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The Coherent Covering Location Problem

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Abstract

In a location-allocation system, it is necessary to obtain not only an efficient set of facility locations, but also an efficient relationship among the catchment areas. This is specially important in services that are hierarchical in nature and where there is a strong relationship between the different levels. In seeking an effective relation among the different levels, all the areas assigned to a particular facility at one hierarchical level should belong to one and the same district in the next level. This property leads to a "coherent" districting structure. In this paper the concept of coherence is introduced within a network location covering framework: the Coherent Covering Location model locates two hierarchical facility levels by maximizing the coverage while ensuring coherence. Adaptations of the model to different location patterns and computational experience are provided.

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1. Introduction.

Space has a profound impact on the organization of economic and social activities. The discipline of location-allocation theory addresses questions related to the spatial organization of activities. A myriad of different formulations have been designed in the past decades to locate many types of public and private sector facilities by optimizing some specified measurable criteria. Some of these models have studied the location of facilities that are hierarchical in nature, such as health care services, bank services or postal facilities. While most hierarchical location models were designed to obtain an optimal location of resources in space, little or no importance was given to the spatial organization obtained once the locations were found.
The division of a region into districts according to some specific criteria also plays a very important role in the spatial organization of services. To achieve an efficient and effective hierarchical system, it is necessary to obtain not only an efficient set of locations but also an efficient districting of the areas served, since these areas will be the ones to benefit from the services provided by the located facilities. In seeking a more responsive and effective relation among the different levels in a hierarchy, all the districts corresponding to a given hierarchical level should belong to one and the same district in the next level of the hierarchy. This property leads to a "coherent" districting structure. Coherent districts and linkages can define optimal capacities for the facilities, integrated planning units for socio-economic studies, and administrative regions for better management.

The research presented here concerns the optimal location and districting of hierarchical facilities. Among the formulations proposed to solve location-allocation problems, two of these, the P-Median Problem (ReVelle and Swain, 1970) and the Maximal Covering Location Problem (MCLP, Church and ReVelle, 1974) have played particularly dominant roles. The key difference between p-median models and covering models is that while p-median models define a clear assignment of the demand areas to the nodes where services are located, covering models determine only the sites for services, and not the assignment. In a covering model a node can be covered by more than one facility, and assignment to the closest facility is done after solving the model - even if the demand area is not covered, or is not within a given distance to any facility.

The first atempt known to the author to combine hierarchical location and coherent districting with a network location framework has recently been studied in the PQ-Median Model (Serra and ReVelle, 1992). The pq-median problem relies on the assignment characteristic of the p-median to obtain coherent locations and districts. The Coherent Covering Location Problem (CCLP), presented in Section 3, will relax the strict assignment constraints of the pq-median and attempt to obtain coherence to the maximum extent possible. As in the pq-median problem, facility location in a two-level hierarchy will be addressed.

The CCLP basically integrates two traditional maximal covering formulations, one for each level of the hierarchy, together with a constraint to achieve coherence. This model attempts to locate facilities in a two-level hierarchical system so that the maximum population is covered within some distance threshold, given the achievement of covering coherence. The number of facilities to be located in the first level of the hierarchy will be larger than the number of facilities in the second level. The CCLP can be reformulated using a p-median objective in one of the hierarchical leverl. Therefore, the p-median q-covering problem described in Section 4 will use a p-median formulation for one of the hierarchical levels, and a maximal covering formulation for the other hierarchical level, together with a constraint on coherence. Computational experience and results for both models are described in section 5.

In fact, some type of services do not require a strict assignment. Covering models give excellent results when locating services such as health care emergency services and fire services, since the main objective is to cover the maximum population within a given distance or travel time. In these types of services, assignment does not play a crucial role. On the other hand, the characteristic of coherence can also be important in these types of services. Although assignment and districting are not crucial in the siting of these services, it may be necessary to cover a given are by two types of facilities that are related to each other. Coherence, in covering terms, is achieved when all areas covered by a facility at level one are also covered at least by one and the same facility at level two. Observe that in this case, we do not specify that the area has to be covered by only one and the same facility. Coherence coverage allows a node to be covered by more than one facility, but it stresses coverage of all nodes covered by a type A facility by one and the same type B facility.

Several considerations must be taken into account to utilize the concept of covering with coherence. First of all, covering models derived from the MCLP attempt to cover as many people or as much area as possible within a distance standard, but this objective admits that not everybody or all areas will be covered. Coverage depends on the distance standard used and the number of facilities located. The larger the distance standard and the greater the number of facilities, the more likely it is that coverage will be complete. Second, it may be possible that coherence is not fully achieved in both the CCLP and the p-median q-covering model, since it depends on the covering distances chosen at each hierarchical level. The formulation and its explanation will clarify the idea of coherence in the covering models.

The concept of coherence has already been considered in the early works of Christaller (1933). The theory of Central Places defines a hexagonal spatial pattern where there are well defined relations between size and type or function of the centers and their distances. In this context, coherence is achieve in a k-7 hierarchy where a hexagonal area corresponding to a lower center is completely within the hexagonal area associated with the next higher hierarchical center.

2. Hierarchical Covering Location Models.

Hierarchical systems generally apply to large regions or countries, as well as urban areas. The minimum number of levels within the hierarchy is two. The literature on network location contains only several articles on hierarchical systems were the models are generally integrated, i.e., the solution to the model is provided for all levels. Another characteristic is that, in general, lower hierarchical levels have more service centers than higher ones. The number of services increases as we go up the hierarchy to the higher levels.

The concept of a hierarchy has been invoked frequently in the locational modeling of urban service systems and though much analysis has been performed within multilevel frameworks, most studies have presented primarily structural elaborations of more basic single level location problems such as the p-median or MCLP (O'Kelly and Storbeck 1984). The MCLP has been successfully used to locate any given number of hierarchical facilities so that the area or population covered within a given distance or time standard is maximized. Covering problems are very well suited to the location of hierarchical services.

Banergi and Fisher (1974) integrated the Set Covering Problem and the p-median problem to locate different hierarchical services (hospitals, schools, markets, post-offices. etc.) in a region of rural India. As they point out, "these two algorithms may be combined provided that violations of the maximum travel distance standards are acceptable for some settlements"(p.55). Hence, the Set Covering Model determined the minimum number of centers to cover all settlements, and the p-median located the centers minimizing users costs. First, they solved for the location for the highest level in the hierarchy taking into account existing centers following several ranking techniques based on service-area populations and other parameters. Then, after fixing the location of the highest-level facilities, the next lower level was solved using the same procedure. In the solution, changes from the p-median solution did not violate to a large extent the maximum distance chosen for the Set Covering Problem.

Charnes and Storbeck (1980) formulated a goal programming model to locate emergency medical services. Two levels of vehicles (services) were defined: Basic Life Support (BLS) and Advanced Life Support (ALS). The objective was to cover all the demand area by vehicles. If demand cannot be covered by any vehicle within a given distance criterion, then at least the BLS service should cover it.
Moore and ReVelle (1982) defined a nested hierarchical maximal covering model for locating clinic and hospital facilities. The services were nested; hospitals provide clinic services as well as hospital services. No referral patterns were considered between the levels. Demand areas were conceptually assigned to the closest clinic and closest hospital for the level of services required, but there is no spatial relation such as coherence whatsoever between the different facilities. The integer model was solved by relaxed LP and was applied in Honduras. The authors extended the model to take into account the case where coverage by less than all types of facilities is also relevant (Moore and ReVelle 1983).

