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A theory of front-line management

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Abstract

Mid- and low-level managers play a significant role within the organizational hierarchy, far beyond monitoring. It is often their responsibility to respond to opportunities and threats within their units by adjusting their subordinates' assignments. Most such managers, however, lack the authority to adapt their subordinates' wages. Instead, they rely on other, more restrictive incentive schemes. We study the interaction between a front-line manager and worker, and characterize the "managerial style" as a function of the players' relative patience and information.

Keywords: Front-line management, Perishable incentives, Asymmetric discounting.
JEL Classifications: D21, D82, D86.

1 Introduction

Large organizations generally adopt a hierarchical structure. Indeed, due to limitations on managers' effective span of control and the need to execute parallel processes requiring different skills, such a structure becomes almost unavoidable. If labor contracts were complete (in the sense that they could address every possible contingency), the role of

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managers in the lower parts of the hierarchy would mainly consist of monitoring activities. However, complete contracts are more of a theoretical idealization than a practical benchmark. The organization's uncertainty is typically too complex to be accurately understood, and some events, such as creative ideas down the road, are not even ex ante describable. Moreover, as the organizational knowledge is dispersed and evolves over time, it is sometimes more practical and efficient to solve problems within the relevant unit as they arise.[1] In this respect, mid- and low-level managers fulfill a fundamental role within organizations beyond the supervisory function discussed in, for example, Williamson (1967), Calvo and Wellisz (1978, 1979), Tirole (1986), and Laffont (1988). Namely, they can become an effective means of *complementing* workers' incomplete labor contracts: if given sufficient discretion, they can effectively adapt the assignments of their subordinates to provide timely responses to unexpected opportunities and threats.[2]

Empirical evidence highlights that front-line managers have a substantial influence on organizational performance. For example, Bandiera, Barankay and Rasul (2007) show that team performance is highly responsive to the introduction of performance pay for front-line managers, increasing workers' output by more than 20%. Similarly, Lazear, Shaw and Stanton (2015) show that replacing a bottom-decile manager with a top-decile one raises team output by roughly as much as adding an extra worker to a nine-person team.[3]

Despite their importance, front-line managers are often limited in their authority to provide additional incentives outside their subordinates' baseline contracts.[4] Instead, they incentivize their workers through other discretionary tools: assigning tasks and roles, scheduling and allocating hours, permitting remote work, coaching, and small-scale recognition of employee effort.[5] These tools are local, adjustable at high frequency,

[1] Indeed, Williamson (1975) and Hart (1995) argue that incomplete contracting is necessary for a cogent theory of the firm.

[2] The importance of providing timely responses to such shocks has been noted, for example, by Williamson (1967, p. 125) who writes that "the firm is required to adapt to circumstances which are predictable in the sense that although they occur with stochastic regularity, precise advance knowledge of them is unavailable. ... Coordination in these circumstances is thus essential."

[3] Fenizia (2022); Metcalfe, Sollaci and Syverson (2023); Minni (2023) document additional evidence that "good" front-line managers significantly increase profits and productivity relative to "bad" ones.

[4] For example, survey evidence collected by Marriott, Perkins and Cotton (2019) suggests that the majority of front-line managers have minimal involvement in the design and implementation of workers' pay.

[5] The use of such tools is documented in survey data collected by the CIPD (Purcell and Hutchinson 2007) and is concisely summarized by the following illustrative quote from a manager in Wiltshire County Council: "I have limited influence over rewarding performance but there's more than one way to skin a rabbit, so those in my team who have done well will be rewarded in other ways, for example, through secondment, development, and access to training." The value that employees place on such alternative compensation is also well documented. See, e.g., Mas and Pallais (2020); Barrero, Bloom and Davis (2021); Aksoy et al. (2022); Bloom, Han and Liang (2022), or a 2022 Gallup poll.

and often symbolic, which helps explain why firms delegate them even when wage setting is centralized.

We argue that these incentivization tools have a substantially different structure compared to the traditional model of monetary incentives. In particular, the per-period incentive budget is typically *small* (e.g., allowing one day of working from home may be insufficient compensation for substantial effort), and the incentives available to the manager are usually *perishable* (e.g., the option to grant work-from-home on a given day is lost if unused by that day).

In this paper, we study the interaction between a front-line manager and a worker, and obtain a rich set of qualitative results regarding the dynamics at the bottom of the organizational hierarchy. Our results connect the players' relative patience and information structure to a variety of commonly observed managerial practices. We also demonstrate how these findings can be applied to the broader organizational level (beyond the interaction between the front-line manager and the worker).

We propose a tractable model in which the (front-line) manager can offer only small and perishable per-period compensation, which she uses to incentivize the worker's "extra" effort that is needed on random occasions. The interpretation is that, occasionally, there are peaks at work or events that require special attention and effort above and beyond the regular work requirements. For example, workers may sometimes need to stay past their usual work hours to prepare urgent presentations or comply with an unexpected demand from a major client, deal with unexpected breakdowns, or exert special effort to implement creative ideas that increase output. We refer to such events as "opportunities." In some cases, the manager observes the arrival of opportunities (e.g., when she receives an urgent task from her superiors), while in other cases, the worker observes the arrival of opportunities privately (e.g., his creative ideas).

As the manager's per-period compensation budget is small, there is structural asynchronicity between work and compensation in our model: the manager is unable to *instantaneously* compensate the worker for his effort. This asynchronicity, in turn, gives rise to a dynamic spillover between opportunities since new opportunities may arrive while the worker is still receiving compensation for his previous effort. While most papers focus on the case where the players discount the future at the same rate, there is no compelling reason to assume that, within their specific interaction, the worker and the manager consider the future identically. For example, a recently promoted manager (or a newly hired worker) may wish to quickly create a positive impression on her superiors. Likewise, a manager or a worker who plans to hold her position for only a short while might be impatient (or a short-termist) compared to her counterpart who expects to stay in the same position for a very long time. In general, each party can be more or less patient than the other and, to an extent, this can be endogenously determined by the organization's personnel policies.

We characterize the optimal contracts for any pair of the players' discount rates, both for the case where the arrival of opportunities is observable and for the case where opportunities can be concealed by the worker. The optimal contracts generate distinct qualitative features for different specifications of the model that naturally correspond to various "managerial styles." For example, what degree of flexibility do workers enjoy at work and what affects this flexibility? How do the workers' work requirements change over time? Do managers maintain equal status among similar workers or do they generate artificial differences within their teams? If a perk is granted, is it permanent or temporary?

Summary of Results. We begin by considering the case in which the arrival of opportunities is observed by the manager. When the manager is *patient* relative to the worker, the manager treats workers identically, regardless of their past work or tenure. When an opportunity arrives, the manager requires the worker's effort and promises all perks within her discretion for a given amount of time. These promises, however, are "conditional" and do not accumulate: each promise will be nullified upon the arrival of the next opportunity to make room for a new conditional promise. It is therefore a question not of *how much* but rather *whether* some work was done in the recent past.

When the manager is *impatient*, on the other hand, the interaction features completely different dynamics. Initially, the worker has a *junior* status and is expected to exert effort on every opportunity that arrives, without enjoying any perks. At some point, the worker moves up to an *intermediate* status, in which he still needs to work whenever an opportunity arrives, but now he also enjoys all the perks at the manager's discretion. Finally, he attains a *senior* status, in which he enjoys the maximal level of perks without exerting any effort (beyond his unmodeled baseline duties). The transition times between the different statuses are (essentially) fixed and do not depend on the actual amount of effort the worker exerted over time. Hence, the manager adopts a "*tenure-based seniority system*" – a substantially different managerial style from that of the patient manager.

Under the above contracts (both for the patient and the impatient manager), the arrival of an opportunity is typically bad news for the worker. Hence, such contracts cannot be used to incentivize opportunities that the worker can conceal. In the case of *concealable* opportunities, optimal contracts will have the *perfect bookkeeping* property: on every path of play, the discounted value (using the worker's discount rate) of granted perks is equal to the discounted cost of the effort exerted.

As in the case of observable opportunities, the relative patience of the players affects the managerial style when opportunities are concealable. If the manager is patient, the worker enjoys perks immediately after exerting effort. Due to perfect bookkeeping, these promises of future compensation accumulate. Since the manager is patient relative to the worker, it is suboptimal for her to increase the promises of future compensation without limit. As a result, the required effort may change nonmonotonically over time.

When the manager is only slightly impatient, the dynamics of effort and compensation are similar to those of a patient manager. The only difference is that such a manager will continue to incentivize maximum effort until she completely runs out of incentives. Thus, she will eventually promise all the perks under her control indefinitely, at which point the worker will stop exerting effort on new opportunities. This can be interpreted as simple two-tier seniority system.

A more impatient manager will institute a three-tier seniority system; however, the seniority system when opportunities are concealable is *performance-based* rather than tenure-based (as was the case when opportunities are observable). The *junior* and *senior* statuses are similar under both of these seniority systems: junior workers exert effort without receiving any perks, whereas senior workers receive all the perks without exerting any effort when opportunities arrive. The *intermediate* status, on the other hand, has a different structure: when opportunities are concealable, the worker enjoys a *partial* level of perks as a guaranteed baseline, and whenever he exerts effort he enjoys an immediate *temporary* increase in perks.[6] The seniority system is performance-based in that transitions between statuses are related directly to the realized amount of work over time.

Table 1 summarizes our main findings about the qualitative features of the manager–worker interaction for varying information structures and relative patience.

Opportunities	**Manager**	
	Patient	**Impatient**
Observable	*Conditional* promises (Finite)	*Tenure*-based seniority system
Concealable	*Accumulating* promises (Finite)	*Performance*-based seniority system

Table 1: Dynamics at the bottom of the organizational hierarchy.

Our characterization of the bilateral interaction between front-line managers and their subordinates contributes to understanding decentralized managerial challenges in large organizations. Specifically, it can be considered as the analysis of a "continuation game"

[6] This intermediate status exists so long as the manager is not extremely impatient.

in a more general interaction in which an organization first shapes certain organizational policies. In Section 5, we illustrate examples of such applications that offer insights into the organization's preference over the manager's discretion, level of technical expertise, and discount rate. We believe that these insights can provide new perspectives into how organizations can effectively utilize front-line managers, and that our framework can be further extended and adapted to study other managerial challenges in large organizations. Examples of future directions include managing teams of workers, promotions and lateral movements within organizations, endogenous arrival of opportunities, and asymmetric information about, for example, the degree of impatience or the profitability of opportunities.

Related Literature

This paper contributes to the literature that has provided various explanations for the prevalence of hierarchical structures in large organizations. Williamson (1967) and Calvo and Wellisz (1978,1979) argue that hierarchies arise due to limitations on the number of employees that a manager can effectively control and monitor.[7] Rosen (1982) suggests that hierarchies enable highly talented senior managers to increase the productivity of their subordinates. Garicano (2000) and Harris and Raviv (2002) propose the idea that hierarchies enable the efficient utilization of expert knowledge within the firm, whereas Rajan and Zingales (2001) argue that hierarchies can also prevent employees from stealing a firm's core knowledge. Hart and Moore (2005) show that hierarchies can be an efficient method for allocating resources within the firm. See Mookherjee (2013) for an extensive review of this literature.[8]

Two papers that, like ours, assume that the front-line manager is responsible for determining the responsibilities of her workers are McAfee and McMillan (1995) and Melumad, Mookherjee and Reichelstein (1995). These papers differ from ours in that they consider a static problem in which there is no stochasticity in the tasks that the worker must perform, and the manager can offer monetary incentives to the worker. Moreover, they focus on deriving conditions under which adding an intermediate level between the principal and the worker is beneficial.

Our paper also complements works that study the optimal timing of compensation

[7] Tirole (1986) and Laffont (1988) show that, in this case, hierarchical structures are susceptible to collusion between the workers and the low-level managers that monitor them. More recently, Letina, Liu and Netzer (2020) show that if low-level managers care about their workers' welfare, but do not collude with them outright, then the firm should induce a contest between the workers.

[8] In the broader context of organizational design, Rantakari (2008) studies how the need to coordinate a firm's activities affects the optimal allocation of decision rights within a two-tier hierarchy.

(e.g., Lazear 1981; Carmichael 1983) and, in particular, those that analyze the mixture between short- and long-term incentives. The literature has primarily focused on the dynamic effects of moral hazard and screening. For example, Sannikov (2008) proposes a canonical continuous-time moral hazard framework and derives rich abstract dynamics of compensation, retirement and retention dynamics. Garrett and Pavan (2012, 2015) focus primarily on the effects of dynamic screening of persistent private information about the agent's productivity. Our paper shows that rich dynamic structures of effort and compensation arise in the absence of the above, simply due to the combination of capped periodic compensation and differences in relative patience.[9]

Our work also contributes to the recent literature on optimal contracting under different discount factors. Opp and Zhu (2015) study relational contracting in a repeated moral hazard setting, Frankel (2016) studies dynamic delegation, Hoffmann, Inderst and Opp (2021) study a one-shot moral hazard problem in which there is a delay in the arrival of information, and Krasikov, Lamba and Mettral (2023) and Knoepfle and Salmi (2024) analyze adverse selection models with heterogeneous discount rates. We contribute to this literature by studying a model in which per-period compensation is limited and perishable. These natural features lead to a tractable model of contracting with different discount rates that exhibits rich and realistic dynamics.

Finally, our work is related to the growing literature that studies principal–agent interactions with randomly arriving "projects." Forand and Zápal (2020) and Bird and Frug (2021) consider optimal contracting under symmetric information: Forand and Zápal (2020) study a model with no transfers in which projects of different types arrive randomly over time, whereas Bird and Frug (2021) study a canonical employment model in which the agent's productivity of effort varies over time. Li, Matouschek and Powell (2017), Bird and Frug (2019), and Lipnowski and Ramos (2020) consider transfer-free environments with asymmetric information. More specifically, Li, Matouschek and Powell (2017) derive the optimal relational contract when the agent has private information on project availability. Bird and Frug (2019) derive the optimal contract under full commitment in a setting where the agent privately observes the stochastic arrival of different types of projects as well as compensation opportunities. Lipnowski and Ramos (2020) characterize efficient equilibria when the agent has private information on project payoffs. Methodologically, we contribute to this literature by incorporating different discounting which requires novel arguments and proof strategies. In terms of new qualitative fea-

[9] Acharya, Lipnowski and Ramos (2024) study the role of limited incentives in a dynamic moral hazard problem in the context of political accountability. Their work differs from ours as they assume both players use the same discount factor and consider a setting in which the principal's only periodic choice is whether to irreversibly terminate the interaction.

tures, the case of a relatively patient principal leads to non-monotonic dynamics of effort and compensation under full commitment. Whereas when the principal is relatively impatient, we obtain contracts that exhibit rich seniority systems (with only a single type of project) and a combination of front- and back-loading phases.

