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Does Paternity Leave Promote Gender Equality within Households?*

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Abstract

We consider a non-cooperative model of the household, in which the husband and wife decide on parental leave and the allocation of time between child rearing and the labor market. They can choose the non-cooperative outside option or cooperate by reaching an agreement of specialization in which the wife specializes in raising kids (home production) while the husband works and transfers consumption to his wife. The model identifies three distinct groups of couples: Egalitarian couples (with a sufficiently low gender wage gap), Intermediate-gap couples (with an intermediate gender wage gap) and high-gap couples (with a sufficiently high gender wage gap). Our model predicts that while egalitarian couples never specialize and always share home production, those with intermediate and high gaps do have such an agreement. An expansion in paternity leave reduces the net benefits from the agreement and moves the intermediate-gap couples to their outside option where women work more and men do more home production. As a result, the cost of raising children increases and fertility declines. Assuming a loss of utility from children in the case of divorce, lower fertility increases the probability of divorce. Using Spanish data and RDD analysis, we confirm our model's predictions. Specifically, while we don't find systematic effects of paternity leave expansion on egalitarian and high-gap couples, we find that, among intermediate-gap couples, the two-week paternity leave introduced in 2007 resulted in a reduction in fertility by up to 60%, an increase in the probability to divorce by 37%, and an increase in father's childcare and housework time as much as 2-3 hours per day.

Keywords: Gender equality, specialization, fertility, divorce, time allocation
JEL: D13, J12, J13, J16

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1 INTRODUCTION

Maternity leave has been an important policy tool to boost fertility (Raute (2019)). It allows mothers to take better care of their children and may improve children's outcomes (Rossin-Slater (2018)), and it helps women to have children without giving up a successful career (Lalive and Zweimüller (2009); Lalive et al. (2014)). To further encourage married couples to combine family life with careers, many countries now offer paternity leave.[1] The aim of this policy is to subsidize children and allow partners to share childcare activity. While subsidizing children is expected to increase the number of children, sharing childcare may increase female employment and produce more equal outcomes among spouses.

Recent empirical contributions find that paternity leave has a significant impact on families and their choices. In particular, Avdic and Karimi (2018) find that paternity leave increases the probability of separation in Sweden, and Farré and González (2019) find that the introduction of paternity leave in Spain led to a reduction in fertility. This is somewhat puzzling as parental leave is expected to reduce the cost of raising children and loosen parents' budget constraints, which is expected to increase fertility and tighten couples' relationships. Does paternity leave allow for more flexibility in sharing the childcare cost? What accounts for the reduction in fertility, and why do marriages dissolve?

Our theory provides one possible reasoning for these seemingly puzzling empirical results. The model predicts that some couples have an informal agreement, in which they trade time for consumption between themselves. Specifically, in couples with a positive gender wage gap (the husband out-earns the wife), the wife specializes in home production and the husband specializes in market work and transfers consumption for an ex-ante agreed price.[2] The agreement to specialize has two separate benefits. First, it increases productivity in raising kids and thus reduces their cost. Second, it increases the household's potential income due to the gender wage gap and returns to experience. However, the agreement also has a cost. We assume limited commitment between spouses regarding the re-

[1] We focus here on a non-transferable paternity leave, in which a quota is assigned to the father only.

[2] The model is symmetric and can account for a reversed specialization where the husband specializes in home production and the wife specializes in market work when the wife's wage is higher than the husband's. However, we limit our theoretical analysis to the more common case of positive gender wage gap. The literature on gender norms finds that couples tend to avoid a situation in which the wife out-earns her husbands. Moreover, this literature concludes that the bargaining theory is inconsistent with the behavior of such couples. Finally, the share of households, in which the wife out-earns her husband are 27% in US for the period 2008-2011 (Bertrand et al. (2015)) and 11% in Spain in 2007 (Survey of Income and Living Conditions).

distribution of consumption. Specifically, we assume two types of husbands: A fair husband who makes the transfer ex-post, and an unfair one, who shirks and does not transfer the agreed consumption to his wife. We show that, in our setting, the unfair husband can always mimic the fair one and proposes to his wife the same agreement proposed by a fair husband. The model thus generates a pooling equilibrium.

The model shows that once paternity leave is introduced or expanded, the net benefits from the agreement decline. Therefore, the marginal couple moves from a state of agreement to a state of no-agreement. That is, they stop specializing and the father starts taking paternity leave. As a result, the cost of raising children goes up, and fertility declines. Assuming that divorce yields a loss of utility from kids (Weiss and Willis (1985), Browning et al. (2014)), a reduction in fertility increases the probability of divorce. Put differently, the transition from a state of agreement to a state of disagreement triggers separation, which works in our model through the under provision of the public good, namely, fertility.

The intuition for the dissolution of agreements is as follows. An increase in paternity leave, which is a subsidy for raising children, pushes the household's solution towards more children. Given an agreement, in which the wife is specializing in raising kids, the price of compensation in terms of consumption that the wife requests increases by more than the amount that the husband is willing to pay, due to the risk of shirking by unfair husbands. As a result, they don't agree on the price, and the agreement to specialize stops being optimal.[3]

In our model, each agent derives utility from private consumption, the number of children, and a non-pecuniary value that reflects the match quality of the relationship. They face exogenous wages and decide on their labor supply and the allocation of time to raising their children.

The spouses make their economic choices in two stages. In the first stage, each agent decides non-cooperatively on their own labor supply, which implies the remaining time that each one will allocate to raising kids. Each one can also decide whether to take parental leave. This will define the outside option of each agent. The couple may choose the outside option or reach an agreement, in which they trade time for consumption between themselves. Specifically, the husband proposes to reduce the time for raising kids, which may include avoiding paternity leave, and make a transfer to his wife. Once they make their first stage choices (la-

[3]The intuition describes the situation in which couples have an agreement in which the husband does not take paternity leave. In case he does, his loss of experience at work and his learning by doing in raising kids act as other forces affecting his willingness to compensate his wife.

bor supply and childcare time), they are committed to this time allocation, as the labor market commitment is binding. In the second stage, the quality of the relationship is revealed. At this stage, couples with a sufficiently low match quality (but not too low) may save their marriage by redistributing consumption again (Becker et al. (1977)). Thus, the time allocation within the household is driven by a trade-agreement between spouses with an outside option given by a noncoopertaive game in the first stage (Lundberg and Pollak (1993)) and a divorce threat in the second stage (Manser and Brown (1980); McElroy and Horney (1981)).

Our model predicts that the household's choice depends on the gender potential wage gap between the spouses. The model distinguishes between three distinct groups. First, couples with a sufficiently low gender wage gap, who do not reach an agreement, and thus choose the outside option in which they share childcare costs. Henceforth, we will label this group as "egalitarian" couples. Although an expansion in paternity leave may have some effect on their time allocation, fertility, and consumption, these couples will keep choosing their outside option. Second, for a sufficiently high gender wage gap the range of prices (compensation) that sustains an agreement in equilibrium is very large, such that a paternity leave expansion changes the agreement's conditions but will keep its existence optimal. Henceforth, we will label thus group as "high-gap" couples. Third, in between these two corner groups comes the group that is most affected by the paternity leave policy. For this group, the range of prices that sustains an agreement in equilibrium is small enough that small expansions in paternity leave will make this range empty. Namely, couples cannot find a price of time that will make at least one of them better off. In this case, couples will move from a state of agreement to their outside option. Henceforth, we will label this group as "intermediate-gap" couples.

While the effect of paternity leave for the two corner groups is small and continuous, it is discrete and discontinuous in the intermediate one. The intermediate-gap couples move from a state of agreement in which they specialize, to the outside option in which they share childcare. Thus, the model predicts that the main effect of paternity leave expansions is expected to be found among intermediate-gap couples, while paternity leave policies will merely lead to small income and price (substitution) effects for egalitarian and high-gap couples.

For intermediate-gap couples, our model has five predictions concerning the effect of an introduction or expansion of paternity leave: (1) an increase in take-up of paternity leave; (2) a reduction in fertility; (3) an increase in the probability to divorce; (4), an increase in women's employment at the expense of childcare time; (5) an increase in men's childcare time at the expense of their employment.

We test the predictions of the model in the setting of Spain, which introduced two weeks of paternity leave in 2007. Eligibility was based on the date of birth of the child. We study the effect of paternity leave on household outcomes following a regression discontinuity design, such that we compare families who had a child very close to the cutoff date, and are thus very similar on average along all dimensions except paternity leave eligibility. We study take-up using the *Survey on the Use of Parental Leave and their Labor Consequences*, conducted between January and June 2012 in the metropolitan area of Madrid. The survey targeted parents living in Madrid with a child aged 3 to 7 at the time of the survey. The Madrid Survey (MS) provides information on the month and year of birth of the youngest child, as well as data on parental leave take-up. To analyze the effects of the 2007 extension on parental labor supply, fertility and divorce, we merge *Labor Force Survey* (LFS) data for all quarters of 2008-2010. Finally, to analyze the effects of paternity leave on fathers' (and mothers') time-use, and in particular the time devoted to childcare and housework, we use the *Spanish Time-Use Survey*, conducted between October 2009 and September 2010.

A non-trivial question is how to classify families as egalitarian, intermediate-gap and high-gap couples. Since potential wages are both endogenous and unobserved, the literature uses the gap in age and/or educational attainment between the spouses to proxy or predict potential wages (Folke and Rickne (2020); Bertrand et al. (2015)).[4] Therefore, one possible way is to follow this literature and use the gap in age and/or education between partners to proxy for the three distinct groups that our model identifies. However, any exogenous choice will be somewhat arbitrary. While our model predicts that fathers' take-up time of parental leave is barely affected in egalitarian and high-gap couples, it predicts that fathers' response in the intermediate-gap couples is discrete and significant. Thus, we run a first stage, in which we calibrate the age-gap and education-gap thresholds between the three groups of couples by targeting some moments in the take-up data. Specifically, we target a very small take-up response to paternity leave introduction for the corner groups while maximizing the effect for the intermediate one. That is, we sacrifice the take-up data to calibrate and endogenize the classification of couples into the three groups.

We find that the take-up data gives rise to such a pattern: Very small and insignificant effects of paternity leave in the corners and full effect (around 13 days) in the middle, which supports the model per-se. Then, in a second stage, we use this endogenous classification to test our model predictions with regard to sub-

[4] While Folke and Rickne (2020) proxy for the type of couple by the gender age gap, Bertrand et al. (2015) use race, age and education to predict potential wages.

sequent fertility, time-use and separation. We find that the 2007 paternity leave introduction affects the intermediate-gap couples more than egalitarian or high-gap ones. Specifically, for middle group couples, it takes longer to have another child: the probability of having an additional birth decreases by about 50%-60% up to 15 months after the policy change. The time-use data shows that fathers in this group devote more of their time to childcare (44-80 minutes per day) and housework (88-117 minutes per day), which makes their input in home production close to mothers'. They are also more likely to break up. The probability of separation increase by about 37% for mothers in the intermediate-gap group. These effects are not found in egalitarian or high wage gap couples. Our empirical results thus strongly support the main predictions of the model.

Our theory relates to several literatures. First, the literature in household economics on the importance of the gender wage gap in understanding household choices abounds (Becker et al. (1977)). Within a unitary model, Galor and Weil (1996) were the first to establish a relationship between fertility, female employment and the gender wage gap. Later contributions examine the role of the gender wage gap in a variety of important outcomes such as women's empowerment (Duflo (2012); Doepke and Tertilt (2019)); the marketization of childcare (Hazan and Zoabi (2015a); Gobbi et al. (2018); Bar et al. (2018)); international trade (Sauré and Zoabi (2014); Do et al. (2016)) and the gender educational gap (Chiappori et al. (2009); Becker et al. (2010); Hazan and Zoabi (2015b)). Second, limited commitment plays a crucial role in our theory. Within a dynamic setting, Voena (2015) argues that a unilateral divorce yields lack of commitment, which produces distortion in household asset accumulation, Gobbi (2018) finds that limited commitment between spouses produces an underinvestment in childcare, and Doepke and Kindermann (2019) argue that disagreement over having children stems from lack of commitment. Similarly, in our theory, limited commitment produces a cost of specialization, and causes partners to disagree. Finally, Meier and Rainer (2017) study some theoretical aspects of paternity leave and argue that paternity leave solves the hold-up problem and may increase fertility.

Efficiency in our model is reached through trade as our model assumes a noncooperative outside option (Lundberg and Pollak (1993)), which describes the final solution among egalitarian couples. While noncooperation gives rise to an inefficient allocation of resources, the literature on family economics argues that since partners interact on a daily basis, they learn how to reach efficiency (Browning et al. (2014)). However, our model shows that the lack of commitment sometimes prevents this efficiency from being achieved. The under provision of the public good (fertility) driven from the noncooperative element is sufficient but not necessary for our story to hold. Any partial specialization in the outside option gives the same qualitative results. What is crucial for our story to hold is the absence of any sort of transferable utility in the outside option, which is reasonable to assume. Moreover, assuming a cooperative framework will not alter any of our results, as it increases the provision of the public good when couples depart from their outside options. Put differently, moving from a cooperative solution (or an agreement) to a noncooperative outside option gives the same predicted change in fertility, time allocation, and divorce. The only difference is that, while the cooperative framework focuses on preferences our mechanism focuses on the production side. Finally, the non-cooperative outside option is tractable as it clearly explains the the lack of specialization, which plays an important role in our results on fertility, divorce and time allocation that are consistent with the data.[5]

Our empirical analysis also connects to the literature that has evaluated the effects of paternity leave extensions in different countries. Several papers have documented large effects on take-up in the US, Norway, Sweden and Canada (Bartel et al. (2018); Cools et al. (2015); Dahl et al. (2014); Ekberg et al. (2013); Patnaik (2019)). Few studies have analyzed the effect of paternity leave on fertility, and most of those report zero effects on average. Cools et al. (2015), Dahl et al. (2014) and Kotsadam and Finseraas (2011) found no effects of paternity leave extensions on fertility in Norway, and Bartel et al. (2018) reported similar results for the US. On the other hand, Farré and González (2019) and Fontenay and Tojerow (2020) found negative effects on fertility in Spain, and Belgium, respectively.

Recent work has also reported that an increase in fathers' share of parental leave increased marital separation rates in Sweden (Avdic and Karimi (2018)), although Dahl et al. (2014) and Cools et al. (2015) found no effect of paternity leave on marital stability in Norway, and Olafsson and Steingrimsdottir (2020) find a positive effect on marital stability in Iceland.

Regarding the effects of paternity leave on parents' labor supply and fathers' involvement in childcare and housework, the evidence is somewhat mixed, with some papers finding zero or very small effects, and others finding positive effects on mothers' employment and on fathers' childcare time (Cools et al. (2015); Dahl et al. (2014); Dunatchik and Özcan (2017); Ekberg et al. (2013); Kluve and Tamm (2013); Patnaik (2019); Rege and Solli (2013); Tamm (2018)).

Our model is thus able to reconcile many of these empirical findings. We show

[5]For a deep discussion about the validity of the non-cooperative game and the separate budget constraints within the family, see Doepke and Tertilt (2019), who analyze the effect of mandated transfers on the public good provision.

that small or zero average effects can coexist with large effects for the subgroup of more affected, “intermediate wage gap” families.

Overall, our theory and empirical examination show that introducing or expanding paternity leave pushes some couples from preserving some traditional roles (the husband is the breadwinner and wife is homemaker) to act as egalitarian ones. That is, paternity leave does induce more couples to share home production while developing their careers. Our model thus expresses the idea that while maternity leave was designed to help women, it actually increases women’s relative advantages in home production and thus pushes some of them into the traditional role, which makes them carry the major burden of child raising and home production. Our model shows that mothers know that fathers take paternity leave in the outside option. Therefore, in a non-cooperative framework, mothers’ best response is to reduce their time input in home production and work more. This is the final effect for egalitarian couples. However, for couples with agreements, this results in a larger redistribution of consumption from the father to the mother. Thus, paternity leave increase the bargaining power of women, which empowers women.

Recent literature argues that labor market penalties associated with motherhood are the main obstacle for closing the gender earnings gap (Kleven et al. (2019a,b); Bertrand (2020); Titan et al. (2021)). Our paper thus argues that balancing maternity leave with a non-transferable paternity leave mitigates this problem. Paternity leave seems to be an instrumental tool in promoting gender equality.[6]

The paper proceeds as follows. Section 2 presents the setup of the model. Section 3 characterizes the equilibrium and provides the main results. Section 4 presents our empirical analysis. Section 5 concludes, and Figures, Tables and our proofs appear in the Appendix.

[6]The literature finds that not only does equality in treatment promote gender equality but it also promotes economic development (Doepke and Tertilt (2009); Hazan et al. (2019)).

2 THE MODEL

Consider a married couple, which is composed of a man (father), m and woman (mother), f. Each agent derives utility from private consumption, the number of children and a non-monetary value that reflects the match quality of the relationship. They face exogenous wages and decide on their labor employment and the allocation of time in raising their kids. The utility of agent $i \in \{m, f\}$ is given by

$$U_i(c_i, n, \theta_i) = \log c_i + \log n + \theta_i \tag{1}$$

where $c_i \geq 0$ is the private consumption of an agent i, and $n \geq 0$ is the couple's number of children. θ_i is a non-monetary shock to marriage that is revealed by the end of the first period (Weiss and Willis (1993; 1997), Browning et al. (2014)). θ_i is drawn from a given distribution with zero mean and positive variance: $\theta_i \sim (0, \sigma^2)$.

Figure 1 sketches the sequence of events that the couple faces. They make their economic choices in two stages. In the first stage, each agent decides non-cooperatively on their own labor supply, which implies the remaining time that each one will allocate for raising children. Each one can also decide whether to take parental leave. This will define the outside option of each agent. The couple may choose the outside option or reach an agreement, in which they trade time for consumption between themselves. Specifically, the husband proposes to reduce the time for raising kids, which may include avoiding paternity leave, and make a transfer to his wife. Once they make their first stage choices - labor supply and childcare time - they are committed to this time allocation as the labor market commitment is binding.[7] In the second stage the quality of the relationship is revealed. At this stage, couples with a sufficiently low match quality, but not too low, may save their marriage by redistributing consumption again.

[Figure 1]

Raising children requires parents' time, and the number of children n is given by:

$$n(t_m + \tau_m, t_f + \tau_f) = (t_m + \tau_m)^a + (t_f + \tau_f)^a \tag{2}$$

[7]This assumption is sufficient and the necessary assumption is that the labor market commitment is stronger than the commitment between opuses in redistributing consumption. This assumption captures the fact that once the mother specializes in raising children, she loses future income and thus consumption.

