ECONOMICS WORKING PAPERS SERIES

Economics Working Paper 25

Indeterminacy of Equilibria in a Hyperinflationary World: Experimental Evidence

Ramon Marimon*
and
Shyam Sunder†

July 1990
Revised: August 1992

UNIVERSITAT POMPEU FABRA

Balmes, 132
Telephone (343) 484 97 00
Fax (343) 484 97 02
08008 Barcelona
e- mail econwp@upf.es

ECONOMICS WORKING PAPERS SERIES

Economics Working Paper 25

Indeterminacy of Equilibria in a Hyperinflationary World: Experimental Evidence

Ramon Marimon*
and
Shyam Sunder†

July 1990
Revised: August 1992

UNIVERSITAT POMPEU FABRA

Balmes, 132
Telephone (343) 484 97 00
Fax (343) 484 97 02
08008 Barcelona
e- mail econwp@upf.es

Abstract

We design and study an OLG experimental economy where the government finances a fixed real deficit through seigniorage. In this economy there is a continuum of non- stationary rational expectations equilibria and two stationary rational expectations equilibria. In our experiments, we do not observe non-stationary rational expectations paths. We observe paths that tend to converge close to, or somewhat below, the Low Inflation Stationary State (Low ISS). We compare our results with those produced by optimizing agents who use adaptive learning algorithms. The adaptive learning hypothesis is consistent with our data in selecting the Low ISS rational expectations equilibrium as a long-run stationary equilibrium. Nevertheless, simple adaptive learning models do not capture the market uncertainty or the biases observed in the data.

1 Introduction

Equilibrium models help us understand economic environments and provide a framework for the design of economic policies. In this sense, economic models are specially useful when they yield unique policy prescriptions for specified policy goals. Unfortunately, many competitive models have multiple equilibria, and different equilibria often prescribe different policies for the same goal. Uncertainty about which of the multiple equilibria -if any- the economy may be at, forces theorists to be cautious in their policy recommendations. The pressing need for decisions may induce economic policy makers to discard such caution and to act as if economic theory yielded a unique policy prescription.

Price stabilization policies in economies that experience high government deficits and inflation often present such a problem. Deficit reduction is the "classical" policy recommendation to stabilize prices in such economies. In contrast, the rational expectations version of Cagan's (1956) model of hyperinflation shows that the set of equilibria in such economies may be very large (*i.e.,* a continuum); a Laffer-type curve causes two stationary equilibrium inflation rates to exist. An increase in deficit causes the lower stationary inflation rate to rise while the higher stationary inflation rate falls. And while the low-inflation stationary state equilibrium is consistent with the "classical" policy prescription the high-inflation stationary equilibrium is not (see, for example, Sargent and Wallace (1987)). The "classical" policy recommendation would be defensible if we knew that the low inflation stationary state is most frequently observed of all the theoretically possible equilibria,; or, even better, if we knew that it is most frequently observed because it has some robustness property that the other equilibria do not share. Unfortu-

nately, the empirical evidence is not available, but, in spite of this, many policy-makers adhere to the "classical" prescription (see, for example, Bruno (1989)).

Determining which equilibria are more often observed in historical economies is a difficult endeavour. Ex post realizations of aggregate economic series can be consistent with many different equilibrium paths depending on the underlying parameters of the economy. These parameters are only known with uncertainty. Furthermore, if we allow for the possibility that the data may deviate from equilibrium paths, we might not even be able to distinguish between equilibrium and non equilibrium paths. On the other hand, the results of deliberate policy experiments –such as reducing the deficit in order to lower inflation– can be more conclusive, since different equilibria might have different comparative dynamics. Unfortunately, the political and economic consequences of such experiments, and their high social cost, render them infeasible.

A possible -and less costly- way to generate the relevant data is to study the empirical relevance of multiplicity problems in an economic laboratory. The experimenter is free to select the parameters of the economy, and can therefore identify its equilibria. This knowledge enables the experimenter to determine which, if any, of the equilibrium (or non-equilibrium) paths is observed.

In this paper, we study an experimental version of Cagan's (1965) model of hyperinflation. More specifically, we use the overlapping generations structure described in Sargent and Wallace's (1987). This economy has one non-storable good, fiat money as the only financial asset, and a government that finances a fixed real deficit collecting seigniorage. The structure of the economy and the level of deficit are assumed to be common knowledge, and agents

are assumed to observe current and past prices. Leaving aside the possibility of sunspot equilibria, this economy has two stationary equilibria and a continuum of non-stationary rational expectations equilibria. The low inflation stationary state (Low ISS) is also called the "classical" stationary equilibrium since it matches the conventional wisdom that stationary inflation decreases when the level of monetized deficit is reduced.

It is important to notice that even if there is a continuum of first-period prices, for which the rational expectations equilibrium paths have a long-run High ISS, this does not mean that the economy is more likely to be in a High ISS in the long–run. Equilibrium theory has no implications about the distribution of equilibria. It only states that any particular equilibrium, such as the Low ISS, requires a high degree of coordination between the agents' beliefs about the future evolution of prices and of other relevant variables. Rational expectations equilibrium theory is also silent about the behavior of the agents when they observe non-equilibrium price sequences.

The existence of multiple equilibria means that there are many –possibly an uncountable many– agents' beliefs that can be mutually consistent. To reduce this set of mutually consistent beliefs, specific assumptions about the ways in which agents form and coordinate their beliefs must be imposed. More precisely, when maximizing agents use specific forecasting rules, their behavior is well defined for any observed path and the set of "stable" equilibria can become fairly small, or even unique point, making the long-run behavior of the economy determinable.

This brings the issue of selecting among different equilibria into the broader perspective of the issue of how agents learn to form expectations and learn to coordinate them. There is an extensive literature in which maximizing agents explicitely form their beliefs and coordinate them by learning

from past experience (see, for example, Bray (1982, 1983), Blume and Easley (1982), Marcet and Sargent (1989a, 1989b)) Some authors have postulated the "stability of learning rules" as a selection procedure in models with multiple equilibria (see, for example, Evans (1983, 1988), Fudenberg and Kreps (1988) and Marimon and McGrattan (1992)). However, little is known about the ways in which people learn in competitive environments. Lucas (1987), for instance, suggested that experimental data can identify whether agents behave according to the rational expectations hypothesis or according to some form of adaptive learning –such as the least-squares learning postulated by Bray (1982) and by Marcet and Sargent (1989). With such learning rules, the rational expectations hypothesis is usually satisfied in the long-run. In addition, it is possible to use stability theory to show that, for a large class of adaptive learning rules (which includes least-squares learning, and the adaptive rules postulated by Friedman (1957) and Cagan (1956)) and for a large set of initial inflation rates, the rate of inflation converges to the Low ISS (see, Marcet and Sargent (1989b)). One can also use stability theory to show that, for certain learning rules, even the Low ISS may be locally unstable (see, Grandmont and Laroque (1989)). Our experimental data suggest that this type of stability analysis might be the missing piece in equilibrium models with indeterminacy problems. Using these data we can determine whether or not inflation paths tend to cluster in a neighborhood of the Low ISS, and we can also analyze the agents' forecasting rules to find out how the agents' behavior enforces the stability properties of the Low ISS.

This same problem of how to make predictions in models with multiple equilibria also arises in game theoretical models. Simultaneous with our experimental work with overlapping generations economies there has been some interesting experimental work on subject behavior in coordina-

tion games, that has been directed to the study of equilibrium outcomes in repeated normal form games when the one–stage game has multiple Pareto–ranked equilibria (see, for example, Van Huyck, Battalio and Beil (1990) and Cooper, DeJong,Forsythe and Ross (1990)). These authors suggest that, while all equilibrium outcomes are not equally likely to ocurr in their experimental setting, Pareto dominance is not necessarily the best criterion for equilibrium selection. Ex-post it has been shown that their experimental results can be explained better by studying the stability properties of adaptive evolutionary learning rules (see, Crawford (1991), Marimon and McGrattan (1992); Kandori, Mailath and Rob (1991) and Young (1991)).

Our results with OLG economies resemble the game-experimental findings in highlighting the usefulness of adaptive learning as a selection criterion when there are multiple equilibria. Both sets of experiments strengthen the view that economic agents are more prompt to follow inductive reasoning based on common observed data rather than deductive reasoning based on future expectations. In spite of these similarities, an important difference should also be noted. Coordination-game experiments study the "selection" of equilibria in one-stage games. The experiments are dynamic, but there is no explicit dynamic theory to be tested. Either there are strategic interactions among a fixed finite set of players, in which case the set of equilibria of the repeated games is too large (the so called *Folk Theorem*), or individual agents' actions have no effect -or it is not explicitely recognized- on the future evolution of the game, in which case the main candidates for equilibrium processes are the repeated play of equilibria of the one-stage game. In contrast, in this paper, we study a simple dynamic model in which rational expectations dynamics and adaptive learning dynamics can be unambiguously set apart and tested.

To study an infinite-horizon dynamic model in an experimental environment we have had to develop a new experimental methodology (partially introduced by Lim, Prescott and Sunder (1986)). These innovations require certain departures from the theoretical model under study and the possible impact of such departures must be taken into account. We discuss our experimental design in Section 3, but there are three experimental innovations that deserve to be mentioned here. First, we have a fixed number of subjects in any given experimental session. A subset that is randomly selected enters the market in each period and remains in the market for two consecutive periods. In other words, our subjects live several "lives" over the many periods of a particular economy. Assets can not be carried from one "life" to the next but memory and experience obviously are. This is more like an OLG model in which parents are not allowed to bequest assets to their children, but may pass on their experience. Second, to be able to end the laboratory economy in a finite number of periods, without eliminating the indeterminacy problem of the model under study, we introduce a forecasting contest. Each subject, who is not currently in the market, submits a price forecast for the current period. The winner (the one with a smallest *ex-post* forecast error) receives a previously specified prize (in dollars). This prediction game has no other repercussions through the economy. After gathering price predictions at the beginning of a not-previously-announced period, the experimenter states that the economy has reached its end and the mean predicted price is used to convert the monetary assets held by subjects into real assets. Third, the market institution to set prices and quantities is a supply game in which subjects entering the market submit supply schedules (points linearly interpolated by the computer) conditional on foreseen future prices, but not on foreseen future inflation rates.

These experimental features introduce elements of uncertainty and of strategic behavior that are not present in the simple deterministic OLG model used as a benchmark. We argue that these departures do not distort the main features of the model. In particular, in Section 3 we show the equivalence between the rational expectations equilibrium set and the Nash equilibrium set of the anonymous game of our experimental design. We also show that if repeat entry of the same individuals into the economy generates strategic opportunities, such opportunities are also present in the OLG game where there is no rebirth. Given that our game is played by a small number of players, we also use our data to study whether subjects behave as competitive agents. We do not detect evidence of strategic behavior. Finally, although we design our experiments according to a deterministic model, market uncertainty is clearly present in our experimental economies. This may help explain the bias towards higher-than-competitive cash balances observed in our data.

The rest of the paper is organized as follows. In Section 2 we describe the competitive deterministic model. In Section 3 we discuss the experimental implementation of the model. In Section 4 we present our data on inflation paths for nine experimental economies. We present data on agents' forecasts in Section 5. In Section 6 we compare our data against benchmark adaptive learning rules. Finally, we conclude the paper in Section 7.

2 Indeterminacy and Hyperinflation in an OLG Economy

The overlapping generations structure is as follows. Each generation has n agents and each agent in generations born after period zero lives for two

periods. An agent i of generation $t, t \geq 1$ has a two–period endowment of a unique perishable good $(\omega^1_{t,i}, \omega^2_{t,i}) = (\omega^1, \omega^2)$, $\omega^1 > \omega^2 > 0$, and his preferences over consumption are represented by $u_i(c^1_t, c^2_t) = \ln(c^1) + \beta_i \ln(c^2)$ where the superscript denotes the period in the agent's life. An agent i of the initial generation that exits in Period 1 only lives for one period, and is endowed with $\omega_{0,i} = \omega^2$ of the consumption good. He also has an endowment of fiat money of h_0 and his preferences are represented by $U_i(c_0) = \ln(c_0)$.

Given a sequence of consumption good prices $\{p_t\}_{t=0}^{\infty}$, an agent i of generation $t, t \geq 1$, solves the problem,

$$\begin{aligned} \max \quad & \ln c^1_t + \beta_i \ln c^2_t \\ \text{s.t.} \quad & p_t(c^1_t - \omega^1) + p_{t+1}(c^2_t - \omega^2) \leq 0. \end{aligned}$$

Let $\pi_{t+1} = p_{t+1}/p_t$, and $\pi^e_{t+1} = \mathbf{E}_{t-1}\pi_{t+1}$ (i.e., expectation at the beginning of period t about the rate of inflation between periods t and $t+1$). If $\omega^1 - \omega^2$ is large enough, the agent's supply in the first period of his life is

$$s_{it} = (\beta_i \omega^1 - \pi^e_{t+1}\omega^2)/(1 + \beta_i).$$

The per capita aggregate supply is

$$s_t = \alpha\omega^1 - \pi^e_{t+1}\gamma\omega^2 \tag{1}$$

where

$$\alpha = \frac{1}{n}\sum_{i=1}^{n}(\beta_i/(1+\beta_i)), \text{and} \gamma = \frac{1}{n}\sum_{i=1}^{n}(1+\beta_i)^{-1}.$$

Let h_t be the *per capita* money supply in period t. The government finances a constant level of deficit d through seigniorage, and, therefore, the supply of money follows the process

$$h_t = h_{t-1} + p_t d,$$

$$\text{or } m_t = m_{t-1}/\pi_t + d, \tag{2}$$

where $m_t = h_t/p_t$ is the per capita money supply in real terms. The equilibrium condition is

$$m_t = s_t \tag{3}$$

Equations (1)–(3) define the equilibrium restrictions of the model. They can be integrated into the equilibrium map

$$T(\pi^e_{t+1},\, \pi^e_t,\, \pi_t) = 0, \quad \text{i.e.,} \tag{4a}$$

$$\pi^e_{t+1} - c - \frac{\pi^e_t - b}{\pi_t} = 0. \tag{4b}$$

where $b = \frac{\alpha\omega^1}{\gamma\omega^2}$ and $c = b - \frac{d}{\alpha\omega^2}$. Stationary solutions satisfy $T(\bar{\pi}, \bar{\pi}, \bar{\pi}) = 0$ and if $(c+1)^2 > 4b$, there are two stationary solutions $(\pi^L,\, \pi^H)$. Given that, for $\pi^e_t \neq b$,, $\partial_3 T(\cdot) = (\pi^e_t - b)/(\pi_t)^2 \neq 0$, by the Implicit Function Theorem we have

$$\pi_t - \phi(\pi^e_{t+1},\, \pi^e_t) = 0, \quad \text{where} \tag{5a}$$

$$\phi(\pi^e_{t+1},\, \pi^e_t) = \frac{b - \pi^e_t}{c - \pi^e_{t+1}} \tag{5b}$$

Equation (5) describes the equilibrium dynamics of the economy; actual inflation as a function of expected inflation for the current and the following period. We can close the equilibrium condition by postulating the *rational expectations hypothesis.* That is

$$\pi_t = \pi^e_t \tag{6}$$

then rational expectations equilibrium paths, for $\pi_t \in (0,\, b)$, are given by the difference equation

$$\pi_{t+1} = R(\pi_t), \quad \text{i.e.,} \tag{7a}$$

$$\pi_{t+1} = (c+1) - \frac{b}{\pi_t} \tag{7b}$$

If $\pi_0 \in (\pi^L, \pi^H)$, then the nonstationary equilibrium path $\{\pi_t\}_{t=0}^{\infty}$ satisfies $\pi_t \to \pi^H$ exponentially. Figure 1 shows the $R(\cdot)$ map and an arbitrary nonstationary equilibrium path.

