OPTIMAL REGULATION OF A FULLY INSURED DEPOSIT BANKING SYSTEM [1]

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Abstract

We analyze risk sensitive incentive compatible deposit insurance in the presence of private information when the market value of deposit insurance can be determined using Merton's (1997) formula. We show that, under the assumption that transferring funds from taxpayers to financial institutions has a social cost, the optimal regulation combines different levels of capital requirements combined with decreasing premia on deposit insurance. On the other hand, it is never efficient to require the banks to hold riskless assets, so that narrow banking is not efficient. Finally, chartering banks is necessary in order to decrease the cost of asymmetric information.

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1 INTRODUCTION

One of the main characteristics of banks is that they issue liabilities that the agents view as money. This characteristic of demand deposits has often been stressed, at times emphasizing their role in the transfer of property rights (Fama (1980)) and at times insisting upon the liquidity they provide (Diamond and Dybvig (1983), Gorton and Pennacchi (1990)). Thus for the banking system to create money **in a credible way**, a clear commitment of the monetary authorities is required, and, although through history this commitment has taken different forms, it is nowadays tantamount to the existence of deposit insurance. This point is perfectly taken by Eisenbeis (1986, p.174) when he writes: "What makes banks 'special' today is not that they offer a wide range of services and perform varied functions but rather that they have access to government deposit insurance".

The benefits of deposit insurance are well known (in particular since Diamond-Dybvig (1983) and Dewatripont and Tirole (1994)). Still if deposit insurance has social benefits, the recent financial distress experienced by financial institutions in the U.S. as well as in Europe has made clear that deposit insurance had a cost for taxpayers, that may well have been underestimated.

The literature on deposit insurance has addressed this issue only partially. Deposit insurance has been extensively studied, both from a theoretical (Berlin et al.,1991) and empirical standpoint (Avery and Berger (1991)) and the pricing of deposit insurance has been analyzed, following (Merton (1977)), with controversial results (Buser, Chen and Kane (1981), Marcus and Shaked (1984)). Also, the effect of deposit insurance on the banks risk taking behaviour has been stressed. Finally, the existence of a flat deposit insurance premium for wide classes of assets has been widely criticized, and the need to extract private information to obtain risk adjusted pricing of premium has been recognized (Pyle (1984), Fama (1985), James (1987), Lucas and Mc Donald (1987, 1992), Chan, Greenbaum and Thakor (1992)).

Restrictions on the structure of assets and liabilities is a way to decrease the cost of deposit insurance and the risk taking behaviour of the banking system, and therefore have to be seen as an important part of banking regulation. The debate confronting advocates and opponents to free banking, narrow banking or the Glass-Steagall Act shows that the asset and liabilities regulation issues in banking are far from being settled. Traditionally, a rationale for restricting banking activities has been found in historical events, justified on the one hand by the concern of bank power, and on the other hand by the need for the banking system to be operated in a safe and sound way (Eisenbeis, 1986). More recently, the prudential angle has been stressed, and the search for worldwide uniform banking rules, as the BIS proposal for risk-based capital standards, has resulted in a generalization of this type of restrictions. But, in spite of the widespread trend towards deregulation of the financial services industry these restrictions have been left unchanged[1]; on the contrary, the recent Financial Institutions Reform Recovery and Enforcement Act of 1989 has strengthened these regulations by imposing tighter capital requirements and a ban on investments in certain classes of financial assets, thus deviating from the trend towards "universal banking" characteristic of European countries.

Indeed it is clear that asset liabilities regulation will decrease the cost of deposit insurance, as it has been suggested (by Buser, Chen and Kane, 1981 among others). Still, it would be inconsistent to look at the deposit

[1]The Depository Institution Deregulation and Monetary Control Act of 1980 and the Garn-St-Germain Act of 1982 dismantle the interest rate and product specialization restrictions, but did not tackle this issue

