On Optimal Location with Threshold Requirements*

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Abstract

The optimal location of services is one of the most important factors that affects service quality in terms of consumer access. On the other hand, services in general need to have a minimum catchment area so as to be efficient. In this paper a model is presented that locates the maximum number of services that can coexist in a given region without having losses, taking into account that they need a minimum catchment area to exist. The objective is to minimize average distance to the population. The formulation presented belongs to the class of discrete P-median-like models. A tabu heuristic method is presented to solve the problem. Finally, the model is applied to the location of pharmacies in a rural region of Spain.

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1 Introduction

Location theory and modeling have been a burgeoning topic in the last decades in both public and private sectors. A myriad of models have been formulated to address different issues related to the spatial organization of activities. These models can be broadly classified in two groups: (1) decision models, where the main focus is to find a given set of locations so as to optimize one or several objectives; and (2), normative models, where the main purpose is to examine the strategic behavior of firms in a competitive environment. Most of the formulations presented in the first group are related to public-sector decision making, and the objectives in general are related to the optimization of some measure of service quality in terms of access (e.g., maximizing service coverage, minimizing average distance to the service). An exception is plant location models, where the main objective is to find the optimal location of plants and/or warehouses that minimizes costs or maximizes profits. In general the spatial representation of these models has been a two-dimensional plane or a network.

The second group, closer to the private sector, is more related to normative models that examine location and price equilibria given a set of assumptions on the strategic behavior of competing firms. In general, their spatial representation is quite simple (a line or a circle), since it easily yields mathematical results related to price setting, production levels and locational strategies.

Recently, several models have appeared to try to link behavioral models with decision models ([14][25][15][17][22]). That is, to use more realistic spatial representations in order to analyze the spatial behavior of firms.

In this paper a decision model is presented to address the problem of locating the maximum number of services in a region to achieve the best possible service given a threshold level below which a given service will not be operational. For example, when locating emergency services we would like to maximize the access to the population given a minimum service level. Another area of application is related to the provision of services that are considered merit goods, but that are serviced by the private sector. This is specially relevant for merit services that have been publicly owned or controlled in several countries and are being transferred to the private sector, such as postal services, gas stations, fire departments and pharmacies. While the planner wants to maintain a good service quality by keeping a balanced spatial distribution of services, these need to have a minimum service threshold level that will allow them to survive. Other fields of applications can be found in the private sector, especially in franchising. The franchiser may want as many franchisees as possible, but needs to ensure that they are able to obtain a minimum level of profits.

The spatial representation is a discrete network, where both demand and potential sites are on the network nodes. The number of services to be located is endogenous and the price is exogenous and the same for all consumers across services.

The paper is organized as follows. In section 2 the decision model is presented. In section 3 a meta-heuristic based on tabu search is developed. Section 4 presents some computational experience in different sized networks. In section 5 the model is applied to the spatial organization of pharmacies in a rural region of Spain within the context of de-regulation. Finally section 6 concludes the paper.

2 The Model

As mentioned before, space is represented by a discrete network, where consumers are located at specific points, such as vertices of the network. Potential locations for the services are also pre-specified.

Suppose that a_i is a measure of demand located on node i. We assume that demand is totally inelastic, that is, that consumers decide to buy a given amount of the good q. The good sold is homogeneous and consumers will purchase it at the closest facility.

Price is set exogenously, and consumers bear transportation costs. The servers have a unit variable cost c related to the amount of service, and a

fixed cost f. We will assume that the cost does not depend on the location of the service, even though the model can be easily modified to take this aspect into account.

Let V be the service volume and I the revenues in a given period. The break-even point to determine profitability is given by f + cV = I. If the average expense per consumer is known, the minimum number of consumers that will make the service profitable can be easily determined.

