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Platform tying and its effects

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Platform Tying and its Effects*

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Abstract

We study tying by an integrated platform controlling a primary access point while competing in an adjacent market against a rival with a direct channel. In a two-sided setting, tying improves integrated quality but shifts mediated demand away from the rival, reducing its seller participation. This degrades the rival's direct channel through network externalities. As a result, tying can lower consumer surplus and welfare. We also show that tying can act as a substitute tool for rent extraction when access fees are constrained. Finally, we compare tying against demotion and non-discriminatory access policies.

JEL codes: D42, K21, L12, L41

Keywords: abuse of dominance, tying, self-preferencing, network effects, digital platforms.

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1 Introduction

In 2017, the European Commission (EC) fined Google EUR 2.42 billion for favouring its own price-comparison service in general search results: Google Shopping was displayed prominently, while rival comparison sites such as Kelkoo, Idealo, and PriceRunner were demoted to lower positions and lost substantial traffic.[1] In 2024, the EC fined Meta EUR 797 million for tying Facebook Marketplace to Facebook, thus giving it an automatic reach advantage over rival classified-ad platforms such as eBay, Vinted, and Subito. Similar concerns have also arisen, among others, in relation to Microsoft for tying Teams with Office 365, Meta for tying its AI assistant within WhatsApp, Apple for favouring iCloud over rival cloud storage services, Amazon for alleged *self-preferencing* on its Buy Box, or Booking.com's promotion of its own services inside the booking funnel.

These cases differ in legal and technological details, but share a common structure: a firm that controls a primary access point favours its own adjacent service over rival alternatives, via defaults, ranking, prominence, or technical integration. Such tying conduct may create immediate user benefits, for example, by reducing inconvenience costs or improving the user experience. At the same time, it can divert the demand away from rivals and reduce their scale. The central question is whether and when the efficiency gains from integration might be outweighed by any indirect competitive harm.

In many digital markets, rivals retain a direct route to users outside the dominant ecosystem. Comparison shopping services sought access to Google Search, but in principle consumers could also access them directly on their websites. eBay and Vinted compete with the Facebook Marketplace within Meta's social network, but also serve users through their websites.[2]

The existence of a direct channel is sometimes taken to weaken concerns about tying. If users can still reach the rival directly, the argument goes, the conduct within the integrated platform should not eliminate the rival's competitive constraint. This paper shows that this reasoning can be misleading: when network effects are at play, the rival's direct channel is not independent of the mediated channel. A conduct such as tying, that reduces rival traffic within the integrated platform, might affect the rival's overall demand, including from its direct channel users.

Let us be more precise about how this mechanism works within our two-sided market (such as in *Google Shopping* or *Facebook Marketplace*) baseline model.[3] In our model, an integrated platform controls a primary access point and also operates a specialised service,

[1] The Google Shopping decision was definitively upheld by the Court of Justice of the European Union (CJEU), on September 10, 2024.

[2] Similarly, communication and collaboration apps such as Zoom, Webex, or Slack; AI chatbots such as ChatGPT and Claude; or cloud services such as Dropbox, Proton Drive and many other specialised services are both intermediated by dominant platforms and reachable through their own apps or websites.

[3] With within-group network effects (more in line with *Microsoft Teams*, *Meta AI* or *Apple iCloud*) similar results hold, as we show in an extension.

which faces a competing service. A group of users can reach both specialised services through the primary platform (mediated channel), while other users can reach the rival directly (direct channel). The specialised services charge sellers, but (for simplicity) not users. Seller participation affects consumer demand through indirect network effects: a larger seller base makes a service more attractive.

We model tying as an increase in the utility from the integrated platform's service, for instance by making it easier to reach and use through defaults, prominence or technical integration. Therefore, tying *directly* benefits consumers, but it also shifts demand away from the rival. The loss of mediated demand reduces seller participation on the rival service, which in turn weakens the rival in both channels, imposing a negative externality on its consumers on the direct channel. Thus, the effects of tying are a priori ambiguous.

To illustrate the general model and study welfare effects, we use a Hotelling real-line model. We show there that welfare effects are non-monotonic in the size of the direct channel. When it is small, the beneficial effect on the mediated channel dominates and tying is welfare-improving. As the direct channel expands, the negative externality imposed by tying carries greater weight, resulting in lower overall consumer, seller and total surplus. Once the direct channel is large enough to attract almost all sellers regardless of mediated demand, however, tying has a negligible impact on the rival's seller base and consumers in both channels are effectively independent: by raising surplus in the mediated channel, tying will have a positive effect overall. Thus, consumer harm arises for intermediate levels of the direct channel size.

The integrated platform's incentive to tie depends on its ability to extract the rival's rents through access fees. For simplicity, in the baseline model we assume away such fees (like in *Google Shopping* or *Facebook Marketplace*), and tying is always profitable. We then show that if fees are endogenous and allow it to extract all of its rival's profits (perfect rent extraction), the integrated platform ties only when joint platform profits rise. For intermediate cases, when access fees are positive but set exogenously to the specialised service (think of platforms that set the same fees for hundreds of different products and services) tying is profitable as long as fees are not too high. In this case of imperfect rent extraction, tying becomes an alternative means of capturing the rival's rents, and there can exist substitutability between access fees and tying.

Another form of self-preferencing, *demotion*, degrades the user's experience with the rival in the mediated channel, for instance by making it less visible or by adding interface frictions. As for tying, this reduces the rival's seller base and weakens its direct channel. But while tying improves the integrated service and raises the utility for its consumers, demotion reduces the convenience of the rival service and *lowers* the utility from using it — and hence lacks this offset. We show in our Hotelling example that welfare consequences are accordingly different: whereas the effects of tying are a priori ambiguous, we are unable to find situations where demotion is welfare-improving.

We also study a common (or non-discriminatory) access policy, in line with the EU Digital Markets Act (DMA) and analogous regulations,[4] which aim to avoid preferential treatment within a gatekeeping platform's environment. This policy differs from tying because it imposes an improvement in access to *both* the integrated and rival service. In our model, this expands seller participation in both platforms and strengthens the direct channel of the rival through the same network mechanism that makes tying potentially harmful. While still profitable for the platform, and therefore an incentive-compatible policy, non-discriminatory access eliminates possible anti-competitive effects.

Finally, we show that the main results continue to hold in a one-sided setting with consumer-side network effects (more in line with cases such as *Meta AI*): tying shifts mediated consumers towards the integrated service, reducing the rival's scale and hence its value to users in the direct channel.

Related Literature Our paper is related to several strands of literature. First, it is related to papers on anti-competitive tying, in particular those motivated by imperfect rent extraction.[5] Indeed, in our model, tying is always profitable when the exogenously given access price is sufficiently low, which can be due to the platform design or access rules,[6] a regulatory cap or a previous business decision by the incumbent.[7]

Other papers find an imperfect-rent extraction rationale for tying in two-sided markets.[8] In Choi and Jeon (2021), perfect extraction would require negative prices, whereas prices are constrained to be non-negative.[9] In Choi et al. (2026), consumers are heterogeneous and some are left with surplus. Tying allows the dominant platform to leverage this unexploited surplus by converting those consumers into a quasi-installed base in the tied market, which is characterised by network externalities. Whereas in most of this literature tying operates on consumers, in de Cornière, Jerath and Taylor (2025), the platform imposes restrictions on sellers (third-party merchants or developers). To relax price competition, sellers would not symmetrically adopt the same ancillary service of-

[4]See e.g., the UK's Digital Markets, Competition and Consumers Act 2024 (DMCCA).

[5]In Carlton and Waldman (2012), the product at issue does not yet exist and contingent contracts cannot be written. In de Cornière and Taylor (2024), there is downstream competition in the complementary-product market, and the input monopolist sets prices through secret bilateral contracts. See Fumagalli, Motta and Calcagno (2018) for a survey. Our paper also shares with Carlton and Waldman (2002) or Whinston (1990) the idea that tying may lead to (partial or total) foreclosure by depriving the rival of sales or profits. Note that in our model, tying is profitable whether or not it induces full exclusion.

[6]For instance, in *Google Shopping* any website could be listed in the organic searches; in *Meta AI*, users could have third-party AI agents interact in WhatsApp chats; As for *Microsoft Teams*, note that in a Windows environment, one could download any communication app.

[7]For instance, Motta (2023) argues that a platform such as Amazon or Apple might commit to favourable terms and conditions to attract third-party sellers or software developers to its marketplace or app store, but later might want to engage in tying or other conduct to extract more rents from them.

[8]Rochet and Tirole (2003, 2006), Armstrong (2006), Caillaud and Jullien (2003). See Jullien, Pavan and Rysman (2021) for a survey.

[9]See also Amelio and Jullien (2012), where tying is an indirect way to subsidise adoption when prices are non-negative.

fered by the platform. The platform therefore ties its core service to the ancillary service to reduce seller differentiation. This increases its earnings and drives down prices for end consumers.

We show that the rationale for tying can survive under perfect rent extraction. When the integrated platform can optimally set the price for access to the mediated channel, it will tie if and only if doing so raises the two platforms' joint profit.

Our result that in some circumstances (i.e., for intermediate size of the direct channel) access fees and tying might be substitutes, makes our paper also related to recent research that stresses that platforms can resort to different monetisation tools (Franck and Peitz, 2023; Bar-Isaac, 2026; Bar-Isaac and Shelegia, 2026), implying that policy restrictions on one such tool might be neutralised by a platform by using a substitutable tool.

One can also see the mechanism analysed here as a form of *platform envelopment* (see Eisenmann, Parker and Van Alstyne 2011): by offering a better integrated service, the "platform enveloper" takes business and network effects away from the rival. A growing literature examines (data-driven) envelopment and ecosystem advantages (Condorelli and Padilla, 2024; Krämer and Shekhar, 2025; Carballa Smichowski, 2025; Rhodes, Zhou and Zhou, 2026). Our analysis departs from these works in two dimensions. First, existing models treat rivals as standalone single-product firms, whereas our rival relies on the integrated platform to reach a subset of its consumers. This intermediation enables conduct within the mediated channel to spill over onto the rival's direct channel. Second, while prior work generates consumer harm through market foreclosure (Condorelli and Padilla, 2024) or price–quality tradeoffs where price increases outweigh quality gains (Krämer and Shekhar, 2025; Rhodes, Zhou and Zhou, 2026), harm in our setting occurs even though the rival remains active and the integrated service strictly improves.

Our paper also belongs to the large literature on hybrid platforms, and in particular on *self-preferencing* and the related practice of steering, in which a hybrid platform favours its own products or services through biased ranking, recommendation, or default design, or through imitation of the rivals' products. This literature includes both theoretical and empirical contributions.[10] Relative to it, we identify a mechanism based on network externalities (either cross-group or within-group) through which such conduct could have anti-competitive effects even when it generates a direct convenience benefit for consumers. A key difference is that we explicitly model a direct channel through which the rival reaches consumers outside the integrated platform. This channel changes the welfare effects: through network effects, harm in the mediated channel spills over to consumers in the direct channel rather than remaining confined to the integrated platform.[11]

[10]For theoretical contributions, see, for example, de Cornière and Taylor (2019); Padilla, Perkins and Piccolo (2022); Zennyo (2022); Hagiu, Teh and Wright (2022); Motta (2023); Anderson and Bedre Defolie (2024); Hervas-Drane and Shelegia (2025). For empirical contributions, see, for example, Farronato, Fradkin and MacKay (2023); Chen and Tsai (2024); Waldfogel (2024).

[11]In a previous version of the paper, we showed that a similar effect arises when overall scale affects

There also exists a literature that models endogenous consumer arbitrage across sales channels (e.g., Edelman and Wright, 2015; Wang and Wright, 2020; Hagiu and Wright, 2024), which is complementary to our framework. While these papers analyse how consumers move between platforms and direct channels in response to price parity, search costs, or platform leakage, we assume that two different groups of consumers are assigned to the mediated or the direct channel exogenously. Apart from the possible empirical justification that consumers often face channel inertia due to search costs and platform lock-in, this assumption allows us to ignore possible demand-reallocation effects and show that conduct in the mediated channel can harm consumers purely through network externalities. We also conjecture it provides a conservative lower bound on harm: if consumers could freely arbitrage, the degradation of the rival's direct channel quality would induce further migration towards the integrated platform, possibly raising consumer harm.

The remainder of the paper is organised as follows. Section 2 presents the main model of tying in the presence of cross-group network externalities between sellers and consumers. Section 3 studies the effects of tying. Section 4 considers the case of demotion. Section 5 examines a common-access policy. Section 6 extends the analysis to a one-sided environment with within-consumer network effects. Section 7 concludes.

2 Baseline model: setup

Our environment is described in Figure 1 and captures most of the specificities of landmark cases such as *Google Shopping* or *Facebook Marketplace*. We consider a two-sided market in which the *integrated platform* 1 owns A_1, the primary access point, and B_1, its integrated specialised service. The *rival platform* (platform 2) operates B_2, which competes with B_1 for consumers and sellers. Throughout, A_1, B_1 and B_2 denote the services themselves, and platform $i = 1, 2$ denotes their owner; since only B_1 and B_2 set fees, "platform i chooses r_i" always refers to the fee set for service B_i. A group of consumers can reach both specialised services through the mediated channel controlled by A_1. Another group of consumers can reach B_2 through a direct channel outside the integrated platform. As discussed in the previous section, this captures environments where consumers face costs of switching across channels, and allows us to isolate the effects of tying in a simple way.

Each specialised platform B_i charges a listing fee r_i, for $i = 1, 2$, to sellers, but does not charge consumers.[12] Seller participation affects consumer demand through indirect network effects: a larger seller base makes a platform more attractive to consumers.

investment incentives. By reducing the rival's expected scale and profits, tying weakens its incentive to invest, lowering its equilibrium quality in both the mediated and direct channels.

[12]This simplification is not unrealistic, and fits several of the abovementioned antitrust cases. See Section 6 for a slightly different model where consumers pay the platforms.

In the baseline analysis, we assume for simplicity that B_2 does not pay an access fee to platform A_1 (in line with several antitrust cases mentioned in the Introduction). We relax this assumption in Sections 3.6 and 3.7.

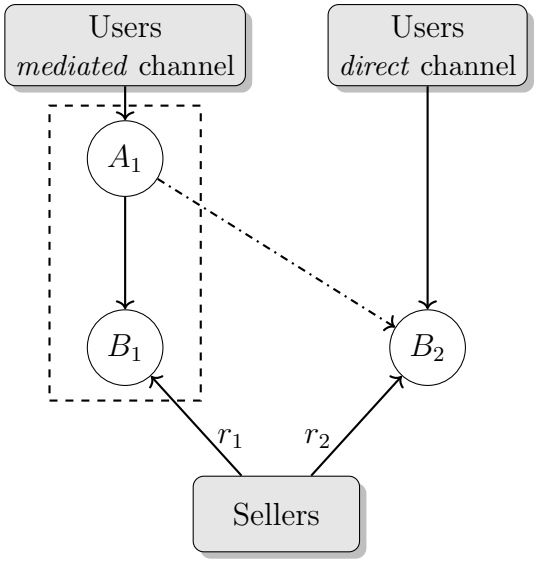


Figure 1: Market configuration

2.1 Primitives

The functions f_{1M} and f_{2M} below describe consumer demand for B_1 and B_2 through the mediated channel. The function f_{2D} describes demand for B_2 through its direct channel.

Let s_1 and s_2 denote the masses of sellers active on B_1 and B_2, respectively. The parameter b_i measures the convenience benefit associated with service B_i within the mediated channel. Equivalently, one can think of b_i as the inverse of the cost consumers incur to access service B_i while on the core platform A_1. There is no such cost for consumers who access B_2 on the direct channel. Our analysis focuses on marginal changes in the platform's convenience parameters. We interpret an increase in b_1 as the reduced-form effect of tying, which raises the convenience of accessing B_1 through the mediated channel. We interpret a decrease in b_2 as demotion of B_2 in the mediated channel.