insurance problem only from the angle of the cost which directly or indirectly will be born by the taxpayer. Indeed, the social benefits of the banks' operations have also to be taken into consideration, whether we think of the benefits the firms get from the loans the banks grant them or of the benefits derived from the existence of a payment system. In order to encompass both the costs and benefits of banking we have to take a global approach, and analyze the regulation that maximizes a global measure of the net surplus, thus considering simultaneously the benefits of regulation which decreases the cost of insurance and the costs it generates by making banks operation more expensive. This is the approach we adopt here: we consider a simplified asymmetric information framework and look for the optimal regulation when banks are confronted simultaneously with different combinations of capital requirements and deposit insurance premium, using the type of models developed to address the regulatory issues. We do not consider the constraints on optimal bank regulatory policies which are introduced due to a potential divergence of interests between taxpayers and regulators ((Kane 1990), Campbell, Chan and Marino (1992), Boot and Thakor (1993)). Indeed this potential divergence are mainly due to a moral hazard problem associated to the regulator's discretion (such as closure policies). In our model we assume rigid regulatory rules. Moreover we consider only prudential ratios based on quite easily observable variables such as the banks' capital and the banks holdings of riskless assets -this last point weakens even more the scope for moral hazard- therefore we can look for a regulation that maximizes social welfare. Our model is related to Chan, Greenbaum and Thakor (1992), but since we allow here for deposit insurance premia that are not actuarially fair, we are able to address the question of the optimal tax or subsidy the deposit insurance may include. Our model is also related to Giammarino, Lewis and Sappington (1993) and Bensaid, Pagès and Rochet (1995), but unlike these authors we took as a starting point that the value of deposit insurance is given by the Black–Scholes–Merton formula, and did not address the potential moral hazard issues.

Addressing the question of regulation from an optimization standpoint may help to clarify the debate on banking regulation, establishing which regulations may be inferior options, and which may be optimal in a specific environment. This view may also help us to understand the differences in banking regulations we observe across countries *with similar deposit insurance policies*. To make our point clear we disregard the moral hazard issue that is the usual justification for capital requirements. Therefore, if capital requirements or holdings of non–risky assets are obtained in the optimal mechanism, they come into play only in order to decrease the cost of deposit insurance.

It is well known that regulation act as "an implicit premium on deposit insurance" (Buser et al., 1981); our investigation centers on the trade off between explicit premia and the different types of implicit premia.

Our results show that under perfect information it is optimal to have an actuarially fair deposit insurance and no capital requirements. Still, under imperfect information the optimal mechanism entails a subsidy to the banking industry. It implies that the banks are not required to hold riskless assets but may be subject to capital requirements, thus implying a strong rejection of narrow banking and confirming the traditional view that reserve requirements are justified by liquidity reasons and not by solvency considerations. It finally implies that low quality banks pay high deposit insurance premia while being submitted to no capital requirements while high quality banks accept to be submitted to a capital requirement in exchange for a reduction in their premia.

2 THE MODEL

We consider a simplified framework in which each bank k with assets A_k chooses to invest L_k in a portfolio of loans with maturity[2], T and $A_k - L_k$ in a riskless asset with a continuous rate of return equal to r. To fund these operations, the bank will use two sources: it will attract an amount D_k of deposits and will issue securities with a market value of $A_k - D_k$. Except for deposits, we assume that financial markets are perfect, so that we need not specify whether the banks' additional funding is made through the issue of equity or bonds, since the Modigliani-Miller theorem implies that the risk adjusted cost of funds is the same. Nevertheless, in order to compare with standard capital requirements, we will consider that this residual funding is made through equity.

If deposits are fully insured, as we will assume, use of this source of funds means that the bank benefits from a subsidy on its borrowing rates and simultaneously pays a deposit insurance.

Banks, that behave as price takers, will differ in their skill to create net wealth by investing in safe and sound projects. The parameter θ will measure the per dollar value of the portfolio of loans, so that the informational rent of the bank is $\theta - 1$ per dollar invested. Thus, if bank k has a characteristic θ, the market value of its portfolio of loans under perfect information would be θL_k.

We will assume that θ is unknown to the regulator, although she knows its distribution function $F(\theta)$ and its density, function $f(\theta)$. We assume

[2]Alternatively, T can be interpreted as the period until the next bank examination, as in Merton (1977)

that the level of assets of a bank, A_k, gives no additional information on the distribution of θ, i.e., that the size and the quality θ of a bank are independently distributed.

In an unregulated world, a bank k would maximize its value by investing all its assets in a portfolio of loans, and its profit would be:

$$U_s = (\theta - 1)A_k$$

If a bank with characteristic θ chooses to become a specialized financial institution refusing at the same time the costs and benefits of having access to deposit insurance, then it will obtain the level of utility U_s. In what follows, we will call the banks choosing this option the unregulated banking sector.