The paper proceeds as follows. In Section 2 we present the model. In Sections 3 and 4 we analyze the cases where opportunities are observable and concealable, respectively. In Section 5 we present some organizational implications of our analysis, and in Section 6 we provide concluding remarks and discuss the role of selected modeling assumptions. All proofs are relegated to the Appendix.

2 Model

We consider an infinite-horizon continuous-time interaction between a manager (she) and a worker (he), in which opportunities arrive stochastically over time according to a Poisson process with arrival rate $\mu > 0$. The no-effort action, $\alpha = 0$, is always available to the worker. When an opportunity arrives, and only then, in addition to the no-effort action, the worker can exert effort $\alpha \in (0, 1]$.[10] The worker's effort $\alpha \in [0, 1]$ induces a (lump-sum) benefit of $\alpha \cdot B$ to the manager and a (lump-sum) cost of $\alpha \cdot A$ to the worker, where $B > A > 0$. At each instant, the manager chooses a flow compensation $\varphi \in [0, 1]$. We assume that both the worker's marginal utility from compensation and the manager's marginal cost of compensation are constant, and equal to 1.[11]

The players maximize expected discounted payoffs. We denote the worker's discount rate by $r_w > 0$ and focus on the case where there is no fundamental shortage of incentives. That is, we assume that the worker's discounted payoff from receiving $\varphi = 1$ indefinitely exceeds his expected discounted cost of full-intensity work, $\alpha = 1$, on all opportunities that arrive, even if one is currently available.[12] Formally,

Assumption 1.

$$A + \frac{\mu A}{r_w} < \frac{1}{r_w}.$$

We denote the manager's discount rate by $r_m > 0$ and refer to the manager as *patient* if $r_m < r_w$ and as *impatient* if $r_m > r_w$.

[10] We discuss the case of storable opportunities in Section 6.

[11] This assumption is a normalization. Were we to multiply the manager's cost of providing compensation and her benefit from effort (B) by the same positive constant, our result would remain unchanged.

[12] Allowing for the opposite inequality would add trivial cases with corner solutions that would not add much of substance but would needlessly impede the exposition.

Information. Throughout the paper we assume that the worker's effort is perfectly observed by the manager, but we vary our assumptions about whether or not she observes the arrival of opportunities.[13] If the manager does observe the arrival of opportunities ("*observable opportunities*"), then a public history h_t specifies for every $s < t$ whether or not an opportunity was available and the worker's choice of effort. On the other hand, if the manager does not observe the arrival of opportunities ("*concealable opportunities*"), then a public history h_t contains only the worker's effort choices. Given the Poisson arrival of opportunities, any private information that the worker has about the availability of opportunities in the past is irrelevant, and so there is no need to keep track of his private information. Hence, to reduce notation and terminology we refer to a public history as a history under both information structures we consider. We denote the set of all histories of length t by H_t and the set of all finite histories by $H = \cup_{t\in\mathbb{R}_+} H_t$.

Contracts. Our objective is to capture formal as well as informal arrangements between a manager and a worker in a large organization, and also the asymmetry in their commitment abilities. Our solution concept is the manager's optimal contract (i.e., we assume that the manager has full commitment power and the worker has none). In Section 6 we argue that the main qualitative features of our results would also hold under a relational contracting approach.

Formally, we assume that at the beginning of the interaction, the manager specifies a *work schedule*

$$\alpha : H \to [0,1],$$

which assigns a required effort to every history should an opportunity arrive at that history, and she commits to a *compensation policy*

$$\varphi : H \to [0,1],$$

which maps histories into a flow compensation. Note that the manager's policy is deterministic, and so all randomness stems from the stochastic arrival of opportunities. A pair $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is referred to as a *contract*. Without loss of optimality, we assume that after the manager detects a deviation from the required work schedule, she provides no compensation indefinitely (and does not require any additional effort).

We say that the contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is *incentive compatible* if it is measurable[14] and,

[13] The assumption that effort is perfectly observed is made to abstract away from standard moral hazard problems that are not the focus of this paper.

[14] The contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is measurable if, at every history, the worker's continuation utility and manager's continuation value are well defined. This occurs whenever the expectation $\mathbb{E}\left(\int_{s=t}^{\infty} e^{-(s-t)} \left(\varphi(h_s) - \mu\alpha(h_s)\right) ds | h_t\right)$, exists for every $h_t \in H$. Recall that we assumed the $\phi(\cdot) = \alpha(\cdot) = 0$

for every $h_t \in H$, it is optimal for the worker to choose $\alpha = \alpha(h_t)$ (conditional on the availability of an opportunity), given the continuation of the contract. If opportunities are observable, then any deviation by the worker will be detected and result in a continuation utility of zero. Hence, in this case, incentive compatibility requires that the worker's cost of effort is weakly less than his continuation utility (excluding the current cost of effort) from the contract. If, on the other hand, opportunities are concealable, then the worker can exert no effort on an available opportunity and receive the continuation utility he would have obtained had the opportunity not arrived. Hence, in this case, incentive compatibility requires that exerting effort increases the worker's continuation utility by at least the cost of effort. We state the incentive compatibility constraints formally in Appendix A.1. Since the worker can guarantee himself a payoff of zero by never exerting effort, there is no need to impose an explicit individual rationality constraint. In what follows, we restrict attention to incentive-compatible contracts.

Timing of effort and compensation

In our analysis and discussion of the results, we often compare contracts in terms of the timing of effort/compensation. We use the following relations. A work schedule $\alpha(\cdot)$ *postpones* effort relative to $\alpha'(\cdot)$ at a history h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha(h_s)|h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha'(h_s)|h_t\right) \tag{1}$$

with an equality for $\tau = \infty$ and a strict inequality for some τ.[15] Similarly, a compensation policy $\varphi(\cdot)$ *postpones* compensation relative to $\varphi'(\cdot)$ at h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi(h_s)|h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi'(h_s)|h_t\right) \tag{2}$$

with an equality for $\tau = \infty$ and a strict inequality for some τ. Analogous definitions for *expediting* effort and compensation are obtained by reversing the inequalities in (1) and (2). Note that the above definitions use the worker's discount factor.

The manager-discounted marginal cost of providing the worker with a worker-discounted util t units of time from now is $e^{-r_m t}\frac{1}{e^{-r_w t}} = e^{(r_w - r_m)t}$ and, similarly, the manager-discounted marginal benefit from the worker exerting one worker-discounted util of effort t units of time from now is $\frac{B}{A}e^{(r_w-r_m)t}$. Whether these expressions are increasing or decreasing in t

for any history that is off the path of play, and so contracts are measurable for all off-path histories.

[15]The latter requirement implies that a work schedule does not postpone effort relative to itself.

is fully pinned down by whether the manager is patient or impatient.

Observation 1.

1. *Expediting compensation and postponing effort are both profitable for a patient manager.*
2. *Postponing compensation and expediting effort are both profitable for an impatient manager.*

3 Observable Opportunities

We begin our analysis by studying the case where the manager observes the arrival of opportunities (e.g., assignments allocated to the front-line manager by her superiors).

Patient manager: ***"Have you done anything for me lately?"***

When the manager is patient ($r_m < r_w$), increasing the lag between effort and compensation in a manner that keeps the worker indifferent is detrimental for the manager. It is therefore easy to see that to maximize the gain from the *first* opportunity that arrives, the manager would have to provide the worker with maximal compensation, $\varphi = 1$, immediately after his work, for a deterministic interval of time that is just long enough to compensate for his cost of effort. However, were the manager to do so, it would not be possible for her to provide prompt compensation for any additional opportunities that arrive within that time interval. Hence, maximizing the gain from the first opportunity reduces the potential gain from further opportunities. Thus, a patient manager faces a complex optimization problem where she endeavors to provide timely compensation for the worker's effort on each of the randomly arriving opportunities. The main result of this section shows that the solution to this problem is simple and qualitatively appealing: under the optimal contract, the worker exerts the same effort α^* on all opportunities, and receives a flow compensation of $\varphi = 1$ if he exerted effort in the last S^* units of time.

This form of compensation can be understood as "conditional promises"; following the worker's effort on a given opportunity, the manager promises a fixed periodic compensation for a given time interval, but this promise is nullified upon the arrival of the next opportunity. The randomness in the arrival of the next opportunity implies that the time lag between effort and compensation on a given opportunity varies across histories, which, in turn, implies that the marginal cost of compensation is higher in some histories than in others. However, note that under the compensation policy above, all of the compensation budget (across all histories) within S^* units of time from the arrival of an opportunity is guaranteed to be fully utilized (to provide compensation for effort on one opportunity or another). Hence, it is impossible to reduce the cost of compensation for a given opportunity without increasing, by at least as much, the cost of compensation for other opportunities. The complete nullification of the manager's obligations to the worker upon the arrival of a new opportunity frees incentivization resources precisely when they are needed, and enables the manager to incentivize effort on every opportunity via a new conditional promise. In essence, this compensation method "shifts" compensation budget from histories with few opportunities to those with many opportunities.

To complement the above intuition, one needs to argue that it is optimal for the manager to incentivize the same level of effort on all opportunities, which we establish formally in the proof. Roughly speaking this is a consequence of the fact that a patient manager does not want to accumulate debt to the worker between opportunities.

Assumption 1 implies that it is possible to induce full effort on all opportunities. However, doing so need not be optimal for the manager. To see why this is the case, recall that the manager's cost of providing the worker with one worker-discounted util t units of time in the future is $e^{(r_w - r_m)t}$. As the manager's profit from a util worth of effort exerted by the worker is $\frac{B}{A}$, it follows that, for a patient manager, the *maximal profitable lag* between effort and compensation is T^*, where T^* is defined implicitly by

$$e^{T^*(r_w - r_m)} = \frac{B}{A}.$$

In optimum, the manager will require the worker to exert the maximal effort that can be incentivized via a conditional promise of length (at most) T^*.

To formally characterize the optimal contract, let σ denote the random amount of time that will pass until the arrival of the next opportunity. Then, we can denote the maximal effort that the worker is willing to exert on an opportunity in exchange for a conditional promise of length T^* (i.e., setting $\varphi = 1$ until either T^* units of time have passed or an opportunity arrives) by

$$\alpha^* \equiv \min\{\frac{1}{A}\mathbb{E}\left(\int_0^{\min\{T^*,\sigma\}} e^{-r_w t} dt\right), 1\}.$$

If $\alpha^* = 1$, providing the worker with a conditional promise of length T^* overcompensates him for his effort. Hence, we will denote by S^* the length of the conditional promise needed to exactly compensate the worker for incurring an effort cost of $\alpha^* A$, i.e.,

$$\alpha^* A = \mathbb{E}\left(\int_0^{\min\{S^*,\sigma\}} e^{-r_w t} dt\right).$$

Note that if $S^* < T^*$ then $\alpha^* = 1$, whereas $\alpha^* < 1$ implies $S^* = T^*$. Finally, let $\sigma_{-1}(h_t)$ denote the supremum of opportunity arrival times along h_t. In all subsequent results, equalities and uniqueness statements should be interpreted as holding almost surely.

Proposition 1. *Assume that the manager observes the arrival of opportunities. If $r_m \le r_w$, then*

$$\alpha(h_t) = \alpha^* \quad ; \quad \varphi(h_t) = \begin{cases} 1 & \text{if } t - \sigma_{-1}(h_t) \le S^* \\ 0 & \text{if } t - \sigma_{-1}(h_t) > S^* \end{cases}$$

is an optimal contract. Moreover, this is the unique optimal contract if $r_m < r_w$.

When $r_w = r_m$ there are multiple optimal contracts. Intuitively, in this case, postponing compensation does not alter the manager's value or violate any of the worker's incentive compatibility constraints. Thus, any contract that results from postponing compensation relative to the optimal contract characterized in Proposition 1 is also optimal. Assumption 1 implies that if $r_w = r_m$, then under the optimal contract described in Proposition 1, $\alpha^* = 1$ and $S^* < \infty$. Hence, postponing compensation is feasible.

Impatient manager: *"Tenure-based seniority"*

By Observation 1, postponing compensation and expediting effort (according to the worker's discount factor) are both profitable for an impatient manager. The first observation underlying the characterization in this section is that neither postponing compensation nor expediting effort violates incentive compatibility. A direct implication of this is that *within* each history, the worker will exert full effort on opportunities that arrive before some (history-dependent) date and enjoy compensation from some other date onward. However, it turns out that the manager is able to postpone compensation and expedite effort *across* histories as well. In particular, in *all* histories that share the same arrival time for the first opportunity the worker will be required to exert effort for exactly the same duration of time, and will begin receiving compensation at exactly the same point in time.

We now build an intuition for why an impatient manager uses a tenure-based seniority system: i.e., the worker's status, which determines his level of compensation and required intensity of work, is determined by the amount of time that has passed since the arrival of the first opportunity. Assume for a moment that an opportunity is available at $t = 0$. It is optimal for the impatient manager (and feasible by Assumption 1) to incentivize maximal effort on that opportunity. Consider an arbitrary incentive-compatible contract $\mathcal{C}$ that does so, and denote by X and Y the worker's future discounted expected effort and compensation, respectively. Incentive compatibility of $\mathcal{C}$ implies that $Y \ge X + A$.

Now consider modifying this continuation contract by fully expediting effort and postponing compensation. That is, define τ_X^O and τ_Y^C implicitly by

$$\int_0^{\tau_X^O} \mu A e^{-r_w t} dt = X \quad , \quad \int_{\tau_Y^C}^{\infty} e^{-r_w t} dt = Y,$$

and consider the contract where the worker: i) is required to exert full effort on all opportunities that arrive before τ_X^O and no effort afterwards, and ii) will receive no compensation until τ_Y^C and maximal compensation thereafter. Note that this modification is incentive compatible due to the Poisson arrival of opportunities, and profitable for the manager due to Observation 1. Hence, if an opportunity is available at $t = 0$, the optimal contract must have a threshold structure as described above.