[Insert Figure 1]

As can be seen from (7), the two steady state rates of inflation move in opposite directions when a parameter such as the real deficit d is changed. The low inflation steady state, π^L, (known as classical) decreases with a decrease in deficit. On the contrary, such a decrease raises the level of inflation at the high inflation steady state, π^H.

3 The OLG Experimental Environment

Thirteen experimental economies, numbered chronologically for reference in this paper, have been conducted in seven sessions.[3] A fixed number of subjects (N) participate in each session. Subjects know the approximate duration of the session but not of a particular economy. For each period of an economy, agents are assigned specific roles: n subjects act as young consumers, n as old consumers, and the remaining ($N - 2n > n$) wait as interested onlookers in the market. At the beginning of each period n of the $(N-2n)$ players who are outside in the previous period are randomly selected to enter the market. Each player is informed whether he/she enters the market or stays out. Once an agent enters as a young consumer, he/she stays the next period as an old consumer and must spend the following period outside the market.

Consumers receive a higher endowment of chips (ω^1 units) when young and they may offer to sell some or all of these chips to the old consumers. To

do this they must submit a supply schedule consisting of a reservation price for each integer quantity i, $i = 0, 1, \ldots, \omega^1$. A continuous supply schedule is computed by linear interpolation. Once the market clearing price is determined, they automatically sell the quantity defined by their continuous supply. Young consumers carry the francs (label for units of Fiat money in laboratory) they receive in exchange for the chips they sell to their old age in the next period.

Old consumers add to their endowment of chips $\omega^2 < (\omega^1)$ by simply offering all their franc holdings in exchange for more chips. The number of chips held at the end of the young period, c^1, and at the end of the old period, c^2, a constant and known conversion rate k, and the individual discount rate β_i determine the dollar amount $k \cdot [\log c^1 + \beta_i \log c^2]$ earned by the subject when he/she leaves the market at the end of the old period. This dollar amount is accumulated and the total is paid to subjects at the end of the experiment.

When subjects re-enter the market as young in a subsequent generation they cannot use dollars from this account; they re-enter as new subjects. The total number of subjects (N) is chosen to be sufficiently large ($N > 3n$) to ensure that each subject sits out for a random number of periods (≥ 1) between leaving and re-entering the economy. In economies with heterogeneous agents in each generation, they know in advance all the n possible values β_i may take; when a subject enters the economy as a young agent, he or she is randomly assigned one of these values so the realized distribution of β_i in every generation is identical. Each subject sees the specific value of β_i assigned to him or her on the screen. In economies with homogeneous agents, β_i remains fixed at 1 for all.

It is *common knowledge* that the experimenter buys d chips every period

at the market clearing price and that, therefore, the amount of money (francs) in circulation grows. The market clearing price is computed and announced each period. The past history of prices is displayed on the computer screen.

Table 1 shows some important features of the thirteen overlapping generations economies conducted in seven separate sessions and highlights some of the important features of these economies. Gross inflation rates, per capita sale of chips and dollar earnings predicted by the Low ISS and High ISS equilibria, and by constant consumption behavior of agents are shown in Table 2. Further details about the design and rationale of these economies are discussed in the next section along with the results. Summary of procedures and a sample set of instructions for subjects for one of the economies (Economy 3) is enclosed as an Appendix to this paper. Instructions used in all other economies were variations on this basic form. Complete details are available from the authors on request.

[Insert Tables 1 and 2]

3.1 The Terminal Condition

The OLG model has an infinite horizon and, in a strict sense, cannot be cast in an experimental environment. The experimenter's choice of a form to end the economy may affect the set of equilibria. We use a procedure introduced by Lim, Prescott and Sunder (1986) to end each economy. During the experiment, players outside the market play a forecasting game: At the beginning of each period, they are asked to forecast the equilibrium price for the period; the player(s) whose prediction turns out to be the best ex–post receive(s) a prize (in dollars) that is added to their dollar accounts. This forecasting game has no direct effect on the evolution of the market. The

winning forecast is announced and displayed on all computer screens at the end of each period.

Without any previous announcement, and after forecasts for the period $(T+1)$ have been submitted, the experimenter declares that the period just ended (T) is the last period of the economy. It is then that the forecasting game plays a role. Money (franc) holdings of agents who entered the economy in period T are converted into chips using *the average of predicted market prices for period* $T+1$ *by outside-market participants.* This procedure for ending the game is announced and explained to subjects at the outset as part of the instructions.

3.2 The OLG–Forecasting Game

Since this forecasting game is not a feature of the OLG economy, one may ask if its use in laboratory introduces an important distortion of the OLG model. In this section, we show the equivalence between the standard OLG model and the *anonymous* game played between agents of different generations and an outside group of forecasters.

The set of players in this game consists of n players per generation $t, t \geq 1$ plus a group of m infinitely lived players who play the forecasting game. We shall show that the results are not affected if we considered a sequence of short-lived forecasters instead. In period t there is a probability ρ_t that the economy will be terminated and the above mentioned *terminal condition* will be applied. Let Φ^t be the set of all possible publicly-observed histories up to period t. Observed histories include the initial money supply and the realized sequence of prices. A strategy for agent i of generation t is a map $s_{i,t} : \Phi^t \mapsto \Re$ defining his/her savings. A strategy for the j the forecaster

is a sequence of price forecasts $\{f_{j,t}\}_{t=1}^{\infty}$, where $f_{j,t} : \Phi^t \times \Re_+^t \mapsto \Re_+$. Let σ denote a joint pure strategy profile, the payoff for a player i of generation t is given by: $U_{i,t}(\sigma) = k \cdot E[\ln(\omega^1 - s_{i,t}) + \ln(\omega^2 + s_{i,t}/\pi_{t+1})]$; notice that for simplicity we assume $\beta_i = 1$. Let $x_{j,t}$ be the prize-payoff to forecaster j at t. That is, $x_{j,t} = z \cdot m/w_t$ if j wins the forecasting contest and has to split the dollar prize $z \cdot m$ among w_t winners, and $x_{j,t} = 0$ otherwise. Then, $U_j(\sigma) = \sum_{t=0}^{\infty}(\prod_{\tau=0}^{t-1}(1-\rho_\tau))x_{j,t}$. In addition, the money is created and introduced into the game as in the OLG economy. It is assumed that the game is competitive in the sense that individual deviations do not affect price expectations or average forecasts (i.e., n and m are large enough). This ends the description of the game and we can now describe its equilibria. Let $g_{t+1}(\sigma)$ denote the inflation rate expected by a player i of generation t when $\sigma = (s_{i,t}, \sigma_{-i})$ is being played and the market remains open, and $\bar{f}_{t+1}(\sigma)$ the corresponding average forecast price. Let $q_{t+1} = \frac{f_{t+1}}{p_t^e}$, where p_t^e is the expected price at the beginning of period t by i when σ is being played. Then, for $\omega^1 >> \omega^2 > 0$, $s_{i,t}$ is the best response by player i of generation t to σ_i if and only if

$$s_{i,t} = \frac{1}{2}[\omega^1 - (\rho_t q_{t+1} + (1-\rho_t)g_{t+1})\omega^2] \qquad (1')$$

The forecaster has to find the forecast that maximizes his expected payoff. All forecasters have reason to choose a symmetric strategy profile $\{f_t\}_{t=1}^{\infty}$, then, for all t, $f_{j,t+1} \equiv f_{t+1} = \bar{f}_{j,t+1}$. That is, at the termination date they all share the prize. Now suppose $(1-\rho_t)\cdot z \cdot m > \rho_t \cdot z$, then given $\sigma = (s_t, f_t, \sigma_{-t})$ and the observed history $\phi^t = (h_{-1}, p_0, h_0, \ldots, p_{t-1}, h_{t-1})$, the t^{th} period price is uniquely determined by $p_t = h_{t-1}/(s_t - d)$. Therefore, the best forecast for player j at the beginning of period t and the best prediction for a player

(i,t) in his entry period is

$$f_{j,t} = p_t^e = \frac{h_{t-1}}{s_t - d} \tag{8}$$

Since all forecasters share the same public information and have identical preferences, $\bar{f}_t = f_{j,t}$. Note that without the above restriction on prizes, any strategy for t that has all forecasters submitting the same forecast is a best response at t. Similarly, taking the strategies s_{t+1} as given and ϕ^t, $p_{t+1} = \frac{h_{t-1}+p_t \cdot d}{s_{t+1}-d}$ From these price equations we obtain,

$$\pi_{t+1} = \frac{s_t}{s_{t+1} - d.} \tag{9}$$

It follows that when player (i,t) takes the per capita supplies s_t and s_{t+1} as part of the strategy profile σ being played, his beliefs about the next period's inflation rate are uniquely defined ($g_{t+1}(\sigma) = \pi_{t+1}$). The best response of (i,t) takes the simple form $B_{(i,t)}(\sigma|\phi^t) = B_{(i,t)}(s_t, s_{t+1}, \bar{f}_{t+1}|h_{t-1})$. Furthermore, since the optimal forecasts imply $q_{t+1} = \pi_{t+1}$ we have that $(\rho_t q_{t+1} + (1-\rho_t)g_{t+1} = \pi_{t+1}$. In other words, the rational expectations hypothesis (6) is satisfied and the *Nash equilibrium* inflation paths are described by (7); including the terminal inflation rate. It should be noted that the best response of the forecasters also takes a simple form: $B_{(j,t)}(\sigma|\phi^t) = B_{(j,t)}(s_t|h_{t-1})$.

We have shown that the Nash (and Sequential) equilibrium paths are REE paths. To show the converse result it is enough to expand the sequence of actions $\{s_{i,t}\}$ to strategies of the OLG-Forecasting game. The sequence $\{s_{i,t}\}$ together with h_{-1} defines an equilibrium sequence of money supplies $\{h_t\}$. Using the above simplified form of the best response functions, the strategies for all players are uniquely defined, and by the above argument these strategies are equilibrium strategies of the OLG-Forecasting game. This completes the proof of the following proposition.

Proposition 1. Assume $(1-\rho_t)\cdot m > \rho_t$ for all t. Then $(\{s_{i,t}\}_{t=0}^{\infty}, \{h_t\}_{t=-1}^{\infty}, \{p_t\}_{t=0}^{\infty})$ is a *Rational Expectations Equilibrium* of the OLG economy iff $(\{s_{i,t}\}_{t=0}^{\infty}, \{f_{j,t} = p_t\}_{t=0}^{\infty})$ is a *Nash (and Sequential) equilibrium* of the OLG-Forecasting game with initial condition h_{-1}.

We still have a gap between the above OLG-Forecasting game and the game played in our experimental environment. However, this gap is small and can be covered. First, the above analysis can be extended to the case where actions of the young agents consist of supply schedules (as a function of the current period price) instead of quantities supplied. This follows from the fact that the price for the current period, p_t, is implicitly computed in determining the best response $s_{(i,t)} = B_{(i,t)}(s_t, s_{t+1}, f_{t+1}^{\Phi}|h_{t-1})$. Second, even if we do not use an explicit random device to determine the terminal period, it would appear to the participants that such a mechanism (with a time dependent ρ) is in effect. (*Ex-post*,it is not clear that the implicit ρ_t always satisfies the restriction on prizes).[4]

Two more features of the experiment deserve comment. One is the effect of having a small number of subjects who go through many "lives" during the course of an economy, and become forecasting players when they are not"alive". The second is the introduction of *market uncertainty* due to the fact that there are only a few participants in each generation, and they can make mistakes in computing their optimal supplies.

3.3 The"Repeated Game" that Subjects Play

For individual subjects to be reborn in the economy is, apparently, an important distortion of the OLG environment since it opens up the possibility that subjects take into account the future strategic effect of their current

actions (as they might do in any repeated game). In order to behave strategically, it is necessary that individual subjects be able to affect the publicly observed outcomes such as prices; otherwise in the anonymous competitive game there is no room for strategic considerations (in the game theoretical language, there is an *anti-folk theorem*). If individual subjects can influence prices because of small generation size, then we have a larger set of equilibria. However, this is also true of the game in which subjects enter the economy only once. To see this, consider an OLG economy with homogeneous patient agents. Let $\bar{s}$ be the savings level of a two-period lived agent that maximizes constant consumption. That is, $\bar{s} = (1/2)(\omega^1 + \omega^2 - d)$ and let s^L be the competitive savings when the inflation rate is π^L,the Low ISS. The constant consumption path is Pareto optimal, but it is not competitive since at $\pi^c = \bar{s}/(\bar{s} - d)$ individual agents have an incentive to deviate by reducing their supply. Now, consider the following strategies: $s_{i,t} = \bar{s}$ if either $t = 0$ or the past history of prices is consistent with the constant per capita supply $\bar{s}$ (which is uniquely defined by (8) and (9)), otherwise $s_{i,t} = s^L$. The following result is proved in Appendix I:

Lemma 1. Let $\beta_i = 1$ for all i. The maximal constant consumption path is a *sub-game perfect* equilibrium of the OLG game without "rebirth", provided that changes in individual saving decisions are reflected in changes in prices.

In summary, we should be alert to the strategic possibilities open to our subjects. However, as the above result shows, our laboratory implementation may depart from the OLG model mainly due to imperfection of competition, not due to the repeated entry of the same subjects into the economy. Of course, to reach some equilibrium, such as the maximal constant consumption, subjects will have to understand the game and learn to coordinate

their strategies; their repeat experience in, and continuous observation of the economy may promote such a tendency. Our computerized laboratory prevents forms of communication that can help to coordinate strategies and our procedures for forming generations preclude persistence of cohorts. The existence of strategic behavior, however, is an empirical issue. We deal with it in Subsection 4.1.

3.4 A Stochastic Environment for a Deterministic Model

The model described in Section 2 is deterministic and we follow it closely in designing our experiment. Nevertheless, since we do not (and cannot) impose *perfect foresight* or *perfect rationality* on subjects' decisions, and since the number of subjects is small (i.e., individual mistakes can not be smoothed out by the law of large numbers), randomness appears naturally in our experimental data. The qualitative features of the model described in Section 2 are robust to the existence of a small amount of randomness, and the equilibrium realizations of the stochastic model are in close neighborhoods of the deterministic equilibria. However, it is important to remark that if agents of generation t face a random inflation rate π^e_{t+1}, then their optimal supply, $\tilde{s}$, is higher than the supply for the deterministic inflation rate $E_{t-1}\pi^e_{t+1}$. This is due to risk-aversion and the introduction of a precautionary motive for holding the only existing asset. In summary, since our subjects not only predict future prices but also make savings decisions, we can expect a bias toward higher cash balances relative to a forecasting model in which agents made point-forecasts only and their optimal savings were determined by these forecasts.)

4 Inflationary Patterns in Experimental OLG Economies

According to the theoretical model described in Section 2, nonstationary rational expectations paths with an initial inflation above the Low ISS (π^L) converge, in the long–run, to the High ISS (π^H). The initial price and inflation are endogenous to our experimental economies. Do we observe inflation paths that tend to cluster around the High ISS? In Figure 2 we have summarized the data from all nine experimental economies in which the level of real deficit was held constant. [5] It provides a striking negative answer to this question.

[Insert Figure 2]

Figure 3 shows the evolution of inflation rates and real balances for three representative Economies (4A, 4B, and 7C). Figure 3 also shows, as reference benchmarks, the two paths corresponding to the high and low inflation stationary equilibria, and a path consistent with REE law of motion from the *observed* initial price. This REE path can only be taken as a reference theoretical path, it has explanatory power.[6] As Figure 2 shows, the absence of paths converging to the High ISS is a general characteristic of our experimental economies.

[Insert Figure 3]

One might argue that it is difficult for agents to predict inflation rates in a range that they have not previously experienced, or that the initial inflation rates are close enough to the Low ISS and agents follow an *approximately* stationary REE path. We designed several experiments to test these potential explanations. For example, Economy 7C was conducted immediately following Economies 7A and 7B in the same session. In Economies 7A and

7B, agents were subjected to inflation rate near and above the High ISS of Economy 7C, even if they started with inflation rates below Low ISS.[7] We also designed Economy 4 in an attempt to generate initial conditions favorable to the High ISS. Even though its subjects experienced many periods of net inflation rates near or above 200 percent per period in Economies 7A and 7B, Economy 7C does not settle down in the vicinity of High ISS, as can be seen from Figure 3.

