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*Information, Liquidity and Asset Trading in a
Random Matching Game*

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Information, Liquidity and Asset Trading in a Random Matching Game *

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Abstract

This paper shows that the extent of information available in the market about a security can crucially affect its liquidity. Following Kiyotaki and Wright (1989), exchange is modeled as a sequential random matching game. Agents who want to consume relatively early optimally choose to exchange their initial assets for a new asset that has lower expected payoff but is more liquid in subsequent trading. This incentive to pay a premium for liquidity is further analyzed by allowing agents to credibly disclose more about their assets at a cost. Alternative intermediated mechanisms for increasing liquidity such as certification of assets by an investment bank, pooling assets in mutual funds, and introducing commercial banks are also discussed.

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1 Introduction

A fundamental question in finance is what makes some assets more liquid than other assets. The notion of liquidity relates to how easily a financial security can be traded. Our hypothesis is that the extent to which market participants are informed about the returns of different assets will affect their tradeability. This paper studies that hypothesis in the context of a model with heterogeneous assets and asymmetrically informed agents.

Kiyotaki and Wright (1989) use a random matching model to study the frictions in the exchange of commodities that arise from the lack of double coincidence of needs. In a similar context, Williamson and Wright (1991), analyze the consequences for trading of private information about the quality of commodities. As in these models, ours is one where mutually beneficial trades can take place but trades are made difficult by the imperfect information agents have about asset returns. We derive demand for assets that is not solely based on the actual pecuniary payoffs they promise but also on their ability to reduce the negative impact of frictions in the trading environment.¹ If assets differ in the number of traders that are informed about their returns, one may expect that the ones about which information is more widespread will be easier to trade. Such assets should command a premium in the market since they are more tradeable. This is the notion of liquidity that will be analyzed here.

A three period world is studied where heterogeneity of preferences, captured by different rates of time preference, provides an incentive for trades to take place. Risk neutral agents are endowed with assets in the initial period. The assets are heterogeneous in two respects; the expected payoffs differs across assets, and the extent of information available about each asset varies. It is useful to think of the second characteristic as a measure of how easily the expected payoffs of an asset can be verified. A security which is hard to verify could for example be one which is in small supply, is traded infrequently, or for which not much information is disclosed in financial reports and through public announcements. Opportunities for trades arise as a result of random matching. The owner of an asset knows its type, but the trading partner does not necessarily know its expected payoff. To the extent that agents are reluctant to trade unless they know the quality of an asset, this feature introduces a friction in the trading environment.

Liquidity *per se* is not a precisely defined concept in the literature. The basic idea is that a liquid asset can be bought or sold at low cost without substantial delay. Potential ways to measure these costs are direct transactions costs (fees and bid-ask spreads), indirect costs such as price impact of (large) trades, and ease of finding someone to take the other side.² Liquidity has often been introduced in models of financial markets via exogenously specified transaction costs. Williamson (1991) is a good example of a model that emphasizes the relationship between (fixed) transactions costs, liquidity, and market

¹In fact, one equilibrium in Kiyotaki and Wright (1989) bears some resemblance to the notion of liquidity that is emphasized in this paper. The authors call an equilibrium a speculative equilibrium if agents sometimes trade a lower- for a higher-storage-cost commodity because they believe this to be more marketable in the future.

²Kyle (1985) suggests that market liquidity can be measured by the "'tightness' (the cost of turning around a position over a short period of time), 'depth' (the size of an order flow innovation required to change prices a given amount), and 'resiliency' (the speed with which prices recover from a random, uninformative shock)" of a market (p.1316).

participation. To specify transactions costs in this manner is a useful shortcut to study some implications that liquidity has for asset returns and trading volume in financial markets. It does, however, beg the question of what determines the transactions costs in the first place.

The literature on rational expectations and asymmetric information provides a rationale for securities being associated with different transactions costs measured in terms of bid-ask spreads.³ Due to the adverse selection problem, the bid-ask spread is larger for assets where there are more informed traders in the market. Although our model is far simpler in terms of its institutional structure than models in the market microstructure literature, we share its focus on the role that *asymmetric* information has for increasing transactions costs. We emphasize, instead, the role that the distribution of information about an asset has for the ability of executing trades. This probability is generally larger for assets whose future payoffs the market is more informed about. For thinly traded securities there simply might not be someone available who is willing to take the other side.

We provide conditions under which information always makes an asset more tradeable. As a consequence, agents are willing to hold these assets even if they have a lower rate of return. In a series of papers, Amihud and Mendelson (1986, 1989, 1991) provide empirical support for the hypothesis that liquidity affects asset pricing of both stocks and Treasury securities. They argue that one can approximate the value loss due to illiquidity by assessing the discounted value of expected future transactions costs. More illiquid assets should require a higher expected return. In the most recent paper, Amihud and Mendelson (1991), Treasury bills and Treasury notes with the same time to maturity are compared. Treasury bills have lower transactions costs,⁴ and should based on this have a lower yield to maturity than the corresponding notes. Interestingly, the evidence shows that notes offer a higher yield to maturity even after controlling for bid-ask spreads and brokerage fees. Amihud and Mendelson interpret this as implying that "investors are willing to pay a yield concession for the *option* to liquidate their holdings before maturity at lower costs." (p.1417) It is this kind of option that gives additional value to assets in our model. Specifically, since impatient agents might want to sell assets before maturity they value an asset more highly that can be sold more easily in the second period. In our world, this corresponds to an asset whose expected return is better known.

The decentralized trading environment described above makes trades quite difficult. Two ways of potentially improving on the trading technology are explored here. The first one involves allowing agents to choose to invest in verifiability *ex ante*, thus improving the verifiability of their assets at some cost per unit of improvement. The second one consists of introducing three more centralized trading mechanisms which might serve to circumvent both the problems due to asymmetric information and the matching technology. These are investment banks, mutual funds, and commercial banks. In the case of the latter two institutions, a decentralized market will in general remain in existence and only a

³A summary of the literature is given in Admati (1989). Diamond and Verrecchia (1991) is a recent paper that addresses the issue of disclosure and liquidity in the context of a market microstructure model.

⁴The authors cite a brokerage fee for bills between \$12.5 and \$25 per \$1 million, compared to \$78.125 per \$1 million for notes, and typical bid-ask spread on bills of an order of 1/128 of a point compared with 1/32 on notes (both per \$100 face value).

subset of the securities participate in the centralized market. The choice between search and centralized trade was explored by Pagano (1989). It was shown there that traders who expect to execute large trades will choose the decentralized market. We instead characterize which types of assets will be traded in a decentralized fashion and which ones will participate in more centralized exchange.

The paper proceeds as follows. Section 2 describes the economic environment. Equilibria where more information makes assets more liquid are derived and analyzed in Section 3. Section 4 illustrates equilibria where there are incentives to hide the quality of assets. Agents are given the opportunity to invest in verifiability, which captures the notion of improving financial reporting and disclosure about the asset, in Section 5. We address alternative intermediated market mechanisms, such as investment banks, mutual funds, and commercial banks in Section 6. Summary and conclusions are given in Section 7.

2 Economic Environment

Consider the following three period economy. The market has a continuum of three period lived agents with unit mass. There are two types of agents, patient and impatient, who are distinguished by their rate of time discount. Preferences for an agent of type i are given by

$$E_0 \left\{ \sum_{t=2}^3 \beta_i^t C_t^i \right\}, \quad (1)$$

where $0 < \beta_I < \beta_P < 1$ are the rates of time discount for impatient and patient agents respectively, and C_t^i is consumption of agent i in period t . Agents only consume once and then leave the market. A fraction μ_P of the agents are patient and a fraction $\mu_I = (1 - \mu_P)$ are impatient. Type is observable by everyone.

In the first period, each agent is endowed with an asset, let us call it a tree. Each tree is indivisible and distinguished by two features; the quality of the tree, and the extent to which that quality is verifiable upon inspection. Trees can either give a large payoff (L) or a small payoff (S). Payoffs are indivisible, and can usefully be thought of as a fruit of different size. Quality is measured by the probability, q , that the tree gives a large payoff (L). For future reference, let us define the expected payoff from a tree as $\pi(q) = S + q(L - S)$. Verifiability of a tree is parameterized by the probability, n , that the quality of tree (q, n) is observed upon inspection. With probability $(1 - n)$, the quality is not observed upon inspection. Should a trade still take place, the quality of the tree is revealed to the buyer after the trade is completed. The verifiability of a tree is observable to everyone. Verifiability can usefully be thought of as the class of asset to which the tree belongs. For each agent, the type (q, n) of the tree he is endowed with is drawn independently from a common distribution $F(q, n)$.

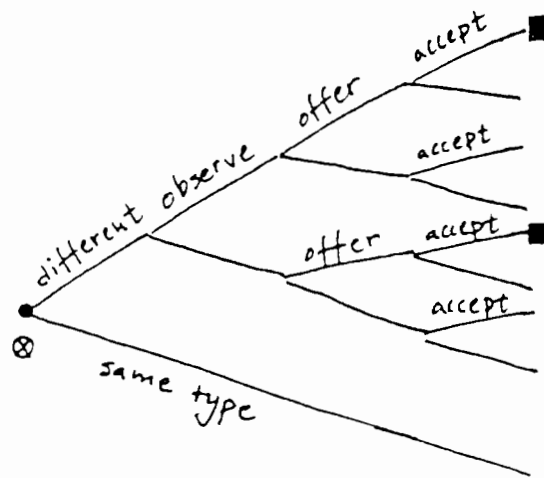
Assumption 1. F is continuous for all n .

Agents are asymmetrically informed about the quality of each tree; whereas the owner of the asset has perfect information, potential buyers may not be informed about the

quality of the tree at the time of purchase. The verifiability of a particular tree measures how informed potential buyers are about the quality of that tree, i.e. the extent of asymmetry of information for that specific tree. Since the quality of a tree is always revealed after trading, at the extremes assets whose quality is unobservable, $n = 0$, are pure experience goods while those which are always observed, $n = 1$, are pure search goods.

Trees give fruit in the second period with probability δ and all trees that did not give fruit in the second period will do so in period three. The size of fruit carried by an agent is observable. Though in the pool of assets, some are ex post more long lived than others, they have ex ante the same expected maturity. Maturity is independent of the other characteristics of the asset; its quality and verifiability. In consequence, no ex ante mutually beneficial asset trades result from the maturity structure per se.

Each period, agents are matched randomly in pairs. For each match, both agents decide whether or not to trade. Consider the potential trades. In the first period, no assets have yet paid off and therefore only asset trades are possible, i.e. an asset can only be traded for another asset.