The relation between the two thresholds under the optimal contract, τ^O and τ^C, can be identified by two simple conditions. First, because compensation is postponed and effort is expedited, the only relevant incentive compatibility constraint is the one at $t = 0$ (the arrival time of the first opportunity). As compensation is costly to the manager, in optimum, this constraint will be binding

$$A + \int_0^{\tau^O} \mu A e^{-r_w t} dt = \int_{\tau^C}^{\infty} e^{-r_w t} dt. \tag{3}$$

Second, note that $\tau^O > \tau^C$ as otherwise the manager can require the worker to exert effort at τ^O and provide compensation at the (weakly) later time of τ^C. This modification is profitable for an impatient manager. Indeed, in optimum, $\tau^O - \tau^C$ is such that the net surplus generated by effort is offset by the manager's relative impatience over $\tau^O - \tau^C$ units of time,

$$\frac{B}{A} = e^{(r_m - r_w)(\tau^O - \tau^C)}. \tag{4}$$

Since incentivizing more effort (increasing τ^O) requires more compensation (decreasing τ^C), the difference $\tau^O - \tau^C$ is increasing in τ^O. Thus, there is a unique solution to Equations (3)-(4) and, hence, the optimal contract is unique.

Returning to the original interaction (which does not begin with an opportunity), it is straightforward that providing compensation to the worker before he exerts effort for the first time is suboptimal. Furthermore, the contracting problem from the time the first opportunity arrives is the same for all first-arrival times, and, in particular, is identical to the problem described above. Hence, the continuation contract from every possible first-arrival time is identical to the contract derived above. In other words, in the optimal contract, "the clock is set to zero" at the arrival of the first opportunity, and the optimal contract is, in essence, history-independent. To characterize the optimal contract formally, let $\sigma_1(h_t)$ denote the infimum of the arrival times of opportunities along the history h_t.

Proposition 2. *Assume that the manager observes the arrival of opportunities. If $r_m > r_w$, then the unique optimal contract is*

$$\alpha(h_t) = \begin{cases} 1 \text{ if } t \leq \sigma_1(h_t) + \tau^O \\ 0 \text{ if } t > \sigma_1(h_t) + \tau^O \end{cases} \quad \text{and} \quad \varphi(h_t) = \begin{cases} 0 \text{ if } t \leq \sigma_1(h_t) + \tau^C \\ 1 \text{ if } t > \sigma_1(h_t) + \tau^C \end{cases},$$

where τ^C, τ^O are the unique solution to (3) *and* (4).

Proposition 2 suggests that tenure-based seniority systems arise naturally if the front-line manager is impatient. Under the contract characterized in the proposition, initially, the worker exerts effort on all opportunities but does not receive compensation; at some point ($\sigma_1 + \tau^C$), he begins to receive compensation but still has to work whenever an opportunity arrives; and finally (at $\sigma_1 + \tau^O$), he receives compensation without being required to exert further effort.

Furthermore, Proposition 2 yields clear predictions about the factors affecting the time required for a worker to attain a senior status (i.e., enjoy full compensation without exerting further effort) and the corresponding discounted effort exerted until that point. Formally,

Corollary 1.

- *As the manager becomes more impatient relative to the worker, the worker attains a senior status more quickly and his expected effort decreases.*
- *As the profitability of opportunities ($\frac{B}{A}$) increases, the worker attains a senior status more slowly and his expected effort increases.*

Propositions 1 and 2 are visualized in Figure 1 for a typical sequence of opportunity arrivals. The upper panel depicts the worker's effort (in red) and compensation (in blue) when the manager is patient, and the lower panel depicts the same outcomes when the manager is impatient.

4 Concealable Opportunities

In the optimal contracts derived in the previous section, the arrival of opportunities typically leads to an immediate decrease in the worker's continuation utility. In settings

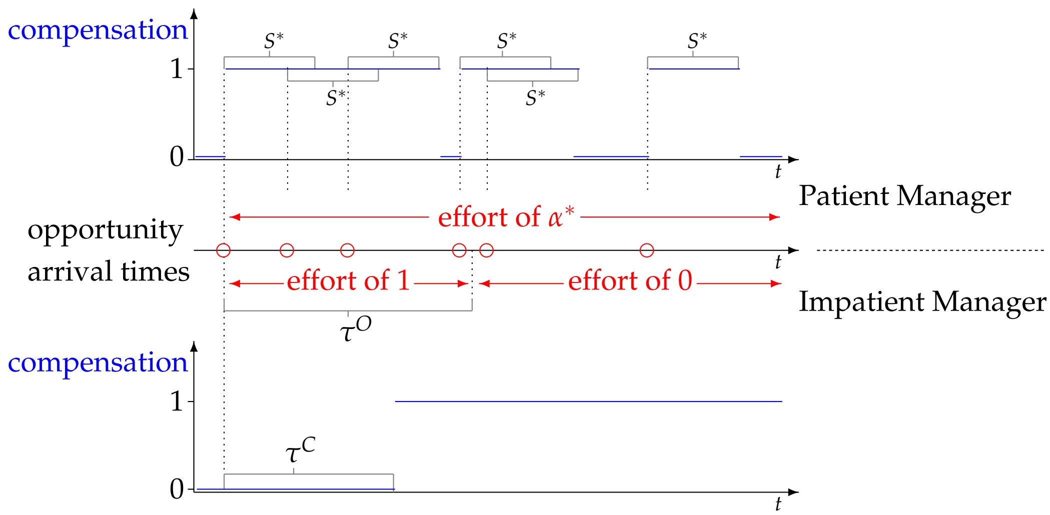

Figure 1: Qualitative dynamics of effort and compensation: Observable opportunities.

where the arrival of opportunities is observed only by the worker, such contracts are not incentive-compatible as the worker prefers to conceal opportunities from the manager. Indeed, to provide incentives for the worker to reveal when opportunities become available, the arrival of opportunities must never be "bad news" for the worker.[16]

The need to make the arrival of opportunities not bad news for the worker means that the manager must keep track of both his compensation and effort. This, in turn, suggests that when opportunities are concealable it is convenient to characterize the optimal contract using the worker's continuation utility as a state variable.[17] In Appendix A.2 we argue formally that the optimal contract can be characterized using recursive techniques. Moreover, we use the recursive representation to obtain two distinct objectives.

First, we derive several properties that an optimal contract must satisfy. While some of these properties are technical, this analysis also reveals a general property of the managerial style of a front-line manager that does not observe the arrival of opportunities: she engages in *perfect bookkeeping* wherein, path by path, the worker-discounted compensation and effort are equal. Note that this property is stronger than standard promise keeping which requires that, from every history, the worker receives his promised continuation utility *in expectation*. The perfect bookkeeping property, therefore, stands in contrast to the low correlation between work and compensation that arises under the optimal

[16]Recall that we formally define the incentive compatibility constraint in Appendix A.1.

[17]For completeness, in Appendix A.2.1 we provide a recursive representation of the optimal contracts when opportunities are observable.

contract when opportunities are observable.

Second, we derive the HJB equation that characterizes an optimal contract and use it to show that the optimal contract is generically unique and is defined by two thresholds, u^C and u^O, such that the worker is required to exert effort if his continuation utility is less than u^O and receives the maximal flow compensation if it is above u^C. Formally,

Lemma 1. *If opportunities are concealable, then the optimal contract is generically unique. Moreover, there exist thresholds $u^C, u^O \in [0, \frac{1}{r_w}]$ such that the optimal contract is given by*[18]

$$\alpha(u) = \min\{1, \frac{(u^O - u)^+}{A}\} \quad ; \quad \varphi(u) = \begin{cases} 1 & \text{if } u > u^C \\ r_w u & \text{if } u = u^C \\ 0 & \text{if } u < u^C \end{cases}.$$

While the optimal contract exhibits this two-threshold form regardless of whether the manager is relatively patient or impatient, the values of the thresholds and the resulting dynamics of work and compensation depend critically on the players' relative patience. Before exploring the qualitative properties of the optimal contract as a function of the players' relative patience, we clarify how these thresholds govern the general dynamics.

The threshold u^O dictates the dynamics of work. The worker's continuation utility is bounded from above by his utility from receiving maximal compensation indefinitely without being required to exert additional effort – an outcome that provides him with a continuation utility of $1/r_w$. Hence, the threshold value $u^O = \frac{1}{r_w}$ corresponds to a work schedule in which the manager instructs the worker to fully exploit every opportunity that arrives until all of her compensation budget is exhausted. For lower values of u^O, the manager will sometimes forgo opportunities even though her compensation budget is not exhausted.

The threshold u^C dictates the dynamics of compensation. In particular, it captures the degree of back/front-loading of compensation. So long as the worker's continuation utility is below u^C, compensation is deferred to the future. Setting the compensation threshold at the maximal possible value, $u^C = \frac{1}{r_w}$, corresponds to full back-loading: when the worker's continuation utility reaches that level, it is necessary to set $\varphi = 1$ indefinitely. At the other extreme, the compensation threshold $u^C = 0$ corresponds to full front-loading because, in this case, the manager provides the maximal compensation whenever the worker's promised continuation utility is positive.

For $u^C \in (0, \frac{1}{r_w})$, the compensation dynamics consists of two phases. In the beginning,

[18] For $y \in \mathbb{R}$, we use the notation $y^+ = \max\{y, 0\}$.

the *back-loading phase* takes place. So long as $u < u^C$, the worker exerts effort and accumulates promises of future compensation but does not receive any compensation. When his continuation utility attains (or exceeds) u^C, the *front-loading phase* begins. In this phase, the worker receives a permanent base compensation of $r_w u^C$ – which can be thought of as compensation for effort exerted in the earlier back-loading phase – and a temporary additional compensation of $1 - r_w u^C$ whenever $u > u^C$. If $u = u^C$, then the base compensation of $r_w u^C$ maintains the worker's continuation utility at that level. Hence, once the worker's continuation utility reaches u^C, it never drops below this level again.

4.1 Patient Manager: Accumulating Promises; Nonmonotonic Effort

When the manager is more patient than the worker it is beneficial for her to front-load compensation: doing so not only decreases the direct cost of providing the worker with his promised utility, but it also increases her capacity to incentivize additional effort in the future. Hence, each instance of the worker's effort is met with a promise of immediate compensation for some duration. However, due to the perfect bookkeeping property, these compensation promises accumulate rather than reset (as they do when opportunities are observable).

Due to the manager's relative patience, the accumulation of compensation promises raises the effective cost of incentivizing current work. As a result, after several effort instances in a relatively short period, the manager finds it optimal to temporarily reduce effort requirements, even though she still has compensation budget available. Interestingly, she stops incentivizing full effort before effort becomes unprofitable, in the sense that effort requirements are decreased even though the marginal cost of providing compensation is less than marginal profit from effort. Intuitively, the manager deliberately holds back in order to preserve the option to use her limited compensation budget in the future when there will be a shorter lag between effort and compensation.

To state the formal result, recall that the maximal profitable lag between effort and compensation is given by $T^* = \frac{\log(B/A)}{r_w - r_m}$, and let $u(T^*)$ denote the worker's continuation utility from maximal compensation for T^* units of time.

Proposition 3. *Suppose that $r_m < r_w$. Under the optimal contract:*

1. *Maximal compensation is offered whenever the worker has positive continuation utility, i.e., $u^C = 0$.*

2. *Effort is incentivized as long as the worker's continuation utility remains below a threshold $u^O \in (0, u(T^*))$.*

The above contract generates distinct dynamics from those that arise under observable opportunities. Recall that when the arrival of opportunities is observable, the worker exerts the same effort on all opportunities. By contrast, when opportunities are concealable, periods of full effort are followed by periods in which the worker receives compensation without fully seizing available opportunities. Once enough of the accumulated promises have been fulfilled (e.g., for a while, the worker enjoyed compensation without exerting effort), the cycle restarts. These dynamics are depicted in Figure 2 below for a typical sequence of opportunity arrivals.

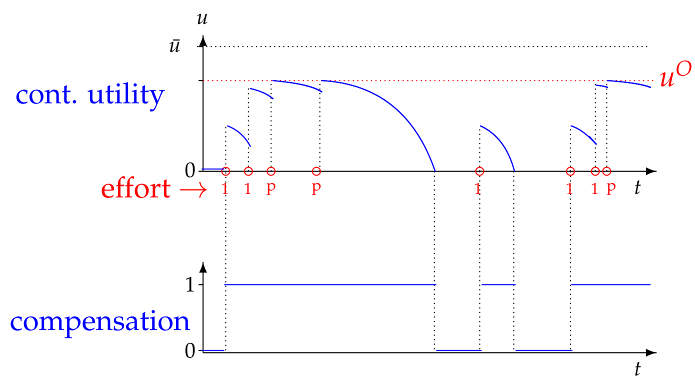

Figure 2: Qualitative dynamics of effort and compensation: Patient manager. Red circles indicate the arrival of opportunities, 1 indicates full effort, and p indicates partial effort.

4.2 Impatient Manager: Performance-Based Seniority System

When the manager is more impatient than the worker, the optimal contract takes the form of a performance-based seniority system. The worker's status, which determines the level of compensation and the required intensity of work, evolves with the accumulation of past effort. While the structure of the seniority system varies with the degree to which the manager is more impatient than the worker, eventually the worker attains senior status, where he receives maximal compensation forever without exerting any additional effort.

The driver behind this result is that the relatively impatient manager wants to expedite effort, and so seizes every opportunity that arrives. That is, she continues to incentivize effort as long as any incentivization budget is available. A qualitative implication of this result is that – unlike in the case where the manager is patient – there are no waves of effort: the worker exerts effort on every opportunity, until the point where he stops entirely.

Proposition 4. *If $r_m \geq r_w$, then $u^O = \frac{1}{r_w}$.*

We now turn to describing the dynamics of compensation and how they vary with the degree of the manager's relative impatience. For a very impatient manager, the optimal contract can be interpreted as having two levels of status. A junior worker exerts effort without receiving any compensation. This gradually increases the manager's debt (i.e., worker's continuation utility), which eventually reaches a point where the only way to respect past promises is to grant the worker the senior status (where he stops working and receives indefinite maximal compensation).