4.1 The Bias Toward Constant Consumption

In some economies with homogeneous patient agents (i.e., payoffs without time discount of the form: $\log c_t^1 + \log c_t^2$). we detected a bias toward a steady-state inflation rate below the Low ISS. This bias is most pronounced in Economy 2, shown in Figure 4, is the one with the most explicit bias. The "constant consumption" inflation rate seems to explain the data better than either of the two stationary equilibria. As we have seen in Subsection 3.3, this constant consumption path can be achieved as a sub-game perfect equilibrium of the OLG game. Should we reject the null-hypothesis that our subjects behaved competitively?

[Insert Figures 4]

In Figure 4c we include the theoretical earnings of an agent who uses ordinary least squares on observed prices to predict future inflation and submits competitive supplies. (We show in Section 5 that these forecasts approximate the actual forecasts submitted by the agents.) Figure 4c shows that the marginal earnings from using competitive supplies are small. We design several experiments to explore the sources of this bias. In the first step, we

design and conduct Economies 4 and 5 with heterogeneous agents (with respect to discount rate β_i) in which constant consumption is no longer a Pareto optimal outcome. In the second step, we conduct economies (6B, 6C, and 7C) in which subjects had prior experience with "target rate" economies. In target rate economies (6A and 7A), real deficit level was adaptively changed to target a fixed rate of inflation (see footnote 6 above). These "target rate" economies allowed subjects to gain experience in submitting competitive supplies.

The general result is that, with experience, subjects learn to behave more competitively, and the constant consumption bias tends to disappear (see Figures 5 and 6). This finding is reinforced by the supplies submitted by agents in their young period. These supplies (see Figure 7 for three examples) show that even in economies with a clear bias toward constant consumption,(such as Economy 2) agents did not try to achieve this level of consumption by submitting very inelastic supplies (submitting inelastic supplies is an easy way to influence prices in the OLG game). They also show how more experienced subjects (such as those in Economy 7C) learn to submit competitive supplies. Table 1 summarizes the prior experience of our subjects in each economy.

[Insert Figures 5, 6 and 7]

Experience with a competitive environment and different rates of inflation may enable subjects to capture the marginal gain that accrues to those who behave competitively. Some form of experience and adaptive or evolutionary learning (in addition to the uncertainty effect discussed in Subsection 3.4) may explain the oversupply bias. For example, in Economy 5B with heterogeneous agents, one observes oversupply on average. However, individual

supplies exhibit a larger deviation from competitive levels when the discount factor (β_i) is low. A lower discount factor also means a lower dollar reward. This means that subjects tend to behave more competitively when incentives to do so are greater. They behave as if, remembering from their past high β_i "lives" that high supplies (that were approximately competitive in those cases) were associated with high payoffs, they tend to oversupply even when their β_i is low.

5 Forecasting and Competitive Behavior in an Experimental Environment

The competitive supply schedule is a function of the expected rate of inflation, π^e_{t+1}. In our experimental environment, at the beginning of their entry period, subjects submit a supply schedule (several points of their supplies) in terms of the expected price for the period, π^e_t. By submitting a supply schedule instead of a quantity, s_t, subjects can partially insure themselves against price fluctuations. Nevertheless, price forecasting is a basic component in their decision process.

We do not collect direct information on expected prices from agents *in* the market. With the prediction game, however, we have information from agents *outside* the market. Since the same subjects randomly enter and exit the market and observe all data, it is reasonable to assume that the predictions of the insiders are well represented by the predictions of the outsiders.[8]

Our subjects learned rapidly to make accurate price predictions. Figures 8a and 8b show the evolution of prices (in natural logarithms) and predicted prices for two representative economies. They also show the price predictions that would be obtained by using ordinary least squares on past (levels of)

prices.

[Insert Figure 8]

We can calculate predicted inflation rates implied by predicted prices (expected inflation = (expected price)/(observed price), i.e., $\pi^e_{t+1} = p^e_{t+1}/p_t$). We asked subjects to predict prices, not inflation rates, and we cannot rule out the possibility that the forecasts of inflation implicit in price predictions could be different from directly solicited predictions of inflation.[9]

The magnitude of inflation prediction errors (Figures 8c and 8d) in these economies is not negligible, although in economies with a longer and more stationary inflationary process (e.g., Economy 3), prediction errors tend to dampen. We test these predictions to determine if they make efficient use of the available information; in particular, whether the prediction errors are orthogonal to past information.[10] We regress prediction errors $(\pi^e_{t+1} - \pi_{t+1})$ on lagged inflation rates and π_t, π_{t-1}; there was no gain in explanatory power from using more lags.

Table 3 reports the results for five economies. With the exception of Economy 5B, prediction errors are not orthogonal to information available to traders; they show a positive dependence on past inflation rates. That is, predictions of subjects tend to "overshoot" at high inflation rates. However, Figures 8c, 8d and Table 3 suggest that our subjects are not the only ones to err; their prediction errors are hardly distinguishable from the prediction errors generated by ordinary least squares on past price levels. We return to this point in the next section.

[Insert Table 3]

Since price predictions define the terminal condition it is important to check whether they might have had some "real" effect. Figure 9 reports the

cross-sectional summary statistics for errors in the first 15 and the last 15 periods of the nine economies. In the last period, the standard deviation of the average prediction error is small, and the magnitude of error in the last period follows the trend from the preceding periods. Our procedure for terminating the economies does not seem to have any real effect on the market, in the sense that it does not create any perturbation in the agents' behavior toward the end of these economies. This is consistent with our analysis of the *OLG-Forecasting* game in Subsection 3.3. Recall that subjects do not learn that period T is the last period until after the outsiders have submitted their forecasts for period (T+1).

[Insert Figure 9]

Figure 9 shows that our subjects' predictions were close to the predictions of an econometrician who uses least squares predictors from observed data. Figures 10a and 10c compare realized mean predictions with least squares predictions from observed data for Economies 2 and 5B. Figures 10b and 10d compare the realized volume of trade with the theoretical volume of trade that would have been achieved if the agents had performed least squares on observed data and submitted competitive supplies for the deterministic model.

Figure 11 makes the same comparisons for Economies 4 and 6B and 6C where savings were closer to the prediction of the deterministic model. In particular, Economy 6B & C show how fairly experienced subjects learned not only to forecast but also to behave competitively.

[Insert Figures 10 and 11]

6 Adaptive Learning and Equilibrium Selection

In the last section, we have seen that the accuracy and biases of inflation rates (prices) predicted by subjects are comparable to those achieved by an econometrician using ordinary least squares. OLS is a particular adaptive rule, in this section we study a more general class of learning rules, we estimate the forecasting rules of our experimental subjects and we analize the local stability properties of the estimated models.

Alternative forecasting rules differ in their functional form and/or the the variable being forecasted. Expected inflation is the relevant variable that determines competitive supplies, therefore price forecasts must be translated into inflation forecasts(11) $\mathrm{E}_{t-1}\pi_{t+1} = \frac{p^e_{t+1}}{p^e_t}$. In our experimental environment, however, subjects submit supply schedules and their competitive supplies take the contingent form,

$$s(p_t) = \alpha\omega^1 - \frac{p^e_{t+1}}{p_t}\gamma\omega^2 \quad (1')$$

For this reason, most of the forecasting rules that we estimate take $\pi^e_{t+1} = \frac{p^e_{t+1}}{p_t}$ as the forecasted variable. Following a long tradition of adaptive forecasting models (Friedman, Cagan, *et al.*), we can consider rules of the form,

$$\pi^e_{t+1} = \pi^e_{t-1} + \alpha(\pi_{t-1} - \pi^e_{t-1}). \quad (10)$$

The second-order forecasting scheme (10) is the standard adaptive model where future expectations are equal past expectations corrected by a fraction of the forecast error (It is assumed that $\alpha \in (0, 1)$.[12]). It differs from the more standard first-order scheme (where $t-1$ is replaced by t) because π_t is unknown when π_{t+1} is forecasted. But this process of recursive updating of

forecasts can also be realized with a modified first-order scheme of the form

$$\pi^e_{t+1} = \pi^e_t + \alpha(\pi_{t-1} - \pi^e_t). \tag{11}$$

As in the first-order scheme, (11) defines a distributed lag process

$$\pi^e_{t+1} = \sum_{n=0}^{\infty} \alpha(1-\alpha)^n \pi_{t-1-n}$$

The stability of the inflation process depends on the interaction between the equilibrium map $\phi(\cdot, \cdot)$ of equation (5) and the forecasting map used by agents. For example, if agents use (11) as their forecasting rule, then expected inflations follow the difference equation,

$$\pi^e_{t+1} = \pi^e_t + \alpha(\phi(\pi^e_t, \pi^e_{t-1}) - \pi^e_t). \tag{12}$$

The local stability of a stationary equilibrium depends on whether the process $\pi_t = \phi(\pi_{t-1}, \pi_{t-2})$ is *asymptotically stable* around the steady state and whether this stability property is reinforced by local stability of the forecasting rule. Linearizing (5) around the steady state $\bar{\pi}$ we obtain the characteristic equation

$$z^2 - \phi_1 \cdot z - \phi_2 = 0, \tag{13}$$

where $\phi_1 = \frac{(b-\bar{\pi})}{(c-\bar{\pi})^2}$ and $\phi_2 = -(c-\bar{\pi})^{-1}$. In our economies $\phi_1 < 1$ at $\bar{\pi} = \pi^L$ and (12) has two complex roots of modulus less than one. Now, if (11) describes the forecasting rule of the agents, the local behavior of (12) around the steady state is caracterized by the roots, (λ^1, λ^2) of

$$z^2 - ((1-\alpha) + \alpha\phi_1) \cdot z - \alpha\phi_2 = 0, \tag{14}$$

That is, $(\lambda^1 + \lambda^2) = ((1-\alpha) + \alpha\phi_1)$ and $(\lambda^1\lambda^2) = -\alpha\phi_2$. Since $\phi_1 \in (0,1)$ (or for low enough values of α when $\phi^1 > 1$) and $\alpha_2 \in (-1,0)$ it follows that

both roots have modulus less than one. The same is true for the second-order forecasting scheme. However, for an arbitrary forecasting rule (or if α is unrestricted) the resulting process may be locally unstable. As Grandmont and Laroque (1990) have shown, if agents' forecasting rules are sensitive enough to variations in inflation rates then local perturbations around the steady state generate local instability.

A similar forecasting rule can be derived when agents follow the myopic belief that the inflation rate is constant at β, (i.e.,$\pi^e_{t+1} = \pi^e_t = \beta$). In this case, the "true" inflation rate is given by

$$S(\beta) \equiv \phi(\beta, \beta).$$

If agents use least squares forecasting rules (on inflation rates) to forecast their -believed to be constant- inflation rate β, then their recursive updating of β takes the form

$$\pi^e_{t+1} = \pi^e_t + \alpha_t(S(\pi^e_t) - \pi^e_t). \tag{15}$$

with α_t converging to zero at the rate of $\frac{1}{t}$. Since at π^L, $S' < 1$ local stability is guaranteed by the assumptions of the model. Marcet and Sargent (1989b) postulate an "expected constant inflation" and least squares forecasting on prices. This case is also of the same form, although α_t converges to a nonzero constant (in (0,1)). It should be noted, however, that the model is deterministic and the use of least squares estimates is appropriate only as long as the market uncertainty discussed in Section 3 persists.

Now we use the data from our experimental economies to estimate forecasting rules and analize the corresponding local stability of the resulting low of motion. In our estimations, we use as the expected price variable, to compute π^e_{t+1}, the mean prediction of the forecasters. In this sense, we are looking at the forecasting rule of a representative agent who is always present in the

economy. Unfortunately, our data is not complete enough as to identify all the individual forecasts. Our time series are relatively short and estimations of finite linear forecasting rules ($\pi^e_{t+1} = \beta_0 + \beta_1\pi_{t-1}, \ldots \beta_n\pi_{t-n}$) give poor results. A better fit is obtained by estimating modified versions of a version of Cagan's model. Table IV summarizes some of these results. The economies seem to be devided in two groups. For Economies 2, 3 and 6B, Equation $\{a\}$, a version of equation (11), is the best fit. For Economies 1, 4A, 5B and 7C, however, a second-order schemes (Equations $\{c\}$) and $\{d\}$) provides a better estimate. In particular, when we impose the coefficient restriction implied by (10) (Equation $\{c\}$. We also estimate if our subjects where using Least Squares learning rules (on prices) as the discussion of Section 5 suggested. In Table V we report the results of regressing the mean forecasted inflation (*i.e.*, $\frac{p^e_{t+1}}{p_t}$) on predicted inflation according to least squares estimates. Only for Economies 2, 4A and 6C we improve our previous estimates.

We can then use our estimated forecasting rules to analyze the local stability around the Low ISS. As we have already said, local stability is guaranteed when it is assumed that agents believe inflation to be constant and update their estimates using least squares. Similarly, local stability is guranteed when the coefficients are restricted as in rules (10) and (11) (*i.e.*, $\alpha \in (0, 1)$). For unresticted values of the parameters, however, local stability is not guaranteed. For example, we can construct confidence intervals using our -unrestricted- estimates. That is, for a given estimated equation we can find the largest confidence interval for which all the roots of the corresponding characteristic equation are in the unit interval (*e.g.*, the roots of equation (12) for estimation $\{a\}$), then for higher significance levels the local unstability hypothesis can not be rejected. Table VI shows the results of this test for iur economies. As expected, at a high enough significance level, local unstability

can not be rejected.Furthermore, in general, the stability confidence interval shrinks when more -possibly insignificant- lags are added in the estimation (*e.g.*, for estimation $\{b\}$).

In summary, while our data is consistent with -broadly defined- adaptive forecasting rules, it can not discriminate between alternative formulations. While our best estimates result in different forecasting rules for different economies there is nothing intrinsicly different about these economies that may explain subtle differences on learning behavior. Even if most of the estimated forecasting models are locally stable, the local unstability analyzed by Grandomont and Laroque (1990) can not be rejected at sufficiently high significance level (or for alternative forecasting rules).

7 Conclusion

This paper summarizes the results of a relatively large number of experiments. Our approach and our data bears on five related issues (see also, Marimon and Sunder (1990) and Marimon, Spear and Sunder (1992)).

First, it provides data on how agents learn to form beliefs and to coordinate them in a dynamic competitive environment. More specifically, our data supports the hypothesis that agents behave adaptively, but does discriminate in favour of any specific learning rule.

Second, addresses the problem of multiplicity or indeterminacy of equilibria. For example, we do not observe nonstationary paths converging toward the High ISS in our laboratory economies. Our data suggest that the indeterminacy problem encountered in rational expectations equilibrium models may not be such an acute problem in historical economies.

The observed clustering of data around the Low ISS is consistent with

imposing adaptive learning as a behavioral assumption. One could argue that the results might have been different if the agents had been more sophisticated. However, Calvo (1988) has shown that the aggregate dynamics of an economy can be determined by a small group of adaptive learners, even when a large fraction of the agents behave according to the rational expectation hypothesis and they also know that a small group of agents acts adaptively. The known presence of a small group of adaptive learners is sufficient to coordinate the beliefs of all the other agents. In other words, we should expect the same type of results when experiments include subjects who are knowledgeable about the model.

Third, since our subjects had to make savings decisions, in addition to forecasting future prices, our data reflect the market uncertainty and the difficulties of learning to make optimal decisions in a way that is not captured by standard forecasting models, such as point-forecasting models that postulate automatic optimal decisions. Our data suggest a need for learning models that capture the complexity of the decision process.