Consumers. Consumer demand is given by

$$q_{1M} = f_{1M}(b_1, b_2, s_1, s_2), \qquad q_{2M} = f_{2M}(b_1, b_2, s_1, s_2), \qquad q_{2D} = \alpha f_{2D}(s_2),$$

where $\alpha > 0$ measures the relative size of the direct channel. We impose the following restrictions on demand primitives.

Assumption 1 (Demand primitives)**.** *The demand functions f_{1M}, f_{2M}, and f_{2D} are continuously differentiable. For $i, j \in \{1, 2\}$, $i \neq j$,*

(i) for all relevant arguments,

$$\frac{\partial f_{iM}}{\partial b_i} > 0, \qquad \frac{\partial f_{iM}}{\partial b_j} < 0, \qquad \frac{\partial f_{iM}}{\partial s_i} > 0, \qquad \frac{\partial f_{iM}}{\partial s_j} < 0, \qquad \frac{\partial f_{2D}}{\partial s_2} > 0.$$

(ii) Own effects weakly dominate *cross effects in the mediated channel, that is,*

$$\frac{\partial f_{iM}}{\partial b_i} \geq \left|\frac{\partial f_{jM}}{\partial b_i}\right|, \qquad \frac{\partial f_{iM}}{\partial b_i} \geq \left|\frac{\partial f_{iM}}{\partial b_j}\right|, \qquad \frac{\partial f_{iM}}{\partial s_i} \geq \left|\frac{\partial f_{jM}}{\partial s_i}\right|.$$

Assumption 1 imposes standard monotonicity restrictions on demand. Part (i) requires that a higher own convenience benefit increases own demand and reduces the rival's demand in the mediated channel. It also captures indirect network effects from seller participation to consumer demand: an increase in seller participation on one platform raises demand for that platform and lowers demand for the competing platform in the mediated channel. Part (ii) requires own effects to weakly *dominate* cross effects.

Sellers. Sellers are heterogeneous in their fixed cost of participation, κ, which is distributed according to the cdf G, with density $g > 0$ in $[0, \bar{\kappa}]$. A seller active on platform i obtains payoff

$$\pi_i^S(\kappa) = mq_i - r_i - \kappa, \qquad i = 1, 2,$$

where $q_1 = q_{1M}$ and $q_2 = q_{2M} + q_{2D}$, the parameter $m > 0$ denotes the gross seller margin per consumer and r_i is a lump-sum listing fee charged to active sellers on platform i. A seller participates on platform i if and only if $\pi_i^S(\kappa) \geq 0$. Thus, the number of sellers on platform i is given by $s_i = G(mq_i - r_i)$. Note that a seller's participation decision on one platform does not depend on whether it is also active on the other: a seller of type κ joins B_i whenever $mq_i - r_i \geq \kappa$, for $i = 1, 2$, so sellers can possibly multi-home.[13]

Given seller fees (r_1, r_2), there is a fixed point (S_1, S_2) satisfying

$$\begin{aligned} S_1 &= G(mf_{1M}(b_1, b_2, S_1, S_2) - r_1)\,, \\ S_2 &= G(m\left[f_{2M}(b_1, b_2, S_1, S_2) + \alpha f_{2D}(S_2)\right] - r_2)\,. \end{aligned}$$

We assume that this system admits a locally unique solution.

In most of the analysis, we focus on the case where seller participation is interior. In Section 3.4, we study the case where the direct channel is so large that all sellers will want to be on platform B_2.

[13]Merchants might list on both Google Shopping and on comparison sites, and on both Facebook Marketplace and on eBay or Vinted.

Equilibrium quantities are

$$\begin{aligned} Q_{1M}(b_1, b_2, r_1, r_2, \alpha) &= f_{1M}(b_1, b_2, S_1, S_2), \\ Q_{2M}(b_1, b_2, r_1, r_2, \alpha) &= f_{2M}(b_1, b_2, S_1, S_2), \\ Q_{2D}(b_1, b_2, r_1, r_2, \alpha) &= \alpha f_{2D}(S_2). \end{aligned}$$

For expositional convenience, we let $g_1 \equiv g(mQ_{1M} - r_1)$ and $g_2 \equiv g(m(Q_{2M} + Q_{2D}) - r_2)$.

We impose the following regularity conditions.

Assumption 2 (Regularity of seller-side feedbacks)**.**

(i) For all relevant arguments,

$$1 - mg_1 \frac{\partial f_{1M}}{\partial s_1} > 0, \qquad 1 - mg_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2} > 0,$$

(ii) $\Delta_S \equiv \left(1 - mg_1 \frac{\partial f_{1M}}{\partial s_1}\right)\left(1 - mg_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2}\right) - m^2 g_1 g_2 \frac{\partial f_{1M}}{\partial s_2}\frac{\partial f_{2M}}{\partial s_1} > 0.$

Assumption 2.(i) requires the feedback effect on the seller side not to be too large. Assumption 2.(ii) is a standard determinant condition, $\Delta_S > 0$, which guarantees well-defined comparative statics.

2.2 Platform payoffs and timing

Each platform i earns revenue from the seller fees paid by sellers active on B_i. The platform profits are $\Pi_i = r_i S_i$. (Sections 3.6 and 3.7 consider the case in which platform 1 can also extract rents from transactions on the mediated channel by its rival.)

Throughout the analysis, we impose the following conditions.

Assumption 3 (Regularity of the fee game)**.** *For all relevant arguments,*

(i) $\frac{\partial^2 \Pi_1}{\partial r_1^2} < 0, \ \frac{\partial^2 \Pi_2}{\partial r_2^2} < 0,$

(ii) $\Delta_r \equiv \frac{\partial^2 \Pi_1}{\partial r_1^2}\frac{\partial^2 \Pi_2}{\partial r_2^2} - \frac{\partial^2 \Pi_1}{\partial r_1 \partial r_2}\frac{\partial^2 \Pi_2}{\partial r_2 \partial r_1} > 0.$

Assumption 4 (Super- and sub-modularity of fees in b)**.** *At the equilibrium, for $J, K \in \{1, 2\}$ with $J \neq K$:*

(i) (own convenience) $\dfrac{\partial^2 \Pi_J}{\partial r_J \, \partial b_J} > 0;$

(ii) (rival convenience) $\dfrac{\partial^2 \Pi_J}{\partial r_J \, \partial b_K} < 0.$

Assumption 3 is a standard assumption on profit concavity and ensures stability. Assumption 4 states that a platform's marginal return to raising its seller fee increases with the convenience of its own specialised service and decreases with that of its rival. Greater accessibility of B_J makes its mediated demand more valuable to sellers, relaxing platform J's fee-setting trade-off; greater accessibility of the rival instead diverts mediated demand from B_J and may further reduce its direct demand through lower seller participation.

In Section 3, tying (an increase in b_1) invokes part (i) for platform 1 and part (ii) for platform 2. In Section 4, demotion (a decrease in b_2) invokes part (i) for platform 2 and part (ii) for platform 1.

The game. The timing is as follows. In the first stage, platforms simultaneously choose seller fees r_1 and r_2. In the second stage, sellers and consumers decide whether to participate on each platform. Sellers and consumers have fulfilled expectations about seller participation and consumer demand on both sides of the market. The equilibrium concept is subgame perfect Nash equilibrium with fulfilled expectations.

3 Tying

We now characterise the comparative effects of tying—an increase in b_1, due, for example, to better integration or lower inconvenience from accessing B_1 through the mediated channel—on equilibrium seller fees, seller participation, consumer demand across channels, and platform profits. We shall also discuss the role played by the direct channel, and illustrate the analysis with a specific demand example that will allow us to study welfare effects. Finally, we shall also explore the effects of the case where the integrated platform charges a positive access fee to rival one.

3.1 The effect of tying on seller fees

We first study how an increase in b_1 affects equilibrium seller fees. Throughout this section, all derivatives of Q_{1M}, Q_{2M}, and Q_{2D} are evaluated at the equilibrium seller-participation levels induced by $(r_1, r_2, b_1, b_2, \alpha)$.

The first-order conditions for an interior equilibrium in seller fees are[14]

$$\frac{\partial \Pi_1}{\partial r_1} = S_1 + r_1 g_1 \left(m \frac{\partial Q_{1M}}{\partial r_1} - 1 \right) = 0, \tag{1}$$

$$\frac{\partial \Pi_2}{\partial r_2} = S_2 + r_2 g_2 \left[m \left(\frac{\partial Q_{2M}}{\partial r_2} + \frac{\partial Q_{2D}}{\partial r_2} \right) - 1 \right] = 0. \tag{2}$$

[14] As we discuss below, when S_2 approaches full market coverage, depending on the distribution of κ, there may exist an equilibrium in mixed strategies in which platform 1 randomises over fees, occasionally lowering r_1 to attract demand away from platform 2.

The terms S_i capture infra-marginal seller-fee revenue, whereas the remaining terms capture the loss of seller participation and the resulting change in the amount of transactions in the mediated channel.

Using the implicit function theorem, we sign the effect of b_1 on equilibrium fees (under Assumption 4):

$$\frac{dr_1^*}{db_1} = \frac{-\frac{\partial^2\Pi_1}{\partial r_1 \partial b_1}\frac{\partial^2\Pi_2}{\partial r_2^2} + \frac{\partial^2\Pi_2}{\partial r_2 \partial b_1}\frac{\partial^2\Pi_1}{\partial r_1 \partial r_2}}{\Delta_r}, \quad \frac{dr_2^*}{db_1} = \frac{-\frac{\partial^2\Pi_2}{\partial r_2 \partial b_1}\frac{\partial^2\Pi_1}{\partial r_1^2} + \frac{\partial^2\Pi_1}{\partial r_1 \partial b_1}\frac{\partial^2\Pi_2}{\partial r_2 \partial r_1}}{\Delta_r}. \tag{3}$$

where $\Delta_r > 0$ by Assumption 3. Thus, the sign of the expressions (3) is the same as the sign of the numerator. Here, tying affects equilibrium seller fees through two channels. The first is a *direct marginal profitability effect*: tying changes each platform's own incentive to raise its seller fee. The second is a *strategic feedback effect*: once one platform's fee incentive changes, the rival adjusts its own fee, which feeds back into the platform's optimal fee.

Under Assumption 4, tying raises platform 1's marginal incentive to increase its seller fee and lowers platform 2's. The reason is that a higher b_1 increases the value to sellers of participating on B_1, making platform 1's seller base less sensitive to a fee increase. Conversely, tying reduces the attractiveness of B_2 by lowering its demand in the mediated channel and, through seller participation, weakening its demand in the direct channel too. Platform 2 therefore has a stronger incentive to lower its fee to retain sellers.

The equilibrium response also depends on the strategic interaction between seller fees. The following lemma summarises the result.

Lemma 1 (Effect of tying on seller fees)**.** *Under our maintained assumptions:*

(i) if r_1 and r_2 are strategic substitutes, then tying raises platform 1's equilibrium seller fee and lowers platform 2's equilibrium seller fee, that is $dr_1^*/db_1 > 0$, $dr_2^*/db_1 < 0$;

(ii) if r_1 and r_2 are strategic complements, the same conclusion holds under the diagonal-dominance condition reported in the Appendix.

Proof. See Appendix. □

The intuition is as follows. Tying strengthens the position of the integrated platform and weakens that of the rival. Platform 1 therefore has an incentive to increase the seller fee whereas platform 2 has an incentive to reduce it. Under strategic substitutability, the strategic-feedback terms in (3) reinforce the direct effect of tying, which pushes platform 1 towards a higher seller fee and platform 2 towards a lower seller fee. Under strategic complementarity, by contrast, the strategic-feedback terms work against the direct effect. The diagonal dominance condition (reported in the Appendix) ensures that the direct marginal effects dominate these countervailing feedbacks, and that the direct effect of tying determines the sign of the equilibrium fee response: r_1^* rises and r_2^* falls.

3.2 The effect of tying on seller participation

Differentiating the seller-entry conditions yields:

$$\frac{dS_1^*}{db_1} = g_1\left(m\frac{dQ_{1M}^*}{db_1} - \frac{dr_1^*}{db_1}\right), \qquad \frac{dS_2^*}{db_1} = g_2\left(m\frac{dQ_{2M}^*}{db_1} + m\frac{dQ_{2D}^*}{db_1} - \frac{dr_2^*}{db_1}\right). \tag{4}$$

Two forces affect the response of seller participation. The first is an *indirect network effect*: tying changes consumer demand, which in turn affects seller participation. The second is a *strategic pricing effect*: under the conditions in Lemma 1, tying raises platform 1's seller fee and lowers platform 2's seller fee.

On the consumer side, the response of demand reflects three forces: the direct effect of tying on mediated demand, and two cross-side network effects operating through seller participation on each platform:

$$\frac{dQ_{1M}^*}{db_1} = \overbrace{\frac{\partial f_{1M}}{\partial b_1}}^{\text{direct effect}} + \overbrace{\frac{\partial f_{1M}}{\partial s_1}\frac{dS_1^*}{db_1} + \frac{\partial f_{1M}}{\partial s_2}\frac{dS_2^*}{db_1}}^{\text{cross-side network effect}}, \tag{5}$$

$$\frac{dQ_{2M}^*}{db_1} = \overbrace{\frac{\partial f_{2M}}{\partial b_1}}^{\text{direct effect}} + \overbrace{\frac{\partial f_{2M}}{\partial s_1}\frac{dS_1^*}{db_1} + \frac{\partial f_{2M}}{\partial s_2}\frac{dS_2^*}{db_1}}^{\text{cross-side network effect}}, \tag{6}$$

$$\frac{dQ_{2D}^*}{db_1} = \underbrace{\alpha\frac{\partial f_{2D}}{\partial s_2}\frac{dS_2^*}{db_1}}_{\text{cross-side network effect}}. \tag{7}$$

Tying shifts demand towards platform 1 and away from platform 2, which tends to increase S_1^* and reduce S_2^*. The fee responses work in the opposite direction: under Lemma 1, platform 1 raises its seller fee, whereas platform 2 lowers it. To determine the net effect on seller participation, we solve the system simultaneously. Applying Cramer's rule yields:

$$\frac{dS_1^*}{db_1} = \frac{1}{\Delta_S}\left\{g_1\left(m\frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1}\right)\left(1 - mg_2\frac{\partial f_{2M}}{\partial s_2} - \alpha mg_2\frac{\partial f_{2D}}{\partial s_2}\right) + mg_1\frac{\partial f_{1M}}{\partial s_2}\,g_2\left(m\frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1}\right)\right\}, \tag{8}$$

$$\frac{dS_2^*}{db_1} = \frac{1}{\Delta_S}\left\{\left(1 - mg_1\frac{\partial f_{1M}}{\partial s_1}\right)g_2\left(m\frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1}\right) + mg_2\frac{\partial f_{2M}}{\partial s_1}\,g_1\left(m\frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1}\right)\right\}, \tag{9}$$

where $\Delta_S > 0$ by Assumption 2(ii).[15] For the rest of the analysis, we introduce the

[15]Stability conditions in Assumption 2(ii) ensure that the relevant terms in determining the sign of equations (8) and (9) are those in Condition 1.

following sufficient condition.[16]

Condition 1 (Dominance of direct effects)**.** Direct demand effects of tying dominate strategic fee effects *if*

$$m\,\frac{\partial f_{1M}}{\partial b_1} \;>\; \frac{dr_1^*}{db_1} \;>\; 0 \qquad \textit{and} \qquad m\left|\frac{\partial f_{2M}}{\partial b_1}\right| \;>\; \left|\frac{dr_2^*}{db_1}\right|. \tag{10}$$

Let us explain the meaning and the role of Condition 1. An increase in b_1 makes platform 1 more attractive and platform 2 less attractive to consumers in the mediated channel. These changes affect sellers because seller participation depends on the demand that sellers expect to reach. Holding fees fixed, tying raises the value of joining platform 1 and lowers the value of joining platform 2.

