variable for people that were somewhat independent of the match they are in.

It is interesting to note that turnover is almost unaffected by many changes in policy parameters. This is due to the fact that with Nash bargaining termination of the match is efficient and creation of matches is more affected than destruction. There is some separation between production and match technology. This feature would disappear if there were a resource that were used by both technologies, say in a general equilibrium model where capital could be used to improve the matching technology so that there were certain substitutability between production and matching.

Another extension to the model would contemplate the state of the match as a result of learning about its intrinsic quality, not of shocks to productivity. That leads to a stochastic process for the state of the match that is a martingale.

Finally, it would be interesting to know what are the effects of policy in a context with risk averse agents. This is important because for some types of policy, like the unemployment insurance or the layoff compensation the insurance motive appears to be very important, and an adequate assessment of the impact of this policy should consider agents who are risk averse.

6 Appendix

Proof of Proposition 1:

The first step in showing that a stationary equilibrium exists is to show that the value functions are continuous and well defined.

With assumptions (W1), (W2) and (W3) we obtain $w(x)$ as a continuous function of θ, x^*, and U if $m(\theta)$ is continuous.

Lemma 1 Assume that $F(x'|x)$ has the Feller property. Then the $V(x)$ and $W(x)$ are well defined, continuous functions of x.

Proof:

We can write the operator that defines $V(x)$ as

$$(TV)(x) = \int_{x'>x^*} H(x, x', V(x'))dF(x'|x), \quad \forall x \in X \tag{17}$$

where $H(x, x', V(x')) = \frac{\lambda}{\lambda+r+\delta}(\pi^f(x, w(x))/(1-F(x^*|x))+V(x')+T)$. $H : X \times X \times \Re \to \Re$ is a continuous function, which is also continuously differentiable in V and $H_3(x, x', y) = \frac{\lambda}{\lambda+r+\delta} < 1$. Since F has the Feller property, all the assumptions for Lemma 17.1 in Stokey, Lucas (1989) are satisfied and so the operator we use to define V has a unique fixed point in the space of continuous functions. Thus V is well defined and continuous. The proof for W is analogous.□

Notice also that $V(x)$ is a continuous function of x^* and U (it depends on U through the salary $w(x)$) and $W(x)$ is a continuous function of x^* and U. Since $V(x)$ and $W(x)$ are continuous so are $V_s(y)$ and $W_s(y)$.

By equation (4) we can write

$$U(e) = \frac{1}{r + m(\theta, e)}\left[b - e + m(\theta, e)\left[\int_{y>y^*} W_s(y)dG_s(y) + G_s(y^*)U\right]\right] \tag{18}$$

$U(e)$ is a bounded, continuous function of e and therefore it has a maximum. It is also continuous in θ, x^*, y^*, and U by the continuity of $W_s(y)$ in those variables. These continuity is inherited by the maximum. Let $D(x^*, y^*, \theta, U) = \max_e U(e)$.

Lemma 2 Assume that $F(x'|x)$ has the Feller property, $m(\theta, e)$ is a continuous function such that $\lim_{\theta\to\infty} \frac{m(\theta,e)}{\theta} = 0$, and $X = [\underline{x}, \overline{x}]$, there are values $\theta \in [0, \infty)$, $y^* \in Y$, $x^* \in X$ and U that satisfy jointly their definitions.

Proof:

Let first $\theta \in [0, \overline{\theta}]$, with $\overline{\theta}$ such that for all e

$$\frac{m(\overline{\theta}, e)}{\overline{\theta}} \frac{\pi^f(\bar{x}, 0)}{r+\delta} < c. \tag{19}$$

The $\overline{\theta}$ exists because $\pi^f(\bar{x}, 0)$ is finite and because of the assumption on $m(\theta, e)/\theta$.