We assume that $a > 1$, which reflects "*learning by doing*" in raising children. $\tau_i \geq 0$ is the maximum time that the government can finance, $t_i \geq 0$ is the time individual i spends on raising children.

Agents accumulate experience at work. The budget constraint of an individual i is given by

$$c_i(t_i, \tau_i) = w_i(1 - t_i)(1 - \tau_i - t_i) \tag{3}$$

where w_i is an exogenous wage per unit of time. $(1 - t_i)$ is the labor supply and $(1 - t_i - \tau_i)$ is the return to experience. We assume a non-transferable parental leave, which is paid to compensate fully for the labor income forgone.[8] We also assume that the wage of the male parent is more than the wage of the female parent.

2.1 Outside option

In the outside option in the first stage parents choose the amount of time that they spend for raising children and the parental leave that they take. It is assumed that in this case of non-cooperation, the solution is given by a Nash-Cournot game, i.e. the couple solves the maximization problems:

$$\max_{t_m, \tau_m} \mathbb{E}(U_m)$$
$$\max_{t_f, \tau_f} \mathbb{E}(U_f)$$

with the constraints $t_i \geq 0$, $\tau_i \geq 0$, $i \in \{m, f\}$. And also the time constraint $\tau_i + t_i \leq 1$.

Denote the optimal outside option solution by t_m^0, t_f^0, τ_m^0, τ_f^0.

2.2 First stage

In the first stage of the game, the father chooses an optimal amount of time which he wants to "buy" from his spouse (t_m, τ_m).[9] Then the couple bargains over redistributing consumption which is given by a transfer T.

[8] The full wage compensation is just a simplifying assumption. Any partial payment delivers the same qualitative results.

[9] Transferring τ means that the husband does not take paternity leave but buys this time from his wife as well.

An *agreement* exists if there exists a set (t_m, τ_m, T) that both agents gain from agreeing upon transferring time for consumption, i.e. the expected utility of co-operating is greater than in the outside option for both parents. If it exists, we call this set (t_m, τ_m, T) an *agreement*. By having an agreement, the father increases his working time at the market by t_m or $t_m + \tau_m$, and pays the transfer to his spouse. The mother thus increases her time spent on raising children by t_m, or $t_m + \tau_m$.[10]

We assume two types of men: fair and unfair. The fair father follows the rules of the agreement, and transfers the arranged T in full. The unfair father shirks and sends only a part of the transfer T if the agreement occurs (assume for simplicity that for the unfair case $T = 0$).

Denote by c_{fj} the mother's consumption and by c_{mj} the man's consumption for the type of a father $j \in \{f, u\}$ - fair or unfair. Since $T_f = T$ and $T_u = 0$, the budget constraints for both spouses are:

$$c_{mj} = w_m(1 - t_m^0 + t_m)(1 - t_m^0 - \tau_m^0 + t_m + \tau_m) - T_j$$
$$c_{fj} = w_f(1 - t_f^0 - t_m - \tau_m)(1 - t_f^0 - \tau_f^0 - t_m - \tau_m) + T_j$$

Thus, the number of kids is:

$$n = (t_m^0 + \tau_m^0 - t_m^* - \tau_m^*)^a + (t_f^0 + \tau_f^0 + t_m^* + \tau_m^*)^a$$

In the second stage, which will be discussed below, the spouses can decide whether to get a divorce upon the realization of a non-monetary shock θ. Denote the ex-ante probability of divorce in the second period by p_d. Then the spouses maximize expected utility for both periods.

The expected utility of a father of type $j \in \{f, u\}$:

$$\mathbb{E}(U_{mj}(t_m, \tau_m, T)) = (1 - p_d)U_{mj}^M + p_d U_{mj}^D$$

Where U_{mj}^M is the utility in case of stable marriage without divorce:

$$U_{mj}^M = \log c_{mj} + \log n + \theta$$

And U_i^D is the utility in case of getting a divorce:

$$U_{mj}^D = \log c_{mj} + d \log n$$

[10] As will be analyzed below, our model allows for two types of agreements: one in which the fathers take a paternity leave and this is the only time they spend raising children and the other in which they do not take paternity leave and make a higher compensation to their spouses.

We follow the literature by assuming that in case of a divorce the spouses do not suffer from non monetary shock, but they receive less utility from children.

Note that it is always profitable for an unfair father to pursue an agreement, as he benefits from specialization, and does not send back anything. A fair father cannot separate himself from an unfair one. Since a mother would not make an agreement with an unfair father, the optimal strategy for an unfair father is to mimic a fair one. Thus, the model generates a pooling equilibrium. We consider the problem of a fair father. We also assume that the mother knows the distribution of types - $0 < \beta < 1$ share of fathers are of type f, and $1 - \beta$ are of type u - and that she believes that her husband is fair with probability β.
Therefore, the mother's expected utility:

$$\mathbb{E}(U_f(t_m, \tau_m, T)) = \beta \left[(1 - p_d)U^M_{ff} + p_d U^D_{ff}\right] + (1 - \beta) \left[(1 - p_d)U^M_{fu} + p_d U^D_{fu}\right]$$

where U_{ff} is the mother's utility in the case of a realized "fair" spouse, and U_{fu} stands for an "unfair" one.

The father of type j chooses $t_m,\ \tau_m,\ T$ to maximize $\mathbb{E}(U_{mj}(t_m, \tau_m, T))$ subject to budget constraints.[11] The solution of this problem are optimal $t^*_m,\ \tau^*_m$ and maximum price that the father is ready to pay, T_m.

The optimization problem for the mother is the following. Given $t^*_m,\ \tau^*_m$ she chooses T to maximize $\mathbb{E}(U_f(t^*_m, \tau^*_m, T))$ subject to budget constraints. Denote the minimum transfer that the mother is ready to accept in an agreement by T_f.

The equilibrium transfer is the weighted average between T_m and T_f:

$$T = \alpha T_m + (1 - \alpha)T_f,\ \alpha \in [0, 1] \tag{4}$$

The *agreement* exists if there exist T such that having non-zero transfers of time is profitable:

$$\mathbb{E}U_f(t_m, \tau_m, T) > \mathbb{E}U^0_f$$
$$\mathbb{E}U_m(t_m, \tau_m, T) > \mathbb{E}U^0_m$$

where U^0_i is the utility in the outside option for individual i.

Denote the optimal utility levels at this stage by $U^1_i(t_m, \tau_m, T), i \in \{m, f\}$.

[11] We also assume that the mother does not exhaust all of her time upon making an agreement: $t^0_f + \tau^0_f + t^0_m + \tau^0_m \leq 1$.

2.3 Second stage

In the second stage, the non-monetary shock θ_i is realized and the individuals can divorce. They choose to divorce if their utility from staying married (defined in the first stage) is less than from divorcing. The utility of an agent $i \in \{m, f\}$ for a type $j \in \{f, u\}$ of a father in case of divorce is:

$$U_i^D = \log c_{ij} + d \log n$$

The time spent for children t is fixed due to the commitment in labour time. That is, even in case of divorce the agents spend the same amount of time for work as in the marriage. In turn, it implies that the private consumption is also fixed. We follow Browning et al. (2014) and assume that after a divorce the agents' utility from children is depreciated, as captured by d $(d < 1)$.[12]

At this stage the agents can choose the transfer T^M to prevent divorce. The divorce does not occur if there exists T^M such that both agents find it more appealing to keep the marriage:

$$U_m^M(t_m, \tau_m, T, T^M) > U_m^D$$
$$U_f^M(t_m, \tau_m, T, T^M) > U_f^D$$

where U_m^M and U_f^M are the utilities of a father and a mother in case of keeping their marriage:

$$U_m^M = \log(c_{mj} + T^M) + \log n + \theta$$
$$U_f^M = \log(c_{fj} - T^M) + \log n + \theta, \; j \in \{f, u\}$$

3 Solution

Firstly, we solve for the outside option:

$$\max_{t_m, \tau_m} \mathbb{E}(U_m)$$
$$\max_{t_f, \tau_f} \mathbb{E}(U_f)$$

with the constraints $\tau_i \geq 0, \; t_i \geq 0, \; \tau_i + t_i \leq 1$ for $i = \{m, f\}$.

[12]This drop in utility from children comes to capture the idea that since parents are not living together after divorce, they lose part of the control they had over children. Weiss and Willis (1985) argue that "divorce causes [parents] to reveal a reduced interest in the welfare of their children".

Lemma 1. *In any agreement $T \geq 0$*

Proof. See the proof in the Appendix II. ∎

It immediately follows that whenever a mother can observe the unfair spouse, she chooses the outside option. We now show that in any agreement an unfair father mimics a fair one, thus constituting a pooling equilibrium.

Proposition 1. *In any agreement, a weakly dominant strategy of an unfair father is to imitate a fair father.*

Proof. See the proof in the Appendix II. ∎

In the outside option, the utilities of fair and unfair fathers are the same, so if there is an agreement, then an unfair father can benefit from imitating a fair father with the agreement still being beneficial.
This result allows us to consider solely a problem of a fair father for the solution of the first stage of the game.

Proposition 2. *In equilibrium in the outside option, agents always prefer full paid leave.*

Proof. The agents can choose the paid parental leave of $\tau_i \leq \bar{\tau}_i$. Assume by contradiction that $\tau < \bar{\tau}$. Consider $\tilde{\tau}_i = \tau_i + \Delta$, $\tilde{t}_i = t_i - \Delta$. Then:

$$\mathbb{E}\tilde{U}_i = \log(1 - t_i + \Delta) + C$$

where C includes all the terms which do not depend on Δ. It is clear that for positive Δ the expected utility increases, so it is profitable to choose $\tau_i = \bar{\tau}_i$. ∎

The outside option solution is the set: $(t_m^0, \tau_m^0, t_f^0, \tau_f^0)$.

In other words, since in the outside option t is interior and since it is assumed that the time required for raising children is far above paternity leave, τ paternity leave policy becomes a free subsidy granted to fathers as they can reduce t by the size of τ.

Now let us move on to solving the first stage of the game for cooperation. First of all, we show that we do not need to solve the model in terms of expectations as it suffices to solve it only for the case $\theta = 0$.

Lemma 2. *The problems*

$$\max_{t_m, \tau_m} \mathbb{E}(U_m)$$
$$\max_{t_f, \tau_f} \mathbb{E}(U_f)$$

and

$$\max_{t_m,\tau_m} U_m(\theta = 0)$$
$$\max_{t_f,\tau_f} U_f(\theta = 0)$$

subject to identical constraints:

$$\tau_i \geq 0$$
$$t_i \geq 0$$
$$\tau_i + t_i \leq 1$$

for $i = f, m$
have coinciding solutions

Proof.

$\mathbb{E}(U_m) = \log w_m(1 - t_m)(1 - \tau_m - t_m) + \log n + \mathbb{E}(\theta) = U_m|_{\theta=0}$
$\mathbb{E}(U_f) = \log w_f(1 - t_f)(1 - \tau_f - t_f) + \log n + \mathbb{E}(\theta) = U_f|_{\theta=0}$

■

In order to solve the first stage,let us define the utilities from the second stage. In the second period, the choice is between getting divorced and keeping the marriage. The divorce does not occur if there exists T^M such that:

$$U_m^M(t_m, \tau_m, T, T^M) > U_m^D$$
$$U_f^M(t_m, \tau_m, T, T^M) > U_f^D$$

where U_i^M and U_i^D are the utilities for sustaining marriage and divorce respectively for the individual i.

Assume that the realized shock is identical for both agents. The following proposition provides the equilibrium p_d.

Proposition 3. *The stability of the marriage does not depend on the transfer T. The agents choose to divorce if and only if*

$$\theta < (d - 1) \log n$$

Proof. See Appendix II for proof. ■

In fact, the first-stage transfer does not affect the probability of divorce since in the second stage for sufficiently high match quality the couple is always able to agree on a transfer to keep their marriage.

Corollary. *Assume the uniform distribution of* $\theta \in [x_1, x_2]$ *s.t.* $x_1 < (d-1)\log n_0 < x_2$*. Then the probability of divorce is*

$$p_d = \frac{x_1 - (d-1)\log n}{x_1 - x_2}$$

Proof. See Appendix II for proof. ∎

Having obtained these results, we can find the solution for the first stage. If the agents decide to have an agreement, then in equilibrium it is optimal to transfer all the time with the children, i.e. the father chooses full specialization:

Proposition 4. *If it is optimal to transfer a single unit of time, then it is optimal to transfer the full amount of time. Formally, one of the following constraints always binds in the maximization problem in the first stage:*

$$t_m^0 \geq t_m$$
$$t_m \geq 0$$

Proof. See Appendix II for proof.[13] ∎

Proposition 5. *If it is optimal to transfer a single unit of parental leave, then it is optimal to transfer the full parental leave. Formally, one of the following constraints always binds in the maximization problem in the first stage:*

$$\tau_m^0 \geq \tau_m$$
$$\tau_m \geq 0$$

Proof. See Appendix II for proof.[14] ∎

Intuitively, both propositions state that whenever it is optimal to transfer a single unit of time, it is optimal to transfer the full amount of time. This is because once the mother devotes more time to childcare, she becomes better by the assumption of learning by doing. Similarly, the more the father specializes in work, the larger his wage becomes due to the assumption of positive return to experience. both assumptions increase the benefit from an agreement the larger the traded amount of time is, which leads to corner solutions.

Hence, it suffices to compare 4 cases:

- Agents choose an outside option

[13]The idea of the proof is convexity of a maximization problem with respect to t_m for both agents. Thus, the optimal solution is one of two corner solutions.

[14]The idea of this proof is also convexity of a maximization problem w.r.t. τ_m.

- Agents choose an agreement in both t and τ: $(t_m = t_m^0, \tau_m = \tau_m^0, T)$
- Agents choose an agreement in t: $(t_m = t_m^0, \tau_m = 0, T)$
- Agents choose an agreement in τ: $(t_m = 0, \tau_m = \tau_m^0, T)$

We can further reduce the number of cases:

Proposition 6. *The agreement $(t_m = 0, \tau_m = \tau_m^0, T)$ is not optimal*

Proof. See Appendix II for proof. ■

The intuition is that since t^0 is interior and since τ is paid by the government it is always cheaper to trade in t before τ.

Thus, to solve the model we need to consider three cases. The first is when we do not have agreement at all. That is, for any T the agents derive higher utililty from the outside option[15]:

$$U_m(c_m(0, \tau_m^0 - \tau_m) - T, n(\tau_m^0 - \tau_m, \tau_f^0 + t_m^0 + t_f^0 + \tau_m), \theta) < U_m(c_m(t_m^0, \tau_m^0), n(\tau_m^0 + t_m^0, \tau_f^0 + t_f^0), \theta)$$
$$\beta(U_f(c_{ff}(t_f^0 + t_m^0, \tau_f^0 + \tau_m) + T, n(\tau_m^0 - \tau_m, \tau_f^0 + t_m^0 + t_f^0 + \tau_m), \theta))+$$
$$(1-\beta)(U_f(c_{fu}(t_f^0 + t_m^0, \tau_f^0 + \tau_m), n(\tau_m^0 - \tau_m, \tau_f^0 + t_m^0 + t_f^0 + \tau_m), \theta) <$$
$$U_f(c_f(t_f^0, \tau_f^0), n(\tau_m^0 + t_m^0, \tau_f^0 + t_f^0), \theta)$$

where $\tau_m = \{\tau_m^0, 0\}$, i.e. neither agreement only on t_m or on both $t_m,\ \tau_m$ is profitable.

Secondly, if the previous conditions are not satisfied, we have an agreement. Moreover, the optimal agreement is an agreement[16] in both t_m and τ_m if for any T there exists T_1:

$$U_m(c_m(0, \tau_m^0) - T, n(\tau_m^0, \tau_f^0 + t_m^0 + t_f^0), \theta) < U_m(c_m(0,0) - T_1, n(0, \tau_f^0 + t_f^0 + \tau_m^0 + t_m^0), \theta)$$
$$U_f(c_f(t_f^0 + t_m^0, \tau_f^0) + T, n(\tau_m^0, \tau_f^0 + t_f^0 + t_m^0), \theta) < U_f(c_f(t_f^0 + t_m^0, \tau_f + \tau_m^0) + T_1, n(0, \tau_f^0 + t_f^0 + \tau_m^0 + t_m^0), \theta)$$

Otherwise an optimal agreement will be an agreement in t_m only. These conditions conclude the solution of the model.

We now use the results of the model for a comparative analysis. We start with how the change in the wage gap affects the existence of an agreement.

Proposition 7. *For sufficiently high gender wage gap there always exists an agreement in both t, τ. For sufficiently low gender wage gap there exist some parameters under which there is no agreement. As the gender wage gap increases, there can be a switch only from no agreement to some agreement.*

[15] See Appendix I for the conditions for having the outside option as an optimal choice

[16] See 5.1 for the conditions for having an agreement in both $t,\ \tau$

Proof. See the Proof in the Appendix II. ■

The intuition for this result is that for sufficiently low gender wage gap, the ability of the father to redistribute consumption by transferring consumption to his spouse is rather low. Given that the mother is unaware of the type of her husband and takes into consideration that with probability $1-\beta$ he is an unfair spouse, the compensation that she requires in any agreement is higher than what the husband is willing to pay. This makes the range of prices for any potential agreement empty. On the contrary, for a sufficiently high gender wage gap, the ability of the husband to redistribute consumption is relatively high. Therefore, couples can always find a price to agree on.

3.1 Comparative statics

In this section we will show that an increase in paternity leave τ_m can only lead in the direction of no agreement.

Proposition 8. *Consider the parameters of the model (τ_m, τ_f, a, α, β, w_m, w_f) and divorce parameters s.t. there is no agreement. Then if τ_m increases, then*
(i) the agents cannot switch to an agreement in t_m.
(ii) the agents cannot switch to an agreement in t_m and τ_m.

Proof. See the Proof in the Appendix II ■

Propositions 7 and 8 summarize the main results of the model. While Proposition 7 identifies different types of families in equilibrium, Proposition 8 examines the effect of an introduction or extension of paternity leave policy on that equilibrium.

These results state that the model identifies three distinct groups of families. Moreover, they imply that only the intermediate gender wage gap group experiences a transition from a state of agreement to a state of no agreement. This final important result requires a word of intuitive explanation. Paternity leave policy actually is a subsidy for raising children as it is a payment conditional on having children. This subsidy, ceteris-paribus, decreases the cost of raising children and thus biases the household's optimization towards more of them. For a family that has an agreement of specialization, this subsidy requires mothers to spend more of their time in raising their children. As a result, mothers demand higher consumption redistribution from fathers who buy, according to the agreement, time from their spouses. Thus, for a gender wage gap group, couples agree on a higher price in terms of consumption redistribution. However, for an intermediate gender wage gap group, since part of the male population is unfair and the model generates a pooling equilibrium, the increase in the redistribution that the mother demands is more than what the father is willing to pay. As a result, the agreement stops being optimal and they switch to sharing childcare.