On the other hand, equilibrium fee responses move seller participation in the opposite direction. Platform 1 raises its seller fee, which discourages participation on B_1. Platform 2 lowers its seller fee, which helps preserve participation on B_2.

Condition 1 ensures that the "direct" demand-side effect of tying is stronger than the "strategic" effect due to this fee response.[17] Were both inequalities in Condition 1 reversed—so that on each platform the fee response outweighed the direct demand effect—tying would reduce seller participation on platform 1 and raise it on platform 2: the practice would strengthen the rival, and it would likely not be profitable for the integrated platform.

As long as seller participation moves in this direction, the effects on consumer demand are strengthened by cross-side network effects: more sellers on platform 1 further increase demand for B_1, whereas fewer sellers on platform 2 further reduce demand for B_2. This is true not only in the mediated channel. Because the direct channel demand for B_2 also depends on the mass of sellers on B_2, the reduction in S_2^* lowers direct demand Q_{2D}^* as well. Thus a practice implemented inside the mediated channel spills over to the rival's direct channel through seller participation. The next proposition summarises these results for interior seller participation.[18]

Proposition 1 (Effect of tying on seller participation and quantities)**.** *Under our maintained assumptions and Condition 1, tying leads to:*

(i) higher seller participation on platform 1 and lower seller participation on platform 2, i.e., $dS_1^*/db_1 > 0$, $dS_2^*/db_1 < 0$.

(ii) higher consumer demand on platform 1's mediated channel, lower consumer demand

[16] In the Supplementary material, we show that these conditions are always satisfied in our Hotelling setting.

[17] Under Condition 1, $m\,\partial f_{1M}/\partial b_1 > dr_1^*/db_1$ and $m\,\partial f_{2M}/\partial b_1 < dr_2^*/db_1$. Then the signs in (8)–(9) are straightforward and, thus, seller participation on platform 1 increases and on platform 2 decreases.

[18] When seller participation is full, sellers no longer respond to tying. See discussion in Section 3.4.

in platform 2's mediated and direct channels, i.e., $dQ^*_{1M}/db_1 > 0$, $dQ^*_{2M}/db_1 < 0$, $dQ^*_{2D}/db_1 < 0$.

Proof. See Appendix. □

The effect on mediated and total quantities We have identified conditions under which tying increases mediated demand and seller participation for B_1, while reducing demand for B_2 across both the mediated and direct channels. How tying affects total quantity cannot be signed in general. Even the effect on total quantity in the mediated channel is ambiguous. One might expect the dominance of own effects over cross effects in Assumption 1 to imply that the increase in Q^*_{1M} exceeds the reduction in Q^*_{2M}. That reasoning is incomplete, because it ignores equilibrium feedback through seller participation. If tying induces a sufficiently large reduction in S^*_2, the fall in Q^*_{2M} may dominate the increase in Q^*_{1M}, so quantity in the mediated channel may decline.

Let total quantity in the mediated channel be $Q_M \equiv Q_{1M} + Q_{2M}$. Then, summing (5)–(6), the effect of tying on the quantity in the mediated channel is given by

$$\frac{dQ^*_M}{db_1} = \underbrace{\left(\frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial b_1}\right)}_{\text{direct effect (+)}} + \underbrace{\left(\frac{\partial f_{1M}}{\partial s_1} + \frac{\partial f_{2M}}{\partial s_1}\right)\frac{dS^*_1}{db_1}}_{\text{network-related effect on platform 1 (+)}} + \underbrace{\left(\frac{\partial f_{1M}}{\partial s_2} + \frac{\partial f_{2M}}{\partial s_2}\right)\frac{dS^*_2}{db_1}}_{\text{network-related effect on platform 2 (-)}}. \tag{11}$$

Under Assumption 1, the first term in brackets, which is the *direct net market expansion* of tying in the mediated channel, holding seller participation fixed, is positive; the second and third terms, which are respectively the net effects on total mediated demand of an additional seller on B_1 and on B_2, are positive and negative, respectively.

Under the conditions of Proposition 1, $dS^*_1/db_1 > 0$ and $dS^*_2/db_1 < 0$. Hence, the sign of dQ^*_M/db_1 cannot be determined unambiguously. Tying raises total quantity in the mediated channel *unless* the rival's seller-base reduction is large enough to outweigh both the direct expansion and the gain on the primary platform—which is when seller-side network effects are strong and the rival's loss of scale is most consequential.

In Section 3.5 we present a specific functional form example to better understand the factors at play, and to study the welfare effects of tying.

3.3 The effect of tying on profits

We now consider the profit effects of tying when seller participation is interior. Since $\Pi^*_i = r^*_i S^*_i$, the equilibrium total derivatives with respect to b_1 are (using Lemma 1 and Proposition 1)

$$\frac{d\Pi^*_1}{db_1} = S^*_1\frac{dr^*_1}{db_1} + r^*_1\frac{dS^*_1}{db_1} > 0, \qquad \frac{d\Pi^*_2}{db_1} = S^*_2\frac{dr^*_2}{db_1} + r^*_2\frac{dS^*_2}{db_1} < 0.$$

Indeed, under Condition 1, by Lemma 1 and Proposition 1 we have $dr_1^*/db_1 > 0$, $dr_2^*/db_1 < 0$, $dS_1^*/db_1 > 0$, and $dS_2^*/db_1 < 0$: both the fee-level and the participation term are positive for platform 1, and both are negative for platform 2. The following summarises this discussion.

Proposition 2 (Effect of tying on profits)**.** *Under our maintained assumptions and Condition 1, tying is profitable and harms the rival, i.e.,* $d\Pi_1^*/db_1 > 0$ *and* $d\Pi_2^*/db_1 < 0$.

Proof. Follows from the discussion. □

It is important to note that Proposition 2 is obtained under the assumption that the access fee is zero. We shall study profitability of tying with positive access fees in Sections 3.6 and 3.7 below.

3.4 The role of the direct channel

The direct channel, whose size is measured by α, plays a critical role. At $\alpha = 0$ there is no direct channel and, therefore, no harm. Tying still harms B_2 consumers through the mediated channel (that is, $dQ_{2M}^*/db_1 < 0$) and reduces seller participation on platform 2 ($dS_2^*/db_1 < 0$), but no consumers outside the integrated platform are affected.

For $\alpha > 0$, the response of the direct-channel demand to tying is

$$\frac{dQ_{2D}^*}{db_1} \;=\; \alpha \frac{\partial f_{2D}}{\partial s_2} \;\times\; \frac{dS_2^*}{db_1},$$

where $\partial f_{2D}/\partial s_2 > 0$ by Assumption 1(i) and $dS_2^*/db_1 < 0$ by Proposition 1. The sign of dQ_{2D}^*/db_1 is negative.

The magnitude of the harm is the product of two terms. First, a *scale effect*: a larger α increases the mass of consumers in the direct channel exposed to any given fall in S_2^*. Holding $|dS_2^*/db_1|$ fixed, a marginal increase in α increases the harm by $(\partial f_{2D}/\partial s_2)|dS_2^*/db_1| > 0$. Second, a *participation-response effect*: $|dS_2^*/db_1|$ itself varies with α. When seller participation is interior, a larger direct channel raises the rival's total scale and thereby reinforces the feedback between seller participation and consumer demand on platform 2, so that a given loss of mediated demand translates into a larger contraction of S_2^*. (In the Hotelling specification of Section 3.5, this amplification dominates, and both terms increase in α.)

More generally, however, these two effects need not vary monotonically with the size of the direct channel. While the scale effect tends to magnify the impact of tying as α increases, the participation response may eventually weaken as seller participation becomes less responsive to changes in mediated demand. The overall harm to the rival's direct channel may therefore be non-monotonic in α. The functional-form illustration in Section 3.5 shows that this possibility can indeed arise.

Because the participation cost κ is distributed on the bounded support $[0, \bar{\kappa}]$, seller participation on platform 2 may eventually become full as the size of the direct channel increases. This creates a complication in the analysis. Consider a value of α at which the candidate equilibrium pair is $(\tilde{r}_1^*, \tilde{r}_2^*)$ and the marginal seller joins B_2. Platform 1 might have an incentive to deviate from it: by slightly reducing r_1, it would raise seller participation on B_1, which in turn would shift demand towards it and away from B_2. Hence, Q_{1M} increases and Q_{2M} falls, which in turn will result in the marginal seller not join B_2, contradicting the condition of full participation. Conversely, the interior pure-strategy equilibrium may break down before S_2 reaches one: when S_2 is already close to full participation, platform 1 may profitably raise r_1, since the resulting demand shift towards B_2 can induce only a limited further increase in seller participation on B_2. In the Hotelling specification of Section 3.5 in the Supplementary Material, with a uniform distribution of κ, we show that for a very large α and $S_2 \approx 1$ a mixed-strategy equilibrium may arise, where platform 1 randomises between two fees and seller participation is close to, but not exactly at, its upper bound. In this interval, tying has a negligible effect on the direct channel and hence its sign effectively coincides with the positive effect on the mediated channel.[19]

We therefore delimit the region in which the interior characterisation applies. Let $\big(\widetilde{r}_1(\alpha), \widetilde{r}_2(\alpha)\big)$ denote the solution to the interior first-order conditions (1)–(2), and let $\widetilde{S}_2(\alpha)$ be the seller participation on platform 2. We denote by $\widehat{\alpha}$ the upper bound of the region over which this interior solution constitutes a Nash equilibrium:

$$\widehat{\alpha} \equiv \sup \left\{ \alpha \geq 0 : \big(\widetilde{r}_1(\alpha), \widetilde{r}_2(\alpha)\big) \text{ is a Nash equilibrium in pure strategies} \right\}. \tag{12}$$

We do not characterise the equilibrium for $\alpha \geq \widehat{\alpha}$ in the general model. In the stylised illustration below, however, the post-threshold equilibrium can be solved explicitly.

Proposition 3 (Effect of tying on the direct channel)**.** *Under our maintained assumptions and Condition 1, tying weakly reduces demand in the rival's direct channel for any $\alpha < \widehat{\alpha}$: it is absent at $\alpha = 0$, strictly negative for $\alpha \in (0, \widehat{\alpha})$.*

Proof. See Appendix. □

Proposition 3 shows how tying affects users on the direct channel depending on the size of the latter. At $\alpha = 0$, only the mediated channel exists, and since own effects dominate cross effects (by Assumption 1(ii)), tying has a positive net effect on (mediated channel) users. As the size of the direct channel starts increasing from zero, tying has a negative effect on direct channel users since there is a contraction of seller participation on platform

[19]Note that when the distribution is non-uniform and there is sufficiently little mass around $\bar{\kappa}$, only a pure-strategy equilibrium exists. Intuitively, in this case a small reduction in r_1 attracts too few additional sellers to generate a sufficiently large reduction in platform 2's mediated demand, eliminating platform 1's incentive to deviate from the full-participation equilibrium.

2. Eventually, though, the direct channel will be so large that conduct in the mediated channel has almost no impact on it. For $\alpha > \widehat{\alpha}$, the interior characterisation no longer applies. Nevertheless, as the direct channel becomes sufficiently large, the dependence of seller participation on mediated demand may weaken, so that direct-channel consumers become progressively insulated from the effects of tying. Therefore, tying is beneficial since only the mediated channel — where own effects dominate cross effects — is affected.

One implication of Proposition 3 is that the overall effects of tying can be non-monotonic in the size of the direct channel, as we shall see in the following Section.

3.5 An illustrative example

We illustrate our baseline model with a specific functional form example where two specialised platforms compete à la Hotelling. Consumers are distributed over $\theta \in \mathbb{R}$ with unit density[20] and platforms B_1 and B_2 are located at $l_1 = 0$ and $l_2 = 1$. Consumers in the direct channel, with density α, can buy B_2 obtaining

$$U_{2D}(\theta) = 1 + \omega s_2 - t|1 - \theta|,$$

whereas consumers on the mediated channel B_J obtain

$$U_{JM}(\theta) = 1 + b_J + \omega s_J - t|l_J - \theta|.$$

Here $t > 0$ measures disutility from their ideal variety, s_J is seller participation on B_J, and $\omega > 0$ measures cross-network effects.

Figure 2 illustrates consumer allocation in the mediated channel (for the hypothetical case $b_2 + \omega s_2 > b_1 + \omega s_1$). The marginal consumers are θ_1, θ_{12}, and θ_2, yielding[21]

$$q_{JM}(b_J, b_{\neg J}, s_J, s_{\neg J}) = \frac{t + 2 + 3b_J - b_{\neg J} + 3\omega s_J - \omega s_{\neg J}}{2t}, \qquad J = 1, 2.$$

In the direct channel, the marginal consumers satisfy $U_{2D}(\theta) = 0$, so that[22]

$$q_{2D}(s_2) = \frac{2\alpha(1 + \omega s_2)}{t}.$$

There is a mass 1 of sellers with fixed costs $\kappa \sim U[0, \overline{\kappa}]$. We assume that for each quantity sold in the market, a seller obtains a margin m. Thus, the number of sellers joining each platform is

$$s_J \;=\; \min\left\{\frac{\kappa_J^*}{\overline{\kappa}}, 1\right\}, \qquad J = 1, 2, \tag{13}$$

[20] We assume θ is i.i.d. and independent of whether a consumer uses the mediated or direct channel.
[21] Formally, $\theta_1 = (1 + b_1 + \omega s_1)/t$, $\theta_2 = 1 + (1 + b_2 + \omega s_2)/t$, and $\theta_{12} = (t + b_1 - b_2 + \omega s_1 - \omega s_2)/2t$.
[22] The corresponding cutoffs are $\theta_D^{\pm} = 1 \pm (1 + \omega s_2)/t$.

where the upper bound binds when $\kappa_J^* \geq \overline{\kappa}$, that is, when all sellers join platform J. The interior pure-strategy equilibrium applies up to $\widehat{\alpha}$. Beyond this threshold, for the admissible parameter region reported in the Supplementary Material, the uniform-cost specification admits a semi-mixed fee equilibrium: platform 1 randomises between two fees whereas platform 2 chooses a pure fee. The Supplementary Material derives the two supports, the equilibrium mixing probability, and the conditions under which neither platform has a profitable deviation.