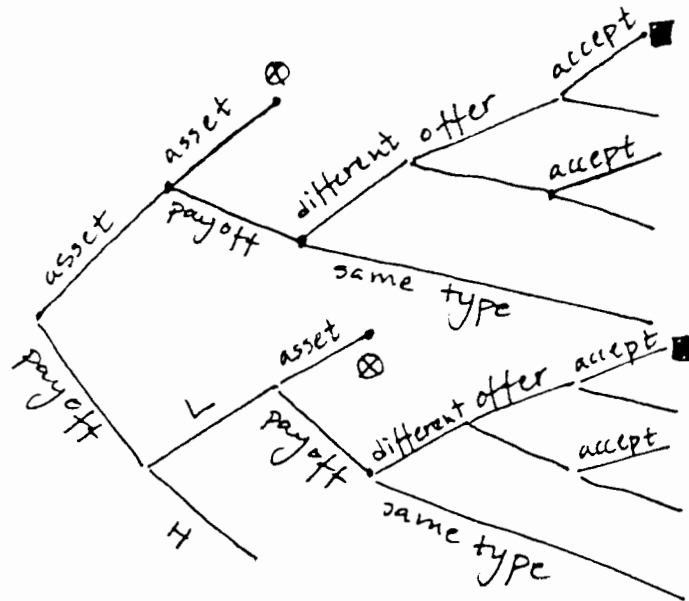


■ Trade takes place

The decision tree illustrates market activity in the first period for an agent. Think of a patient agent (the event tree is identical for an impatient agent). The lower node represents the event that he meets another patient agent. No mutually beneficial trades are possible and both agents keep their assets. The upper node is the event that he meets an impatient agent. The next issue is whether or not the patient agent verifies the asset held by the impatient agent. In either case, the patient agent decides whether or not to offer his asset for trade. Only when the impatient agent accepts the offer does a trade actually take place. All other outcomes involve each agent holding on to their original

asset.

In the second period, some assets will have paid off. Consequently, matches can occur between agents carrying both an asset, both the consumption good, or one an asset and the other one a consumption good. Goods are assumed to be indivisible. This is an assumption of convenience which eliminates any bargaining between participants in second period markets. The following decision tree describes the alternatives.



■ Trade takes place

If the asset of a patient agent (argument is parallel for an impatient) has paid off, we are on the lower node. If the patient agent received a large fruit (L), he consumes it. In the event that the fruit is small (S), the patient agent might want to trade. In case he meets someone with an asset, the decision tree is identical to that of the first period asset market starting at \otimes . If the other party carries a fruit, there is no inference problem so trades take place if agents are of different types and both are willing to trade. The upper node represents the event that the asset did not pay off in period two. The patient agent can either meet someone carrying an asset, the upper branch, or someone carrying a fruit, the lower branch. In the former case, the structure from first period trading is repeated starting at \otimes . In case the other party carries a fruit, the verification is automatic and the tree otherwise follows the same structure as before.

In the third period, agents who left the second period with assets simply consume the fruit and no trades take place.

Each agent chooses a trading strategy to maximize the expected discounted utility from consumption, taking the strategies of other agents as well as the distribution of assets in the population, \mathbf{d} , as given. The trading strategy of an agent will be a function of the asset that agent is carrying, $i = (q, n)$, and the perceived asset of the agent with whom he is matched, $j = (q', n')$. Notice that this allows situations where the agent

decides to offer an asset for trade although the asset of the other party is not observed, in which case asset j is a distribution of quality of assets conditional on n' . Let $\tau_k^t(i, j) = 1$ if an agent of type k in period t wants to trade what he is carrying, i , for what the counterparty is perceived to be carrying, j , and zero otherwise. It is clear that agents of the same type will never trade, $\tau_k^t \tau_k^t \equiv 0$, and a double coincidence of wants is needed for a trade to take place, $\tau_k^t(i, j) \tau_l^t(j, i) = 1$. The equilibrium is defined as follows.

Definition 1. *An equilibrium is a set of trading strategies $\tau_k^t \rightarrow \{0, 1\}$, one for each type of agent $k \in \{P, I\}$, together with distributions of assets $\mathbf{d} = \text{dist}\{(q'_P, n'_P), (q'_I, n'_I)\}$ that satisfy*

- (i) *maximization, so that each agent chooses trading strategies that maximizes expected utility given the trading strategies chosen by other agents and given a second period distribution of assets, \mathbf{d} ,*
- (ii) *rational expectations, so that the strategies chosen, $\{\tau_k^t\}$, produce the second period distribution of assets, \mathbf{d} ,*
- (iii) *Bayesian updating, so that agents revise their priors on the distribution of assets \mathbf{d} taking any new information into account.*

We now proceed to characterize the equilibrium for two scenarios. The first one is an equilibrium in which trade only takes place when and if both parties have full information. In the second case, trades take place in spite of an asymmetry of information between the buyers and sellers of assets.

3 Informed Trading

In this section we will consider situations where an asset whose quality is more likely to be observed will command a premium in the first period asset market. Assets that have a higher probability of being verified are valued more highly because they are more liquid in the second period market. To understand which trades will take place in each period, let us start by studying second period trading activity. Any agent whose asset has given a large payoff in the second period will clearly not want to trade her food for an asset, however high the quality of that asset might be. Time discounting will make her better off simply consuming the crop and leaving the market. Trade will not take place when two agents of the same type meet, regardless of whether they both have fruit or assets, or if one carries an asset and the other one carries fruit. A double coincidence of wants is required for trades to take place. If there is no difference between agents' preferences, no mutually beneficial trades are possible. It is also easy to see that no trades will take place between an impatient agent with a small fruit and a patient agent with an asset.

When it comes to pure asset trades between agents of different types, three situations need to be discussed. Everyone knows that all trees still around in the second period will pay off in the final period. This implies that no agent is willing to pay a premium for verifiability in the context of pure asset trades in the second period market. It is obvious that if both agents have verified each others asset, no such trades will take place in the second period. Trades asset for asset are also precluded when only one agent is able to observe the other party's asset. The fully informed agent will only want to trade

if the partially informed agent's asset is of higher quality. That is a signal to the partially informed agent that she is being ripped off. If neither agent sees the quality of the other party's asset, trades are precluded by the standard rational expectations argument.

Consequently, the only trades that may occur in the second period are those between a patient agent with a small fruit (S) and an impatient agent with an asset. Suppose that the quality of the asset of the impatient agent is observed. If the quality of the asset of the impatient agent is sufficiently low, it might be better for the patient agent to consume the small fruit. Similarly, if the quality of the asset is sufficiently high, it might be better for the impatient agent to hold on to the asset.

Definition 2. *The second period asset market trading set, conditional upon an asset's quality being verified, is defined as*

$$T \equiv \{q \mid \underline{q} \equiv \frac{S}{L-S} \frac{(1-\beta_P)}{\beta_P} \leq q \leq \min(1, \frac{S}{L-S} \frac{(1-\beta_I)}{\beta_I}) \equiv \bar{q}\}.$$

If an impatient agent with an asset with $q \in T$ meets a patient with S , and the asset is observed, a mutually beneficial trade would take place.

Should the quality of the asset of the impatient agent not be observed, however, the patient agent makes his trading strategy contingent on the expected quality of assets among impatient agents, conditional on trade being suggested. We will allow for first period trading which changes the distribution of assets across the two groups of agents. Let us therefore define the distributions of assets after initial asset market trading for patient agents as $G_P(q, n)$ and for impatient agents as $G_I(q, n)$. It will be shown later that $\int_0^1 q dG_I(q, n) \leq \int_0^1 q dF(q, n)$. A sufficient condition to preclude trade without verification is that the expected quality in the initial distribution conditional on any n is lower than \underline{q} . Thus, we shall assume this to be the case.

Assumption 2. $\int_0^1 q dF(q, n) \leq \underline{q}$ for all n .

Prior to matching in the second period, the expected value to a patient agent of his asset depends on the likelihood that a successful trade takes place. The expected value, $W_P(q)$, of an asset of quality $q \in T$ carried by a patient agent is then

$$W_P(q) = (\delta + (1 - \delta)\beta_P)\pi(q) + \mu_I(1 - \delta)\delta(1 - q) \int_{q \in T} n'[\beta_P\pi(q') - S] dG_I(q', n'), \quad (2)$$

where (q', n') represents an impatient agent's asset.

The first part is the expected value of an asset that does not get traded. The second part is the expected value of the option to trade the asset in period two. The probability of meeting an impatient agent with whom a successful trade can take place is $\mu_I(1 - \delta)$. Trading only takes place when the asset pays off a low amount in the second period, which happens with probability $\delta(1 - q)$. The integral takes into account the fact that a trade only takes place if the asset in the hands of the impatient agent is of sufficient quality (that is $q \in T$) and is verified. Such a trade gives the patient agent an asset with the discounted value $\beta_P\pi(q')$ in return for the small payoff S .

We will now derive an expression for the expected value of an asset (q, n) to an impatient agent. Let $W_I(q, n)$ denote the expected value of an asset with quality $q \in T$

and verifiability n in the hands of an impatient agent,

$$W_I(q, n) = (\delta + (1 - \delta)\beta_I)\pi(q) + \mu_P\delta(1 - \int_0^1 q' dG_P(q', n'))n(1 - \delta)[S - \beta_I\pi(q)]. \quad (3)$$

The first part reflects the expected value of an asset that does not get traded. The second part represents the option value of second period trading. Such a trade will only take place if the asset did not pay off in the second period, which happens with probability $(1 - \delta)$. For convenience of notation, let $\alpha \equiv \mu_P\delta(1 - \int_0^1 q dG_P(q, n))$ be the probability that an impatient agent meets a patient agent with a small fruit in the second period. Note that the expected quality among patient agents in the second period, depends on first period trading and consequently α will have to be determined in equilibrium. The probability of a successful trade is αn and the trade gives the impatient agent S in return for the expected discounted value of the asset $\beta_I\pi(q)$. Notice that $\partial^2 W_I(q, n)/\partial n \partial q < 0$, so the marginal contribution of verifiability to the expected value is lower for higher quality assets.

Now step back one period to study pure asset trades in the first period. Recall that the initial distribution of quality and verifiability of assets is independent of type. Two agents of the same type will thus not trade, regardless of whether the quality of their assets are observed or not. Only if two agents value the same assets differently are there gains to be made from trade. Since impatient agents will only be able to exchange an asset for a payoff in the second period if that asset is verified by the patient agent, impatient agents will generally care about the verifiability of an asset. They might accept a lower quality tree in the initial asset market provided that it has a higher level of verifiability. This difference in valuation forms the basis for the existence of an initial asset market where assets are traded for other assets.

Trades thus take place exclusively between patient and impatient agents also in the initial asset market. The willingness of impatient agents to give up quality for verifiability clearly depends on the probability that they will be able to execute a successful trade in the second period. Patient agents do not value verifiability of purchased assets in first period trading since they will only trade in the second period if their asset has paid off.

Assumption 2 implies that a patient agent will never offer to exchange an asset of quality higher than \underline{q} unless they observe the quality of the impatient agent's asset. The impatient agent might, however, be willing to give up an asset of quality above \underline{q} even though she cannot observe the quality of the patient agent's asset. This would occur if the expected benefit from improving the verifiability is sufficiently large to offset the expected loss in quality.

Recall that the verifiability of assets is observable. Let B_I be the expected benefit from trade to an uninformed impatient agent conditional on the verifiability of the asset proposed for trade, n' ,

$$B_I(q, n; n') = \int_0^q [W_I(q', n') - W_I(q, n)] f(dq'|n') \quad (4)$$

where f is the marginal probability distributions of q' conditional on n' . Consider an impatient agent with a non-verifiable asset, $n = 0$. The best this agent can do is to obtain an asset that is fully verifiable, $n' = 1$. The expected benefit would be

$$B_I(q, 0; 1) = \int_0^q W_I(q', 1) f(dq'|1) - W_I(q, 0). \quad (5)$$

To maximize the benefits from verifiability, set $\alpha = 1$. Note that an asset of quality below \underline{q} has zero second period option value. The above expression can then be written as

$$B_I(q, 0; 1) \leq \int_0^q (\delta + (1 - \delta)\beta_I)(\pi(q') - \pi(q))f(dq'|1) + \int_{\underline{q}}^q (1 - \delta)(S - \beta_I\pi(q'))f(dq'|1). \quad (6)$$

A sufficient condition to prevent the impatient agent from trading an asset $q > \underline{q}$ without information is that $B_I(q, 0; 1) \leq 0$. We shall assume this to be the case.

