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Returns to labor mobility

Layoff costs and quit turbulence

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Returns to Labor Mobility *

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Abstract

Returns to labor mobility have too often escaped the attention they deserve as conduits of important forces in macro-labor models. These returns are shaped by calibrations of productivity processes that use theoretical perspectives and data sources from (i) labor economics and (ii) industrial organization. By studying how equilibrium unemployment responds to (a) layoff costs, and (b) likelihoods of skill losses following quits, we tighten calibrations of macro-labor models.

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Keywords: labor mobility, quits, turnover, layoff cost, turbulence, unemployment, human capital, matching model, search model, search-island model.

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1 Introduction

Although returns to labor mobility are important intermediating forces in all modern macroeconomic models with frictional labor markets, sources of evidence about stochastic processes that determine productivities of new and ongoing employment relationships and thereby influence those returns differ across studies. Thus, as inputs to calibrations, some leading macro-labor models have used worker flows and unemployment experiences, including patterns of how different government policies have been related to hazard rates for job-finding and job-separating. Other macro-labor models have used evidence about firm size dynamics assembled by students of industrial organization to restrict calibrations that support structural interpretations of how shocks that ultimately reshape labor reallocations are intermediated through production technologies. By studying how model-implied returns to labor mobility transcend these distinct theoretical perspectives and data sources, this paper sheds new light on workable calibrations of some celebrated macro-labor models.

Popular frameworks for studying frictional unemployment are: (1) matching models in the Diamond-Mortensen-Pissarides tradition; (2) equilibrium versions of McCall (1970) search models; and (3) search-island models in the spirit of Lucas and Prescott (1974). Calibrated versions of all three types of models have fit data on labor market flows well and have also generated plausible responses of unemployment rates to government policies like unemployment insurance and layoff taxes. We deploy some of these models here, focusing on their implications about returns to labor mobility and associated predictions for two distinct "computational experiments" in the spirit of Kydland and Prescott (1996): (1) effects of increases in layoff taxes, and (2) increases in workers' exposure to risks of human capital losses at times of voluntary quits ("quit turbulence").

Two leading frameworks for studying effects of layoff taxes on unemployment are the matching model of Mortensen and Pissarides (1999, henceforth MP), who calibrate productivity processes to unemployment statistics and outcomes in an unemployment insurance system; and the search-island model of Alvarez and Veracierto (2001, henceforth AV), who enlist establishment data on firm and worker turnover (Davis and Haltiwanger, 1990) to calibrate firm size dynamics that offer us different perspectives. Thus, AV's growth model intermediates productivity shocks through a neo-classical production function and gives rise to large returns to labor mobility that are robust to perturbations of parameters. MP's parameterization also yields the high returns to labor mobility that are compatible with the observation that high layoff taxes do not eliminate substantial labor reallocation in welfare states. But we have discovered a previously undetected fragility in MP's calibration that is associated with elements of a ridge traced out by two key parameters that, although they have very different implications for returns to labor mobility, can generate the same unemployment statistic targeted by MP.

More generally, in macro-labor models not quantitatively motivated by evidence on firm size dynamics and shocks to productivity that are intermediated through production functions, it is important to verify that parameter values yield high enough returns to labor mobility to be consistent with evidence on the substantial labor reallocation observed across market economies that deploy government policies that impose quite different costs and rewards to reallocating labor across firms.

We also approach returns to labor mobility from a different angle by studying the effects on unemployment of "turbulence," by which we mean increased hazard rates of human capital losses at times of job separations. When those skill losses occur at times of *involuntary* layoffs ("layoff turbulence"), Ljungqvist and Sargent (1998) show that increased turbulence causes unemployment to increase in a welfare state with generous unemployment benefits that are indexed to past earnings. den Haan, Haefke and Ramey (2005) added possible human capital losses coincident with *voluntary* separations ("quit turbulence"). By reducing workers' incentive to churn among jobs as they search for better opportunities, exposures to that risk exert downward pressure on unemployment. This channel provides another lens through which we can study returns to labor mobility. Thus, in the presence of quit turbulence Baley, Ljungqvist and Sargent (2023) showed that a positive turbulence-unemployment relationship requires returns to labor mobility that are high enough to be consistent with evidence that substantial labor reallocation occurs even in economies with significant layoff costs. In this paper we show how high returns to labor mobility are also required to accompany empirically plausible responses of unemployment to variations in layoff costs within the models of MP and AV. Within the same two models, those high returns to labor mobility also sustain a positive turbulence-unemployment relationship when quit turbulence is present.

Section 2 sets forth a benchmark model based on Ljungqvist and Sargent's (2007, henceforth LS) matching model into which we shall project productivity processes that we gather from versions of the MP and AV models. Sections 3 and 4 study outcomes of computational experiments that are intimately affiliated with returns to labor mobility. These sections also discuss how inferences about returns to labor mobility depend on whether we deduce them from theoretical perspectives and data coming from labor economics or from industrial organization. Section 5 offers some concluding remarks. Auxiliary materials appear in an online Appendix.

2 Benchmark model

We use a single benchmark model as our platform for bringing to bear diverse sources of evidence about the determinants of the returns to labor mobility. It is a standard matching model to which we add human capital dynamics that incorporate turbulence. Specifically, we adopt the LS matching model that has layoff turbulence in the form of worse skill transition probabilities for workers who suffer involuntary layoffs. We augment the model to include quit turbulence in the form of worse skill transition probabilities for workers who experience voluntary quits.

2.1 Environment

Workers There is a unit mass of workers who are either employed or unemployed. Workers are risk neutral, value consumption, and have preferences ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t. \tag{1}$$

They discount future utilities at a rate $\beta \equiv \hat{\beta}(1-\rho^r)$, where $\hat{\beta} \in (0,1)$ is a subjective time discount factor and $\rho^r \in (0,1)$ is a constant probability of retirement. A retired worker exits the economy and is replaced by a newborn worker.

Worker heterogeneity Besides employment status, workers differ along two dimensions: a current skill level i that can be either low (l) or high (h) and a skill level j that determines a worker's entitlement to unemployment benefits. An employed worker has $j = i$, but for an unemployed worker, j is the skill level during her last employment spell. Workers gain or lose skills depending on their employment status and instances of layoffs and quits. We assume that all newborn workers enter the labor force with low skills and a low benefit entitlement. In this way, each worker bears two indices (i, j), the first denoting current skill and the second denoting benefit entitlement.

Firms and matching technology There is free entry of firms who can post vacancies at a cost μ per period. Aggregate numbers of unemployed u and vacancies v are inputs into an increasing, concave and linearly homogeneous matching function $M(v, u)$. Let $\theta \equiv v/u$ be the vacancy-unemployment ratio, also called market tightness. The probability $\lambda^w(\theta) = M(v,u)/u = M(\theta, 1) \equiv m(\theta)$ that an unemployed worker encounters a vacancy is increasing in market tightness. The probability $M(v,u)/v = m(\theta)/\theta$ that a vacancy encounters an unemployed worker is decreasing in market tightness.

Worker-firm relationships and productivity processes A job opportunity is a productivity draw z from a distribution $v_i^o(z)$ that is indexed by a worker's skill level i. We assume that the high-skill distribution first-order stochastically dominates the low-skill distribution: $v_h^o(z) \leq v_l^o(z)$. Wages are determined through Nash bargaining, with π and $1-\pi$ as the bargaining weights of a worker and a firm, respectively.

Idiosyncratic shocks within a worker-firm match determine an employed worker's productivities. Productivity in an ongoing job is governed by a first-order Markov process with a transition probability matrix Q_i, also indexed by the worker's skill level i, where $Q_i(z, z')$ is the probability that next period's productivity becomes z', given current productivity z. Specifically, an employed worker retains her last period productivity with probability $1 - \gamma^s$, but with probability γ^s draws a new productivity from the distribution $v_i(z)$. As in the case of the productivity distributions for new matches, the high-skill distribution in ongoing jobs first-order stochastically dominates the low-skill distribution: $v_h(z) \leq v_l(z)$. Furthermore, an employed worker's skills may get upgraded from low to high with probability γ^u. A skill upgrade is accompanied by a new productivity drawn from the high-skill distribution $v_h(z)$. A skill upgrade is realized immediately, regardless of whether the worker remains with her present employer or quits.

We can now define our notions of layoffs and quits.

(i) **Layoffs:** At the beginning of each period, a job is exogenously terminated with probability ρ^x. We call this event a layoff. An alternative interpretation of the job-termination probability ρ^x is that productivity z becomes zero and stays zero forever. A layoff is involuntary in the sense of offering no choice.

(ii) **Quits:** As a consequence of a new productivity draw on a job and possibly a skill upgrade, a relationship can continue or be endogenously terminated. We call separation after such an event a voluntary quit because a firm and a worker agree to separate after Nash bargaining.

Turbulence We define turbulence as the risk of losing skills after a job separation. High-skilled workers might become low-skilled workers. Two types of turbulence shocks depend on the reason for a job separation, namely, a layoff or a quit. Upon a layoff, a high-skilled worker experiences a skill loss with probability $\gamma^{d,x}$. We call this risk *layoff turbulence*. Upon a quit, a high-skilled worker faces the probability γ^d of a skill loss. We call this risk *quit turbulence*.

Turbulence shocks are timed as follows. At the beginning of a period, exogenous job terminations occur and displaced workers face layoff turbulence. Employed workers can experience new productivity draws on the job and skill upgrades; if they quit, they are subject to quit turbulence. All separated workers join other unemployed workers in the matching function where they might or might not encounter vacancies next period.

Government policy The government provides unemployment compensation. An unemployed worker who was low (high) skilled in her last employment receives a benefit b_l (b_h).[1] Unemployment benefit b_i is calculated as a fraction ϕ of the average wage of employed workers with skill level i. The government imposes a layoff tax Ω on every job termination except for retirements.

The government runs a balanced budget by levying a flat-rate tax τ on production. If layoff tax revenues fully cover payments of unemployment benefits, the government sets $\tau = 0$ and returns any surplus as lump-sum transfers to workers. Since the latter will not happen in our analyses, we omit such lump-sum transfers in our expressions below.

2.2 Match surpluses

A match between a firm and a worker with skill level i and benefit entitlement j that has drawn productivity z will form an employment relationship, or continue an existing one, if a match surplus is positive. The match surplus for a new job $s^o_{ij}(z)$ or a continuing job $s_{ij}(z)$ is given by the after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$ minus the outside values of the match that consist of the worker's receiving unemployment benefit b_j and a future value ω^w_{ij} associated with entering the unemployment pool in the current period; and the firm's value ω^f from entering the vacancy pool in the current period. For notational simplicity, we define $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.