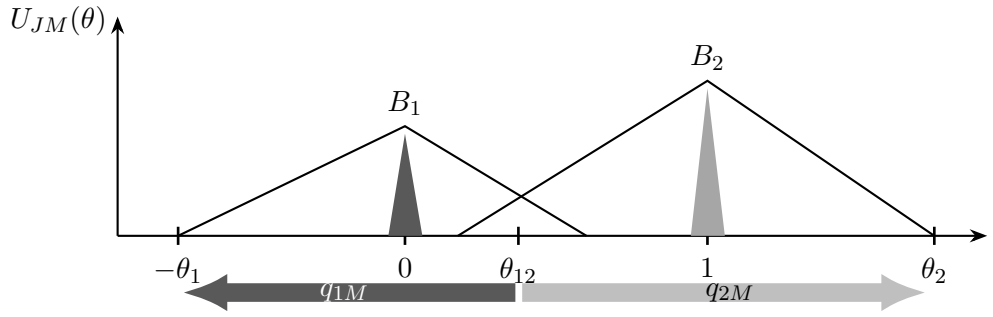


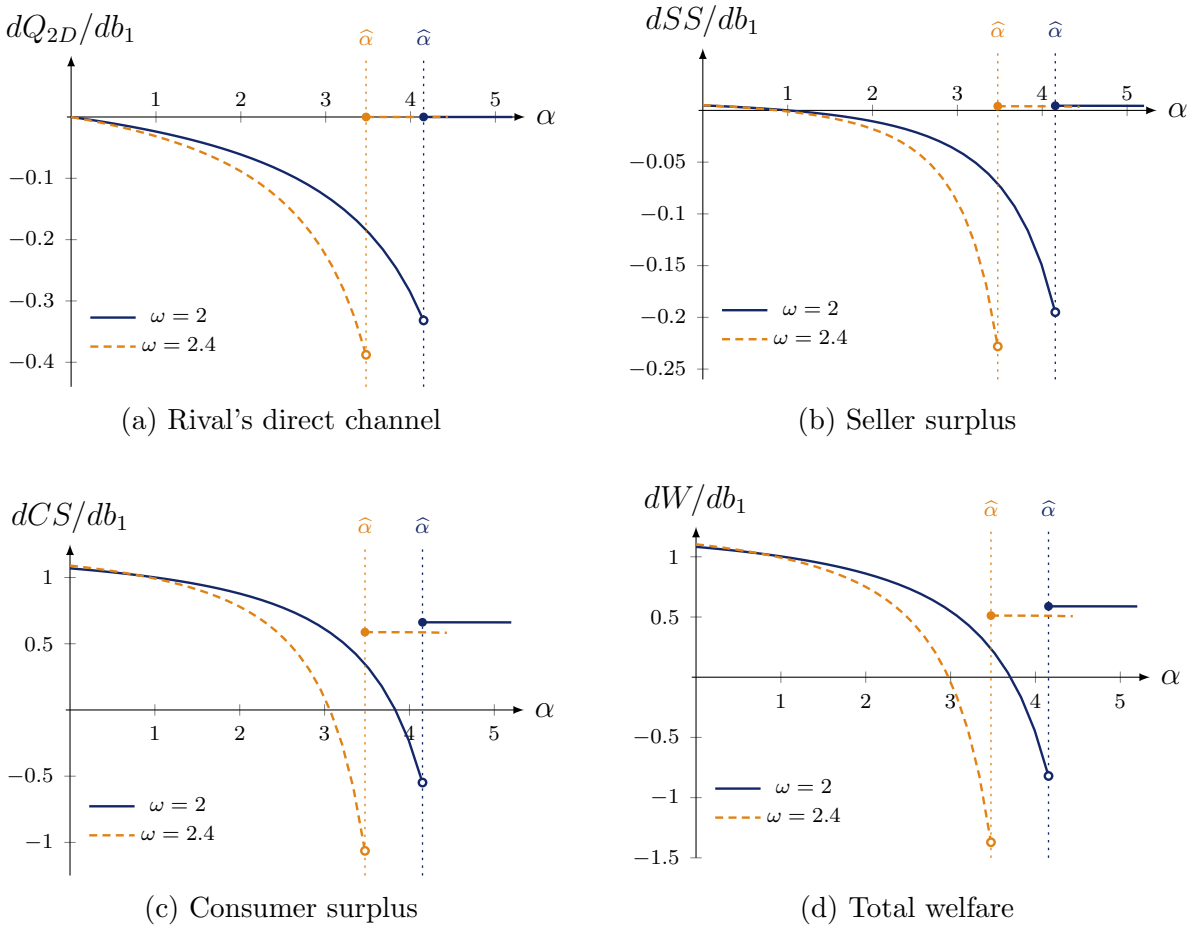
Figure 2: The mediated channel in the Hotelling real line

Figure 3 provides graphical evidence of the welfare effects of tying. Each panel reports the marginal effect of tying, measured by the derivative with respect to b_1, for two values of the network effect. Panel (a) shows that tying reduces demand in the direct channel whenever $\alpha > 0$, due to lower seller participation on the rival platform. This negative effect becomes larger as the direct channel grows. At $\widehat{\alpha}$, however, almost all sellers are present on B_2 and the interior equilibrium ceases to apply. So, in the semi-mixed region, the equilibrium response of seller participation to tying is substantially smaller, and hence the negative effect on the direct channel tends to disappear (see discussion after Proposition 3 and the Supplementary Material). This explains the discontinuity in the effects of tying.

Panel (b) shows the effect on seller surplus. When the direct channel is small, tying benefits sellers overall because the gains to sellers on B_1 are stronger than the losses to sellers on B_2. As the direct channel becomes larger, the losses on the rival increase and eventually dominate. Once α exceeds $\widehat{\alpha}$, tying has a negligible impact on sellers on platform 2, whereas sellers on B_1 continue to benefit from tying due to the positive effect on the quantities in the mediated channel.

Panel (c) reports the effect on consumer surplus, which shows similar non-monotonicity. Thus, there exists an intermediate region of the direct channel where tying harms consumers. Panel (d) shows that similar non-monotonicity arises also for total welfare.

Note that a stronger network effect (measured by ω) works as an amplifier of harm for interior solutions, but it also lowers the point where pro-competitive effect emerges (by lowering $\widehat{\alpha}$). We summarise these results as follows.



(a) Rival's direct channel

(b) Seller surplus

(c) Consumer surplus

(d) Total welfare

Figure 3: Welfare effects of tying. Parameters: $b_1 = b_2 = 0.2$, $m = 0.4$, $t = 2.5$, $\overline{\kappa} = 4$. For $\alpha < \widehat{\alpha}$ the curves are computed from the pure interior fee equilibrium; for $\alpha \geq \widehat{\alpha}$ they are equilibrium expectations under the semi-mixed equilibrium. See Supplementary Material.

Remark 1. *In our Hotelling example, there exist thresholds α^{CS}, α^{SS} and α^{W} such that $dCS/db_1 < 0$ for $\alpha \in (\alpha^{CS}, \widehat{\alpha})$, $dSS/db_1 < 0$ for $\alpha \in (\alpha^{SS}, \widehat{\alpha})$ and $dW/db_1 < 0$ for $\alpha \in (\alpha^{W}, \widehat{\alpha})$, with (i) $\alpha^{SS} < \alpha^{W} < \alpha^{CS} < \widehat{\alpha}$, and (ii) α^{h}, $h = \{CS, SS, W\}$, decreasing in ω.*

3.6 Partial rent extraction

The baseline model assumes that each platform earns revenues only from seller fees. We now allow platform A_1 to extract rents from its rival through a per-transaction access fee $a \geq 0$. Platform profits are given by

$$\Pi_1 = r_1 S_1 + a Q_{2M}, \qquad \Pi_2 = r_2 S_2 - a Q_{2M}, \tag{14}$$

where the access fee applies only to platform 2 sales on A_1. Throughout this subsection, the access fee is exogenous. In the next one, we shall endogenise it.

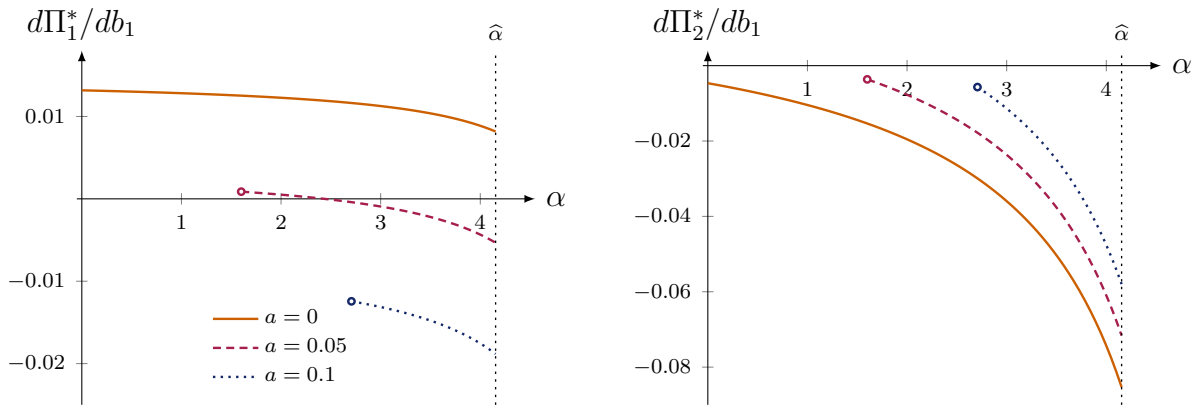
The access fee creates an additional trade-off in the tying decision. On the one hand, tying continues to increase seller demand for platform 1, thereby raising seller-fee revenues as shown in the previous subsection. On the other hand, tying diverts transactions away from the rival platform, reducing the volume on which access fee is collected.

Differentiating equilibrium profits with respect to b_1 yields

$$\frac{d\Pi_1^*}{db_1} = S_1^* \frac{dr_1^*}{db_1} + r_1^* \frac{dS_1^*}{db_1} + a \frac{dQ_{2M}^*}{db_1}. \tag{15}$$

The first two terms coincide with those obtained in the baseline model where $a = 0$. The last term captures the direct opportunity cost of tying, which is negative (by Proposition 1). Since tying is profitable at $a = 0$ (Proposition 2), by continuity it is also profitable for sufficiently small a. When the access fee grows larger, the loss of access-fee revenues obtained from the rival reduces the profitability of tying.

Figure 4 shows that tying can become unprofitable when the access fee is substantial and the direct channel is large enough. Panel (a) shows that a higher a lowers the profitability of tying for platform 1. Panel (b) shows that tying always harms platform 2.[23]



(a) Integrated platform (b) Rival platform

Figure 4: Effects of tying on profits for different access fees. Parameters: $b_1 = b_2 = 0.2$, $m = 0.4$, $t = 2.5$, $\overline{\kappa} = 4$, $\omega = 2$.

3.7 Endogenous access fee

Consider now the case in which the access fee is endogenous. Suppose the integrated platform sets a at the highest level consistent with the rival remaining on the mediated channel. Let Π_2^{out} denote the profit platform 2 earns only from its direct channel. Then,

[23] Curves are drawn only for $\alpha \geq \underline{\alpha}(a)$, the size of the direct channel above which platform 2 prefers to remain on the mediated channel ($\Pi_2 \geq \Pi_2^{\text{out}}$).

platform 1 will set the fee $\bar{a}(b_1)$ at which platform 2's participation constraint binds:

$$\Pi_2^*\big(b_1, \bar{a}(b_1)\big) = \Pi_2^{\text{out}}. \tag{16}$$

Recall from (14) that Π_2^* is already net of the access-fee payment, $\Pi_2^* = r_2^* S_2^* - aQ_{2M}^*$; condition (16) therefore says that platform 2's total equilibrium profit is driven down to its outside option. Note that Π_2^{out} depends neither on the access fee a nor on the convenience parameter b_1, because it is earned outside the mediated channel.

Define $\widehat{\Pi}_i(b_1) \equiv \Pi_i^*\big(b_1, \bar{a}(b_1)\big)$, that is, platform i's equilibrium profit under perfect extraction, evaluated at the perfect-extraction fee $a = \bar{a}(b_1)$. By definition, $\widehat{\Pi}_2(b_1) = \Pi_2^{\text{out}}$ for every b_1 in the relevant neighborhood. Since Π_2^{out} is constant, $\widehat{\Pi}_2'(b_1) = 0$.

Here $\bar{a}(\cdot)$ is a function of b_1: as b_1 changes, the access fee is continuously re-optimised so that platform 2's equilibrium profit remains at Π_2^{out} for every b_1 in the relevant neighbourhood. Because $\widehat{\Pi}_2'(b_1) = 0$, the increase in platform 1's profits from engaging in tying is equal to the effect of tying on the two platforms' joint profit:

$$\widehat{\Pi}_1'(b_1) = \widehat{\Pi}_1'(b_1) + \widehat{\Pi}_2'(b_1) = \frac{d}{db_1}\left[\Pi_1^*\big(b_1, \bar{a}(b_1)\big) + \Pi_2^*\big(b_1, \bar{a}(b_1)\big)\right]. \tag{17}$$

The reason is the following. The access payment aQ_{2M}^* is a transfer between platforms.[24] The transfer cancels from the two platforms' joint profit, which by (14) reduces to $\Pi_1^* + \Pi_2^* = r_1^* S_1^* + r_2^* S_2^*$; consequently, $\widehat{\Pi}_1'(b_1)$ is positive if the right-hand side of (17) is.

Proposition 4 (Tying under endogenous fee)**.** *Suppose the access fee is endogenous. Then, tying is profitable for the integrated platform if and only if it raises the two platforms' joint profit,* $\Pi_1 + \Pi_2 = r_1 S_1 + r_2 S_2$.

Proof. Follows from discussion. □

The intuition is that, once the integrated platform has extracted all the surplus that keeps the rival on the mediated channel, it becomes the residual claimant on that surplus. A marginal increase in b_1 can no longer benefit the integrated platform merely by shifting rents away from the rival: the rival is already held at Π_2^{out}. Tying is profitable if and only if it increases the joint profit generated by the two platforms, taking into account the induced changes in seller fees, seller participation, and quantities. If tying reduces joint

[24] To see why, note that

$$\frac{d\widehat{\Pi}_1}{db_1} = \left(S_1^* \tfrac{dr_1^*}{db_1} + r_1^* \tfrac{dS_1^*}{db_1}\right) + \underbrace{\bar{a}\, \tfrac{dQ_{2M}^*}{db_1}}_{\text{per-transaction}} + \underbrace{Q_{2M}^*\, \bar{a}'(b_1)}_{\text{fee adjustment}}, \tag{18}$$

$$\frac{d\widehat{\Pi}_2}{db_1} = \left(S_2^* \tfrac{dr_2^*}{db_1} + r_2^* \tfrac{dS_2^*}{db_1}\right) - \bar{a}\, \tfrac{dQ_{2M}^*}{db_1} - Q_{2M}^*\, \bar{a}'(b_1) = 0, \tag{19}$$

platform profit, the loss is borne by the integrated platform through the lower access fee required to keep platform 2 on the mediated channel.

Figure 4 also shows that, under partial rent extraction, as α increases, platform 2's direct business becomes more important, whereas tying reduces both its mediated and, through seller participation, the quantity on the direct channel. At the same time, Π_2^{out} is increasing in α, reducing the rents that platform 1 can extract through the mediated channel. This explains why tying might turn unprofitable for the integrated platform.

These results suggest that tying and access fees are, to some extent, substitutable rent extraction tools. Thus, as suggested by Franck and Peitz (2023) and Bar-Isaac (2026), a policy which restricts access fees (such as caps) might induce a platform to use tying.

4 Demotion of the rival platform

We now consider an alternative conduct, which we call *demotion*. Formally, demotion is modelled as a marginal *decrease* in b_2.[25]

The mechanism through which demotion propagates is the mirror image of the tying mechanism. A lower b_2 contracts consumer demand for rival platform B_2 in the mediated channel, which in turn reduces sellers' incentive to participate on B_2. Through the seller-side cross-network effect, the contraction of S_2 further depresses q_{2M} *and*, crucially, depresses q_{2D} through $\partial f_{2D}/\partial s_2 > 0$. Of course, the effects of demotion on the integrated platform are of opposite sign: lowering b_2 shifts consumer demand towards platform B_1 and raises seller participation S_1, further raising q_{1M}. This is the main mechanism for given fees. In the Appendix, we also show that, under conditions mirroring those in the tying case, a demotion (i.e., a reduction in b_2) leads to a higher fee for platform 1 sellers and a lower one for platform 2 sellers: $-dr_1^*/db_2 > 0$ and $-dr_2^*/db_2 < 0$.

Also in this case, we focus on the case where there is *dominance of direct effects over strategic effects* (this is the equivalent of Condition 1 for the case of demotion, see Appendix — Condition 2). We state these results for interior seller participation and pure strategy Nash equilibrium as follows, while relegating all algebra to the Appendix.