Church and Eaton (1987) presented two 2-level hierarchical formulations addressed to the location of health care facilities which take into account referral patterns. The two models depend on the relative importance of (1) health professional referrals (or top-down referrals) from higher to lower levels, and (2) patient referrals (or bottom up referrals) from lower to higher health care levels. Both models used a multiobjective integer approach. The first model maximizes the number of people covered by clinics and minimizes the number of clinics uncovered by a hospital. The second model maximizes population covered by clinics and maximizes referral coverage.

3. The Coherent Covering Location Problem.

In this section, a formulation for the CCLP is offered in the context of coherence. The mathematical formulation of the problem is derived from the Hierarchical Service Location Problem of Moore and ReVelle (1982). This model locates two types of facilities such that coverage by both levels is achieved within some distance thresholds. The CCLP locates type A and type B facilities such that coverage at each level is maximized and coherence is obtained.

$$Max\ Z_A = \sum_{i \in I} a_i r_i \qquad Max\ Z_B = \sum_{i \in I} a_i s_i$$

Subject to:

$$r_i \leq \sum_{j \in MA_i} u_j + \sum_{k \in MB_i} v_k \qquad \forall i \in I \qquad (1)$$

$$s_i \leq \sum_{k \in NB_i} v_k \qquad \forall i \in I \qquad (2)$$

$$u_j \leq \sum_{k \in O_j} v_k \qquad \forall j \in J \qquad (3)$$

$$\sum_{j \in J} u_j = p \qquad (4)$$

$$\sum_{k \in K} v_k = q \qquad (5)$$

$$r_i\ s_i\ u_j\ v_k = (0,\ 1) \qquad \forall i \in I \quad \forall j \in J, \quad \forall k \in K$$

where:

$MA_i \quad = \{j \in J \,/\, d_{ij} \leq S^{IA}\}$

$MB_i \quad = \{k \in K \,/\, d_{ik} \leq S^{IB}\}$

$NB_i \quad = \{k \in K \,/\, d_{ik} \leq T^{IB}\}$

$O_j \quad = \{k \in K \,/\, d_{jk} \leq S^{AB}\}$

S^{IA} = Threshold distance for type A facilities offering type A services

S^{IB} = Threshold distance for type B facilities offering type A services

T^{IB} = Threshold distance for type B facilities offering type B services

S^{AB} = Maximum distance from a type A to a type B facility

r_i = 1, if area i is covered by a type A facility; 0, otherwise

s_i = 1, if area i is covered by a type B facility; 0, otherwise

u_j = 1, if there is a type A facility at j; 0 otherwise

v_k = 1, if there is a type B facility at k; 0 otherwise

The objectives are to maximize coverage by both type A and type B facilities. The first set of constraints state that a node cannot be covered for type A services if there is not a type A facility within S^{IA} or a type B facility within SIB. The parameters S^{IA} and S^{IB} are the threshold distances for type A and type B facilities respectively, since type B facilities also offer type A services. Observe that both distance standards do not need to be equal, even though they refer to the same type of services. This is due to the nature of the facilities. For example, in the field of health care delivery, people may be willing to travel further to large facilities than to small facilities to obtain the same type of services, such as the assistance of a nurse, which are offered in both types of centers.

The second set of constraints allow a node i to be covered for type B services if there is a type B facility within T^{IB}. In this case only one distance is used, since type B services are offered only by type B facilities.

The third set of constraints enforces coherence. It states that a type A facility has to be within a given distance standard S^{AB} from any type B facility. If this constraint was not included, results such as the one in Figure 1 are very likely. Observe that nodes 1 and 2 are covered by the same type A facility for type A services, but they are covered by different facilities for type B services. Constraints (3) try to avoid this situation. The closer both types of facilities are from each other, the more likely that all areas covered by a type A facilities will be covered by one and the same type B facility.

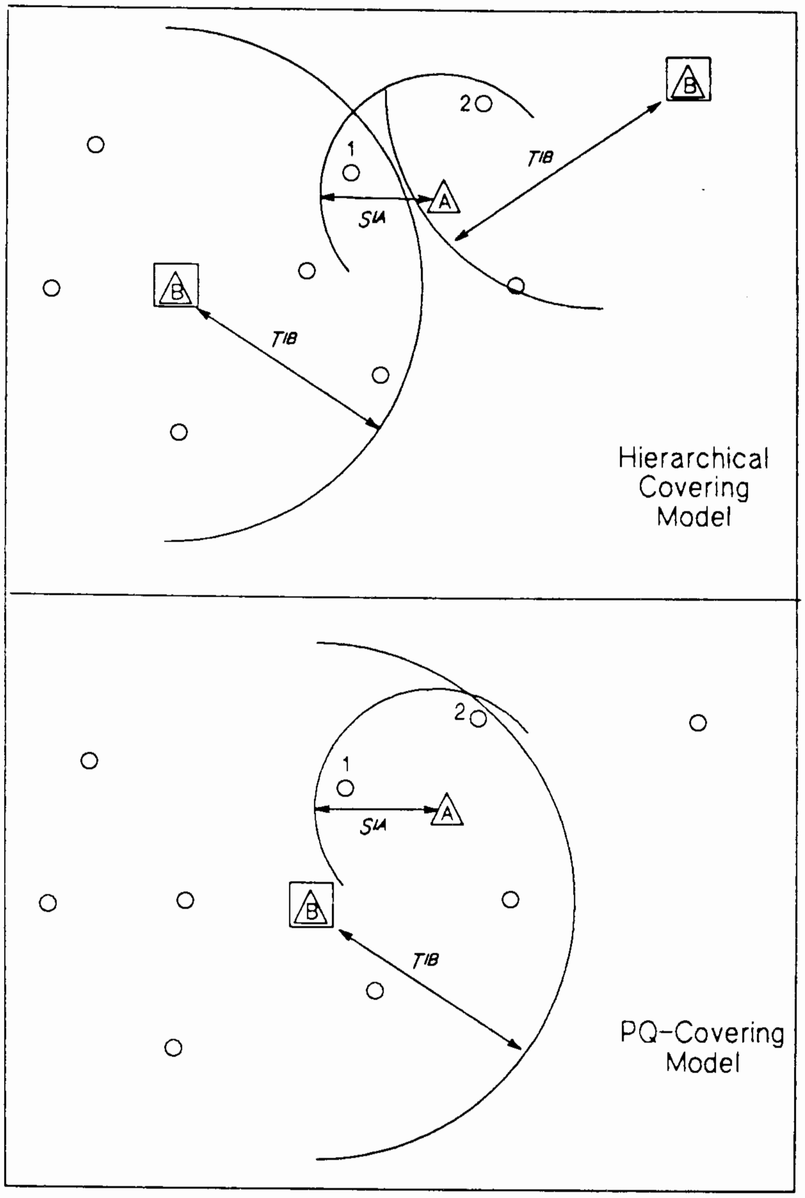
The bottom part of Figure 1 shows a possible solution of the hierarchical covering model with coherence constraints. Both nodes are covered by the same type A facility and the same type B facility. Therefore, coherence is observed.

One must be careful when choosing S^{AB}. It is an arbitrary distance based on how much coherence one wants to enforce. If S^{AB} were equal to 0, type B facilities would be forced to locate on top of type A facilities. Since $T^{IB} > S^{IA}$, all areas covered by a type A facility would be also covered by a type B facility.