The match surplus for a new job $s^o_{lj}(z)$ or a continuing job $s_{lj}(z)$ with a low-skilled worker with benefit entitlement j is given by

$$s^o_{lj}(z) \;=\; s_{lj}(z) \;=\; (1-\tau)z + g_l(z) - [b_j + \omega_{lj}], \qquad j = l, h. \tag{2}$$

To compute the match surplus for jobs with high-skilled workers, we must distinguish between new and continuing jobs. The match surplus when forming a new job with an unemployed high-skilled worker, s^o_{hh}, involves outside values without any risk of skill loss if the match does not result in employment:

$$s^o_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \omega_{hh}]. \tag{3}$$

In contrast, the match surplus for a continuing job with a high-skilled worker or for a job with an earlier low-skilled worker who gets a skill upgrade that is immediately realized involves quit

[1] As mentioned above, newborn workers are entitled to b_l. Also, for simplicity, we assume that a worker who receives a skill upgrade and chooses to quit, is entitled to high benefits.

turbulence:

$$s_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d \omega_{lh}}_{\text{quit turbulence}}]. \tag{4}$$

Reservation productivities and rejection rates A worker and a firm split the match surplus through Nash bargaining with outside values as threat points. The splitting of match surpluses ensures mutual agreement whether to start (continue) a job. For a new (continuing) match, the reservation productivity $\underline{z}^o_{ij}$ $(\underline{z}_{ij})$ is the lowest productivity that makes a match profitable and satisfies

$$s^o_{ij}(\underline{z}^o_{ij}) \;=\; 0 \qquad \Big(s_{ij}(\underline{z}_{ij}) \;=\; -\Omega\Big). \tag{5}$$

Note that in a continuing match the surplus must fall to the negative of the layoff tax before a job is terminated.

Given the reservation productivity $\underline{z}^o_{ij}$ $(\underline{z}_{ij})$, let ν^o_{ij} (ν_{ij}) denote the rejection probability, which is given by the probability mass assigned to all draws from productivity distribution $v^o_i(y)$ $(v_i(y))$ that fall below the threshold:

$$\nu^o_{ij} \;=\; \int_{-\infty}^{\underline{z}^o_{ij}} dv^o_i(y) \qquad \left(\nu_{ij} \;=\; \int_{-\infty}^{\underline{z}_{ij}} dv_i(y)\right). \tag{6}$$

To simplify formulas below, we define

$$E_{ij} \equiv \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_i(y)]\; dv_i(y). \tag{7}$$

2.3 Joint continuation values

Consider a match between a firm and a worker with skill level i. Given a current productivity z, $g_i(z)$ is the joint continuation value of the associated match. We now characterize value functions for low- and high-skilled workers.

High-skilled worker The joint continuation value of a match of a firm with a high-skilled worker with current productivity z, denoted $g_h(z)$, is affected by future layoff turbulence if the worker is laid off or by future quit turbulence if a productivity switch is rejected:

$$\begin{aligned}
&\text{Exogenous separation:} && g_h(z) = \beta\Big[\rho^x(b_h + \underbrace{(1-\gamma^{d,x})\omega_{hh} + \gamma^{d,x}\omega_{lh}}_{\text{layoff turbulence}}) \\
&\text{Productivity switch:} && \quad + (1-\rho^x)\gamma^s(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}})) \\
&\text{No changes:} && \quad + (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z))\Big]. \qquad (8)
\end{aligned}$$

Low-skilled worker The joint continuation value of a match of a firm with a low-skilled worker takes into account the following contingencies: no changes in productivity or skills, an exogenous separation, a productivity switch, and a skill upgrade. When a skill upgrade occurs, a worker immediately become entitled to high unemployment benefits, even if the worker quits. Furthermore, a skill upgrade coincides with a new draw from the high-skill productivity distribution v_h. Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z is

$$\begin{aligned}
&\text{Exogenous separation:} && g_l(z) = \beta\Big[\rho^x(b_l + \omega_{ll}) \\
&\text{Immediate skill upgrade:} && \quad + (1-\rho^x)\gamma^u(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}})) \\
&\text{Productivity switch:} && \quad + (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll} + \nu_{ll}(b_l + \omega_{ll})) \\
&\text{No changes:} && \quad + (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z))\Big]. \qquad (9)
\end{aligned}$$

2.4 Outside values

Value of unemployment An unemployed worker with current skill level i and benefit entitlement j receives benefits b_j and has a future value ω^w_{ij}. Recall that the probability that an unemployed worker becomes matched next period is $\lambda^w(\theta)$.

A low-skilled unemployed worker with benefit entitlement j obtains $b_j + \omega^w_{lj}$, where

$$\omega^w_{lj} = \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{lj}}^{\infty} \pi s^o_{lj}(y)\, dv^o_l(y)}_{\text{match + accept}} + \underbrace{b_j + \omega^w_{lj}}_{\text{outside value}}\Big] \qquad j = l, h. \qquad (10)$$

A high-skilled unemployed worker with benefit entitlement h, obtains $b_h + \omega^w_{hh}$, where

$$\omega^w_{hh} = \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{hh}}^{\infty} \pi s^o_{hh}(y)\, dv^o_h(y)}_{\text{match + accept}} + \underbrace{b_h + \omega^w_{hh}}_{\text{outside value}}\Big]. \qquad (11)$$

Value of a vacancy A firm that searches for a worker pays an upfront cost μ to enter the vacancy pool and thereby obtains a fraction $(1-\pi)$ of the match surplus if an employment relationship is formed next period. Let $\lambda^f_{ij}(\theta)$ be the probability of filling the vacancy with an unemployed worker of type (i,j). Then a firm's value ω^f of entering the vacancy pool is:

$$\omega^f \;=\; -\mu + \beta\Big[\underbrace{\sum_{(i,j)} \lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty}(1-\pi)s^o_{ij}(y)\;dv^o_i(y)}_{\text{match + accept}} \;+\; \underbrace{\omega^f}_{\text{outside value}}\Big]. \tag{12}$$

2.5 Market tightness and matching probabilities

Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j. The total number of unemployed workers is $u=\sum_{(i,j)} u_{ij}$. The probability $\lambda^w(\theta)$ that an unemployed worker encounters a vacancy is function only of market tightness θ; the probability $\lambda^f_{ij}(\theta)$ that a vacancy encounters an unemployed worker with skill level i and benefit entitlement j also depends on the worker composition in the unemployment pool. Free entry of firms implies that a firm's expected value of posting a vacancy is zero. Equilibrium market tightness can be deduced from equation (12) with $w^f=0$. We summarize these labor market outcomes as follows:

$$\omega^f \;=\; 0 \tag{13}$$

$$\mu \;=\; \beta(1-\pi)\sum_{(i,j)}\lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\;dv^o_i(y) \tag{14}$$

$$\lambda^w(\theta) \;=\; m(\theta) \tag{15}$$

$$\lambda^f_{ij}(\theta) \;=\; \frac{m(\theta)}{\theta}\frac{u_{ij}}{u}\,. \tag{16}$$

2.6 Wages

When computing wages, we assume standard Nash bargaining between a worker and a firm each getting their shares of the match surplus in every period.[2] Given a productivity draw z in a new match with a positive match surplus, the wage $p^o_{lj}(z)$ of a low-skilled worker with benefit entitlement $j=l,h$ and the wage $p^o_{hh}(z)$ of a high-skilled worker, respectively, solve the

[2]An implication of the Nash bargaining assumption is that workers pay part of the layoff tax upon a job separation. An alternative assumption is that once a worker is hired, firms are the only ones liable for the layoff tax. This generates a two-tier wage system à la Mortensen and Pissarides (1999). Risk neutral firms and workers would be indifferent between adhering to period-by-period Nash bargaining or a two-tier wage system. Ljungqvist (2002) showed that the two-tier wage system affects the wage profile, not the allocation. Match surpluses, reservation productivities, and market tightness remain the same. Under the two-tier wage system, an initial wage concession by a newly hired worker is equivalent to her posting a bond that equals her share of a future layoff tax.

following maximization problems:

$$\max_{p^o_{lj}(z)} \quad \left[(1-\tau)z - p^o_{lj}(z) + g^f_l(z) - \omega^f\right]^{1-\pi} \left[p^o_{lj}(z) + g^w_l(z) - b_j - \omega^w_{lj}\right]^{\pi} \tag{17}$$
$$\max_{p^o_{hh}(z)} \quad \left[(1-\tau)z - p^o_{hh}(z) + g^f_h(z) - \omega^f\right]^{1-\pi} \left[p^o_{hh}(z) + g^w_h(z) - b_h - \omega^w_{hh}\right]^{\pi},$$

where $g^w_i(z)$ and $g^f_i(z)$ are future values obtained by the worker and the firm, respectively, from continuing the employment relationship;[3] and ω^f and $b_j + \omega^w_{ij}$ are outside values defined in (10), (11), and (12). The solution to the wage determination problems sets the sum of the worker's wage and continuation value equal to the worker's share π of the match surplus plus her outside value:

$$\begin{aligned} p^o_{lj}(z) + g^w_l(z) &= \pi s^o_{lj}(z) + b_j + \omega^w_{lj} \qquad j = l, h \\ p^o_{hh}(z) + g^w_h(z) &= \pi s^o_{hh}(z) + b_h + \omega^w_{hh}, \end{aligned} \tag{18}$$

where the worker continuation values are

$$\begin{aligned} g^w_l(z) &= \beta(1-\rho^x)\pi \left\{(1-\gamma^u)\left[(1-\gamma^s)s_{ll}(z) + \gamma^s \int_{\underline{z}_{ll}}^{\infty} s_{ll}(y)\, dv_l(y)\right] + \gamma^u \int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\, dv_h(y)\right\} \\ &+ \beta(\rho^x + (1-\rho^x)(1-\gamma^u))\left(b_l + \omega^w_{ll}\right) + \beta(1-\rho^x)\gamma^u\left(b_h + (1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}\right) \\ g^w_h(z) &= \beta(1-\rho^x)\pi\left[(1-\gamma^s)s_{hh}(z) + \gamma^s \int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\, dv_h(y)\right] \\ &+ \beta\rho^x\left(b_h + (1-\gamma^{d,x})\omega^w_{hh} + \gamma^{d,x}\omega^w_{lh}\right) + \beta(1-\rho^x)\left(b_h + (1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}\right). \end{aligned} \tag{19}$$

For ongoing employment relationships, the wages $p_{ll}(z), p_{hh}(z)$ satisfy counterparts of the above equations that use the corresponding match surpluses $s_{ll}(z)$ and $s_{hh}(z)$:

$$\begin{aligned} p_{ll}(z) + g^w_l(z) &= \pi s_{ll}(z) + b_l + \omega^w_{ll} \\ p_{hh}(z) + g^w_h(z) &= \pi s_{hh}(z) + b_h + \underbrace{(1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}}_{\text{quit turbulence}}, \end{aligned} \tag{20}$$

where the latter expression for the high-skilled wage now involves quit turbulence on the right side.