Another important result that our model generates is that even for egalitarian couples an increase in paternity leave increases the welfare of mothers as it increases their bargaining power. Proposition 2 states that it is always optimal for any father to take a full leave in the outside option. Since the mother knows her husband's optimal behavior, her best response is to reduce her time input in childcare. While, this response directly improves the welfare of women in egalitarian couples, it also improves the welfare of other women, who can reach a higher price in any potential agreement.

4 Empirical Analysis

We exploit the introduction of two weeks of paternity leave in Spain in March 2007. The model predicts heterogeneous effects of a paternity leave extension, depending on the potential wage gap in the couple. In particular, we expect that paternity leave extensions will not affect behavior in couples that are either *egalitarian* (who were already sharing market and household work before the extension) or *high potential wage gap* couples (who were specializing before and who continue to specialize after the reform). However, we expect a decrease in specialization (division of labor within the couple in terms of housework and market work) in an *intermediate* group of couples that are neither *egalitarian* nor *high wage gap* (in terms of comparative advantage in market work). More specifically, we expect that this middle group, which we label as *intermediate* wage gap couples, will react to extensions in paternity leave with increases in the length of leave taken by the father, a decrease in subsequent fertility, and a potential increase in divorce. The model also predicts an increase in fathers' involvement in childcare and housework beyond paternity leave in these couples, as well as a positive effect on maternal labor supply.

We test these predictions by conducting an RDD analysis, where we compare couples who had a child shortly before the paternity leave extension with those who had a child shortly after, using several data sources, including the Spanish Labor Force Survey (LFS) and Time Use Survey (TUS). We allow the effect of paternity leave to vary as a function of the characteristics of couples.

4.1 Data and descriptive statistics

4.1.1 Take-up

There is one data source that allows us to study the take up of paternity leave among eligible families. The *Survey on the Use of Parental Leave and their Labor Consequences* (which we will refer to as the *Madrid Survey* or MS) was conducted between January and June 2012 in the metropolitan area of Madrid. The survey targeted parents living in Madrid with a child aged 3 to 7 at the time of the survey. The MS provides information on the month and year of birth of the youngest child, as well as data on parental leave take-up, sociodemographic characteristics of the family, labor supply, and child-related time-use of both parents, for 1,130 children. Out of these 1,130 observations, there are 1,101 observations that have information on the full date of birth (month and year), and 94.5% of the children were born between January 2005 and December 2008. Our final sample includes 1,094 observations.

We use this data set to analyze the take-up of paternity leave, as well as the effect of its introduction on the total number of leave days taken by fathers surrounding childbirth. Before paternity leave introduction in 2007, fathers could take 2 days of leave after the birth of a child. They could also take vacation days, unpaid leave, or even use up some of the maternity leave time. After March 23, 2007, on top of all that they also had two weeks (13 days) of paternity leave (with 100% wage replacement rate)[17].

Our dependent variables are: a binary indicator for paternity leave take-up, the number of days of paternity leave taken, the number of additional days off (vacation, etc) taken around the birth of a child, and the total number of days off around the birth of a child. Descriptive statistics are shown in Table 1 (Panel A). After the introduction of paternity leave, overall take-up among fathers was 66%.

4.1.2 Subsequent fertility, and divorce

To analyze the effects of the 2007 extension on parental labor supply, fertility and divorce, we merge LFS data for all quarters of 2008-2010 (i.e. between 4 and 15 quarters after the policy change). We select households with a male-female couple, with a child born close to March 2007. For effects on completed fertility, we merge all quarters of data for 2017-19 (10-12 years after the reform).

The main sample includes couples with a child born between October 2006 and September 2007 (6 months before and after the policy change). We also use alter-

[17] Maternity leave was 16 weeks both before and after the reform.

native samples that include 3 and 9 months before and after.

The main outcomes are: subsequent fertility and couple separation or divorce. Descriptive statistics are shown in Table 1 (Panels B, C).

4.1.3 Childcare and housework time

To analyze the effects of paternity leave on fathers' (and mothers') time-use, and in particular the time devoted to childcare and housework, we use the Spanish Time-Use Survey, conducted between October 2009 and September 2010. We restrict the sample to include only different-sex parents living in a couple whose youngest child was born 3 years before and after the reform (2004-2010). The final sample includes 941 fathers and 1,047 mothers (the survey interviews only one adult per household).

The survey includes detailed information on the daily minutes devoted to several activities, such as childcare and housework, as well as several socioeconomic indicators, the region where the survey was conducted, and the month and year of birth of all the interviewee's children. We use the number of daily minutes devoted to each task as dependent variables. Descriptive statistics are shown in Table 1 (Panel D). On average, fathers in the sample report devoting slightly more than 3.5 hours per day to childcare and housework, compared with almost 7 for mothers.

4.2 Empirical strategy

We follow a regression discontinuity approach, where the running variable is the date of birth of each couple's child, and the threshold is the date of birth that determines eligibility for paternity leave. The underlying assumption is that, close enough to the threshold, control and treated families are comparable in all dimensions but paternity leave eligibility, or at least there is no discontinuous jump for other reasons exactly at the threshold. We estimate the following equation:

$$Y_{i\tau t} = \alpha + \beta T_{\tau} + \delta_1 m + \delta_2 I[T = 1]m + \gamma X_{i\tau t} + \varepsilon_{i\tau t} \qquad (5)$$

where Y is the dependent variable of interest (e.g. subsequent fertility) for family i who had a child in month τ and is observed in quarter t, T is an indicator for paternity leave eligibility (i.e. the couple having had a child after the paternity leave introduction), m is the running variable (month of birth of the child, normalized so it takes value 0 in April 2007, -1 in March 2007, etc.), and X are control variables (such as mother's age and educational attainment). The coefficient of interest is β, which captures a discrete jump in the outcome variable coinciding with paternity leave eligibility.

We estimate this equation in the full sample (which will give us the average treatment effect), and also separately for couples that vary in terms of the gap in potential wages between the partners. We classify couples in terms of the age and education gap which proxy potential wage gap. In order to detect potential heterogeneous effects, we split couples into three groups: egalitarian, intermediate and high wage gap couples.

As a validity check, Table 2 reports our tests for balance in covariates, both in the full sample and by family type. We run regressions of the form of (5) without controls, where we use the control variables one by one as the dependent variable, to detect any possible discontinuities in family characteristics coinciding with the policy threshold.

4.3 Results

4.3.1 Take-up

Table 3 presents the results for the effect of the introduction of paternity leave in 2007 on time off from work for the full sample of fathers surrounding the birth of their child.[18] The first row shows that overall take-up rate was between 63 and 68%, depending on the bandwidth. On average, fathers took 8 days of paternity leave after the reform. For the full sample, we find that fathers reduced time off from other sources, such as vacation days, so that in the end, the number of days off right after the birth of the child increased between 6 and 8 days, due to the reform.[19]

4.3.2 Endogenous classification

In order to detect heterogenous treatment effects of paternity leave eligibility, we divide couples into 3 groups in terms of their potential wage gap: *egalitarian*, *intermediate* and *high wage gap*. We proxy the potential wage gap by age gap com-

[18] Panel A of Table 2 reports the results of our tests for balance in covariates. Out of 16 covariates, we find a significant discontinuity (at the 95% confidence level) in one.

[19] Notably, the average increase of 8 days in paternity leave by fathers and the paternity take-up of 63-64% correspond to a full length (13 days) for those who actually take it.

bined with education gap.[20] We calibrate the age and education gap thresholds that defines the three groups by targeting some moments in the data.

Our model predicts that while father's take-up time of parental leave is barely affected in egalitarian and high-gap couples, fathers' response in the intermediate-gap couples is discrete and significant. Thus, we run a first stage, in which we calibrate the age-gap and education-gap thresholds between the three groups of couples by targeting a very small take-up response to paternity leave introduction for the corner groups while maximizing the effect for the intermediate one. That is, we sacrifice the take-up data to calibrate and endogenize the classification of couples into the three groups.

The resulting classification defines the three groups of couples as follows (age and education gaps are calculated as husband's age (schooling) minus wife's age (schooling) in years). The resulting classification is depicted in Figure 2, which gives the following thresholds:

- *Egalitarian*
  (i) age gap up to 1 year and education gap up to 4 years or (ii) age gap up to 3 years and education gap up to -2 years

- *Intermediate wage gap*
  (i) age gap between 1 and 3 years and education gap between -2 and 4 years or (ii) age gap up to 1 year and education gap more than 4 years

- *High wage gap*
  (i) age gap more than 3 years or (ii) age gap more than 1 year and education gap more than 4 years

Figure 3 shows the first stage coefficients for the three groups. The *intermediate* group of couples is characterized by a positive significant effect of paternity leave eligibility on the total leave length by fathers, while the effect is not significant for *egalitarian* and *high wage gap* couples. At the same time, as shown in **??** there is a significant difference between the effects of paternity leave eligibility on the leave length of fathers in intermediate couples and the other groups. Although these are targeted moments, the significant positive effect of paternity leave on father take-up length for the middle group and the zero effect for the corner groups is pretty reinforcing.

Table 4 shows the results for the effect on total time off by fathers surrounding the birth of a child on the subsamples of *egalitarian*, *intermediate* and *high wage gap*

[20] The distribution of age and education gap among couples included into the calibration exercise (with a child born between 12 months before and after the reform) are given in Figure 4.

couples.[21] The table shows that not only is the effect found for the intermediate group, but its magnitude accounts for the full leave, which gives support to the theoretical prediction of the model summarized in Proposition 5.

4.3.3 Subsequent fertility results

Next, we estimate the effect of paternity leave on couples' subsequent fertility. The dependent variable is an indicator for the presence of a child under age 1 in the household. Households are surveyed in 2009-10, i.e. between two and three years after the birth of the previous child (and the introduction of paternity leave).

First, we run the fertility regressions for the full sample (all families). We find that paternity leave eligibility led to lower subsequent fertility (Table 5), as already documented in Farré and González (2019). Eligible families are 1 to 4 percentage points less likely to have had another child, when surveyed 2-3 years after the introduction of paternity leave (for an average of 9.2%, see Panel B of Table 1).

We then stratify the analysis by family type according to the endogenous classification calibrated using the model of total parental leave takeup. The results are presented in Table 6. Our main result is that the negative effect on subsequent fertility is driven by *intermediate* wage gap couples: the couples classified as *intermediate* are 5 to 7 percentage points less likely to have had another child 2-3 years after the introduction of paternity leave, whereas there is no significant effect for *egalitarian* couples and *high* wage gap couples.

The magnitude of the effect can be seen in Table 7. The probability of having another child 2-3 years after the paternity leave reform decreases by 54-66% (compared to the baseline of pre-treatment mean) depending on the bandwidth around the cutoff date of the introduction of paternity leave.

4.3.4 Childcare and housework time

We next analyze the effect of paternity leave introduction in 2007 on fathers' contribution to childcare and housework, beyond the paternity leave period (about 3 years after the birth of the child). The results are shown in Table 8 and Table 9.

Table 8 presents the estimated effects on fathers' time-use for the average family, for different subsamples that vary in the bandwidth around the birth-date

[21] See Table 2 (Panel A) for test of covariates in each group of families.

determining eligibility.[22] We find mostly positive insignificant effects for daily minutes devoted to childcare and housework by fathers, and mostly negative insignificant coefficients for market work and leisure, also imprecisely estimated.

We next explore heterogeneity across types of families using the endogenous classification of couples as described in subsubsection 4.3.2, the results are presented in Table 9. The first row shows the results for daily minutes of childcare by fathers for egalitarian, intermediate and high wage gap families. The second row presents the results for housework time by fathers.

We find no significant effect of paternity leave introduction on fathers' time spent on childcare or housework for egalitarian and high wage gap families (columns 1 to 3 and 7 to 9). The coefficients for egalitarian couples are mostly negative, unstable across different bandwidths, and never significantly different from zero. For high wage gap couples, the coefficients are mostly positive, also unstable and insignificant.

We do find significant positive effects for fathers in the intermediate group of couples (columns 4 to 6 in Table 9). Father in this group started spending significantly more time on childcare and housework after the introduction of paternity leave. Fathers in intermediate wage gap couples increased the time they devote to childcare by more than an hour per day, and the time they devote to housework by almost 2 hours per day (15- and 12-months bandwidth).

The magnitudes of the coefficients across groups of families can be seen in Table 10. Compared to the baseline of pre-treatment mean, fathers increased the time on childcare to the level that correspond to mothers' pre-treatment mean in the respective group of couples: an increase by 70-80 minutes daily from the baseline of 80. As for housework time, the increase by 110-120 minutes daily from the baseline of 66-86 minutes also corresponds roughly to the amount of time that mothers spent on housework before treatment.

4.3.5 Divorce results

Next we study the effect of the introduction of paternity leave on couple separation. The sample is now composed of all women living with a child born close to the threshold, including those who do not live with a partner at the time of the survey. 8.3% of women living with a child born close to the cutoff were separated

[22] Panel B of Table 2 shows that there was no significant discontinuity at the threshold in any of the 13 covariates considered.

when surveyed in 2008-10 (see panel D of Table 1).

We find (Table 11) no overall effect on the probability of separation or divorce for the full sample of women, as is also found in Farré and González (2019). The coefficients are all very small in magnitude, and none are significant at the 95% confidence level.

For heterogenous treatment effect analysis, we cannot apply the same classification of couples as before because we do not observe husband characteristics for separated women. We now classify mothers based on their own age and educational attainment only (i.e. high vs. low predicted potential wage of women).
The following classifications are used:

- Mother's schooling:
    1. Mother college educated
    2. Mother medium educ. attainment (high school degree, no college)
    3. Mother low-educated (primary or lower secondary)
- Mother's age and schooling:
    1. Mother college educated or aged 40+ at the first childbirth
    2. Mother medium educ. and aged 27-39 at the first childbirth
    3. Mother low-educated or aged <27 at the first childbirth

We find (Table 12) an increase in the probability of divorce for women with a medium education level (see columns 3-4, first row), and no significant increase for those with low or high educational attainment (in fact we find negative effects among highly educated mothers). We also find an increase in separation in eligible couples labelled as *intermediate* based on a combination of the mother's age and educational attainment but not in more egalitarian (highly educated mothers) or in families with low-educated mothers.

5 Conclusions

We consider a non-cooperative model in which the husband and wife decide on parental leave and the allocation of time between home production and the labor market. They can choose the non-cooperative outside option or reach an agreement of keeping traditional roles, in which the wife specializes in home production (raising kids) while the husband works and transfers consumption to his wife. The model shows that *egalitarian* couples (with a sufficiently small gender wage gap) do not specialize and play the outside option, while *intermediate-gap* (with a medium gender wage gap) and very *high-gap* (with a sufficiently high gender wage gap) couples do have such an agreement. An expansion in paternity leave reduces the net benefits from the agreement and moves *intermediate-gap* couples to their outside option where women work more and men do more home production. As a result, the cost of raising children increases and fertility declines. Assuming a loss of utility from children in the case of divorce, lower fertility increases the probability of divorce.

Using Spanish data and RDD analysis, we confirm our model's predictions. In a first stage, using our model's predictions about fathers' take-up responses, we calibrate the thresholds for age and education gap to classify couples into *egalitarian*, *intermediate-gap* and *high-gap* couples. In a second stage, we use this endogenous classification to examine the impact of paternity leave on subsequent fertility, time-use and separation. While we don't find systematic effects of paternity leave expansion on *egalitarian* and *high-gap* families, we find that the probability of having an additional birth decreases by about 50%-60% up to 15 moths after the policy change. The time-use data shows that fathers devote more of their time to childcare (44-80 minutes per day) and housework (88-117 minutes per day), which makes their input in home production close to women's. These couples are also more likely to break up. The probability of separation increases by about 37% for mothers in the *intermediate-gap* group. Our empirical results thus strongly support the main predictions of the model.

Our theory and empirical results show that introducing or expanding paternity leave produces more equality within couples by pushing some couples to the *egalitarian* group. Our theory and empirical analysis show that not only do these fathers start taking 13 days of paternity leave, but they also start sharing a big part of the home production burden by spending up to three hours more in different home production activities. This new reality expressed by a discontinuous change in the equilibrium for these couples expresses something deep that has changed in these couples' relationship. Our model captures it by an agreement for traditional roles that many couples have. Our model thus expresses the idea that while maternity leave was designed to help women, it actually increases women's relative advantages in home production and thus pushes some of them into the traditional group and makes them carry the major burden of child raising and home production. Recent literature argues that labor market penalties associated with motherhood are the main obstacle for closing the gender earnings gap (Kleven et al. (2019a,b); Bertrand (2020); Titan et al. (2021)). Our paper thus argues that balancing maternity leave with paternity leave, mitigates this problem. Paternity leave has the potential to be an instrumental tool in promoting gender equality.

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Figures and Tables

Table 1: Descriptive statistics
Panel A. Madrid survey

Variables	Number of observations	Mean	Standard deviation	Min	Max
Paternity leave take-up	1 094	0.34	0.470	0	1
Paternity leave length (days)	1 094	4.49	6.330	0	30
Total leave in days	1 094	11.24	20.290	0	477
Age of father (at child's birth)	473	33.97	5.255	15	55
Age of mother (at child's birth)	520	31.49	4.882	16	42
High school or more (father)	465	0.660	0.474	0	1
High school or more (mother)	513	0.735	0.442	0	1
Foreign (father)	467	0.180	0.384	0	1
Foreign (mother)	515	0.194	0.396	0	1
Government employee (father)	456	0.173	0.379	0	1
Government employee (mother)	404	0.191	0.393	0	1
Self-employed (father)	456	0.132	0.338	0	1
Self-employed (mother)	404	0.079	0.270	0	1
Employed (father)	470	0.970	0.170	0	1
Employed (mother)	518	0.724	0.447	0	1
Permanent contract (father)	470	0.970	0.170	0	1
Permanent contract (mother)	404	0.724	0.447	0	1
Married	512	0.691	0.462	0	1
First born	522	0.525	0.500	0	1

Source: Madrid survey (sample of fathers). The sample includes fathers living in a couple with a child born between 12 months before and after March 2007.