For the regulated sector, the bank that chooses to enter the deposit banking system will not only receive the value of a put on its insured deposits, in exchange for the insurance premium it pays, but also abide by some asset-liability regulatory rules. We consider here that the regulator is able to fix the ratios $l_k = L_k/A_k$ of loans to assets (or equivalently the ratio $A_k - L_k/A_k$ of riskless assets to total assets) and $d_k = D_k/A_k$ of deposits to assets the bank's managers have to fulfill[3].

A **regulatory mechanism** will therefore specify for each θ the amount of loans $L(\theta)$, deposits $D(\theta)$ and the premium it has to pay, $T(\theta)$. Still, since the size measured by the level of assets A is independent of θ, the mechanism can be defined from the corresponding ratios $l(\theta) = L(\theta)/A$ and $d(\theta) = D(\theta)/A$ while $-t(\theta) = T(\theta)/A$ is the (per unit of assets) premium a bank with characteristic θ pays.[4]

[3]In fact our results can be interpreted in terms of the maxima and minima ratios that the bank has to respect, more in line with usual the asset-liabilities regulation.

[4]Notice, though, that it could be possible to take the size as part of the mechanism, in such a way as to give a larger part of the market to the best performing banks, that is, those with a higher θ. This is an interesting research direction we do not explore here.

A regulated bank θ that is financed with a percentage $d(\theta)$ of insured deposits and is lending a fraction $l(\theta)$ of its assets will have a profit per unit of asset equal to:

$$U(\theta) = l(\theta)(\theta - 1) + P(v(\theta), l(\theta), d(\theta)) + t(\theta) \tag{1}$$

where $l(\theta)(\theta-1)$ is the net wealth created in the loan market, $P(v(\theta), l(\theta), d(\theta))$ is the market value of the deposit insurance per dollar of asset when $v(\theta)$ is the value of the bank and $t(\theta)$ is the market value of the transfer the bank will receive at the end of the period.

The fact that the deposit insurance is a put is a standard assumption. In the present context, its value is given by Black and Scholes formula. Since $P(v(\theta), l(\theta), d(\theta))$ is homogeneous of degree one, it does not depend on A. The value for $P(v(\theta), l(\theta), d(\theta))$ can be easily obtained once we define the stochastic process that is followed by the θ bank portfolio of loans:

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t$$

where dW_t is a Wiener process. The volatility σ is known and the banks cannot affect it, a strong simplifying assumption that allow us to focus on adverse selection problems and disregard the moral hazard issue.

The boundary conditions are then given by:

$$P(V_T, l(\theta), d(\theta)) = \max(d(\theta)e^{rT} - V_T, 0)$$

where

$$V_T = l(\theta)\theta_T + (1 - l(\theta))e^{rT}$$

so that

$$P(V_T, l(\theta), d(\theta)) = \max(l(\theta) + d(\theta) - 1)e^{rT} - l(\theta)\theta_T, 0) \tag{2}$$

Dropping the dependence on θ, we therefore obtain, using $(d+l-1)e^{rT}$ as the striking price and $l\theta$ as the underlying [5]:

$$P(\theta l, l+d-1) = \begin{cases} (l+d-1)[1-N(x-\sigma\sqrt{t})] - l\theta[1-N(x)] & \text{if } l+d-1>0 \\ 0 & \text{if } l+d-1 \leq 0 \end{cases} \quad (3)$$

where

$$x(l,d) = \frac{\ln(l\theta) - \ln(l+d-1)}{\sigma\sqrt{t}} + \frac{\sigma\sqrt{t}}{2}$$

and $N(\,)$ is the cumulative probability of the normal distribution $\mathcal{N}$ (0,1).

The fact that the variable $d(\theta)$ affects the banks profit only via its effect on the value of the put is justified by the perfect market assumption. On the other hand, since the safe asset and deposits are perfect substitutes, each individual bank's supply of deposits is infinitely elastic.

For the sake of simplicity, we choose to make the payment of the deposit insurance premium out of the bank's equity [6] . Since we want to obtain the optimal mechanism, we cannot know beforehand the sign of $t(\theta)$.

We now proceed to state precisely the regulation problem.

First, the regulator chooses the rules that will govern *deposit banking*. Yet, it cannot prevent a bank from developing its loan activity while fully financed through the capital markets. The per dollar profit the θ bank obtains in the **unregulated banking** sector [7] $\theta - 1$, is, therefore, the minimum value it has to be offered in order to accept to become a deposit bank. In other words, the banks reservation profit level is here $\theta - 1$, so that the individual

[5] Notice that although $P(\theta, l, d)$ is a continuous function its derivative at a point (l, d) such that $l + d = 1$ is not continuous, and this will imply a more involved mathematical solution.