On the other hand, the distance standard between type A and type B facilities should not exceed T^{AB} - S^{IA} if <u>complete</u> coherence is needed, that is,

$$S^{AB} < T^{AB} - S^{IA}$$

Figure 1: Coherent and Non-Coherent Covering Hierarchies



S^{IA} Distance Threshold for Type A Services

T^{IB} Distance Threshold for Type A Services

This can be seen in Figure 2. This figure shows two different situations. The first one is such that the maximum distance S^{AB} from both facilities is larger than the difference between the distance thresholds S^{IA} and T^{AB} ,i.e.,

$$S^{AB} > T^{AB} - S^{IA}$$

Observe that while in this case constraint set (3) is met and there is a type B facility within S^{AB} from the type A facility, node i is not covered by type B services. The second situation shows that if $S^{AB} < T^{AB}$ - , then node i will be covered by both facilities with coherence enforced.

If S^{AB} is strictly less than $T^{AB} - S^{IA}$, coherence is observed, but by increasing S^{AB} it is possible to obtain a better solution in the objective without violating coherence. As S^{AB} increases, the number of candidate nodes for a type B facility that are within S^{AB} from a type A facility increases. Therefore the NB set in constraint (2) is larger and the constraint is less tight. As a consequence, in order to achieve complete coherence, that is, all nodes covered by a type A facility are covered at least by one and the same type B facility, the maximum distance S^{AB} between type A and type B facilities has to be equal to the difference between the distance thresholds of both type A and type B facilities, that is,

$$S^{AB} = T^{AB} - S^{IA}$$

Observe that if S^{AB} is strictly smaller $T^{AB} - S^{IA}$, coherence will be achieved but at greater cost than necessary.

The assumption of coherence can be relaxed if it is sufficient that most nodes covered by a type A facility are also covered by at least one and the same type B facility. In this case, the maximum distance S^{AB} is allowed to be slightly larger than $T^{AB} - S^{IA}$, that is,

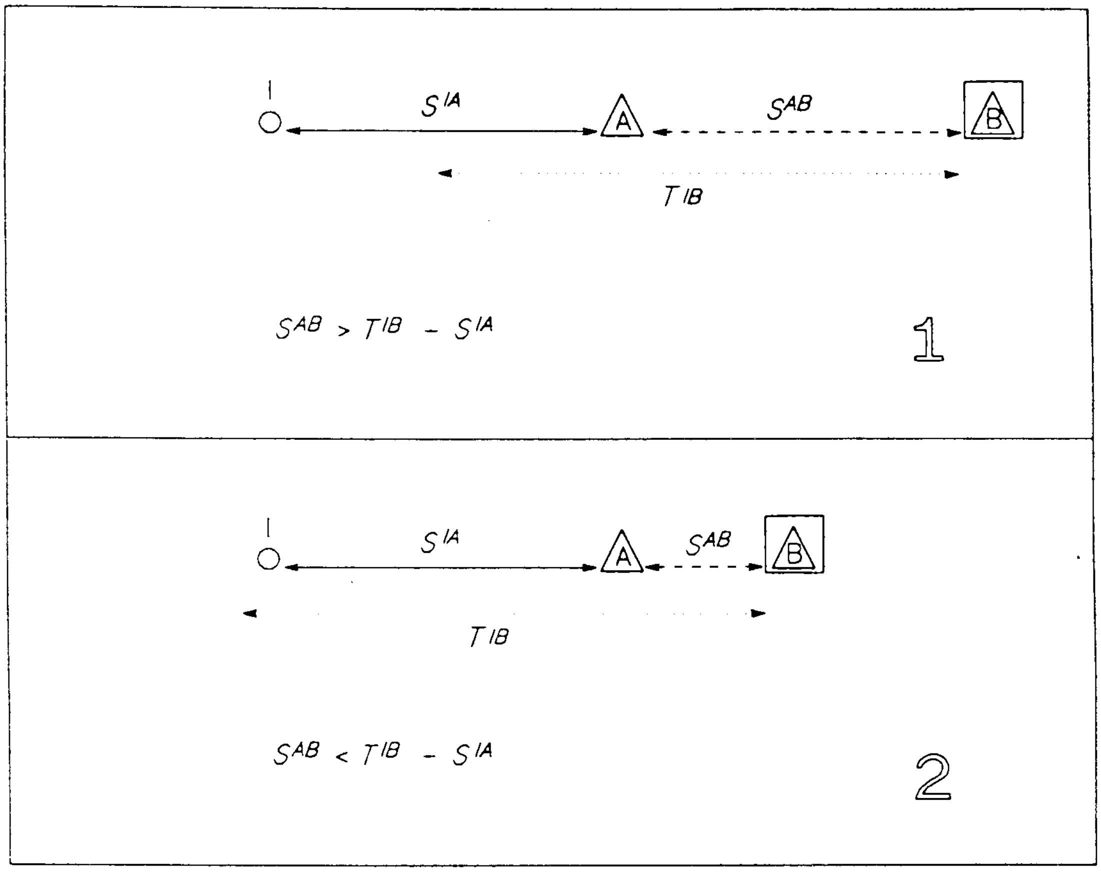
$$S^{AB} = T^{AB} - S^{IA} + O$$

where O is small enough that most such nodes have the needed coverage.
A formal definition of two types of coherence can be now stated. Strong coherence is achieved when all nodes that are covered by a type A facility are covered at least by one and the same type B facility. This will be best achieved when S^{AB} is equal to $T^{AB} - S^{IA}$.

Weak coherence is achieved when as many nodes as possible, but not necessary all, that are covered by a type A facility have also to be covered by at least one and the same type B facility. In this case S^{AB} is strictly larger than $T^{AB} - S^{IA}$ by some small amount.

Figure 3 shows the three scenarios described above. The first one corresponds to $S^{AB} < T^{AB} - S^{IA}$. The circle around the type A facility is the area covered for type A services. The large circle (truncated by the picture) corresponds to the covering

Figure 2: Strong and Weak Coherence



S^{IA}	Distance Threshold for Type A Services
T^{IB}	Distance Threshold for Type B Services
S^{AB}	Max. Distance Between type A and Type B Facilities

area for type B services, its center being the type B facility. All nodes covered by the type A facility are also covered by the same type B facility. Therefore coherence is observed. But if one allows S^{AB} to increase from S^{AB} to $S^{AB'}$, the type B facility can be located in node 2, and then node 3 will be also covered (and therefore the B objective will improve), while coherence is maintained. This situation is depicted in the second scenario. In this case $S^{AB'}$ is equal to T^{AB} - S^{AA} and node 3 is covered by B.

The bottom part of Figure 3 corresponds to $S^{AB''} > T^{AB} - S^{AA}$. Now the type B facility is allowed to move from node 2 to node 5 in order to cover node 4, assuming that node 4 has a larger population than node 1. The objective B will improve, but strong coherence is not observed because node 1 is no longer covered by type B services.