Panel B. Labor Force Survey, Subsequent Fertility Sample

Child under 1 in household	10 207	0.092	0.289	0	1
Pat. leave eligibility	10 207	0.503	0.500	0	1
Month of birth (m)	10 207	−0.500	3.476	−6	5
Quarter	10 207	150.100	1.997	147	153
N. of children	10 207	1.749	0.733	1	8
Age (father)	10 207	36.850	5.286	19	64
Age (mother)	10 207	34.520	4.886	18	53
Foreign (father)	10 207	0.108	0.311	0	1
Foreign (mother)	10 207	0.113	0.316	0	1
Married (mother)	10 207	0.857	0.350	0	1
Primary school or less (father)	10 207	0.120	0.325	0	1
Lower secondary (father)	10 207	0.282	0.450	0	1
Higher secondary (father)	10 207	0.237	0.426	0	1
Higher technical (father)	10 207	0.134	0.340	0	1
University (father)	10 207	0.227	0.419	0	1
Primary school or less (mother)	10 207	0.096	0.294	0	1
Lower secondary (mother)	10 207	0.244	0.429	0	1
Higher secondary (mother)	10 207	0.218	0.413	0	1
Higher technical (mother)	10 207	0.130	0.336	0	1
University (mother)	10 207	0.313	0.464	0	1

Sample: Couples with a child born between October 2006 and September 2007 (6 months before and after the policy change), surveyed by the LFS in the third trimester of 2009 or in 2010, that still live together.

Panel C. Labor Force Survey, Divorce Sample (2007 reform)

Separated	18 700	0.083	0.275	0	1
Pat. leave eligibility	18 700	0.503	0.500	0	1
Month of birth (m)	18 700	−0.507	3.485	−6	5
Quarter	18 700	147.600	3.466	142	153
N. of children	18 700	1.740	0.785	1	80
Age (mother)	18 700	31.850	5.147	16	52
Foreign (mother)	18 700	0.059	0.235	0	1
Primary school or less (mother)	18 700	0.123	0.329	0	1
Lower secondary (mother)	18 700	0.245	0.430	0	1
Higher secondary (mother)	18 700	0.224	0.417	0	1
Higher technical (mother)	18 700	0.116	0.321	0	1
University (mother)	18 700	0.302	0.459	0	1

Sample: Mothers living with a child born between October 2006 and September 2007 (6 months before and after the policy change), surveyed by the LFS in 2008-2010.

Panel D. Time-use survey (fathers)

Childcare min. per day	329	108.459	107.547	0	540
Housework min. per day	329	111.155	119.243	0	560
Age (father)	314	35.180	5.917	18	69
Age (mother)	315	32.820	5.204	16	51
Primary school (father)	317	0.404	0.491	0	1
Secondary school (father)	317	0.356	0.480	0	1
College (father)	317	0.240	0.428	0	1
Primary school (mother)	317	0.312	0.464	0	1
Secondary school (mother)	317	0.391	0.489	0	1
College (mother)	317	0.281	0.450	0	1
Foreign (father)	317	0.161	0.368	0	1
Foreign (mother)	317	0.164	0.371	0	1
Married	317	0.899	0.302	0	1
Workday	317	0.647	0.479	0	1
First born	317	0.574	0.495	0	1

Sample: Spanish Time-Use Survey 2009-10. The sample includes fathers living in a couple with a child born between 12 months before and after March 2007.

Table 2: Balance of covariates by sample

A. Take-Up Sample

		Age and Education gap		
	Full Sample	**Egalitarian**	**Intermediate**	**High**
Age of father	0,03	0,87	-0,57	0,29
	(2,31)	(1,34)	(1,69)	(5,49)
Age of mother	0,27	1,35	-0,53	-0,15
	(1,29)	(1,38)	(1,73)	(1,72)
High school or more (father)	-0,18***	-0,22	-0,24*	-0,07
	(0,06)	(0,13)	(0,11)	(0,10)
High school or more (mother)	0,03	0,04	-0,01	-0,03
	(0,04)	(0,10)	(0,09)	(0,12)
Foreign (father)	-0,04	-0,05	-0,03	-0,01
	(0,04)	(0,10)	(0,09)	(0,07)
Foreign (mother)	-0,03	-0,02	0,06	-0,06
	(0,05)	(0,08)	(0,12)	(0,10)
Government employee (father)	0,03	-0,01	0,15	-0,02
	(0,05)	(0,13)	(0,13)	(0,08)
Government employee (mother)	-0,05	-0,02	-0,10	-0,04
	(0,06)	(0,12)	(0,17)	(0,07)
Self-employed (father)	-0,06	-0,01	-0,11	-0,05
	(0,04)	(0,06)	(0,09)	(0,09)
Self-employed (mother)	-0,04	-0,12	0,02	0,02
	(0,04)	(0,08)	(0,07)	(0,07)
Employed (father)	-0,01	0,03	-0,17**	0,05
	(0,05)	(0,03)	(0,07)	(0,11)
Employed (mother)	0,00	0,04	-0,10	-0,06
	(0,06)	(0,10)	(0,15)	(0,12)
Permanent contract (father)	-0,01	-0,03	-0,06	0,04
	(0,06)	(0,12)	(0,11)	(0,10)
Permanent contract (mother)	0,10	0,08	0,17	0,05
	(0,07)	(0,11)	(0,14)	(0,15)
Married	0,09	0,13*	0,18	-0,08
	(0,07)	(0,07)	(0,16)	(0,15)
First born	-0,11*	-0,06	-0,18*	-0,19
	(0,06)	(0,09)	(0,10)	(0,13)
N	522	190	133	199

The sample includes fathers living in a couple with a child born 12 months before and after March 2007. The table shows coefficients on a dummy variable equal to 1 if the child was born after the reform, for a regression where the outcome variable is the one listed in the first column. All regressions control for a linear trend in the running variable (month of birth) and allow for different trends before and after the reform. Robust standard errors are below in parentheses. Columns 2-4 show coefficients when the sample is limited by type according to the mother's educational level. *** p<0.01, ** p<0.05, * p<0.1.

B. Time-Use Sample

		Age and Education gap		
	Full Sample	**Egalitarian**	**Intermediate**	**High**
Age of father	-1,90	-2,75	1,81	-4,59
	(1,55)	(1,66)	(1,50)	(3,80)
Age of mother	0,07	-2,13	1,83	-2,72
	(1,08)	(1,37)	(1,50)	(3,18)
Foreign (father)	-0,13*	0,00	0,00	-0,29*
	(0,07)	(0,12)	(0,17)	(0,17)
Foreign (mother)	-0,11*	-0,04	0,07	-0,21
	(0,06)	(0,13)	(0,14)	(0,19)
Primary school (father)	-0,08	0,06	-0,18	-0,25
	(0,12)	(0,15)	(0,13)	(0,23)
Secondary school (father)	0,05	0,14	0,02	0,06
	(0,12)	(0,16)	(0,21)	(0,24)
College (father)	0,02	-0,20	0,16	0,19
	(0,09)	(0,17)	(0,17)	(0,13)
Primary school (mother)	-0,09	0,08	-0,39***	0,03
	(0,09)	(0,16)	(0,13)	(0,19)
Secondary school (mother)	0,06	0,06	0,32*	-0,23
	(0,13)	(0,24)	(0,18)	(0,24)
College (mother)	0,05	-0,35	-0,04	0,32**
	(0,06)	(0,23)	(0,16)	(0,12)
Married	0,07	-0,07	0,09	0,10
	(0,07)	(0,16)	(0,09)	(0,08)
First born	-0,05	-0,52***	-0,20	0,41**
	(0,07)	(0,16)	(0,15)	(0,19)
Work day	-0,12	-0,27	0,01	-0,08
	(0,09)	(0,22)	(0,14)	(0,21)
N	317	119	101	97

The sample includes fathers living in a couple with a child born 12 months before and after March 2007. The table shows coefficients on a dummy variable equal to 1 if the child was born after the reform, for a regression where the outcome variable is the one listed in the first column. All regressions control for a linear trend in the running variable (month of birth) and allow for different trends before and after the reform. Robust standard errors are below in parentheses. Columns 2-4 show coefficients when the sample is limited by type according to the mother's educational level. *** p<0.01, ** p<0.05, * p<0.1.

C. Labor Force Survey, Subsequent fertility sample

		Age and Education gap		
	Full Sample	Egalitarian	Intermediate	High
Age of father	-0,76***	-1,00***	-0,64**	-0,10
	(0,21)	(0,28)	(0,32)	(0,42)
Age of mother	-0,64***	-1,20***	-0,62*	-0,23
	(0,19)	(0,28)	(0,32)	(0,37)
Foreign father	0,00	-0,03*	-0,01	0,05
	(0,01)	(0,02)	(0,02)	(0,03)
Foreign mother	-0,02	-0,04**	-0,02	0,03
	(0,01)	(0,02)	(0,02)	(0,03)
Married mother	-0,05***	-0,05**	-0,06***	-0,03
	(0,01)	(0,02)	(0,02)	(0,03)
Primary school or less (father)	-0,01	-0,05**	-0,06***	0,08***
	(0,01)	(0,02)	(0,02)	(0,03)
Lower secondary (father)	0,06***	0,15***	0,03	-0,03
	(0,02)	(0,03)	(0,03)	(0,04)
Higher secondary (father)	-0,01	0,00	-0,01	-0,03
	(0,02)	(0,03)	(0,03)	(0,03)
Higher technical (father)	0,00	-0,03	0,04	-0,01
	(0,01)	(0,02)	(0,03)	(0,02)
University (father)	-0,03*	-0,07**	-0,01	-0,01
	(0,02)	(0,03)	(0,03)	(0,03)
Primary school or less (mother)	0,00	-0,03**	-0,05**	0,08***
	(0,01)	(0,01)	(0,02)	(0,03)
Lower secondary (mother)	0,04**	0,09***	0,03	-0,01
	(0,02)	(0,02)	(0,03)	(0,03)
Higher secondary (mother)	-0,01	-0,01	0,03	-0,04
	(0,02)	(0,02)	(0,03)	(0,03)
Higher technical (mother)	0,02	0,00	0,05**	0,01
	(0,01)	(0,02)	(0,02)	(0,02)
University (mother)	-0,05**	-0,06*	-0,07*	-0,04
	(0,02)	(0,03)	(0,03)	(0,03)
N	10,207	4,055	2,921	3,231

The sample includes couples living together with a child born 6 months before and after March 2007, interviewed in the Labor Force Survey between 2008 and 2010. The table shows coefficients on a dummy variable equal to 1 if the child was born after the reform, for a regression where the outcome variable is the one listed in the first column. All regressions control for a linear trend in the running variable (month of birth) and allow for different trends before and after the reform. Robust standard errors are below in parentheses. Columns 2-4 show coefficients when the sample is limited by type according to the mother's educational level. *** p<0.01, ** p<0.05, * p<0.1.

D. Labor Force Survey, Divorce sample

		Wife's schooling		
	Full Sample	**Egalitarian**	**Intermediate**	**High gap**
Age of mother	-0.27*	0.32*	-0.54**	-0.28
	(0.15)	(0.20)	(0.24)	(0.29)
Foreign (mother)	-0.01	-0.02*	-0.03*	0.01
	(0.01)	(0.01)	(0.02)	(0.02)
Primary school or less (mother)	0.01			0.01
	(0.01)			(0.02)
Lower secondary (mother)	0.01			-0.01
	(0.01)			(0.02)
Higher secondary (mother)	0.00		-0.04	
	(0.01)		(0.02)	
Higher technical (mother)	0.02**		0.04	
	(0.01)		(0.02)	
University (mother)	-0.03**			
	(0.01)			
N	18,699	5,649	6,363	6,687

The sample includes mothers with a child born 6 months before and after March 2007, interviewed in the Labor Force Survey between 2008 and 2010. The table shows coefficients on a dummy variable equal to 1 if the child was born after the reform, for a regression where the outcome variable is the one listed in the first column. All regressions control for a linear trend in the running variable (month of birth) and allow for different trends before and after the reform. Robust standard errors are below in parentheses. Columns 2-4 show coefficients when the sample is limited by type according to the mother's educational level. *** p<0.01, ** p<0.05, * p<0.1.

Table 3: Effect of paternity leave (2007 reform) on total leave length (full sample)

	Effect on paternity leave		
	(1)	(2)	(3)
Paternity leave take-up (binary)	0.676***	0.638***	0.633***
	(0.026)	(0.028)	(0.023)
Total leave in days	6.21***	8.40***	7.06***
	(1.15)	(2.18)	(1.30)
Paternity leave length (days)	8.83***	8.35***	8.32***
	(0.368)	(0.351)	(0.281)
Length other types of leave	−2.63**	0.043	− 1.26
	(1.09)	(2.10)	(1.35)
Bandwidth in months (+/-)	Full sample	15	12
N	1094	669	522

Sample: Couples with a child born in 2005-2008 (Madrid Survey). Each coefficient comes from a different regression. Other types of leave include the 2-day leave following birth, additional leave days taken after birth, vacation days used to extend leave, maternity days used by father, days used from breastfeeding leave, and unpaid leave. *** p<0.01, ** p<0.05, * p<0.1

Table 4: Effect of paternity leave (2007 reform) on total leave length (by type of couple)

	Potential wage gap								
	Egalitarian			**Intermediate**			**High**		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Total leave in days	−0.22	5.52*	6.31	12.37***	17.88***	18.29***	3.44**	5.48	2.62
	(3.42)	(3.03)	(3.69)	(4.43)	(5.42)	(5.11)	(1.68)	(3.36)	(1.64)
Bandwidth in months	24	15	12	24	15	12	24	15	12
N	359	238	190	249	181	133	361	250	199

Sample: Couples with a child born in 2005-2008 (Madrid Survey). Each coefficient comes from a different regression. All regressions control for a linear trend in the running variable (month of birth), and allow for different trends before and after the reform. All regressions include indicators for whether the parents were married, of foreign nationality, whether each parent had at least a high school education, and the age of each parent when the child was born as controls, as well as dummies for missing values in the covariates. Columns 1-3 show results using the sample of egalitarian couples, columns 4-6 use intermediate wage gap couples, and columns 7-9 use high wage gap couples, as defined by the endogenous classification. Robust standard errors clustered at the month of birth level in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 5: Effects of paternity leave (2007 reform) on subsequent fertility (full sample)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Subsequent fertility	−0.0414**	−0.0246*	−0.0088	-0.0057	−0.0288***	−0.0371**	−0.0264**	−0.0105	-0.0081	−0.0296***
	(0.0181)	(0.0135)	(0.0120)	(0.0110)	(0.0096)	(0.0179)	(0.0134)	(0.0120)	(0.0110)	(0.0095)
N	5,020	8,442	10,207	11,975	15,315	5,020	8,442	10,207	11,975	15,315
Controls	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Bandwidths in months	3	5	6	7	9	3	5	6	7	9

Sample: Coresident couples with a child born in 2006 or 2007 (between 3 and 9 months before and after the policy change), surveyed by the LFS in the third quarter of 2009 or 2010. A subsequent birth is captured by a dependent variable that takes value 1 if there is a child under age 1 in the household. Each column shows the results from a different regression. All regressions control for a linear trend in month of birth of the child interacted with the threshold for paternity leave eligibility. Regressions in columns 4 to 6 also include indicators for whether the parents were married, were of foreign nationality, and their age and level of education as controls, as well as dummies for the quarter in which the survey was conducted. Robust standard errors in parentheses.*** p<0.01, ** p<0.05, * p<0.1

Table 6: Effects of paternity leave (2007 reform) on subsequent fertility (by type of couple)

	Potential wage gap								
	Egalitarian			**Intermediate**			**High**		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Subsequent fertility	−0.0177	0.00058	−0.0260	−0.0644***	−0.0534**	−0.0757**	−0.0094	0.0188	0.0113
	(0.0151)	(0.0186)	(0.0278)	(0.0180)	(0.0229)	(0.0328)	(0.0167)	(0.0215)	(0.0328)
Bandwidth	9	6	3	9	6	3	9	6	3
N	6,008	4,055	2,052	4,432	2,921	1,452	4,875	3,231	1,516

Note: Sample now includes coresident couples with a child born in 2006 or 2007 (between 3 and 9 months before and after the policy change), surveyed by the LFS in 2009-10. A subsequent birth is captured by a dependent variable that takes value 1 if there is a child under age 1 in the household. Each column shows the results from a different regression. Robust standard errors are shown in parentheses. Controls include mother and father age and educational attainment, as well as dummies for the quarter in which the survey was conducted. We also control for a linear trend in month of birth of the child interacted with the threshold for paternity leave eligibility. Columns 1-3 show results using the sample of egalitarian couples, columns 4-6 use intermediate wage gap couples, and columns 7-9 use high wage gap couples, as defined by the endogenous classification. *** p<0.01, ** p<0.05, * p<0.1

Table 7: Effect of paternity leave (2007 reform) on subsequent fertility (intermediate wage gap couple)

	Intermediate wage gap		
	(1)	(2)	(3)
Subsequent fertility	-0.0644***	-0.0534**	-0.0757**
	(0.0180)	(0.0229)	(0.0328)
Mean before treatment	0.0982	0.0997	0.114
	(0.00627)	(0.00820)	(0.0120)
Bandwidth	9	6	3
N	4,432	2,921	1,452
N before treatment	2,287	1,497	765

Note: A subsequent birth is captured by a dependent variable that takes value 1 if there is a child under age 1 in the household. Robust standard errors are shown in parentheses. Controls include mother and father age and educational attainment, as well as dummies for the quarter in which the survey was conducted. We also control for a linear trend in month of birth of the child interacted with the threshold for paternity leave eligibility. *** p<0.01, ** p<0.05, * p<0.1

Table 8: Effect of paternity leave (2007 reform) on time-use of fathers (full sample)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Childcare	10.3	−10.9	20.7	7.0	6.7	3.8	18.7	6.7
	(11.3)	(81.4)	(23.8)	(25.7)	(11.2)	(85.6)	(24.8)	(27.8)
Housework	4.1	1.9	15.6	29.6	3.2	13.7	16.7	34.0
	(14.1)	(75.2)	(25.1)	(28.0)	(13.6)	(75.4)	(24.8)	(27.5)
Market work	−10.5	−0.6	3.7	−4.6	−14.6	−62.8	−23.5	−34.4
	(29.8)	(169.9)	(56.5)	(63.1)	(32.5)	(169.3)	(56.2)	(63.9)
Leisure	−3.8	21.4	−56.3	−40.6	0.3	46.0	−31.5	−20.1
	(18.8)	(88.7)	(36.1)	(40.3)	(19.2)	(84.5)	(35.1)	(39.0)
Bandwidth in months	Full sample	24	15	12	Full sample	24	15	12
Controls	No	No	No	No	Yes	Yes	Yes	Yes
N	990	402	423	329	940	388	404	317