Fourth, methodologically this paper closes the gap between the dynamic structure of the OLG model and the more limited horizon of the experimental environment. It has repeatedly been shown in prior experimental work that markets with a relatively small number of agents often exhibit competitive conditions. Through analysis of the OLG-Forecasting game, and through data, we have shown that our experimental construct is a good proxy for the OLG model. Nevertheless, the performance of an experimental design is an empirical issue. Our data support the chosen design as a starting point for the development of experimental macroeconomics.

Fifth, we have used the hyperinflation model with a constant deficit as a particular policy regime. Whether an economy with inflationary pressures

is more likely to be around the Low ISS or the High ISS is of more than academic interest. Low or High are not mere quantitative statements; only the Low ISS is consistent with the classical prescription that it is possible to reduce the inflation level by reducing the seigniorage. Our data supports the classical prscription. (In Marimon and Sunder (1991) we explore other policy regimes).

Universitat Pompeu Fabra and University of Minnesota

Carnegie Mellon University

APPENDIX I

Proof of Lemma 1: If $d = 0$, the result is obvious because the competitive allocation is the maximal constant consumption allocation. Assume $d > 0$.We only need to show that for any generation a deviation from the constant consumption strategy is prevented by the reaction of the next generation. Let

$$\begin{aligned} f(s,\, \pi) &= -(\omega^1 - s)^{-1} + (\omega^2\pi + s)^{-1} \\ h(s;\, n) &= -(\omega^1 - s)^{-1} + (\omega^2\pi(s;\, n) + s)^{-1} \cdot p(s;\, n), \\ \text{where } \pi(s;\, n) &= \frac{\left[\left(\frac{n-1}{n}\right)\bar{s} + \frac{1}{n}\, s\right]}{(s^L - d)} \\ \text{and } p(s;\, n) &= \frac{((n-1)\bar{s})}{((n-1)\bar{s} + s)}. \end{aligned}$$

The first order condition when an agent behaves competitively is $f(s,\, \pi) = 0$, for example, $f(s^L,\, \pi^L) = 0$, and the first order condition when an agent behaves strategically, given the specified strategies, is $h(s;\, n) = 0$. Fix n and let $\tilde{s}$ satisfy $h(\tilde{s};\, n) = 0$. The proof reduces to showing that there exists a $\hat{s} < s^L$, such that, for all $s \geq \hat{s}$, $h(s;n) < f(s, \pi^L)$. Then, since by concavity of $u(\cdot, \cdot)$, $f'(\cdot,\, \pi) < 0$ and $h'(\cdot;\, n) < 0$, it follows that $\tilde{s} < s^L$, $h(s^L;n) < f(s^L, \pi^L) = 0$, and $0 = h(\tilde{s};n) < f(\tilde{s}, \pi^L)$. These facts imply that $u(\tilde{c}^1,\, \tilde{c}^2) < u(c^1(\pi^1),\, c^L(\pi^L)) < u(\bar{c},\, \bar{c})$, where $\tilde{c}$ is the consumption derived from $\tilde{s}$ and $\pi(\tilde{s})$, $c(\pi^L)$ is the Low ISS consumption, and $\bar{c}$ is the maximal constant consumption.

Now, $h(s;n) < f(s, \pi^L)$ iff $(\omega^2\pi(s;n) + s)/p(s;n) > \omega^2\pi^L + s$.

$$\frac{\omega^2\pi(s;n) + s}{p(s;n)} = \omega^2 \cdot \frac{(\frac{n-1}{n}\bar{s} + \frac{2}{n}s)}{s^L - d} + \omega^2 \cdot \frac{(s)^2}{n(n-1)\bar{s}(s^L - d)} + s + \frac{(s)^2}{(n-1)\bar{s}}$$

$$\text{and} \quad \omega^2\pi^L + s = \omega^2 \cdot \frac{s^L}{s^L - d}$$

It follows that if $s \geq \hat{s} = \max\{0, \frac{n}{2}s^L - (n-1)\bar{s}\}$, then $h(s;n) < f(s,\pi^L)$. Note that, since $\bar{s} > s^L$, $s^L > \hat{s}$. Q.E.D.

APPENDIX II

Instructions

This is an experiment in decision-making. Various research foundations have provided funds for this research. The instructions are simple; if you follow them carefully, you might earn a considerable amount of money which will be paid to you in cash.

We shall operate a market in which you may buy and sell chips in a sequence of market periods.

The type of currency used in this market is francs. The only use of this currency is to buy and sell chips. The money you take home with you is in US dollars. The procedures for determining the number of dollars you take home with you is explained later in these instructions.

You will participate in the market for two consecutive periods at a time: your entry period and your exit period. Different individuals may have different entry and exit periods. You may be asked to enter and exit more than once.

At the beginning of your entry period, you will be given 7 chips. You may sell some of the chips to others in exchange for francs. The number of units you sell is determined by the prices at which you express your willingness to sell various number of units, and by the prevailing price of chips during that period. The number of chips "consume" at the end of this period will be 7 minus the number you sell. The francs you receive from selling any of your chips will be carried over into your exit period.

In your exit period, you will be given 1 chip. In addition, you can use the francs carried over from your entry period to buy more chips. The number of units you can buy is determined by the prevailing market price of chips

in that period and the number of francs that you hold at the beginning of your exit period. Francs have no use for you after you exit. They can not be traded outside the market or saved for some future use. Thus the number of chips you "consume" in your exit period is 1 plus the number of chips your francs will buy.

The number of dollars you earn at the end of your exit period is determined by the following formula:

$$\text{Earnings} = \text{maximum } \{0,\ \$2.50 \times \log((c_1 \times c_2)\,/\,(w_1 \times w_2))\}$$

where

w^1 = the number of chips you are given in entry period,

w^2 = the number of chips you are given in the exit period,

c_1 = the number of chips you "consume" (w_1 − what you sell) in your entry period, and

c_2 = the number of chips you "consume" (w_2+ what you buy) in your exit period.

The earnings formula makes sure that your earnings will not be negative. All chips are forfeited at the end of each period.

For some of you the first period of the market will be an exit period. You will receive the exit period endowment of 1 chip plus F francs. Your computer will automatically use all your francs to purchase chips. If for example, the price of a chip in this first period is 2,078 francs per chip, then you will purchase (F/2078) chips and "consume" (1 + F/2078) chips. Your dollar earnings for this period will be determined by the following formula:

$\$1.25 \times \log$ (___ / 2078 + 1)

The experimenter also buys D chips each period by issuing the necessary number of francs.

In every period, the market price is determined by the "willingness" of entry participants to sell, the number of francs in the hands of the exit participants (their ability to buy) and the experimenter's demand for chips. The central computer calculates this price and displays it on your screen.

If a given period is not your entry or your exit period, then you are "outside" the market in that period. In such periods you are asked to predict the market price for that period. Each period a $1.00 prize is given to the participant whose prediction is the closest to the actual market price.

After the outside participants have entered their price forecasts, the experimenter may announce that the period just concluded was the last period of the current experiment. The francs being held by the exit participants are transformed into chips using the "average predicted price" provided by the outside participants.

Thus the specific rules are:

(1) All entry–period players are sellers and all exit–period players are buyers.

(2) All franc holdings of exit–period players will be used up to buy chips at the market price.

(3) At the beginning of each period, you have to enter prices at which you are willing to sell up to $0, 1, 2, 3, 4, 5, or 6$ chips. The price at which you are willing to sell a larger number of chips must be no less than the price for a smaller number of chips.

(4) After considering the amount of francs in the hands of the exit–period players, selling offers made by the entry–period players and the Experimenter's need to buy D chips each period, the computer calculates and informs you about the market clearing price and your transactions and balances.

(5) The computer determines the number of chips purchased by each exit–player and the number of dollars earned by each of these players after considering the chips held at the end of entry and exit periods according to the formula given earlier.

(6) At the beginning of each period, each outside player is prompted by the computer for a market price prediction. At the end of each period, the computer informs you about the average predicted market price and *the winner(s) of the $1.00 prediction prize.*

(7) At the end of the experiment, francs held by all entry–period players are converted into chips using the average of predicted market prices by outside–market players.

(8) At the end of the experiment, your cumulative profit will be paid to you in cash.

Computer Screens

The enclosed sheets show the layout of the three computer screen available to you and the keys necessary to call up each screen:

Notes

[1] A part of the experimental data presented here (Economies 1 and 2) was first reported in "Rational Expectations vs. Adaptive Behavior in a Hyperinflationary World: Experimental Evidence," Center for Economic Research, University of Minnesota, Disc. Paper No. 247, July 1988. Most of this work has been incorporated here. Financial support for this work from the Graduate School of the University of Minnesota, the National Science Foundation (SES–8912552), and Richard M. and Margaret Cyert Family Funds is gratefully acknowledged. [2] We wish to thank those who commented on our work, specially Javier Diaz-Giménez, Jean Michael Grandmont, Robert Lucas, Albert Marcet, Thomas Sargent, Neil Wallace, Michael Woodford and two anonymous referees. We also wish to thank Vijay Rajan for developing the software for the computerized experimental environment and Jackie Cuccaro and Dhananjay Gode for their research assistance. [3] Some of the results are reported in Marimon and Sunder (1991). Experimental sessions 1–2 were performed at the University of Minnesota, and sessions 3–7 were conducted at Carnegie Mellon University's computerized laboratory. Most subjects were undergraduate business or economics majors. [4] Strictly speaking, the OLG-forecasting game analyzed above has a positive probability to last forever, even if ρ_t is arbitrarily close to one after certain period. There are certain modifications that can make the game finite. For example, experimenter can announce that the game will not last for more than T periods. This announcement can be public or private (with randomly selected T_js from a finite set). If it is public, then in the unlikely event that $T-1$ is reached, any final price can be part of a Nash equilibrium. But if beliefs about other agents' forecasts are conditioned on past histories of prices, then

the forecast "as if the market remained open" may be a focal equilibrium move; *ex-post* we can test if this focal point is realized. If the information is private, and the lowest $T_j - 1$ is reached then agent j, knowing that for other forecasters termination is not a certain event, should behave as in the analyzed OLG-forecasting game. Notice that this modifications could not be introduced without distortions if we simply introduced a random termination date without the forecasting game; which, in addition, would introduce an implicit discounting rate for market participants.

[5] We have normalized inflation rates using the transformation $\hat{\pi}_t = (\pi_t - \pi^L)/(\pi^H - \pi^L)$. Table 1 summarizes the description of the experimental economies.

[6] We do not consider here the possibility of 'sunspot' equilibria since, among other things, these require an even higher degree of coordination of beliefs. We study the existence of these equilibria in experimental environments in Marimon, Spear and Sunder (1992).

[7] In Economies 7A and 7B the level of deficit was adjusted every period in order to keep the net expected (rational expectations equilibrium) inflation to be 200% in the following period by using formula $d_t = \frac{(\pi^*-1)}{\pi^*}\frac{h_{t-1}}{p_{t-1}}$ where π^* is the target gross inflation rate. The details of these economies are discussed in Marimon and Sunder (1991).

[8] Figure 7, in fact, is constructed using this assumption. The theoretical competitive supplies are constructed using the reported prediction, p^e_{t+1}, of the outsiders at the beginning of the period $t + 1$. The three supplies correspond to the min, average, and max p^e_{t+1}.

[9] Recall that price predictions were gathered from the outsiders each period in order to implement a terminal condition for the economy. [10] We thank Hassim Pesaran for his insistence that we apply statistical methods to

our experimental data.

[11] In our hyperinflationary economies, it is more appropriate to use least squares estimates on inflation rates rather than on price. In Figure 3 (and Figure 6) we compare realized paths with the *theoretical least squares* paths that would have been followed if agents had been using least squares prediction on price (with initial conditions given by the observed initial prices) and solving for their competitive supplies. That is, the LSQ paths (marked with an inverted triangle) are the "Marcet-Sargent paths" from our observed initial conditions.

[12]In general α can be time dependent and in a stochastic content must satisfy $\alpha_t \to 0, \sum_t \alpha_t = +\infty$, to guarantee convergence.

(13) See, Marimon, Spear and Sunder (1992) for an experimental environment where first ans second order schemes have different stability properties.

(14) Equation *b* shows the standard result that by adding non significant lags the estimation can be improved.

[15] As a historical reference (due to T. Sargent) it should be noticed that A.W. Phillips suggested to Friedman the adaptive hypotesis later adopted by Cagan. This hypothesis is also present in Koyck (1954) and in the early work of Arrow and Nerlove (see, Griliches (1967)).

REFERENCES

Aliprantis and C. Plott (1991): "Competitive Equilibria in Overlapping Generations Experiments," CalTech SSWP 746.

Arifovic, Jasmine (1990): "Learning by Genetic Algorithms in Economic Environments." Santa Fe Institute. Working Paper 90-001.

Blume, L.E., and D. Easley (1982): "Learning to Be Rational," *Journal of Economic Theory*, 26, 340–351.

Blume, L.E., M.M. Bray, and D. Easley (1982): "Introduction to the Stability of Rational Expectations Equilibrium," *Journal of Economic Theory*, 26, 313–317.

Bray, M. (1982): "Learning, Estimation and the Stability of Rational Expectations," *Journal of Economic Theory*, 26, 318–339.

Bruno, M. (1989): "Econometrics and the Design of Economic Reform," *Econometrica*, 57(2), 275-306.

Cagan, P. (1956): "The Monetary Dynamics of Hyperinflation," in *Studies in the Quantity Theory of Money*, ed. by M. Friedman. Chicago: University of Chicago Press.

Calvo, G. (1988): "Passive Money and Idiosyncratic Expectations," Working Paper, May, University of Pennsylvania.

Cooper, R., D. DeJong, R. Forsythe, and T. Ross. (1990): "Selection Criteria in Games," *American Economic Review*, 80:1 (March).

Crawford, V.P. (1991): "An 'Evolutionary' Interpretation of Van Huyck, Battalio, and Beil's Experimental Results on Coordination," *Games and Economic Behavior*, 3, 25–59.

Dornbusch, R., and S. Fischer (1986): "Stopping Hyperinflations: Past and Present," *Weltwirtschafliches Archiv*, 122, 1–47.

Evans, G. (1983): "The Stability of Rational Expectations in Macroeconomics Models," in *Individual Forecasts and Aggregate Outcomes*, ed. by R. Frydman and E. Phelps. Cambridge: Cambridge University Press. ——- (1988): "The Fragility of Sunspot and Bubbles," IMSSS, Stanford University, forthcoming in *Journal of Monetary Economics.*

Friedman, M. (1957): *The Theory of the Consumption Function.* New Jesey, Princeton University Press.

Fudenberg, D., and D. Kreps (1988): "A Theory of Learning, Experimentation, and Equilibrium in Games," Unpublished, M.I.T. and Stanford University.

Grandmont, J.M., and G. Laroque (November 1989): "Economic Dynamics with Learning: Some Instability Examples," Unpublished.

Gray, P., and C.R. Plott (1986): "Notes on Experimental Evidence on Overlapping Generations Economies," Unpublished, Cal Tech.

Imrohoroglu, S. (1989): "Testing for Sunspots in the German Hyperinflation." Working Paper, University of Southern California.

Kandori, M., G. Mailath, and R. Rob (1991): "Learning, Mutation and Long Run Equilibrium in Games," Unpublished, University of Pennsylvania.

Lim, S.S., E.C. Prescott, and S. Sunder (1986): "Stationary Solution to the Overlapping Generations Model of Fiat Money: Experimental Evidence," Working Paper, June, University of Minnesota.

Lucas, R.E. Jr. (1986): "Adaptive Behavior and Economic Theory," *Journal of Business*, 59 (4 part 2, October), 5401–5426.

Marcet, A., and T.J. Sargent (1989a): "Converges of Least Squares Learning Mechanisms in Self Referential, Linear Stochastic Models," *Journal of Economic Theory.*

——- (1989b): "Least Squares Learning and the Dynamics of Hyperinflation," in *Economic Complexity, Chaos, Sunspots, Bubbles and Nonlinearity*, ed. by Barnett, Geweke and Shell. Cambridge: Cambridge University Press.