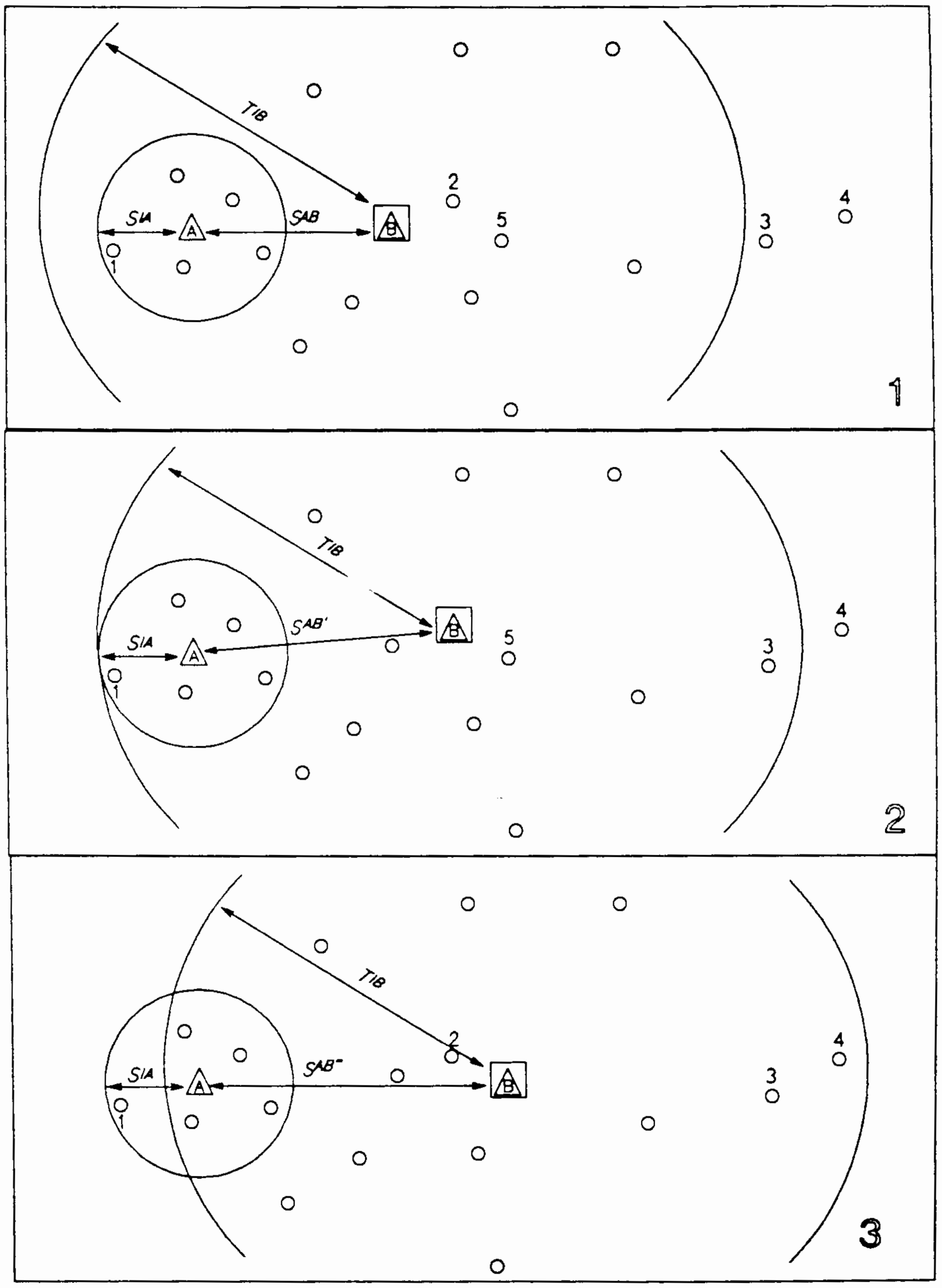
As Figure 3 shows, strong coherence does not prevent a node from being exclusively covered by one or more type B facilities. The weights chosen in the objective will more or less enforce this situation. The first objective, even though it explicitly maximizes type A coverage, also implicitly maximizes coherent coverage, since if a node is covered for type A services it will be also covered for type B services. If the second objective, B coverage, is heavily weighted, the model will tend to enforce B coverage to the detriment of coherent coverage.

In the case of strong coherence the model can be slightly modified and the number of variables reduced. Instead of using two objectives, one for each level, one can use a single objective: the maximization of the population covered by both type A and type B facilities. Variables r_i and s_i can be replaced in constraint sets (1) and (2) by a single one, e_i, where e_i is equal to 1 if area i is covered by at least once by both type A and type B services, and 0 otherwise. In this modification there is not a trade-off between coverage at both levels.

The objective can also be modified so that only type A coverage is maximized, that is, the B objective, maximization of coverage by type B facilities, is eliminated, together with its associated constraint (constraint 2). By virtue of the coherence constraint (constraint 3), type B facilities will be located within a given distance from the nodes covered by a type A facility. Therefore, the node covered by a type A facility will be also covered by the type B facility if the distance standard for the type B facilities is equal to $S^{AA} + S^{AB}$. This relaxation of the problem might lead to alternate solutions for the B level. The inclusion of the the type B objective with a small weight and the constraint on B coverage may eliminate some of them. Finally, constraints (4) and (5) specify the number of facilities to locate in each level. All variables are to be 0-1 integer for the problem to have meaning.

Following the classification proposed by Narula (1985), this covering location formulation locates coherent hierarchical facilities with successively inclusive services. If the facilities have exclusive services, constraint set (1) can be replaced by the following one:

Figure 3: Results Obtained with Different S^{AB} Distances



S^{AB} Maximum Distance between Type A and Type B Facilities $S^{AB} > S^{AB'} > S^{AB''}$

S^{IA} Distance Threshold for Type A Services

T^{IB} Distance Threshold for Type B Services

$$r_1 \le \sum_{j \in MA_i} u_j \qquad \forall i \in I \qquad (7)$$

Constraint (7) states that a node can be covered by type A facilities only for type A services, since in this case type B facilities do not offer type A services. On the other hand, if locally inclusive services are considered, constraint set (1) can be replaced by the following ones:

$$r_i \le \sum_{j \in MA_i} u_j \qquad \forall i \in (I - K) \qquad (8)$$

$$r_i \le v_i + \sum_{j \in MA_i} u_j \qquad \forall i \in (I \cap K) \qquad (9)$$

The first set of constraints (8) allows node i to be covered by a type A facility only, since the constraints are defined for all nodes in I - K. The second constraint set (9) allows node i to be covered by a type A facility located at a distance at most equal to or if there is a type B facility located in the same node i, since type B facilities can locate at these nodes.

4 The P-Median Q-Covering Model.

The CCLP can be reformulated using a p-median criterion for the first level. The statement of the problem is as follows: locate p type A facilities and q type B facilities such that (1) the average distance from demand areas to A services is minimized (2) the population with type B services is maximized within a distance standard, and (3) coherence is observed. At level A a p-median objective is used. Level B uses a Maximal Covering objective.

In the p-median q-covering problem, coherence has to be redefined. While in the pq-median problem it is possible to have all areas that are assigned to a type A facility to be assigned to only one and the same type B facility, and similarly, with the CCLP to have strong coherence enforced, the p-median q-covering model will be unlikely to achieve this full coherence. The p-median problem at the first level of the hierarchy assigns every demand area to a facility. The second level of the hierarchy uses the Maximal Covering Location Problem to cover areas within a threshold distance. If this threshold distance is not extremely large it may happen that not all areas assigned to a type A facility are covered within this distance to a type B facility. Therefore, coherence cannot be fully enforced. In this case, coherence is observed when a demand area assigned to a type A facility is covered by at least one and the same type B facility within some distance standard. The model will try to maximize the coverage by type B facilities, but such that as many areas covered as possible belong to (are assigned to) the same type A facility. This coherence, as in the coherent covering location problem, is called weak coherence.