Source: Couples interviewed in the Spanish Time-Use Survey 2009-10 with children born between 2004 and 2010. Time spent on each activity is measured in daily minutes. All regressions control for a linear trend in month of birth of the child interacted with the threshold for paternity leave eligibility, and region fixed effects. Columns 5-8 also include indicators for whether the parents were married, were of foreign nationality, and their age and level of education as control. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 9: Effect of paternity leave (2007 reform) on father's time spent on childcare and housework (by type of couple)

	Potential wage gap								
	Egalitarian			**Intermediate**			**High**		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Childcare minutes	0.553	6.499	−56.47	43.75	72.39*	81.72*	30.48	6.932	41.81
	(38.32)	(74.55)	(80.91)	(30.06)	(42.35)	(47.37)	(29.40)	(37.02)	(43.43)
Housework minutes	−23.75	−48.47	−35.63	88.11***	111.8***	117.1***	−20.24	1.896	17.75
	(35.38)	(54.87)	(66.22)	(27.24)	(37.71)	(40.86)	(34.20)	(44.98)	(53.89)
Bandwidth in months	24	15	12	24	15	12	24	15	12
N	238	149	118	190	122	100	201	130	94

Source: Couples interviewed in the Spanish Time-Use Survey 2009-10 with children born between 2004 and 2010. Time spent on housework is measured in daily minutes. All regressions control for a linear trend in month of birth of the child interacted with the threshold for paternity leave eligibility, and region fixed effects. All regressions also include indicators for whether the parents were married, were of foreign nationality, and their age and level of education as control. Columns 1-3 show results using the sample of egalitarian couples, columns 4-6 use intermediate wage gap couples, and columns 7-9 use high wage gap couples, as defined by the endogenous classification. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 10: Effect of paternity leave (2007 reform) on father's time spent on childcare and housework (intermediate wage gap couples)

	Intermediate wage gap		
	(1)	(2)	(3)
Childcare minutes	43.75	72.39*	81.72*
	(30.06)	(42.35)	(47.37)
Mean before treatment (fathers)	80.22	77.89	78.48
Mean before treatment (mothers)	143.1	152.6	154.0
Housework minutes	88.11***	111.8***	117.1***
	(27.24)	(37.71)	(40.86)
Mean before treatment (fathers)	88.24	85.96	66.52
Mean before treatment (mothers)	221.6	217.7	211.7
Bandwidth in months (+/-)	24	15	12
N	190	122	100

Source: Couples interviewed in the Spanish Time-Use Survey 2009-10 with children born between 2004 and 2010. All regressions control for a linear trend in month of birth of the child interacted with the threshold for paternity leave eligibility, and region fixed effects. All regressions also include indicators for whether the parents were married, were of foreign nationality, and their age and level of education as control. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 11: Effects of paternity leave (2007 reform) on parental separation (full sample)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Paternity leave eligibility	0.0040	0.0091	−0.0024	−0.0100	0.0030	0.0033	−0.0067	−0.0101
	(0.0118)	(0.0091)	(0.0075)	(0.0066)	(0.0116)	(0.0089)	(0.0073)	(0.0064)
Observations	9,167	15,470	21,997	28,038	9,167	15,470	21,997	28,038
Controls	No	No	No	No	Yes	Yes	Yes	Yes
Bandwidth in months	3	5	7	9	3	5	7	9

Sample: Women living with a child born between October 2006 and September 2007 (6 months before and after the policy change), surveyed by the LFS in 2008-10. Parental separation is measured as a dummy equal to 1 if a woman is not living with a partner. Each column shows the results from a different regression. All regressions control for a linear trend in month of birth of the child interacted with the threshold for paternity leave eligibility. Columns 5-8 also include indicators for whether the parents were married, were of foreign nationality, and their age and level of education as controls, as well as dummies for the quarter in which the survey was conducted. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 12: Effects of paternity leave (2007 reform) on parental separation (by type of couple)

	Egalitarian		**Intermediate**		**High gap**	
	(1)	(2)	(3)	(4)	(5)	(6)
Wife's schooling	−0.0217**	−0.0284*	0.0282**	0.0582***	−0.00622	−0.0174
	(0.00984)	(0.0150)	(0.0131)	(0.0178)	(0.0169)	(0.0255)
Wife's schooling & age	−0.00806	−0.0172	0.0240*	0.0222	−0.00641	0.0123
	(0.00985)	(0.0147)	(0.0134)	(0.0185)	(0.0162)	(0.0244)
Bandwidth in months	6	3	6	3	6	3
N	5,648	2,857	6,365	3,165	6,687	3,144

Sample: Women living with a child born between October 2006 and September 2007 (6 months before and after the policy change), surveyed by the LFS in 2008-10. Parental separation is measured as a dummy equal to 1 if a woman is not living with a partner. Columns 1-2 show results using the sample of egalitarian couples, columns 3-4 use traditional couples, and columns 5-6 use very traditional couples, as defined by the classification listed in the first column. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Figure 1: The Sequence of Events

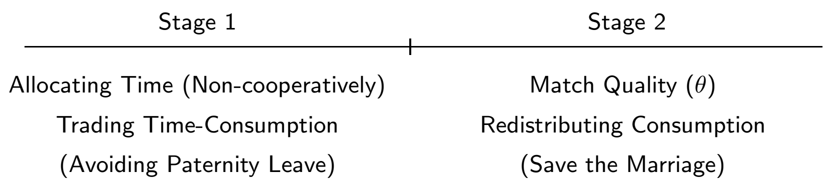


Figure 2: Endogenous classification of couples based on age and education gap

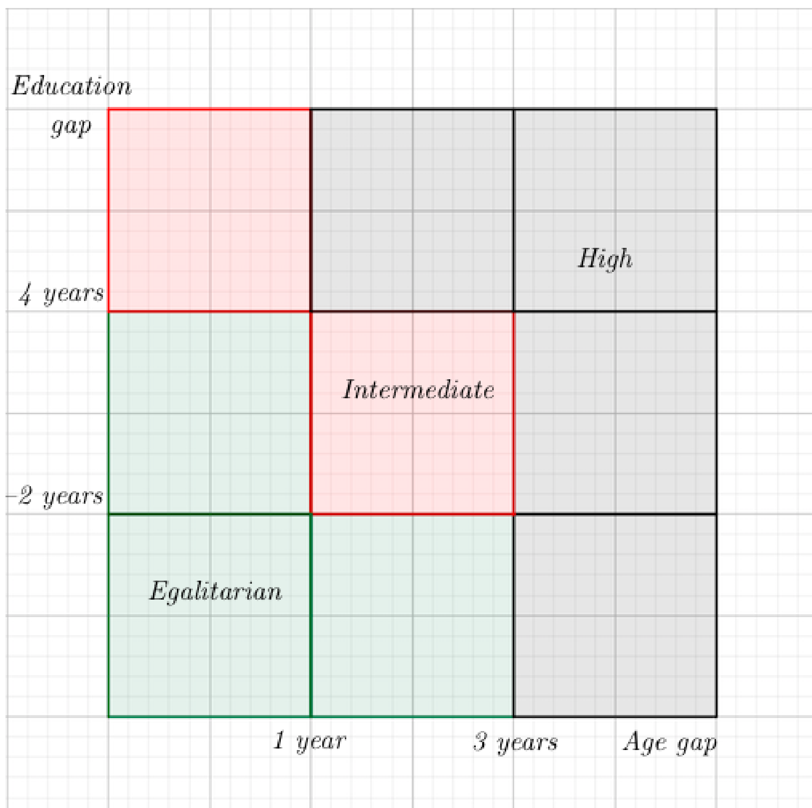
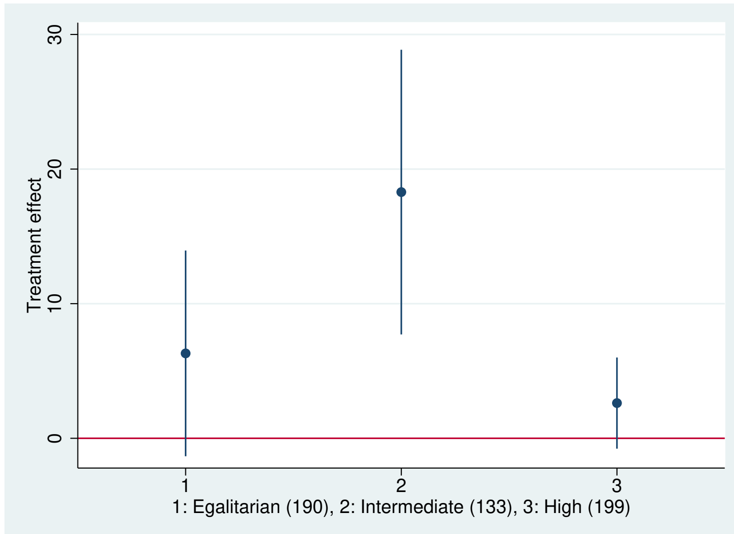
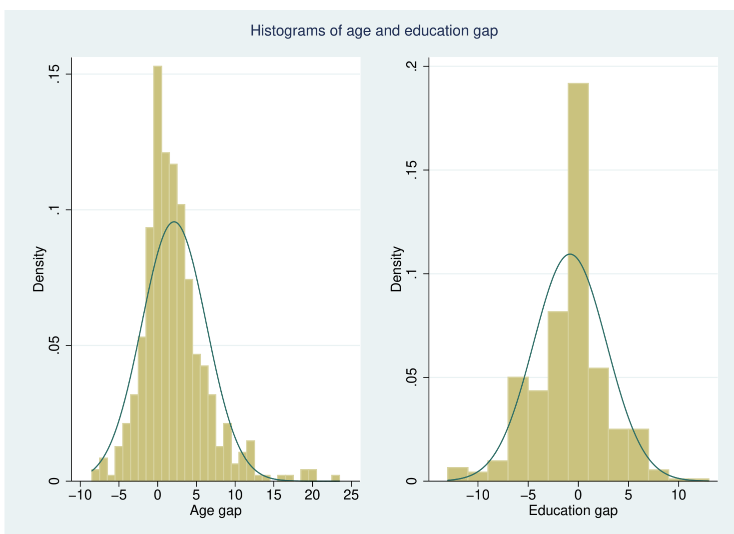


Figure 3: Effect of of paternity leave eligibility on total length of leave taken by fathers in days (by type of couple)



Note: Coefficients with 95% confidence intervals from separate regressions. The sample is limited to a 12-month bandwidth before and after the reform.

Figure 4: Age and education gap



Note: Couples with a child born in 2005-2008 (Madrid Survey). The sample includes couples with a child born between 12 months before and after March 2007.

APPENDICES

5.1 Appendix I

Conditions for the optimality of the outside option

In this appendix we derive the conditions for not having an agreement. The idea of the proof is to find T_m and T_f such that $T_m < T_f$, meaning that the maximum price the husband is ready to pay is less than the minimum that the wife is ready to accept. We will show the conditions under which neither an agreement in both t_m, τ_m nor an agreement only in t_m are preferred to an outside option.

Firstly, let us find T_f. It is defined by $\mathbb{E}(U_f) = U_f^0$, so that the wife is indifferent between an agreement and the outside option:

$$\begin{aligned}
\mathbb{E}(U_f) =&\beta \left[(1-p_d)(\log c_{ff} + \log n) + p_d(\log c_{ff} + d\log n)\right] + \\
&(1-\beta)\left[(1-p_d)(\log c_{fu} + \log n) + p_d(\log c_{fu} + d\log n\right] = \\
&\beta \log c_{ff} + (1-\beta)\log c_{fu} + (1-p_d)\log n + p_d d \log n = \\
&\beta \log\Big(w_f(1-t_f^0 - t_m - \tau_m)(1-t_f^0-\tau_f^0 - t_m - \tau_m) + T\Big) + \\
&(1-\beta)\log\Big(w_f(1-t_f^0 - t_m - \tau_m)(1-t_f^0-\tau_f^0 - t_m - \tau_m)\Big) + \\
&(1-p_d)\log n + p_d d\log n
\end{aligned}$$

$$U_f^0 = \log(w_f(1-t_f^0)(1-t_f^0-\tau_f^0)) + (1-p_d+p_d d)\log n_0$$

Then indifference between an agreement and the outside option for wife implies:

$$\begin{aligned}
&\beta \log\Big(w_f(1-t_f^0 - t_m - \tau_m)(1-t_f^0-\tau_f^0 - t_m - \tau_m) + T\Big) + \\
&(1-\beta)\log\Big(w_f(1-t_f^0 - t_m - \tau_m)(1-t_f^0-\tau_f^0 - t_m - \tau_m)\Big) + \\
&(1-p_d)\log n + p_d d\log n = \\
&\log(w_f(1-t_f^0)(1-t_f^0-\tau_f^0)) + (1-p_d+p_d d)\log n_0
\end{aligned}$$

$$\Big(w_f(1-t_f^0 - t_m - \tau_m)(1-t_f^0-\tau_f^0 - t_m - \tau_m) + T\Big)^\beta = \frac{c_f^0 \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta}\cdot n^{1-p_d+p_d d}}$$

$$T_f = \left[\frac{c_f^0 \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta}\cdot n^{1-p_d+p_d d}}\right]^{1/\beta} - w_f(1-t_f^0 - t_m - \tau_m)(1-t_f^0-\tau_f^0 - t_m - \tau_m)$$

Let us now find T_m. Similarly, it is defined by $\mathbb{E}(U_m) = U_m^0$:

$$\mathbb{E}(U_m) = (1-p_d)U_m^M + p_d U_m^D = (1-p_d)\left(\log c_m + \log n\right) + p_d\left(\log c_m + d\log n\right) =$$
$$= \log\Big(w_m(1-t_m^0+t_m)(1-t_m^0-\tau_m^0+t_m+\tau_m) - T\Big) + (1-p_d)\log n + p_d d\log n$$
$$U_m^0 = \log c_m^0 + (1-p_d+p_d d)\log n_0$$

Then indifference between an agreement and the outside option for the husband implies:

$$\log\Big(w_m(1-t_m^0+t_m)(1-t_m^0-\tau_m^0+t_m+\tau_m) - T\Big) + (1-p_d)\log n + p_d d\log n =$$
$$= \log c_m^0 + (1-p_d+p_d d)\log n^0$$
$$T_m = w_m(1-t_m^0+t_m)(1-t_m^0-\tau_m^0+t_m+\tau_m) - \frac{c_m^0 \cdot n_0^{1-p_d+p_d d}}{n^{1-p_d+p_d d}}$$

The agreement does not exist whenever $T_m < T_f$, i.e. when the lower bound of the interval for price of the agreement exceeds the upper bound.

Condition 1: agreement only on t_m is not profitable:

$$w_m(1-\tau_m^0) - \frac{w_m(1-t_m^0)(1-t_m^0-\tau_m^0)\cdot\Big((t_m^0+\tau_m^0)^a + (t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{\Big((\tau_m^0)^a + (t_f^0+\tau_f^0+t_m^0)^a\Big)^{1-p_d+p_d d}} <$$
$$\left[\frac{(w_f(1-t_f^0)(1-t_f^0-\tau_f^0))\cdot\Big((t_m^0+\tau_m^0)^a + (t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{(w_f(1-t_f^0-t_m^0)(1-t_f^0-\tau_f^0-t_m^0))^{1-\beta}\cdot\Big((\tau_m^0)^a + (t_f^0+\tau_f^0+t_m^0)^a\Big)^{1-p_d+p_d d}}\right]^{1/\beta} -$$
$$- w_f(1-t_f^0-t_m^0)(1-t_f^0-\tau_f^0-t_m^0)$$

Condition 2: agreement on both t_m and τ_m is not profitable:

$$w_m - \frac{w_m(1-t_m^0)(1-t_m^0-\tau_m^0)\cdot\Big((t_m^0+\tau_m^0)^a + (t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{\Big((t_f^0+\tau_f^0+t_m^0+\tau_m^0)^a\Big)^{1-p_d+p_d d}} <$$
$$\left[\frac{(w_f(1-t_f^0)(1-t_f^0-\tau_f^0))\cdot\Big((t_m^0+\tau_m^0)^a + (t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{(w_f(1-t_f^0-t_m^0-\tau_m^0)(1-t_f^0-\tau_f^0-t_m^0-\tau_m^0))^{1-\beta}\cdot\Big((t_f^0+\tau_f^0+t_m^0+\tau_m^0)^a\Big)^{1-p_d+p_d d}}\right]^{1/\beta} -$$
$$- w_f(1-t_f^0-t_m^0-\tau_m^0)(1-t_f^0-\tau_f^0-t_m^0-\tau_m^0)$$

Conditions for the optimality of the agreement in both $(t,\ \tau)$

In this appendix we provide the conditions for which an agreement in both $t,\ \tau$ is profitable.
The agreement on both $t_m,\ \tau_m$ should be a Pareto-improvement for any other agreement. That is, it should be at least as good as the outside option:

$$T^0_{mt,\tau} > T^0_{ft,\tau}$$

Where $T^0_{mt,\tau}$ and $T^0_{ft,\tau}$ are the upper and lower bounds of the interval for prices in case when we compare an agreement in both $t_m,\ \tau_m$ with the absence of an agreement as the outside option.
And this agreement should be at least as good as the agreement only on t_m:

$$T^1_{mt,\tau} > T^1_{mt,\tau}$$

Where $T^1_{mt,\tau}$ and $T^1_{ft,\tau}$ are the upper and lower bounds of the interval for prices in case when we compare an agreement in both $t_m,\ \tau_m$ with an agreement only in t_m as the outside option.
Calculate $T^0_{ft,\tau}$. It is such that $\mathbb{E}(U_f) = U^0_f$ (we take it from Appendix I):

$$T^0_f = \left[\frac{c^0_f \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta} \cdot n_{t,\tau}^{1-p_d+p_d d}} \right]^{1/\beta} - w_f(1 - t^0_f - t^0_m - \tau^0_m)(1 - t^0_f - \tau^0_f - t^0_m - \tau^0_m)$$

Calculate T^0_m. It is such that $\mathbb{E}(U_m) = U^0_m$ (we take it from Appendix I I as well):

$$T^0_m = w_m - \frac{c^0 \cdot_0^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Where $n_{t,\tau} = (t^0_m + \tau^0_m + t^0_f + \tau^0_f)^a$ is the number of kids in case of agreement in both $t_m,\ \tau_m$
Calculate $T^1_{ft,\tau}$. It is such that $\mathbb{E}U_f(t^0_m, \tau^0_m) = \mathbb{E}U_f(t^0_m, 0)$, i.e. when an agreement in t_m is an outside option. Using calculations from appendix C1:

$$\begin{aligned}
&\beta \log \left(w_f(1 - t^0_f - t^0_m - \tau^0_m)(1 - t^0_f - \tau^0_f - t^0_m - \tau^0_m) + T^1_{ft,\tau} \right) + \\
&(1-\beta) \log \left(w_f(1 - t^0_f - t^0_m - \tau^0_m)(1 - t^0_f - \tau^0_f - t^0_m - \tau^0_m) \right) + \\
&(1 - p_d) \log n_{t,\tau} + p_d d \log n_{t,\tau} = \\
&\beta \log \left(w_f(1 - t^0_f - t^0_m)(1 - t^0_f - \tau^0_f - t^0_m) + T^0_{ft} \right) + \\
&(1-\beta) \log \left(w_f(1 - t^0_f - t^0_m - \tau^0_m)(1 - t^0_f - \tau^0_f - t^0_m - \tau^0_m) \right) + \\
&(1 - p_d) \log n_t + p_d d \log n_t
\end{aligned}$$

Where $n_{t,\tau} = (t_f^0 + \tau_f^0 + t_m^0 + \tau_m^0)^a$, $n_t = (\tau_m^0)^a + (t_f^0 + \tau_f^0 + t_m^0)^a$ and T_{ft}^0 is the equilibrium transfer in case of an agreement only in t_m when an absence of an agreement is an outside option. Denote by $c_{f1} = w_f(1 - t_f^0 - t_m^0 - \tau_m^0)(1 - t_f^0 - \tau_f^0 - t_m^0 - \tau_m^0)$ and by $c_{f2} = w_f(1 - t_f^0 - t_m^0)(1 - t_f^0 - \tau_f^0 - t_m^0)$ Then:

$$T_{ft,\tau}^1 = \left[\frac{(c_{f2} + T_{ft}^0)^\beta \cdot c_{f2}^{1-\beta} \cdot n_t^{1-p_d+p_d d}}{c_{f1}^{1-\beta} n_{t,\tau}^{1-p_d+p_d d}} \right]^{1/\beta} - c_{f1}$$

Calculate $T_{mt,\tau}^1$. It is such that $\mathbb{E}U_m(t_m^0, \tau_m^0) = U_m(t_m^0, 0)$:

$$\log \left(w_m - T_{mt,\tau}^1 \right) + (1 - p_d + p_d d) \log n_{t,\tau} = \log \left(w_m(1 - \tau_m^0) - T_{mt}^0 \right) + (1 - p_d + p_d d) \log n_t$$

$$T_{mt,\tau}^1 = w_m - \frac{\left(w_m(1 - \tau_m^0) - T_{mt}^0 \right) n_t^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Then the agreement on (t_m, τ_m) is an equilibrium agreement when two conditions are satisfied:

$$\begin{cases} T_{mt,\tau}^0 > T_{ft,\tau}^0 \\ T_{mt,\tau}^1 > T_{ft,\tau}^1 \end{cases}$$

5.2 Appendix II

In this appendix we present the proofs.