Marimon, R., E. McGrattan (1992): "On Adaptive Learning in Strategic Games," in *Learning and Rationality in Economics*, ed. by A. Kirman and M. Salmon. Basil Blackwell (Forthcoming).

Marimon, R., S. Spear, and S. Sunder (1992): "Expectationally–Driven Market Volatility: An Experimental Study," *Journal of Economic Theory* (forthcoming).

Marimon, R., and S. Sunder (1988): "Rational Expectations vs. Adaptive Behavior in a Hyperinflationary World: Experimental Evidence," Discussion Paper No. 247, July, Center for Economic Research, University of Minnesota.

——- (1991): "Some Puzzles from Experimental Monetary Econo- mies,"Carnegie Mellon University and IAE Universitat Autonoma de Barcelona.

Sargent, T.J., and N. Wallace (1987): "Inflation and the Government Budget Constraint" in *Economic Policy in Theory and Practice*, ed. by Assaf Razin and Efraim Sadka, Macmillan.

Van Huyck, J.B., R.C. Battalio, and R.O. Beil (1990): "Tacit Coordination Games, Strategic Uncertainty and Coordination Failure," *American Economic Review*, 80(1), 234–248.

Woodford, M. (1986): "Indeterminacy of Equilibrium in the Overlapping Generations Model: A Survey," Unpublished, Columbia University.

TABLE I

Design of Experimental Overlapping Generations Economies

Economy No.	No. of Subjects in Economy and in Generation (N,n)	Prior Experience	Endowment of Chips Young ω^1	Endowment of Chips Old ω^2	Endowment of Money h_0	Govt. Deficit (Per Capita) d	Periods T	Discount Rate β
1	(14,4)	None	7	1	10	0.5	1-19	1
2[a]	(12,3)	3 inexperienced 9 experienced	7	1	3.722	1.25	1-17	1
3	(10,3)	None	7	1	10	0.42	1-31	1
4A[b]	(14,4)	2 from Econ. 3 12 None	7	1	0.5	0.975	1-20	0.6,0.6,1.7,1.7
4B[b]	(14,4)	Econ. 4A	7	1	1	0.975	1-8	0.6,0.6,1.7,1.7
5A[c]	(7,2)	Econ. 3 or 4	6	1	1	$(1/3)h_{t-1}/p_{t-1}$	1-12	0.6,1.0
5B	(12,3)	7 from 5A 5 Inexperienced	6	1	1	1	1-28	0.6,1.0,1.7
6A[d]	(8,2)	7 Experienced 1 None	7	1	1	$0.355h_{t-1}/p_{t-1}$	1-19	1
6B	(15,4)	8 Econ. 6A	7	1	0.1	1.3	1-14	1
6C		7 None	7	1	0.1	1.3	1-17	1
7A[e]	(14,4)	Experienced	6	1	1	$(2/3)h_{t-1}/p_{t-1}$	1-14	1
7B[f]	(14,4)	Econ. 7A	6	1	6.225	$(2/3)h_{t-1}/p_{t-1}$	1-6	1
7C	(14,4)	Econ. 7A,B	6	1	1	1	1-18	1

[a]Econ. 2: At the outset of this economy, subjects were informed that there will be no parameter changes between the beginning and termination.

[b]Econ. 4: One of the four possible values of $\beta_i \in (0.6, 0.6, 1.667, 1.667)$ was randomly assigned to the four members of each generation. While the discount rate was random for each individual, it was the same for every generation.

[c]Econ. 5A: Per capita real deficit was adjusted each period using formula $d_t = h_{t-1}(\pi^* - 1)/\pi^* p_{t-1}$, where h_t is per capita money supply in period t and π^* is the target rate of inflation. π^* was set equal to 1.5, the same as constant consumption rate of inflation in Economy 5B with d=1. In Economy 5A, first period deficit was set $d_1=1$.

[d]Econ. 6A: This economy was similar to 5A except that the target inflation rate π^* was 1.55--same as the constant consumption inflation in Economies 6B and C. First period deficit in 6A was $d_1=1.3$.

[e]Econ. 7A: This economy is similar to 5A and 6A except that the target rate of inflation π^* was equal to the High ISS rate of 3.00. First period deficit in 7A was $d_1=1$.

[f]Econ. 7B: This economy is effectively a continuation of 7A. H_0 in this economy was set to one millionth of the value of h_{14} in economy 7A. Also, initial deficit d was set to 1.115 on the basis of price and money supply in the final period of 7A.

TABLE II

Stationary Equilibria of Experimental Economies

Economy No. (Period)	Low ISS Equilibrium			High ISS Equilibrium			Constant Consumption Equil. or Target Levels[a]		
	π	s	u	π	s	u	π	s	u
1 (1-19)	1.21	2.90	3.73	5.79	0.60	2.66	1.18	3.25	3.75
2 (1-17)	2.00	2.50	3.18	3.50	1.75	2.81	1.53	3.63	3.38
3	1.17	2.92	1.78	6.00	0.50	0.015	1.15	3.21	1.80
4A&B	1.56	2.72	1.91	4.49	1.25	0.40	1.39	3.49	1.95
5A(Target Econ.[a])	---	---	---	---	---	---	1.50	3.00	2.57
5B	2.00	2.00	1.97	3.00	1.50	0.99	1.5	3.00	2.52
6A(Target Econ.[a])	---	---	---	---	---	---	1.55	3.65	2.36
6B&C	2.16	2.42	1.63	3.24	1.88	0.72	1.55	3.65	2.36
7A&B(Target Econ.[a])	---	---	---	3.00	1.50	0.94	---	---	---
7C	2.00	2.00	2.30	3.00	1.50	0.94	1.5	3.00	3.24

[a]In economies 5A, 6A, and 7A and 7B the deficit is changed following a simple adaptive rule with a target rate of inflation. In 5A the target rate of inflation was the same as the constant consumption inflation level of 5B; the target rate in 6A was the same as the constant consumption inflation of 6B and 6C; the target rate in 7A and 7B was equal to the High ISS of Economy 7C.

π = gross inflation,

s = per capita real balances, and

u = per capita utility payoff.

TABLE III

Dependence of Inflation Predictions Errors on Past Inflation

(Standard Errors of Estimation given in parentheses)

$$(\hat{\pi}_{t+1}-\pi_{t+1})*100 = \beta_0 + \beta_1(\pi_t-1)*100 + \beta_2(\pi_{t-1}-1)*100 + \epsilon_{t+1}$$

Economy	Source of Predictions	Regression Coefficients β_0	β_1	β_2	R^2	Std. Dev. of Estimation Error	Degrees of Freedom
3	Subjects	-15.26	0.23 (0.16)	0.48 (0.18)	0.26	8.15	25
3	LSQ	-13.63	0.37 (0.16)	0.49 (0.17)	0.32	7.97	25
4A	Subjects	-71.72	0.80 (0.37)	0.24 (0.37)	0.25	23.30	14
4A	LSQ	-78.17	0.94 (0.27)	0.41 (0.27)	0.47	17.06	14
5B	Subjects	-23.59	0.074 (0.29)	0.19 (0.29)	0.03	14.35	22
5B	LSQ	-37.36	0.35 (0.21)	0.30 (0.20)	0.24	10.18	22
6C	Subjects	-182.24	0.80 (0.32)	0.46 (0.41)	0.42	44.09	9
6C	LSQ	-217.42	1.09 (0.30)	0.63 (.38)	0.60	41.71	9
7C	Subjects	-123.74	1.08 (0.28)	0.48 (0.28)	0.57	22.87	12
7C	LSQ	-132.96	0.98 (0.26)	0.61 (0.26)	0.59	21.16	12
7C	LSQ[a]	-187.89	1.66 (0.17)	0.71 (0.18)	0.90	13.97	11

[a]On gross inflation rates

Table IV
Estimated Forecast Equations
(Estimated standard errors of coefficients given in parentheses)

Economy	Forecast Equation (a) a_0	a_1	a_2	R^2	DF	Forecast Equation (b) b_0	b_1	b_2	b_3	R^2	DF	Forecast Equation (c) c_0	c_1	R^2	DF	Forecast Equation (d) d_0	d_1	d_2	R^2	DF
1	-0.13 (0.05)*	-0.06 (0.32)	0.12 (0.24)	0.02	12	-0.13 (0.05)*	0.01 (0.35)	0.13 (0.26)	0.04 (0.24)	0.03	10	-0.02 (0.03)	0.39 (0.17)*	0.30	13	-0.10 (0.06)	0.30 (0.45)	0.13 (0.23)	0.04	12
2	-0.15 (0.13)	0.59 (0.33)	0.14 (0.28)	0.54	10	-0.22 (0.10)*	0.27 (0.37)	0.10 (0.26)	0.23 (0.22)	0.61	8	-0.01 (0.03)	0.53 (0.19)	0.41	11	-0.15 (0.13)	0.73 (0.22)*	0.39 (0.22)	0.52	10
3	-0.02 (0.02)	0.74 (0.14)*	0.09 (0.10)	0.56	22	-0.01 (0.01)	0.68 (0.18)*	0.15 (0.09)*	0.21 (0.09)*	0.68	20	0.00 (0.01)	0.13 (0.11)	0.05	23	-0.03 (0.02)	0.65 (0.18)*	0.02 (0.12)	0.41	22
4A	-0.17 (0.05)*	-0.48 (0.35)	0.08 (0.23)	0.16	13	-0.17 (0.03)	-0.48 (0.36)	0.06 (0.23)	-0.16 (0.18)	0.19	11	-0.03 (0.05)	0.33 (0.16)*	0.24	14	-0.15 (0.04)*	-0.68 (0.45)	-0.28 (0.20)	0.15	13
4B	Insufficient Data																			
5A	Target Economy																			
5B	-0.40 (0.12)*	0.05 (0.24)	0.22 (0.19)	0.09	21	-0.42 (0.13)*	0.04 (0.24)	0.13 (0.20)	0.05 (0.18)	0.04	19	-0.03 (0.02)	0.48 (0.14)*	0.35	22	-0.34 (0.15)*	0.39 (0.79)	0.25 (0.17)	0.10	21
6A	Target Economy																			
6B	-0.04 (0.18)	0.65 (0.26)*	0.33 (0.11)*	0.73	7	0.04 (0.17)	0.78 (0.34)*	0.10 (0.18)	0.15 (0.14)	0.80	5	0.01 (0.05)	0.28 (0.15)	0.29	8	-0.19 (0.25)	0.73 (0.35)*	0.29 (0.16)	0.56	7
6C	-0.02 (0.15)	0.58 (0.65)	0.31 (0.29)	0.12	9	-0.01 (0.17)	0.54 (0.70)	0.35 (0.32)	0.02 (0.25)	0.16	7	0.04 (0.08)	0.10 (0.18)	0.03	10	-0.07 (0.14)	0.30 (0.71)	-0.08 (0.25)	0.09	9
7A	Target Economy																			
7B	Target Economy																			
7C	-0.21 (0.05)*	0.25 (0.23)	-0.37 (0.26)	0.17	10	-0.32 (0.08)*	-0.16 (0.36)	-0.32 (0.26)	-0.19 (0.19)	0.34	8	-0.05 (0.08)	0.55 (0.23)*	0.33	11	-0.23 (0.05)*	-0.33 (0.25)	-0.17 (0.18)	0.16	10

*Significant at 5 per cent level.

Forecast Equation (a): $\hat{\pi}^{e}_{t+1} = a_0 + a_1\hat{\pi}^{e}_{t} + a_2\hat{\pi}_{t-1}$; where $\hat{\pi}_t = \frac{p_t}{p_{t-1}} - \pi^L$,

Forecast Equation (b): $\hat{\pi}^{e}_{t+1} = b_0 + a_1\hat{\pi}^{e}_{t} + b_2\hat{\pi}_{t-1} + b_3\hat{\pi}_{t-2}$; $\hat{\pi}^{e}_{t} = \frac{p^{e}_{t}}{p_{t-1}} - \pi^L$, and

Forecast Equation (c): $(\hat{\pi}^{e}_{t+1} - \hat{\pi}^{e}_{t-1}) = c_0 + c_1(\hat{\pi}_{t-1} - \hat{\pi}^{e}_{t-1})$; π^L = Low Inflation Stationary State.

Forecast Equation (d): $\hat{\pi}^{e}_{t+1} = d_0 + d_1\hat{\pi}^{e}_{t-1} + d_2(\hat{\pi}_{t-1} - \hat{\pi}^{e}_{t-1})$;

Table V
Regressions of Subjects' Forecasts on Least Squares Estimators
(Estimated standard errors of coefficients given in parantheses)
Forecast Equation (e)

Economy	e_0	e_1	R^2	DF
1	-0.08 (0.03)*	0.89 (0.31)*	0.39	13
2	-0.10 (0.07)	0.90 (0.14)*	0.80	11
3	-0.04 (0.02)*	0.73 (0.20)	0.36	23
4A	-0.12 (0.02)*	0.57 (0.21)*	0.34	14
4B	Insufficient Data			
5A	Target Economy			
5B	-0.28 (0.10)*	0.54 (0.21)*	0.24	22
6A	Target Economy			
6B	-0.37 (0.11)*	0.64 (0.23)*	0.49	8
6C	-0.19 (0.04)*	0.67 (0.13)*	0.73	10
7A	Target Economy			
7B	Target Economy			
7C	-0.04 (0.06)	0.67 (0.26)*	0.38	11

*Significant at 5 per cent level.

Forecast Equation (e): $\hat{\pi}^{e}_{t+1} = \theta_0 + \theta_1(\beta_t - \pi^L)$; ;

where $\hat{\pi}^{e}_{t+1} = \dfrac{p^{e}_{t+1}}{p_t} - \pi^L$,

$\beta_t = \dfrac{\sum_{s=1}^{t-1} p_{s+1}\, p_s}{\sum_{s=1}^{t-1} p_s^2}$, and

π^L = Low Inflation Stationary State.

Table VI

Local Stability
(Estimated standard errors of estimated coefficients given in parantheses)

Economy Number	Equilibrium Learning Model*	Equilibrium Coefficients ϕ_1	ϕ_2	Estimated Forecasting Coefficients* a_1	a_2	c_1	R^2	DF	Local Stability
1	(c)	0.23	-0.21			0.39 (0.17)	0.30	13	If $c \in (0,1)$, yes; or at 2.5%, not 1%
2	(a)	0.80	-0.40	0.59 (0.33)	0.14 (0.30)		0.54	10	at 25%, not 10%
3	(a)	0.23	-0.20	0.74 (0.14)	0.94 (0.10)		0.56	22	at 5%, not 2.5%
4A	(c)	0.45	-0.29			0.33 (0.16)	0.24	14	If $c \in (0,1)$, yes; or at 5%, not 2.5%
4B	Insufficient Data								
5A	Target Economy								
5B	(c)	1.00	-0.50			0.48 (0.14)	0.35	22	If $c \in (0,1)$, yes; or at 0.25%, not 0.1%
6A	Target Economy								
6B	(a)	0.97	-0.45	0.65 (0.26)	0.33 (0.11)		0.73	7	yes at 40%, not 25%
6C	(a)	0.97	-0.43	0.58 (0.65)	0.31 (0.29)		0.12	9	yes at 40%, not 25%
7A	Target Economy								
7B	Target Economy								
7C	(c)	1.00	-0.50			0.55 (0.23)	0.33	11	If $c \in (0,1)$; or at 2.5%, not 1%

*From Table IV.

Equilibrium Learning Model (a): $\pi^e_{t+1} = a_0 + a_1\pi^e_t + a_2\phi(\pi^e_t, \pi^e_{t-1})$;

$\psi_A(z) = z^2 - (a_1 + a_2\phi_1)z - a_2\phi_2$.