At the other extreme, when the manager is almost as patient as the worker, the structure is similar, except that a worker of junior status is now compensated immediately after each instance of effort. Specifically, following effort, the worker receives maximal compensation for a deterministic interval of time, which is updated upon every new instance of effort. While at the junior status, a sufficiently long period without effort will cause compensation to stop, and it will resume only after additional effort is exerted. Note that in this case the (slightly) impatient manager fully front-loads the worker's compensation. Why would an impatient manager ever choose to do so? The reason is that compensating earlier generates additional budget for future opportunities – budget that would otherwise remain unavailable. Since this benefit is realized only in the future, the trade-off between early compensation and additional incentivization budget leads to full front-loading only if the manager is only slightly impatient relative to the worker.

A richer seniority system arises between these two extremes. For intermediate levels of the manager's relative impatience, a three-tier seniority system emerges: a junior status that features work without (immediate) compensation, an intermediate seniority level where the worker still exerts effort but enjoys immediate compensation, and finally there is, of course, the senior status where indefinite compensation is granted without any additional effort.

Unlike in the tenure-based seniority system, where the times at which status changes are essentially fixed, transition times under concealable opportunities are governed entirely by the realized path of effort. The seniority system is therefore *performance-based*. This shift reflects the fact that the manager can no longer pool different realization paths ex ante to improve *average* efficiency: each opportunity must be individually incentivized as it arrives, due to the perfect bookkeeping property.

There is only one way to sustain past promises when at the senior status – provide maximal compensation forever. Hence, the structure of compensation at the senior level coincides throughout the levels of relative impatience, as well as across the observable and concealable opportunities cases. In the junior level of the three-tier performance-based seniority system, the worker gets no immediate compensation. This too mirrors the junior level compensation structure under the tenure-based seniority system used in the observable opportunity case.

The difference in terms of compensation between the two seniority systems is in the intermediate status. Once that level is reached, a positive flow of compensation is guaranteed to the worker, forever, irrespective of the realized arrival of opportunities. In the observable opportunities case, this is the maximal compensation $\varphi = 1$. Under concealable opportunities, an interior level $0 < \varphi < 1$ of periodic compensation is guaranteed, and whenever effort is exerted, maximal compensation is temporarily granted. Hence, while at the intermediate seniority level, the worker's compensation will jump between these two levels (as a function of realized effort) until he attains senior status. The dynamics for the three-tier performance-based seniority system are depicted in Figure 3 below for a typical sequence of opportunity arrivals.

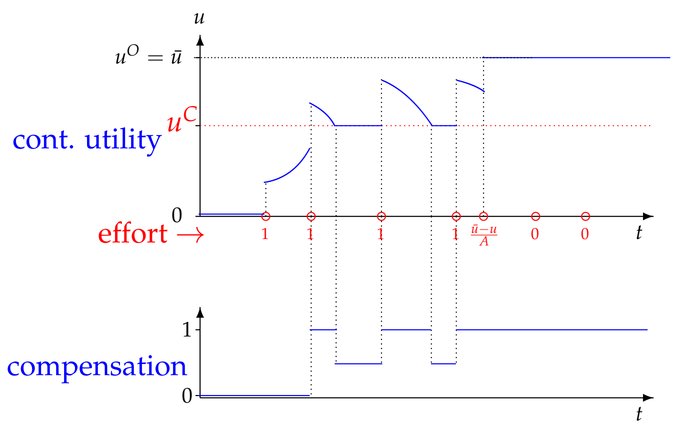

Figure 3: Qualitative dynamics of effort and compensation: Impatient manager.

The transition to the intermediate status is governed by an intuitive trade-off. To minimize the direct cost of compensation, the impatient manager should fully back-load compensation. However, as mentioned above, back-loading compensation decreases the manager's capacity to induce additional effort in the future. The transition occurs when the manager's accumulated debt to the worker balances these two forces.

To conclude the full description of the performance-based seniority system, observe how the two-tier systems that arise at the extremes actually fit within the structure above. At very high levels of manager impatience, the base periodic compensation of the intermediate status reaches the maximal level to allow the manager to maximally postpone compensation. This effectively eliminates the intermediate rank. At the other extreme – when the manager is only slightly more impatient than the worker – the base periodic compensation of the intermediate rank drops to zero, eliminating the junior rank instead.

The discussion above is formalized in the following proposition.

Proposition 5. *Assume that opportunities are concealable. Fix A, B, μ, and r_w and suppose that $r_m \geq r_w$. There exists $r''_m > r'_m > r_w$ such that*

$$u^C \in \begin{cases} \{0\} & \text{if } r_m \leq r'_m \\ (0, \frac{1}{r_w} - A] & \text{if } r_m \in (r'_m, r''_m) \\ \{\frac{1}{r_w}\} & \text{if } r_m > r''_m \end{cases} .$$

Proposition 5 describes the qualitative implications of different levels of managerial impatience. We can in fact provide a stronger result and compare two managers – both more impatient than the worker – where one is more impatient than the other. The former manager prefers to postpone compensation for longer, while the latter one begins compensating earlier to make room for more future effort opportunities. As a result, the transition to intermediate status occurs later under the more impatient manager, and the base periodic compensation granted at that level is correspondingly higher.[19]

Having described the qualitative features of the optimal contract across different levels of the manager's relative patience, we now illustrate how the thresholds u^C and u^O vary across players' time preferences. The figure below presents this relationship across the full range of values for r_m, holding the worker's discount rate r_w (and other parameters of the model) fixed.

[19]This result is derived formally in Lemma B.4 as part of the proof of Proposition 5.

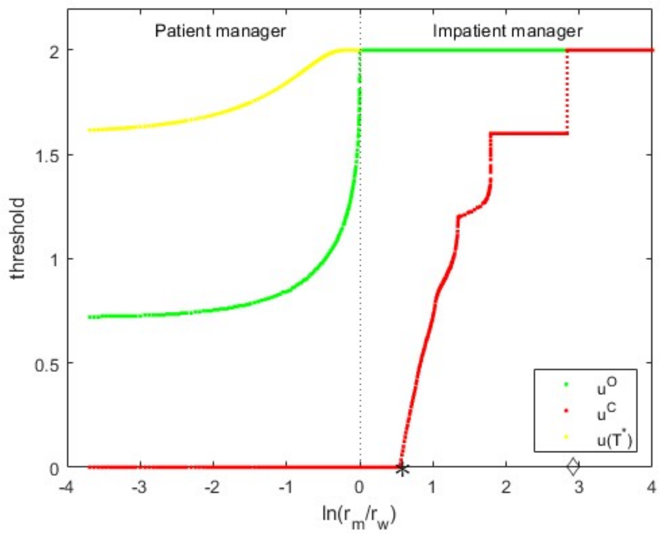

Figure 4: u^O, u^C, and $u(T^*)$ as a function of $\ln(\frac{r_m}{r_w})$, for $A = \frac{2}{5}, B = 2, r_w = \frac{1}{2}$, and $\mu = 2$. The middle of the x-axis corresponds to $r_m = r_w$. The $*$ symbol corresponds to the value of r'_m, and the $\diamondsuit$ symbol corresponds to the value of r''_m.[20]

Note that the behavior of a manager who is only slightly more impatient than the worker closely resembles that of a patient manager. When $r_w < r_m < r'_m$, compensation is fully front-loaded, just as it is when $r_m < r_w$. The key distinction is that, unlike in the latter case, the slightly impatient manager never skips effort opportunities. However, as can be seen in the figure, we also have that, for a patient manager, u^O converges to its upper bound as the relative patience converges to zero. This suggests that, unlike in the observable opportunities case, the transition between different types of contracts is continuous when the manager's and worker's discount rates are nearly equal. Taken together with the discussion of the different instances of the performance-based seniority system for different levels of the manager's relative impatience, it shows that all optimal contracts under concealable opportunities are part of a single, essentially continuous system.[21]

5 Organizational Implications

We conclude the paper by illustrating how our characterization of the bilateral contracts can be used as a tool to address broader questions at the organizational level. In par-

[20] The optimal thresholds were derived via Monte Carlo simulations. Note that on the extreme right of the figure the green dots are obscured by the red ones as $u^C = u^O = \frac{1}{r_w}$ when r_m is sufficiently large.

[21] As can be seen in Figure 4 (and Proposition 5), there is a discontinuity in the optimal compensation threshold at $r_m = r''_m$. This discontinuity, as well as the kinks and abrupt changes in the curvature of u^C as a function of r_m, arises from the discrete arrival of opportunities.

ticular, we show how it can provide insights into the organization's preference over the manager's discretion, information structure, and discount rate.

Discretion vs. Opportunity Volatility

One aspect of a manager's discretion is her authority to approve additional compensation for her workers. In our model, this discretion is captured by the fixed cap on the periodic compensation; which we normalized to one. In a more general model, this cap could be selected optimally by the organization based on the characteristics of the manager–worker interaction, and the cost of providing such discretion.

A natural dimension along which organizations can differ is in the volatility of opportunity arrival. To provide insight into the optimal level of managerial discretion, we analyze the impact of that volatility on the manager's ability to extract value from the worker, holding her discretion fixed. While the manager's value need not be perfectly aligned with that of the organization, it is plausible to assume that, in some cases, the organization can design the manager's incentives so that the two values are sufficiently close, in which case the manager's ability to effectively complement the worker's baseline contract is key for the organization at large.

We analyze the impact of volatility by comparing settings where the frequency and magnitude of opportunities vary, while the total value of expected opportunities stays constant. We say that the opportunities represented by (A_1, B_1, μ_1) are *lumpier* than those represented by (A_0, B_0, μ_0) if there exists $\lambda > 1$ such that $A_1 = \lambda A_0, B_1 = \lambda B_0$, and $\mu_1 = \frac{\mu_0}{\lambda}$. That is, opportunities are lumpier if their arrival process is more volatile.

In the special case where $r_w = r_m$ and opportunities are observable, the lumpiness of opportunities does not affect the manager's value. However, even though common discounting is the standard assumption, the irrelevance of the degree of lumpiness in this case is a knife-edge result. In general, lumpiness is detrimental to the manager.

Proposition 6. *The manager's value strictly decreases when opportunities become lumpier, unless opportunities are observable and $r_m = r_w$.*

Proposition 6 shows that it is harder for a manager who operates in a more volatile environment to extract value from the worker. This implies that organizations should take the volatility of the environment into account when deciding on a manager's level of discretion. This relation has been observed in Aghion et al. (2021) who use large micro-data to show that firms that delegated more authority to local plant managers performed better in periods of increased turbulence in the markets. Similarly, Dessein, Lo and Minami

(2022) analyze a single large retailer and find that local volatility is associated with more decentralization (when there is no need to coordinate between sub units).

When Ignorance Helps: Value of Information Frictions

Even if the organization evaluates periodic payoffs in the same way as the manager, it may use a different discount rate to evaluate streams of effort and compensation: r_o rather than r_m. We now demonstrate via an example that such intertemporal misalignment can lead the organization to value information frictions between the manager and worker (i.e., prefer that opportunities be concealable).

Example 1. *Assume that $A = 1, B = 1.2, \mu = 0.5, r_m = 0.11, r_w = 0.1$, and $r_o = 0.09$. For these parameter values, the organization's value is higher under concealable opportunities than under observable ones: in the latter case the organization's value is* 0.477, *while in the former case the organization's value is* 1.031.

To understand this example recall that, when opportunities are observable, there is a discrete change in the induced streams of effort and compensation around $r_m = r_w$. As the manager selects the worker's contract optimally, *her* value is continuous in r_m, despite the discrete change in managerial style.[22] This is not true for the organization that evaluates the induced streams of effort and compensation using a different discount rate. Indeed, as the organization is strictly more patient than the manager, its value drops discontinuously due to the back-loading of compensation and front-loading of effort that occurs once the manager becomes slightly impatient. By contrast, when opportunities are concealable, the payoffs of both the manager and the organization, are continuous around $r_m = r_w$. Example 1 demonstrates how significant the drop in the organization's value around $r_m = r_w$ can be in the former case. Specifically, it shows that the magnitude of this drop may be greater than the loss of value due to information frictions between the manager and worker.[23]

One interpretation of the example is that, under some circumstances, the organization may prefer to hire professional managers that lack the technical expertise needed to observe the arrival of opportunities, even if they do not possess superior managerial skills.[24] Alternatively, this result suggests that organizations may benefit from imposing constraints on the manager's ability to "punish" workers for not exerting extra effort.

[22] This follows from Berge's theorem of the maximum.

[23] In fact, it is easy to construct examples in which the front-line manager will generate profit for the organization only if she does not observe the arrival of opportunities.

[24] At a high level, a similar observation about the value of information frictions at the bottom of the managerial hierarchy has been made by Castro-Pires (2023). He considers a static model and shows that

Such restrictions, in essence, turn observable opportunities into concealable ones, and can thus be beneficial in mitigating the impact of inter-temporal incentive misalignment between the manager and the organization.

Harnessing the Manager's Impatience

A central result of the model is that the worker's effort and compensation streams are governed by the relative patience between the manager and the worker. In practice, the organization may be able to affect this relative patience by various personnel policies (e.g., lateral moves, fixed-term project appointment, or the rate of wage-increases for various levels). When the organization is more patient than a worker, a natural question is whether it may benefit from pushing the manager's time preference from those of the organization ($r_m \approx r_o$) towards those of the worker ($r_m \approx r_w$).

To illustrate this, we consider a setting with observable opportunities and assume that the organization internalizes only a fraction $\theta \in [0, 1]$ of the manager's cost of compensating the worker. By Proposition 1 the manager will induce a constant effort of α^* on all opportunities. Moreover, there exists $\underline{r} < r_w$ such that the manager will choose $\alpha^* = 1$ (full effort) for any $r_m \in [\underline{r}, r_w]$. To avoid trivial cases we suppose that $r_o < \underline{r}$ and restrict the organization choice to $r_m \in [r_o, \underline{r}]$.[25] Let $r_m^*(\theta)$ denote the optimal manager's discount rate from the organization's perspective as a function of θ.