Assumption 3. $\int_0^q [(\delta + (1 - \delta)\beta_I)(\pi(q') - \pi(q))f(dq'|1) + \int_{\underline{q}}^q (1 - \delta)(S - \beta_I\pi(q'))f(dq'|1)] \leq 0$.

Thus, the only trades that can take place without full information is when at least one uninformed agent has an asset below \underline{q} . Since preferences coincide in this set, no such trades will take place.

Proposition 1. *Under Assumptions 2 and 3, no trades take place unless both agents observe the quality of each others' asset.*

We emphasize that all trades in the initial market require the quality of both assets to be observed. The value of an asset in the initial market depends on the likelihood of a successful trade in the following period. The set of assets that are eligible for trade in initial markets is thus restricted to $q \in T$. If the impatient agent carries a tree with quality $q_I \geq \bar{q}$, she will not be willing to give up quality for verifiability since she optimally holds on to such an asset in period one. If, on the other hand, $q_I \leq \underline{q}$, no patient agent will take the tree for fruit in period one and verifiability is not an issue. It is only for the assets in the set T that agents value the characteristic of the assets differently and can thus engage in mutually beneficial trades.

For trades to be incentive compatible, we require that the patient agent gets a tree of higher quality and that the impatient agent is better off with the acquired combination of quality and verifiability than she is with her old asset.

Definition 3. *The set of potential trades between a patient agent with (q_P, n_P) and an impatient agent with (q_I, n_I) in initial period asset markets is defined as*

$$C(\alpha) \equiv \left\{ (q_I, n_I, q_P, n_P) \mid \begin{array}{l} (i) \quad q_I \geq q_P \\ (ii) \quad W_I(q_P, n_P) \geq W_I(q_I, n_I) \\ (iii) \quad q_P \in T \end{array} \right.$$

where the dependence of W_I , on α is understood. Equation (3) can be used to define the indifference curves of an impatient agent in (q, n) -space. The indifference curve of the impatient agent is negatively sloped and concave below \bar{q} . As mentioned above, the patient agent only cares about the quality of the asset which makes his indifference curves horizontal. Figure 1 illustrates a section of the set $C(\alpha)$ for an impatient agent with (q_I, n_I) .

Equilibrium

With first period asset trades taking place, the second period distributions of quality and verifiability of assets across types of agents are endogenously determined. Decision rules of agents in the first period depend on the likelihood of meeting a person with whom a trade can take place in the second period. This likelihood is a function of the endogenously determined second period distribution of quality of assets, which in turn depends on the trading rules for initial period trading. Proposition 2 ensures that an equilibrium with liquidity trading exists.

Proposition 2. *Under Assumptions 2 and 3, there exists a unique equilibrium with liquidity trading.*

Proof: Consider a patient agent with asset (q, n) , $q \in T$, at the beginning of the first trading period. His end of period asset, (q', n') , will be a random variable with joint distribution that depends on trading strategies. For given $\alpha > 0$, let $D(q, n; \alpha) = \{(q', n') | (q', n', q, n) \in C(\alpha)\}$, where the dependence of C on α follows from Definition 2. This gives the set of possible end of period asset holdings for this agent, should asset trading take place. The larger $D(q, n; \alpha)$ is the more matches will result in trades. Since each trade results in $q' \geq q$, this increases the average end of period q' . Denote this conditional expectation $E(q'|q, n; \alpha)$.

Figure 2 plots the set $D(q, n; \alpha)$, which is given by the intersection of the lower contour set of (q, n) for an impatient agent with the set $\{q' \geq q\}$. It is straightforward to verify from Definition 2 that as α increases, the indifference curves for an impatient agent rotate as shown in Figure 2, thus enlarging the trading set. So, $E(q'|q, n; \alpha)$ is nondecreasing in α and as shown in Lemma 1 (Appendix), it is also continuous. As a result, \hat{q} , the second period average q for patient agents is also continuous and nondecreasing in α . Denote this function $\hat{q} = Q(\alpha)$. The probability of a successful match depends on the proportions of agents in the market being eligible for trade: $\alpha = \mu_P \delta (1 - \hat{q})$ defines a decreasing mapping $P(\alpha) = \mu_P \delta (1 - Q(\alpha))$ on $[0, 1]$ whose fixed points determine equilibria. Since P is continuous and decreasing, there exists a unique equilibrium point. \square

Note that trading in the initial stock market takes place because people are forward looking. They take future trading opportunities into account when assessing the value of an asset. The only heterogeneity across agents is their rate of time preference. This causes agents to evaluate characteristics of assets differently. An impatient agent is more concerned about being able to liquidate an asset early on and thus puts a higher price on the feature of assets that make them more liquid - verifiability. The outcome of the initial stock market is a redistribution of assets among the groups of patient and impatient agents. Specifically, the average verifiability of assets among impatient agents improves while the average quality of their assets falls. Note that this implies that $\int_0^1 q dG_I(q, n) \leq \int_0^1 q dF(q, n)$, which together with Assumption 3 justifies the conjecture that no uninformed trading takes place in the second period. For patient agents, on the other hand, the average quality of assets improves through trading while the average verifiability of assets fall.

Return Dominance

To derive a notion of asset return it is convenient to have an expression for the expected utility of patient and impatient agents with an asset of characteristics (q, n) , denoted $U_P(q, n)$ and $U_I(q, n)$ respectively, measured in terms of second period consumption

$$U_P(q, n) = W_P(q) + \mu_I n \int_{\{(q', n', q, n) \in C\}} n' [W_P(q') - W_P(q)] dF(q', n'), \quad (7)$$

$$U_I(q, n) = W_I(q, n) + \mu_P n \int_{\{(q, n, q', n') \in C\}} n' [W_I(q', n') - W_I(q, n)] dF(q', n'). \quad (8)$$

In equations (7) and (8), the first term measures the conjectured value if no trades take place in the first period asset market while the second term measures the value to each agent from having access to the stock market in the first period.

The standard notion of the expected return to an asset is not defined in an environment where there are no market prices. We can, however, measure the expected shadow return for an asset using the indirect utility function of agents. This shadow return depends on who is initially endowed with the asset in question. In general, the expected shadow return $(1 + r_k)$ for an agent of type $k \in \{P, I\}$ will be defined as the internal rate of return that equates the expected payoffs to an asset with the indirect utility from holding the asset.

$$U_k(q, n) = \pi(q)[\delta + (1 - \delta)/(1 + r_k)], \quad (9)$$

where the left hand sides are equations (7) and (8) for a patient and an impatient agent respectively. It follows that the shadow rate of return is increasing in $(\frac{U_k(q, n)}{\pi(q)})^{-1}$. Since $\pi(q)$ is independent of n and $U_k(q, n)$ increasing in n for $q \in T$, the shadow rate of return is decreasing in n regardless of the type of its owner. The analytical expressions for the (inverse of) the shadow rates of return for assets in T are given by:

$$(1 + r_P)^{-1} = \beta_P + \mu_I \delta \frac{(1 - q)}{\pi(q)} \int_{q' \in T} n' [\beta_P \pi(q') - S] dG_I(q', n') + \Delta_P(q, n, q', n'). \quad (10)$$

$$(1 + r_I)^{-1} = \beta_I + \alpha n \left[\frac{S}{\pi(q)} - \beta_I \right] + \Delta_I(q, n, q', n'). \quad (11)$$

where $\Delta_k \geq 0$ comes from the improvement in expected utility for agent $k \in \{P, I\}$ due to first period trading. With no first period trading, which is represented by the third term in each equation above, it is easy to verify that the shadow returns for both the patient and the impatient agent will be increasing in the quality of the asset.

An asset that is not tradable has an expected shadow return of $(1 + r_k) = 1/\beta_k$. An asset in the trading set, $q \in T$, will have a lower expected shadow return (or a higher shadow price) because of the option value of trading in the future. Since the value of an asset to an agent depends on the opportunities for exchanging the asset in the future, the shadow return also depends on the observability of the asset. We summarize the properties of the expected shadow return in Proposition 3.

Proposition 3. *The expected shadow rate of return on a tradable asset, $q \in T$, is always decreasing in n . With no first period trading, the rate of return is also increasing in q irrespective of the type of its initial owner.*

Tradable assets have a liquidity premium due to the option value of future potential trades. This liquidity premium is decreasing in the quality of the asset. Since successful trades require full information for both the buyer and the seller, an asset whose quality is more easily observed is more valuable. Such an asset has a higher liquidity premium, and correspondingly a lower expected shadow rate of return.

The equilibrium also has implications for the relationship between liquidity premia and trading volume. Even without considering first period trading, assets with higher n will on average trade more in the second period than those with lower n . Moreover, since assets with higher n are needed as "store of liquidity" they are more likely to be traded in the first period. Assets with a liquidity premium are thus also assets that will tend to be traded more frequently in equilibrium.

Welfare

Second period trading unambiguously improves welfare in this economic environment. To see this, assume that $\mu_P = \mu_I = \mu$. The total social gain from second period trading is given by $\mu\delta(1 - \delta)(1 - \int qdF(q, n)) \int_{q' \in T} n'(\beta_P - \beta_I)\pi(q')dF(q', n') \geq 0$. It is, however, not necessarily the case that first period trading unambiguously improves welfare of both types of agents. First period trading affects the option value of trades in the second period by impacting on the trading opportunity set. The following example shows that impatient agents as a group can suffer from the first period asset market being open.

Example 1: Let half of the patient agents each have an asset $(q_P^1, n_P) = (\frac{1}{2}, 1)$, while the rest have an asset $(q_P^2, n_P) = (0, 1)$. Similarly, half of the impatient agents each have an asset $(q_I^1, n_I) = (\frac{281}{514}, \frac{1}{2})$, while the rest have an asset $(q_I^2, n_I) = (0, \frac{1}{2})$. The level of q_I^1 has been chosen such that an impatient agent with an asset q_I^1 is indifferent between this asset and q_P^1 when allowed to trade. She does not gain anything from first period asset trade *per se*. The rest of the parameters take the values: $H = 3, L = 2, \delta = \frac{1}{2}, \mu_P = \mu_I = \frac{1}{2}, \beta_P = 0.99$, and $\beta_I = 0$. When there is no asset trade in the first period, $\alpha = 0.1875$ and the expected utility of an impatient agent of type 1 is $U_I^1 = 1.3671$, while the impatient agent of type 2 has an expected utility of $U_I^2 = 1.00$. When first period trading is allowed, $\alpha = 0.1868$ and $U_I^1 = 1.3667$ and $U_I^2 = 1.00$. The impatient agents are worse off because of first period asset trading ($1.1834 < 1.1835$). For the patient agents, the expected utility when no asset trade is allowed ($\gamma = \frac{1}{16}$ and $\hat{\pi} = 2.5467$) are $U_P^1 = 2.4956$ and $U_P^2 = 1.99$ which is to be compared to the expected utility when trading is permitted ($\gamma = \frac{17}{128}$ and $\hat{\pi} = 2.5438$), $U_P^1 = 2.5103$ and $U_P^2 = 1.99$. Patient agents as a group are thus better off due to trading ($2.2502 > 2.2428$). Assuming that each agent gets the same weight in the social welfare function, the total welfare of participants in the market is larger with asset trading, 1.7168, than without, 1.7132.

Example 1 illustrates the externality that might make first period asset trading detrimental to impatient agents. Since each agent takes α as given, they do not account for the reduction in the probability of future successful trades that results from their desire to increase the liquidity of their own asset through first period trading.