Proposition 5 (Effect of demotion on seller participation and quantities)**.** *Under our maintained assumptions and Condition 2, for any $\alpha < \widehat{\alpha}$ demotion leads to:*

(i) higher seller participation on platform 1 and lower seller participation on platform 2, i.e., $-dS_1^*/db_2 > 0$, $-dS_2^*/db_2 < 0$;

[25] Of course, in some environments, tying and demotion might take place simultaneously. For instance, if a search engine platform artificially changes the ranking of links, placing its own website ahead of the rival's, this can be seen as both tying and demotion. In other environments they can be distinct. For instance, interoperability could be degraded, or users might be misled into thinking that use of the rival might create security issues. In any event, treating them as conceptually different can be insightful, as we shall see below.

(ii) higher consumer demand on platform 1's mediated channel, lower consumer demand in platform 2's mediated and direct channels, i.e., $-dQ_{1M}^*/db_2 > 0$, $-dQ_{2M}^*/db_2 < 0$, *and* $-dQ_{2D}^*/db_2 < 0$.

Proof. See Appendix. □

The effect of demotion on mediated and total quantities As for tying, the net effect of demotion on mediated and total quantities cannot be established a priori. (See discussion following Proposition 1 above.)

The effect of demotion on profits The profit effects of demotion are obtained by the same product-rule decomposition as under tying. Since demotion is a decrease in b_2, the relevant effect is the negative of the derivative with respect to b_2. Hence platform 1's seller-fee revenue increases and platform 2's seller-fee revenue decreases. That is,

$$\frac{d\Pi_1^*}{db_2} = S_1^* \frac{dr_1^*}{db_2} + r_1^* \frac{dS_1^*}{db_2}, \qquad \frac{d\Pi_2^*}{db_2} = S_2^* \frac{dr_2^*}{db_2} + r_2^* \frac{dS_2^*}{db_2}.$$

Since $-dr_1^*/db_2 > 0$, $-dr_2^*/db_2 < 0$, $-dS_1^*/db_2 > 0$, and $-dS_2^*/db_2 < 0$, the effects of demotion on profits carry the same signs as those of tying. Thus, demotion is profitable and harms platform 2. Therefore, we conclude the following.

Proposition 6 (Effect of demotion on profits)**.** *Under our maintained assumptions and Condition 2, demotion is profitable for platform 1 and harms platform 2.*

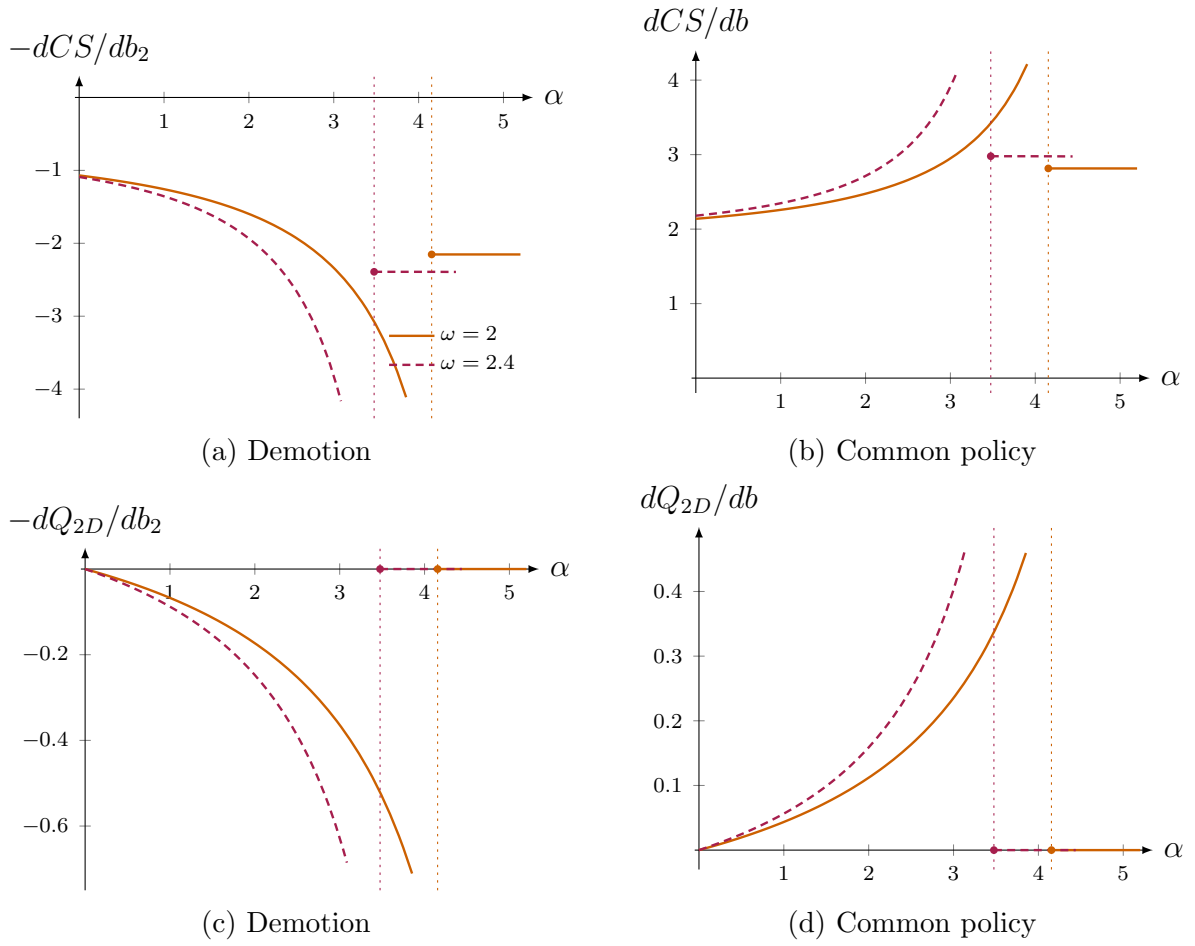
Proof. Follows from the discussion. □

The left column of Figure 5 (Panels (a) and (c)) shows that demotion always harms consumers in our Hotelling illustration. Because the effect on seller surplus is negative at lower values of α than consumer surplus, demotion also always reduces seller surplus and total welfare.

5 A common-access policy

We now turn to a policy that, rather than favouring one specialised service, improves access to *both* services within the mediated environment. Such symmetric access obligations are the structure of many ex ante remedies imposed under the DMA in the EU. Formally, define $b_1 = b_{01} + b$, $b_2 = b_{02} + b$, and study the effect of a marginal increase in b.

By raising both b_1 and b_2 by the same amount, this policy reduces frictions symmetrically for all services on the integrated platform, in contrast to tying, which favours the integrated service alone. In the two-sided environment, the policy operates not only on



(a) Demotion (b) Common policy

(c) Demotion (d) Common policy

Figure 5: Demotion and common policy. Parameters: $b_1 = b_2 = 0.2$, $m = 0.4$, $t = 2.5$, $\overline{\kappa} = 4$. For $\alpha < \widehat{\alpha}$ the curves are computed from the pure interior fee equilibrium; for $\alpha \geq \widehat{\alpha}$ they are equilibrium expectations under the semi-mixed equilibrium. See Supplementary Material. In each panel the interior branch is truncated at the axis boundary.

consumers in the mediated channel but also, through cross-side network effects, on seller participation, and from there on demand in the direct channel.

By Assumption 1.(ii), $\partial f_{JM}/\partial b_1 + \partial f_{JM}/\partial b_2 \geq 0$ for $J = 1, 2$: for given fees and seller participation, a common increase in b weakly raises the demand in the mediated channel for *both* services. The induced expansion of seller participation on both platforms then feeds back positively into consumer demand.

The analysis is quite intuitive because a common increase in b is the sum of two shifts we have already studied. Since $b_1 = b_{01} + b$ and $b_2 = b_{02} + b$, the chain rule decomposes the policy response of any equilibrium object X^* into a *tying component* (its response to b_1) and a *raising-b_2 component* (its response to b_2, the opposite of demotion),

$$\frac{dX^*}{db} = \frac{dX^*}{db_1} + \frac{dX^*}{db_2}. \tag{20}$$

By Lemma 1 and Propositions 1 and 5, the two components carry opposite signs.

Whether the favourable component prevails depends on how each platform's equilibrium responses to its own convenience benefit compare with its responses to the rival's. We impose Condition 3 (see Appendix), which is the counterpart of Conditions 1 and 2, used for tying and demotion respectively. In practice, this condition also implies that the own-convenience effect on mediated quantities dominates the corresponding cross-platform effect:

$$\frac{dQ_{1M}^*}{db_1} \geq \left|\frac{dQ_{1M}^*}{db_2}\right|, \qquad \frac{dQ_{2M}^*}{db_2} \geq \left|\frac{dQ_{2M}^*}{db_1}\right|.$$

Thus, an improvement in a platform's own convenience has a larger effect on its mediated quantity than an equally sized improvement in the rival's convenience. The cross-platform effect may operate in either direction, but cannot overturn the own-convenience effect. Consequently, under a symmetric improvement in convenience, mediated quantities weakly increase on both platforms. Focusing on the region in which seller participation is interior and the fee equilibrium is characterised by the first-order conditions above, we conclude the following.

Proposition 7 (Effect of the policy on seller participation and quantities)**.** *Under our maintained assumptions and Condition 3, for any $\alpha < \widehat{\alpha}$ the common-access policy leads to:*

(i) an increase in seller participation on both platforms, that is, $dS_1^*/db > 0$, $dS_2^*/db > 0$;

(ii) an increase in consumer demand in the mediated channel, that is, $dQ_{1M}^*/db > 0$, $dQ_{2M}^* db > 0$, and an increase in the direct channel, that is, $dQ_{2D}^*/db > 0$.

Proof. See Appendix. □

The contrast with tying (Proposition 1) is the main result of this section. As under both tying and demotion, seller participation and quantities rise for the integrated platform but fall for the rival. Under a policy that forces the integrated platform to improve accessibility (or reduce costs) symmetrically for both services it intermediates, seller participation and quantities instead rise for *both* platforms, and in particular the rival's direct channel expands too. In this dimension the policy is preferable to tying.

The effect of the policy on profits We now ask the question of whether the integrated platform would willingly adopt such a policy: if the symmetric remedy lowered its profit, it might pre-empt the remedy by forgoing the convenience improvement altogether. In the baseline environment the platforms earn only seller-fee revenue, $\Pi_i^* = r_i^* S_i^*$, so the

product rule gives the decomposition

$$\frac{d\Pi_1^*}{db} = \underbrace{S_1^* \frac{dr_1^*}{db}}_{\geq 0} + \underbrace{r_1^* \frac{dS_1^*}{db}}_{\geq 0}, \qquad \frac{d\Pi_2^*}{db} = \underbrace{S_2^* \frac{dr_2^*}{db}}_{\geq 0} + \underbrace{r_2^* \frac{dS_2^*}{db}}_{\geq 0}, \tag{21}$$

where each fee response dr_i^*/db and each participation response dS_i^*/db is non-negative under Condition 3 (in the Appendix) and Proposition 7. We conclude the following.

Proposition 8 (Effect of the policy on profits)**.** *Under our maintained assumptions and Condition 3, the common-access policy leads to an increase of both platforms' profits, i.e.,* $d\Pi_1^*/db \geq 0$, $d\Pi_2^*/db \geq 0$.

Proof. See Appendix. □

The policy is a Pareto improvement on the supply side: unlike tying, which raises the integrated platform's profit while lowering the rival's (Proposition 2), a non-discriminatory access improvement benefits *both* platforms. This matters for its implementability: because the integrated platform is not made worse off, it has no incentive to pre-empt the remedy by forgoing the convenience improvement in the first place.

Consumer surplus, seller surplus, and welfare. The general model is specified in terms of reduced-form demand and therefore does not, without additional structure on consumer utility, permit us to sign consumer surplus from quantity responses alone. The common-access policy does, however, weakly increase seller surplus. In the Hotelling specification, where consumer surplus is explicitly defined from consumer utility, Figure 5 (Panels (b) and (d)) shows that the policy also raises consumer surplus and total welfare.

6 A model with consumer-side network effects

In the baseline model of Sections 2–5 we studied a two-sided market. Such a model fitted the facts of cases such as *Google Shopping* and *Facebook Marketplace*. We now consider an environment in which there are no sellers, platforms charge consumers, and direct network effects operate *among users* on each specialised service. With this setting, we capture recent high-profile cases such as *Microsoft Teams*, *Meta AI* or *Apple iCloud*. Despite the differences, the mechanisms at work and the qualitative results are remarkably similar.

6.1 Setup

The market retains the same structure as in Section 2; what changes is the pricing instrument (platforms receive prices p_J from consumers rather than seller fees) and the carrier of network externalities (own total usage n_J rather than seller participation).

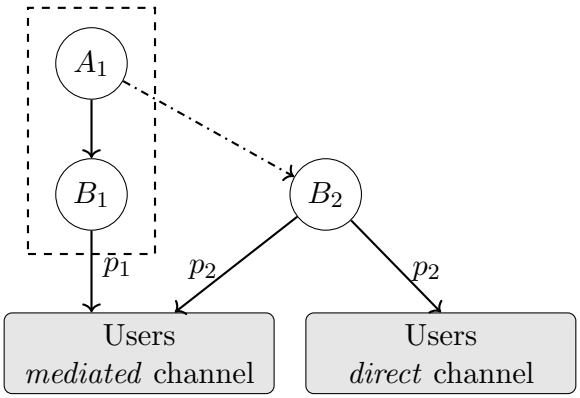


Figure 6: Market structure with consumer-side network effects

Figure 6 provides a graphical representation of our environment. Let q_{1M} denote demand for B_1 and q_{2M} demand for B_2 through the mediated channel, and q_{2D} demand for B_2 through the direct channel:

$$\begin{aligned} q_{1M} &= f_{1M}(b_1, b_2, p_1, p_2, n_1, n_2), \\ q_{2M} &= f_{2M}(b_1, b_2, p_1, p_2, n_1, n_2), \\ q_{2D} &= \alpha f_{2D}(p_2, n_2), \end{aligned} \tag{22}$$

where p_1 and p_2 are the prices, and n_1 and n_2 denote total usage levels of services, for B_1 and B_2 respectively, with $n_1 = q_{1M}$, $n_2 = q_{2M} + q_{2D}$. Marginal costs are normalised to zero, so the platforms' payoffs are $\Pi_1 = p_1 Q_{1M}$ and $\Pi_2 = p_2 (Q_{2M} + Q_{2D})$.[26]

We provide details on the main assumptions in the Appendix, reflecting sign restrictions and stability conditions already made in Section 3. The timing is the same as in the baseline model. In the first stage, platforms 1 and 2 simultaneously choose prices p_1 and p_2. In the second stage, given these prices, consumers take their decisions, and market quantities are determined as a fixed point of the demand system. The equilibrium concept is subgame perfect Nash equilibrium with fulfilled expectations.

We start by characterising market demand for given prices:

$$n_1 = f_{1M}(b_1, b_2, p_1, p_2, n_1, n_2), \qquad n_2 = f_{2M}(b_1, b_2, p_1, p_2, n_1, n_2) + \alpha f_{2D}(p_2, n_2). \tag{23}$$

Under Assumption 6 (see Appendix) and solving for the fixed point, there exists a locally

[26]Note that platform 2 charges the same price across the mediated and direct channels. Uniform pricing across channels is consistent with standard "most-favored-customer" or anti-steering rules on integrated platforms, and is the empirically relevant case for the policy debate.

unique pair of reduced-form fixed-point usage levels,

$$N_1 = \mathcal{N}_1(b_1, b_2, p_1, p_2, \alpha), \tag{24}$$

$$N_2 = \mathcal{N}_2(b_1, b_2, p_1, p_2, \alpha). \tag{25}$$

By substitution, we obtain the following reduced-form quantities as a function of prices (as only endogenous parameters):

$$\begin{aligned} Q_{1M}(b_1, b_2, p_1, p_2, \alpha) &= f_{1M}(b_1, b_2, p_1, p_2, N_1(\cdot), N_2(\cdot)), \\ Q_{2M}(b_1, b_2, p_1, p_2, \alpha) &= f_{2M}(b_1, b_2, p_1, p_2, N_1(\cdot), N_2(\cdot)), \\ Q_{2D}(b_1, b_2, p_1, p_2, \alpha) &= \alpha f_{2D}(p_2, N_2(\cdot)). \end{aligned}$$

By construction, $N_1 = Q_{1M}$ and $N_2 = Q_{2M} + Q_{2D}$. Under the stability and dominance conditions in the Appendix, we have: $\partial N_i/\partial p_i < 0$, $\partial N_i/\partial p_j > 0$, $\partial N_1/\partial b_1 > 0$ and $\partial N_2/\partial b_1 < 0$.