Proposition 7. *There exists $\underline{\theta} \in (0, 1)$ such that, if $\theta \leq \underline{\theta}$, then $r_m^*(\theta) = \underline{r}$. Moreover, $r_m^*(\theta)$ is strictly decreasing and continuous, for $\theta \in [\underline{\theta}, 1]$, reaching the minimal value of r_o at $\theta = 1$.*

Proposition 7 states that if the organization incurs the full cost of compensating the worker, it would prefer the manager to share its discount rate. However, when the compensation of the worker is cheaper from the perspective of the organization, it can benefit from distorting the manager's time preference towards r_m. To understand why, recall that a manager who is sufficiently more patient than the worker will induce partial effort, and that the level of effort decreases as the manager becomes more patient. Hence, as the organization's perceived cost of compensating the worker decreases, it benefits from inducing a greater similarity in time preferences of the worker and the manager.

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the principal may benefit from a monitor observing less accurate information about the worker's effort due to the monitor incentivizing excessively high worker's effort.

[25]This assumption is made for expositional simplicity: if $\underline{r} < r_o$ all managers will require full effort on every opportunity, and if $r_m = \underline{r}$ is optimal for the organization then any $r_m \in [\underline{r}, r_w]$ is also optimal.

6 Conclusion

In this paper we analyzed how front-line managers can complement the incomplete labor contracts of their subordinates: specifically, how they can use their discretion to allocate perks to a worker in order to induce extra effort on random opportunities/needs that fall outside formal job descriptions. A central insight is that relative patience between the manager and the worker shapes managerial style: less patient managers are more likely to create informal status differences among similar workers, while more patient managers maintain steadier, more uniform treatment.

The relative patience within a given manager–worker pair can be shaped by both individual and organizational factors. Beyond intrinsic time preferences or differences in the structure of wage trajectories, relative patience may also reflect different beliefs about the expected duration of the relationship. For example, if a manager believes the worker is overly optimistic about outside opportunities, she may act more patiently than the worker.

Organizational design can further shape managerial patience. A manager who believes that impressing her superiors early could lead to rapid promotion – or to lateral moves that count toward future promotions – may act impatiently, exploiting every opportunity to demonstrate results while postponing rewards to her subordinates. By contrast, if the organization delivers managerial rewards through instruments other than promotions or lateral moves – for example, team-performance bonuses or recognition – while deliberately keeping managers in the same position longer, managers are more likely to act patiently, distributing rewards more evenly over time and avoiding the buildup of costly implicit promises.[26] Large organizations can use this lever strategically by adjusting the frequency of lateral moves, thereby influencing (perhaps asymmetrically) the patience of managers and workers in their interactions.

Turnover patterns also matter. Workers at the extremes of the age distribution often place a lower value on the future within the firm: young workers because they anticipate higher outside mobility, and older workers because they are closer to retirement.[27] In such cases, managers may adjust their patience accordingly. When dealing with older workers, time preferences are often more closely aligned—managers are inclined to offer more immediate rewards, recognizing the short horizon of the interaction. With younger workers, managers may display somewhat more patience, especially if they believe that

[26]The literature on psychological contracts provides evidence that even reneging on unwritten promises made by managers is costly for the organization (e.g., it leads to a reduction in worker productivity and higher turnover). See Zhao et al. (2007) for a meta-analysis of such studies.

[27]See the Census Bureau's Quarterly Workforce Indicators for data on turnover as a function of age.

some of these workers will stay longer than the workers themselves expect. In both of these cases, managers are more likely to be relatively patient compared to their interactions with workers of intermediate age, where less even treatment may be observed through differences in informal status and the buildup of future promises.

There is also substantial variation in turnover rates across sectors.[28] If these sectoral differences are driven more by worker mobility than by managerial turnover, this suggests that in low-turnover sectors (e.g., financial services and government service) managers are more likely to act impatiently. In such environments, perks are allocated unevenly, and informal seniority distinctions play a stronger role in guiding allocation.

We conclude by discussing several key features and assumptions in our model.

Complete vs. Relational Contracting

There is an intrinsic asymmetry between workers and managers along many dimensions, including their ability to commit to future actions. As the interaction between the manager and the worker takes place in an organization, reneging on promises made to the worker also impacts the organization at large. This, in particular, adds credibility to the manager's promises. Some perks, once granted by the manager, are formally authorized by senior management or HR (e.g., the allocation of a privileged parking space). Moreover, as mentioned above, reneging on unwritten promises made by managers is costly for the organization (Zhao et al., 2007). For these reasons, we view full commitment on the part of the manager as an appropriate modeling assumption to capture this asymmetry, especially in light of the manager's limited discretion.

An alternative approach is to restrict the manager to self-sustaining promises. That is, to use the relational contracting approach (as in, e.g., MacLeod and Malcomson, 1989; Ray, 2002; Levin, 2003). While this alternative approach would alter specific details of the optimal contracts, it does not substantially affect our main qualitative insights.

If the manager is patient, then, under the optimal contracts characterized in Propositions 1 and 3 she requires effort on all opportunities and refrains from accumulating a large debt to the worker. Hence, in most cases of interest, i.e., when players are not too myopic and the profit from opportunities is not too low, the optimal contracts characterized above are also relational contracts. Even if these conditions do not hold, the optimal relational contracts are identical to the optimal contracts, apart from inducing slightly lower effort requirements.

[28] See the U.S. Bureau of Labor Statistics Current Population Survey.

If the manager is impatient, she is unable to implement exactly the seniority systems described in Proposition 2 and Propositions 4–5, since they induce a senior status in which workers receive compensation without exerting effort. Under observable opportunities, the optimal relational contract still consists of a three-tier, tenure-based seniority system, albeit one in which senior workers are required to exert a low level of effort. Under concealable opportunities, the optimal relational contract generates a two-tier performance-based seniority system, in which juniors exert full effort and do not receive compensation, and non-juniors receive compensation and exert some effort on all opportunities.

The Structure of Compensation

To capture the key features of the manager's compensation devices – namely their limited and perishable nature – we assumed that she can provide a flow compensation of $\phi \in [0, 1]$ at each instance. This is a stylized simplification that allows for tractable analysis, but it inevitably abstracts from potential intertemporal consequences of granting particular perks. In reality, granting certain perks may have lasting effects: some perks can only be granted a limited number of times (e.g., a given training program cannot be offered repeatedly), while others may alter the future of the relationship itself (e.g., a worker who completes an MBA sponsored by the manager may return to a different role within the organization, or repeated praise and increased visibility may raise the worker's chances of promotion).

If certain perks can be used only a limited number of times, then after such perks are "maxed out" the manager's compensation budget would decrease. Similarly, granting a perk that may hasten the end of the relationship reduces the expected value of promising future perks. Thus, introducing such intertemporal connections could lead to a new trade-off: using the perks provides immediate benefits but shrinks the manager's effective budget to provide incentives in the future. In terms of our model, this could be represented by a reduction in the effective range of feasible ϕ's available in later periods.

While the precise implications of such intertemporal considerations will depend on the particular form of the perks in question, we believe that the main qualitative patterns highlighted by our analysis remain even under additional constraints of this type.

Storable Opportunities

The focus of our analysis was on applications in which opportunities are wasted if they are not acted upon immediately. However, our analysis is also relevant for some applications where it is possible to store opportunities.

If the manager is impatient, expediting effort is profitable. Hence, storing opportunities is suboptimal for an impatient manager.

By contrast, if the manager is patient, it is profitable for her to postpone effort and reduce compensation accordingly since it reduces the time lag between effort and compensation. Observe that requiring effort in t units of time in exchange for a util of compensation provided at the same time generates a current benefit of $e^{-r_m t}\frac{B}{A}$ for the manager. Whereas requiring immediate effort in exchange for a util of compensation provided in t units of time generates a current benefit of $e^{-r_w t}\frac{B}{A} < e^{-r_m t}\frac{B}{A}$. Intuitively, by requiring immediate effort (rather than storing the opportunity) the manager is, in essence, using the worker's higher discount rate to discount future benefits.

This suggests that if opportunities are perfectly storable a patient manager would want to slice arriving opportunities into infinitesimal bits, and have the worker exert a constant flow effort in return for immediate compensation. However, as opportunities oftentimes represent random events that require immediate action, they may depreciate or become obsolete over time. Indeed, if stored opportunities depreciate at a rate greater than $r_w - r_m$, then the cost of depreciation outweighs the manager's gain from reducing the time lag between effort and compensation. Hence, our analysis holds also for applications in which storing opportunities is sufficiently costly.

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Appendices

A Technical Appendix

In the first part of this appendix we formally define the incentive compatibility constraints. In the second part we derive a recursive representation of the contracting problem, establish several properties of optimal contracts, and provide the recursive representation of the contracts characterized in Propositions 1 and 2.

A.1 Incentive Compatibility

Note that a deviation to a non-zero level of effort is detected by the manager under both information structures, and recall that we assumed (without loss of optimality) that following a detectable deviation the worker receives a continuation utility of zero. Since exerting effort is costly for the worker, such a deviation provides the worker with a strictly lower discounted payoff than exerting no effort on the current opportunity and then receiving a nonnegative continuation payoff. Hence, a (measurable) contract is incentive compatible if and only if the worker (weakly) prefers to follow $\alpha(\cdot)$ rather than to deviate to $\alpha = 0$ at some history.

If opportunities are observable, then a deviation to zero effort will lead to a continuation utility of zero. Hence, the incentive compatibility constraints when opportunities are observable are given by

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, O, \alpha(h_t))\right) \geq 0 \quad \forall h_t \in H, \quad (IC_{obs})$$

where $(h_t, O, \alpha(h_t))$ is the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$.

If opportunities are concealable, then a deviation to zero effort will lead to the same continuation utility as if the opportunity had not arrived. Hence, the incentive compatibility constraints when opportunities are concealable are given by

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, O, \alpha(h_t))\right) \geq$$
$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, N)\right) \quad \forall h_t \in H, \quad (IC_{conc})$$

where (h_t, N) denotes the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity does not arrive; and, as before, $(h_t, O, \alpha(h_t))$ denotes the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$ on that opportunity.

A.2 Technical Analysis: Recursive Contracts

It is well known in the dynamic contracting literature that if the environment is stationary, then the agent's continuation utility can be used as a state variable for deriving the optimal contract (see Spear and Srivastava 1987 and Thomas and Worrall 1990). The argument behind this result relies on the property that the continuation payoffs of an efficient contract must always lie on the constrained Pareto frontier. This, in turn, follows from two observations: first, if the agent receives a continuation utility u via an inefficient continuation contract, then the principal can increase her value by replacing that continuation contract with a different one that provides the agent with u utils; second, since the agent is indifferent between the original and modified continuations of the contract, this change has no impact on earlier incentive compatibility constraints. Notice that these observations do not depend on the assumption that the players share the same discount rate and, thus, they are valid in our model where the players use different discount factors.

We denote, respectively, by $\alpha(u)$, $\varphi(u)$, and $V(u)$ the recursive work schedule, the recursive compensation policy, and the manager's value as a function of the worker's continuation utility, u. Notice that $u \in [0, \frac{1}{r_w}]$ as, from any point in time onward, the worker can guarantee himself a nonnegative payoff by exerting no effort, and his payoff from receiving the maximal compensation indefinitely is $\int_0^\infty 1 \cdot e^{-r_w t} dt = \frac{1}{r_w}$. The manager's value function satisfies two properties that help characterize the optimal contract:

Lemma 2. *$V(u)$ is strictly decreasing and weakly concave.*

Lemma 2 has two important consequences. First, it directly implies that the worker's expected utility from an optimal contract is zero. Second, it implies that if opportunities are concealable, then the incentive compatibility constraint at every history is binding regardless of the relative patience of the players.

Corollary 2. *Assume that opportunities are concealable. Under an optimal contract:*

1. *The worker's expected utility (at the beginning of the interaction) is zero.*
2. *All the incentive compatibility constraints are binding.*

The second part of Corollary 2 reveals a general property of the managerial style of a front-line manager that does not observe the arrival of opportunities: she engages in *perfect bookkeeping* wherein, path by path, compensation and effort discounted according to the worker's discount rate are equal. Note that this property is i) stronger than standard promise keeping which requires that *in expectation* the worker receives his promised continuation utility in every continuation history, and ii) is in contrast to the low correlation between work and compensation when opportunities are observable.

By Corollary 2, the worker's continuation utility at the beginning of the interaction is zero and after he exerts effort $\alpha(u)$ his continuation utility increases by exactly $\alpha(u)A$. Moreover, the drift in the worker's continuation utility while no opportunities arrive is

$$du = r_w u - \varphi(u). \tag{5}$$

Hence, the optimal contract is given by the solution of the HJB equation:

$$\sup_{\varphi(u),\alpha(u)\in[0,1]} \{-r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) \\ +\mu\left(\alpha(u)B + V\left(u+\alpha(u)A\right) - V(u)\right)\} = 0, \quad (HJB)$$

subject to (5), where $V'(u)$ exists almost everywhere since $V(\cdot)$ is concave (Lemma 2).

A.2.1 Recursive representation for optimal contract with observable opportunities

To ease the comparison between the optimal contracts under the cases of observable and concealable opportunities, we provide the recursive representation of the contracts characterized in Propositions 1 and 2.

Patient manager: Recall that in the contract characterized in Proposition 1, the worker exerts a constant effort of α^* on all opportunities, and following each instance of effort he receives a conditional promise of length S^*. It follows that, for this contract,

$$\alpha(u) = \alpha^* \; ; \varphi(u) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{else.} \end{cases}$$

The dynamics of u are as follows. The worker's initial continuation utility is zero, and following the exertion of effort it is set to $u_e \equiv \alpha^* A$. For $u > 0$, the drift of u is given by $du = (\mu + r_w)u - 1$. To see this, note that the recursive representation of the worker's continuation utility over ϵ units of time is

$$u = (1-e^{-\epsilon\mu})\int_0^{\epsilon}(\frac{1-e^{-r_w s}}{r_w} + e^{-r_w s}(u_e - \alpha^* A))\frac{1}{\epsilon}ds + e^{-\epsilon\mu}(\frac{1-e^{-r_w\epsilon}}{r_w} + e^{-r_w\epsilon}u_\epsilon) + O(\epsilon^2),$$

where the first term represents the case where an opportunity arrives, and the second term the case where it does not. The above representation of du is derived from this expression via standard techniques.

Note that du is less steep under observable opportunities than under concealable ones. This difference arises since it takes more time to provide a given level of utility via conditional promises (which are used in this contract) than via unconditional ones (which are used when opportunities are concealable).