For patient agents, the externality caused by first period trading is not unambiguously negative. First period asset trading improves the likelihood of a successful trade in the second period market. At the same time, the average quality of assets that patient agents

can receive in return for S is poorer due to first period trading. Note that an asset can change hands at most twice in this world. A relatively poor quality asset that was exchanged for a better quality asset in first period trading can make its way back to the patient agents via second period trading. On net, the trades should still be beneficial for the patient group as a whole. The better quality asset received in the first market is expected to give a higher crop and it might do so already in the second period. The poor quality asset will return to the patient group only in exchange for a low payoff and it will affect only third period consumption. Due to discounting, the patient agents gain from asset market trading. We conjecture that patient agents as a group never lose from having access to an asset market.

Numerical Example

To illustrate the equilibrium of the model, a numerical example assuming uniform and independent distributions for q and n is used. It implies that the unconditional distribution of quality, $E[q|n] = E[q] = \frac{1}{2}$. The payoffs are $L = 1.0$ and $S = 0.9$. $\mu_P = \frac{1}{2}$ of the population is assumed to be patient, and $\mu_I = \frac{1}{2}$ are impatient. The probability of second period payoff $\delta = \frac{1}{2}$. A patient agent has a time discount factor of $\beta_P = 0.945$ while the impatient one has a time discount factor $\beta_I = 0.90$. This implies that the trading set are all assets with $q \in (0.52, 1.0)$.

The endogenously determined probability of an impatient agent with a tradable asset meeting a patient agent with S in the second period is $\alpha = 0.12499$. The new level of expected quality of assets among patient agents after first period trading is $\hat{q}_P = 0.50005$.

Figure 3 illustrates the option value of the second period market as a function of the quality of an asset for three levels of verifiability: $n \in \{0.1, 0.5, 0.9\}$. In the top panel the results for an impatient agent are given while the bottom panel illustrates the situation for the patient agent. The functions jump at $q = 0.52$, where assets become tradable. The figure shows that the value of being able to trade in the second period is lower the higher the quality of, and larger the higher the verifiability of an impatient agent's asset. For a patient agent, quality alone determines the value of having access to the second period market. This value is decreasing in the quality of a patient agent's asset.

Figure 4 illustrates the option value of the first period asset market for assets of three different levels of quality: $q \in \{0.55, 0.75, 0.95\}$. Again, the top graph represents the impatient agent and the bottom one the patient agent. This option value is orders of magnitude smaller than the option value of second period trading for both agents. It is a direct consequence of making initial trading very difficult by requiring that both assets are verified for trades to take place and by assuming that only one match is allowed. The first period option value is concave-convex in verifiability for an impatient agent while the same option value is convex in verifiability for a patient agent. Impatient agents endowed with assets of low verifiability are those that benefit the most from the initial asset market. Similarly, it is the patient agents with assets of high verifiability that have the most to gain from initial trading. The value of the first period asset market is more important for low quality assets. An impatient agent with a high quality asset will be less willing to trade, while a patient agent with a high quality asset faces a smaller set of potential trades in initial markets.

Finally, in Figure 5 we study the gross shadow return on assets as a function of their quality for three levels of verifiability: $n \in \{0.1, 0.5, 0.9\}$. The top panel is for the impatient agent and the bottom panel for the patient agent. Note that the expected shadow return of an asset is decreasing in verifiability for an impatient agent while verifiability has a negligible effect for a patient agent. This follows since it is the effect of verifiability of an impatient agent's asset on second period trading that dominates the smaller effect of verifiability on the option value of first period trading for both patient and impatient agents. For both types of agents, the expected shadow return is increasing in the quality of the assets for assets in the trading region. The liquidity premium is thus decreasing in the quality of an asset and increasing in the asset's verifiability.

4 Uninformed Trading

Our previous analysis established that when trading only takes place between informed agents, verifiability of an asset enhances its tradeability and thus its shadow price. This in turn implies that the expected rate of return on less verifiable assets is higher to compensate for the lack of tradeability. In this section we study the case where second period transactions occur in absence of asset quality verification.

Let \hat{q}_I be the expected value of q for impatient agents trading assets in the second period. A patient agent will trade in absence of verification provided that $\hat{q}_I > \underline{q}$. Since \hat{q}_I not only depends on the initial distribution but also on first period trades, it is not enough to assume this property for the unconditional mean. However it is simple to impose restrictions on the distribution of agents types and on preferences to derive this result. As an example, if $\beta_P = 1$, and $\underline{q} = 0$ the condition immediately follows.

Proposition 4. *If $\hat{q}_I > \underline{q}$ there exists an equilibrium with uninformed trading.*

Proof: See Appendix.

What trades will occur in the first period? All assets with $q > \underline{q}$ are fully tradeable in the second period, while assets with $q < \underline{q}$ will be traded only if not verified. Consequently, only impatient agents with $q \in [0, \underline{q})$ will trade in the first period. The purpose of these trades will be to *lower verifiability*, thus increasing the probability of exchange in the following period. Incidentally, note that since the maximum reduction in q that could take place in the first period is \underline{q} , the assumption in the above proposition will be satisfied if, for instance, $E[q|n] > 2\underline{q}$. Will uninformed trading take place in the first period? Again, it is easy to find conditions under which this will always occur. For example, if the initial distribution of assets is uniform on $[0, 1] \times [0, 1]$, such trades will take place.

We now turn to the analysis of return dominance. For assets in $[\underline{q}, 1]$, verifiability has no effect on trading and thus the rate of return is independent of n . In contrast, for those assets in $[0, \underline{q})$ higher n will make the asset *less tradeable*. The shadow price for these assets will be decreasing in n and consequently their rate of return increasing. Agents with low quality assets will only be able to trade if their assets remain undetected. When uninformed trading takes place, it is thus the "lemons" or low quality assets which are less known in the market that carry a liquidity premium. Liquidity trading in the first

period will again imply that assets which have a liquidity premium will be traded more frequently.

To illustrate the kinds of equilibria that arise when uninformed trading takes place, two examples are developed below. In each case, the crucial feature of the equilibrium is that there are some assets whose expected returns are close but for which the amount of information in the market is quite different. This opens up room for trades to take place in the first period which deteriorate the welfare of patient agents and under some circumstances even reduces total welfare.

Welfare

Since second period trading is voluntary, it always generates ex-ante welfare gains for both types of agents. The same is true ex-post for all impatient agents. Furthermore, since these trades result in earlier consumption for the impatient agent, they always result in ex-post gains in total welfare if we consider equal weights for both type of agents. First period trading is also voluntary. But as these trades change the distribution of asset holdings, they may affect the incentives agents have to trade in the second period and this may result in a welfare loss. The following examples illustrate this point. Example 2 shows that first period trading can harm patient agents. Example 3 shows that it is possible for both equilibria with informed and equilibria with uninformed trading to exist. Furthermore, the equilibria with uninformed trading has no first period trading but produces higher average utility for agents.

Example 2. Let us assume that there are three assets $\{(q_1, n_1), (q_2, n_2), (q_3, n_3)\}$ with the following properties:

- (i) $\bar{q} > q_1 > \underline{q} > q_2 > q_3 = 0$
- (ii) $n_1 = n_3 = 0$ and $n_2 = 1$.
- (iii) $\iota_2 = \iota_1 = \frac{1}{2}$, and $\iota_3 = 0$; $p_1 = p_2 = 0$, and $p_3 = 1$.
- (iv) $\iota_1 q_1 + \iota_2 q_2 + \iota_3 q_3 > 2\underline{q}$,

where ι_i , and p_i are the initial distributions of asset i of impatient agents and patient agents respectively. These assumptions imply the following properties for trading: second period trading will occur without verification; impatient agents with q_1 will not trade in the first period; and patient agents who all have assets q_3 will trade in the first period.

If an impatient agent carrying an asset of type q_2 meets a patient agent, a trade will occur provided that

$$W_I(q_3, n_3) - W_I(q_2, n_2) = (\pi(q_3) - \pi(q_2))(\delta + (1 - \delta)\beta_I) + \alpha(1 - \delta)(L - \beta_I\pi(q_3)) > 0.$$

Assume that this expression is positive, e.g. that $q_2 - q_3$ is small. Under this assumption the only trades that take place the first period are between patient agents with asset type q_3 and impatient agents with q_2 . This is clearly the unique equilibrium for this environment.

Case 1. No trading allowed in first period. Given that $n_2 = 1$, $n_1 = 0$ and $\iota_3 = 0$, patient agents will trade in the second period only if they do not verify the quality of an asset offered for trade, in which case it follows that the asset is of type q_1 . It is easy to calculate that the expected gains for patient agents from this trading will be: $\frac{1}{4}\delta(1 - \delta)[\beta_P\pi(q_1) - S]$.

Case 2. Trading occurs first period. If trading is permitted the first period, then all patients that meet an impatient with asset q_2 will trade. This will occur with probability $\frac{1}{4}$ for patients and $\frac{1}{2}$ of the impatient with this type of asset will trade. Since $n_3 = n_1 = 0$ and $n_2 = 1$, the conditional distribution of quality after meeting an impatient agent in the second period and *not* observing her asset is $\{P(q_1) = \frac{2}{3}, P(q_3) = \frac{1}{3}\}$. Finally, assume

$$(vi) \frac{2}{3}q_1 + \frac{1}{3}q_3 > \underline{q}$$

so that such matches result in trade. The low quality assets *contaminate* the distribution, and the benefits from second period trading for impatient agents are now much smaller than those obtained in Case 1. For values $q_2 - q_3$ small enough, the benefits obtained by those patient agents that trade in the first period will not offset the lower benefits for the group of patient agents in the second period, resulting in a net welfare loss.

Example 3. Let there be three assets $\{(q_1, n_1), (q_2, n_2), (q_3, n_3)\}$ with the following properties:

$$(i) \underline{q} > q_1 > q_2 > \underline{q} > q_3 = 0,$$

$$(ii) n_1 = \frac{1}{2}, n_2 = 1, \text{ and } n_3 = 0,$$

$$(iii) \iota_1 = \iota_3 = \frac{1}{2}, \text{ and } \iota_2 = 0, p_2 = 1, \text{ and } p_1 = p_3 = 0,$$

where ι_i denotes the initial distribution of impatient agents asset holdings and p_i the corresponding one for patient agents.

Case 1. No trading in the first period. Consider second period trading conditional on no observation. The posterior distribution for the asset held by the impatient agent is: $\{P(q_1) = \frac{1}{3}, P(q_3) = \frac{2}{3}\}$. The following assumption ensures that trading takes place.

$$(iv) \frac{1}{3}q_1 + \frac{2}{3}q_3 > \underline{q}.$$

Trades occur with probability $\frac{1}{4}\delta(1 - \delta)(\frac{3}{4})$ and generates an average net welfare gain of $(\frac{1}{3}\pi(q_1) + \frac{2}{3}\pi(q_3))(\beta_P - \beta_I)$. It is interesting to note that if patient agents are willing to exchange without verifying in the second period, no first period trades can benefit impatient agents and thus this is an equilibrium for the matching game.

Case 2. Trading in the first period. We will construct another equilibrium where as a result of first period trading, the distribution of second period asset holdings of impatient agents is downgraded enough to kill uninformed trading. The equilibrium will involve only informed trading in the second period, provided $q_1 - q_2$ is not too large which we shall assume to be the case. Since $n_2 > n_1$, impatient agents will now be willing to trade in the first period.