6.2 The effect of tying

In the Appendix, we show that — under similar assumptions and conditions as in the baseline model — the effects of tying are qualitatively the same as those discussed above.

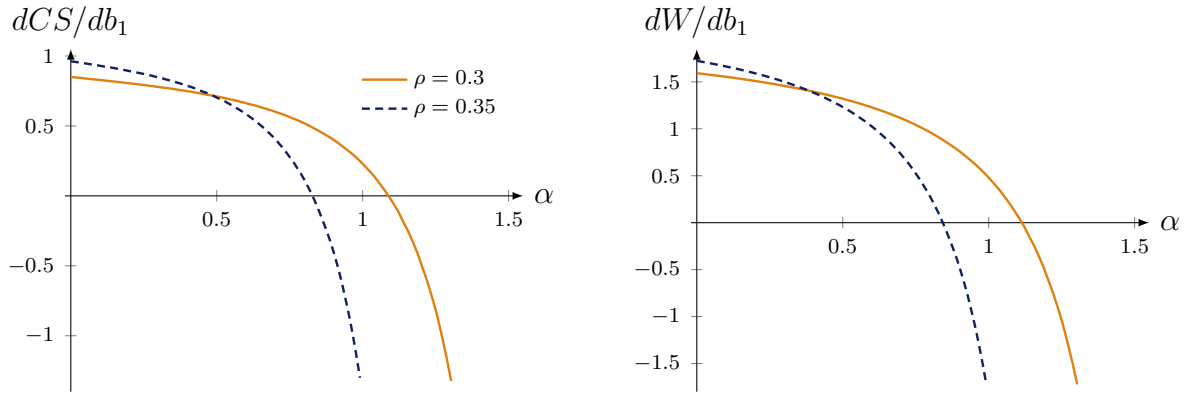
Firstly, at equilibrium, tying raises the price of platform B_1 and lowers that of platform B_2. Intuitively, tying strengthens platform 1's position and weakens platform 2's in the mediated channel, inducing the former to raise its price and the latter to cut it, thereby partially offsetting the direct demand reallocation. Because platform 2 charges a common price across channels, its price cut also tends to benefit direct-channel consumers, an effect absent from the two-sided model.

Next, consider that tying has the direct effect of raising B_1's demand and lowering B_2's demand in the mediated channel. Network externalities reinforce this reallocation by reducing platform 2's total demand, although the induced reduction in p_2 may partly offset this effect for consumers in direct channel. We show in the Appendix that, as long as the strategic price effects are dominated by the direct and the network-related effects, at equilibrium platform 1's quantity increases and platform 2's decreases.

Figure 7 provides a graphical illustration of the effect of tying in the usual Hotelling setting (albeit with the modifications required to deal with direct rather than indirect network effects). When the direct channel (whose size is measured by α) is small, tying raises consumer surplus through its positive effects on the mediated channel consumers. As the direct channel grows, more consumers are exposed to platform 2's loss of scale, eventually reducing consumer surplus. Stronger network effects amplify the cost of reduced scale, and hence lower the value of α at which the sign of the effect changes. Total

surplus follows a qualitatively similar pattern.

Note that the effects diverge from the two-sided model as the direct channel grows. There, seller participation was bounded: once the direct channel were sufficiently large, all sellers joined the rival platform and participation would no longer respond to tying, so the negative direct-channel effect disappeared, generating the non-monotonicity described in Section 3. Here, instead, consumer participation is unbounded and there is no corner solution, so platform 2's usage continues to respond to tying for every admissible α.



(a) Consumer surplus

(b) Total welfare

Figure 7: Effects of tying in the one-sided model. Parameters: $b_1 = b_2 = 0.2$, $t = 1.5$.

7 Conclusions

Digital companies often integrate into their core platforms new specialised services, or make more visible and accessible services they already offer, typically in competition with other companies. This practice, which is a form of tying and self-preferencing, reduces the inconvenience costs of the core platform users who also consume those services, thus providing a direct benefit to them.

However, it might also have a detrimental impact. In our baseline two-sided model, tying tilts demand in the mediated channel (that is, of consumers on the primary platform) away from the rivals and towards the platform's own specialised service, thus reducing the attractiveness for sellers to be on the rival platform. To the extent that the rival also serves consumers via a direct channel, this will decrease rival's demand in that channel as well, and will feed back negatively on its seller participation. The size of the rival's direct channel influences the effects of tying. When the direct channel is negligible, tying benefits consumers on the integrated platform, and hence its effect is positive overall. As the direct channel grows, sellers on the rival platform are harmed by a contraction in consumer demand, which may in turn reduce consumer surplus. But once the direct channel is large enough to make seller participation on the rival platform less responsive to tying, the pro-competitive effect of tying is restored, since harm becomes negligible

or zero on the direct channel, whereas the net effect on consumers (and sellers) on the mediated channel is beneficial.

A similar mechanism might also take place in other environments, as we show in a one-sided version of the model with network externalities: tying shifts demand away from the rival, and this scale effect impacts negatively on consumers in the direct channel, and feeds back into the mediated channel by reinforcing the initial effect. Likewise, it might also affect investment decisions (not formalised in the current version of the paper): the fall in rival's demand in the mediated channel will reduce the incentive of the rival to invest in product improvement. To the extent that it also sells via a direct channel, the lower investment will be harmful to consumers directly accessing the rival's services. Again, tying's beneficial effect on consumers in the mediated channel should be traded-off with this source of negative effects.[27,28]

Tying is a practice that *reduces* inconvenience costs of the users of the services of the integrated core platform, and as such it might give rise to a trade-off between beneficial and anti-competitive effects. Instead, another form of self-preferencing, demotion, which consists of *increasing* the inconvenience costs of *rivals* (for instance, by making them less visible and more cumbersome to use, or degrading their interoperability with the integrated platform) will tend to have unambiguous effects, decreasing consumer and total surplus both directly and indirectly.

We have also studied a non-discriminatory remedy to tying, often used by antitrust agencies. For instance, in *Google Shopping*, the European Commission initially considered a remedy whereby Google committed to giving access to its 'Shopping unit' (that is, the box where links and pictures of merchants appear prominently on Google's Search Engine Results Pages) not only to its own service, but also to selected rival Comparison Shopping Services. In *Facebook Marketplace*, to comply with the European Commission's abuse decision, Meta launched a commitment program consisting in the inclusion of listings from other platforms (e.g., eBay) directly within Marketplace.[29]

This type of remedy is aimed at avoiding asymmetric treatment between the integrated and the independent service within the integrated platform' s ecosystem. In our model, it amounts to increasing the benefit for (or reducing the inconvenience costs in-

[27]We have assumed that the rival is already in the market. One could extend the model by having an initial stage where the rival decides whether to enter by paying a fixed sunk cost. Since tying by the integrated platform is optimal (due to imperfect rent extraction) even when the entrant is already in the market, the expectation of lower profits might deter entry, even absent any pre-commitment on tying.

[28]On the other hand, in some circumstances the effect on the direct channel might also be positive. Imagine an environment where scale and network effects are insignificant, and that the rival sets uniform pricing across channels. Then the reduction in demand in the mediated channel will prompt the rival to lower its prices in both channels, resulting in a positive —rather than negative — externality for its direct consumers.

[29]In the same vein, in response of the European Commission's scrutiny for self-preferencing its own products, assigning to them the Buy Box with higher frequency in its Marketplace platform, Amazon introduced a second Buy Box in its European marketplace, displaying an alternative seller's offer alongside the default one.

curred by) consumers in the mediated channel for both the integrated platform and its rival's specialised platforms. We show that this policy increases welfare and raises both platforms' profits relative to the status quo, so the integrated platform has no incentive to pre-empt the remedy by abandoning integration altogether.

Our setting could be modified to account for other real-world situations, or to explore other issues. For instance, we could use our theoretical framework to analyse the acquisition of adjacent platforms to enrich the ecosystem, rather than assuming that the specialised service is already offered by the core platform, and examining the resulting impact of tying.

Appendix

Two-sided model: proofs

Proof of Lemma 1

Differentiating the first-order conditions (1) and (2) with respect to b_1 and applying Cramer's rule yields

$$\frac{dr_1^*}{db_1} = \frac{-\partial^2\Pi_1/\partial r_1\partial b_1 \cdot \partial^2\Pi_2/\partial r_2^2 + \partial^2\Pi_2/\partial r_2\partial b_1 \cdot \partial^2\Pi_1/\partial r_1\partial r_2}{\Delta_r},$$

$$\frac{dr_2^*}{db_1} = \frac{-\partial^2\Pi_2/\partial r_2\partial b_1 \cdot \partial^2\Pi_1/\partial r_1^2 + \partial^2\Pi_1/\partial r_1\partial b_1 \cdot \partial^2\Pi_2/\partial r_2\partial r_1}{\Delta_r},$$

where $\Delta_r > 0$ is the standard regularity determinant of the fee game. Under Assumption 4, $\partial^2\Pi_1/\partial r_1\partial b_1 > 0$ and $\partial^2\Pi_2/\partial r_2\partial b_1 < 0$. Under strategic substitutes both terms in the numerator of dr_1^*/db_1 are non-negative and at least one is strictly positive, and both terms in the numerator of dr_2^*/db_1 are non-positive and at least one is strictly negative, so $dr_1^*/db_1 > 0$ and $dr_2^*/db_1 < 0$.

Under strategic complementarity, the same conclusion follows under a *diagonal-dominance condition* on the fee-game Jacobian[30] :

$$\frac{-\partial^2\Pi_1/\partial r_1^2}{\partial^2\Pi_2/\partial r_2\partial r_1} > \frac{\partial^2\Pi_1/\partial r_1\partial b_1}{-\partial^2\Pi_2/\partial r_2\partial b_1} > \frac{\partial^2\Pi_1/\partial r_1\partial r_2}{-\partial^2\Pi_2/\partial r_2^2}.$$

[30]The condition above means that each platform's own-fee curvature ($\partial^2\Pi_i/\partial r_i^2$, the slope of its own best-response, which is large in absolute value when own-fee marginal profitability falls steeply) outweighs the cross-fee strategic response ($\partial^2\Pi_i/\partial r_i\partial r_j$, the slope of i's best response in j's fee). This is the same diagonal strict concavity condition of Rosen (1965).

Proof of Proposition 1

We proceed by steps.

Part (i). Under Assumption 1-2, and Condition 1 we can sign (8) and (9):

$$\frac{dS_1^*}{db_1} = \underbrace{\frac{1}{\Delta_S}}_{(+)} \left\{ \underbrace{g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right)}_{(+) \text{ by Condition 1}} \underbrace{\left(1 - m g_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2} \right)}_{(+) \text{ by Assumption 2.(i)}} \right.$$
$$\left. + \underbrace{m g_1 \frac{\partial f_{1M}}{\partial s_2}}_{(-) \text{ by Assumption 1.(i)}} \underbrace{g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right)}_{(\text{-}) \text{ by Condition 1}} \right\}$$

and

$$\frac{dS_2^*}{db_1} = \underbrace{\frac{1}{\Delta_S}}_{(+)} \left\{ \underbrace{\left(1 - m g_1 \frac{\partial f_{1M}}{\partial s_1} \right)}_{(+) \text{ by Assumption 2(i)}} \underbrace{g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right)}_{(\text{-}) \text{ by Condition 1}} + \underbrace{m g_2 \frac{\partial f_{2M}}{\partial s_1}}_{(-) \text{ by Assumption 1(i)}} \underbrace{g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right)}_{(+) \text{ by Condition 1}} \right\},$$

where $\Delta_S > 0$ by Assumption 2(ii).

Part (ii). Substituting (8)–(9) into (5)–(7) yields

$$\frac{dQ_{2D}^*}{db_1} = \alpha \frac{\partial f_{2D}}{\partial s_2} \frac{1}{\Delta_S} \left\{ \left(1 - m g_1 \frac{\partial f_{1M}}{\partial s_1} \right) g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) + m g_2 \frac{\partial f_{2M}}{\partial s_1} g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \right\}, \tag{A-1}$$

$$\begin{aligned} \frac{dQ_{1M}^*}{db_1} = \frac{\partial f_{1M}}{\partial b_1} &+ \frac{\partial f_{1M}}{\partial s_1} \frac{1}{\Delta_S} \left\{ g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \left(1 - m g_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2} \right) \right. \\ &\left. + m g_1 \frac{\partial f_{1M}}{\partial s_2} g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) \right\} \\ &+ \frac{\partial f_{1M}}{\partial s_2} \frac{1}{\Delta_S} \left\{ \left(1 - m g_1 \frac{\partial f_{1M}}{\partial s_1} \right) g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) + m g_2 \frac{\partial f_{2M}}{\partial s_1} g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \right\}, \end{aligned} \tag{A-2}$$

$$\begin{aligned} \frac{dQ_{2M}^*}{db_1} = \frac{\partial f_{2M}}{\partial b_1} &+ \frac{\partial f_{2M}}{\partial s_1} \frac{1}{\Delta_S} \left\{ g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \left(1 - m g_2 \frac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \frac{\partial f_{2D}}{\partial s_2} \right) \right. \\ &\left. + m g_1 \frac{\partial f_{1M}}{\partial s_2} g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) \right\} \\ &+ \frac{\partial f_{2M}}{\partial s_2} \frac{1}{\Delta_S} \left\{ \left(1 - m g_1 \frac{\partial f_{1M}}{\partial s_1} \right) g_2 \left(m \frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1} \right) + m g_2 \frac{\partial f_{2M}}{\partial s_1} g_1 \left(m \frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1} \right) \right\}, \end{aligned} \tag{A-3}$$

where $\Delta_S > 0$.

The total effect of tying on each of the above quantities is determined by two objects:

$$m\frac{\partial f_{1M}}{\partial b_1} - \frac{dr_1^*}{db_1}, \qquad m\frac{\partial f_{2M}}{\partial b_1} - \frac{dr_2^*}{db_1},$$

whose signs are established under Condition 1: the first is positive and the second negative. Under the maintained assumptions, both braces in (A-1) are then negative (as shown in the proof of part (i)), and since $\alpha\,\partial f_{2D}/\partial s_2 > 0$ and $\Delta_S > 0$, we obtain $dQ_{2D}^*/db_1 < 0$. In (A-2), the first brace is positive and is multiplied by $\partial f_{1M}/\partial s_1 > 0$, whereas the second brace is negative and is multiplied by $\partial f_{1M}/\partial s_2 < 0$; adding the direct effect $\partial f_{1M}/\partial b_1 > 0$ yields $dQ_{1M}^*/db_1 > 0$. Symmetrically, in (A-3) the first brace (positive) is multiplied by $\partial f_{2M}/\partial s_1 < 0$ and the second brace (negative) by $\partial f_{2M}/\partial s_2 > 0$; adding $\partial f_{2M}/\partial b_1 < 0$ yields $dQ_{2M}^*/db_1 < 0$.