We have shown that W and V are well defined continuous functions. Let the function $A(x^*, \theta, U) = x^* - \min\{\max\{V(x^*) + T, x^* - \overline{x}\}, x^* - \underline{x}\}$, $B(x^*, y^*, \theta, U) = \theta + \min\{\max\{V_e - \theta, 0\}, \overline{\theta} - \theta\}$, $C(x^*, y^*, \theta, U) = y^* - \min\{\max\{V_s(y^*), y^* - \overline{x}\}, y^* - \underline{x}\}$, and $D(x^*, y^*, \theta, U)$ as defined above. A fixed point of these mappings exists by Brouwer's fixed point theorem.□

The only thing that remains to show the existence of a stationary equilibrium is that $\tau(x^*)$ is finite . Since there is a rate δ of exogenous termination at every point in time $\tau(x^*) < \frac{1}{\delta}$.□

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TABLE A.0: Parameters for benchmark case

INSTITUTIONAL PARAMETERS

Payroll tax paid by firm	=	0.075
Payroll tax paid by worker	=	0.075
Quarters of insurance received	=	2.00
Average replacement ratio	=	0.250
Severance payment	=	0.000
Experience rating	=	0.600

PARAMETERS FROM MORTENSEN-MILLARD

β	=	0.300
λ	=	0.250
δ	=	0.015
r	=	0.010
c	=	0.330
k	=	0.275

PARAMETERS OF OUR CALIBRATION

Utility from leisure	=	0.000
Variance of innovation	=	0.650
Persistence	=	0.950
Mean of innovation	=	0.001
Variance of initial distribution	=	0.250
Mean of initial distribution	=	0.004

TABLE A.1: Benchmark

RESULTS

Turnover rate	=	0.059705
Mean wage	=	0.84324
Mean duration	=	0.99224
Rate of unemployment	=	0.055929
Mean shock	=	1.0955
Mean value	=	86.996
Mean output (net of taxes)	=	0.96905

Year	Hazard rate	Mean wage	Wage growth	Coef of variation wage
1.0000	0.33643	0.59901	0.11748	0.31731
2.0000	0.28054	0.66939	0.10009	0.25690
3.0000	0.23567	0.73639	0.080430	0.24494
4.0000	0.20242	0.79562	0.063279	0.25951
5.0000	0.17878	0.84596	0.049812	0.28154
6.0000	0.16216	0.88810	0.039597	0.30302
7.0000	0.15040	0.92327	0.031852	0.32174
8.0000	0.14194	0.95268	0.025896	0.33748
9.0000	0.13572	0.97735	0.021234	0.35057
10.000	0.13106	0.99810	0.017524	0.36143