Equilibrium Learning Model (c): $\pi^e_{t+1} = c_0 + c_1\phi(\pi^e_t, \pi^e_{t-1}) + (1-c_1)\pi^e_{t-1}$;

$\psi_C(z) = z^2 - c_1\phi_1 z - (c_1\phi_2 + (1 - c_1))$.

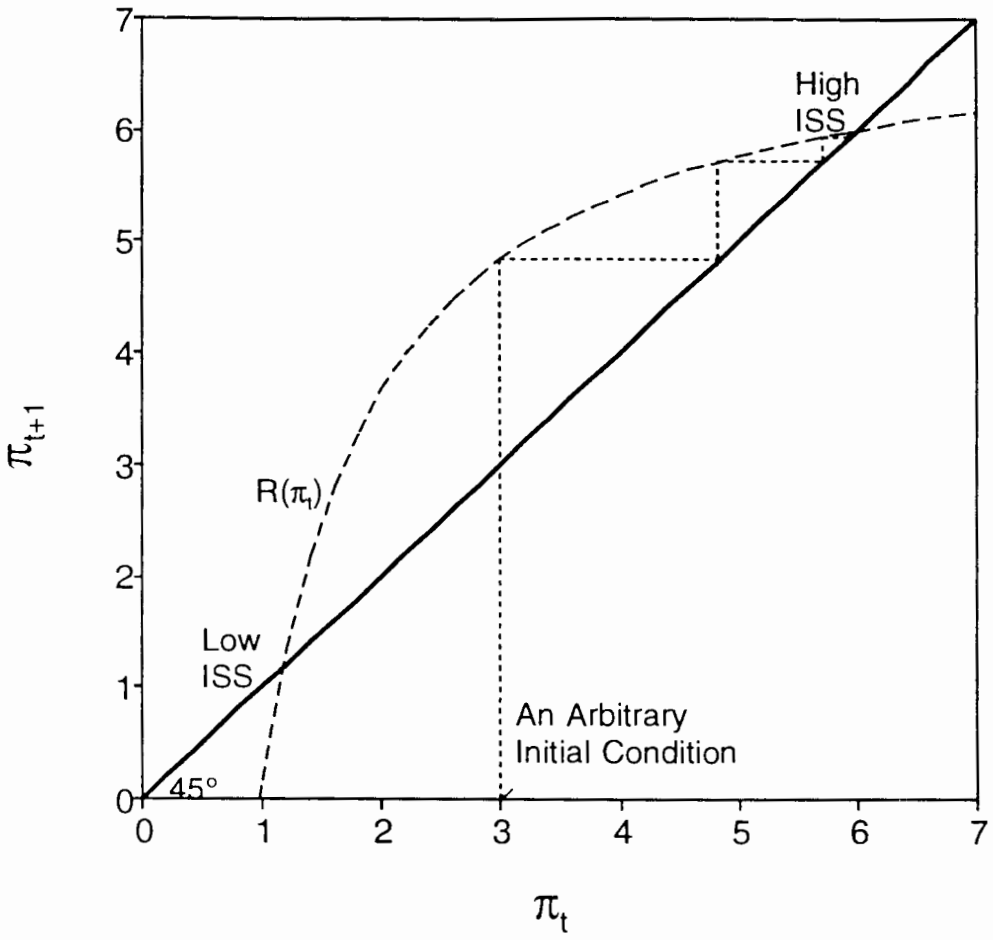


Figure 1.--Rational Expectations Equilibrium Dynamics for Inflation (Eqn. 7b).
(Low ISS = Low Inflation Stationary State,
High ISS = High Inflation Stationary State).

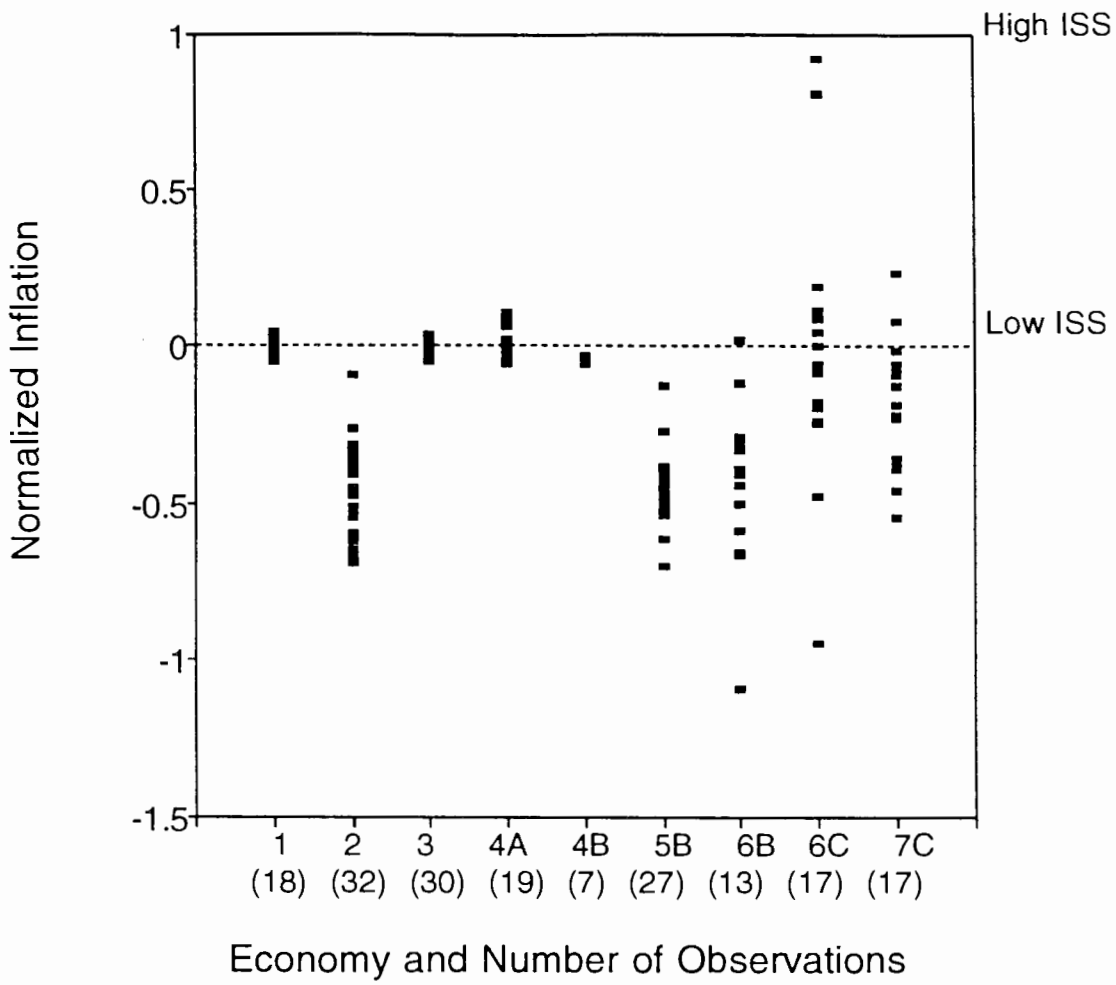
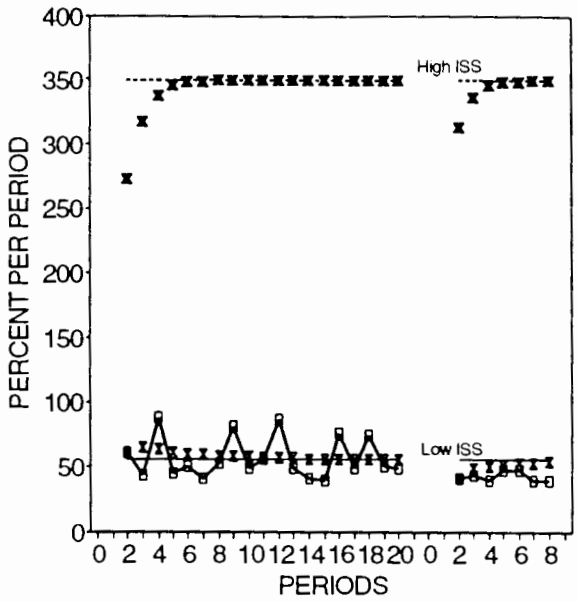


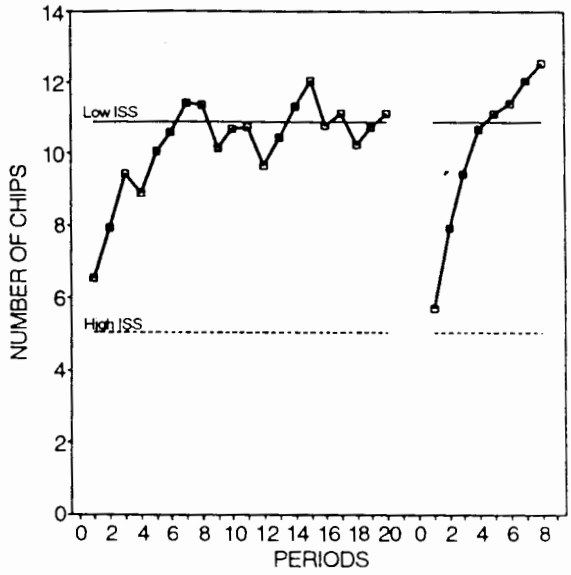
Figure 2.--Observed Inflation Normalized between High and Low Inflation Stationary State:

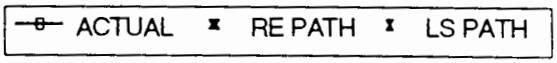
$$\hat{\pi}_t = (\pi_t - \pi^L) / (\pi^H - \pi^L) .$$

Panel A: Inflation (Econ. 4A & B)

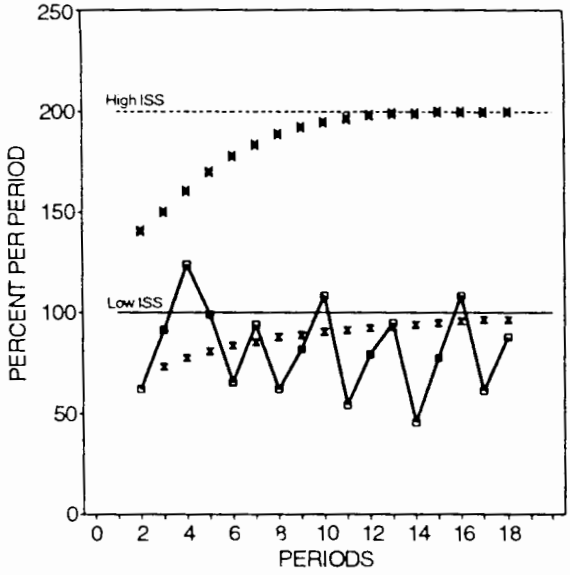


Panel B: Real Balances (Econ. 4A & B)





Panel C: Inflation (Econ. 7C)



Panel D: Real Balances (Econ. 7C)

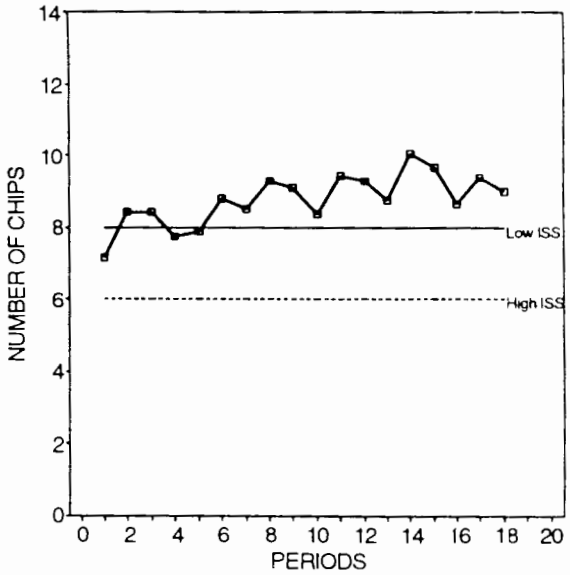
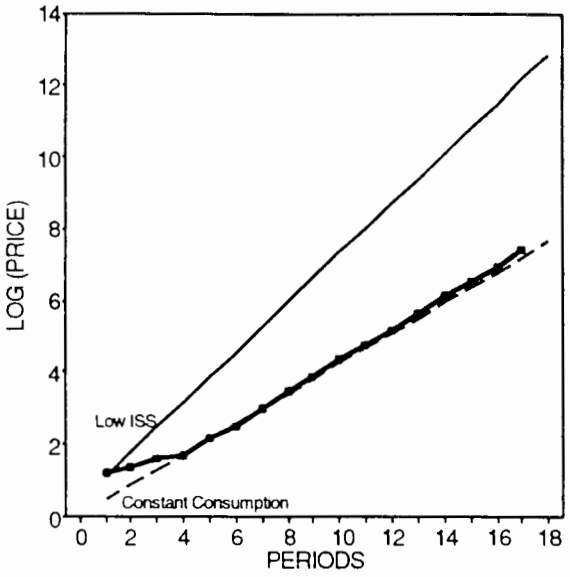
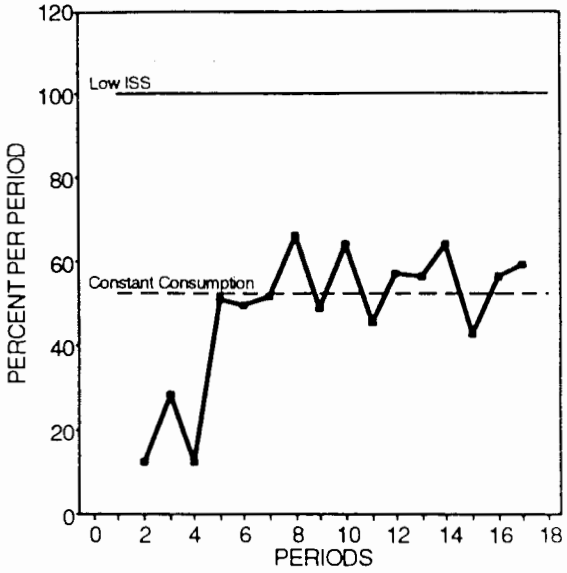


Figure 3.--Inflation and Real Balances for Economies 4A, 4B, and 7C.

Panel A: Price Level (Money/chip)



Panel B: Inflation



Panel C: Money Earned

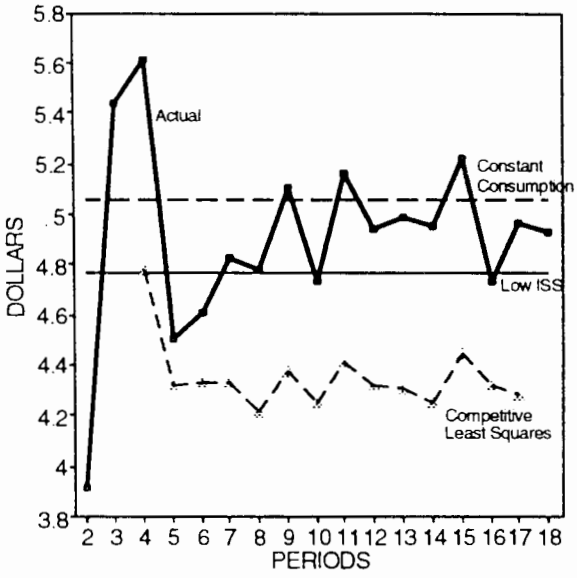


Figure 4.--Evidence on Bias Toward Constant Consumption in Economy 2.