Let τ denote the fraction of impatient agents holding assets of type q_1 that end up trading in the first period. Consider second period trading conditional on no observation. The corresponding distribution for the asset held by the impatient agent is now: $\{P(q_1) = \frac{2}{3-\tau}, P(q_3) = \frac{1-\tau}{3-\tau}\}$. With the following assumption no uninformed second period trading will take place.

$$(v) \frac{1-\tau}{3-\tau}q_1 + \frac{2}{3-\tau}q_3 < \underline{q}.$$

Note that this assumption is not inconsistent with all of the above and that this is also an equilibrium for the matching game. The absence of uninformed trading the second period results in a welfare loss, which is partly offset by the trades that take place the first period. These gains will not be enough to offset the losses, provided $q_1 - q_2$ is not too large. In that case, the equilibrium described in Case 1 gives higher welfare.

For the remainder of this paper, we will focus on the case when only informed trading

takes place. Thus, more information is a good and there are no incentives to hide the quality of assets.

5 Investment in Verifiability

Consider introducing a technology whereby the verifiability of an asset can be improved at a cost per unit of increase in n . To enable the agents to pay this cost, let us endow them each with a perishable perfectly divisible consumption good in the initial period. To make things simple, assume that this good enters linearly into the agents' preferences. Since increased verifiability improves future trading opportunities for tradable assets, agents with potentially tradable assets would be willing to pay something at the outset for being able to increase the verifiability of their specific asset. This investment can be thought of as improving accounting techniques, and implementing disclosure practices which increase the transparency of the asset to a potential buyer.

Which agents will invest more in verifiability? Since most of the benefits associated with higher n derive from second period trading, let us first concentrate on these trades. The corresponding option value for a tree (q, n) to an impatient agent is given by $\alpha n(1 - \delta)(S - \beta_I \pi(q))$, which is linear in n . The marginal value of n is $\alpha(1 - \delta)(S - \beta_I \pi(q))$ which is decreasing in q . Agents with higher quality trees are thus less interested in trading them and thus have less of an incentive to invest in verifiability.

Higher verifiability increases the likelihood of a trade taking place upon a match. Since the surplus obtained by this transaction will generally be shared between the two agents, the private incentives for investment will generally not produce the socially optimum level. This is a standard issue in search models. For our particular model, one may expect investment to be too small. Again this can be easily seen by focusing on the benefits derived from second period trading. The marginal gains from investment for an impatient agent are $\alpha(1 - \delta)(S - \beta_I \pi(q))$ while the social marginal gain is $\alpha(1 - \delta)(\beta_P - \beta_I)\pi(q)$. Since for a trade $q > \underline{q}$, $\beta_P \pi(q) > S$, so $(\beta_P - \beta_I)\pi(q) > S - \beta_I \pi(q)$. At an interior equilibrium, investment would fall short of the optimum. This could rationalize minimal disclosure requirements.

In the general case of both first and second period trading, the results depend in complicated ways on the specific distributional assumptions. For the case of uniform and independent distributions, one may show that there are equilibria where no patient agent invests in verifiability while all impatient agents invest up to some $n^* \leq 1$.⁵

6 Intermediated Trading

Under the decentralized exchange mechanism (matching) studied above, mutually beneficial exchanges are left unexecuted. We now discuss the possibility of some alternative and more centralized mechanisms for exchange. These are all highly stylized constructs with limited resemblance to the actual institutions who lend their names to our intermediated structures. Our purpose is to capture some important features of intermediation in order

⁵Details are available from the authors on request.

to highlight how reducing the frictions in the previously described trading environment might impact on trading opportunities and the welfare of agents. First, we consider the possibility of having a firm, call it an investment bank, determine the quality of an asset at a fixed cost and stamp it. Second, we introduce a firm, call it a mutual fund, which for a fixed cost pools assets in the initial period and offers its participants the possibility of choosing the exact timing for their consumption. Finally, we study a firm that serves as an intermediary for the exchange of payoffs for assets in the second period. We will call this institution a commercial bank. We will assume free entry so that the fee charged for verifying assets in each case is equal to the verifying cost. To simplify the analysis, first period asset market trading is ruled out for these examples. Since second period trading is unambiguously welfare improving, this allows us to focus on the role of intermediation in reducing the frictions associated with the trading environment. As in the previous section, we will assume that agents have a perfectly divisible perishable first period good which enters additively in the utility function. This good is used to pay the fee for obtaining the benefits of participating in the described institutions.

Investment Bank

Suppose that we introduce an investment bank, which at a cost is willing to examine assets and give the owner a credible certificate indicating the quality of the asset. Since only impatient agents care about verifiability, they will be the ones purchasing such services from the investment bank. The investment bank makes the asset perfectly verifiable, providing a gain for an impatient agent with asset (q, n) of $B_I(q, n) = \alpha(1 - n)(1 - \delta)(S - \beta_I \pi(q))$. Letting c denote the fixed cost of verification for the bank and thus the price of its stamping service, all impatient agents for which $B_I(q, n) > c$ will obtain this service. Since $B_I(q, n)$ is decreasing in q and n , the agents with less verifiable assets and lower quality (within the trading set T) will obtain this service. Notice also that $B_I(q, n)$ is increasing in α so, ceteris paribus, an increase in the quality of assets of patient agents will reduce the option value of trading and consequently will decrease the demand for the bank's services. Similar results are obtained for an increase in the verifiability of assets held by impatient agents.

Mutual Fund

Suppose now that agents can take their assets to a firm which at a fixed cost c gives in exchange a payoff at a fixed time period: for impatient agents in the second period and for patient agents in the third period. A contract specifies an interest rate $\frac{1}{\beta}$ where $\beta_I < \beta < \beta_P$ such that an agent with an asset of quality q can choose to consume a lottery with expected value $(\delta + (1 - \delta)\beta)\pi(q)$ in the second period or one with expected value $(\delta + (1 - \delta)\beta)\pi(q)/\beta$ in the third period. Obviously impatient agents will choose to consume in the second period and patient agents in the third. We now establish how the interest rate is determined.

Assume first that the set of agents (q, n) that participate in this trade are A_P for patient agents and A_I for impatient. Let $\pi_P = \mu_P \int_{A_P} \pi(q) dF(q, n)$ and $\pi_I = \mu_I \int_{A_I} \pi(q) dF(q, n)$. The resource constraint for the mutual fund is given by $(\delta + (1 - \delta)\beta)\pi_I = \delta(\pi_I + \pi_P)$

where the left hand side represents the demand for second period payoffs and the right hand side the supply.⁶ From this constraint we obtain the expression:

$$\beta = \frac{\delta}{1 - \delta} \frac{\pi_P}{\pi_I} \quad (12)$$

Note that in spite of the linearity of preferences and the different discount rates, the interest rate is pinned down by the participation of agents. We now turn to the joint determination of β and the participation sets A_P and A_I .

The benefits from participation are:

$$B_I(q, n) = (\beta - \beta_I)(1 - \delta)\pi(q) - \mu_P \delta(1 - \delta) \left(1 - \int_{\{B_P(q') \leq c \text{ and } q' \in T\}} q' dF(q', n')\right) n(S - \beta_I \pi(q)), \quad (13)$$

$$B_P(q) = \frac{(\beta_P - \beta)}{\beta} \delta \pi(q) - \mu_I \delta(1 - \delta)(1 - q) \int_{\{B_I(q', n') \leq c \text{ and } q' \in T\}} n'(\beta_P \pi(q') - S) dF(q', n'), \quad (14)$$

where the second term in each equation reflects the expected opportunity cost of giving up the right to participate in the second period asset market.

For $\beta = \beta_I$, $B_I(q, n) \leq 0$ so $\pi_I = 0$ and for $\beta = \beta_P$, $B_P(q) \leq 0$ so $\pi_P = 0$. Note also that B_I is increasing in β and B_P decreasing in that same variable. Assuming there exists some β for which both π_P and π_I are nonzero, then $\frac{\pi_P}{\pi_I} = \infty$ when $\beta = \beta_I$ and $\frac{\pi_P}{\pi_I} = 0$ when $\beta = \beta_P$. As shown in the Appendix, π_I and π_P vary continuously with β , so there exists an equilibrium with positive participation in the mutual fund.

As before, the benefits of participation are decreasing in n , so it is the less verifiable assets that will be traded through this mechanism. In contrast with the previous case, both B_I and B_P are increasing in q , so the higher quality assets will participate in the mutual fund. Consider now an increase in the quality of assets held by impatient agents. The direct effect is an increase in π_I . There is also an effect on π_P which is ambiguous, since though less trades will be available in second period matching, quality will be higher. Provided π_P does not increase by too much, the net effect will lead to a decrease in β , i.e. an increase in the interest rate. Similarly, an increase in q for patient agents is likely to result in a lower interest rate. Finally, an increase in the verifiability of assets held by impatient agents will tend to reduce their participation in the fund and consequently make the interest rate decrease.

Commercial Bank

In the mutual fund, all participants were charged a fee at the outset to take part in the fund's activities. Ex post, only a subset of those agents will in fact prefer to use the services provided by the fund; the patient agents with a small payoff in the second period and the impatient agents which did not get a harvest in the second period.

⁶A similar constraint can be obtained for third period consumption. However only one is needed since they are linearly dependent.

Retaining the assumption that each agent has been given an endowment of a divisible consumption good to pay any fees with, we may ask the question if there could be a viable commercial bank in the second period. The contract offered by such a bank would promise patient agents an expected payoff of S/β in the third period and an impatient agent $\beta\pi(q)$ in the second period. Since it is only the impatient agents who need the verification services, they are assumed to be the ones to bear the fixed cost of participating in the commercial bank, c . What needs to be determined is the interest rate.

Suppose that all patient agents go to the bank. This implies that there can be no other trading in the second period. Notice that some impatient agents, specifically those with relatively poor assets, might choose to hold on to their assets if the fixed cost, c , is sufficiently high. The demand for asset, or second period deposits, would then be $\delta\mu_P(1 - \int q'dF(q', n'))S$. Impatient agents expect to get $\beta_I\pi(q)$ by staying in the market while the bank offers them $\beta\pi(q) - c$. Define $q(\beta) \equiv \{q | \beta_I\pi(q) \equiv \beta\pi(q) - c\}$. The supply of assets by impatient agents is increasing in β . It is given by $(1 - \delta)\mu_I \int_{q(\beta)}^1 \pi(q) dq$, which in equilibrium must equal withdrawals by impatient agents. Pick a $\beta \in (\beta_I, \beta_P)$. If demand for assets exceeds supply, an increase in β , i.e. a reduction in the interest rate, will increase supply. Similarly, if supply exceeds demand an increase in the interest rate will balance the demand for funds in the bank.

What would happen if at this interest rate some patient agents choose to go to the market instead? Then the equilibrium must involve an interest rate β such that patient agents are indifferent between obtaining S/β and the expected benefits from being in the market. Let A_I be the set of impatient agents with assets in the second period who choose to use the market, and a_P be the fraction of patient agents with a small payoff in the second period who remain in the market. The expected benefits of participating in the commercial bank for a patient and an impatient agent respectively are

$$B_P(\beta) = S(1 - \beta)/\beta - \mu_I(1 - \delta) \int_{A_I} n'(\beta_P\pi(q') - S)dF(q', n'), \quad (15)$$

$$B_I(q, n) = (\beta - \beta_I)\pi(q) - c - \mu_P\delta(1 - \int_0^1 q'dF(q', n'))a_P n(S - \beta_I\pi(q)). \quad (16)$$

The first part of B_P is decreasing in β while the first part of B_I is increasing in that same variable. The second term represents the opportunity cost of not being able to use the second period market when participating in the bank. An increase in β makes the bank more attractive to impatient agents. The set A_I will shrink as a result which makes this cost for patient agents smaller.