The multiobjective formulation for the p-median q-covering problem is as follows. The notation used is the same as in the CCLP.

$$Min\ Z_A = \sum_{i\in I}\sum_{j\in J} a_i d_{ij} x_{ij} \qquad\qquad Max\ Z_B = \sum_{i\in I} a_i s_i$$

Subject to:

$$\sum_{j\in J} x_{ij} = 1 \qquad \forall i \in I \qquad (10)$$

$$s_i \le \sum_{k\in NB_i} v_k \qquad \forall i \in I \qquad (11)$$

$$x_{ij} \le u_j \qquad \forall i\in I \quad \forall j \in (J-K) \qquad (12)$$

$$x_{ij} \le u_j + v_j \qquad \forall i \in I \quad \forall j \in (J \cap K) \qquad (13)$$

$$x_{ik} \le v_k \qquad \forall i\in I \quad \forall k \in (K-J) \qquad (14)$$

$$u_j \le \sum_{k\in O_j} v_k \qquad \forall j \in J \qquad (15)$$

$$\sum_{j\in J} u_j = p \qquad (16)$$

$$\sum_{k\in K} v_k = q \qquad (17)$$

$$r_i\ s_i\ u_j\ v_k \quad = (0,\ 1) \qquad \forall i \in I,\ \forall j \in J,\ \forall k \in K$$

where x_{ij} is equal to 1 if demand node i is assigned to a facility at j for type A services and 0 otherwise.

The first group of equations (10) forces each demand area to be assigned to only one type A facility (p-median constraints). The second group of constraints (11) correspond to the q-covering part of the model. The elements in set NB_i represent the potential type B locations that are within a given distance standard T^{IB} from area i. Therefore, an area i will be covered for type B services ($s_i = 1$) if there is a type B facility at a potential location j that is within T^{IB} from it, i.e., if v_j equals 1, and j is in the set NB_i.

Constraints (12), (13), and (14) state that area i cannot be assigned to a node j for type A services if there is not a type A or a type B facility located there. Constraint set (12) says that if demand area i is assigned for level A services to node j ($j \in J - K$), then this node must a have a type A facility. Observe that this node is not a candidate for a type B facility as it is capable of housing only a type A facility. If node j is candidate for both type A and type B facilities, then constraint type (13) is used; this constraint allows a node i to assign to a type A or B facility for type A services. That is, demand nodes may assign to a type B facility for type A services if this assignment gives a better overall objective than assigning to a type A facility for the same services. If node j can only have type B facilities ($j \in K - J$) then node i will be free to assign to it for type A services only if a type B facility is sited there (constraint group (14)). Observe that constraints (12), (13) and (14) put together account for all candidate nodes to have a facility, that is, $(J \cup K) - (J \cap K)$.

Constraints (15) enforce coherence. The constraint states that a type A facility must always have at least one type B facility within a given distance. The set O_j has as elements all potential type B locations that are within a given distance S^{AB} from a potential type A facility sited at node j. Coherence is enforced in this way as follows: if there is a type B facility at k within S^{AB} from a type A facility at j, and all areas within T^{IB} are covered by B, then all areas assigned to the type A facility that are within T^{IB} - S^{AB} from the type B facility will be covered for type B services. This does not imply that some areas assigned to the type A facility but that are further than T^{IB} - S^{AB} are not covered by a type B facility, since it is T^{IB} and not T^{IB} - S^{AB} that determines B coverage.

This can be seen in Figure 4. Node 2 is assigned to a type A facility and covered by a type B facility, but it is located at a further distance than T^{IB} - S^{AB}. On the other hand node 1 is assigned to the same facility but it is not covered by the type B facility, since it is located at a place further than T^{IB} from B. All nodes within T^{IB} - S^{AB} from A and assigned to it will be covered by the same type B facility. Therefore, it is the arbitrary maximum distance S^{AB} chosen that will determine the rigidity in coherence, as in the CCLP. The smaller this distance is, the more coherence is enforced, since more nodes assigned to a type A facility will be covered by the same type B facility. Therefore, in the p-median q-covering model the maximum distance S^{AB} between type A and type B facilities should be chosen taking into consideration the coverage of nodes by both type A and type B facilities, even though first level coverage is not explicit in this model. As in the CCLP, S^{AB} should be equal to T^{IB} - S_{IA} to obtain the best possible coherence, even though, unlike in the CCLP, strong coherence is not necessarily obtained.

The p-median q-covering model can be transformed to consider coherent facilities with successively exclusive services. Constraint sets (12), (13) and (14) can be replaced by the following constraint set:

$$x_{ij} \leq u_j \qquad \forall i \in I \quad \forall j \in J \qquad (19)$$

By including constraint set (19) areas will be allowed to assign to type A facilities only for type A services.

Similarly, the p-median q-covering model can be adapted if the services offered at the first level are locally inclusive. Constraint sets (12), (13) and (14) can be replaced by the following constraints:

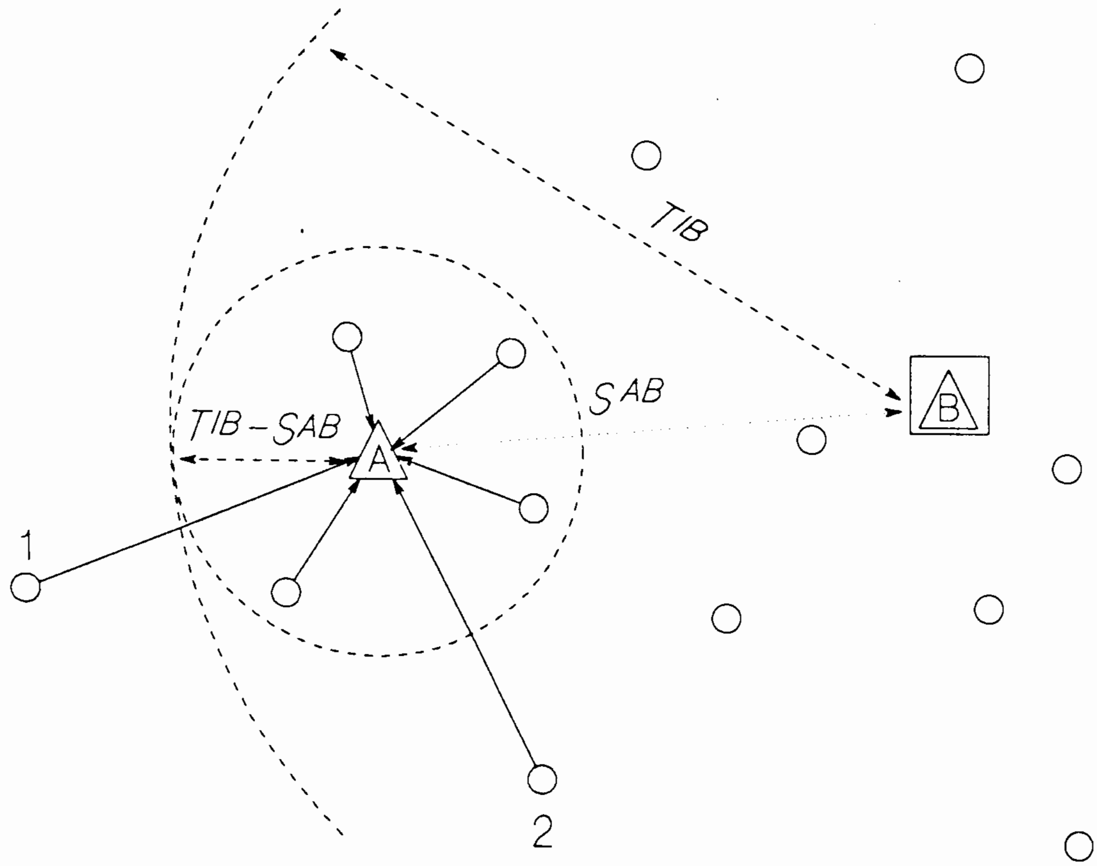
$$x_{ij} \leq u_j \qquad \forall i \in (I-K) \quad \forall j \in J \qquad (20)$$

$$x_{ij} \leq u_j + v_i \qquad \forall i \in (I \cap K) \quad \forall j \in J \qquad (21)$$

Area i can be assigned to a type A facility or to a type B facility if this one is located there, by virtue of constraint (21). On the other hand, if area i is not a canditate for type B facilities, then it can assign only to type A facilities for type A services by virtue of constraint (20).