[3] *Joint* continuation values defined in (8) and (9) equal sums of the individual continuation values: $g_i(z) = g^w_i(z) + g^f_i(z),\ i = l, h$.

2.7 Government budget constraint

Unemployment benefits Benefit entitlement j awards an unemployed worker benefit b_j equal to a fraction ϕ of the average wage $\bar{p}_j$ of employed workers with skill level j. Therefore, total government expenditure on unemployment benefits amounts to

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})). \tag{21}$$

Layoff taxes Let Ξ be total separations excluding retirements, which are equal to

$$\Xi = (1-\rho^r)\Big[\rho^x(e_{ll}+e_{hh}) + (1-\rho^x)[(1-\gamma^u)\gamma^s\nu_{ll} + \gamma^u\nu_{hh}]e_{ll} + (1-\rho^x)\gamma^s\nu_{hh}e_{hh}\Big]. \tag{22}$$

Then government revenue from layoff taxation equals $\Omega\,\Xi$.

Income taxes Output is taxed at a constant rate τ. Let $\bar{z}_i$ be the average productivity of employed workers with skill level i. Hence, total tax revenue equals $\tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh})$, where e_{ll} (e_{hh}) is the number of employed workers with low skills and low benefit entitlement (high skills and high benefit entitlement).

Balanced budget The government runs a balanced budget. The tax rate τ on output is set to cover the expenditures on unemployment benefits described in (21) net of layoff tax revenues $\Omega\,\Xi$:

$$\phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})) - \Omega\,\Xi = \tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh}). \tag{23}$$

For computations of average wages $\bar{p}_i$ and average productivities $\bar{z}_i$, see Appendix A.2.

2.8 Worker flows

Workers move across employment and unemployment states, skill levels, and benefit entitlement levels. Here we focus on a group of workers at the center of our analysis: low-skilled unemployed with high benefits. (Appendix A.1 describes flows for other groups of workers.)

Inflows to the low-skilled unemployed with high benefits u_{lh} occur in the following situations. Layoff turbulence affects high-skilled workers e_{hh} who get laid off; with probability $\gamma^{d,x}$, they become part of the low-skilled unemployed with high benefit entitlement. Quit turbulence affects high-skilled workers e_{hh} who reject productivity switches, as well as low-skilled workers e_{ll} who get skill upgrades and then reject their new productivity draws. All of those quitters face probability γ^d of becoming part of the low-skilled unemployed with high benefit entitlement. Outflows from unemployment occur upon successful matching function encounters and retirements. Thus, the net change of low-skilled unemployed with high benefits (equalling zero in a steady state) becomes:

$$\Delta u_{lh} = (1-\rho^r)\Big\{ \underbrace{\rho^x \gamma^{d,x} e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^d \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} \Big\} - \rho^r u_{lh}. \quad (24)$$

Terms numbered 1 and 3 in expression (24) isolate the forces behind the positive turbulence-unemployment relationship in a welfare state in the LS model. Although more layoff turbulence in term 1 – a higher probability $\gamma^{d,x}$ of losing skills after layoffs – has a small effect on equilibrium unemployment in a laissez-faire economy, it gives rise to a strong turbulence-unemployment relationship in a welfare state that offers a generous unemployment benefit replacement rate on a worker's earnings in her last employment. After a layoff with skill loss, those benefits are high relative to a worker's earnings prospects at her now diminished skill level. As a consequence, the acceptance rate $(1-\nu^o_{lh})$ in term 3 is low; because of the relatively high outside value of a low-skilled unemployed with high benefits, fewer matches have positive match surpluses, as reflected in a high reservation productivity z^o_{lh}. Moreover, given those suppressed match surpluses, equilibrium market tightness θ falls to restore firm profitability enough to make vacancy creation break even. Lower market tightness, in turn, reduces the probability $\lambda^w(\theta)$ that a worker encounters a vacancy, which further suppresses successful matches and contributes to the positive turbulence-unemployment relationship.

The assumption of quit turbulence adds the term numbered 2 in expression (24) that exerts a countervailing force against the positive turbulence-unemployment relationship described above. When higher turbulence is associated with voluntary quits that are also subject to risks of skill loss, there will be a lower incidence of voluntary quits in turbulent times because the risk of skill loss makes high-skilled workers more reluctant to quit. This makes the rejection rate ν_{hh} in term 2 become low in turbulent times. That lower rejection rate causes lower inflows into low-skilled unemployed with high benefits u_{lh} as well as into high-skilled unemployed with high benefits u_{hh}. This force might reverse the positive turbulence-unemployment relationship.

2.9 Steady state equilibrium

A steady state equilibrium consists of measures of unemployed u_{ij} and employed e_{ij}; a labor market tightness θ, probabilities $\lambda^w(\theta)$ that workers encounter vacancies and $\lambda^f_{ij}(\theta)$ that vacancies encounter workers; reservation productivities $\underline{z}^o_{ij}, \underline{z}_{ij}$, match surpluses $s^o_{ij}(z), s_{ij}(z)$, future values of an unemployed worker ω^w_{ij} and of a firm posting a vacancy ω^f; wages $p^o_{ij}(z), p_{ij}(z)$; unemployment benefits b_i, a layoff tax Ω, and a tax rate τ; such that

a) Match surplus conditions (5) determine reservation productivities.

b) Free entry of firms implies zero-profit condition (14) in vacancy creation that pins down market tightness.

c) Nash bargaining outcomes (18) and (20) set wages.

d) The tax rate balances the government's budget (23).

e) Net worker flows, such as expression (24), are all equal to zero: $\Delta u_{ij} = \Delta e_{ij} = 0, \quad \forall\, i, j.$

2.10 Parameterization

Apart from considering alternative assumptions about the productivity process and different values of the layoff tax, the benchmark model shares the remaining parameterization with LS, in conjunction with a codification of quit turbulence contributed by den Haan, Haefke and Ramey (2005), as reported in Table 1. The model period is half a quarter.

Preference parameters Given a semi-quarterly model period, we specify a discount factor $\hat{\beta} = 0.99425$ and a retirement probability $\rho^r = 0.0031$, which together imply an adjusted discount of $\beta = \hat{\beta}(1 - \rho^r) = 0.991$. The retirement probability implies an average time of 40 years in the labor force.

Worker skills and productivity Low-skilled and high-skilled workers' skills are $i \in \{1, 2\}$. Exogenous layoffs occur with probability $\rho^x = 0.005$, on average a layoff every 25 years. We set a probability of upgrading skills $\gamma^u = 0.0125$ so that, on average, it takes 10 years to move from low to high skill, conditional on no job loss. The probability of a productivity switch on the job equals $\gamma^s = 0.05$, so a worker expects to retain her productivity for 2.5 years. Idiosyncratic productivity distributions for new $v_i^o(z)$ and ongoing matches $v_i(z)$ are central to our study of returns to labor mobility and will be described in sections 3 and 4.

Layoff and quit turbulence We parametrize quit turbulence as a fraction ϵ of layoff turbulence, and we vary it from zero – only layoff turbulence – to one – the two types of turbulence are equal: $\gamma^d = \epsilon \gamma^{d,x}$. This specification captures the fact that two types of job leavers differ in their labor market prospects. Workers who suffer involuntary layoffs face higher risks of skill losses than do workers who choose to leave their jobs. And job quitters had a stay-on-a-job option that victims of layoffs do not have.

Table 1: Parameterization of benchmark model

Parameter	Definition	Value
Preferences		
$\hat{\beta}$	discount factor	0.99425
ρ^r	retirement probability	0.0031
$\beta = \hat{\beta}(1-\rho^r)$	adjusted discount	0.991
Sources of risk		
ρ^x	exogenous breakup probability	0.005
γ^u	skill upgrade probability	0.0125
γ^s	productivity switch probability	0.05
$\gamma^{d,x}$	layoff turbulence	$[0, 1]$
$\gamma^d = \epsilon\gamma^{d,x}$	quit turbulence	$\epsilon \in [0, 1]$
Labor market institutions		
π	worker bargaining power	0.5
ϕ	replacement rate	0.7
Ω	layoff tax	0
Matching function		
A	matching efficiency	0.45
α	elasticity of matches w.r.t. u	0.5
μ	cost of posting a vacancy	0.5

Labor market institutions We set a worker's bargaining power to be $\pi = 0.5$. We set the replacement rate in unemployment compensation at $\phi = 0.7$ and the layoff tax at $\Omega = 0$ (where the latter is to be perturbed in our investigation of returns to labor mobility).

Matching We assume a Cobb-Douglas matching function $M(v, u) = Au^{\alpha}v^{1-\alpha}$, which implies that the probability of a worker encountering a vacancy and the probability of a vacancy encountering a particular worker type, respectively, are:

$$\lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}. \tag{25}$$

The elasticity of matches with respect to unemployment is specified to be $\alpha = 0.5$ in accordance with a consensus about plausible values falling in the mid range of the unit interval (e.g., see the survey of Petrongolo and Pissarides (2001)). We adopt LS's parameterization of the matching efficiency $A = 0.45$ and the cost of posting a vacancy $\mu = 0.5$.[4]

The analysis presented in the following two sections provide insights from two distinct perspectives and associated sources of data: one from labor economics in section 3, another from the economics of industrial organization in section 4.

3 Returns to labor mobility in MP

In this section, we explore MP's matching model, a celebrated labor-macro vehicle for studying the consequences of layoff costs. In section 4 we explore another celebrated model, AV's search-island model. From the perspective of bringing the high returns to labor mobility required to succeed in our two computational experiments (i.e., layoff taxes and quit turbulence, respectively), we shall discover that the MP approach of using Labor data is more fragile to choice of parameter values than is AV's using IO data. Nevertheless, even though outcomes are sturdier in the AV framework, there exist plausible parameter values of the MP model that succeed in generating (1) weak effects of layoff costs on equilibrium unemployment and (2) a positive turbulence-unemployment relationship in our computational experiments.