Proof of Lemma 1

Lemma. In any agreement $T \geq 0$

Proof. It follows from an assumption that neither husband nor wife exhaust all of their time on raising children ($t_m^0 + t_f^0 + \tau_m^0 + \tau_f^0 < 1$). Assume by contradiction that there exists a agreement with $T < 0$. That is, given optimally chosen (t_f^0, τ_f^0), the wife is willing to give up some of her private consumption to dedicate more time to childcare. If so, then initial t_f^0, τ_f^0 were chosen suboptimally: in the outside option, an increase in t_f resulting in an equivalent loss of private consumption would be beneficial, as it is beneficial in an agreement. This is a contradiction, and t_f^0 is not optimally chosen. As we assumed arbitrary $T < 0$, this suggests that in any agreement it must be the case that $T \geq 0$. ■

Proof of proposition 1

Proposition. *In any agreement, a weakly dominant strategy of an unfair father is to imitate a fair father.*

Proof. Assuming that the wife always specializes, we get $T > 0$ in any equilibrium.
Note that there never exists a separating equilibrium. Any separating equilibrium would mean that the wife is able to distinguish between the fair and unfair agent. And she would not agree to have an agreement with an unfair father, as she would not receive a transfer. Thus, it is only rational for an unfair father to mimic a fair one.
Now consider a pooling equilibrium. The utility of the fair male partner is:

$$U_{m,f} = \log(c_m - T) + \log(n) + \theta$$

The outside option utility is:

$$U^0_{m,f} = U^0_{m,u} = \log(c^0_m) + \log(n^0) + \theta$$

The utility of the unfair male partner in case of an agreement is:

$$U_{m,u} = \log(c_m) + \log(n) + \theta$$

If it is profitable to have an agreement then:

$$U_{m,f} > U^0_{m,f}$$

Note that

$$U_{m,u} > U_{m,f} > U^0_{m,f} = U^0_{m,u}$$

Then it is profitable for an unfair male partner to imitate a fair one.
When there is no agreement, the unfair male partner is indifferent.
Hence, it is a weakly dominant strategy to imitate a fair father.

■

Proof of proposition 3

Proposition. *The stability of the marriage does not depend on the transfer T. The agents choose to divorce if and only if*

$$\theta < (d - 1) \log n$$

Proof. The no-divorce condition is that given some T^M:

$$U_m^M(T + T^M) \geq U_m^D$$
$$U_f^M(T_r + T^M) \geq U_f^D$$

where T_r can be either T or 0.
Expand it using the definition of the utility function:

$$\log(w_m(1 - t_m^0 + t_m)(1 - t_m^0 + t_m + \tau_m - \tau_m^0) - T - T^M) + \log(n) + \theta \geq$$
$$\log(w_m(1 - t_m^0 + t_m)(1 - t_m^0 + t_m + \tau_m - \tau_m^0) - T) + d\log(n)$$

$$\log(w_f(1 - t_f^0 - t_m - \tau_m)(1 - t_f^0 - t_m - \tau_m - \tau_f^0) + T_r + T^M) + \log(n) + \theta \geq$$
$$\log(w_f(1 - t_f^0 - t_m - \tau_m)(1 - t_f^0 - t_m - \tau_m - \tau_f^0) + T_r) + d\log(n))$$

Rearrange the terms and simplify:

$$\log(w_m(1 - t_m^0 + t_m)(1 - t_m^0 + t_m + \tau_m - \tau_m^0) - T - T^M) + \theta \geq$$
$$\log(w_m(1 - t_m^0 + t_m)(1 - t_m^0 + t_m + \tau_m - \tau_m^0) - T) + (d-1)\log(n)$$

$$\log(w_f(1 - t_f^0 - t_m - \tau_m)(1 - t_f^0 - t_m - \tau_m - \tau_f^0) + T_r + T^M) + \theta \geq$$
$$\log(w_f(1 - t_f^0 - t_m - \tau_m)(1 - t_f^0 - t_m - \tau_m - \tau_f^0) + T_r) + (d-1)\log(n)$$

Which is equivalent to:

$$\log(c_m - T^M) + \theta \geq \log(c_m) + (d-1)\log(n)$$
$$\log(c_f + T^M) + \theta \geq \log(c_f) + (d-1)\log(n)$$

$$(c_m - T^M)e^\theta \geq c_m n^{d-1}$$
$$(c_f + T^M)e^\theta \geq c_f n^{d-1}$$

$$c_m(1 - \frac{n^{d-1}}{e^\theta}) \geq T^M$$
$$-c_f(1 - \frac{n^{d-1}}{e^\theta}) \leq T^M$$

Hence, T^M satisfies

$$-c_f(1 - \frac{n^{d-1}}{e^\theta}) \leq T^M \leq c_m(1 - \frac{n^{d-1}}{e^\theta})$$

1. $1 - \frac{n^{d-1}}{e^\theta} \geq 0$

$$-c_f(1 - \frac{n^{d-1}}{e^\theta}) \leq T^M \leq c_m(1 - \frac{n^{d-1}}{e^\theta})$$
$$-c_f(1 - \frac{n^{d-1}}{e^\theta}) \leq 0$$
$$c_m(1 - \frac{n^{d-1}}{e^\theta}) \geq 0$$

2. $1 - \frac{n^{d-1}}{e^\theta} < 0$

$$0 > c_m(1 - \frac{n^{d-1}}{e^\theta}) \geq T^M$$
$$0 < -c_f(1 - \frac{n^{d-1}}{e^\theta}) \leq T^M$$

There does not exist such T^M that the marriage is stable for the second case. As for the first case, for $T^M = 0$ the marriage is stable. Hence, the marriage is not stable if and only if $1 - \frac{n^{d-1}}{e^\theta} > 0$.

$$1 - \frac{n^{d-1}}{e^\theta} < 0 \ <=> \ \theta < (d-1)\log n$$

■

Proof of corollary

Corollary. *Assume the uniform distribution of $\theta \in [x_1, x_2]$ s.t. $x_1 < (d-1)\log n_0 < x_2$. Then the probability of divorce is*

$$p_d = \frac{x_1 - (d-1)\log n}{x_1 - x_2}$$

Proof. In the first stage the agents have rational beliefs regarding the probability of divorce. Assuming a uniform distribution $\theta \sim U[x_1, x_2]$ s.t. $x_1 < (d-1)\log n_0 < x_2$ we have

$$\Pr(\theta < (d-1)\log n) = \frac{x_1 - (d-1)\log n_1}{x_1 - x_2}$$

Where $\theta < (d-1)\log n_1$ is the condition for having divorce.

■

Proofs of propositions 4 and 5

Proposition. *If it is optimal to transfer a single unit of time, then it is optimal to transfer the full amount of time. Formally, one of the following constraints always binds in the maximization problem in the first stage:*

$$t_m^0 \geq t_m$$
$$t_m^0 \geq 0$$

Proof. The utility of male partner:

$$\begin{aligned} U_m = &\log((1 - t_m^0 + t_m)(1 - t_m^0 - \tau_m^0 + t_m + \tau_m) - T) + \\ &\log((t_m^0 + \tau_m^0 - t_m - \tau_m)^a + (t_f^0 + \tau_f^0 + t_m + \tau_m)^a) \end{aligned}$$

Where the first term is logarithm of private consumption and the second term is logarithm of number of kids. Both terms under the logarithms are convex w.r.t t_m. Their product is convex as well.The optimal solution of the convex problem is the corner solution: either $t_m = 0$ or $t_m = t_m^0$. Monotonic transformation does not change the optimal solution.

The utility of the female partner:

$$\begin{aligned} U_f = &\mathbb{E} \log((1 - t_f^0 - t_m - \tau_m)(1 - t_f^0 - \tau_f^0 - t_m - \tau_m) + T_r) + \\ &\log((t_m^0 + \tau_m^0 - t_m - \tau_m)^a + (t_f^0 + \tau_f^0 + t_m + \tau_m)^a) \end{aligned}$$

Apply the exponentiation to the utility function. The resulting problem is convex w.r.t. t_m, If the female partner agrees to a non-zero t_m, meaning that for a given t_m there exists T_f s.t. $T_f < T_m$, then due to convexity of the utility function w.r.t. t_m every additional $\Delta > 0$ costs less for the female partner in terms of utility. At the same time every additional $\Delta > 0$ brings more utility to the male partner. Thus if they can agree upon a non-zero t_m (so that there exists T_f s.t. $T_f < T_m$), then they can agree upon $t_m + \Delta$, making an interior solution not optimal. The monotonic logarithmic transformation does not change the optimal solution.

■

Proposition. *If it is optimal to transfer a single unit of parental leave, then it is optimal to transfer the full parental leave. Formally, one of the following constraints always binds in the maximization problem in the first stage:*

$$\tau_m^0 \geq \tau_m$$
$$\tau_m^0 \geq 0$$

The proof is identical to the previous proposition.

Proof of proposition 6

Proposition. *The agreement* $(t_m = 0, \tau_m = \tau_m^0, T)$ *is not optimal*

Proof. The utility of male partner:

$$U_m = \log((1 - t_m^0)(1 - t_m^0) - T) + \log((t_m^0)^a + (t_f^0 + \tau_f^0 + \tau_m^0)^a)$$

To prove that the agreement is not optimal it suffices to show that a little deviation is profitable. Consider $\tilde{t}_m = t_m + \Delta$, $\tilde{\tau}_m = \tau_m - \Delta$

$$\begin{aligned}\tilde{U}_m =& \log((1 - t_m^0 + \Delta)(1 - t_m^0 + \Delta - \Delta) - T) + \log((t_m^0 + \Delta - \Delta)^a + (t_f^0 + \tau_f^0 + \tau_m^0 + \Delta - \Delta)^a) = \\ & \log((1 - t_m^0 + \Delta)(1 - t_m^0) - T) + \log((t_m^0)^a + (t_f^0 + \tau_f^0 + \tau_m^0)^a) > U_m\end{aligned}$$

The utility of female partner:

$$\begin{aligned}U_f =& \mathbb{E}\log((1 - t_f^0 - t_m - \tau_m)(1 - t_f^0 - \tau_f^0 - t_m - \tau_m) + T_r) + \\ & + \log((t_m^0 + \tau_m^0 - t_m - \tau_m)^a + (t_f^0 + \tau_f^0 + t_m + \tau_m)^a)\end{aligned}$$

It is easy to see that $\tilde{U}_f = U_f$. Then there is Pareto-improvement, and the initial agreement is not optimal.

■

Proof of Proposition 7

Proposition. *For sufficiently high wage gap there always exists an agreement in both* t, τ. *For sufficiently low gender wage gap there exist some parameters under which there is no agreement. As the gender wage gap increases, there can be a switch only from no agreement to some agreement.*

Proof. We start the proof with Lemma 3.

Lemma 3. *For sufficiently high wage gap there always exists an agreement in both* t, τ

Proof: Let us use the results from Appendices I and II in the proof.
$(T^0_{mt,\tau}, T^0_{ft,\tau})$, $(T^1_{mt,\tau}, T^1_{ft,\tau})$ are the sets of upper and lower bounds of the interval for prices for an agreement in both (t_m, τ_m) in case when 1) lack of agreement is the outside option; 2) the agreement only on t_m is the outside option. Then an agreement in both (t_m, τ_m) is the equilibrium choice if the following conditions are satisfied:

$$\begin{cases} T^0_{mt,\tau} > T^0_{ft,\tau} \\ T^1_{mt,\tau} > T^1_{ft,\tau} \end{cases}$$

Where

$$T^0_{ft,\tau} = \left[\frac{c^0_f \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta} \cdot n_{t,\tau}^{1-p_d+p_d d}}\right]^{1/\beta} - w_f(1-t^0_f - t^0_m - \tau^0_m)(1 - t^0_f - \tau^0_f - t^0_m - \tau^0_m)$$

$$T^0_{mt,\tau} = w_m - \frac{w_m(1-\tau^0_m)(1-t^0_m - \tau^0_m)\cdot n_0^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Where $n_0 = (t^0_m + \tau^0_m)^a + (t^0_f + \tau^0_f)^a$ is the number of kids without an agreement, $n_{t,\tau} = (t^0_m + \tau^0_m + t^0_f + \tau^0_f)^a$ is the number of kids with an agreement in both $(t_m,\ \tau_m)$. The other variables are defined in the Appendix I.

$$T^0_{ft} = \left[\frac{c^0_f \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta} \cdot n_t^{1-p_d+p_d d}}\right]^{1/\beta} - w_f(1-t^0_f - t^0_m)(1 - t^0_f - \tau^0_f - t^0_m)$$

$$T^0_{mt} = w_m(1-\tau^0_m) - \frac{w_m(1-\tau^0_m)(1-t^0_m - \tau^0_m)\cdot n_0^{1-p_d+p_d d}}{n_t^{1-p_d+p_d d}}$$

$$T^1_{ft,\tau} = \left[\frac{(c_{f2} + T^0_{ft})^\beta \cdot c_{f2}^{1-\beta} \cdot n_t^{1-p_d+p_d d}}{c_{f1}^{1-\beta} n_{t,\tau}^{1-p_d+p_d d}}\right]^{1/\beta} - c_{f1}$$

$$T^1_{mt,\tau} = w_m - \frac{\left(w_m(1-\tau^0_m) - T^0_{mt}\right) n_t^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Note that due to loglinearity of the utility function, the w_m and w_f do not affect the outside option solutions of the husband's and wife's problems. As the number of kids with an agreement is always greater than without it (due to specialization in kids, $a > 1$), we have $n_t > n_0$, $n_{t,\tau} > n_0$. Also $(1-\tau^0_m)(1-t^0_m-\tau^0_m) < 1$. Hence $T^0_{mt,\tau}$ is increasing in w_m. At the same time an increase in w_m does not affect $T^0_{ft,\tau}$. Thus for any given parameters we can always find w_m s.t. $T^0_{mt,\tau} > T^0_{ft,\tau}$. Rewrite $T^1_{mt,\tau}$:

$$T^1_{mt,\tau} = w_m\left(1 - \frac{\left(1-\tau^0_m\right) n_t^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}\right) + \frac{T^0_{mt} n_t^{1-p_d p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Note that due to specialization in raising kids $n_{t,\tau} > n_t$. Also $(1-\tau^0_m) < 1$, hence the first term is increasing in w_m. Note that T^0_{mt} is also increasing in w_m. Thus

$T^1_{mt,\tau}$ is increasing in w_m. An increase in w_m does not affect $T^1_{ft,\tau}$. Then for any given set of parameters there exists such w_m that $T^1_{mt,\tau} > T^1_{ft,\tau}$.

$\square$

Lemma 4. *For sufficiently low gender wage gap there exist some parameters under which there is no agreement.*

Proof: Let us find the condition on β s.t. there is no agreement in (t_m, τ_m): $T^0_{mt,\tau} < T^0_{ft,\tau}$:

$$w_m - \frac{w_m(1-\tau^0_m)(1-t^0_m-\tau^0_m)\cdot n_0^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}} <$$
$$\left[\frac{c^0_f \cdot n_0^{1-p_d+p_d d}}{c_{fu,t,\tau}^{1-\beta}\cdot n_{t,\tau}^{1-p_d+p_d d}}\right]^{1/\beta} - w_f(1-t^0_f-t^0_m-\tau^0_m)(1-t^0_f-\tau^0_f-t^0_m-\tau^0_m)$$

Rearranging the terms, the condition on β is:

$$\tilde{\beta} < \frac{\log(c^0_f n_0^{1-p_d+p_d d}) - \log(c_{fu,t,\tau} n_{t,\tau}^{1-p_d+p_d d})}{log[w_m(1-(1-\tau^0_m)(1-t^0_m-\tau^0_m)(n_0/n_{t,\tau})^{1-p_d+p_d d}) + c_{fu,t,\tau}] - \log c_{fu,t,\tau}}$$

Let us now find the condition on β s.t. there is no agreement in t_m: $T^0_{mt} < T^0_{ft}$:

$$w_m(1-\tau^0_m) - \frac{w_m(1-\tau^0_m)(1-t^0_m-\tau^0_m)\cdot n_0^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}} <$$
$$\left[\frac{c^0_f \cdot n_0^{1-p_d+p_d d}}{c_{fu,t}^{1-\beta}\cdot n_{t,\tau}^{1-p_d+p_d d}}\right]^{1/\beta} - w_f(1-t^0_f-t^0_m)(1-t^0_f-\tau^0_f-t^0_m)$$

Rearranging in terms of β:

$$\bar{\beta} < \frac{\log(c^0_f n_0^{1-p_d+p_d d}) - \log(c_{fu,t} n_t^{1-p_d+p_d d})}{log[w_m((1-\tau^0_m)-(1-\tau^0_m)(1-t^0_m-\tau^0_m)(n_0/n_1)^{1-p_d+p_d d}) + c_{fu,t}] - \log c_{fu,t}}$$

Then there is no agreement if $\beta \leq \min[\tilde{\beta}, \bar{\beta}]$
It is easy to see that for $a = 1$ $\tilde{\beta}$ and $\bar{\beta}$ are positive. For $a = 1$ $n_0 = n_t = n_{t,\tau}$ because there is no specialization in kids. And the outside option consumption is higher than consumption of a wife married to an unfair husband.