A candidate equilibrium is a (a_P, β) such that $B_P = 0$.⁷ Since B_I is unambiguously increasing in q , there is a q^* for given (a_P, β) such that impatient agents with $q < q^*$ go to the market while those with $q \geq q^*$ have the commercial bank verify their assets and exchange them for second period consumption. Notice that it is the assets with low q and high n that remain in the market. These are the ones for which the cost of verifying assets is most onerous.

⁷It is fairly straightforward to show the existence of such an equilibrium. The proof is available from the authors upon request.

7 Summary and Conclusions

This paper has studied sequential asset trading in a random matching game. We have focused on the situation that arises when agents are not equally informed about the returns of different assets. The model endogenously determines trading strategies for the agents and, in particular, whether an uninformed agent will be willing to trade or not. These trading strategies imply that in equilibrium some assets are more tradeable than others and through initial trading become reallocated to agents with higher preference for liquidity. Consequently, these assets are more heavily traded and, since they are preferred by agents, have lower rates of return. Our model is thus consistent with the negative relationship between volume traded and asset returns.

In contrast with other models of asset trading, uninformed agents rationally decide whether to engage in a particular trade or not. Though an agent may be uninformed about the returns of a particular asset offered for trade, he nonetheless has well defined priors. These priors are part of a sequential equilibrium and thus consistent with the sequential trading strategies of the population. If in equilibrium no uninformed agent is willing to trade, it is always good for an asset to be better known. However, the opposite occurs with low return assets if in equilibrium uninformed agents are willing to take them.

Because of the sequential nature of trading, transactions that take place at an earlier stage affect the distribution of asset holdings at later stages and thus the corresponding trading strategies. Yet it is the trading opportunities in the later stages that determine trading strategies in the early ones. This has some interesting theoretical consequences. Firstly, there can be both, equilibria with only informed trading and equilibria with uninformed trading for the same environment. Furthermore, as shown in an example, the latter can lead to higher welfare. Secondly, in equilibrium all trades are individually rational and thus *ex ante* beneficial to agents. But since agents are atomistic, they do not take into account the effect that their individual trades have on overall trading opportunities in the future. Consequently, it is possible, as indicated by examples in the paper, that even those agents that obtain the largest surplus from these trades could be better off if they were prohibited.

As in all models of decentralized exchange, when deciding to participate in a trade agents take into account their private surplus only. Thus the private incentives to invest in making assets more tradeable are not necessarily aligned with the social ones. In our model this shows as underinvestment in information, suggesting the potential benefits of minimum disclosure regulations.

Our model has been useful to study some characteristics of asset trading that arise from the sequential nature of trades with informational and trading frictions. In order to do so, we have relied on an extremely decentralized trading structure. Two particular implications of this modeling strategy deserve consideration. Firstly, the equilibrium leaves many opportunities of mutually beneficial exchange unexploited. This clearly points to the benefits and need of analyzing more centralized institutions of trade. We have only explored a few in the paper. Secondly, the assumed indivisibilities in exchange, implicitly set the terms of trade exogenously. Though this simplifies the analysis considerably, it leaves out many interesting issues. We believe that it is important to extend the type of model considered here along these lines.

References

- ADMATI, A. R. (1989). "Information in financial markets: The rational expectations approach. In Battacharaya and Constantinides, editors, *Financial Markets and Incomplete Information Frontiers of Modern Financial Theory*, volume 2. Rowman and Littlefield.
- AMIHUD, Y. AND H. MENDELSON (1986), "Asset pricing and the bid-ask spread," *Journal of Financial Economics*, 17:223-249.
- (1989), "The effects of Beta, bid-ask spread, residual risk and size of stock returns," *Journal of Finance*, 44:479-486.
- (1991), "Liquidity, maturity, and the yields of u.s. treasury securities," *Journal of Finance*, 46(4):1411-1425.
- DIAMOND, D. W. AND R. E. VERRECHIA (1991), "Disclosure, liquidity and the cost of capital," *Journal of Finance*, 46(4):1325-1359.
- KIYOTAKI, N. AND R. WRIGHT (1989), "On money as a medium of exchange," *Journal of Political Economy*, 97(41):927-954.
- KYLE, A. S. (1985), "Continuous auctions and insider trade," *Econometrica*, 53:1315-1335.
- PAGANO, M. (1989), "Trading volume and asset liquidity," *Quarterly Journal of Economics*, pages 255-274.
- WILLIAMSON, S. D. (1991). "Liquidity and market participation. Mimeo, University of Western Ontario.
- WILLIAMSON, S. D. AND R. WRIGHT (1991). "Barter and monetary exchange under private information," Research Department Staff Report 141, Federal Reserve Bank of Minneapolis.

Appendix

Proposition 2: Continuity of conditional expectation, \hat{q}'_P .

Let $E[q'|q, n; \alpha] = \mu_I n \int_{C(q, n; \alpha)} q' n' F(dn', dq') + [1 - \mu_I n \int_{C(q, n; \alpha)} n' F(dn' dq')] q$.

Lemma 1. *The conditional expectation $E[q'|q, n; \alpha]$ is continuous in α .*

Proof: For $q \notin T$, $E[q'|q, n; \alpha] = q$ for all α . So assume $q \in T$. From Definition 2 it is easy to verify that for any α , $C(q, n; \alpha)$ is closed and that for any decreasing sequence $\alpha_m \rightarrow \alpha$, $\bigcap_m C(q, n; \alpha_m) = C(q, n; \alpha)$ and for any increasing sequence $\alpha_m \rightarrow \alpha$, $\bigcup_m C(q, n; \alpha_m) \supset \text{int}(C(q, n; \alpha))$. Since $F(q, n)$ is continuous, the boundary of $C(q, n; \alpha)$ has probability zero, so $E[q, n; \alpha]$ is continuous in α for each (q, n) . \square

Lemma 2. *Let $f(x, z)$ be a uniformly bounded continuous function and $g(x, z)$ another continuous function, where $x \in X \subset \mathbb{R}^n$ and $y \in Y \subset \mathbb{R}^m$ and both X and Y are compact. Let μ be a continuous measure on X and assume $\mu(g^{-1}(\{0\})) = 0$. Then $H(z) \equiv \int_{\{x: g(x, z) \geq 0\}} f(x, z) \mu(dx)$ is continuous in z .*

Proof: Suppose $z_n \rightarrow z$. Let χ_n° be the indicator function of $g(x, z_n) > 0$ and χ_n the indicator of $g(x, z_n) \geq 0$. Define similarly χ° and χ . Then

$$\int_{\{g(x, z_n) \geq 0\}} f(x, z_n) \mu(dx) = \int \chi_n(x) f(x, z) \mu(dx) = \int \chi_n^\circ(x) f(x, z) \mu(dx)$$

But $\inf \chi_n^\circ(x) f(x, z) \geq \chi^\circ(x) f(x, z)$ and $\limsup \chi_n(x) f(x, z) \leq \chi(x) f(x, z)$. Applying Fatou's lemma, $\int_{\{g(x, z_n) \geq 0\}} f(x, z_n) \mu(dx) \rightarrow \int_{\{g(x, z) \geq 0\}} f(x, z) \mu(dx)$ \square .

Proposition 4: Existence of Equilibria with Uninformed Trading.

Proof: Let

$$\begin{aligned} G_I(\pi, n, n'; \alpha) &= \lambda(n, \pi) \int_{\{\pi' \geq \pi\}} W(\pi', n'; \alpha) + (1 - \lambda(n, \pi)) x_1 - W(\pi, n; \alpha) \\ G_P(\pi, n, n'; \alpha) &= \varphi(n, \pi) (E\pi' | W(\pi', n'; \alpha) \leq W(\pi, n; \alpha)) + (1 - \varphi(n, \pi)) x_2 - \pi \end{aligned} \quad (17)$$

where

$$\begin{aligned} \lambda(n, \pi) &= \frac{n \int_{\pi' \leq \pi} F(d\pi', dn')}{n \int_{\pi' \leq \pi} F(d\pi', dn') + (1-n)y_1} \\ \varphi(n, \pi) &= \frac{n \int_{W(\pi', n') \leq W(\pi, n)} F(d\pi', dn')}{n \int_{W(\pi', n') \leq W(\pi, n)} F(d\pi', dn') + (1-n)y_2} \\ x_1 &= \int_{G_P(\pi, n) \geq 0} W(\pi, n) F(d\pi, dn) / y_1 \\ y_1 &= \int_{G_P(\pi, n) \geq 0} F(d\pi, dn) \\ x_2 &= \int_{G_I(\pi, n) \geq 0} \pi F(d\pi, dn) / y_2 \\ y_2 &= \int_{G_I(\pi, n) \geq 0} F(d\pi, dn) \end{aligned}$$

$G_I(\pi, n, n'; \alpha)$ gives the expected gain for an impatient agent with asset (π, n) from trading with a patient agent with an asset with verifiability n' and unknown quality. The first term in the right hand side involves the expected value of the asset obtained if the

patient agent is *informed*, while the second term gives the expected value if the patient agents is also *uninformed*. In turn, $G_p(\pi, n, n'; \alpha)$ gives the expected increase in π an uninformed patient agent would get from trading.

Define the following trading sets:

$$\begin{aligned}
A_{11} &= \{(q, n, q', n') | W(q, n; \alpha) \geq W(q', n'; \alpha) \text{ and } q' \geq q\} \\
A_{10} &= \{(q, n, q', n') | W(q, n; \alpha) \geq W(q', n'; \alpha) \text{ and } G_P(\pi, n, \pi', n'; \alpha) \geq 0\} \\
A_{01} &= \{(q, n, q', n') | G_I(\pi', n', \pi, n; \alpha) \geq 0 \text{ and } q' \geq q\} \\
A_{00} &= \{(q, n, q', n') | G_I(\pi', n', \pi, n; \alpha) \geq 0 \text{ and } G_P(\pi, n, \pi', n'; \alpha) \geq 0\}
\end{aligned}$$

A_{11} is the set of trades between informed agents; A_{10} those between an informed impatient and uninformed patient; A_{01} between an uninformed impatient and informed patient and A_{00} between both uninformed. The dependence of these sets on α should be understood.

Since the points at which $G_I = 0$ and $G_P = 0$ are generically regular, $P(G_I(\pi, n, n'; \alpha) = 0) = 0$ and $P(G_P(\pi, n, n'; \alpha) = 0) = 0$. So by applying the above lemma repeatedly one can establish that G_I and G_P are continuous in α . We now account for the endogeneity of α .