The formulation of the model is as follows:

$$\min Z = \sum_{i \in I} \sum_{j \in J} a_i d_{ij} x_{ij}$$

Subject to:

$$\sum_{i \in J} x_{ij} = 1 \qquad i \in I \quad (1)$$

$$x_{ij} \leq w_j$$
 $i \in I, \quad j \in J \quad (2)$

$$\sum_{i \in I} a_i x_{ij} \geq C w_j \qquad j \in J \quad (3)$$

$$x_{ij} \in \{0, 1\}, \quad w_j \in \mathcal{N} \qquad i \in I, \quad j \in J$$

where:

i, I = index and set of demand areas

j, J = index and set of potential locations

 a_i = Demand at node i

 $d_{ij} = \text{distance between node } i \text{ and node } j$

C = Minimum amount of customers required to survive

 $x_{ij} = 1$, if node i is served by a facility at node j; 0, otherwise

 w_i = number of facilities located at node j

Variable x_{ij} assigns customers to facilities. Since customers purchase in their closest facility, this variable is binary. Therefore, consumers are assigned

to only one facility. Constraint group (1) forces this to happen. Since each demand node i is assigned to only one facility, the sum of x_{ij} with respect to j has to be equal to 1. But to allow x_{ij} to be one, at least one facility has to be open at node j. The second group of constraints may allow x_{ij} to be one only if $w_j \geq 1$. If $w_j = 0$ assignment will be forbidden. Finally, the last group of constraints determines the possibility (or not) of opening at least one facility at node j. This will be allowed if the total demand assigned to node j is at least equal to the threshold level C.

The objective is to minimize the population weighted average distance.

In this sense it is a P-median objective. As more facilities are opened in different nodes, the lower the objective value will be.

This model is similar to the P-median problem, formulated by [19]. This model seeks where to locate a given number of facilities in a discrete network so as to minimize the weighted average distance. The difference between both models is that while the P-median model assumes that the number of servers is known a priori, in the new model this one is endogenous and is directly related to the minimum threshold level.

The threshold model presented here can be solved using linear programming and branch and bound when necessary (LP+BB) for relatively small networks. But, as it will be seen in the computational experience, this method can become burdensome as the network size increases. First, be-

cause the number of variables and constraints can increase dramatically, even for medium-size networks. For example, if the number of nodes is equal to 50, the problem will have 2500 binary variables x_{ij} and 2600 constraints. If the number of nodes is 100, the number of binary variables jumps to 10000 variables. There are several methods to reduce the number of variables and constraints, but the problem may still be intractable (see [21]). Second, the threshold constraint creates an additional problem in finding integer solutions since the specified parameters are not equal to 0 or 1. this implies that most likely the number of branches can increase dramatically (see [18] on Integer Friendly Programming). Even if a solution of the problem can be obtained using LP+BB, the final assignment of nodes to facilities may not be correct, since we have assumed that consumers go to the closest facility, and some nodes may assign to a facility that is not closest, it is usual in capacitated problems. If this happens, that is, if in the final solution demand node i is assigned to j ($x_{ij} = 1$ and $w_j = 1$) but there is a facility in l that is closer to i than j ($w_l = 1$, $w_j = 1$ and $d_{il} < d_{ij}$), then it is necessary to add the following constraint to the problem:

$$x_{ij} \ge w_j - \sum_{k \in N_{ij}} w_k \qquad (4)$$

where N_{ij} is the set of potential locations that are closer to i than j, and re-solve the model. This constraint works as follows:

if there is no other facility closer to i than j, then $\sum_{k \in N_{ij}} w_k = 0$ and therefore $x_{ij} = 1$. If this is not so, the constraint does not affect the problem ([20] [27]). Nevertheless, by adding this constraint we do not guarantee that an incorrect assignment will not happen again. It may be necessary to add this constraint several times in order to find a final correct assignment.

In the next section we propose a meta-heuristic to solve this combinatorial problem.

3 A Tabu Heuristic to Solve the Model

Several heuristics have been proposed to tackle the P-median problem. These can be grouped in two classes ([13]): construction algorithms and improvement algorithms. In the first ones a solution is obtained from 0. For example, a greedy adding heuristic would belong to the first class. The second class algorithms use a known starting solution and try to improve it. Most algorithms modify the original problem by relaxing some constraints and/or modifying the objective function. Lagrangian relaxation is one of the most widely used methods (see, for example, [5][7]). Other improvement algorithms include exchange heuristics.

Heuristic methods for specific capacitated problems have also been studied by [16][2] (capacitated covering models) and by [3][1] (capacitated plant location problems) among others.