Impatient manager: Recall that in the contract characterized in Proposition 2, the worker exerts full effort on all opportunities that arrive during the τ^O units of time after the arrival of the first opportunity, and receives compensation if more than τ^C units of time have passed since that event. Since $\tau^C < \tau^O$, it follows that

$$\alpha(u) = \begin{cases} 1 & \text{if } u < \frac{1}{r_w} \\ 0 & \text{if } u = \frac{1}{r_w}. \end{cases}$$

The worker's continuation utility when he starts receiving compensation is

$$\int_0^\infty e^{-r_w t} dt - A\mu \int_0^{\tau^O - \tau^C} e^{-r_w t} dt = \frac{1 - A\mu(1 - e^{-r_w(\tau^O - \tau^C)})}{r_w} \equiv u^\dagger.$$

Hence,

$$\varphi(u) = \begin{cases} 0 & \text{if } u < u^\dagger \\ 1 & \text{if } u \geq u^\dagger. \end{cases}$$

Finally, we derive the dynamics of u. Note that these dynamics depend on whether the worker is in the junior status, $u \in (0, u^\dagger)$, or intermediate status ,$u \in [u^\dagger, 1/r_w)$. In the intermediate status, the recursive representation of the worker's continuation utility over ϵ units of time is

$$u = \int_0^\epsilon e^{-r_w s}(1 - \mu A) ds + e^{-r_w \epsilon} u_\epsilon.$$

Hence, at this status, $du = r_w u - (1 - \mu A)$.

At the junior status, for $u > 0$, the worker's continuation utility over ϵ units of time is

$$u = \int_0^\epsilon e^{-r_w s}(-\mu A) ds + e^{-r_w \epsilon} u_\epsilon.$$

Hence, at this status, $du = r_w u + \mu A$.

B Proofs

Proof of Proposition 1. Providing the worker with compensation before he exerts effort is suboptimal. In addition, the worker will not receive compensation if he has not exerted effort in the last T^* units of time since providing compensation outside the maximal profitable lag is suboptimal. Hence, under an optimal contract $\varphi(h_t) = 0$ if $t - \sigma_{-1}(h_t) \geq T^*$.

The first step in the proof establishes that it is suboptimal for the manager to require effort on a given opportunity that will necessitate providing compensation after the next opportunity arrives.

Lemma B.1. *Under an optimal contract $\alpha(h_t) \leq \alpha^*$ for almost all $h_t \in H$.*

Proof. Assume by way of contradiction that under an optimal contract $\alpha(h_t) > \alpha^*$ for a set of histories with positive measure. Since $\alpha^* = 1$ if $r_w = r_m$, within this lemma we consider only the case where $r_m < r_w$.[29] Denote by ν the set of finite histories (of various lengths) such that for each $h \in \nu$, and every h' that is a (proper) prefix of h, $\alpha(h) > \alpha^*$ and $\alpha(h') \leq \alpha^*$.[30] Note that the contract reaches a history in ν with positive probability and that if $h \in \nu$ and $\tilde{h} \in \nu$, then neither history is a prefix of the other. Thus, it is sufficient to construct a profitable modification of the continuation contract conditional on an opportunity arriving at every $h \in \nu$.

Fix $h_t \in \nu$ and assume that an opportunity is available. A conditional promise of length T^* is not enough to compensate the worker for exerting an effort of more than α^* at h_t. Hence, with positive probability some of the compensation for the effort exerted at h_t must be provided after the arrival of the next opportunity. Formally, there exists a set $\nu_1(h_t)$ of continuation histories of h_t (of various lengths) with positive measure such that for every $h_s \in \nu_1(h_t)$ opportunities do not arrive in (t, s), and the worker's continuation utility conditional on an opportunity arriving at h_s is strictly greater than $A \cdot \alpha(h_s)$.

If there exists $\tilde{\nu} \subset \nu_1(h_t)$ with positive measure (conditional on reaching h_t) such that $\alpha(h_s) < 1$ for every $h_s \in \tilde{\nu}$, then postponing effort (according to the worker's discount factor) from h_t to the histories in $\tilde{\nu}$ is strictly profitable for the (patient) manager (Observation 1) and does not violate incentive compatibility.

If, on the other hand, $\alpha(h_s) = 1$ for almost all $h_s \in \nu_1(h_t)$, then for every such h_s there exists a set of continuation histories (of various lengths) with positive measure (conditional on reaching h_s) $\nu_2(h_s)$ such that for every $h_{s'} \in \nu_2(h_s)$: 1) no opportunities arrive in (s, s'), and 2) the worker's continuation utility if an opportunity arrives at $h_{s'}$ is greater than his continuation utility at h_s by at least $(1 - \alpha^*)A$. To see why such histories exist, recall that $\varphi = 0$ if no opportunity arrived in the last T^* units of time, and so the maximal worker-discounted expected compensation that can be provided between two successive opportunities is $\alpha^* A$. Hence, to compensate the worker for exerting an effort of 1 on the opportunity at h_s, at least $(1 - \alpha^*)A$ of this compensation is provided after the next opportunity arrives with positive probability.

If there exists $\tilde{\nu} \subset \nu_2(h_s)$ with positive measure (conditional on reaching h_s) such that $\alpha(h_{s'}) < 1$ for every $h_{s'} \in \tilde{\nu}$, then postponing effort from h_s to the histories in $\tilde{\nu}$ does not violate incentive compatibility. Moreover, if effort can be postponed in this manner from a subset of $\nu_1(h_t)$ with positive measure, then doing so increases the manager's value at h_t. Otherwise, for almost all $h_s \in \nu_1(h_t)$, it holds that $\alpha(h_{s'}) = 1$ for almost all $h_{s'} \in \nu_2(h_s)$. In this case, with strictly positive probability the worker's continuation utility at $h_{s'}$ is greater

[29] If $r_w = r_m$, then $T^* = \infty$, and so it is both profitable and feasible (Assumption 1) to incentivize full effort on all opportunities.

[30] A *prefix* of $\{x_t\}_{t \leq T}$ is $\{x_t\}_{t \leq S}$ where $S \leq T$, and the prefix is *proper* if $S < T$.

than his continuation utility at h_t by at least $(1-\alpha^*)2A$. Continuing in an iterative manner shows that the manager's value can be increased by postponing effort, as otherwise the worker's continuation utility increases without bound with positive probability (which cannot be the case as it is bounded from above by $\frac{1}{r_w}$). ■

The next part of the proof establishes that, under an optimal contract, the manager will require the maximal effort that can be required without accumulating debts or exceeding the maximal profitable lag.

Lemma B.2. *Under an optimal contract $\alpha(h_t) = \alpha^*$ for almost all $h_t \in H$.*

Proof. By Lemma B.1, $\alpha(h_t) \leq \alpha^*$ for almost all $h_t \in H$. Assume by way of contradiction that under an optimal contract $\alpha(h_t) < \alpha^*$ on a set of histories with positive measure. Let n be the minimal element of $\mathbb{N}$ for which the worker's effort on the n^{th} opportunity to arrive is strictly less than α^* with positive probability. Let ν denote the set of histories (of various lengths) that end at the arrival of the $n+1-$th opportunity such that the required effort on the n-th opportunity is strictly less than α^*. Finally, for $k \in \{1, \ldots, n\}$, define $NOC(k)$ to be the set of prefixes of histories in ν along which exactly k opportunities have arrived and the $k-$th opportunity arrived at most S^* units of time ago. We say that maximal compensation is provided in $NOC(k)$ if $\varphi(h) = 1$ for almost all $h \in NOC(k)$.

By construction, maximal compensation in $NOC(k)$ for all $k \in \{1, \ldots, n\}$ is sufficient to incentivize effort α^* on the first n opportunities to arrive. Hence, if maximal compensation is provided in $NOC(k)$ for all $k \in \{1, \ldots, n\}$, compensation can be decreased in $NOC(n)$ without violating any of the incentive-compatibility constraints. Otherwise, there exists a maximal $\bar{k} \leq n$ such that maximal compensation is not provided in $NOC(\bar{k})$. If $\bar{k} = n$, an improvement can be reached by increasing the required effort on the n-th opportunity and increasing the compensation within $NOC(n)$, without affecting any of the earlier or later IC constraints. Finally, if $\bar{k} < n$, since the required effort on opportunity $\bar{k}$ is α^*, some compensation for effort on that opportunity must be postponed until after future opportunities have arrived. Hence, the IC constraint at the arrival of the $\bar{k}+1$ opportunity is not binding with positive probability. Since maximal compensation is provided within $NOC(\bar{k}+1)$, the principal can reach an improvement by expediting compensation from $NOC(\bar{k}+1)$ to $NOC(\bar{k})$. ■

To conclude the proof, note that the contract described in the proposition is the only incentive compatible contract under which $\alpha(\cdot) \equiv \alpha^*$ and the worker does not receive compensation if he has not exerted effort in the last S^* units of time. ■

Proof of Proposition 2. When opportunities are observable, the manager's problem can be solved separately for each possible arrival time of the first opportunity. This is because the manager will not provide compensation prior to the first opportunity, and the worker must have a nonnegative continuation utility at all times.

Consider an arbitrary first arrival time σ_1. By Assumption 1 the manager can incentivize the worker to exert maximal effort on the opportunity at σ_1. As the manager is impatient, she will do so in an optimal contract. Moreover, as we established in the main text, the manager will use a threshold structure. Hence, the manager's objective function (conditional on σ_1) is

$$\max_{\tau^O(\sigma_1),\tau^C(\sigma_1)} e^{-r_m\sigma_1}B + \int_{\sigma_1}^{\sigma_1+\tau^O(\sigma_1)} \mu B e^{-r_m t}dt - \int_{\sigma_1+\tau^C(\sigma_1)}^{\infty} e^{-r_m t}dt$$

$$\text{s.t. } A + \int_{\sigma_1}^{\sigma_1+\tau^O(\sigma_1)} \mu A e^{-r_w t}dt = \int_{\sigma_1+\tau^C(\sigma_1)}^{\infty} e^{-r_w t}dt. \tag{6}$$

Assumption 1 implies that, in optimum, both $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are interior. To see this, note that the constraint (6) is violated if $\tau^C(\sigma_1) = \infty$ or $\tau^C(\sigma_1) = 0$. Furthermore, setting $\tau^O(\sigma_1) = 0$ implies that $\tau^C(\sigma_1) > \tau^O(\sigma_1)$, and so by slightly increasing $\tau^O(\sigma_1)$ (and decreasing $\tau^C(\sigma_1)$ to maintain incentive compatibility) the worker will exert more effort on opportunities for which he will receive compensation after he has exerted effort. As the manager is impatient, this change is profitable. Finally, setting $\tau^O(\sigma_1) = \infty$ implies that the worker continues exerting effort for an arbitrarily long period of time after he begins receiving compensation. Because the manager is impatient, slightly increasing $\tau^C(\sigma_1)$ (and decreasing $\tau^O(\sigma_1)$ to maintain incentive compatibility) is profitable.

The above discussion implies that the optimal thresholds are given by the FOC of the Lagrangian that corresponds to the above (concave) maximization problem. The first-order conditions with respect to $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are, respectively,

$$\begin{aligned} &\mu B e^{-r_m\tau^O(\sigma_1)} - \gamma(\sigma_1)e^{-r_w\tau^O(\sigma_1)}\mu A = 0 \\ &e^{-r_m\tau^C(\sigma_1)} - \gamma(\sigma_1)e^{-r_w\tau^C(\sigma_1)} = 0, \end{aligned}$$

where γ is the Lagrange multiplier. It follows that $\frac{B}{A} = e^{(r_m-r_w)(\tau^O(\sigma_1)-\tau^C(\sigma_1))}$. Hence, $\tau^O(\sigma_1) = \tau^C(\sigma_1) + K$, where $K > 0$ is a constant that does not depend on σ_1. This relation implies that the LHS of (6) is increasing in $\tau^O(\sigma_1)$ while the RHS is decreasing in $\tau^O(\sigma_1)$. Hence, there is a unique solution that does not depend on σ_1. ∎

Proof of Corollary 1. Since σ_1 is exogenous, the change in the time it takes to attain seniority (and the worker's expected effort) can be measured by the change in τ^O.

From Equation (4) it follows that

$$\tau^C = \tau^O - \frac{\log(B/A)}{r_m - r_w}.$$

Plugging this expression into (3) and solving, yields that

$$\tau^O = \frac{1}{r_w} \log \left(\frac{\left(\frac{B}{A}\right)^{\frac{r_w}{r_m - r_w}} + A\mu}{A(\mu + r_w)} \right).$$

This expression is decreasing in r_m and A, and increasing in B. ■

In the analysis that follows we use a technical lemma that states that for every incentive-compatible contract for which $u > 0$, there exists another incentive-compatible contract that implements the same work schedule via a compensation policy that is (pointwise) weakly lower, and strictly lower on a set of histories with strictly positive measure.

Lemma B.3. *Assume that opportunities are concealable. Moreover, assume that under an incentive-compatible contract the continuation contract at h_t, $\langle \alpha(\cdot), \varphi(\cdot) \rangle$, is such that the worker's continuation utility is $u > 0$. There exists $\tilde{u} < u$ such that for every $u' \in (\tilde{u}, u)$ there exists an incentive-compatible contract $\langle \alpha'(\cdot), \varphi'(\cdot) \rangle$ that provides the worker with a continuation value of u', and for which $\varphi'(h_s) \leq \varphi(h_s)$ and $\alpha'(h_s) = \alpha(h_s)$ at every h_s that is a continuation of h_t.*

Proof of Lemma B.3. Consider an incentive-compatible contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ under which the worker's continuation utility is $u > 0$ and normalize the current time to zero. If the worker's expected discounted compensation along the histories in which there are no binding incentive compatibility constraints is positive, then the worker's continuation utility at time zero can be decreased by reducing his compensation along those histories. If this is not the case, then the worker's compensation is almost surely zero prior to a binding incentive compatibility constraint. Hence, concealing all opportunities is a best response for the worker. However, as $\varphi = 0$ before the worker exerts effort, this best response provides a payoff of $0 < u$. ■

Proof of Lemma 2. Let $\langle \hat{\alpha}(\cdot), \hat{\varphi}(\cdot) \rangle$ be an incentive-compatible (continuation) contract under which the worker's expected discounted payoff is $u > 0$. From Lemma B.3 it follows that there exists $\tilde{u} < u$ such that if the worker's continuation utility is in $(\tilde{u}, u)$, then the manager can induce the same work schedule for a lower compensation. Thus, there is an open neighborhood to the left of u for which the manager can obtain a value strictly greater than $V(u)$. The strict monotonicity of $V(\cdot)$ follows from the fact that the choice of u is arbitrary.