Proof of Proposition 3

Demand in the direct channel depends on b_1 only through S_2^*, so

$$\frac{dQ_{2D}^*}{db_1} = \alpha\frac{\partial f_{2D}}{\partial s_2}\frac{dS_2^*}{db_1}.$$

At $\alpha = 0$, the direct channel does not exist, so $Q_{2D}^* = 0$, hence $dQ_{2D}^*/db_1 = 0$.

Consider next $\alpha \in (0, \hat{\alpha})$, where seller participation on platform 2 is interior. By Assumption 1.(i), $\partial f_{2D}/\partial s_2 > 0$, while Proposition 1 implies $dS_2^*/db_1 < 0$. Hence, $dQ_{2D}^*/db_1 < 0$ and tying strictly reduces demand in the rival's direct channel whenever seller participation is interior.

For $\alpha \geq \widehat{\alpha}$, the interior first-order conditions no longer characterise the equilibrium. The Hotelling specification of Section 3.5 characterises the equilibrium in that region; see the Supplementary Material.

Proof of Proposition 5

Let us start by identifying the effect of demotion on equilibrium fees. Following the same steps as in Section 2, we determine the effect of demotion on seller fees. Under Assumptions 1, 2, and 4:

(i) if r_1 and r_2 are strategic substitutes, then demotion raises platform 1's equilibrium seller fee and lowers platform 2's equilibrium seller fee, that is $-dr_1^* db_2 > 0$, $-dr_2^*/db_2 < 0$;

(ii) if r_1 and r_2 are strategic complements, the same conclusion holds under the diagonal-dominance condition analogous to that in Lemma 1.

The proof is as follows. Differentiating the first-order conditions (1) and (2) with respect to b_2 and applying Cramer's rule yields

$$\frac{dr_1^*}{db_2} = \frac{-\dfrac{\partial^2\Pi_1}{\partial r_1 \partial b_2}\dfrac{\partial^2\Pi_2}{\partial r_2^2} + \dfrac{\partial^2\Pi_2}{\partial r_2 \partial b_2}\dfrac{\partial^2\Pi_1}{\partial r_1 \partial r_2}}{\Delta_r}, \qquad \frac{dr_2^*}{db_2} = \frac{-\dfrac{\partial^2\Pi_2}{\partial r_2 \partial b_2}\dfrac{\partial^2\Pi_1}{\partial r_1^2} + \dfrac{\partial^2\Pi_1}{\partial r_1 \partial b_2}\dfrac{\partial^2\Pi_2}{\partial r_2 \partial r_1}}{\Delta_r}, \tag{A-4}$$

where $\Delta_r > 0$ by assumption. Under Assumption 4, a demotion of platform 2 (a reduction in (b_2)) raises platform 1 's marginal incentive to increase (r_1) and lowers platform 2's marginal incentive to increase (r_2).

Under strategic substitutes,

$$\frac{\partial^2\Pi_1}{\partial r_1 \partial r_2} \leq 0, \qquad \frac{\partial^2\Pi_2}{\partial r_2 \partial r_1} \leq 0.$$

Together with own second-order conditions, this implies that both terms in the numerator of dr_1^*/db_2 are non-positive and both terms in the numerator of dr_2^*/db_2 are non-negative. Since $\Delta_r > 0$, then $-dr_1^* db_2 > 0$, $-dr_2^*/db_2 < 0$. This proves part (i).

Under strategic complements, the same sign pattern follows under the diagonal-dominance condition

$$\frac{-\partial^2\Pi_1/\partial r_1^2}{\partial^2\Pi_2/\partial r_2 \partial r_1} > \frac{-\partial^2\Pi_1/\partial r_1 \partial b_2}{\partial^2\Pi_2/\partial r_2 \partial b_2} > \frac{\partial^2\Pi_1/\partial r_1 \partial r_2}{-\partial^2\Pi_2/\partial r_2^2},$$

with all inequalities evaluated at the equilibrium and with the middle ratio well defined.

This condition is the demotion analogue of the diagonal-dominance condition used in the proof of Lemma 1. It requires each platform's own-fee curvature to dominate the cross-fee feedback induced by the rival's response. Hence the direct effect of a change in b_2 on each platform's marginal fee incentive is not reversed by strategic complementarity. Consequently, then $-dr_1^* db_2 > 0$, $-dr_2^*/db_2 < 0$.

Next, we look at the effect of demotion on seller participation. Differentiating the seller-entry conditions with respect to b_2 yields, in analogy with (4),

$$\frac{dS_1^*}{db_2} = g_1\left(m\,\frac{dQ_{1M}^*}{db_2} - \frac{dr_1^*}{db_2}\right), \qquad \frac{dS_2^*}{db_2} = g_2\left(m\,\frac{dQ_{2M}^*}{db_2} + m\,\frac{dQ_{2D}^*}{db_2} - \frac{dr_2^*}{db_2}\right). \tag{A-5}$$

The total derivatives of consumer quantities with respect to b_2 are

$$\frac{dQ_{1M}^*}{db_2} = \frac{\partial f_{1M}}{\partial b_2} + \frac{\partial f_{1M}}{\partial s_1}\frac{dS_1^*}{db_2} + \frac{\partial f_{1M}}{\partial s_2}\frac{dS_2^*}{db_2}, \tag{A-6}$$

$$\frac{dQ_{2M}^*}{db_2} = \frac{\partial f_{2M}}{\partial b_2} + \frac{\partial f_{2M}}{\partial s_1}\frac{dS_1^*}{db_2} + \frac{\partial f_{2M}}{\partial s_2}\frac{dS_2^*}{db_2}, \tag{A-7}$$

$$\frac{dQ_{2D}^*}{db_2} = \alpha\,\frac{\partial f_{2D}}{\partial s_2}\frac{dS_2^*}{db_2}. \tag{A-8}$$

In what follows, we mirror Condition 1 from the tying analysis.

Condition 2 (Dominance of direct effects under demotion)**.** *Direct demand effects of demotion dominate the strategic fee effects if*

$$m\left|\frac{\partial f_{1M}}{\partial b_2}\right| \;>\; \left|\frac{dr_1^*}{db_2}\right|, \qquad m\,\frac{\partial f_{2M}}{\partial b_2} \;>\; \frac{dr_2^*}{db_2}$$

Proposition 1 yields a 2×2 linear system in $(dS_1^*/db_2, dS_2^*/db_2)$ with the same coefficient matrix (determinant $\Delta_S > 0$) but right-hand-side brackets $g_J(m\,\partial f_{JM}/\partial b_2 - dr_J^*/db_2)$. By Assumption 1(i), $\partial f_{1M}/\partial b_2 < 0$ and $\partial f_{2M}/\partial b_2 > 0$; Moreover, by (A-4), $dr_1^*/db_2 \le 0$ and $dr_2^*/db_2 \ge 0$.

Applying Cramer's rule:

$$\frac{dS_1^*}{db_2} = \frac{1}{\Delta_S}\left\{\underbrace{\left(1 - mg_2\tfrac{\partial f_{2M}}{\partial s_2} - \alpha m g_2 \tfrac{\partial f_{2D}}{\partial s_2}\right)}_{(+)\text{ by Assumption 2.(i)}}\underbrace{g_1\left(m\tfrac{\partial f_{1M}}{\partial b_2} - \tfrac{dr_1^*}{db_2}\right)}_{(-)\text{ Condition 2}} + \underbrace{mg_1\tfrac{\partial f_{1M}}{\partial s_2}}_{(-)\text{ by Assumption 1.(i)}}\underbrace{g_2\left(m\tfrac{\partial f_{2M}}{\partial b_2} - \tfrac{dr_2^*}{db_2}\right)}_{(+)\text{ by Condition 2}}\right\} <$$

(A-9)

$$\frac{dS_2^*}{db_2} = \frac{1}{\Delta_S}\left\{\underbrace{\left(1 - mg_1\tfrac{\partial f_{1M}}{\partial s_1}\right)}_{(+)\text{ by Assumption 2.(i)}}\underbrace{g_2\left(m\tfrac{\partial f_{2M}}{\partial b_2} - \tfrac{dr_2^*}{db_2}\right)}_{(+)\text{ Condition 2}} + \underbrace{mg_2\tfrac{\partial f_{2M}}{\partial s_1}}_{(-)\text{ by Assumption 1.(i)}}\underbrace{g_1\left(m\tfrac{\partial f_{1M}}{\partial b_2} - \tfrac{dr_1^*}{db_2}\right)}_{(-)\text{ by Condition 2}}\right\} > 0,$$

(A-10)

since $\Delta_S > 0$. This establishes part (i). For part (ii), substitute $dS_1^*/db_2 < 0$ and $dS_2^*/db_2 > 0$ into (A-6)–(A-8), then

$$\begin{aligned}
\frac{dQ_{1M}^*}{db_2} =& \frac{\partial f_{1M}}{\partial b_2} + \frac{\partial f_{1M}}{\partial s_1}\frac{dS_1^*}{db_2} + \frac{\partial f_{1M}}{\partial s_2}\frac{dS_2^*}{db_2} < 0\\
\frac{dQ_{2M}^*}{db_2} =& \frac{\partial f_{2M}}{\partial b_2} + \frac{\partial f_{2M}}{\partial s_1}\frac{dS_1^*}{db_2} + \frac{\partial f_{2M}}{\partial s_2}\frac{dS_2^*}{db_2} > 0\\
\frac{dQ_{2D}^*}{db_2} =& \alpha\frac{\partial f_{2D}}{\partial s_2}\frac{dS_2^*}{db_2} > 0,
\end{aligned}$$

so $-dQ_{1M}^*/db_2 > 0$, $-dQ_{2M}^*/db_2 < 0$, and $-dQ_{2D}^*/db_2 < 0$.

Proof of Proposition 6

Writing the equilibrium profits as $\Pi_J^* = r_J^* S_J^*$, then

$$\frac{d\Pi_1^*}{db_2} = S_1^* \frac{dr_1^*}{db_2} + r_1^* \frac{dS_1^*}{db_2}, \qquad \frac{d\Pi_2^*}{db_2} = S_2^* \frac{dr_2^*}{db_2} + r_2^* \frac{dS_2^*}{db_2}, \tag{A-11}$$

where dS_J^*/db_2 are the full equilibrium derivatives, incorporating both platforms' fee responses. Then, under Condition 2: $dr_1^*/db_2 \leq 0$, $dr_2^*/db_2 \geq 0$, $dS_1^*/db_2 < 0$, and $dS_2^*/db_2 > 0$. Hence in $d\Pi_1^*/db_2$ both terms are non-positive, with the second strictly negative, and in $d\Pi_2^*/db_2$ both terms are non-negative, with the second strictly positive. Thus, $d\Pi_1^*/db_2 < 0$ and $d\Pi_2^*/db_2 > 0$ and, therefore, $d\Pi_1^*/d(-b_2) > 0$ and $d\Pi_2^*/d(-b_2) < 0$.

Proof of Proposition 7

Since $b_1 = b_{01} + b$ and $b_2 = b_{02} + b$ with b_{01}, b_{02} held fixed, $db_1/db = db_2/db = 1$, so for any equilibrium object X^* the chain rule gives (20), wherein the first term being the *tying component* and the second the *raising-b_2 component*.

The signs of the two components are established elsewhere in the paper: the b_1 component by Lemma 1 (fees) and Proposition 1 (participation and quantities), and the b_2 component by Proposition 5, read with reversed sign because those results are stated for a *decrease* in b_2. Collecting them:

	component in b_1	component in b_2
r_1^* :	$dr_1^*/db_1 > 0$	$dr_1^*/db_2 < 0$
r_2^* :	$dr_2^*/db_1 < 0$	$dr_2^*/db_2 > 0$
S_1^* :	$dS_1^*/db_1 > 0$	$dS_1^*/db_2 < 0$
S_2^* :	$dS_2^*/db_1 < 0$	$dS_2^*/db_2 > 0$
Q_{1M}^* :	$dQ_{1M}^*/db_1 > 0$	$dQ_{1M}^*/db_2 < 0$
Q_{2M}^* :	$dQ_{2M}^*/db_1 < 0$	$dQ_{2M}^*/db_2 > 0$.

Let us first introduce the following condition:

Condition 3 (Dominance of own-convenience effects)**.** Own effects dominate rival-convenience effects *if, for each platform J and $K \neq J$, the responses of platform J's equilibrium seller fee and seller participation to b_J dominate, in absolute value, their responses to b_K:*

$$\frac{dr_J^*}{db_J} \geq \left|\frac{dr_J^*}{db_K}\right|, \qquad \frac{dS_J^*}{db_J} \geq \left|\frac{dS_J^*}{db_K}\right|. \tag{A-12}$$

We now present the main results. *Part (i): seller participation.* By (20), under

Condition 3,

$$\frac{dS_1^*}{db} = \underbrace{\frac{dS_1^*}{db_1}}_{>0} + \underbrace{\frac{dS_1^*}{db_2}}_{<0} \geq 0, \qquad \frac{dS_2^*}{db} = \underbrace{\frac{dS_2^*}{db_1}}_{<0} + \underbrace{\frac{dS_2^*}{db_2}}_{>0} \geq 0.$$

By the same argument applied to fees,

$$\frac{dr_1^*}{db} = \frac{dr_1^*}{db_1} + \frac{dr_1^*}{db_2} \geq 0, \qquad \frac{dr_2^*}{db} = \frac{dr_2^*}{db_1} + \frac{dr_2^*}{db_2} \geq 0,$$

which we use in the proof of Proposition 8.

Part (ii): quantities. For the direct channel, $Q_{2D}^* = \alpha f_{2D}(S_2^*)$, so

$$\frac{dQ_{2D}^*}{db} = \alpha \frac{\partial f_{2D}}{\partial s_2} \frac{dS_2^*}{db} \geq 0,$$

as $\alpha > 0$ and $\partial f_{2D}/\partial s_2 > 0$ (Assumption 1) and $dS_2^*/db \geq 0$ by part (i), with strict inequality if $dS_2^*/db > 0$. For the mediated quantities, equation (20) together with Condition 3 give

$$\frac{dQ_{1M}^*}{db} = \underbrace{\frac{dQ_{1M}^*}{db_1}}_{>0} + \underbrace{\frac{dQ_{1M}^*}{db_2}}_{<0} \geq 0, \qquad \frac{dQ_{2M}^*}{db} = \underbrace{\frac{dQ_{2M}^*}{db_1}}_{<0} + \underbrace{\frac{dQ_{2M}^*}{db_2}}_{>0} \geq 0.$$

Proof of Proposition 8

Differentiating $\Pi_J^* = r_J^* S_J^*$ with respect to b yields:

$$\frac{d\Pi_J^*}{db} = S_J^* \frac{dr_J^*}{db} + r_J^* \frac{dS_J^*}{db}, \qquad J = 1, 2.$$

By Proposition 7, under Condition 3 the fee responses $dr_J^*/db \geq 0$ and the participation responses $dS_J^*/db \geq 0$; and $r_J^*, S_J^* \geq 0$ at an interior equilibrium. Hence both terms are non-negative for $J = 1, 2$, giving $d\Pi_1^*/db \geq 0$ (part i) and $d\Pi_2^*/db \geq 0$ (part ii).

Seller surplus (used in the welfare discussion of Section 5). A type-κ seller on platform J earns $mq_J^* - r_J^* - \kappa$ and participates iff $\kappa \leq \kappa_J^* \equiv mq_J^* - r_J^*$, with $S_J^* = G(\kappa_J^*)$, $q_1 = Q_{1M}^*$, $q_2 = Q_{2M}^* + Q_{2D}^*$. Hence $SS_J = \int_0^{\kappa_J^*} (\kappa_J^* - \kappa)\, g(\kappa)\, d\kappa$, and since the integrand vanishes at $\kappa = \kappa_J^*$,

$$\frac{dSS_J}{db} = \frac{d\kappa_J^*}{db} G(\kappa_J^*) = S_J^* \Big(m \frac{dq_J^*}{db} - \frac{dr_J^*}{db} \Big).$$

Differentiating $S_J^* = G(\kappa_J^*)$ gives $dS_J^*/db = g_J \big(m\, dq_J^*/db - dr_J^*/db \big)$, so $m\, dq_J^*/db - dr_J^*/db = (1/g_J)\, dS_J^*/db$ and

$$\frac{dSS_J}{db} = \frac{S_J^*}{g_J} \frac{dS_J^*}{db} \geq 0,$$

since $S_J^* \geq 0$, $g_J > 0$, and $dS_J^*/db \geq 0$ (Proposition 7). Thus seller surplus on each

platform weakly rises.