MP study how skill dynamics can interact with welfare-state institutions in a matching model. But in contrast to our benchmark model, MP assume that individual workers are permanently attached to their skill levels and focus on the effects of a mean preserving spread of the cross section distribution of skills across workers. To capture "directed search," MP assume separate matching functions for each skill level.

For us, a key object of the MP model is a probability distribution of idiosyncratic productivities that multiply workers' skills in ongoing matches. MP assume that distribution function is uniform on support $[z^{min}, 1]$ so that the cumulative density is $F(z) = (z - z^{min})/(1 - z^{min})$ for all $z \in [z^{min}, 1]$. As in the benchmark model, productivity shocks in ongoing matches arrive at an exogenous rate γ^s. But in contrast to the benchmark model, new matches have productivity equal to the upper support of the distribution.

MP's parameterization in Table 2 gives the same arrival rate of productivity switches as in the benchmark model, i.e., MP's quarterly probability $\gamma^s = 0.1$ is consistent with the semi-quarterly probability $\gamma^s = 0.05$ in Table 1. Because of the narrow range of the support of MP's uniform distribution $[0.64, 1]$, one might expect returns to labor mobility in the MP model to be small. However, all new matches in the MP model have productivity equal to the upper support of the distribution, which enhances returns to labor mobility compared to our assumption that

[4]When calibrating a matching model to an aggregate unemployment rate without including a calibration target for vacancy statistics, selecting the parameter pair (A, μ) is a matter of normalization. LS's calibration target was 5 percent unemployment in the laissez-faire economy, $\phi = \Omega = \tau = 0$, in non-turbulent times, $\gamma^{d,x} = 0$.

a new match draws productivity from the same distribution as continuing matches. Thus, the question is a quantitative one – a question that will also compel us to investigate the calibration approach chosen by MP.

Table 2: MP's parameter values (central to our study)

Parameter	**Definition**	**Value**
z^{min}	minimum productivity	0.64
γ^s	productivity switch probability (at a quarterly frequency)	0.1

3.1 Mapping MP's productivity process into benchmark model

Our criterion for how faithfully we map the MP productivity process into the benchmark model is how closely the resulting economy resembles MP's (1999, Table 2a) findings on how unemployment responds to unemployment insurance and layoff taxes as reproduced in the first panel of our Table 3. Note that our benchmark model has two skill levels while MP choose to conduct their calculations for the case of a single skill level equal to 1. Another difference is that MP assume a training cost while our benchmark model has none.

As an intermediate step, we compute outcomes in a perturbed version of the benchmark model with several features modified to be the same as in MP. Specifically, the perturbed benchmark model has only low-skilled workers (with skills equal to one), no exogenous breakups $\rho^x = 0$, an added value of leisure equal to 0.28, and MP's productivity process with $z^{min} = 0.64$. The efficiency factor on the matching function is calibrated to be $A = 0.66$ in order to keep our target of 5 percent unemployment in the laissez-faire economy. The unemployment outcomes of the perturbed benchmark model in the second panel of Table 3 are almost the same as those of MP in our first panel. However, a noticeable difference is that benchmark model unemployment cannot become zero since there is exogenous retirement with probability $\rho^r = 0.0031$. Hence, the influx of new workers in the benchmark model means that the unemployment rate can never fall below 0.3 percent and will be higher if the average time to find a job for newcomers exceeds one semi-quarterly model period.

Encouraged by the success of our intermediate step in approximating MP's unemployment outcomes, we turn to the full-fledged version of the benchmark model with two skill levels, low-skilled and high-skilled workers with skills equal to 1 and 2, respectively. We restore the exogenous breakup probability $\rho^x = 0.005$ and set the value of leisure to zero. In short, we adopt the exact parameterization of the benchmark model in Table 1 while assuming the MP

Table 3: Unemployment Rate Effects of the UI Replacement Ratio (ϕ) and Layoff Tax (Ω)

Mortensen and Pissarides (1999, Table 2a)

	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$	$\phi = 0.3$	$\phi = 0.4$	$\phi = 0.5$
$\Omega = 0.0$	4.8	5.5	6.2	7.3	9.0	11.9
$\Omega = 0.5$	3.7	4.3	5.0	5.9	7.5	10.3
$\Omega = 1.0$	2.5	2.9	3.5	4.4	5.7	8.4
$\Omega = 1.5$	1.1	1.5	1.9	2.6	3.6	5.9
$\Omega = 2.0$	0.0	0.0	0.0	0.0	1.3	2.9

Perturbed version of benchmark model with only low-skilled workers

	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$	$\phi = 0.3$	$\phi = 0.4$	$\phi = 0.5$
$\Omega = 0.0$	5.0	5.5	6.2	7.2	8.6	11.0
$\Omega = 0.5$	4.2	4.6	5.2	6.0	7.2	9.2
$\Omega = 1.0$	3.2	3.6	4.1	4.8	5.9	7.6
$\Omega = 1.5$	2.2	2.5	2.9	3.5	4.4	5.9
$\Omega = 2.0$	1.1	1.3	1.7	2.1	2.8	3.9
$\Omega = 2.5$	0.4	0.5	0.5	0.6	1.0	1.8

A perturbed version of the benchmark model with only low-skilled workers, no exogenous breakups $\rho^x = 0$, an added value of leisure equal to 0.28, and MP's productivity process with $z^{min} = 0.64$. Matching efficiency is calibrated to $A = 0.66$. Layoff taxes Ω are expressed in terms of quarterly output.

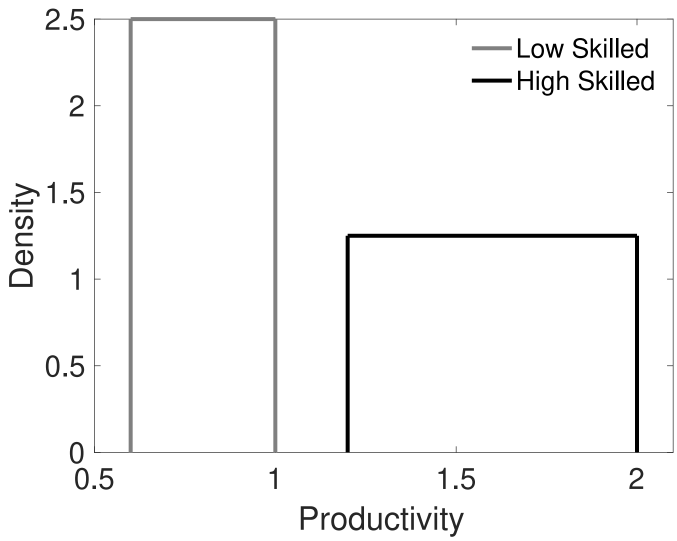
Benchmark model with the MP productivity process

	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$	$\phi = 0.3$	$\phi = 0.4$	$\phi = 0.5$	$\phi = 0.6$	$\phi = 0.7$
$\Omega = 0.0$	5.0	5.4	5.8	6.4	7.0	7.8	8.8	10.2
$\Omega = 1.0$	3.9	4.2	4.5	5.0	5.5	6.2	7.1	8.4
$\Omega = 2.0$	3.0	3.3	3.6	4.0	4.5	5.1	5.9	7.0
$\Omega = 3.0$	2.1	2.3	2.6	3.0	3.4	3.9	4.5	5.5
$\Omega = 4.0$	1.3	1.3	1.5	1.8	2.2	2.6	3.1	3.9
$\Omega = 5.0$	1.3	1.3	1.4	1.5	1.6	1.7	1.8	2.3

The benchmark model with MP's productivity process with $z^{min} = 0.6$. Matching efficiency is calibrated to $A = 0.37$. Layoff taxes Ω are expressed in terms of quarterly output.

productivity process with $z^{min} = 0.6$.[5] Figure 1 shows productivity densities for the MP model. Also, we re-calibrate the efficiency factor on the matching function to be $A = 0.37$ to have 5 percent unemployment in the laissez-faire economy.

Figure 1: MP PRODUCTIVITY DISTRIBUTIONS



The third panel of Figure 3 contains outcomes of our full-fledged version of the benchmark model with the MP productivity process. Now our comparison to MP's outcomes in the first panel has to be more subtle and bring to bear adjustments beyond those to the retirement rate deployed in our intermediate step. First, in our two-skill economy, the steady-state labor force consists of 20 percent low-skilled and 80 percent high-skilled workers. Thus, the layoff tax numbers in the third panel would have to be cut approximately in half to be comparable to the first two panels when expressing layoff taxes relative to workers' output since high-skilled workers who make up the vast majority of the labor force in the third panel are twice as productive as the workers of the first two panels. Because the layoff taxes reported in the third panel are twice as high as those reported in the first two panels, we can compare outcomes line-by-line across panels. Second, the assumption of a value of leisure equal to 0.28 for workers with skill level one in the first two panels lets us convert that into an extra replacement rate in unemployment insurance of 0.3 in the third panel. Thus, a replacement rate ϕ in the first two panels would correspond to a replacement rate of $\phi + 0.3$ in the third panel. Third, the last panel can be thought of as having calibrated a laissez-faire unemployment rate of 6.4 percent, as given by column $\phi = 0.3$ (and no layoff tax), because a replacement rate $\phi = 0.3$ would

[5] Since we do not aim to reproduce MP's unemployment outcomes exactly, we have rounded off the parameter value $z^{min} = 0.6$. We subject this parameter to sensitivity analysis in the next subsection. Low-skilled distribution is $1 \times [z^{min}, 1]$ and high-skilled distribution is $2 \times [z^{min}, 1]$.

represent only the value of leisure according to our conversion argument. A way to correct for this concocted elevated unemployment rate of the laissez-faire calibration is to deduct from each computed unemployment rate an adjustment equal to the difference between the third panel's column $\phi = 0.3$ and column $\phi = 0$, i.e., a single adjustment for each value of the layoff tax. As an illustration, these adjustments would turn the unemployment rates in column $\phi = 0$ into the new numbers of column $\phi = 0.3$.

The preceding three adjustments intended to make the third panel comparable to the first two panels are implemented in Table 4, including a re-labeling of replacement rates to become $\hat{\phi} = \phi - 0.3$ and layoff taxes to become $\hat{\Omega} = 0.5\,\Omega$. Evidently, our mapping of MP into the benchmark model is quite successful when comparing Table 4 to the MP outcomes in the first panel of Figure 3. However, differences appear at higher layoff taxes at which the higher unemployment rates of the benchmark model can largely be attributed to its exogenous rates of retirement $\rho^r = 0.0031$ and of breakups $\rho^x = 0.005$. Since our intermediate step includes the retirement rate but not the exogenous breakup rate, it is understandable that unemployment outcomes at higher layoff taxes in the second panel of Table 3 fall between the lower and higher unemployment rates of MP in the first panel of Table 3 and the benchmark model in Table 4, respectively. Apparently, at such high layoff taxes, endogenous separations have either shut down or are about to in all of the economies so that unemployment becomes driven mostly by exogenous shocks of separation.