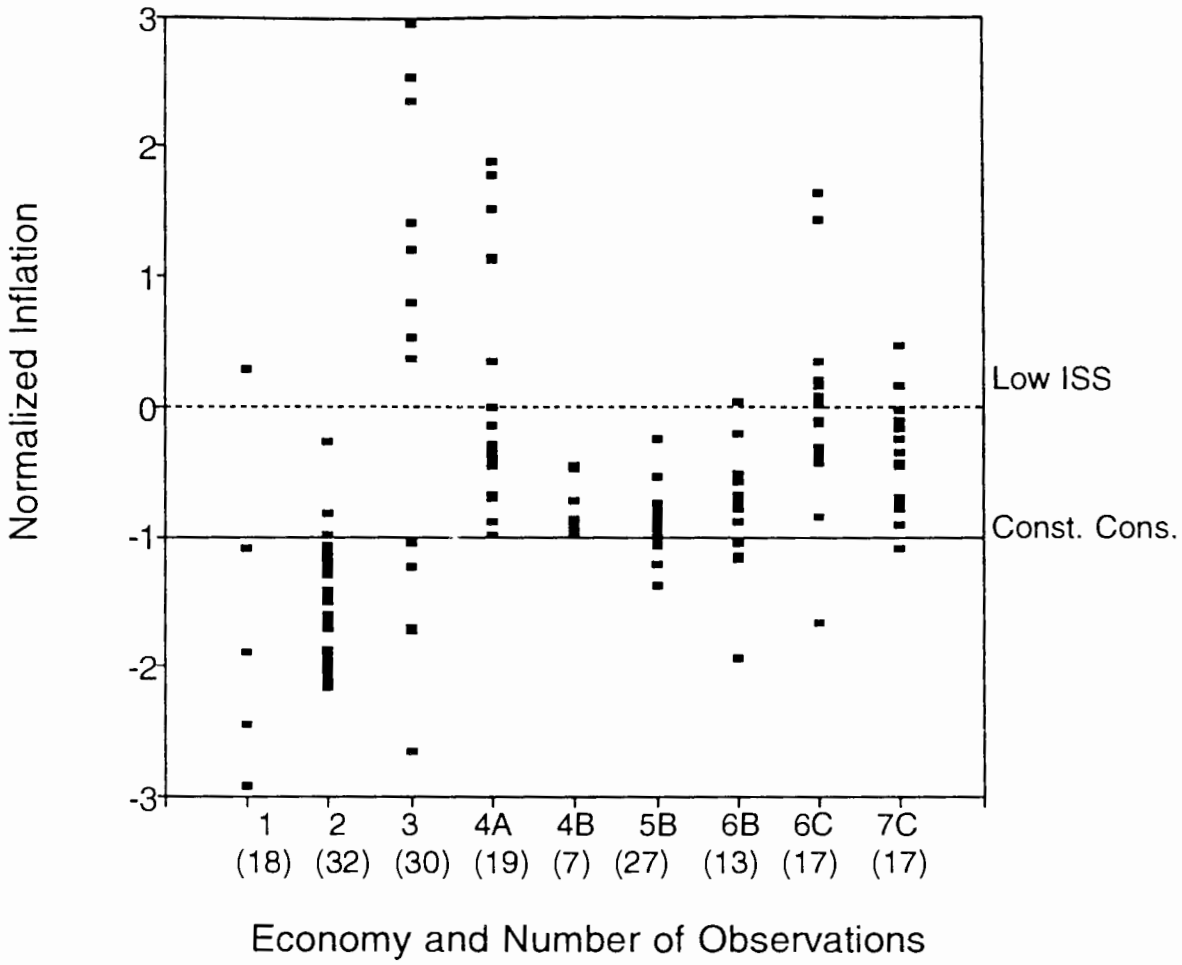


Figure 5.--Observed Inflation Normalized between Low Inflation Stationary State and Constant Consumption Inflation Rate:

$$\hat{\pi}_t = (\pi_t - \pi^L) / (\pi^L - \pi^{CC}) .$$

(Economy 1: Four observations at the top and eight at the bottom are out of scale. Economy 3: Four observations at the top and thirteen at the bottom are out of scale.)

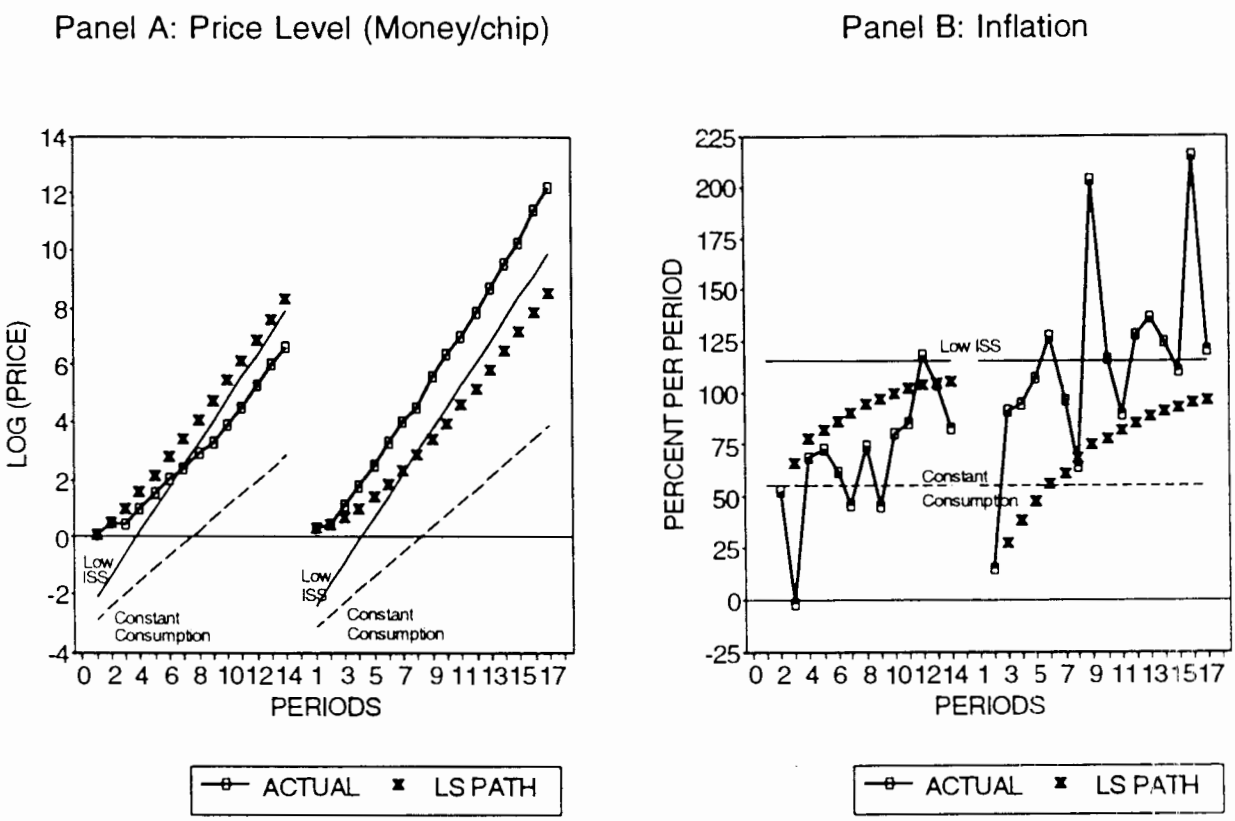
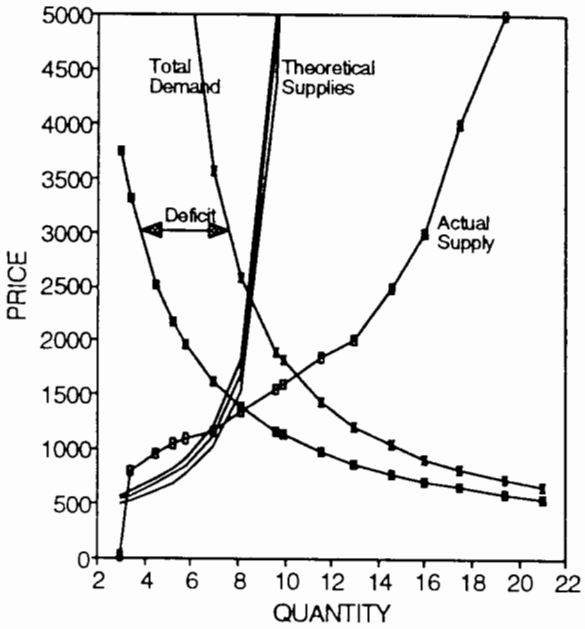
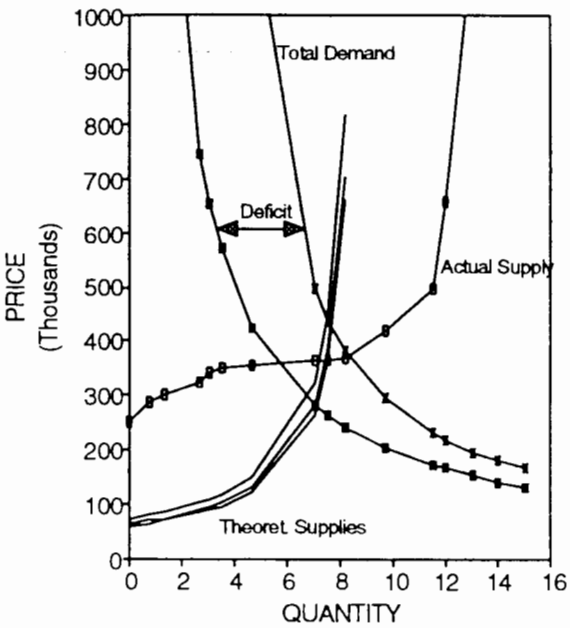


Figure 6.--Price Level and Inflation in Economies 6B and C After Experience with Target Inflation Economy.

Panel A: Economy 2, Period 17



Panel B: Economy 5B, Period 28



Panel C: Economy 7C, Period 18

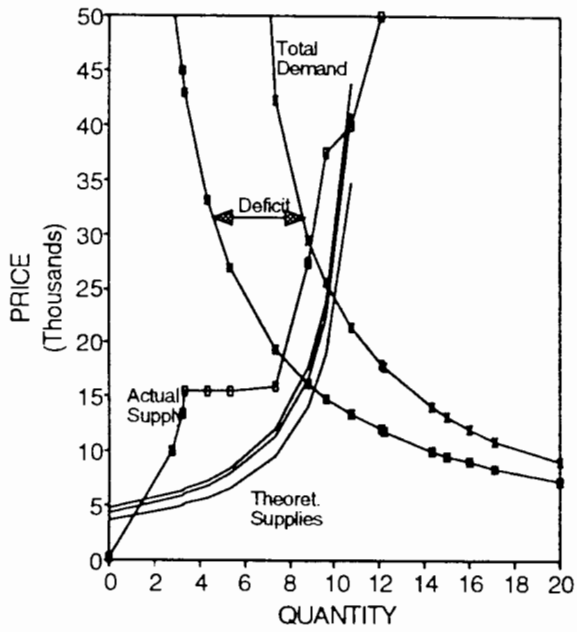
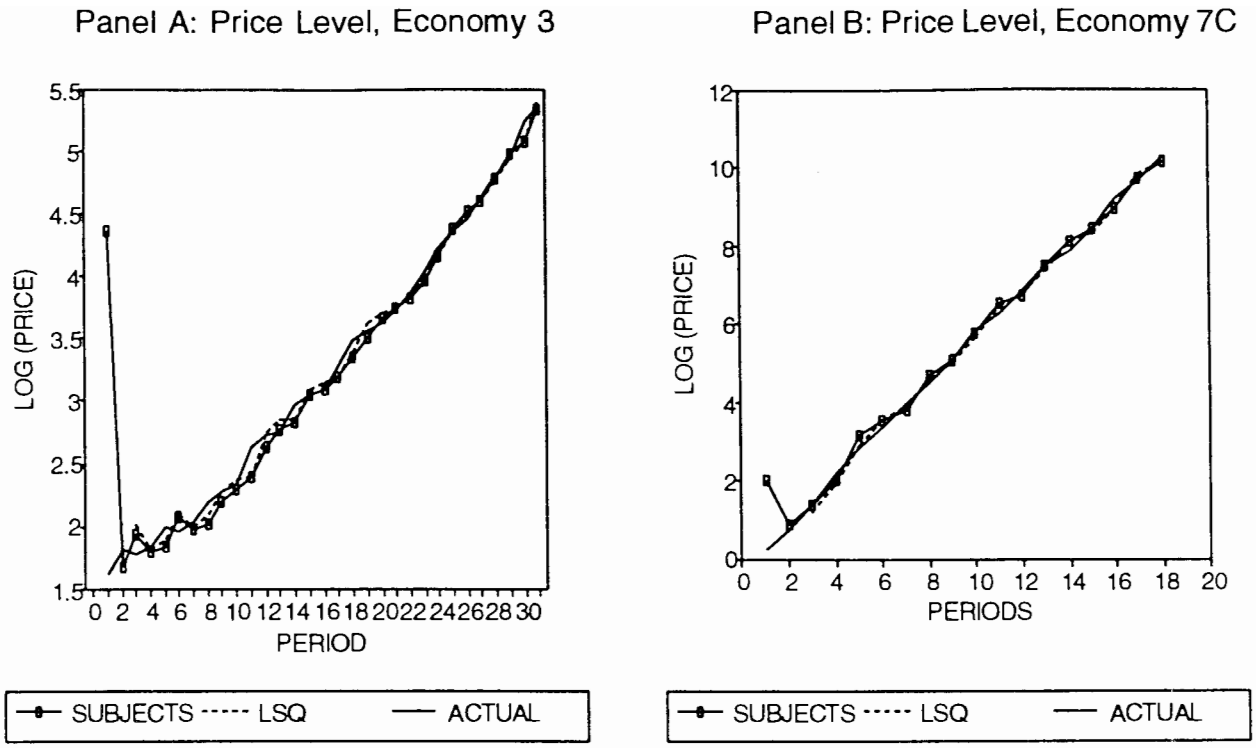


Figure 7.--Demand, and Actual and Theoretical Supplies for Last Periods of Three Economies.



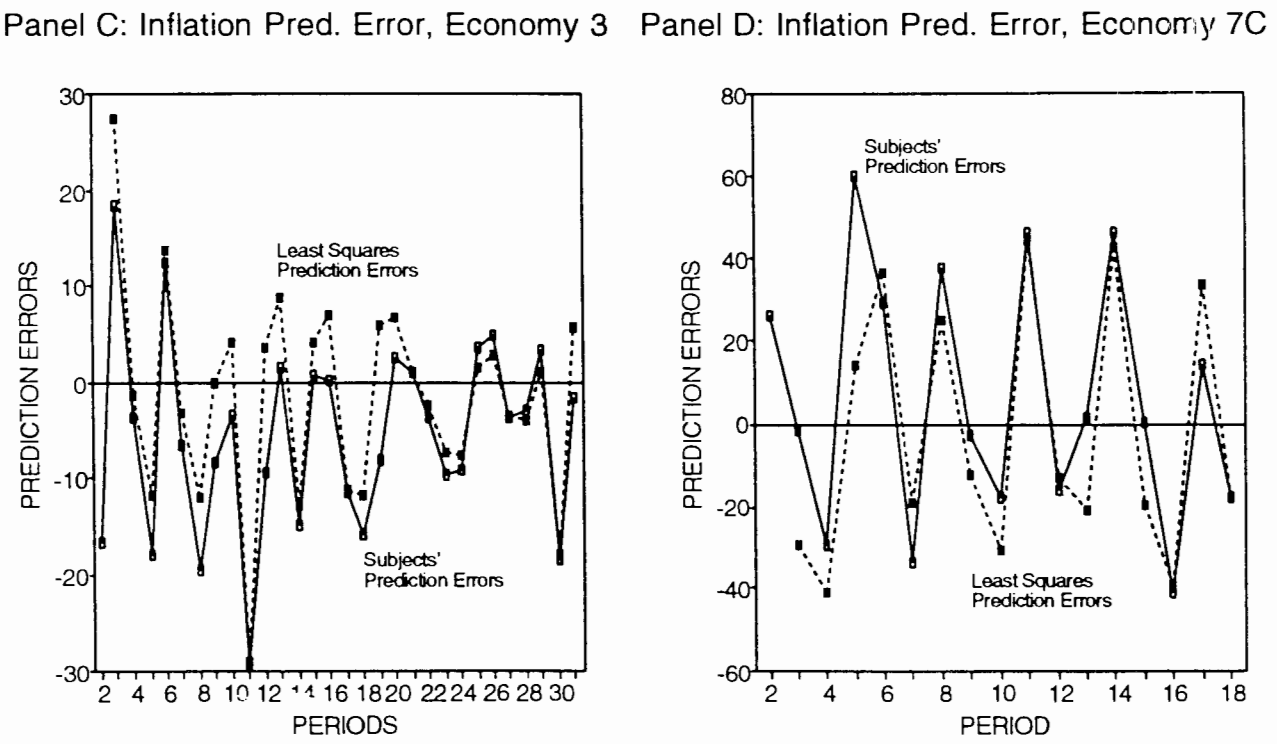


Figure 8.--Accuracy of Subjects' and Least-Squares Predictions of Price and Inflation.