Consumer surplus and total welfare. The preceding argument establishes that seller surplus is weakly increasing. No analogous consumer-surplus conclusion follows from Assumption 1: the reduced-form demand system does not provide a consumer-utility representation. Hence the general model does not sign the effect on consumer surplus and welfare, dCS/db and dW/db. Consumer surplus and total welfare are instead computed in the Hotelling specification in the Supplementary Material.

One-sided model: comparative statics and proofs

Assumptions used in Section 6.1

Assumption 5 (Consumer-demand primitives)**.** *For $i, j \in \{1, 2\}$, $i \neq j$, and for all relevant arguments,*

$$\frac{\partial f_{iM}}{\partial b_i} > 0, \qquad \frac{\partial f_{iM}}{\partial b_j} < 0, \qquad \frac{\partial f_{iM}}{\partial p_i} < 0, \qquad \frac{\partial f_{iM}}{\partial p_j} > 0,$$

$$\frac{\partial f_{iM}}{\partial n_i} > 0, \qquad \frac{\partial f_{iM}}{\partial n_j} < 0, \qquad \frac{\partial f_{2D}}{\partial p_2} < 0, \qquad \frac{\partial f_{2D}}{\partial n_2} > 0.$$

Moreover, own effects weakly dominate cross effects in the mediated channel:

$$\frac{\partial f_{iM}}{\partial b_i} \geq \left|\frac{\partial f_{jM}}{\partial b_i}\right|, \qquad \left|\frac{\partial f_{iM}}{\partial p_i}\right| \geq \frac{\partial f_{jM}}{\partial p_i}, \qquad \frac{\partial f_{iM}}{\partial n_i} \geq \left|\frac{\partial f_{jM}}{\partial n_i}\right|.$$

Assumption 5 is the consumer-network analogue of the seller-side demand assumptions in Section 2. It says that own convenience, lower own prices, and own usage increase demand, whereas rival convenience, lower rival prices, and rival usage reduce it.

Assumption 6 (Stability of consumer-side network feedbacks)**.** *For all relevant arguments,*

(i) $1 > \frac{\partial f_{1M}}{\partial n_1} + \left|\frac{\partial f_{1M}}{\partial n_2}\right|$, $1 > \frac{\partial f_{2M}}{\partial n_2} + \alpha\frac{\partial f_{2D}}{\partial n_2} + \left|\frac{\partial f_{2M}}{\partial n_1}\right|$,

(ii) $\Delta_n \equiv \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right)\left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha\frac{\partial f_{2D}}{\partial n_2}\right) - \frac{\partial f_{1M}}{\partial n_2}\frac{\partial f_{2M}}{\partial n_1} > 0.$

Assumption 6 rules out explosive fulfilled-expectations feedbacks and guarantees local invertibility of the fixed-point system.

Assumption 7 (Regularity of the price game)**.** *At the equilibrium,*

(i) $\frac{\partial^2 \Pi_1}{\partial p_1^2} < 0$, $\frac{\partial^2 \Pi_2}{\partial p_2^2} < 0$,

(ii) $\Delta_p \equiv \frac{\partial^2 \Pi_1}{\partial p_1^2}\frac{\partial^2 \Pi_2}{\partial p_2^2} - \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}\frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} > 0.$

Assumption 8 (Super-modularity and sub-modularity)**.** *At the equilibrium,*

$$\frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} > 0, \qquad \frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} < 0.$$

Assumption 8 states that tying raises platform 1's marginal profitability of increasing its price and lowers platform 2's marginal profitability of increasing its price.

Reduced-form comparative statics

The Jacobian of (23) with respect to (n_1, n_2) is

$$J_n = \begin{pmatrix} 1 - \frac{\partial f_{1M}}{\partial n_1} & -\frac{\partial f_{1M}}{\partial n_2} \\ -\frac{\partial f_{2M}}{\partial n_1} & 1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha \frac{\partial f_{2D}}{\partial n_2} \end{pmatrix}.$$

Its determinant is $\Delta_n > 0$ by Assumption 6. Differentiating the fixed-point system with respect to b_1 gives

$$\frac{\partial N_1}{\partial b_1} = \frac{\left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha \frac{\partial f_{2D}}{\partial n_2}\right) \frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{1M}}{\partial n_2} \frac{\partial f_{2M}}{\partial b_1}}{\Delta_n} > 0,$$

$$\frac{\partial N_2}{\partial b_1} = \frac{\left(1 - \frac{\partial f_{1M}}{\partial n_1}\right) \frac{\partial f_{2M}}{\partial b_1} + \frac{\partial f_{2M}}{\partial n_1} \frac{\partial f_{1M}}{\partial b_1}}{\Delta_n} < 0.$$

Thus, holding prices fixed, tying raises platform 1's demand and lowers platform 2's demand. Similarly, Assumptions 5 and 6 imply

$$\frac{\partial N_1}{\partial p_1} < 0, \qquad \frac{\partial N_2}{\partial p_1} > 0, \qquad \frac{\partial N_1}{\partial p_2} > 0, \qquad \frac{\partial N_2}{\partial p_2} < 0.$$

Price comparative statics under tying

Differentiating the two first-order conditions with respect to b_1 gives

$$\begin{pmatrix} \frac{\partial^2 \Pi_1}{\partial p_1^2} & \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} & \frac{\partial^2 \Pi_2}{\partial p_2^2} \end{pmatrix} \begin{pmatrix} \frac{dp_1^*}{db_1} \\ \frac{dp_2^*}{db_1} \end{pmatrix} = - \begin{pmatrix} \frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} \\ \frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} \end{pmatrix}.$$

Applying Cramer's rule yields

$$\frac{dp_1^*}{db_1} = \frac{-\frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} \frac{\partial^2 \Pi_2}{\partial p_2^2} + \frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}}{\Delta_p}, \qquad \frac{dp_2^*}{db_1} = \frac{-\frac{\partial^2 \Pi_2}{\partial p_2 \partial b_1} \frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial b_1} \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1}}{\Delta_p}.$$

By Assumptions 7 and 8, $\Delta_p > 0$, $\partial^2\Pi_1/\partial p_1^2 < 0$ and $\partial^2\Pi_2/\partial p_2^2 < 0$. Moreover, $\partial^2\Pi_1/\partial p_1\partial b_1 > 0$ and $\partial^2\Pi_2/\partial p_2\partial b_1 < 0$

If prices are strategic substitutes,

$$\frac{\partial^2\Pi_1}{\partial p_1\partial p_2} \leq 0, \qquad \frac{\partial^2\Pi_2}{\partial p_2\partial p_1} \leq 0,$$

then the numerator of dp_1^*/db_1 is positive and the numerator of dp_2^*/db_1 is negative. Thus, $dp_1^*/db_1 > 0$ and $dp_2^*/db_1 < 0$. If prices are strategic complements, the same sign pattern follows under the sufficient diagonal-dominance condition

$$\frac{-\dfrac{\partial^2\Pi_1}{\partial p_1^2}}{\dfrac{\partial^2\Pi_2}{\partial p_2\partial p_1}} > \frac{\dfrac{\partial^2\Pi_1}{\partial p_1\partial b_1}}{-\dfrac{\partial^2\Pi_2}{\partial p_2\partial b_1}} > \frac{\dfrac{\partial^2\Pi_1}{\partial p_1\partial p_2}}{-\dfrac{\partial^2\Pi_2}{\partial p_2^2}}.$$

Effect of tying on quantities with network effects

Let equilibrium quantities be

$$Q_{1M}^* = f_{1M}(b_1, b_2, p_1^*, p_2^*, Q_{1M}^*, Q_{2M}^* + Q_{2D}^*)\,, \tag{A-13}$$
$$Q_{2M}^* = f_{2M}(b_1, b_2, p_1^*, p_2^*, Q_{1M}^*, Q_{2M}^* + Q_{2D}^*)\,,$$
$$Q_{2D}^* = \alpha f_{2D}(p_2^*, Q_{2M}^* + Q_{2D}^*)\,. \tag{A-14}$$

Define the strategic pricing effects as

$$SE_{1M} \equiv \frac{\partial f_{1M}}{\partial p_1}\frac{dp_1^*}{db_1} + \frac{\partial f_{1M}}{\partial p_2}\frac{dp_2^*}{db_1} \leq 0, \qquad SE_{2M} \equiv \frac{\partial f_{2M}}{\partial p_1}\frac{dp_1^*}{db_1} + \frac{\partial f_{2M}}{\partial p_2}\frac{dp_2^*}{db_1} \geq 0,$$

$$SE_{2D} \equiv \alpha\frac{\partial f_{2D}}{\partial p_2}\frac{dp_2^*}{db_1} \geq 0$$

where the sign follows from using $dp_1^*/db_1 \geq 0$, $dp_2^*/db_1 \leq 0$, and Assumption 5. Differentiating (A-13)–(A-14) with respect to b_1 yields:

$$\frac{dQ_{1M}^*}{db_1} = \frac{\partial f_{1M}}{\partial b_1} + \underbrace{\frac{\partial f_{1M}}{\partial p_1}\frac{dp_1^*}{db_1} + \frac{\partial f_{1M}}{\partial p_2}\frac{dp_2^*}{db_1}}_{\equiv\, SE_{1M}} + \underbrace{\frac{\partial f_{1M}}{\partial n_1}\frac{dQ_{1M}^*}{db_1} + \frac{\partial f_{1M}}{\partial n_2}\left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1}\right)}_{\text{network-related effect}},$$

$$\frac{dQ_{2M}^*}{db_1} = \frac{\partial f_{2M}}{\partial b_1} + \underbrace{\frac{\partial f_{2M}}{\partial p_1}\frac{dp_1^*}{db_1} + \frac{\partial f_{2M}}{\partial p_2}\frac{dp_2^*}{db_1}}_{\equiv\ SE_{2M}} + \underbrace{\frac{\partial f_{2M}}{\partial n_1}\frac{dQ_{1M}^*}{db_1} + \frac{\partial f_{2M}}{\partial n_2}\left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1}\right)}_{\text{network-related effect}}, \tag{A-15}$$

$$\frac{dQ_{2D}^*}{db_1} = \underbrace{\alpha\frac{\partial f_{2D}}{\partial p_2}\frac{dp_2^*}{db_1}}_{\equiv\ SE_{2D}} + \underbrace{\alpha\frac{\partial f_{2D}}{\partial n_2}\left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1}\right)}_{\text{network-related effect}}. \tag{A-16}$$

Solving the resulting system yields

$$\frac{dQ_{1M}^*}{db_1} = \frac{1}{\Delta_n}\Bigg\{ \underbrace{\left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha\frac{\partial f_{2D}}{\partial n_2}\right)\frac{\partial f_{1M}}{\partial b_1} + \frac{\partial f_{1M}}{\partial n_2}\frac{\partial f_{2M}}{\partial b_1}}_{\text{network-adjusted direct effect}}$$

$$+ \underbrace{\left(1 - \frac{\partial f_{2M}}{\partial n_2} - \alpha\frac{\partial f_{2D}}{\partial n_2}\right)S_{1M} + \frac{\partial f_{1M}}{\partial n_2}(S_{2M} + S_{2D})}_{\text{network-adjusted strategic-pricing effect}}\Bigg\},$$

$$\frac{dQ_{2M}^*}{db_1} = \frac{1}{\Delta_n}\Bigg\{ \underbrace{\left(1 - \alpha\frac{\partial f_{2D}}{\partial n_2}\right)\left[\frac{\partial f_{2M}}{\partial n_1}\frac{\partial f_{1M}}{\partial b_1} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right)\frac{\partial f_{2M}}{\partial b_1}\right]}_{\text{network-adjusted direct effect}}$$

$$+ \underbrace{\left(1 - \alpha\frac{\partial f_{2D}}{\partial n_2}\right)\left[\frac{\partial f_{2M}}{\partial n_1}S_{1M} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right)S_{2M}\right] + \left[\left(1 - \frac{\partial f_{1M}}{\partial n_1}\right)\frac{\partial f_{2M}}{\partial n_2} + \frac{\partial f_{1M}}{\partial n_2}\frac{\partial f_{2M}}{\partial n_1}\right]S_{2D}}_{\text{network-adjusted strategic-pricing effect}}\Bigg\},$$

$$\frac{dQ_{2D}^*}{db_1} = \frac{1}{\Delta_n}\Bigg\{ \underbrace{\alpha\frac{\partial f_{2D}}{\partial n_2}\left[\frac{\partial f_{2M}}{\partial n_1}\frac{\partial f_{1M}}{\partial b_1} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right)\frac{\partial f_{2M}}{\partial b_1}\right]}_{\text{network-adjusted direct effect}}$$

$$+ \underbrace{\alpha\frac{\partial f_{2D}}{\partial n_2}\left[\frac{\partial f_{2M}}{\partial n_1}S_{1M} + \left(1 - \frac{\partial f_{1M}}{\partial n_1}\right)S_{2M}\right] + \left[\left(1 - \frac{\partial f_{1M}}{\partial n_1}\right)\left(1 - \frac{\partial f_{2M}}{\partial n_2}\right) - \frac{\partial f_{1M}}{\partial n_2}\frac{\partial f_{2M}}{\partial n_1}\right]S_{2D}}_{\text{network-adjusted strategic-pricing effect}}\Bigg\}.$$

The network-adjusted direct effect is positive for Q_{1M}^* and negative for Q_{2M}^* and Q_{2D}^*, whereas the network-adjusted strategic price effects have the opposite signs. Hence, if the former dominate the latter in each channel,

$$\frac{dQ_{1M}^*}{db_1} > 0, \qquad \frac{dQ_{2M}^*}{db_1} < 0, \qquad \frac{dQ_{2D}^*}{db_1} < 0.$$

Effect of tying on profitability. Differentiating equilibrium profits with respect to b_1 yields:

$$\frac{d\Pi_1^*}{db_1} = p_1^* \left(\frac{dQ_{1M}^*}{db_1} - \frac{\partial Q_{1M}}{\partial p_1} \frac{dp_1^*}{db_1} \right), \tag{A-17}$$

$$\frac{d\Pi_2^*}{db_1} = p_2^* \left(\frac{dQ_{2M}^*}{db_1} + \frac{dQ_{2D}^*}{db_1} - \left(\frac{\partial Q_{2M}}{\partial p_2} + \frac{\partial Q_{2D}}{\partial p_2} \right) \frac{dp_2^*}{db_1} \right). \tag{A-18}$$

Since $dp_1^*/db_1 > 0$ and $\partial Q_{1M}/\partial p_1 < 0$, the second term in brackets in (A-17) is positive. Thus, a sufficient condition for platform 1's profit to increase is $dQ_{1M}^*/db_1 > 0$, which holds when network-adjusted direct effects dominate network-adjusted strategic effects. Similarly, since $\partial Q_{2M}/\partial p_2 < 0$, $\partial Q_{2D}/\partial p_2 < 0$, and $dp_2^*/db_1 < 0$, the network-adjusted price-adjustment term in (A-18) is negative. Thus, a sufficient condition for platform 2's profit to decrease is $dQ_{2D}^*/db_1 + dQ_{2M}^*/db_1 \leq 0$, i.e., overall demand decreases.

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