Table 4: Assessing the success of mapping MP into benchmark model

Adjusted version of the benchmark model with the MP productivity process

	$\hat{\phi} = 0.0$	$\hat{\phi} = 0.1$	$\hat{\phi} = 0.2$	$\hat{\phi} = 0.3$	$\hat{\phi} = 0.4$	Adj. factor
$\hat{\Omega} = 0.0$	5.0	5.6	6.4	7.4	8.8	1.4
$\hat{\Omega} = 0.5$	3.9	4.4	5.1	6.0	7.3	1.1
$\hat{\Omega} = 1.0$	3.0	3.5	4.1	4.9	6.0	1.0
$\hat{\Omega} = 1.5$	2.1	2.5	3.0	3.6	4.6	0.9
$\hat{\Omega} = 2.0$	1.3	1.7	2.1	2.6	3.4	0.5
$\hat{\Omega} = 2.5$	1.3	1.4	1.5	1.6	2.1	0.2

3.2 Fragility of MP's calibration

In conducting the quantitative analysis of the preceding subsection, we encountered a fragility in how MP had restricted the calibration of a key parameter that affects returns to labor mobility, namely, the lower support z^{min} of the productivity distribution. We describe that fragility by conducting a quantitative sensitivity analysis with respect to the parameter z^{min} after first describing MP's calibration strategy.

MP (1999, pp. 256-257) describe their calibration strategy as follows:

> "The policy parameters are chosen to reflect the US case. All other structural parameters, except for the value of leisure b and minimum match product $[z^{min}]$ which are chosen so that the steady-state unemployment rate and the average duration of an unemployment spell match the average experience in the United States over the past twenty years are similar to those assumed and justified in Mortensen (1994) and Millard and Mortensen (1997)."

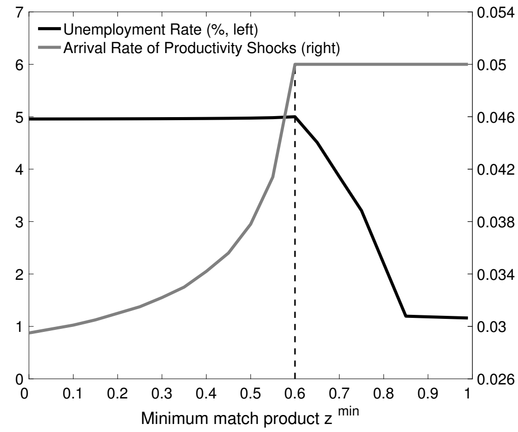
That calibration of values of leisure and z^{min} is confirmed by Millard and Mortensen (1997, p. 555) who say:

> "…two parameters for which there is no direct evidence, the forgone value of leisure b and a measure of dispersion in the idiosyncratic shock denoted as $[z^{min}]$, are chosen to match the average duration of unemployment and incidence of unemployment experienced over the 1983-92 period."

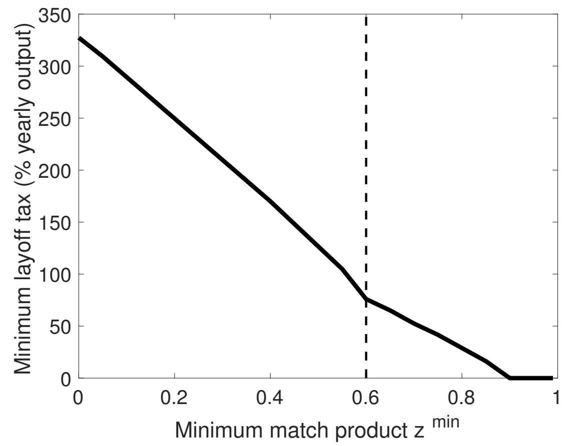
For a given steady-state unemployment rate, calibrations of the average duration of an unemployment spell and the incidence of unemployment are two sides of the same coin. Below, we calibrate to target the incidence of unemployment. However, our most important move is to put on the table another of MP's parameters for which we have no direct evidence, namely, the arrival rate γ^s of productivity shocks.

We use the laissez-faire version ($\phi = \Omega = 0$) of the benchmark model with the MP productivity process in the third panel of Table 3 to explain this important tradeoff associated with the choice of a pair (z^{min}, γ^s). Recall that the economy is parameterized to have $z^{min} = 0.6$ and a productivity switch probability $\gamma^s = 0.005$ in the semi-quarterly model period (which corresponds to MP's quarterly probability 0.1 in Table 2). Now, in accordance with MP's target of a particular incidence of unemployment (or, on the flip side, a particular average duration of an unemployment spell), we 'freeze' the laissez-faire economy's quarterly separation rate of 6.77 percent. Specifically, for each value of $z^{\min} \leq 0.6$, we find an associated value of γ^s that implies an unchanged quarterly separation rate (while adjusting parameter A to keep hitting our target of 5 percent unemployment). The lighter curve in Figure 2a traces out pairs of (z^{min}, γ^s) that attain the targeted quarterly separation rate of 6.77 percent. In our 'normal' parameter range, there is a positive relationship between z^{min} and γ^s, because a higher z^{min} means smaller dispersion of productivity and therefore fewer shocks that call forth endogenous quits, so the exogenous arrival rate of shocks γ^s has to be raised to keep the separation rate

Figure 2: Calibration of benchmark model with MP productivity z^{min}



(a) Arrival rate of productivity shocks, γ^s



(b) Minimum layoff tax to shut down quits

unchanged. The darker line shows that the laissez-faire unemployment rate remains constant at 5 percent throughout these calculations for $z^{\min} \leq 0.6$.

We can also extend these calculations for $z^{\min} > 0.6$ (not shown); but after 0.64 no γ^s can be found to generate as high a quarterly separation rate as 6.77 percent. To see why, notice that the lighter curve in Figure 2a becomes ever steeper as it approaches $z^{\min} = 0.6$ from below. Evidently, this arithmetic must eventually come to a stop, since it would be impossible to maintain *any* endogenous separations as the parameter z^{min} approaches the upper support of 1 where the productivity distribution would become degenerate as a single mass point. Instead of depicting the breakdown of our algorithm, we freeze all the parameters of the economy at $z^{min} = 0.6$, except for the parameter itself as we compute equilibria for higher values of z^{min}. As depicted in Figure 2a for $z^{\min} > 0.6$ and a constant productivity switch probability $\gamma^s = 0.05$, the unemployment starts falling until all endogenous separations come to a halt and the unemployment curve becomes horizontal to reflect exogenous rates of retirement $\rho^r = 0.0031$ and breakups $\rho^x = 0.005$.

For each parameter configuration (z^{min}, γ^s) deduced in Figure 2a (and the associated value of parameter A), we study the unemployment effects of layoff taxes and the associated returns to labor mobility in the following way. Under the assumption of a replacement rate of $\phi = 0.7$, Figure 2b depicts the minimum layoff tax required to shut down all endogenous separations measured in terms of an average worker's annual output in the laissez-faire economy. Notice how the layoff-tax curve flattens out at zero minimum tax at the far right. As in Figure 2a, the flattening occurs because endogenous separations come to a halt at high values of z^{min}, so no layoff tax is required to shut them down.[6] While all parameter configurations to the left of $z^{\min} = 0.6$ in Figure 2 can generate the same unemployment statistic that MP targeted, they have very different implications for returns to labor mobility. Specifically, at higher values of the parameter $z^{\min}$, the implied returns to labor mobility are smaller since a smaller minimum layoff tax then shuts down all endogenous separations in Figure 2b.

The takeaway from Figure 2 is that MP unnecessarily constrained themselves by postulating a quarterly productivity switch probability 0.1 in Table 2. That caused MP to back themselves into a treacherous region of the parameter space in which further increases in $z^{\min}$ would have threatened to render MP's calibration targets unattainable. Furthermore and more problematic, MP's calibration inhabits a parameter region where returns to labor mobility are fragile with respect to perturbations of those parameters: while small increases in $z^{\min}$ would be compatible with attaining the unemployment statistic targeted by MP, they would cause the domain of the uniform distribution to become too small to generate returns to labor mobility that are high enough to describe plausible responses of unemployment to layoff taxes.

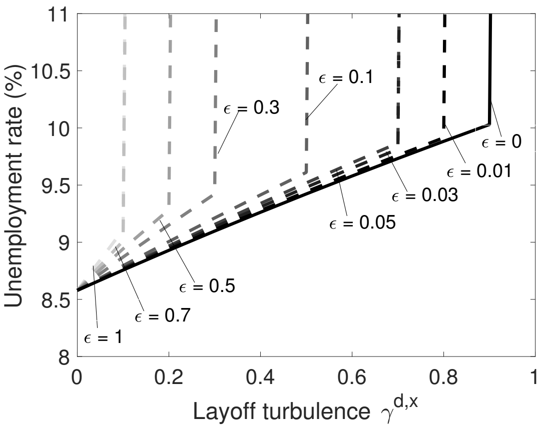
3.3 Turbulence under MP productivity process

Baley *et al.* (2023) verify that another manifestation of high returns to labor mobility is that quit turbulence does not overturn a positive turbulence-unemployment relationship in a welfare state with generous unemployment benefits. Within the benchmark model, we can investigate how high risks of skill losses at times of voluntary separations must be relative to risks of skill losses at times of involuntary separations in order to generate a negative rather than a positive turbulence-unemployment relationship. With a replacement rate $\phi = 0.7$, Figure 3 depicts how unemployment responds to turbulence in four of the calibrated economies from Figure 2, indexed by $z^{min} \in \{0, 0.2, 0.4, 0.6\}$. The two top panels show robust positive turbulence-unemployment relationships for any combination of layoff and quit turbulence.