The p-median q-covering model can become computationally expensive as the number of nodes increases, since it has assignment variables x_{ij} and Balinski type constraints. The number of these variables can be reduced by $n^2 - n(n - p + 1)$ without affecting the formulation, where n is the number of nodes and p the number of type A facilities to locate. Nevertheless, the model can be solved using the weigthing method and linear programming relaxation with branch and bound when necessary for relatively large networks. For example, if $n = 100$ the problem will have 10.300 variables and 10.302 constraints. Observe that in most cases it will not be necessary to declare assignment variables and coverage variables integer, unless there are ties in the coefficients of the weighted objective, since they are upper bounded by the locational variables. By declaring only these, the model will try to set the coverage and assignment variables with the largest coefficient in the objective to one.

Figure 4: P-Median Q-Covering Result



S^{AB} Maximum Distance between Type A and Type B Facilities

T^{lB} Distance Threshold for Type B Services

5. Computational Experience

Both models were solved using the weighting method and linear programming relaxation, with branch and bound when non-integer solutions were found. The software used was MPSX/MIP in an IBM 3091 E-600 mainframe computer. Only locational variables were declared integers in both models. No fractions were ever found for the assignment or coverage variables of both models in any of the runs. Two networks, a 25-node network and a 79-node network, were used.

For the 25-node network the distance threshold to type A services was set to 10 kilometers. The distance to type B services was set to 40 kilometers. Since in the CCLP strong coherence was sought, the maximum distance between type A and type B facilities was set to 30 kilometers. This problem had 100 variables and 77 constraints. To be able to compare both models the same distance thresholds were used in the p-median q-covering problem. This problem had 175 variables and 202 constraints.

Figure 5 shows the trade off between both objectives obtained with the CCLP when (p,q) = (3,2), (4,2), (5,2), (6,2) type A facilities and type B facilities are located respectively in the 25-node network. Figure 6 shows the trade off between both objectives when solving the p-median q-covering problem when the same number of facilities are located. Each curve in both charts represents a given number of type A and type B facilities. Only problems where $q = 2$ are represented. It is interesting to note that in both models, regardless of the number of type A facilities sited, the locations of the type B facilities remained unchanged. Therefore, the trade off curves are parallel in both figures.

Table 1 presents the results for the 25-node network for both models. For each set of facilities located the value of the objectives obtained by changing weights is shown. The first column corresponding to the CCLP gives the values of the first objective: maximize coverage by type A services. The second column represents the second objective of the CCLP: maximization of coverage by type B facilities. The third column gives the average distance to type A facilities, even though it is not an objective of the CCLP. This is done to be able to compare both the CCLP and the p-median q-covering model, since this last model has as its first objective the minimization of average distance to type A facilities. Therefore, the columns corresponding to the p-median q-covering model represent both of its bjectives and the coverage by type A services.

The coverage by type B facilities was the same in most cases for both models since it is a common objective. This was expected specially when the weights used emphasized type B coverage. The p-median q-covering model did not give very good coverage for type A services, but as the number of facilities increased the gap between coverage obtained with the coherent covering location and the p-median q-covering decreased. The average distance to type A services was substantially different depending on the models used.