Let μ denote the product measure of initial asset holdings $F \times F$. Consider first $\bar{q}_P = Eq + \Delta q$, where Δq represents the increase in patient agent's q resulting from first period trading. The latter is given by

$$\begin{aligned}
\Delta q(\alpha) &= \int_{A_{11}} n' n (q' - q) d\mu(q', n', q, n) \\
&+ \int_{A_{10}} (1 - n') n (q' - q) d\mu(q', n', q, n) \\
&+ \int_{A_{01}} n' (1 - n) (q' - q) d\mu(q', n', q, n) \\
&+ \int_{A_{00}} (1 - n') (1 - n) (q' - q) d\mu(q', n', q, n)
\end{aligned}$$

Letting $G(\alpha) = \delta(1 - Eq + \Delta q(\alpha))$, an equilibrium is a fixed point of this function. To prove that this function is continuous we apply the above Lemma. For that purpose, it is convenient to represent the trading set by functions, as follows. Let $g_{11}(q, n, q', n'; \alpha) = \min(W(q, n; \alpha) - W(q', n'; \alpha), q' - q)$, $g_{10}(q, n, q', n'; \alpha) = \min(W(q, n; \alpha) - W(q', n'; \alpha), G_P(\pi, n, \pi', n'; \alpha))$ and define analogously g_{01} and g_{00} . Generically the zeros of these functions will be regular points and since μ is continuous the set of zeros will be μ null sets. Applying the above lemma $\Delta q(\alpha)$ will be continuous and so will function G . Since $G : [0, 1] \rightarrow [0, 1]$, there exists a fixed point \square

Figure 1.

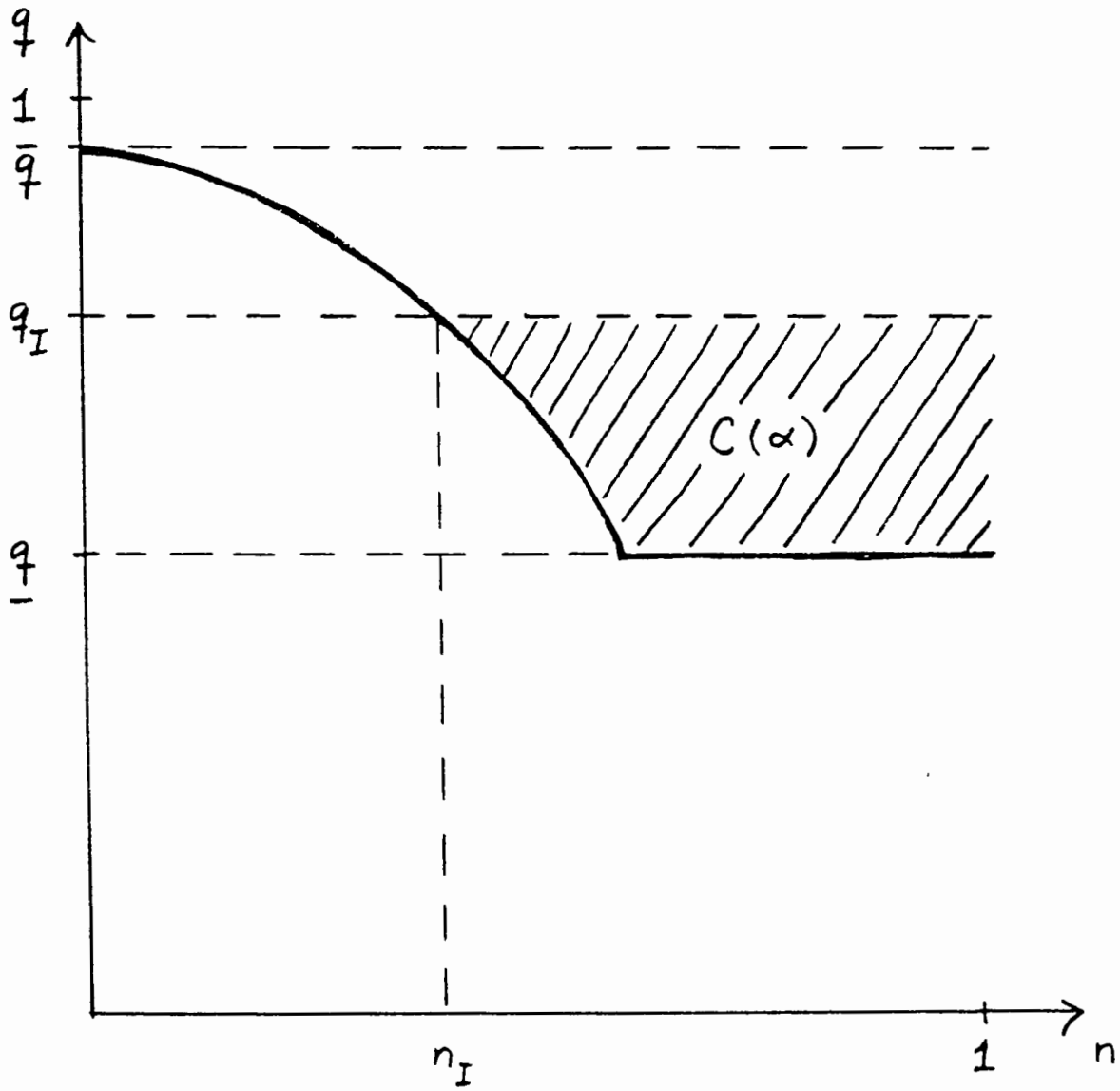


Figure 2.

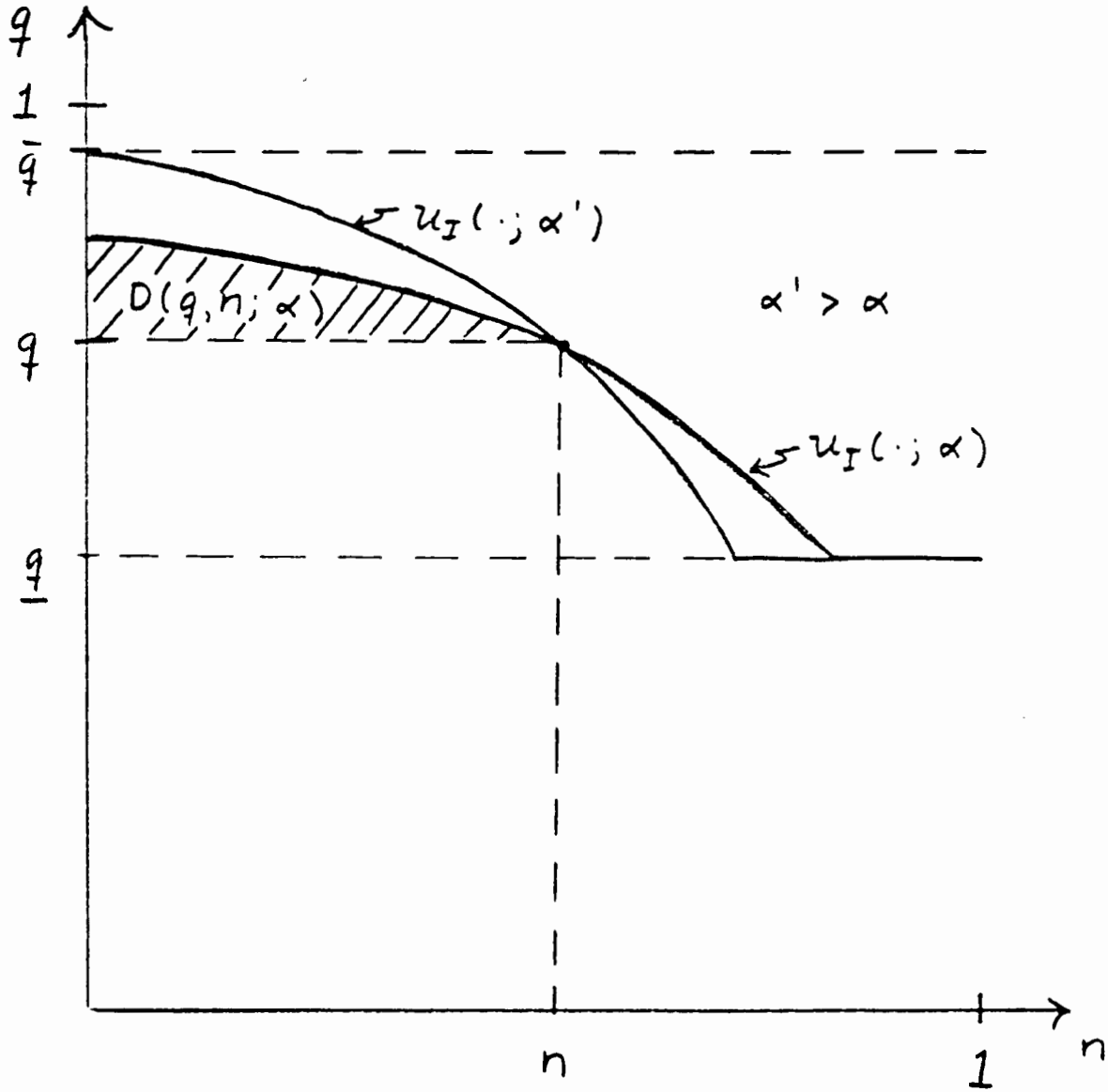


Figure 3.

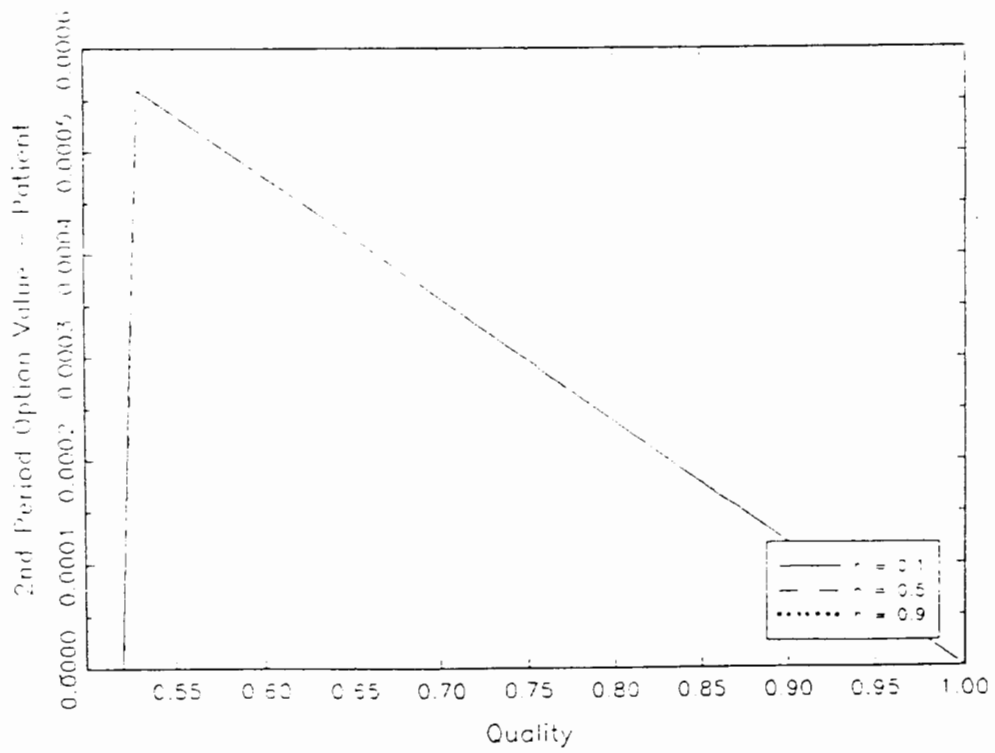
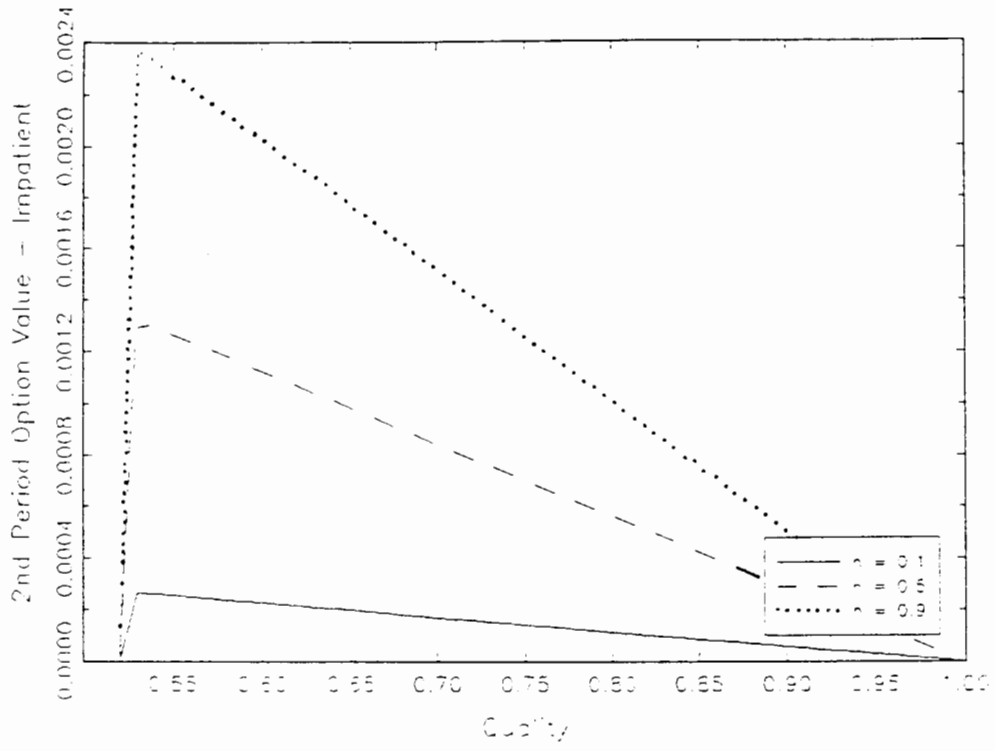