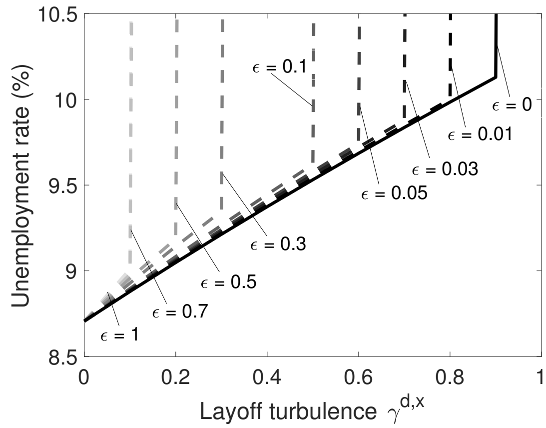
As compared to productivity processes studied by Baley *et al.*, a new feature in Figure 3 is the possibility of a spike that indicates a 'meltdown' that occurs when the unemployment rate soars to a level of $55 - 60$ percent (outside of the graphs). Several forces cause the meltdown. Under MP's assumption that all new jobs start with productivity equal to the upper support of the distribution, a reservation productivity can take only one of two possible values: either the upper support of the distribution is acceptable to a worker-vacancy encounter or it is not. This creates a possible a 'tipping point' at which a change in turbulence moves the economy *from*

[6] That the flattening occurs at a somewhat lower value of z^{min} in the laissez-faire economy of Figure 2a as compared to welfare state outcomes in Figure 2b indicates that the minimum layoff tax required to shut down all endogenous separations in the laissez-faire economy (when $\phi = 0$) would be lower than that of the welfare state (when $\phi = 0.7$). Without unemployment compensation, the gains from quitting and searching for another job are smaller, so it requires a smaller layoff tax to shut down endogenous separations in the laissez-faire economy.

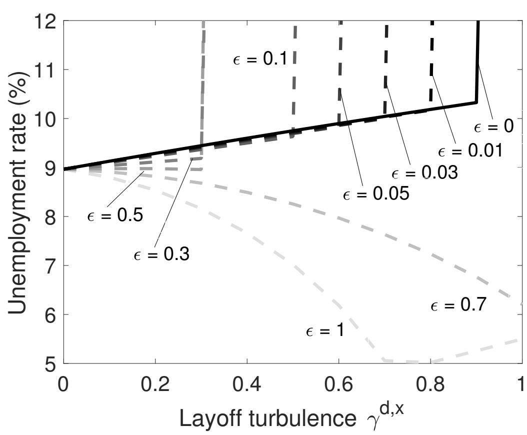
Figure 3: Turbulence with MP productivity $z^{min} \in \{0, 0.2, 0.4, 0.6\}$



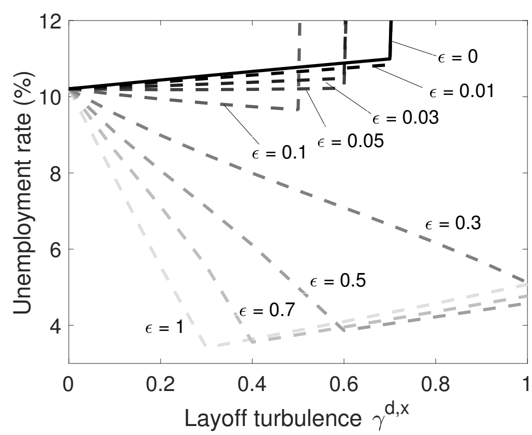
(a) $z^{min} = 0$



(b) $z^{min} = 0.2$



(c) $z^{min} = 0.4$



(d) $z^{min} = 0.6$

an equilibrium in which all worker-vacancy encounters result in matches *to* an equilibrium in which there is no Nash-bargaining solution for some worker-vacancy encounters. This happens at the meltdowns in Figure 3: firms cannot afford to pay a wage to low-skilled workers with high benefits that is high enough to compensate them for surrendering their high benefits. When turbulence reaches that tipping point, the stochastic steady state becomes one in which skill loss leads to an absorbing state of unemployment until retirement – a positive turbulence-unemployment relationship that is 'turbo-charged' .

In the two bottom panels of Figure 3, negative turbulence-unemployment relationships do appear; first only for high levels of quit turbulence and then at lower levels. Successive reductions in implied returns to labor mobility bring outcomes that mirror those in Figure 2b where a higher z^{min} is associated with a lower minimum layoff tax required to shut down all endogenous separations. Evidently, in the MP model the magnitude of returns to labor mobility determine responses of unemployment to quit turbulence and to layoff costs. Baley *et al.* (2023) dub this interrelatedness a "cross-phenonemon restriction" that is intermediated through returns to labor mobility. Outcomes in the last panel of Figure 3 ($z^{min} = 0.6$) provide another perspective on the troublesome region of the MP parameter space in which outcomes are fragile with respect to perturbations of parameters.

4 Returns to labor mobility in AV

To study the effects of firing costs and severance payments in an incomplete markets setting in which rigid wages do not depend on individual firms' states and risk-averse agents self-insure against income risk, AV formulate a search-island model in the tradition of framework of Lucas and Prescott (1974).[7] A state-independent wage and an incentive to self-insure are features that are absent from our section 2 benchmark model in which workers are risk neutral and wages are determined in Nash bargaining between a worker and a firm. For our present purposes, an object of the Alvarez-Veracierto model that especially interests us is the stochastic process governing idiosyncratic productivities that, when transmitted through a production function, determine workers' outputs. AV calibrate a productivity distribution that they coax from establishment data on job creation and destruction (Davis and Haltiwanger, 1990), cast within a model in which outcomes are shaped by a neo-classical production function.

An individual firm's output y_t at time t is given by the production function

$$y_t = x_t \, k_t^{\xi} \, n_t^{\psi} \,, \tag{26}$$

[7]Because they calibrate their model to Davis and Haltiwanger's (1990) establishment data, AV use the term "establishment" instead of "firm".

where $\xi > 0$, $\psi > 0$, $\xi + \psi < 1$, k_t is capital, n_t is labor, and x_t is an idiosyncratic productivity shock. The idiosyncratic shock x_t can take one of three values $\{0, x^{low}, x^{high}\}$ and follows a first-order Markov process with a transition probability matrix Q. Zero productivity is an absorbing state that indicates death of a firm.

The transition probability matrix Q takes the following form:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ \eta & \omega(1-\eta) & (1-\omega)(1-\eta) \\ \eta & (1-\omega)(1-\eta) & \omega(1-\eta) \end{bmatrix}, \tag{27}$$

where $\eta \in (0,1)$ is the probability of a firm's death and, conditional on surviving, $\omega \in (0,1)$ is the probability that a firm's productivity is unchanged from last period. The transition probability matrix Q in (27) treats low and high productivity shocks symmetrically. In addition, initial productivities drawn by new firms have equal probabilities of being low and high. Under these assumptions, there are as many firms with low productivity as with high productivity in a stochastic steady state.

Table 5 lists parts of AV's parameterization that most concern us. The production function is calibrated in a standard way to match commonly used targets: AV calibrate the capital share parameter ξ to match the U.S. capital-output ratio and the labor share parameter ψ to replicate a labor share in national income of 0.64. For a semi-quarterly model period and normalization $x_1 = 1$, AV (2001, p. 488)

> "select the parameters [η], ω and [x_2] to reproduce observations on job creation and job destruction reported by Davis and Haltiwanger (1990): the average job creation and job destruction rates due to births and deaths are both about 0.73 percent a quarter, the average job creation and job destruction rates due to continuing establishments are about 4.81 percent a quarter, and the annual persistence of both job creation and destruction is about 75 percent. We obtained these observations by selecting [x_2] = 2.12, [η]= 0.0037, and $\omega = 0.973$."[8]

Note that AV's empirical targets for quarterly job churning sum to 5.5 percent – 0.73 percent due to births and deaths of establishments and 4.81 percent from job creation and job destruction due to continuing establishments. There is a quantitatively close overlap between the empirical 0.73 percent a quarter attributed to establishment turnover, modelled as an exogenous firm failure rate by AV (i.e., twice the semi-quarterly rate $\eta = 0.0037$ in Table 5), and the exogenous breakup/layoff rate of 1 percent assumed in our benchmark model (i.e., twice

[8] We have corrected AV's (2001, p. 488) erroneous reference to "[η] = 0.037" with the correct number 0.0037, as reported in Table 1 of AV's 1998 working paper (Federal Reserve Bank of Chicago, WP 98-2).

Table 5: AV's parameter values (central to our study)

Parameter	Definition	Value
Technology		
ξ	capital share	0.19
ψ	labor share	0.58
Productivity		
x_2	high productivity	2.12
ω	persistence of productivity	0.973
η	death of firm	0.0037

the semi-quarterly rate $\rho^x = 0.005$ in Table 1). It remains for us to describe how to map the AV productivity process pertaining to production functions with both capital and labor into our matching framework and the productivities of one-worker firms with no physical capital.

4.1 A streamlined AV model

We simplify AV's benchmark economy by assuming an endowment of perpetual firms, and by eliminating a minor firing tax. First, instead of AV's costly creation of new establishments, suppose that the economy is endowed with a fixed measure of firms equal to the steady-state measure in AV's benchmark economy. And whenever a firm dies with probability η, it is replaced by a new firm as in AV's steady state, but now without any cost of creation. We retain AV's assumption that a banking sector owns both the establishments and the capital that they rent. Second, we eliminate a minor firing tax in AV's (2001, p. 487) benchmark economy that represents employers' experience-rated tax to finance the unemployment benefit system, motivated by AV's argument that "these taxes work approximately as firing taxes". Instead, the government could marginally increase the payroll tax by the annuitized expected value of that minor firing tax.[9]

With the firm creation cost and the firing tax gone, a firm's problem is purely static. A firm maximizes profits renting enough capital and labor in spot markets to equate their marginal products to the rental rate on capital r and the before-payroll-tax wage $w^\star$, respectively. In a steady state, there are only two types of firms: firms with low (high) productivity, of which each one rents k_1 (k_2) units of capital and hires n_1 (n_2) workers. In this stationary equilibrium, we can switch from a time subscript on variables to a state subscript: state 1 stands for low productivity, $x_1 = x^{low}$, and state 2 for high productivity, $x_2 = x^{high}$.

[9] According to AV's 1998 working paper (Federal Reserve Bank of Chicago, WP 98-2), the firing tax is equal to only 30 percent of the semi-quarterly before-payroll-tax wage rate.