In this paper a meta-heuristic is presented, based on the well-known Teitz and Bart [24] one-opt heuristic, improved by [4]. The flow process has three phases. In the first phase, an initial solution is obtained using a greedy adding heuristic where at each iteration a facility is located in the node than gives the best marginal improvement in the objective without violating the threshold constraint. Phase one is over when no facility can be added without violating the threshold constraint. This determines the initial number of facilities and their locations. Then, in the second phase, a Teitz and Bart heuristic is used. At each iteration a facility is moved from its current position to another potential facility that does not violate the threshold constraint for any facility already located. The objective is computed and kept as the current solution if it has improved. If this is not so, the solution before the one-opt trade is restored. If at the end of all trades the objective has not improved, the heuristic is over. Otherwise, the process is restarted. The problem with this heuristic is that the solution may not be optimal. In the third phase a tabu process is used, having its initial solution the one found in the second phase. Essentially, the tabu heuristic explores a piece of the solution space through a repeated examination of all solution neighbors, and moving to a neighbor even if the objective value obtained with this value is deteriorated ([9][10][11]). This approach avoids being trapped in a local optimum. In order to avoid cycling solutions that have recently been examined these are inserted in a tabu list that is constantly updated. The exchange is allowed even if the threshold constraint is violated. The objective is increased in proportion to the extent of the violation.

This method has been successfully applied to a wide variety of optimization problems (see, for example, [23][6][12]).

Once the final solution is obtained with the initial number of facilities, the first phase is used to add a new facility in the node that gives the lowest violation of the threshold constraint.

Second and third phases of the algorithm are then executed. If at the end of the third phase there is no feasible solution, the algorithm stops. Otherwise, a new facility is added and the procedure starts again.

As mentioned above, In the third phase of the procedure, violations of the threshold constraint are allowed.

A more detailed description of the algorithm follows. Let W_t be the set of locations $w_j, w_j > 0$.

Phase 1

- 1. Set $W_0 := \emptyset$ and p := 1.
- 2. Set W_p := W_{p-1} ∪ v_k, where v_k represents the index of the node that gives the largest decrease in the average distance without violating the capacity constraint:

$$\min_{v_k \in V} \left[z(W_{p-1} \cup v_k) - z(W_{p-1}) \right]$$

IF $W_{p-1} \cup v_k$ is not feasible, go to the second phase; otherwise, p := p+1 a repeat step 2.

Phase 2

- 1. Set $W^* := W_p$ and $z_W^* := z(W_p)$
- 2. Set t := 0, $z_W^0 = z_W^*$.
- 3. Set $W_t := W_{t-1} v_k + v_l$, where $v_k \in W_p$ and $v_l \in (V W_{t-1})$.
- 4. If $z(W_t) < z_W^*$, $W^* := W_t$ and $z_W^* := z(W_t)$, repeat step 2 until all nodes and facilities have been exchanged.
- 5. If $z_W^* < z(W_0)$, set $z_W^0 := z_W^*$ and repeat step 2. Otherwise, go to phase 3.

Phase 3

- 1. Set again t := 0.
- 2. Set $z_W^0 := z_W^*$. No node is tabu.
- 3. Consider all solutions of adjacent nodes W_t^i of W_t , obtained by exchanging a facility from node $v_i' \in W_t$ to node $v_i'' \notin W_t$. Relabel the solutions

 W_t^i in decreasing order of $z(W_t^i)$. Relabel all vertices accordingly. Set i := i + 1.

- 4. If z(Y_tⁱ) < z_W^{*} or if v_i" is not tabu, set W_{t+1} = W_t, z_W^{*} := z(W_t), declare v_i' tabu until t + θ, where θ is a pre-fixed value, and go to step
 5. Otherwise, set i := i + 1. If i is larger than the number of adjacent solutions, set i equal to the index of the vertex v_i" with the lowest tabu tag t + θ and lift the tabu status of v_i". Repeat step 3.
- 5. Set t := t + 1. If t is less than a pre-fixed upper bound T, go to step 2 of phase 3. Otherwise, set p := p + 1 and go to step 2 of phase 1 if the solution found is feasible. If in the last iteration no feasible solution is found, stop.