Figure 4.

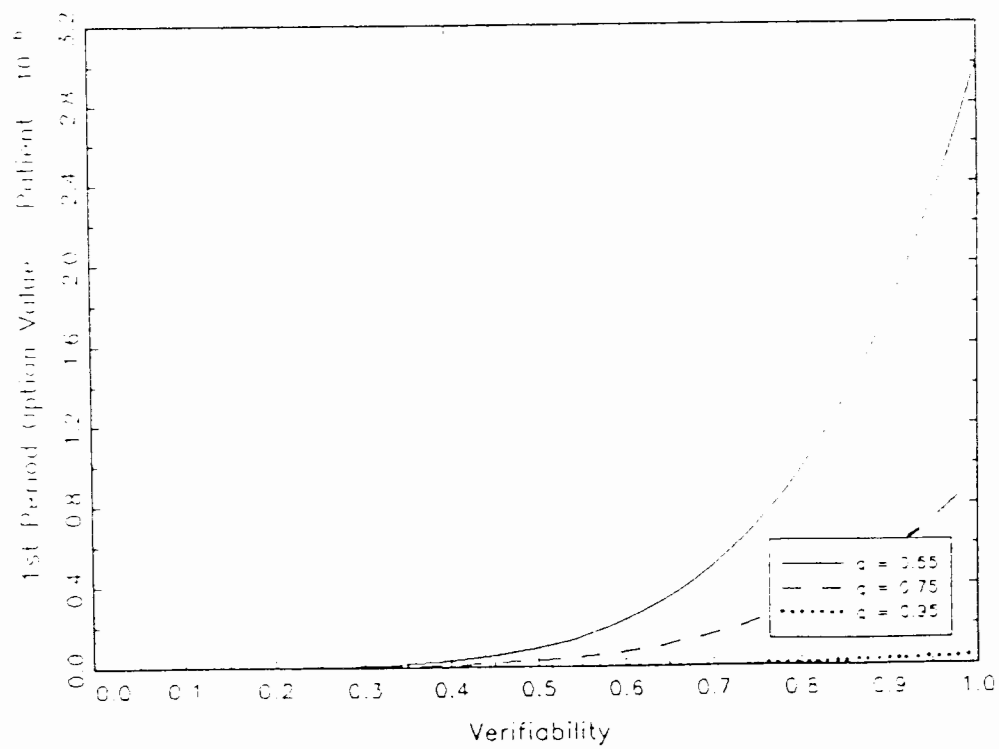
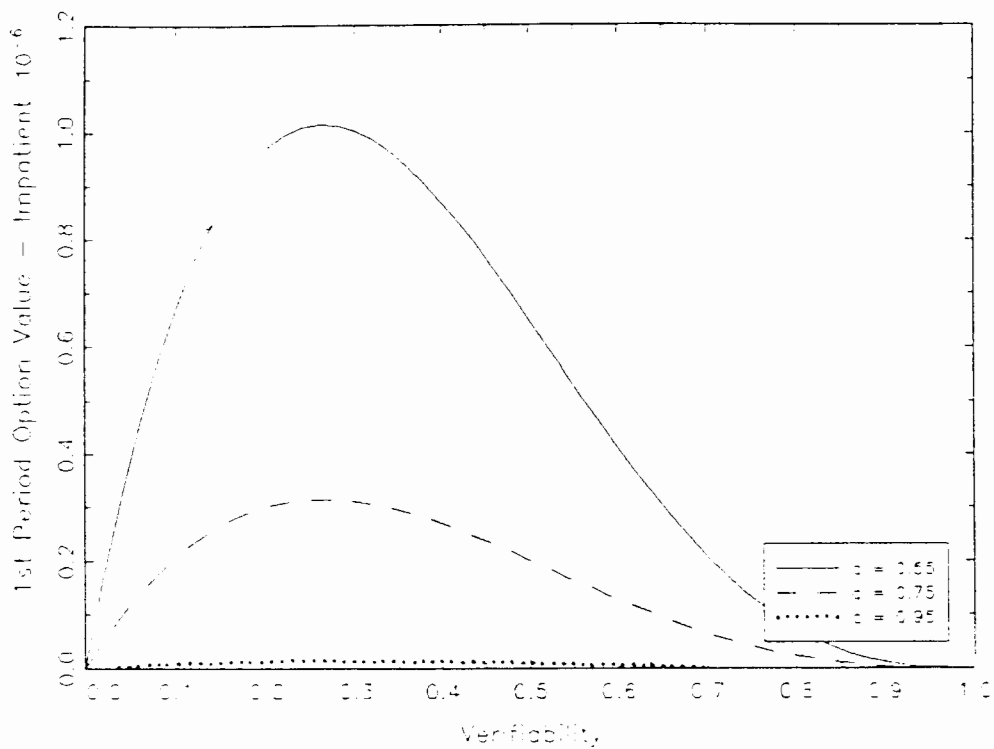
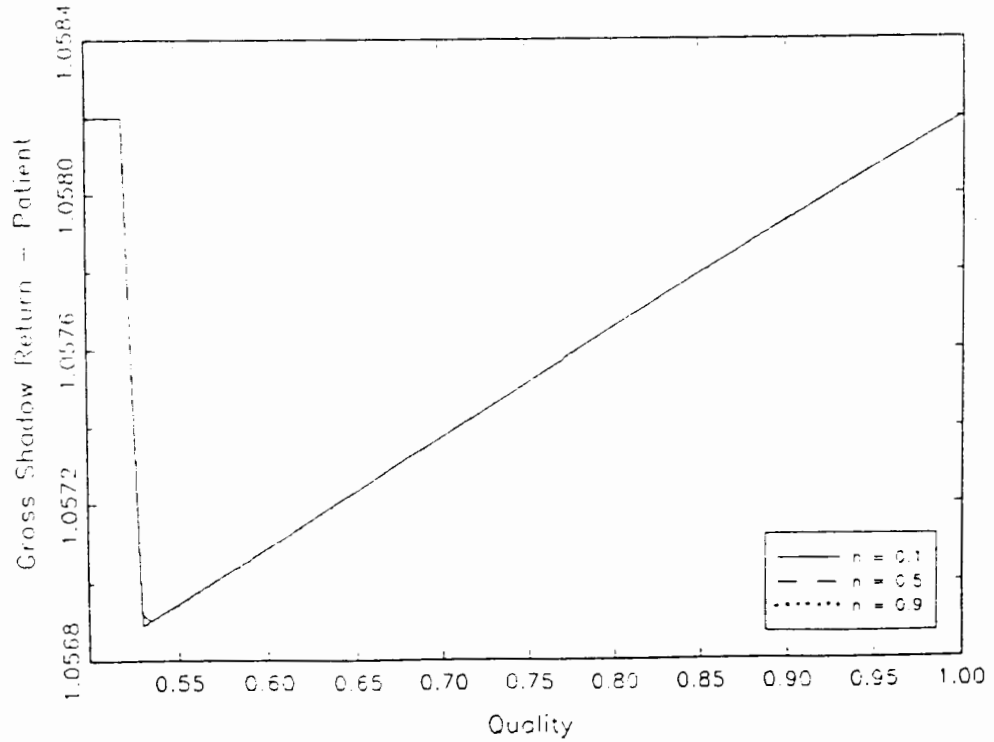
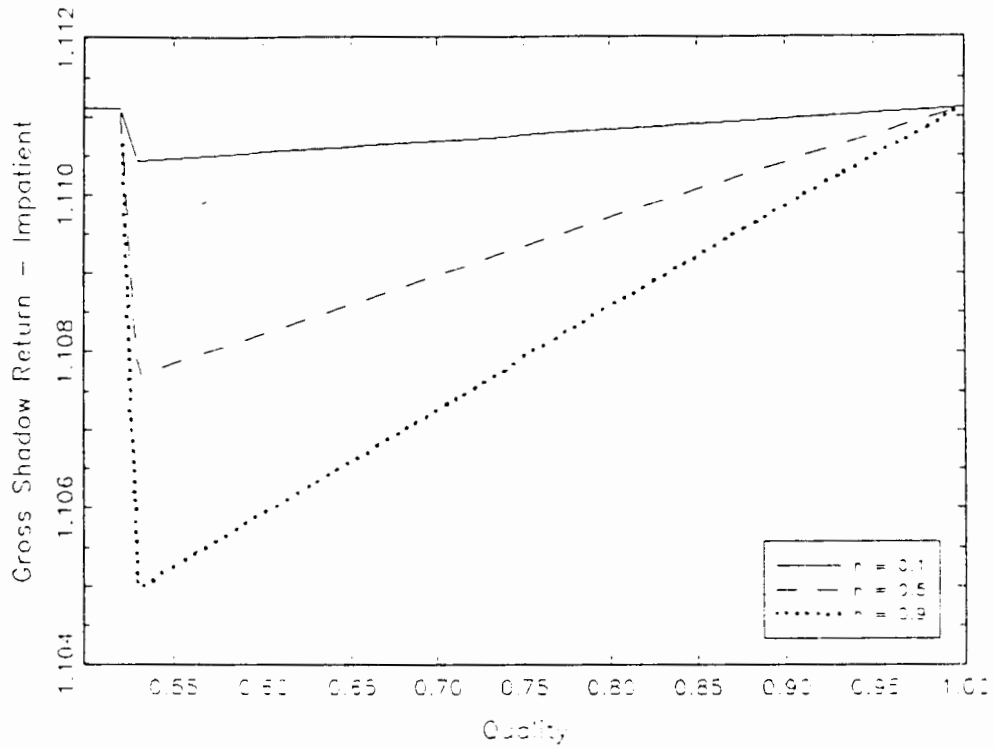


Figure 5.



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