[6] Treating the insurance premium as an upfront deduction implies a much more complex valuation model studied, in particular by Acharya and Dreyfus (1989).

[7] This can also be seen as the activity that characterizes the investment banking sector.

rationality constraints hold as [8]

$$U(\theta) \geq \theta - 1 \tag{4}$$

Second, the social value of banking activity will differ from its private value. Since the deficit generated by the banking sector has to be covered by tax payers, we have to take into account the social costs of levying taxes. In order to introduce such a cost in the simplest way, we use the Laffont and Tirole (1986) approach and consider that a transfer t from the regulatory agency to a bank will generate a social cost of λt. Simultaneously, we consider the role of deposits as part of the system of payments which generates a social value of ν per each deposited dollar [9] where ν is non negative, but may take a zero value.

Consequently, a bank with characteristics θ creates per dollar social benefits equal to

$$\begin{aligned} U_s = \ & l(\theta - 1) + P(\theta, l, d) + t + \nu d \\ & -[1 + \lambda][P(\theta, l, d) + t] \end{aligned} \tag{5}$$

or, replacing the value of the bank's profit, U given by (1),

$$U_s = l(\theta - 1)(1 + \lambda) + \nu d - \lambda U \tag{6}$$

3 PERFECT INFORMATION

To begin with, consider the case where the bank's characteristic, θ, is perfectly known by the regulatory agency. Selecting a mechanism $(l(\theta), d(\theta), t(\theta))$ will allow the regulator to discriminate among the different types of banks.

[8] Notice that Gianmarino et al (1993) choose instead $U(\theta) \geq 0$. This means that implicitly they assume loans could not be granted by financial institutions that are not funded by deposits.

[9] More precisely, ν is the difference in the social benefits of having fully insured deposits versus having non insured deposits.

The regulation problem is the following:

$$\begin{cases} \max l(\theta-1)(1+\lambda)+\nu d-\lambda u \\ u \geq \theta-1 \end{cases}$$

which give us the obvious solution $l^* = 1, d^* = 1, u^* = \theta - 1$ so that $t^* = -P(\theta, 1, 1)$.

We summarize the results in the following proposition:

Proposition 1 *If $\nu > 0$, the perfect information optimal mechanism is one in which*

i. there is no unregulated banking sector

ii. banks have maximum deposits funding and invest all their assets in the loan market

iii. banks pay the actuarial cost of the deposits insurance.

Alternatively, as intuition suggests, for $\nu = 0$ we obtain that the optimal level of deposits is undetermined. But then Modigliani–Miller theorem applies also to deposits and there is no justification for the existence of a banking industry.

4 ASYMMETRIC INFORMATION

We will assume hereafter that the regulatory agency does not know the value of the capability θ of each bank, although it knows the probability distribution on θ. We view this framework of asymmetric information as the relevant one, while the perfect information case will only stand as a benchmark.

In order to characterize the optimal mechanism, we will have to obtain, first, the mechanisms that are incentive compatible. Then, the regulatory agency will maximize its objective function under the constraint that (within the deposit banking industry) the mechanism are incentive compatible and individually rational. Of course, in the unregulated banking sector no restriction is needed. Thus we use control theory techniques that are standard in the analysis of regulation:[10] that is, we will first determine the necessary differential conditions for a mechanism to be incentive compatible, then determine the optimal mechanism under the necessary constraints and then check that the optimal mechanism is incentive compatible.

A regulatory mechanism $l(\theta), d(\theta), t(\theta)$ is incentive compatible if each bank is better off announcing the true value of its characteristic θ rather than any other value $\hat{\theta}$:

$$\begin{gathered} l(\theta)(\theta-1)+P(l(\theta)\theta, l(\theta)+d(\theta)-1)+t(\theta) \geq \\ \geq l(\hat{\theta})(\theta-1)+P(l(\hat{\theta})\theta, l(\hat{\theta})+d(\hat{\theta})-1)+t(\hat{\theta}) \quad \forall \theta, \hat{\theta} \end{gathered} \tag{7}$$