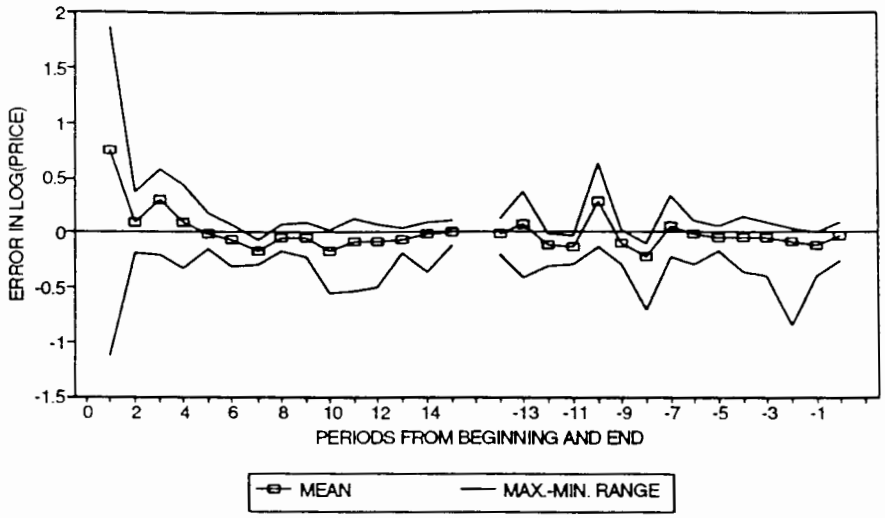
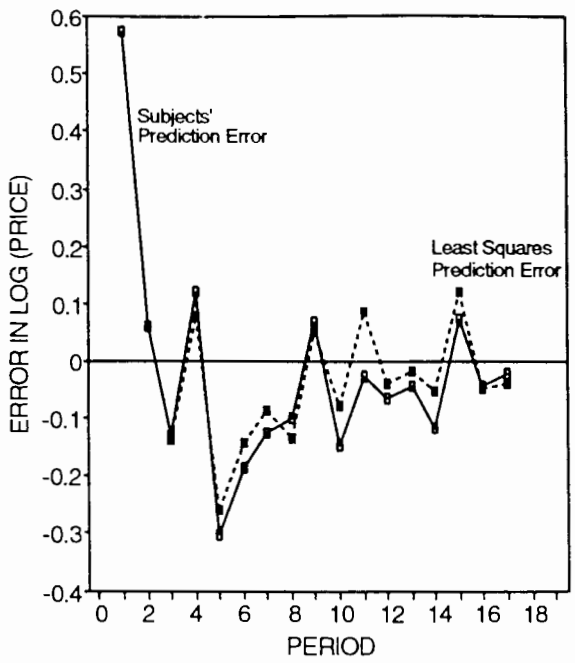
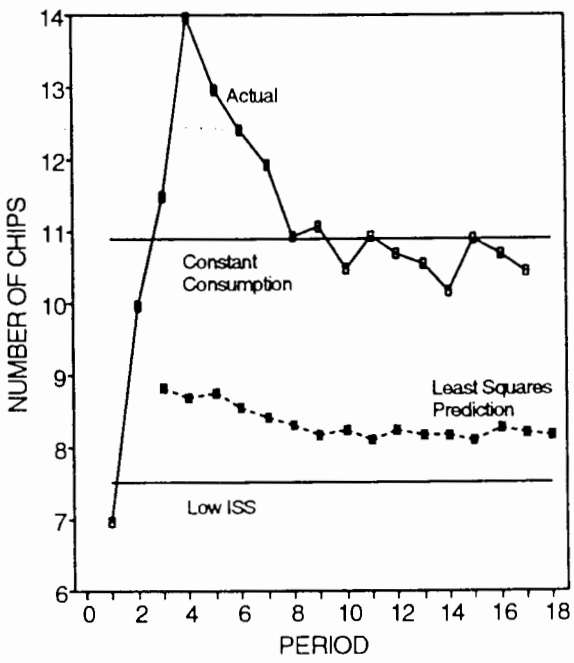


Figure 9.--Cross-Sectional Average of Price Prediction Error Statistics for Economies 1, 2, 3, 4A, 4B, 5B, 6A, 6B, and 7C.

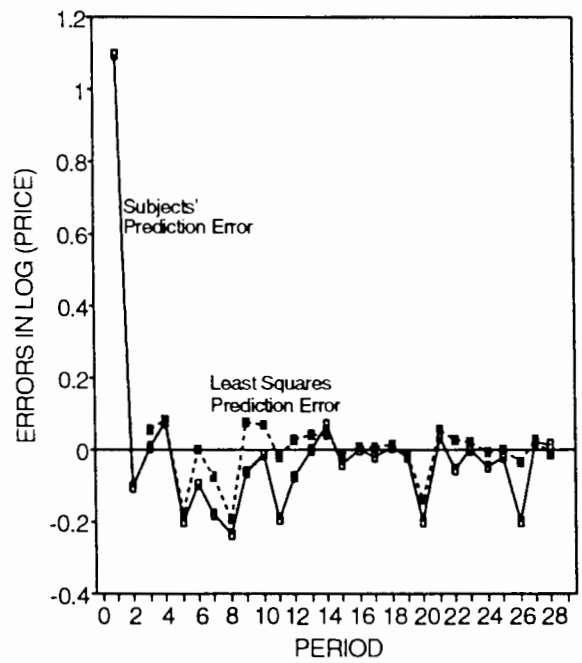
Panel A: Price Prediction Errors, Economy 2



Panel B: Real Balances, Economy 2



Panel C: Price Prediction Errors, Economy 5B



Panel D: Real Balances, Economy 5B

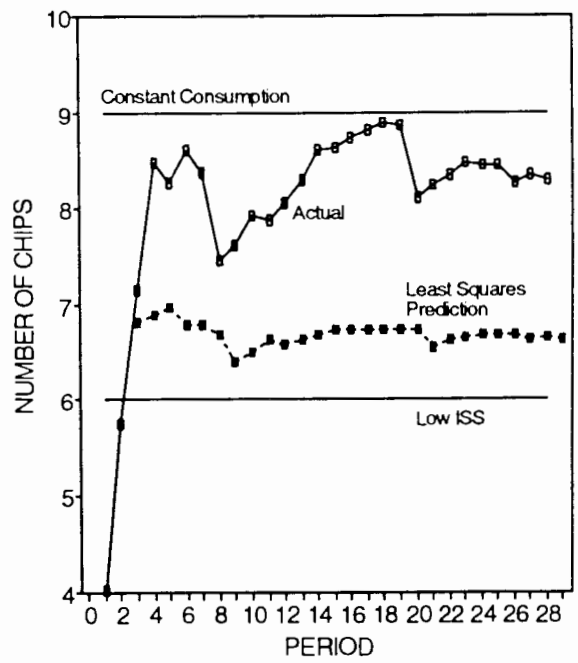
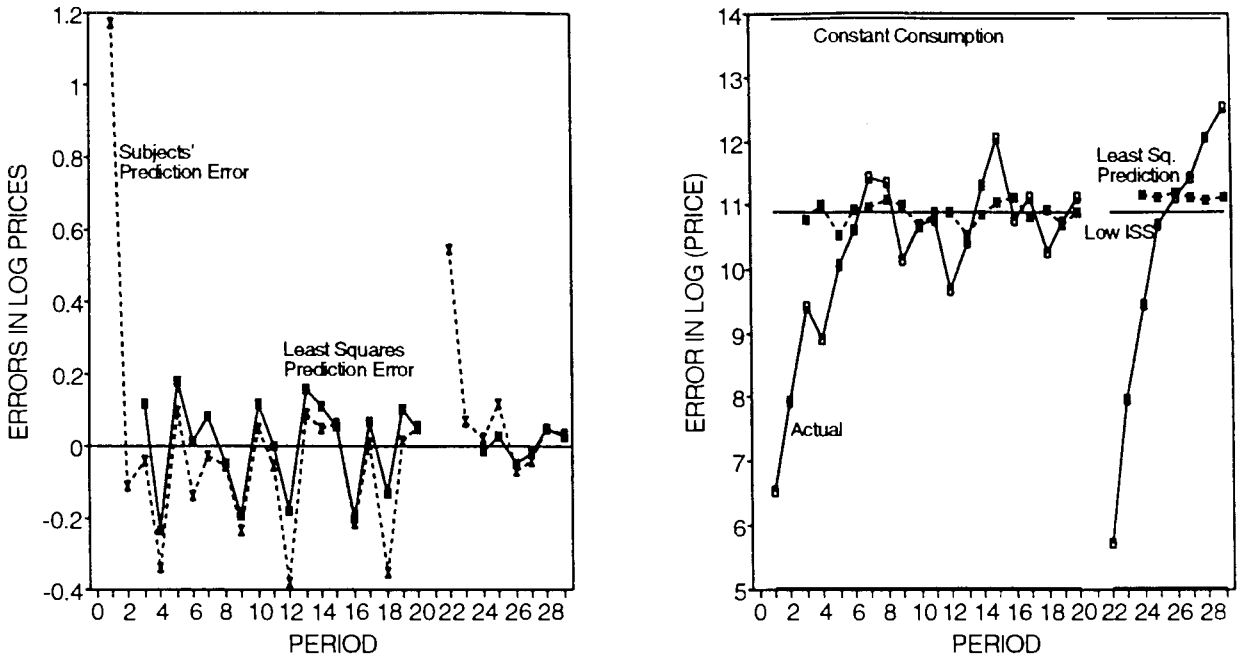


Figure 10.--Price Prediction Errors and Real Balances for Economies 2 and 5B.

Panel A: Price Prediction Errors, Economy 4A,B Panel B: Real Balances, Economy 4A,B



Panel C: Price Prediction Errors, Economy 6B,C Panel D: Real Balances, Economy 6B,C

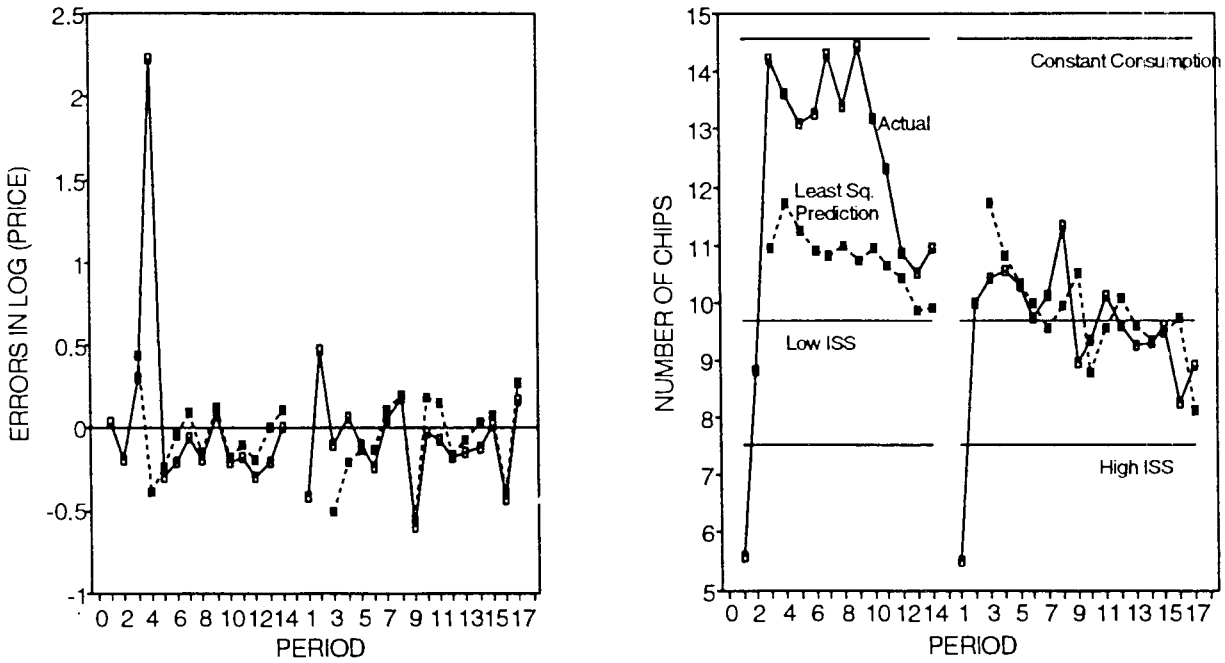


Figure 11.--Price Prediction Errors and Real Balances for Economies 4A, B and 6B,C.

RECENT WORKING PAPERS

1. Albert Marcet and Ramon Marimon
Communication, Commitment and Growth. (June 1991)
[Published in *Journal of Economic Theory* (December 1992)]

2. Antoni Bosch
Economies of Scale, Location, Age and Sex Discrimination in Household Demand. (June 1991)
[Published in *European Economic Review* 35, (1991) 1589-1595]

3. Albert Satorra
Asymptotic Robust Inferences in the Analysis of Mean and Covariance Structures. (June 1991)
[Published in *Sociological Methodology* (1992), pp. 249-278, P.V. Marsden Edt. Basil Blackwell: Oxford & Cambridge, MA]

4. Javier Andrés and Jaume Garcia
Wage Determination in the Spanish Industry. (June 1991)

5. Albert Marcet
Solving Non-Linear Stochastic Models by Parameterizing Expectations: An Application to Asset Pricing with Production. (July 1991)

6. Albert Marcet
Simulation Analysis of Dynamic Stochastic Models: Applications to Theory and Estimation. (November 1991)

7. Xavier Calsamiglia and Alan Kirman
A Unique Informationally Efficient and Decentralized Mechanism with Fair Outcomes. (November 1991)
[Forthcoming in *Econometrica*]

8. Albert Satorra
The Variance Matrix of Sample Second-order Moments in Multivariate Linear Relations. (January 1992)
[Published in *Statistics & Probability Letters* Vol. 15, no. 1, (1992), pp. 63-69]

9. Teresa Garcia-Milà and Therese J. McGuire
Industrial Mix as a Factor in the Growth and Variability of States'Economies. (January 1992)
[Forthcoming in *Regional Science and Urban Economics*]

10. Walter Garcia-Fontes and Hugo Hopenhayn
Entry Restrictions and the Determination of Quality. (February 1992)

11. Guillem López and Adam Robert Wagstaff
Indicadores de Eficiencia en el Sector Hospitalario. (March 1992)
[Published in *Moneda y Crédito* Vol. 196]

23. Ramon Marimon
Adaptive Learning, Evolutionary Dynamics and Equilibrium Selection in Games. (March 1993)
[Published in *European Economic Review* 37 (1993)]

24. Ramon Marimon and Ellen McGrattan
On Adaptive Learning in Strategic Games. (March 1993)
[Forthcoming in *A. Kirman and M. Salmon eds.*"Learning and Rationality in Economics" Basil Blackwell]

25. Ramon Marimon and Shyam Sunder
Indeterminacy of Equilibria in a Hyperinflationary World: Experimental Evidence. (March 1993)
[Forthcoming in *Econometrica*]