Figure 5: Trade off curve: CCLP, 25-node network

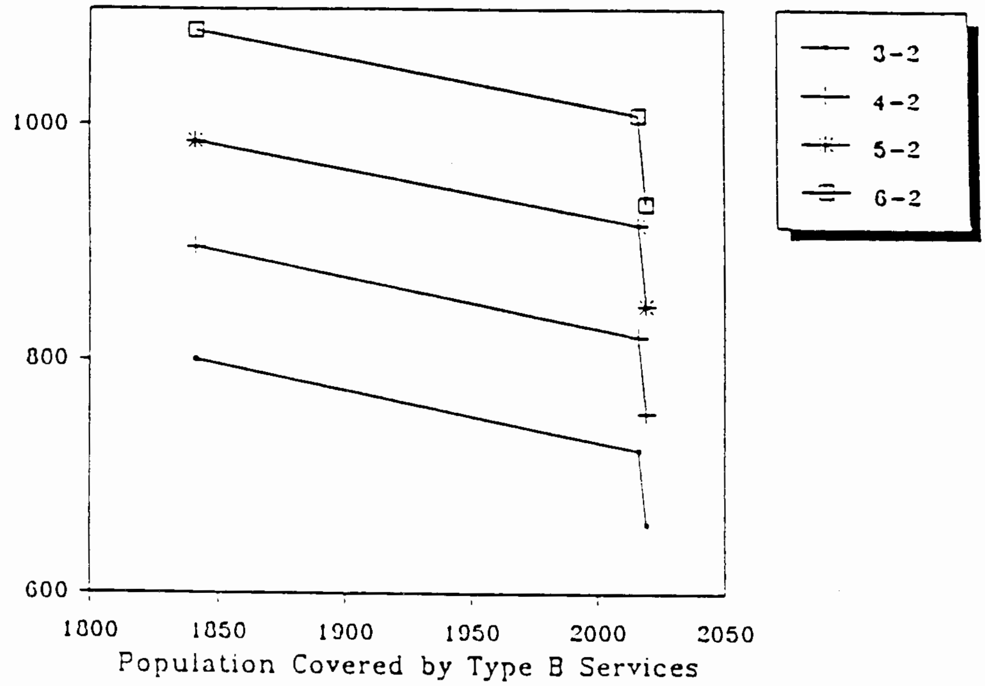


Figure 6: Trade off curve: p-median q-covering, 25-node network

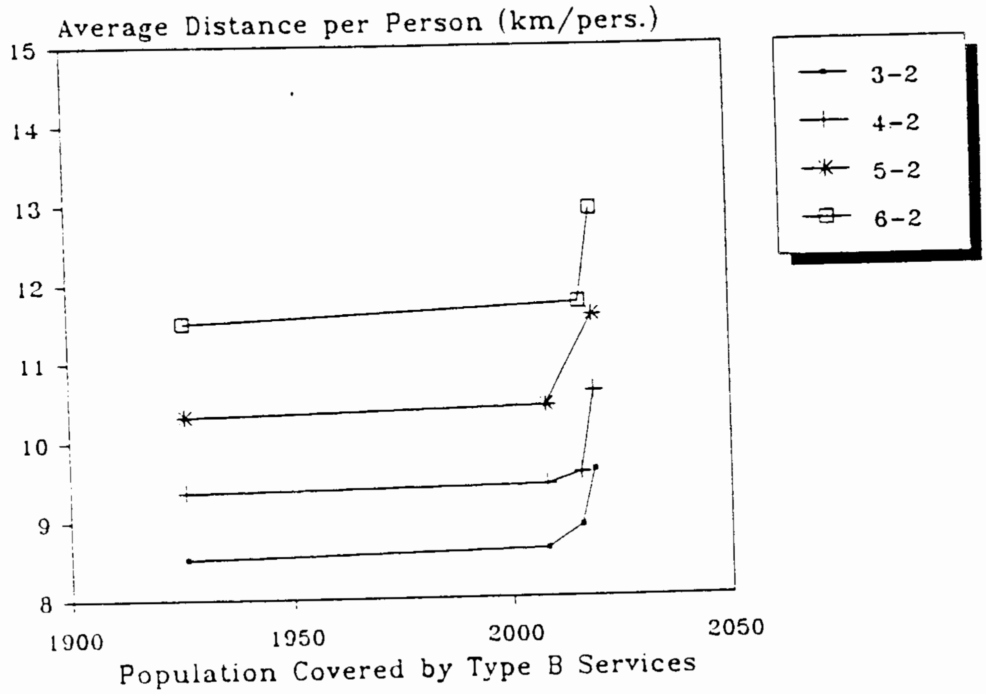


Table 1: Results of the CCLP, 25-node Network

	PQ-Covering Model			*P*-Median *Q*-Covering Model		
# of Facilities Located (p,q)	A Coverage (people)	B Coverage (people)	A p-median (km/pers)	A Coverage (people)	B Coverage (people)	A p-median (km/pers)
(2,1)	545	2097	18.20	275 469 463	1617 1849 2097	14.67 14.81 15.93
(3,2)	800 722 659	1841 2016 2019	14.50 14.40 14.26	565 547 462	1926 2016 2019	11.50 11.74 12.92
(4,2)	895 819 754 754	1841 2016 2019 2019	11.90 12.73 12.59 12.59	642 626 639 639	1926 2008 2019 2019	10.31 10.40 11.57 11.57
(5,2)	986 915 845	1846 2016 2019	11.63 11.71 11.14	813 787 795 725	1926 2008 2016 2019	9.36 9.43 9.56 10.59
(6,2)	1081 1009 933	1846 2016 2019	10.03 11.19 10.08	983 984 878 922	1926 2008 2016 2019	8.52 8.61 8.90 9.60

Figure 7 represents two solutions obtained in the CCLP when 4 type A facilities and 2 type B facilities are located in the 25-node network. The first one (top) correspond to the maximization of type A coverage, with a very small weight on the second objective. The opposite is presented in the bottom part of the picture. Observe that in both cases all areas covered by a type A facility are also covered by the same type B facility. Therefore, coherence was observed. Figure 8 shows two solutions obtained locating the same amount of facilities with the p-median q-covering model and using the same weights. In this case strong coherence is not achieved. Some nodes assigned to type A services are not covered by type B facilities.

Average run times for each model are shown in Tables 2 and 3. In very few instances branch and bound was needed to obtain integer solutions when the results obtained with the linear programming relaxation were non-integer. In the CCLP, of a total of 31 runs, only 7 required some branch and bound. Similarly, in the p-median q-covering model out of 24 runs only 7 needed additional B&B to obtain integer solutions. In any case, for both models the B&B runs did not require a long CPU time to obtain integer solutions.

Both models were also solved in the 79 nodes network using MPSX. The threshold distance for type A services was set to 300 meters and the threshold distance for type B services to 800 meters in both models. Different weights were chosen to generate the tradeoff between both objectives in each model. Again strong coherence was enforced by setting the maximum distance between both types of facilities to 500 meters.

Table 4 shows the results for both models. After doing several runs using different weights for each combination of type A and type B facilities, only two non-inferior points were found in the trade off curve of both models, except for the CCLP when 6 type A and 3 type B facilities were located. On the other hand, observe that, for the CCLP, the population covered by type A services is larger when 6 type A facilities and 3 type B facilities offering type A services are located than when 8 type A and 2 type B facilities are located. This may seem odd, but it is due to the coherence constraint. Since the coherence constraint forces the type A facilities to be located at a given distance threshold from type B facilities, the less there are type B facilities, the more constrained the type A facilities will be in deciding their location. In this case, with 3 type B facilities the type A facilities can "spread" more in the region and hence covering more people than when 2 type B facilities are located.

Run times are displayed for each model in Tables 5 and 6. An interesting feature was common to both models. When the B objective was heavily weighted, little or no additional runs were necessary to obtain solutions after linear relaxation was used to solve the problem. In contrast, when the objective A was heavily weighted, these run times increased substantially, making both models expensive to solve. Or the models would solve quickly (when the weighted objective was favorable to the B objective) or heavy additional branch and bound was necessary to obtain integer solutions.

The CCLP and the p-median q-covering model can be solved using linear programming relaxation and branch and bound when needed for fairly large networks. Nevertheless, heuristic procedures such as the ones developed for the pq-median problem can be used for the solution of the CCLP and the p-median q-covering model. The objective of average distance in the Solution Algorithms for the Hierarchical Problem (SAPHIERS, Serra and ReVelle 1992.b) can be replaced by a maximal covering objective without modifying the one-opt iterative process. In the case of the the CCLP, at each iteration the weighted objective of maximal coverage is computed for both types of services and compared to the one obtained so far. Similarly, in the p-median q-covering model at each iteration the weighted objective is computed after finding the values of the p-median objective for the type A level and the covering objective for the type B level.

Table 2: Average CPU time, CCLP, LP+B&B, 25-Node Network

# of Located Facil. (p,q)	Total Number of Runs	Optimal Integer Solution using LP only	Average Run Time LP (CPU sec)	Average Additional Run Time using B&B (CPU sec)	Total Average Run Time (CPU sec)
(2,1)	4	4	102.6	0.0	102.6
(3,2)	8	8	120.0	0.0	120.0
(4,2)	5	4	141.4	93.0	155.4
(5,2)	6	4	145.4	167.1	197.5
(6,2)	8	4	128.1	197.8	223.7

Table 3: Avge. CPU time, p-median q-covering LP+B&B 25-Node Network

# of Located Facil. (p,q)	Total Number of Runs	Optimal Integer Solution using LP only	Average Run Time LP (CPU sec)	Average Additional Run Time using B&B (CPU sec)	Total Average Run Time (CPU sec)
(2,1)	8	8	352.2	0.0	352.2
(3,2)	4	1	544.2	804.6	1348.8
(4,2)	4	2	858.0	1780.0	2638.0
(5,2)	5	3	575.2	2853.2	3428.4
(6,2)	5	3	846.8	1025.5	1872.3

Table 4: Results of the Covering Models, 79 Nodes Network

# of facilities located (p,q)	Total number of runs	Optimal int. sol. using linear relax. only	Avge. run time linear solution (CPU sec.)	Avge. additional run time using B&B (CPU sec.)	Total avg. run time (CPU sec.)
(2,1)	8	8	352.2	0	352.2
(3,2)	4	1	544.2	804.6	1348.8
(4,2)	4	2	858.0	1780.0	2638.0
(5,2)	5	3	575.2	2853.2	3428.4
(6,2)	5	3	846.8	1025.5	1872.3

Table 5: Average CPU time, CCLP, LP+B&B, 79 nodes

# of Located Facil. (*p*,*q*)	Total Number of Runs	Optimal Integer Solution using LP only	Average Run Time LP (CPU sec)	Average Additional Run Time using B&B (CPU sec)	Total Average Run Time (CPU sec)
(6,3)					
I	3	3	14.0	0	14.0
II	3	0	13.8	274	287.4
(8,2)					
I	3	3	15.0	0	15
II	3	0	14.6	203	217.6

Table 6: Average CPU time, p-median q-covering LP+B&B, 79 nodes

# of Located Facil. (*p*,*q*)	Total Number of Runs	Optimal Integer Solution using LP only	Average Run Time LP (CPU sec)	Average Additional Run Time using B&B (CPU sec)	Total Average Run Time (CPU sec)
(6,3)					
I	3	3	107.4	45.0	152.4
II	3	0	130.2	3501.6	3631.8
(8,2)					
I	3	3	126.0	73.2	199.2
II	3	0	96.0	4107.0	4204.0

I indicates predominant weights on B objective
II indicates predominant weights on A objective