TABLE A.1.a: Stationary distribution of shocks, values and wages

Shock	Density	Value of firm	Walue of worker	Wages	Output
0.012267	1.1075	70.762	70.802	0.54093	-0.068872
0.031151	1.1079	70.934	70.848	0.54620	-0.050779
0.064482	1.0972	71.245	70.932	0.55550	-0.018844
0.11320	1.0574	71.714	71.059	0.56910	0.027834
0.17527	0.97352	72.339	71.227	0.58642	0.087302
0.25269	0.83893	73.161	71.448	0.60803	0.16149
0.34196	0.68048	74.167	71.719	0.63294	0.24702
0.44626	0.53301	75.421	72.057	0.66204	0.34695
0.56063	0.43493	76.892	72.453	0.69396	0.45654
0.68917	0.38335	78.661	72.930	0.72984	0.57970
0.82608	0.35841	80.672	73.472	0.76804	0.71087
0.97558	0.33815	83.007	74.101	0.80976	0.85411
1.1319	0.31494	85.589	74.797	0.85340	1.0039
1.2986	0.28763	88.481	75.576	0.89990	1.1636
1.4708	0.25858	91.602	76.417	0.94798	1.3286
1.6505	0.22912	94.981	77.327	0.99811	1.5008
1.8346	0.20082	98.555	78.290	1.0495	1.6772
2.0229	0.17429	102.31	79.301	1.1020	1.8576
2.2144	0.14986	106.21	80.352	1.1555	2.0411
2.4070	0.12780	110.20	81.427	1.2092	2.2256
2.6009	0.10800	114.27	82.525	1.2634	2.4114
2.7935	0.090605	118.35	83.624	1.3171	2.5960
2.9850	0.075400	122.42	84.721	1.3705	2.7794
3.1733	0.062346	126.43	85.799	1.4231	2.9598
3.3574	0.051257	130.32	86.848	1.4745	3.1363
3.5371	0.041898	134.07	87.859	1.5246	3.3084
3.7093	0.034152	137.61	88.812	1.5727	3.4734
3.8760	0.027694	140.95	89.711	1.6192	3.6331
4.0323	0.022471	143.99	90.530	1.6628	3.7829
4.1818	0.018165	146.78	91.284	1.7045	3.9261
4.3187	0.014760	149.24	91.945	1.7427	4.0573
4.4473	0.011992	151.44	92.539	1.7786	4.1805
4.5616	0.0098543	153.31	93.042	1.8105	4.2901
4.6659	0.0081523	154.93	93.479	1.8396	4.3900
4.7552	0.0068711	156.26	93.836	1.8645	4.4755
4.8326	0.0058830	157.36	94.132	1.8862	4.5497
4.8947	0.0051690	158.20	94.361	1.9035	4.6092
4.9434	0.0046549	158.85	94.534	1.9171	4.6559
4.9767	0.0043257	159.28	94.650	1.9264	4.6878
4.9956	0.0041470	159.52	94.715	1.9316	4.7059

TABLE A.2: Growth rate of wages with tenure

Topel	This paper
0.117	0.117
0.102	0.100
0.088	0.080
0.075	0.063
0.065	0.050
0.055	0.040
0.048	0.032
0.041	0.026
0.036	0.018

TABLE A.3: First policy experiment

CHANGES IN INSTITUTIONAL PARAMETERS

Severance payment	=	0.270

RESULTS

Turnover rate	=	0.059639
Mean wage	=	0.84828
Mean duration	=	1.4266
Rate of unemployment	=	0.078408
Mean shock	=	1.0942
Mean value	=	86.728
Mean output (net of taxes)	=	0.96701

Year	Hazard rate	Mean wage	Wage growth	Coef of variation wage
1.0000	0.33613	0.60399	0.11649	0.31492
2.0000	0.28031	0.67435	0.099315	0.25518
3.0000	0.23549	0.74132	0.079861	0.24341
4.0000	0.20229	0.80052	0.062871	0.25793
5.0000	0.17868	0.85085	0.049517	0.27989
6.0000	0.16208	0.89299	0.039379	0.30130
7.0000	0.15033	0.92815	0.031688	0.31997
8.0000	0.14187	0.95756	0.025770	0.33567
9.0000	0.13566	0.98224	0.021136	0.34874
10.000	0.13101	1.0030	0.017447	0.35958

TABLE A.4: Second policy experiment

CHANGES IN PARAMETERS

Wage tax paid by firm	=	0.0000
Wage tax paid by worker	=	0.0000

RESULTS

Turnover rate	=	0.059668
Mean wage	=	0.90560
Mean duration	=	0.98157
Rate of unemployment	=	0.055328
Mean shock	=	1.0948
Mean value	=	99.111
Mean output (net of taxes)	=	1.0948

Year	Hazard rate	Mean wage	Wage growth	Coef of variation wage
1.0000	0.33626	0.64302	0.11764	0.31790
2.0000	0.28041	0.71866	0.10020	0.25733
3.0000	0.23557	0.79067	0.080508	0.24529
4.0000	0.20234	0.85432	0.063339	0.25981
5.0000	0.17872	0.90844	0.049860	0.28182
6.0000	0.16212	0.95373	0.039637	0.30330
7.0000	0.15036	0.99153	0.031885	0.32201
8.0000	0.14190	1.0232	0.025924	0.33775
9.0000	0.13569	1.0497	0.021258	0.35084
10.000	0.13103	1.0720	0.017545	0.36171