It is possible to prove that incentive compatible mechanisms are differentiable except, at most, in a countable number of points. When the mechanism is differentiable, since inequality (7) implies that the maximum value of the θ bank's profit is obtained setting $\hat{\theta}=\theta$, we obtain the following first and second order conditions

$$\dot{l}(\theta)(\theta-1)+\theta\dot{l}(\theta)P_1+(\dot{l}(\theta)+\dot{d}(\theta))P_2+\dot{t}(\theta)=0 \tag{8}$$

$$\dot{l}(\theta)(1+P_1+\theta l(\theta)P_{11}+l(\theta)P_{21})+\dot{d}(\theta)l(\theta)P_{21} \geq 0 \tag{9}$$

where an upper dot represents the derivative with respect to θ, P_k is the derivative of the put function with respect to the $k-th$ argument, and P_{kl} the second derivative with respect to arguments k and l. (Expression (9)

[10] See Baron and Myerson, 1982 Guesnerie and Laffont, 1984 Laffont and Tirole, 1986. The difficulties introduced by the discontinuity of the derivative are dealt with following Tomiyama's, 1985 suggestions.

is obtained using the derivative of (8) and replacing it in the second order conditions.).

Defining $U(\theta)$ as:

$$U(\theta) = l(\theta)(\theta - 1) + P(l(\theta)\theta, l(\theta) + d(\theta) - 1) + t(\theta)$$

we obtain an equivalent expression for (8):

$$\dot{U}(\theta) = l(\theta)N(x(\theta)) \quad \text{if} \quad d + l - 1 > 0 \tag{10}$$

since from (3) we have $1 + P_1 = N[x(\theta)]$ wherever $d + l - 1 \geq 0$. On the other hand, for the values of d and l for which $P = 0$, we have:

$$\dot{U}(\theta) = l(\theta) \qquad \text{if} \quad d + l - 1 \leq 0 \tag{11}$$

We will now proceed to characterize the optimal regulatory mechanism assuming first full participation and then relaxing this assumption so as to allow for an unregulated banking sector.

The optimal deposit banking regulation will be the regulatory mechanism $(l(\theta), d(\theta), t(\theta))$ that maximizes the net expected social benefits U_s within the class of incentive compatible individually rational mechanisms.

The characteristics of the optimal deposit banking regulation are given by the following proposition, in which we set $\sqrt{s} = \sigma\sqrt{t}$.

Proposition 2 *If* $\log\theta - \sqrt{2s\log(\lambda F(\theta)/\theta f(\theta)\nu\sqrt{2\Pi s})}$ *is decreasing in* θ, *on the subset of* $[\theta, \bar{\theta}]$ *on which it is defined, the optimal banking regulation under full participation implies that the banks hold only loans in their assets and combines two industries.*

i. An unrestricted banking industry with $l^*(\theta) = 1, d^*(\theta) = 1$ *for* θ *in* $[\underline{\theta},\, \theta^*)$

ii. A banking industry with a minimal risk adjusted capital requirement, with

$$l^*(\theta) = 1,\ d^*(\theta) = \theta \exp\left[\frac{-s}{2} - \sqrt{2s \log k(\theta)}\right]$$

for θ *in* $[\theta^*, \overline{\theta}]$

where $k(\theta) = \lambda F(\theta) / \left(\theta f(\theta) \nu \sqrt{2\Pi s}\right)$

Proof: see appendix

Without asset/liabilities regulatory rules, banks would choose $l = 1$ and $d = 1$ maximizing their net informational rents plus the value of the put they receive. Thus in the first sector i.) the regulator need not impose any restriction on deposits to obtain $d = 1$. To implement the optimal level of deposits, d^*, in sector ii.), the regulator has to impose only restrictions that take the form $d \leq d^*$, which we interpret as $1 - d \geq 1 - d^*$, that is, in the optimal regulation the bank is bound to issue equity [11] in the financial markets.

Depending on the parameters affecting the choice of an optimal regulation, we may have as a particular case, $\theta^* = \bar{\theta}$ (no capital requirements). Yet the symmetric case $\underline{\theta} = \theta^*$ (no unrestricted banking) will never occur.

Proposition 3 *Under the assumptions of proposition 2, the optimal banking regulation involves no unregulated banking sector.*

The proof is an obvious implication of Proposition 2.

The optimal regulation involves no unregulated banking sector since there exists no interval where the individual rationality constraint is binding.

[11] although since Modigliani–Miller's theorem applies our model leaves partially undetermined the type of securities that are issued.

Finally, since the optimal mechanism implies a pooling of the different types in the unrestricted banking industry, we also obtain the following result:

Proposition 4 *i. In the unrestricted banking industry the bank pays a constant deposit insurance premium.*

ii. In the restricted banking industry total deposit insurance premium is decreasing with θ.