Figure 7: Solutions for the CCLP, p =4, q = 2

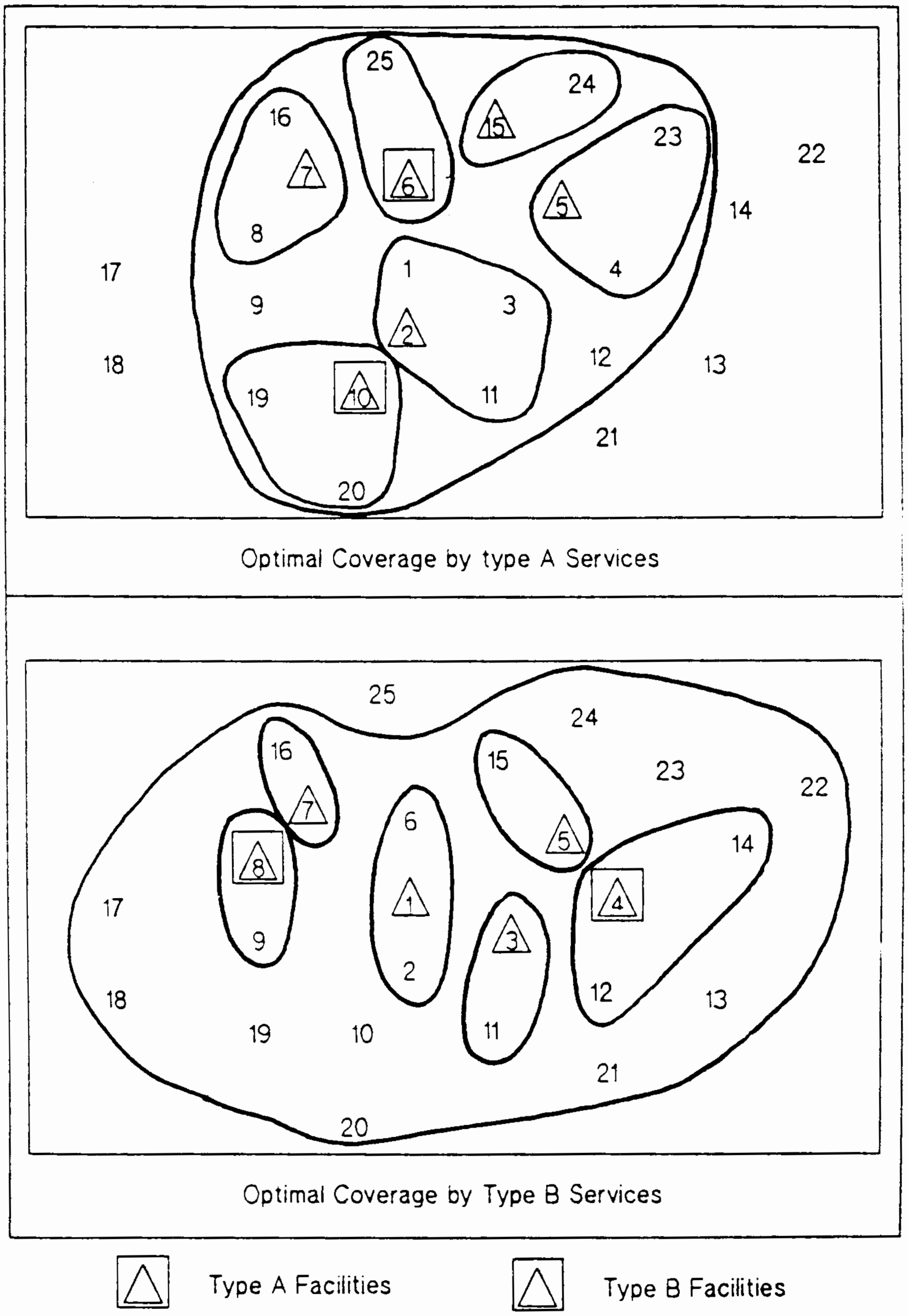
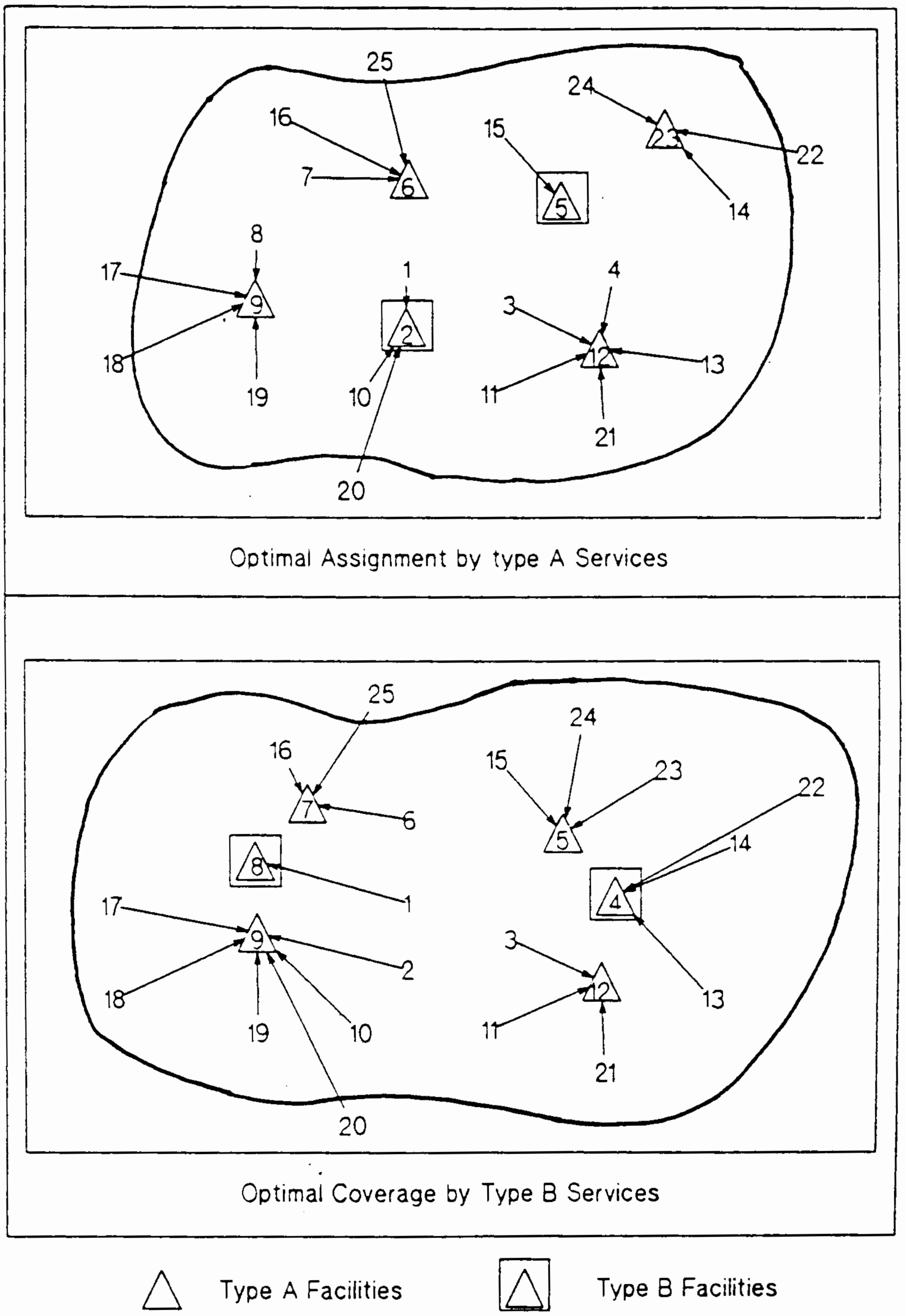


Figure 8: Solutions for the p-median q-covering model, p =4, q = 2



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