Some observations on the heuristics:

- In step 2 of phase 3, the tabu status can be canceled if this implies an improvement in the objective. This rule is known as aspiration criterion.
- In phase 2, only the nodes that satisfy the threshold constraint are visited.
- In phase 3, nodes that violate the threshold constraint are visited, and the corresponding objective is penalized in proportion to the extent of

the violation. This is to avoid being trapped in local optima. This procedure is also performed in other types of heuristics such as simulated annealing (see [26]).

In the following section some computational experience is presented.

4 Computational Experience

The algorithm has been applied to several randomly generated networks. First, for each network, $n \in [0, 50]^2$ nodes were generated following a uniform distribution. The connecting arcs were also generated randomly to ensure the full connectivity of the network, with a minimum of 3 arcs and a maximum of 8 arcs per node. Observe that this figure determines the number of adjacent neighbors in phase 3. Once the arcs were computed, the Euclidean distance between directly connected nodes is computed. Finally, a shortest path algorithm is used to obtain the distance matrix. The demand in each node was generated randomly within the interval [0, 1000] following again a uniform distribution. For each network, several threshold levels were set as a function of $(\sum_{i \in I} a_i/n)\alpha$, were α is a parameter that is modified to see the effects of the threshold on results: the higher α is, the higher the threshold level required to ensure survival. For each n 200 networks were generated and for low values of n heuristic solutions were compared to optimal ones. Optimal solutions were obtained by enumeration. In phase 3 of the algorithm

T was equal to 20 and θ was within [5, 10], following the suggestion by [12]. The algorithm was coded in FORTRAN77 and executed with a Pentium PC 75 with 24mb of RAM. Results are shown in Tables 1 and 2

In column 3 of Table 1 the percentage of times that the optimal solution has been found for each value of α . For example, for $\alpha=4$, 91.9% of the solutions found with the heuristic were optimal. In the fourth column the percentage of times that optimal solutions were found after Phase 2 of the algorithm, making it unnecessary to execute Phase 3. Again, for $\alpha=4$, the percentage of times that Phase 2 found the optimal solution was equal to 40.9%. Finally, the last column indicates the percentage of times that Phase 3 found an optimal solution out of the 200 runs. This one was equal to 5.6% when $\alpha=4$. As Table 1 shows, the proposed heuristic achieved good results in finding solutions, except for $\alpha=2$. As α increases more optimal solutions are found with phase 3, showing that phase 1 and 2 (and specially phase 1) were not very efficient in finding solutions.

Other results from the heuristic are presented in Table 2. In the third and fourth columns the average initial and average final number of facilities respectively are presented for several values of α . In the fifth and sixth columns the percentage of times that Phase 2 improved the initial solution found in phase 1 is shown (column 5), and the solutions that were improved when using Phase 3 after Phase 2 was terminated. All percentages refer to

the 200 runs. As can be seen in this table, Phase 2 achieved good results in improvements, while Phase 3 seems to improve as the threshold level increases. Nevertheless, it may be possible that several solutions obtained in phases 1 and 2 were optional and therefore phase 3 was not able to improve them. Finally, the last column indicates the average execution time.

5 An Example: The Spatial Distribution of Pharmacies in Catalonia, Spain

The pharmaceutical sector at the retail level is regulated by the Catalan and Spanish Governments (Law 31/1991, December 1991, Government of Catalonia). For each product sold there is a fixed margin that the pharmacy can charge (28.3% over the drug cost). The spatial distribution of pharmacies is also regulated. The health map of Catalonia divides the region into health areas that are classified rural, semi-urban and urban. These areas may have one or more municipalities. For each area there is a maximum permissable number of pharmacies determined in relation to the population served. This number is currently set at one pharmacy per 3000 inhabitants. So if the region has 30000 inhabitants, a maximum of 10 pharmacies are allowed to co-exist. Additionally, no pharmacy may be closer than 250 meters from another pharmacy, and 225 meters from a Primary Health Center. Other regulatory measures include the requirement that the owner hold a Degree

in Pharmacy from an approved university and that the pharmacy be manned by such a graduate. Opening hours are also fixed.

The debate in Spain (as in most western countries) is whether or not to spatially de-regulate this sector. The Spanish Antitrust Commission argues that both spatial and price regulations imply that the pharmacy becomes a local monopoly with the consequent loss of efficiency. The Commission argues that de-regulation would increase consumers' welfare.