In an equilibrium, the marginal product of labor in both types of firms equals the wage $w^\star$,

$$w^\star = \psi\, x_1\, k_1^\xi\, n_1^{\psi-1} = \psi\, x_2\, k_2^\xi\, n_2^{\psi-1}\,. \tag{28}$$

After dividing both sides of the last equality by $\psi\, x_1\, k_1^\xi\, n_1^\psi\, n_2^{-1}$, we have

$$\frac{n_2}{n_1} = \frac{x_2}{x_1}\left(\frac{k_2}{k_1}\right)^\xi \left(\frac{n_2}{n_1}\right)^\psi . \tag{29}$$

Likewise, the marginal product of capital equals the rental rate r,

$$r = \xi\, x_1\, k_1^{\xi-1}\, n_1^\psi = \xi\, x_2\, k_2^{\xi-1}\, n_2^\psi\,. \tag{30}$$

After dividing both sides of the last equality by $\xi\, x_1\, k_1^\xi\, n_1^\psi\, k_2^{-1}$, we have

$$\frac{k_2}{k_1} = \frac{x_2}{x_1}\left(\frac{k_2}{k_1}\right)^\xi \left(\frac{n_2}{n_1}\right)^\psi . \tag{31}$$

Since the right-hand sides of expressions (29) and (31) are the same, the capital-labor ratio is the same across all firms,

$$\frac{n_2}{n_1} = \frac{k_2}{k_1} \qquad \Rightarrow \qquad \frac{k_1}{n_1} = \frac{k_2}{n_2}\,. \tag{32}$$

By substituting (32) into expression (29), the ratio of labor employed by the two types of firms is

$$\frac{n_2}{n_1} = \frac{x_2}{x_1}\left(\frac{n_2}{n_1}\right)^\xi \left(\frac{n_2}{n_1}\right)^\psi \qquad \Rightarrow \qquad \frac{n_2}{n_1} = \left(\frac{x_2}{x_1}\right)^{\frac{1}{1-\xi-\psi}} . \tag{33}$$

When using AV's parameterization in Table 5 to evaluate expression (33), a low-productivity firm employs only 3.81 percent as many workers as a high-productivity firm does. Furthermore, since there are equal numbers of the two types of firms, it follows that high-productivity firms account for more than 96 percent of aggregate employment.

4.2 Mapping AV's productivity process into benchmark model

We use two steps to map AV's productivity process into the benchmark model. First, for our simplified AV model in the preceding section, we construct a hypothetical wage schedule of a firm that experiences a switch from high to low productivity, but offers all its workers to remain in the firm at a schedule of different pay. Second, we re-interpret that hypothetical wage schedule as a probability distribution of productivities in our matching framework with one-worker firms.

For the first step, consider a high-productivity firm that has just experienced a shock of low productivity, but instead of reducing its employment by $n_2 - n_1$ workers, the firm randomly orders its current employees and offers the following wage schedule. The first n_1 workers are offered the wage rate $w^\star$, i.e., the market-determined wage rate that all firms pay to their workers and n_1 is the employment level of other low-productivity firms. Then, under a pledge to keep the capital-labor ratio unchanged, the firm offers each successive worker in the randomly arranged order a wage equal to her marginal product. Thus, the wage offered to the worker in position $n \in (n_1, n_2]$ is given by

$$\begin{aligned}\psi\, x_1\, k^\xi\, n^{\psi-1} &= \psi\, x_1\, k^\xi\, n^{\psi-1} \frac{w^\star}{\psi\, x_2\, k_2^\xi\, n_2^{\psi-1}} = \frac{x_1 \left(\frac{k}{n}\, n\right)^\xi n^{\psi-1}}{x_2 \left(\frac{k_2}{n_2}\, n_2\right)^\xi n_2^{\psi-1}}\, w^\star \\ &= \frac{x_1}{x_2}\left(\frac{n}{n_2}\right)^{-(1-\xi-\psi)} w^\star \;\equiv\; \Gamma_{w^\star}\left(\frac{n}{n_2}\right) \qquad \text{for } \frac{n}{n_2} \in \left(\frac{n_1}{n_2}, 1\right], \end{aligned} \tag{34}$$

where the first equality multiplies and divides by the same quantity $w^\star$ while in the denominator imposing that $w^\star$ equals the marginal product of labor in a high-productivity firm, as given by expression (28), and the third equality uses the firm's pledge to keep the capital-labor ratio unchanged; hence, in the numerator and denominator the capital-labor ratio cancels.

The search frictions that workers face in a search-island model would make some workers in our simplified AV model choose to accept wage offers below $w^\star$. But under AV's parameterization, the vast majority would decline such offers and instead enter the pool of unemployed. However, for our purposes, it is useful to proceed as if all workers choose to remain with the firm. Since the argument of wage schedule $\Gamma_{w^\star}(n/n_2)$ is employment position n relative to the employment level of a high-productivity firm, the inverse function $\Gamma_{w^\star}^{-1}(w)$ gives the fraction of workers earning a wage greater than or equal to w and hence, the fraction of workers earning less than or equal to w is given by

$$F_{w^\star}(w) = 1 - \Gamma_{w^\star}^{-1}(w) = 1 - \left[\frac{x_1 w^\star}{x_2 w}\right]^{\frac{1}{1-\xi-\psi}} \qquad \text{for } = w \in \left[\frac{x_1 w^\star}{x_2},\, w^\star\right), \tag{35}$$

and the fraction of workers at the mass point $w = w^\star$ is equal to

$$1 - \lim_{w \to w^\star} F_{w^\star}(w) = \Gamma_{w^\star}^{-1}(w^\star) = \left[\frac{x_1}{x_2}\right]^{\frac{1}{1-\xi-\psi}} \tag{36}$$

which is indeed the same as the equilibrium value of n_1/n_2 in expression (33).

In the second step of our mapping of AV into the benchmark model, we re-interpret the shocks of AV as follows. AV's probability η that a firm dies becomes our probability ρ^x of an exogenous breakup. AV's probability $1 - \omega$ that a firm receives a productivity shock becomes our probability γ^s that a productivity switch hits a continuing firm-worker match. At such a switch, a new productivity z is now drawn from a skill-specific distribution $F_{z_i^{max}}(z)$ where $i = l$ and $i = h$ for a low-skilled and a high-skilled worker, respectively, with cumulative density

$$F_{z_i^{max}}(z) = 1 - \Gamma^{-1}_{z_i^{max}}(z) = 1 - \left[\frac{x_1\, z_i^{max}}{x_2\, z}\right]^{\frac{1}{1-\xi-\psi}} \qquad \text{for } = z \in \left[\frac{x_1\, z_i^{max}}{x_2},\, z_i^{max}\right), \qquad (37)$$

and the probability of mass point $z = z_i^{max}$ is given by expression (36). We take AV's variable $w^\star$ as the upper bound z_i^{max} of our skill-specific productivity distribution. It is a rather direct analogue to the above hypothetical wage schedule in the simplified AV model, but instead of workers being randomly assigned along a wage offer schedule, continuing firm-worker matches in the benchmark model draw productivities from a corresponding distribution. In accordance with AV and similar to MP in the preceding section, the productivity of a newly formed firm-worker match is equal to the upper support of the productivity distribution.

Figure 4: AV productivity distributions

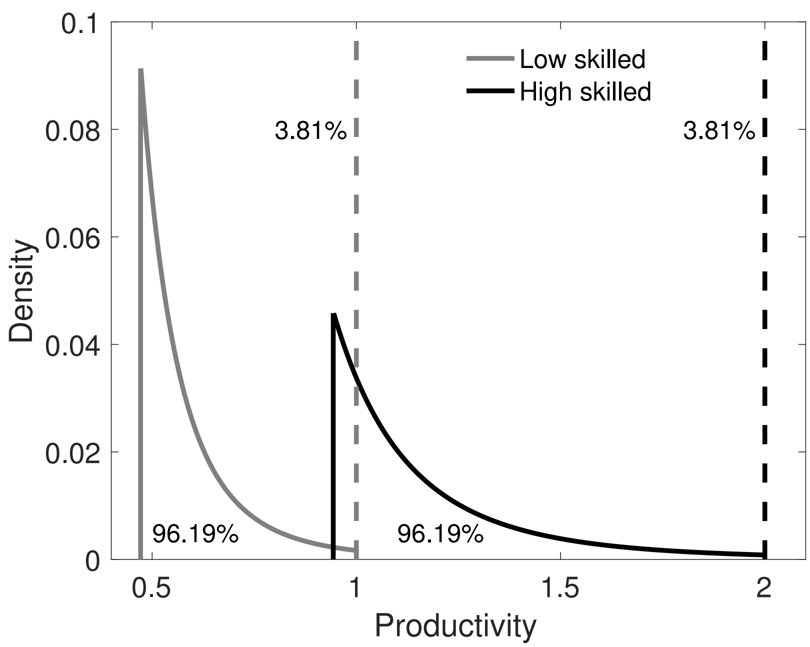


Figure 4 depicts the densities of our two skill-specific productivity distributions when blending AV's parameterization in Table 5 with the assumption of the benchmark model that a low-skilled worker has half the earnings potential of a high-skilled worker, $z_l^{max} = 1$ and $z_h^{max} = 2$. (For comparison, Figure 1 shows productivity densities for the MP model.) The shape of a density in Figure 4 reflects the concavity of AV's production function. In particular, since we imposed a constant capital-labor ratio in the employment perturbations away from an efficient level of operation, the concavity of a firm's output with respect to employment arises from AV's assumption of decreasing returns to scale. The lowest productivity of a distribution in Figure 4 reflects an excessively high employment level of a firm that has not shed its labor force after switching from high to low productivity. Hence, the excessively high employment is far up on a flattening concave production function where a rather small increase in the marginal product of labor would be associated with a relatively long journey down the production surface to significantly lower employment levels that explains the high densities at those low productivities. The reasoning is the opposite for productivities just below the efficient employment level, where the steeper curvature of the concave production function means that a small increase in the marginal product of labor does not have much of an associated change in employment, providing the low densities at high productivities just below the efficient level. The mass point at the upper support reflects that all workers employed at that efficient level are paid the marginal product of labor evaluated at that efficient employment level.