Next, we show that $V(u)$ is weakly concave. Let $u_1 < u_2$ such that $u_1, u_2 \in [0, \frac{1}{r_w}]$. One (unnatural) way the manager can deliver a promise of $\frac{u_1+u_2}{2}$ is to fictitiously split all opportunities and compensation in half and create two (perfectly correlated) fictitious worlds, each of which contains half of the compensation flow and half of each opportunity. Observe that scaling all payoffs by $\frac{1}{2}$ multiplies the players' discounted payoffs by half in any contract, and so any optimal contract in the original non-scaled world is also an optimal contract in each fictitious world. The manager can then use the continuation contract that supports $V(u_1)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_1}{2}$ in fictitious world 1, and the continuation contract that supports $V(u_2)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_2}{2}$ in fictitious world 2. Since using these continuation contracts cannot increase the manager's payoff, $V(\frac{1}{2}(u_1+u_2)) \geq \frac{1}{2}V(u_1)+\frac{1}{2}V(u_2)$, which establishes the concavity of $V(\cdot)$. ■

Proof of Lemma 1. We establish the proposition separately for the case where the manager is patient and the case where she is impatient. In each case, we first derive one part of the optimal contract (the work schedule when the manager is impatient, and the compensation policy when she is patient), and then use the HJB equation to derive the optimal contract and show that it is, generically, unique.

Case 1: impatient manager ($r_m > r_w$). The first step of the proof is to show that under any optimal contract the work schedule is $\overline{\alpha}(u) = \min\{1, \frac{1/r_w - u}{A}\}$.

Assume by way of contradiction that $\alpha(\hat{u}) < \min\{1, \frac{1/r_w-\hat{u}}{A}\}$ for some $\hat{u} \in [0, \frac{1}{r_w}]$. Suppose that the current state is $\hat{u}$ and that an opportunity is currently available. If the worker's expected discounted future effort is zero, then it is both possible and profitable to increase α and increase the worker's compensation in the future without changing his continuation utility. If, on the other hand, the worker's expected discounted future effort is positive, then the manager can expedite effort (in the non-recursive representation of the contract) without altering the compensation policy. By Observation 1 it is profitable for the manager to expedite effort, and, since she does so according to the worker's discount factor, it also relaxes all incentive-compatibility constraints.

The above claim enables us to simplify the HJB equation to

$$(HJB_{imp}) \qquad \sup_{\varphi(u)\in[0,1]} \{-r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) \\ + \mu\left(\overline{\alpha}(u)B + V\left(u+\overline{\alpha}(u)A\right) - V(u)\right)\} = 0.$$

From the FOC of (HJB_{imp}) it follows that $\varphi(u) = 1$ if $V'(u) < -1$ and that $\varphi(u) = 0$ if $V'(u) > -1$. Since $V(\cdot)$ is weakly concave (Lemma 2), there is a (possibly degenerate) interval $I \subset [0, \frac{1}{r_w}]$ over which $V'(u) = -1$. Note that for any $u^\dagger \in I$ the compensation policy given by

$$\varphi_{u^\dagger}(u) = \begin{cases} 1 & \text{if } u > u^\dagger \\ r_w u^\dagger & \text{if } u = u^\dagger \\ 0 & \text{if } u < u^\dagger \end{cases}$$

is an optimal compensation policy.

Next, we show that, generically, I is degenerate. Fix A, B, μ, and r_w, and let $I(r_m)$ denote the interval (or point) for which $V'(\cdot) = -1$ for a manager with discount rate r_m. To establish the generic uniqueness of optimal contracts we will show that if there exist $\tilde{r}_m < \hat{r}_m$ such that both $I(\tilde{r}_m)$ and $I(\hat{r}_m)$ have a positive measure, then these intervals have a disjoint interior. The result then follows from a standard argument about the density of rational numbers.

Assume by way of contradiction that for some $\tilde{r}_m < \hat{r}_m$, the set $I^* \equiv I(\tilde{r}_m) \cap I(\hat{r}_m)$ has a nonempty interior. Select u^* and $\epsilon > 0$ such that $u^*, u^* - \epsilon \in int(I^*)$.

Fix the optimal compensation policy $\varphi_{u^*}(\cdot)$, and let $\Delta\varphi_s = \mathbb{E}(\varphi_s | u_0 = u^* - \epsilon) - \mathbb{E}(\varphi_s | u_0 = u^*)$ and $\Delta\alpha_s = E(\alpha_s | u_0 = u^* - \epsilon) - \mathbb{E}(\alpha_s | u_0 = u^*)$. Since the chosen compensation policy, $\varphi_{u^*}(\cdot)$, is optimal, we have

$$V(u^* - \epsilon) - V(u^*) = \int_0^\infty e^{-r_m s}(\mu B \Delta\alpha_s - \Delta\varphi_s) ds. \tag{7}$$

As path by path u_s is monotone in u_0, and the work schedule and compensation policies are threshold policies, it follows that $\mu B \Delta\alpha_s - \Delta\varphi_s \geq 0$ for all s, with a strict inequality on a set of times with strictly positive measure. Hence, differentiating the RHS of (7) with respect to r_m shows that the RHS of (7) is decreasing in r_m. However, as $V'(u) = -1$ for all $u \in I^*$ it holds that $V(u^* - \epsilon) - V(u^*) = \epsilon$. Hence, (7) can be satisfied for at most one r_m and so the interior of I^* is empty.

It follows that if the interior of $I(r_m)$ is nonempty, then the compensation policies corresponding to elements of $int(I(r_m))$ are suboptimal for any $r'_m \neq r_m$. Thus, we can index every r_m for which the optimal contract is not unique by a rational number from the interior of $I(r_m)$. Hence, the set of manager-discount factors for which the optimal contract is not unique is (at most) countable.

Case 2: weakly patient manager ($r_m \leq r_w$). We begin by showing that the optimal compensation policy is

$$\overline{\varphi}(u) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{if } u = 0. \end{cases}$$

To do so, we show that if $u(h_t) > 0$ then in an optimal contract $\varphi(h_t) = 1$ in the next dt units of time conditional on no opportunity arriving in that interval. If $u(h_t) = \frac{1}{r_w}$, this is immediate. Assume by way of contradiction that $u(h_t) \in (0, \frac{1}{r_w})$, and that the worker does not receive the maximal compensation with probability 1 in the next dt units of time conditional on no opportunity arriving in that interval. By arguments analogous to those used in the proof of Lemma B.3, it is possible to expedite compensation into the interval $[t, t + dt]$ (conditional on no opportunity arriving) without violating the incentive compatibility constraints in any history that is a continuation of h_t. If $r_m < r_w$, then expediting compensation is profitable. If, on the other hand, $r_m = r_w$, expediting compensation is profitable as it enables the manager to require more effort in the future.

The above claim enables us to simplify the HJB equation to

$$(HJB_p) \quad V(u) = \sup_{\alpha(u)\in[0,\min\{1,\frac{\frac{1}{r_w}-u}{A}\}]} \{-r_m V(u) + V'(u)[r_w u - \overline{\varphi}(u)] - \overline{\varphi}(u) \\ + \mu\left(\alpha(u)B + V\left(u+\alpha(u)A\right) - V(u)\right)\} = 0.$$

The FOC of HJB_p with respect to $\alpha(u)$ is $B + V'\left(u+\alpha(u)A\right)A = 0$. Thus, to show that there is a unique optimal contract, it is sufficient to show that $V(\cdot)$ is strictly concave. To do so, we return to the construction used to establish the weak concavity in Lemma 2 and further the analysis by utilizing the structure of $\overline{\varphi}(\cdot)$.

In the event with strictly positive probability where no opportunity arrives for T units of time, where T solves $u_1 = \frac{1-e^{-r_w T}}{r_w}$, the worker's continuation utility in fictitious world 1 is zero while his continuation utility in fictitious world 2 is strictly positive. At this point, the manager can temporarily merge the two fictitious worlds and expedite compensation in world 2 by using the compensation from world 1. By Observation 1 this modification is profitable for a strictly patient manager and, hence, $V(u)$ is strictly concave if $r_m < r_w$. If $r_w = r_m$, then merging the fictitious worlds increases the discounted effort the worker can be incentivized to exert in the future, which also strictly increases the manager's profit. ■

Proof of Proposition 3. In Lemma 1 we established that if $r_m \leq r_w$, then $u^C = 0$ and there is a unique optimal contract. Setting $u^O = 0$ implies that the worker never exerts effort, which, in turn, implies that the manager's value is zero; an outcome that is clearly suboptimal. If the manager is patient, the setting $u^O > u(T^*)$ is suboptimal as when the worker exerts effort that increases his continuation utility to above $u(T^*)$, the manager will have promised compensation more than T^* units of time in the future. By the definition of T^*, this reduces the manager's value. Thus, to establish the proposition it remains to show that setting $u^O = u(T^*)$ is also suboptimal.

Assume towards a contradiction that under the optimal contract $u^O = u(T^*)$, and let τ be the first time at which the worker's continuation utility reaches u^O. Since $u^C = 0$, an opportunity must be available at τ. If the first opportunity to arrive after τ arrives sufficiently quickly, then the worker will not exert full effort on that opportunity. Choose $s \in (0, T^*)$ such that if the first opportunity to arrive after τ arrives at $\tau + s$, then the worker's effort on that opportunity is strictly less than 1, and denote the probability that the first opportunity after τ arrives in $[\tau + s/2, \tau + s]$ by $p > 0$. For $\epsilon > 0$ define the following (non-Markovian) modification of the contract at time τ: reduce the worker's required effort at τ by $u(\epsilon) = \left(\int_{T^*-\epsilon}^{T^*} e^{-r_w t} dt\right) / A$ and refrain from promising compensation in $[\tau + T^* - \epsilon, \tau + T^*]$. Then, use the freed compensation in $[\tau + T^* - \epsilon, \tau + T^*]$ to increase the worker's effort on the first opportunity to arrive after τ, if it arrives in $[\tau + s/2, \tau + s]$. If it does not arrive in this interval, never provide compensation in $[\tau + T^* - \epsilon, \tau + T^*]$.

The manager's loss of value at time τ from this deviation is bounded from above by the manager's value from the worker exerting an effort of $u(\epsilon)$ and receiving compensation in $T^* - \epsilon$ units of time. This upper bound is given by

$$\frac{e^{-r_w T^*}\left(e^{r_w \epsilon} - 1\right)\left(\frac{B}{A} - e^{(r_w - r_m)(T^* - \epsilon)}\right)}{r_w} = e^{-r_w T^*}\left(\frac{B}{A} - e^{(r_w - r_m)T^*}\right)\epsilon + O\left(\epsilon^2\right) = O\left(\epsilon^2\right),$$

since $\left(\frac{B}{A} - e^{(r_w - r_m)T^*}\right) = 0$ by the definition of T^*.

If ϵ is small enough, then the worker will not exert full effort on the first opportunity in $[\tau + s/2, \tau + s]$ after this deviation. In this case, the gain from the additional effort on that opportunity is bounded from below by the product of p and the manager's time τ discounted gain should the next opportunity arrive at $\tau + s/2$ (Observation 1). This bound is given by

$$p\left(\frac{B\left(e^{r_w \epsilon} - 1\right) e^{r_w (s/2 - T^*) - r_m s/2}}{A r_w} - \frac{e^{-r_m T^*}\left(e^{r_m \epsilon} - 1\right)}{r_m}\right)$$
$$= \epsilon p\left(e^{(r_w - r_m)s/2} - 1\right)\left(\frac{B}{A}\right)^{\frac{r_m}{r_m - r_w}} + O\left(\epsilon^2\right).$$

Since $r_w > r_m$ and $s > 0$, the first-order term in the approximation is strictly positive. Hence, for sufficiently small ϵ, the modification is profitable. ∎

Proof of Proposition 4. In Lemma 1 we established that $u^O = \frac{1}{r_w}$ if $r_m > r_w$. Furthermore, when $r_m = r_w$ the manager will use the same threshold, to avoid wasting her limited capacity to compensate the worker. ∎

Proof of Proposition 5. We begin by showing that $u^C = \frac{1}{r_w}$ is an optimal threshold if and only if $r_m \geq r_m'' \equiv r_w + (\frac{B}{A} - 1)\mu$. An upper bound on the manager's marginal net gain from providing the worker with a util at present is attained by the worker exerting a worker-discounted util on the first opportunity to arrive. Note that if $u^C > \frac{1}{r_w} - A$ and $u = u^C$, then this upper bound is attained. The value of this upper bound is given by

$$\int_0^\infty \mu e^{-\mu t} \frac{B}{A} e^{(r_w - r_m)t} dt - 1 = \frac{\mu}{\mu + r_m - r_w} \frac{B}{A} - 1.$$

It is straightforward to show that this expression is positive if and only if $r_m \leq r_m''$. It follows that if $r_m > r''_m$, then providing compensation while $u < \frac{1}{r_w}$ is suboptimal. On the other hand, if $r_m < r''_m$, then it is strictly suboptimal to set $u^C = \frac{1}{r_w}$ since for such discount rates it is profitable to compensate the worker when $u > \frac{1}{r_w} - A$.

Next, we establish that there exists $r'_m > r_w$ for which $u^C = 0$. Let τ denote the random arrival time of the first opportunity on which the worker will not exert full effort under the contract with $u^C = 0$. Note that under any other contract, the worker will not exert full effort (weakly) earlier. It follows that the marginal value from decreasing the worker's continuation utility is at least $E(e^{-r_m \tau})(\frac{B}{A} - 1) > 0$. If $r_w = r_m$ the timing of compensation does not affect the manager's cost of providing compensation. This, in turn, implies that when $r_w = r_m$ the manager's marginal gain from providing compensation is at least $E(e^{-r_m \tau})(\frac{B}{A} - 1) > 0$ for all u. By the continuity of the manager's payoff in r_m, it follows that there exists $r'_m > r_w$ such that $u^C = 0$ is optimal for any $r_m \leq r'_m$.

The result follows from the following lemma that states that u^C is increasing in r_m.