The basic question is that price regulation has an important effect on accessibility to the service. The dilemma is basically the following: The setting of a lower (higher) price implies higher (lower) consumer welfare. On the other hand, it implies a lower (higher) accessibility to the service, since the number of pharmacies will be lower (higher), and the consumer welfare will be decreased (increased). In this section we examine this issue by assuming that regulation is price-based but does not determine spatial distribution. That is, pharmacies can open anywhere but have to follow a price margin. We assume that consumers go to the closest pharmacy, since the product sold is homogeneous.

The region studied is located in the Pyrenees, and consists of 61 towns. Population figures for each town are presented in Table 3. The current number of pharmacies is 70 (column Ph91 of Table 3). As mentioned above, we consider that the market is regulated in price terms but not spatially. The

threshold level is set as follows. In Spain, the per capita expenditure on pharmaceutical products is 20,190 ptas per year (around US\$144, 1991 figures), of which 5,694 correspond to the pharmacy's profit margin. That is, margin is around 39.28% over the cost that pharmacies pay for their products, or 28,2% over final price. Pharmacies cost structure was obtained from [8]. Variable costs represent only 1.79% over total revenues, and they correspond basically to financial costs. Average fixed costs per year across pharmacies is equal to 7,974,532 ptas (around US\$ 57,000).

With this information and once the margin is set, it is possible to know the minimum population threshold so as to at least have 0 profits. Table 3 shows the results obtained given different profit margins. If the margin is set to 28% (close to the actual one) the number of pharmacies that can co-exist is equal to 106, compared to the current 70 pharmacies. As the margin increases more pharmacies are located, since we have assumed that the product is totally inelastic.

6 Conclusions

In this paper a new location model has been presented to study the issue of minimum threshold requirements to survive in a given spatial setting. The basic model is very relevant when planning service delivery where there are merit goods involved, such as, for example, postal offices, pharmacies or tax offices, since it aims to increase social welfare by making the service as close as possible. On the other hand, the threshold constraint can be considered as a search for efficiency, in the sense that each facility needs to have a given thershold area below which it is not viable. A tabu search algorithm has been developed to solve the problem. Finally, it has been applied to the location of pharmacies in a rural region of Spain.

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Table 1: Comparisons between the Heuristic and Enumeration algorithms

n	α	% of	% of	% of	% Average
		Optimal	Optm. sol	Optm. sol	Deviation
		Solutions	T&B	Tabu	from Optm.
15	1	93.0%	35.0%	2.0%	8.0%
	2	76.8%	39.4%	5.1%	7.4%
	4	91.9%	40.9%	23.9%	5.6%
	6	99.3%	54.4%	19.4%	4.7%

Table 2: Results from Heuristic Solutions

n	α	Average nu	mber	% of in	$_{ m nprovements}$	Average
		Beginning	End	T&B	Tabu	Time
15	1	8.6	8.7	47.0%	3.0%	1.29s
	2	4.7	4.9	67.8%	15.4%	0.54s
	4	2.6	2.9	69.2%	28.7%	0.26s
	6	1.9	2.0	61.5%	19.7%	0.19s
25	2	7.5	7.6	68.0&	6.0%	1.73s
	4	4.1	4.4	63.0%	24.0%	0.65s
	6	2.9	3.2	54.0%	25.0%	0.38s
50	2	12.8	12.9	70.0%	0.0%	12.3s
	4	7.2	7.4	67.0%	3.0%	$7.4\mathrm{s}$
	6	5.2	5.5	83.0%	9.0%	6.1s