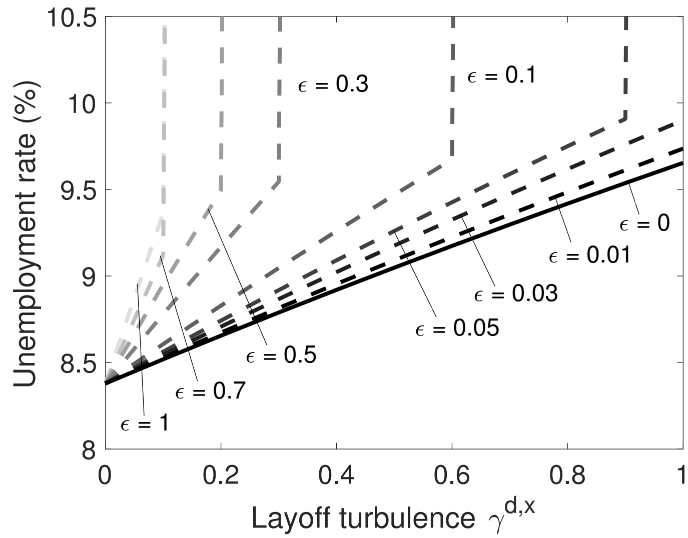
4.3 Turbulence under AV productivity process

As in section 3.3, we import the AV productivity process into the benchmark model to study how unemployment responds to turbulence. Thus, we adopt the AV productivity process as parameterized in Table 5 with the modified productivity distribution in expression (37) while keeping the rest of the parameterization of the benchmark model in Table 1, except for the matching efficiency A that we calibrate to target a laissez-faire unemployment rate of 5 percent in tranquil times.

The turbulence outcomes under the AV productivity process in Figure 5 resemble those under the versions of the MP process in the top two panels of Figure 3 and indicate a strong positive relationship between unemployment and turbulence. Actually, the relationship is even stronger under the AV productivity process given the functional form of the AV probability distribution with densities depicted in Figure 4. That functional form reflects AV's underlying growth model as mirrored in its neo-classical production function. The theoretical structure makes it difficult to imagine how any plausibly parameterized quit turbulence could ever suppress the strong forces for reallocation of workers across establishments that are present in the AV model.

The establishment data on firm and worker turnover from Davis and Haltiwanger (1990) that AV use to calibrate their model, as well as data sets from other countries, provide compelling evidence that extensive reallocations occur within different market economies that operate under different government policies directed at influencing job separations. Our present study of the consequences of alternative labor productivity processes in macro-labor models conveys a message consistent with that evidence: explaining observations on firm turnover, labor mo-

Figure 5: Turbulence with AV productivity



bility, and government policies that aim to arrest firm-worker separations requires theoretical constructs calibrated to imply ample returns to labor mobility. Quantitative models that have meager returns to labor mobility cannot explain these observations.

5 Concluding remarks

Mapping productivity processes from two celebrated quantitative models into our benchmark model has taught us about sources of fragilities of calibrations of parameters that affect the returns to labor mobility that their agents face. In particular, parameterizations of models like AV's in which shocks to productivity are intermediated through neo-classical production functions and parameters are calibrated to fit firm size dynamics have high returns to labor mobility, even when their parameters are perturbed. But other macro-labor models that rely solely on unemployment statistics to calibrate per-worker productivity processes have returns to labor mobility that are fragile with respect to perturbations of parameters that nevertheless continue to fit unemployment outcomes. Thus, we have discovered a previously undetected fragility with respect to small perturbations of MP's calibration that manifests itself in the form of a ridge traced out by two key parameters that can generate the same targeted unemployment statistic although they have very different implications for returns to labor mobility. MP didn't note that their calibration resides at the end of that ridge, close to a region where returns to labor mobility are very sensitive to perturbations of those parameters. Because MP focused on employment effects of layoff taxes, equilibrium outcomes would have led MP to confront this issue only if their calibration had wandered into the region with extremely low returns to mobility. That would probably have prompted them to explore more of their parameter space, since market economies, even those with heavy-handed government interventions designed to suppress it, still exhibit substantial labor reallocation.

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Returns to Labor Mobility

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Online Appendix

A Equilibrium computation

Here we outline the structure of the algorithm that we used to compute equilibria.[10]

A.1 General algorithm structure

The algorithm centers around approximating the joint continuation values $g_i(z)$ by using linear projections on a productivity grid. It employs the following steps:

1. Fix a parameterization and construct productivity distributions over a grid of size N_z.
2. Guess initial values for:
   - ζ_i^k: coefficients for linear approximations $\hat{g}_i(z) = \zeta_i^0 + \zeta_i^1 z$ to $g_i(z)$
   - b_j: unemployment benefits
   - ω_{ij}^w: workers' outside values, not including current payment of benefit
   - $\omega^f = 0$: firms' outside value
   - τ: tax rate
   - u_{ij}, e_{ij}: masses of unemployed and employed workers
3. Given linear approximations $\hat{g}_i(z)$, use (2)–(5) to compute reservation productivities $\underline{z}_{ij}^o, \underline{z}_{ij}$.
4. Given cutoffs $\underline{z}_{ij}^o, \underline{z}_{ij}$, compute rejection probabilities ν_{ij}^o, ν_{ij} using (6) and compute E_{ij} using (7).
5. Compute the expected match surplus of a vacancy that encounters an unemployed worker:
$$\bar{s} \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}_{ij}^o}^{\infty} s_{ij}^o(y)\, dv_i^o(y).$$
6. Compute joint continuation values $g_i(z)$ using (8) and (9). Then update coefficients ζ_i^0, ζ_i^1 described in step 2 by regressing $g_i(z)$ on $[1 \;\; z]$.
7. Update the value of posting a vacancy, market tightness, and matching probabilities:
$$w^f = 0, \qquad \theta = \left(\frac{\beta A(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha}, \qquad \lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda_{ij}^f(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}.$$

[10] We are grateful to Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code. That code was augmented and modified by LS and further by us.

8. Update values ω_{ij}^w of being unemployed using (10) and (11).

9. Compute net changes in worker flows (all must be zero in a steady state)

$$\begin{aligned}\Delta u_{ll} &= \rho^r + (1-\rho^r)\left\{\rho^x + (1-\rho^x)(1-\gamma^u)\gamma^s \nu_{ll}\right\} e_{ll} \\ &- \rho^r u_{ll} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{ll}^o)u_{ll} \end{aligned} \tag{A.1}$$

$$\begin{aligned}\Delta u_{lh} &= (1-\rho^r)\left\{\rho^x \gamma^{d,x} e_{hh} + (1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{lh} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{lh}^o)u_{lh} \end{aligned} \tag{A.2}$$

$$\begin{aligned}\Delta u_{hh} &= (1-\rho^r)\left\{\rho^x(1-\gamma^{d,x})e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{hh} - (1-\rho^r)\lambda^w(\theta)(1-\nu_{hh}^o)u_{hh} \end{aligned} \tag{A.3}$$

$$\begin{aligned}\Delta e_{ll} &= (1-\rho^r)\lambda^w(\theta)\left\{(1-\nu_{ll}^o)u_{ll} + (1-\nu_{lh}^o)u_{lh}\right\} \\ &- \rho^r e_{ll} - (1-\rho^r)[\rho^x + (1-\rho^x)(\gamma^u + (1-\gamma^u)\gamma^s\nu_{ll}]e_{ll} \end{aligned} \tag{A.4}$$

$$\begin{aligned}\Delta e_{hh} &= (1-\rho^r)\left\{\lambda^w(\theta)(1-\nu_{hh}^o)u_{hh} + (1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}\right\} \\ &- \rho^r e_{hh} - (1-\rho^r)[\rho^x + (1-\rho^x)\gamma^s\nu_{hh}]e_{hh} \end{aligned} \tag{A.5}$$

10. Compute average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ as described in Appendix A.2, in order to determine government expenditures for unemployment benefits and government tax revenues using the left side and right side of (23), respectively.

11. Adjust tax rate τ in (23) to balance government budget.

12. Check convergence of a set of moments. If convergence has been achieved, stop. If convergence has not been achieved, go to step 2 and use as guesses the last values computed.

A.2 Average wages and productivities

Our computation of the equilibrium measures of workers in equations (A.1)–(A.5) involve only two groups of employed workers, e_{ll} and e_{hh}, but each of these groups needs to be subdivided when we compute average wages and productivities. For employed low-skilled workers, we need to single out those who gained employment after first having belonged to group u_{lh}, i.e., low-skilled unemployed workers who received high benefits b_h. In the first period of employment, those workers will earn a higher wage $p_{lh}^o(z) > p_{ll}^o(z) \geq p_{ll}(z)$. And even afterwards, namely until their first on-the-job productivity draw, those workers will on average continue to differ from other employed low-skilled workers because of their higher reservation productivity at the time they regained employment, $\underline{z}^o_{lh} > \underline{z}^o_{ll} \geq \underline{z}_{ll}$.

Let e'_{ll} denote the measure of unemployed low-skilled workers with high benefits who gain employment in each period (they are in their first period of employment):

$$e'_{ll} = (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}.$$

Let e''_{ll} be the measure of such low-skilled workers who remain employed with job tenures greater than one period and who have not yet experienced any on-the-job productivity draw:

$$\begin{aligned} e''_{ll} &= (1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)\left[e'_{ll}+e''_{ll}\right] \\ &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\, e'_{ll}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed low-skilled workers and also their average productivity

$$\begin{aligned} \bar{p}_l &= \int_{\underline{z}^o_{lh}}^{\infty}\left[\frac{e'_{ll}}{e_{ll}}p^o_{lh}(y)+\frac{e''_{ll}}{e_{ll}}p_{ll}(y)\right]\frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty}p_{ll}(y)\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})} \\ \bar{z}_l &= \frac{e'_{ll}+e''_{ll}}{e_{ll}}\int_{\underline{z}^o_{lh}}^{\infty} y\,\frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty} y\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})}. \end{aligned}$$

For employed high-skilled workers, we need to single out those just hired from the group of unemployed high-skilled workers u_{hh} who earn a higher wage in their first period of employment, $p^o_{hh}(z) > p_{hh}(z)$. This is because they do not face the risk of quit turbulence if no wage agreement is reached and hence, no employment relationship is formed. For the same reason discussed above, we also need to keep track of such workers until their first on-the-job productivity draw (or layoff or retirement, whatever comes first). Reasoning as we did earlier, let e'_{hh} and e''_{hh} denote these respective groups of employed high-skilled workers;

$$\begin{aligned} e'_{hh} &= (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \\ e''_{hh} &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^s)}\, e'_{hh}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed high-skilled workers and also their average productivity

$$\bar{p}_h = \int_{\underline{z}^o_{hh}}^{\infty} \left[\frac{e'_{hh}}{e_{hh}} p^o_{hh}(y) + \frac{e''_{hh}}{e_{hh}} p_{hh}(y)\right] \frac{dv^o_h(y)}{1 - v^o_h(\underline{z}^o_{hh})} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} p_{hh}(y) \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}$$

$$\bar{z}_h = \frac{e'_{hh} + e''_{hh}}{e_{hh}} \int_{\underline{z}^o_{hh}}^{\infty} y \frac{dv^o_h(y)}{1 - v^o_h(\underline{z}^o_{hh})} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} y \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}.$$