Lemma B.4. *Fix A, B, μ, and r_w and assume that the manager is impatient. If u^C is an optimal threshold for r_m and $\tilde{u}^C$ is an optimal threshold for $\tilde{r}_m > r_m$, then $\tilde{u}^C \geq u^C$.*

Proof. Consider two contracts, $\mathcal{C}_1$ and $\mathcal{C}_2$, that differ in their compensation threshold, $u_2^C > u_1^C$. Denote by $u_{t,i}$ the worker's continuation utility at time t under contract $\mathcal{C}_i$ and let $\overline{\tau} = \sup\{t : u_{t,2} \leq u_2^C\}$. That is, $\overline{\tau}$ is the latest time at which the worker's continuation utility is lower than u_2^C under $\mathcal{C}_2$. Note that $\overline{\tau}$ is finite (almost surely) by the Borel–Cantelli lemma.

Observe that since $u_2^C > u_1^C$, it holds that $u_{t,1} \leq u_{t,2}$ for all t. This implies that the worker exerts the same effort on every opportunity that arrives before $\overline{\tau}$ under both $\mathcal{C}_1$ and $\mathcal{C}_2$. In addition, it implies that the compensation for effort exerted on those opportunities is postponed under $\mathcal{C}_2$ relative to $\mathcal{C}_1$. Denote by $g(r_m)$ the gain from this postponement as a function of r_m. Moreover, from $\overline{\tau}$ onwards, under $\mathcal{C}_2$ the worker exerts weakly less effort than he does under $\mathcal{C}_1$, and he receives a compensation of $\varphi_t = 1$ at all times. Let $d(r_m)$ denote the difference in the time-zero discounted continuation value from $\overline{\tau}$ onward between $\mathcal{C}_1$ and $\mathcal{C}_2$. The net gain from replacing $\mathcal{C}_1$ with $\mathcal{C}_2$ is $g(r_m) - d(r_m)$. Note that $g(\cdot)$ is increasing and $d(\cdot)$ is decreasing. Hence, whenever $g(r_m) \geq d(r_m)$, we also have $g(r'_m) > d(r'_m)$ for all $r'_m > r_m$, which establishes the monotonicity of u^C. ∎

To see why the lemma implies that $r'_m < r''_m$, observe that when $r_m = r''_m$ there exist histories for which under the optimal contract the worker exerts effort on only the first opportunity, whereas, when $r_m = r'_m$ in every history the worker's discounted cost of effort must equal his discounted compensation, which, by Assumption 1, is greater than the cost of effort on a single opportunity. ∎

Proof of Proposition 6. First, we explain why in the special case where $r_w = r_m$ and opportunities are observable, the lumpiness of opportunities does not impact the manager's value, so long as Assumption 1 continues to hold. This follows from three observations. First, since $r_w = r_m$, Assumption 1 implies that, under the contract characterized in Proposition 1, the worker exerts full effort on all opportunities. Second, recall that the worker's expected utility from that contract is zero. Finally, $r_w = r_m$ also implies that the timing of compensation does not affect the manager's cost of providing compensation.

Next, we prove that, in all other cases, lumpiness is detrimental.

Part A: concealable opportunities. To establish this part it is convenient to think of each opportunity as being composed of many "small opportunities." We will show that making opportunities lumpier in the actual model is equivalent to a certain change in the correlation structure of these small opportunities.

First, we consider the case where opportunities become lumpier by a rational factor. Assume that opportunities become lumpier by $\frac{N}{M} > 1$, where $N, M \in \mathbb{N}$. We analyze this change by considering an auxiliary representation of the model in which there are $M \times N$ Poisson processes, each with an arrival rate of $\frac{\mu}{N}$, that govern the arrival of the small opportunities. Moreover, we assume that the payoff from exerting full effort on each small opportunity is $(-\frac{A}{M}, \frac{B}{M})$. Both the original and the lumpy versions of the model correspond to appropriately defined correlation structures of the arrival processes in the auxiliary representation.

To map the auxiliary representation to the original model, divide the Poisson processes into N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. To see why this correlation structure represents the original model, note that when a group of opportunities is available the payoff vector from exerting full effort on all opportunities in the group is $M \times (-\frac{A}{M}, \frac{B}{M}) = (-A, B)$, which is exactly the payoff vector from exerting full effort on a single opportunity in the original model. Moreover, the probability that a given group arrives in an (infinitesimal) interval dt is $\frac{\mu}{N} dt$, and since the groups are independent, the probability that some group arrives in that interval is $\sum_{i=1}^{N} \frac{\mu}{N} dt = \mu dt$.

To map the auxiliary representation to the lumpy model, divide the processes into M groups of N processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. For this correlation structure, the payoff from exerting full effort on all opportunities in a group is $N \times (-\frac{A}{M}, \frac{B}{M}) = (-\frac{N}{M}A, \frac{N}{M}B)$ and the probability that some group of opportunities is available in an interval dt is $\frac{\mu}{N/M} dt$.

Next, we construct a sequence of modifications that begins with the lumpy representation and ends with the original one, such that the first two modifications do not impact the manager's value, and the third modification strictly increases it.

Consider the lumpy representation. The first modification utilizes the idea of splitting the interaction into fictitious worlds introduced in Lemma 2. In particular, we consider N fictitious worlds, denoted by $(1, \ldots, N)$, that each contain $\frac{1}{N}$ of the flow compensation and M arrival processes, one from each group. We denote the processes in fictitious world n by $(P_1^n, \ldots, P_M^n)$. Note that the arrival processes within each fictitious world are independent of one another, and so each fictitious world is a scaled version of the lumpy representation. Hence, by the argument used in Lemma 2, the sum of the manager's values across all fictitious worlds is equal to her value in the lumpy representation.

The second modification is to the correlation structure of the processes across fictitious worlds. Changing the correlation structure of two arrival processes that are assigned to *different* fictitious worlds does not impact the manager's value in either fictitious world. Hence, so long as the processes within each fictitious world are independent of one another, the correlation across fictitious worlds is immaterial. Thus, we can replace the original correlation structure with the following correlation structure: P_m^n and $P_{m'}^{n'}$ are perfectly correlated if $m - n \overset{(\text{mod} N)}{=} m' - n'$, and independent otherwise. This modification maintains the independence of the processes within each fictitious world. To see this note that for any $n \leq N$ and $m, m' \leq M$, such that $m' \neq m$, the fact that $M < N$ implies that $m - n \overset{(\text{mod} N)}{\neq} m' - n$.

The third modification is to re-merge the fictitious worlds. Note that under the correlation structure created in the second modification, there are N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. Thus, merging these fictitious worlds creates the auxiliary representation of the original model. Regardless of the relative patience, there are instances in which the manager benefits from merging two fictitious worlds: if $r_m < r_w$ this occurs when in fictitious world i the worker's continuation is positive while in fictitious world j it is zero, whereas if $r_m \geq r_w$ this occurs when in fictitious world i an opportunity is (partially) forgone while in fictitious world j the worker's continuation utility is below its maximal level. It follows that the sum of the manager's values across all fictitious worlds is strictly less than her value in the original model.

Finally, consider the case where $\lambda \notin \mathbb{Q}$. The manager's value is continuous in λ as i) the distribution of arrival times is continuous in λ, and ii) if opportunities are made slightly lumpier then the manager can instruct the worker to incur the same *cost* of effort on every opportunity that arrives by using the same compensation policy. As the set of rational numbers is dense, this establishes the proposition.

Part B: observable opportunities. We establish this part separately for the case where the manager is patient and the case for which she is impatient.

Case 1: patient manager. We start this proof by establishing the comparative statics of α^* with respect to λ. If the worker's expected utility from a conditional promise of length T^* is strictly greater than A, then the worker exerts full effort on all opportunities. Moreover, this will remain the case if opportunities become marginally lumpier. Thus, we focus on the case where the worker's expected utility from such a promise is at most A.

The worker's expected utility from a conditional promise of length T^*, as a function of the parameter λ, is $\frac{1}{r_w+\mu/\lambda}(1-e^{-T^*(r_w+\mu/\lambda)})$. The marginal increase in the value of such a promise from making opportunities lumpier (i.e., the derivative of this value with regard to λ, evaluated at $\lambda = 1$) is $\frac{\mu}{(\mu+r_w)^2}(1-e^{-T^*(\mu+r_w)}T^*(\mu+r_w))$. Thus, to establish that α^* is decreasing in λ it is enough to show that making opportunities marginally lumpier has a larger impact on the cost of exerting the required effort than on the value of a conditional promise of length T^*.

The worker's cost of exerting effort is $\lambda\alpha^* A$, and so the marginal effect of making opportunities lumpier (i.e., the derivative of this cost with regard to λ, evaluated at $\lambda = 1$) is $\alpha^* A$. Under the assumption that a conditional promise of length T^* does not provide excess compensation, it holds that $A\alpha^* = \frac{1-e^{-T^*(\mu+r_w)}}{\mu+r_w}$. Hence, it is sufficient to show that

$$\frac{\mu}{(\mu+r_w)^2}(1-e^{-T^*(\mu+r_w)}T^*(\mu+r_w)) < \frac{1-e^{-T^*(\mu+r_w)}}{\mu+r_w}.$$

When $\mu + r_w$ is kept constant, this inequality is harder to satisfy for higher values of μ. Thus, it is sufficient to show that it holds for $r_w = 0$, i.e., to show that

$$1-e^{-\mu T^*}(1+\mu T^*) < 1-e^{-\mu T^*},$$

which holds for any $\mu T^* > 0$. Note that the above calculation does not depend on the value of T^*. Hence, the same calculation shows that when it is possible to induce full effort, S^* increases when opportunities become marginally lumpier.

Next, we show that lumpier opportunities are detrimental for the manager. If $\alpha^* = 1$, this is an immediate consequence of S^* being increasing in λ. Assume that $\alpha^* < 1$ and let $f(r,\lambda) = \frac{1-e^{-T^*(r+\frac{\mu}{\lambda})}}{r\lambda+\mu}$ denote the $r-$discounted compensation that is provided via a conditional promise of length T^* as a function of λ. Note that the average cost of providing a util of compensation is $\frac{f(r_m,\lambda)}{f(r_w,\lambda)}$. The cross-derivative of $f(r,\lambda)$ evaluated at $\lambda = 1$ is

$$\frac{\partial^2 f(r,\lambda)}{\partial r \partial \lambda}|_{\lambda=1} = \frac{\mu e^{-T(\mu+r)}\left(T^*(\mu+r)(T^*(\mu+r)+2) - 2e^{T^*(\mu+r)} + 2\right)}{(\mu+r)^3}.$$

The sign of this cross-derivative is the sign of $x(x+2)+2-2e^x$, where $x = T^*(r+\mu)$. As this sign is negative, the cross-derivative is negative. As $f(r,\lambda)$ is positive and decreasing in λ, it follows that the average cost of compensating the worker for his effort is increasing in λ (recall that $r_w > r_m$). As the worker's total effort is also decreasing in λ, we can conclude that making opportunities lumpier reduces the manager's value.

Case 2: Impatient manager. Solving the optimal thresholds for the contract characterized in Proposition 2, τ^O, τ^W, as a function of λ gives

$$\tau^O(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w-r_m}} + 1\right) - \ln(A(\mu+\lambda r_w))}{r_w} + \frac{\ln\left(\frac{B}{A}\right)}{r_m - r_w},$$
$$\tau^C(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w-r_m}} + 1\right) - \ln(A(\mu+\lambda r_w))}{r_w}.$$

Recall that the manager's value is

$$\mathbb{E}\left(e^{-r_m\sigma_1}\left(B\lambda + \frac{B\mu(1-e^{-r_m\tau^O(\lambda)})}{r_m} - \frac{e^{-r_m\tau^C(\lambda)}}{r_m}\right)\right).$$

Plugging in the expressions for the optimal thresholds, differentiating with respect to λ, and evaluating at $\lambda = 1$ shows that making opportunities marginally lumpier changes the manager's value by

$$(r_w - r_m)\frac{\mu\left(\frac{B}{A}\right)^{-\frac{r_m}{r_w-r_m}}\left(B\mu\left(\frac{B}{A}\right)^{\frac{r_m}{r_w-r_m}} + 1\right)e^{-r_m\left(\frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w-r_m}}+1\right)-\ln(A(\mu+r_w))}{r_w} + \frac{\ln\left(\frac{B}{A}\right)}{r_m-r_w}\right)}}{(\mu+r_w)(\mu+r_m)^2}.$$

This expression is negative as $(r_w - r_m) < 0$ and the fraction is clearly positive. ∎

Proof of Proposition 7. Since the optimal contract characterized in Proposition 1 is stationary, it is sufficient to maximize the organization's expected profit from each opportunity. The assumption that $r_m \in [r_o, \underline{r}]$ implies that the manager will provide a conditional promise of length $T^* = \frac{\log(B/A)}{r_w - r_m}$. From Proposition 1 we have that, in such cases,

$$\alpha^*(r_m) = \frac{1}{A}\int_0^{T^*} e^{-t(r_w+\mu)}dt = \frac{1 - \left(\frac{B}{A}\right)^{\frac{\mu+r_w}{r_m-r_w}}}{A(\mu+r_w)}.$$

The organization's cost of providing a conditional promise of length T^* is

$$\kappa(r_m) = \theta \int_0^{T^*} e^{-t(r_o+\mu)} dt = \frac{\theta \left(1 - \left(\frac{B}{A}\right)^{\frac{\mu+r_o}{r_m-r_w}}\right)}{\mu + r_o}.$$

Hence, the organization's objective is to maximize $B\alpha^*(r_m) - \kappa(r_m)$. The First Order Condition of this objective is given by

$$\theta = \left(\frac{B}{A}\right)^{1-\frac{r_w-r_o}{r_w-r_m}}.$$

If $\theta = 1$, then the RHS must also be one, and so $r_m^* = r_0$. The RHS of this FOC is decreasing in r_m and converges to zero as r_m approaches r_w. Hence, this condition will still have an interior solution for $\theta \in (\underline{\theta}, 1)$, where

$$\underline{\theta} = \left(\frac{B}{A}\right)^{1-\frac{r_w-r_o}{r_w-\underline{r}}}.$$

For $\theta \leq \underline{\theta}$, the FOC will hold at the corner solution of $r_m^* = \underline{r}$. ■