Both $\tilde{\beta}$ and $\bar{\beta}$ are decreasing in w_m. Hence, lower w_m is associated with higher threshold for fair male partners in the population for the agreement to be profitable for the female partner.

□

Lemma 5. *There can only be a switch from no agreement to any type of agreement as the gender wage gap increases*

Proof: The agreement is not profitable to make iff $T^0_{mt,\tau} < T^0_{ft,\tau}$ and $T^0_{mt} < T^0_{ft}$. Thus to show that the "switch" from no agreement to an agreement may occur only in one direction, we need to show that the ranges $T^0_{mt,\tau} - T^0_{ft,\tau}$ and $T^0_{mt,\tau} - T^0_{ft,\tau}$ are expanding in w_m/w_f. Normalize $w_f = 1$ without loss of generality.
As previously analyzed in Lemma 5, the range $T^0_{mt,\tau} - T^0_{ft,\tau}$ is expanding in w_m. Using similar arguments, it is easy to see that T^0_{ft} does not depend on w_m, while T^0_{mt} is increasing in w_m. Hence the range $T^0_{mt} - T^0_{ft}$ is expanding in w_m as well.
As a result of an increase in w_m, the "switch" can occur only in one direction: from no agreement to some agreements. We abstain from further analysis of switches between the two types of agreements, as the case of switch from no agreement is of importance here. □
This concludes the proof. ■

Proof of Proposition 8

Proposition. *Consider the parameters of the model (τ_m, τ_f, a, α, β, w_m, w_f) and divorce parameters s.t. there is no agreement. Then if τ_m increases, then*
(i) the agents cannot switch to an agreement in t_m.
(ii) the agents cannot switch to an agreement in t_m and τ_m.

Proof. ***(i) the agents cannot switch to an agreement in t_m.***

We start the proof with the following lemma:

Lemma 6. *The choice between having an agreement in t_m and having no agreement at all does not depend on the bargaining power, α.*

Proof: This follows directly from the conditions of having an agreement: the agents refuse to engage in any sort of agreements iff $T_{m,t} < T_{f,t}$ and $T_{m,t,\tau} < T_{f,t,\tau}$, i.e. the maximum price the male partner is ready to pay is lower than the minimum price the female partner is ready to accept. As these prices do not depend on α, then α does not affect the choice of the agents. □
As the type of an agreement does not depend on α, we can assume it to be 1 without loss of generality, i.e. if there is an agreement, the transfer is such that the male partner is indifferent between the agreement and the outside option.

As the agreement exists if it is a Pareto-improvement of the outside-option, it suffices to show that

$$\frac{\partial(U_f - U_f^0)}{\partial\tau_m} \leq 0$$

As the male partner in case of agreement is indifferent, this condition will ensure that an increase in paternity leave will not lead to switch to agreement from the outside option.
By definition of the utility function this is equivalent to:

$$\frac{\partial\left(f(n) + \log(c_f) - f(n^0) - \log(c_f^0)\right)}{\partial\tau_m} \leq 0$$

Lemma 7. *If Proposition 8 is true for $a = 1$, then it is also true for any $a \geq 1$.*

Proof:
Outside option number of kids:

$$n^0 = (t_m^0 + \tau_m^0)^a + (t_f^0 + \tau_f^0)^a$$

Number of kids for agreement in t_m:

$$n = (\tau_m^0)^a + (t_m^0 + t_f^0 + \tau_f^0)^a$$

Consider the difference in number of kids under an agreement and in the outside option:

$$n - n^0 = (\tau_m^0)^a + (t_m^0 + t_f^0 + \tau_f^0)^a - (t_m^0 + \tau_m^0)^a - (t_f^0 + \tau_f^0)^a$$

Take its derivative with respect to τ_m^0:

$$\frac{\partial(n-n^0)}{\partial\tau_m^0} = a\left((\tau_m^0)^{a-1} + (\frac{\partial t_m^0}{\partial\tau_m^0} + \frac{\partial t_f^0}{\partial\tau_m^0})(t_m^0 + t_f^0 + \tau_f^0)^{a-1} - (1 + \frac{\partial t_m^0}{\partial\tau_m^0})(t_m^0 + \tau_m^0)^{a-1} - \frac{\partial t_f^0}{\partial\tau_m^0}(t_f^0 + \tau_f^0)^{a-1}\right)$$

$$\begin{aligned}\frac{\partial(n-n^0)}{\partial\tau_m^0} &= a((\tau_m^0)^{a-1} - (t_m^0 + \tau_m^0)^{a-1}) + a\frac{\partial t_m^0}{\partial\tau_m^0}((t_m^0 + t_f^0 + \tau_f^0)^{a-1} - (t_m^0 + \tau_m^0)^{a-1}) \\ &+ a\frac{\partial t_f^0}{\partial\tau_m^0}((t_m^0 + t_f^0 + \tau_f^0)^{a-1} - (t_f^0 + \tau_f^0)^{a-1})\end{aligned}$$

Note that $\frac{\partial t_f^0}{\partial\tau_m^0} < 0$ and $\frac{\partial t_m^0}{\partial\tau_m^0} < 0$.

If $a > 1$ than for any $x > 0$, x^{a-1} is an increasing function. Note also that $\tau_f \geq \tau_m$.

Hence, we have

$$(t_m^0 + t_f^0 + \tau_f^0)^{a-1} > (t_f^0 + \tau_f^0)^{a-1})$$
$$(t_m^0 + t_f^0 + \tau_f^0)^{a-1} > (t_m^0 + \tau_m^0)^{a-1})$$
$$(\tau_m^0)^{a-1} < (t_m^0 + \tau_m^0)^{a-1}$$

Which implies that

$$a\frac{\partial t_f^0}{\partial \tau_m^0}((t_m^0 + t_f^0 + \tau_f^0)^{a-1} - (t_f^0 + \tau_f^0)^{a-1}) < 0$$
$$a\frac{\partial t_m^0}{\partial \tau_m^0}((t_m^0 + t_f^0 + \tau_f^0)^{a-1} - (t_m^0 + \tau_m^0)^{a-1}) < 0$$
$$a((\tau_m^0)^{a-1} - (t_m^0 + \tau_m^0)^{a-1} - (t_f^0 + \tau_f^0)^{a-1}) < 0$$

Hence, for $a > 1$ the difference in number of kids is declining in τ_m^0:

$$\frac{\partial(n - n^0)}{\partial \tau_m^0} < 0$$

Now consider the difference in utilities for $a = 1$:

$$n - n^0 = (\tau_m^0) + (t_m^0 + t_f^0 + \tau_f^0) - (t_m^0 + \tau_m^0) - (t_f^0 + \tau_f^0) = 0$$

Partial derivative of this difference with respect to τ_m^0 is also 0.
Utility of the agent is defined by the number of kids and consumption:

$$U_i^j = \log(c_i^j) + \log(n_i^j)$$

Where $i \in \{m, f\}$ defines whether the agent is male or female, j defines whenever there is an agreement or the outside option.

The proposition is true for $a = 1$ (as the first derivative wrt τ_m^0 is 0), thus when τ_m increases there is no switch to an agreement.
Denote by 'A' parameters before the increase in τ_m and by 'B' parameters after the increase. So if initially there was no agreement then: $U_f^A \leq U_f^{0A}$, that is $\log(c_f^{0A}) > \log(c_f^A)$ (because $n = n^0$) and there is an agreement after the increase, that is $U_f^B \geq U_f^{0B}$, or $\log(c_f^B) > \log(c_f^{0B})$.

We already know that for $a > 1$: $\frac{\partial(n-n^0)}{\partial \tau_m^0} < 0$ so because τ_m increases when we moving from 'A' to 'B', hence $(n^B - n^A) > (n^{0B} - n^{0A})$.

If there was no agreement before the increase (f is some increasing and monotonic function. Note that this is not logarithm because of the divorce): $f(n1) + \log(c_f 1) < f(n^0 1) + \log(c_f^0 1)$
So we have:

$$f(n^A) + \log(c_{fA}) < f(n^{0A}) + \log(c^0_{fA})$$

$$(n^B - n^A) > (n^{0B} - n^{0A})$$

$$\log(c_f^B) > \log(c_f^{0B})$$

$$\log(c_f^{0A}) > \log(c_f^A)$$

Hence $\log(n^B) + \log(c_{fB}) < \log(n^{0B}) + \log(c^0_{f2})$ that is there is no agreement after increase. So we cannot switch to agreement in t_m. This concludes the proof. □
Hence, we can consider $a = 1$ without loss of generality. Then $n = n^0$ and our condition is equivalent to:

$$\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} \leq 0$$

By definition:

$$c_f = (w_f(1-t_f-t_m)(1-t_f-\tau_f-t_m)+T)^{\beta}(w_f(1-t_f-t_m)(1-t_f-\tau_f^0-t_m))^{1-\beta}$$

$$c_f^0 = w_f(1-t_f)(1-t_f-\tau_f)$$

T is defined from the fact that male partner is indifferent between agreement and no agreement:

$$U_m = \log(c_m) + f(n) = \log(c_m^0) + f(n^0)$$

Because $n = n^0$ and by definition of consumption of male partner it is equivalent to:

$$w_m(1-t_m)(1-\tau_m-t_m) = w_m(1-\tau_m) - T$$

Then:

$$\log c_f - \log c_f^0 = \beta \log(w_f(1-t_f-t_m)(1-t_f-\tau_f-t_m) - w_m(1-t_m)(1-\tau_m-t_m) +$$
$$+ w_m(1-\tau_m)) + (1-\beta)\log(w_f(1-t_f-t_m)(1-t_f-\tau_f^0-t_m)) - \log(w_f(1-t_f)(1-t_f-\tau_f^0))$$

Let us take the derivative and normalize the wage of female partner to 1 (by definition $t'_m = \frac{\partial t_m}{\partial \tau_m}$, $t'_f = \frac{\partial t_f}{\partial \tau_m}$):

$$\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} = (\beta)((1-t_f-t_m)(1-t_f-\tau_f-t_m) - w_m(1-t_m)(1-\tau_m-t_m)$$
$$+ w_m(1-\tau_m))^\beta + ((1-t_f-t_m)(1-t_f-\tau_f^0-t_m))^{-1}\left(-w_m + w_m(1-\tau_m-t_m)t'_m + \right.$$
$$\left. + w_m(1-t_m)(1+t'_m) + \frac{\partial((1-t_f-t_m)(1-t_f-\tau_f-t_m))}{\partial \tau_m}\right)$$
$$- \frac{\partial((1-\beta)\log(w_f(1-t_f-t_m)(1-t_f-\tau_f^0-t_m)) - \log(w_f(1-t_f)(1-t_f-\tau_f^0)))}{\partial \tau_m}$$

Note that:

$$-1 + (1-\tau_m-t_m)t'_m + (1-t_m)(1+t'_m) = 2(1-t_m)t'_m - t_m - \tau_m t'_m < 0$$

So $(-w_m + w_m(1-\tau_m-t_m)t'_m + w_m(1-t_m)(1+t'_m)$ is decreasing in w_m.
Also:

$$(1-t_m)(1-\tau_m-t_m) < (1-\tau_m)$$

So $((1-t_f-t_m)(1-t_f-\tau_f-t_m) - w_m(1-t_m)(1-\tau_m-t_m) + w_m(1-\tau_m))^\beta + ((1-t_f-t_m)(1-t_f-\tau_f^0-t_m))^{-\beta}$ is also decreasing in w_m (because $0 < \beta < 1$).
Thus, we have that $\frac{\partial(c_f - c_f^0)}{\partial \tau_m}$ is decreasing in w_m. So if we show that $\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} < 0$ for $w_m = 1$, that is the minimum wage we can have (because $1 = w_f \leq w_m$), then for any wage $w_m > 1$ we also have $\frac{\partial(c_f - c_f^0)}{\partial \tau_m} < 0$.
So to prove our statement it is sufficient to show that:

$$\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} \leq 0$$

if $c_f = c_f^0$ and $w_m = w_f = 1$
Up to now we have:

$$((1-t_f-t_m)(1-t_f-\tau_f-t_m) - (1-t_m)(1-\tau_m-t_m)$$
$$+(1-\tau_m))^\beta((1-t_f-t_m)(1-t_f-\tau_f^0-t_m))^{1-\beta} - ((1-t_f)(1-t_f-\tau_f^0)) = 0$$

From this condition we find β and put it into the equation $\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} \leq 0$.

After taking the derivative we receive the following:

$$-(1-\log\left(\frac{(1-\tau_f)(1-\tau_f-t_f)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)})\right)$$
$$\frac{t'_m(1-\tau_f-t_f-t_m)-(1-\tau_f-t_m)(t'_f+t'_m)+1-(1-\tau_m-t_m)-(1-\tau_m)(1-t'_m)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)+(1-\tau_m)-(1-\tau_m)(1-\tau_m-t_m)}-$$
$$-\log\left(\frac{(1-\tau_f)(1-\tau_f-t_f)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)}\right)$$
$$\frac{t'_m(1-\tau_f-t_f-t_m)-(1-\tau_f-t_m)(t'_f+t'_m)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)}-\frac{(1-\tau_f)t'_f}{(1-\tau_f)(1-\tau_f-t_f)}\geq 0 \tag{1}$$

If we can show that (1) is true then we are done with the prove. But for that we have to find t_m, t_f, t'_m, t'_f. We find them in the outside option:

$$\max_{t_m}(1-t_m)(1-\tau_m-t_m)(t_m+\tau_m+t_f+\tau_f)$$

$$\max_{t_m}(1-t_f)(1-\tau_f-t_f)(t_m+\tau_m+t_f+\tau_f)$$

Taking the first order conditions and rewriting them as:

$$15t_m^4+(38\tau_m+8\tau_f-28)t_m^3+(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)t_m^2+ (10\tau_m^3+7\tau_m^2\tau_f-36\tau_m^2-20\tau_m\tau_f+18\tau_m+8\tau_f+4)t_m+\tau_m^4+\tau_m^3\tau_f-6\tau_m^3-5\tau_m^2\tau_f+8\tau_m^2+ 6\tau_m\tau_f+2\tau_m-1=0$$

$$15t_f^4+(38\tau_f+8\tau_m-28)t_f^3+(32\tau_f^2+14\tau_f\tau_m-58\tau_f-16\tau_m+10)t_f^2+ (10\tau_f^3+7\tau_f^2\tau_m-36\tau_f^2-20\tau_f\tau_m+18\tau_f+8\tau_m+4)t_f+\tau_f^4+\tau_f^3\tau_m-6\tau_f^3-5\tau_f^2\tau_m+8\tau_f^2+ 6\tau_f\tau_m+2\tau_f-1=0$$

We can solve these for t_m by the following algorithm:

$$a1=-\frac{(38\tau_m+8\tau_f-28)^2}{600}+\frac{(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)}{15}$$

$$b1=\frac{(38\tau_m+8\tau_f-28)^3}{27000}-\frac{(38\tau_m+8\tau_f-28)(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)}{450}+ \frac{(10\tau_m^3+7\tau_m^2\tau_f-36\tau_m^2-20\tau_m\tau_f+18\tau_m+8\tau_f+4)}{15}$$

$$g1=-\frac{(38\tau_m+8\tau_f-28)^4}{4320000}+\frac{(38\tau_m+8\tau_f-28)^2(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)}{27000}$$

$$-\frac{(38\tau_m+8\tau_f-28)(10\tau_m^3+7\tau_m^2\tau_f-36\tau_m^2-20\tau_m\tau_f+18\tau_m+8\tau_f+4)}{900}+$$
$$\frac{\tau_m^4+\tau_m^3\tau_f-6\tau_m^3-5\tau_m^2\tau_f+8\tau_m^2+6\tau_m\tau_f+2\tau_m-1}{15}$$
$$p=-\frac{a1^2}{12}-g1$$
$$q=-\frac{a1^3}{108}+\frac{a1g1}{3}-\frac{b1^2}{8}$$
$$r=-\frac{q}{2}-(\frac{q^2}{4}+\frac{p^3}{27})^{\frac{1}{2}}$$

$u=r^{\frac{1}{3}}$ Note that here any root is suitable(they give the same results).

$$y=-\frac{5a1}{6}+u-\frac{p}{3u}$$
$$w=(a1+2y)^{\frac{1}{2}}$$
$$t_m=-\frac{(38\tau_m+8\tau_f-28)}{120}+\frac{(\pm w\pm(-(3a1+2y+\frac{2b1}{w}))^{0.5})}{2}$$

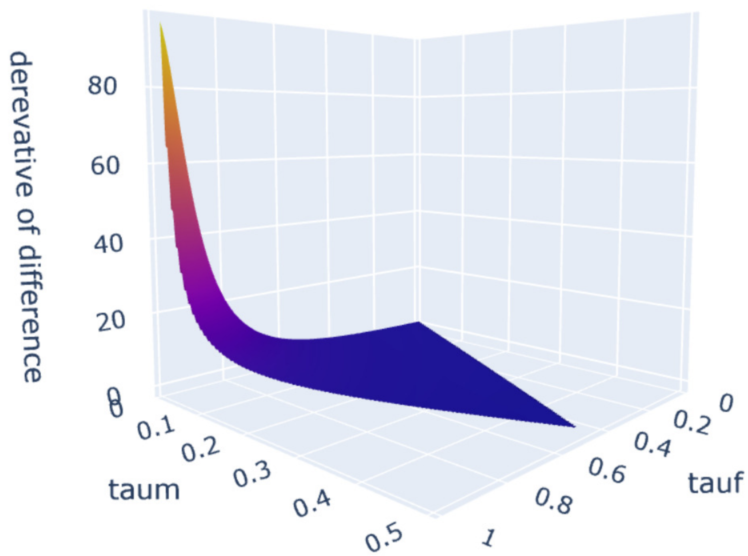
Here we have 4 roots we take the root that is real. If it is less then 0, $t_m=0$. The solution for t_f is the same the only change is that instead of τ_m we have τ_f and instead of τ_f we have τ_m (because of symmetry). So up to now we find the t_m and t_f and the only unknowns are t'_m and t'_f.