Table 3: Number of Pharmacies

30%	0	0	-	П	0	0	П	10	0	0	_	0	0	0	-	0	0	0	0	0	0	2	0	2	0	0	0	2	0	0	3
28%	0	0	Τ	-	0	0	-	10	0	0	_	0	0	0	_	0	0	0	0	0	0	2	0	2	0	0	0	2	0	0	3
36%	0	0	Т	1	0	0	1	6	0	0	Т	0	0	0	Τ	0	0	0	0	0	0	2	0	2	0	0	0	2	0	0	3
24%	0	0	1	1	0	0	1	∞	0	0	_	0	0	0	П	0	0	0	0	0	0		0	2	0	0	0		0	0	2
22%	0	0	-	-	0	0	0	∞	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0	-	0	0	0	-	0	0	2
20%	0	0	1	2	0	0	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0		0	П	0	0	0		0	0	2
18%	0	0	0	ಣ	0	0	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0	-	0	0	0	-	0	0	1
16%	0	0	0	2	0	0	0	ಬ	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0	-	0	0	0	-	0	0	1
14%	0	0	0	2	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0	0	0	-	0	_	1
Phs 1991	0	0		_	0	0	0	വ	_	0	-	0	0	0	-	0	0	0	0	0	0	-	0	-	0	0	0	-	0	-	П
1991 Pop.	32	328	2.129	1.254	308	33	1.665	14.007	546	140	446	41	28	413	1.772	162	148	246	239	138	199	2.330	122	3.388	256	150	429	2.218	373	647	3.896
Municipality	Gisclareny	Saldes	Baga	Guardiola de Berguedà	Vallcebre	Figols i Alinya	Cercs	Berga	Vilada	La Nou de Bergadà	Borreda	Castell de l'Areny	S. Jaume de Frontanyà	Les Lloses	La Pobla de Lillet	Castellar d'Nugh	Toses	Planoles	Gombren	Campelles	Queralbs	Ribes de Fresser	Pardines	Campdevanol	Ogassa	Setcases	Vilallonga	Camprodon	Mollo	S. Pau de Seguries	S. Joan de les Abadesses

Table 3 (continued): Number of Pharmacies

Municipality	1991 Pop.	Phs 1991	14%	16%	18%	20%	22%	24%	26%	28%	30%
La Vall de Bianya	1.051	П	0	0	0	0	0	0	0	0	1
Olot	26.713	14	_∞	6	11	13	14	16	18	21	22
Riudaura	364	0	0	0	0	0	0	0	0	0	0
Vallfogona de Ripollés	252	0	0	0	0	0	0	0	0	0	0
Ripoll	11.334	ы	ಣ	4	4	22	9	9	7	∞	6
Montesquieu	855		П	1		Т	_	0	П	0	0
S. Quirce de Bessora	2.028		0	0	0	0	_	2	П	2	2
S. Agustí de Lluçanes	123	0	0	0	0	0	0	0	0	-	1
Sora	211	0	0	0	0	0	0	0	0	0	0
Alpens	249	0	0	0	0	0	0	0	0	0	0
Lluca	282	0	0	0	0	0	0	0	0	0	0
Sagas	194	0	0	0	0	0	0	0	0	0	0
Prats de Lluganes	2.625		П	1		2	2	2	3	2	ಣ
S. Martí d'Albars	125	0	0	0	0	0	0	0	0	0	0
Olost	096	П	0	0	0	0	0	0	0	0	0
Perafita	338	0	0	0	0	0	0	0	0	0	0
Sobremunt	91	0	0	0	0	0	0	0	0	0	0
S. Hipolit de Voltregà	2.981	П	0	1	-	Т	П	1	1	2	2
Manlleu	16.148	9	ಬ	ы	9	۲-	∞	6	10	11	13
S. Boi de Lluçanes	530		0	0	0	0	0	0	0	0	0
Torelló	11.460	ಬ	3	4	4	ಬ	9	9	-	∞	6
Oris	226	0	0	0	0	0	0	0	0	0	0
Quar	43	0	0	0	0	0	0	0	0	0	0
Olvan	983	-	0	0	0	0	0	0	0	0	П
Gironella	5.009	ಣ	1	2	2	2	3	3	4	4	4
Sta. Maria de Merles	200	0	0	0	0	0	0	0	0	0	0
S. Bartolomeu del Grau	1.121	-	0	0	0	0	0	0	0	0	0
Gurb	1.674	-	0	0	-	1	_	П	_	2	2
Vic	28.736	11	6	10	11	13	15	17	19	21	23
Total	154.989	20	40	48	22	65	14	82	92	106	211
Distancia Media Recorrida			$1,65~\mathrm{Km}.$	1,2 Km.	1,14 Km.	1,12 Km.	1 Km.	$0,7 \mathrm{~Km}.$	0,69 Km.	0,64 Km.	$0,56 \mathrm{~Km}.$