If $t_m=0$ then $t'_m=0$ and if $t_f=0$ then $t'_f=0$ (because unbounded t_m and t_f is continuous). If t_m and t_f are not 0 by implicit derivative theorem:

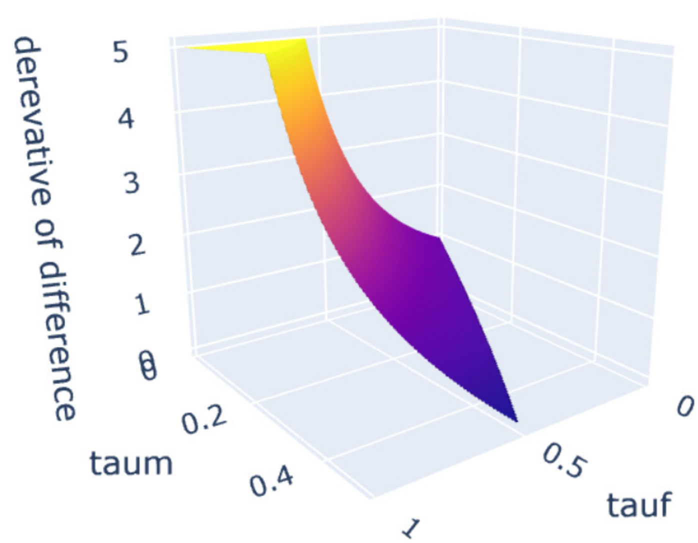
$$t'_m=\frac{38t_m^3+(-58+64\tau_m+14\tau_f)t_m^2+2(9+15\tau_m^2-10\tau_f+\tau_m(-36+7\tau_f))t_m+2+4\tau_m^3+3\tau_m^2(-6+\tau_f)+6\tau_f-2\tau_m(-8+5\tau_f)}{60t_m^3+3(38\tau_m+8\tau_f-28)t_m^2+2(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)t_m+10\tau_m^3+7\tau_m^2\tau_f-36\tau_m^2-20\tau_m\tau_f+18\tau_m+8\tau_f+4}$$

$$t'_f=\frac{8t_f^3+2(-8+7\tau_f)t_f^2+(8-20\tau_f+7\tau_f^2)t_f+\tau_f(6-5\tau_f+\tau_f^2)}{60t_f^3+3(38\tau_f+8\tau_m-28)t_f^2+2(32\tau_f^2+14\tau_f\tau_m-58\tau_f-16\tau_m+10)t_f+10\tau_f^3+7\tau_f^2\tau_m-36\tau_f^2-20\tau_f\tau_m+18\tau_f+8\tau_m+4}$$

So given τ_m and τ_f we found t_m, t_f and then found t'_m and t'_f which can be plugged in (1). Then the plot is as follows:



The same plot but where all values that are more than 5 are set to 5:



It can be seen that for any values of τ_m and τ_f (1) holds. This concludes the proof.

(ii) the agents cannot switch to an agreement in t_m and τ_m.

According to Lemma 6, α does not affect the regime, so we can set it to 1 (female partner has full bargaining power). So if there is agreement, male partner is always indifferent between agreement and no agreement.
Now consider utility of the male partner in case of agreement, U_m, and utility of female partner in case of agreement, U_f; utility of male partner in case of no agreement, U_m^0, and utility of female partner in case of no agreement, U_f^0. The agents have an agreement if the utility from agreement for both agents is more or equal than the utility from no agreement.
The statement from the proposition is equivalent to the following "statement 2": if there is no agreement for some values of parameters, the agreement for the same parameters except bigger τ_m is impossible.
Note that to prove the statement 2 it is sufficient to show: if there exist some set of parameters such that $U_f = U_f^0$, then for that set $\frac{\partial(U_f - U_f^0)}{\partial \tau_m} \leq 0$.
If there is no such set when τ_m increases there is no change in the regime of the agreement because of the continuity of the utilities in τ_m. More strictly, if there is a change in regime, that is for some τ_{m1} there is a agreement ($U_f > U_f^0$), and for another τ_{m2} there is no agreement ($U_f < U_f^0$), then there exists some $\tau_m \in (\tau_{m1}, \tau_{m2})$ such that $U_f = U_f^0$ by the continuity theorem. Note that here we need U_f and U_f^0 to be continuous in τ_m (that is obviously true). By the same theorem, if $\tau_{m1} > \tau_{m2}$ (there is a switch from agreement to no agreement), then there exist τ_m such that $U_f = U_f^0$ and $\frac{\partial(U_f - U_f^0)}{\partial \tau_m} > 0$. So under "statement 2", the switch from no agreement to agreement is impossible.
Hence, it remains to prove the following:

$$\frac{\partial(U_f - U_f^0)}{\partial \tau_m} < 0$$

if $U_f = U_f^0$.
Denote c_f the consumption of the female partner under agreement, c_f^0 — consumption of female partner under no agreement, n number of kids under agreement, n^0 number of kids without agreement.
In the outside option the maximization is as follows:

$$\max_{t_f, t_m}(1 - t_f - \tau_f)(1 - t_f)(1 - t_m - \tau_m)(1 - t_m)((t_f + \tau_f)^a + (t_m + \tau_m)^a)$$

The first order conditions for this maximization problem are:

$$\frac{1}{1 - t_f - \tau_f} + \frac{1}{1 - t_f} = \frac{a(t_f + \tau_f)^{a-1}}{(t_f + \tau_f)^a + (t_m + \tau_m)^a}$$

$$\frac{1}{1-t_m-\tau_m}+\frac{1}{1-t_m}=\frac{a(t_m+\tau_m)^{a-1}}{(t_f+\tau_f)^a+(t_m+\tau_m)^a}$$

Solving the two equations numerically, t_m and t_f can be obtained. Let us find the derivative of the t_m and t_f with respect to τ_m. Denote $t'_m=\frac{\partial t_m}{\partial \tau_m}$ and $t'_f=\frac{\partial t_f}{\partial \tau_m}$. Taking the derivative of first order conditions with respect to τ_m, we get:

$$\frac{t'_f}{(1-t_f-\tau_f)^2}+\frac{t'_f}{(1-t_f)^2}=$$
$$=\frac{t'_f a(a-1)(t_f+\tau_f)^{a-2}}{(t_f+\tau_f)^a+(t_m+\tau_m)^a}-\frac{a^2(t_f+\tau_f)^{a-1}(t'_f(t_f+\tau_f)^{a-1}+(t'_m+1)(\tau_m+t_m)^{a-1})}{((t_f+\tau_f)^a+(t_m+\tau_m)^a)^2}$$
$$\frac{t'_m+1}{(1-t_m-\tau_m)^2}+\frac{t'_m}{(1-t_m)^2}=$$
$$=\frac{(t'_m+1)a(a-1)(t_m+\tau_m)^{a-2}}{(t_f+\tau_f)^a+(t_m+\tau_m)^a}-\frac{a^2(t_m+\tau_m)^{a-1}(t'_f(t_f+\tau_f)^{a-1}+(t'_m+1)(\tau_m+t_m)^{a-1})}{((t_f+\tau_f)^a+(t_m+\tau_m)^a)^2}$$

Note that these derivatives are linear in t'_m and t'_f. To find t'_m and t'_f, denote:

$$x=\frac{1}{(1-t_f-\tau_f)^2}+\frac{1}{(1-t_f)^2}-\frac{a(a-1)(t_f+\tau_f)^{a-2}}{(t_f+\tau_f)^a+(t_m+\tau_m)^a}+\frac{a^2(t_f+\tau_f)^{2a-2}}{((t_f+\tau_f)^a+(t_m+\tau_m)^a)^2}$$

$$y=\frac{a^2(t_f+\tau_f)^{a-1}(\tau_m+t_m)^{a-1}}{((t_f+\tau_f)^a+(t_m+\tau_m)^a)^2}$$

$$z=\frac{1}{(1-t_m-\tau_m)^2}+\frac{1}{(1-t_m)^2}-\frac{a(a-1)(t_m+\tau_m)^{a-2}}{(t_f+\tau_f)^a+(t_m+\tau_m)^a}+\frac{a^2(t_m+\tau_m)^{2a-2}}{((t_f+\tau_f)^a+(t_m+\tau_m)^a)^2}$$

$$f=\frac{1}{(1-t_m-\tau_m)^2}-\frac{a(a-1)(t_m+\tau_m)^{a-2}}{(t_f+\tau_f)^a+(t_m+\tau_m)^a}+\frac{a^2(t_m+\tau_m)^{2a-2}}{((t_f+\tau_f)^a+(t_m+\tau_m)^a)^2}$$

Rewriting the conditions for t'_m and t'_f as: $xt'_f+yt'_m+y=0$ and $yt'_f+zt'_m+f=0$ and solving them, we obtain:

$$t'_m=\frac{fx-y^2}{y^2-zx}$$
$$t'_f=\frac{zy-fy}{y^2-zx}$$

Also note that if $t_m = 0$, then $t'_m = 0$ and if $t_f = 0$, then $t'_f = 0$ (no change for a small increase in τ_m). Hence, t'_m and t'_f are determined by the equations above. Let us now return to the derivative of the difference in utility.

$$\frac{\partial(U_f - U_f^0)}{\partial \tau_m} < 0$$

Note that by definition of U_f and U_f^0:

$$\frac{\partial(U_f - U_f^0)}{\partial \tau_m} = \frac{\partial((c_f + T)^c (c_f)^{1-c} n - c_f^0 n^0)}{\partial \tau_m} = \frac{\partial((1 + \frac{T}{c_f})^c c_f n - c_f^0 n^0)}{\partial \tau_m}$$

Taking the derivative, we obtain:

$$\frac{\partial(c_f n)}{\partial \tau_m}(1 + \frac{T}{c_f})^c - \frac{\partial(c_f^0 n^0)}{\partial \tau_m} + c_f n c \left(1 + \frac{T}{c_f}\right)^{c-1} \frac{\partial(\frac{T}{c_f})}{\partial \tau_m} < 0$$

This equation is equivalent to:

$$\frac{\partial(\frac{c_f}{w_f} n)}{\partial \tau_m}(1 + \frac{T}{c_f})^c - \frac{\partial(\frac{c_f^0}{w_f} n^0)}{\partial \tau_m} + \frac{c_f}{w_f} n c (1 + \frac{T}{c_f})^{c-1} \frac{\partial(\frac{T}{c_f})}{\partial \tau_m} < 0 \tag{1}$$

c is such that female partner is indifferent between agreement and no agreement. So:

$$U_f = U_f^0$$

$$(1 + \frac{T}{c_f})^c c_f n = c_f^0 n^0$$

$$(1 + \frac{T}{c_f})^c = \frac{c_f^0 n^0}{c_f n}$$

$$c = \frac{\log(\frac{c_f^0 n^0}{c_f n})}{\log(1 + \frac{T}{c_f})}$$

Plugging c in the equation (1):

$$\frac{\partial(\frac{c_f}{w_f} n)}{\partial \tau_m} \frac{c_f^0 n^0}{c_f n} - \frac{\partial(\frac{c_f^0}{w_f} n^0)}{\partial \tau_m} + \frac{c_f^0}{w_f} n^0 \frac{\log(\frac{c_f^0 n^0}{c_f n})}{\log(1 + \frac{T}{c_f})}(1 + \frac{T}{c_f})^{-1} \frac{\partial(\frac{T}{c_f})}{\partial \tau_m} < 0 \tag{2}$$

The value of T is such that male partner is indifferent between agreement and no agreement:

$$(c_m - T)n = c_m^0 n^0$$
$$\frac{T}{c_f} = \frac{c_m}{c_f} - \frac{c_m^0 n^0}{nc_f}$$

Plugging this result in (2):

$$\frac{\partial(\frac{c_f}{w_f}n)}{\partial\tau_m}\frac{c_f^0 n^0}{c_f n} - \frac{\partial(\frac{c_f^0}{w_f}n^0)}{\partial\tau_m} + \frac{c_f^0}{w_f}n^0\frac{\log(\frac{c_f^0 n^0}{c_f n})}{\log(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})^{-1}\frac{\partial(\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}{\partial\tau_m} < 0 \tag{3}$$

Let us now find the relationship between the left hand side of (3) and w_f or w_m. Note that $\frac{c_f}{w_f}$, $\frac{c_f^0}{c_f}$, n^0, n do not depend on w_m or w_f. So the only part of (3) that depend on w_f or w_m is

$$\frac{\frac{\partial(\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}{\partial\tau_m}}{\log(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}$$

Note that $\frac{c_m}{c_f} - \frac{c_m^0 n^0}{nc_f}$ is linear in $\frac{w_m}{w_f}$. Let $d = \frac{w_f c_m}{w_m c_f} - \frac{w_f c_m^0 n^0}{w_m nc_f}$. Note that d does not depend on w_f or w_m. Then we can rewrite previous equation as:

$$\frac{w_m}{w_f}\frac{\frac{\partial(a)}{\partial\tau_m}}{\log(1+\frac{w_m}{w_f}a)(1+\frac{w_m}{w_f}a)} \tag{4}$$

Note that $a > 0$ because $a = \frac{w_f T}{w_m c_f}$ and w_m, w_f, T, c_f are positive. Also note that previous equation is monotone in $\frac{w_m}{w_f}$. It can be proved by dividing by constant $\frac{\partial(a)}{\partial\tau_m}$ (it does not depend on $\frac{w_m}{w_f}$) and taking log (monotonic transformation). Then taking a derivative with the respect to $\frac{w_m}{w_f}$, we obtain:

$$\frac{1}{\frac{w_m}{w_f}} + \frac{a}{(1+\frac{w_m}{w_f}a)} + \frac{a}{\log(1+\frac{w_m}{w_f}a)(1+\frac{w_m}{w_f}a)}$$

Note that this equation is positive, motononicity of (4) is proved.
Note also that $\frac{w_m}{w_f} \in [1, +\infty)$. So the minimum and maximum (with the respect

to $\frac{w_m}{w_f}$) of (4) and (3) are achieved for either $\frac{w_m}{w_f} = 1$ or $\frac{w_m}{w_f} \to +\infty$.
For $\frac{w_m}{w_f} = 1$ (4) is equal to

$$\frac{\frac{\partial(a)}{\partial \tau_m}}{\log(1+a)(1+a)}$$

For $\frac{w_m}{w_f} \to +\infty$ (4) is equal to 0.
To prove the proposition, we need to prove that (3) is less than 0 for all possible $\frac{w_m}{w_f}$. Because (4) is linear in (3) it is sufficient to show that (3) is less than 0 for maximum and minimum of (4). Hence, inequality (3) is equivalent to:

$$\frac{\partial(\frac{c_f}{w_f}n)}{\partial \tau_m}\frac{c_f^0 n^0}{c_f n} - \frac{\partial(\frac{c_f^0}{w_f}n^0)}{\partial \tau_m} + \frac{c_f^0}{w_f}n^0 q < 0 \tag{5}$$

where

$$q = \max\{0, \log(\frac{c_f^0 n^0}{c_f n})\frac{\frac{\partial(\frac{c_m}{c_f} - \frac{c_m^0 n^0}{n c_f})}{\partial \tau_m}}{\log(1 + \frac{c_m}{c_f} - \frac{c_m^0 n^0}{n c_f})(1 + \frac{c_m}{c_f} - \frac{c_m^0 n^0}{n c_f})} \text{if } \frac{w_m}{w_f} = 1\}$$

Note that (5) does not depend on $\frac{w_m}{w_f}$. Using the definition of c_f^0, n^0, n, c_m^0, c_f, c_m, we obtain:

$$\frac{\partial((1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a)}{\partial \tau_m}\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a} -$$
$$-\frac{\partial((1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a))}{\partial \tau_m} + (1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)\max\{0, K_1\} < 0$$

where

$$K_1 = \log\left(\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a}\right)$$
$$\partial\log\left(\log\left(1+\frac{1}{(1-\tau_f-\tau_m-t_m-t_f)} - \frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)\right)\Big/\partial \tau_m$$

This inequality comes down to

$$(a(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^{a-1}(1+t'_m+t'_f) - (1+t'_m+t'_f)(\tau_m+t_m+\tau_f+t_f)^a)$$
$$\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a} + t'_f((\tau_m+t_m)^a+(\tau_f+t_f)^a) - (1-\tau_f-t_f)$$
$$a((\tau_m+t_m)^{a-1}(1+t'_m)+t'_f(\tau_f+t_f)^{a-1}) + (1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)\max\{0, K_2\} < 0$$

where

$$K_2 = \log\left(\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a}\right)\left(-\frac{1+t'_m+t'_f}{(1-\tau_f-\tau_m-t_m-t_f)^2}+\right.$$
$$+(1-\tau_m-t_m)((\tau_m+t_m)^a+(\tau_f+t_f)^a)$$
$$\frac{((\tau_m+t_m+\tau_f+t_f)^{a-1}(1+t'_m+t'_f)(1-\tau_f-\tau_m-t_m-t_f))-(\tau_m+t_m+\tau_f+t_f)^a(1+t'_m+t'_f)}{(\tau_m+t_m+\tau_f+t_f)^{2a}(1-\tau_f-\tau_m-t_m-t_f)^2}-$$
$$\left.-\frac{(1-\tau_m-t_m)((\tau_m+t_m)^{a-1}(1+t'_m)+(\tau_f+t_f)^{a-1}t'_f)-(1+t'_m)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)$$
$$\Bigg/\left(\log\left(1+\frac{1}{(1-\tau_f-\tau_m-t_m-t_f)}-\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)\right)$$
$$\left(1+\frac{1}{(1-\tau_f-\tau_m-t_m-t_f)}-\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)$$

Note that this inequality depends only on a, τ_m, τ_f, t_m, t_f, t'_m, t'_f. We previously showed that t_m, t_f, t'_m, t'_f can be expressed as a function of a, τ_m, τ_f. Recall that we should have $t_m+t_f+\tau_m+\tau_f<1$. For any given values of a, τ_m, τ_f, it can be shown that the above inequality hold

Hence, for any fixed a, τ_m, τ_f there cannot be a switch from no agreement to agreement in both t_m and τ_m. ∎