Proof: see appendix

It is important to stress that we have introduced the opportunity cost λ of transfers from public to private sector and this allow us to obtain results on the way a deposit insurance system has to be managed. Our result shows that under imperfect information the optimal deposit insurance management has to face a *chronic deficit*. This deficit decreases when the information improves and is equal to zero for perfect information, as established in Proposition 1. *This is equivalent to stating that it is optimal to set a price for deposit insurance premia inferior to its market value.*[12]

Our result stands in contrast with the empirical literature that tries to assert whether the deposit insurance is fairly priced (see e.g. Buser and Chen and Kane, 1981) where it is implicitly assumed that the deposit insurance company has to break even. The results we obtain correspond to the fact that "underpricing" of deposit insurance is *ex ante* optimal in an asymmetric information setting.

[12]This result is obtained because of the informational rents the banks obtain, given their reservation level outside the regulated banking industry. When we tax or regulate the profit from these alternative activities we are able to decrease the subsidy implicit in the deposit contract.

5 REGULATORY IMPLICATIONS

Although under perfect information we obtain as the optimal mechanism an unrestricted banking scheme, in which banks are allowed to make loans funded by deposits, under imperfect information only some of the characteristics we obtain are part of the banking regulations currently in use. Thus, in our interpretation of the preceding results, we will be careful to point out which of the features we obtain bear some relationship to the regulatory rules that we observe in different countries.

5.1 COMPARISON WITH THE FULL INFORMATION CASE

The intuition behind propositions 2, 3 and 4 is simple. First, the optimal full information regulation is not incentive compatible, because every bank would prefer to pay the minimal deposit insurance premium, and would announce type $\bar{\theta}$. To correct this we have to offer an informational rent, and this is why only $\bar{\theta}$ would be at its reservation level. In addition the informational rents have to be decreasing with θ, so that, if we give an additional rent to an agent θ, we will have to give it to all agents in the interval between $\underline{\theta}$ and θ . (This is why the optimal mechanism depends upon $F(\theta)$). Thus, a first possibility is to stick to the full information mechanism as far as loans and deposits are concerned and to increase the transfers. This is what happens in the unrestricted banking industry.

But transfering informational rents is costly. That is why the optimal regulatory scheme will use other means to give the banks incentive not to overstate their quality. This can be obtained either by increasing the banks holdings of riskless assets (or reserves) or by allowing for a progressive increase in the capital requirement (with θ) and compensate the banks by a decreasing deposit insurance premium. Our model shows that imposing larger holdings of riskless assets is never optimal. Hence capital requirements will be the efficient way to decrease the cost of deposit insurance under asymmetric information. Indeed, a higher capital requirement means a lower cost of deposit insurance and deposit insurance is more valuable for low quality banks than for high quality banks. Overstating the quality of its loans becomes then less attractive for a bank, since it involves less insured deposit in its balance sheet and therefore it implies a lower transfer from the deposit insurance system.

Finally the optimal regulatory scheme involve no distortions on the amounts lent by the banks. Indeed, contrary to deposit, the lending activity is more valuable for high quality banks than for low quality banks. Decreasing the amounts lent by the banks cannot help alleviating the cost of banking.

5.2 COMPARATIVE STATICS

Since we obtain the value of θ^*, it is possible to determine how the size of the unrestricted and restricted banking industries will change when the social cost of transfers, λ or the social benefits of deposits, ν, are modified.

The limit point θ^* is determined by

$$\psi(\theta^*, \lambda, \nu, \sigma) = \theta \exp\left[-\frac{s}{2} - \sqrt{2\log k(\theta)}\right] = 1$$

with

$$k(\theta) = \lambda F(\theta)/(f(\theta)\theta\nu\sqrt{2\Pi s})$$

Clearly,

$$\frac{\partial\psi}{\partial\lambda} < 0, \qquad \frac{\partial\psi}{\partial\nu} > 0$$

As for the sign of $\frac{\partial\psi}{\partial\theta}$, we prove in Lemma 3 that $\frac{\partial\psi}{\partial\theta} < 0$.

Consequently, as intuition suggests, *the optimal size of the restricted banking industry increases (θ^* decreases) with the social cost of transfers and decreases with the social benefits of deposits.*

5.3 SOCIAL BENEFITS OF LOANS

To extend our analysis so as to account for the social benefits of bank lending is straightforward. We assume that lending to the firms may generate additional social benefits since it allows to develop projects generating profits for the firms and employment for the workers. This can be justified because unlike banks, the borrowing firms do not have access to a perfect capital market[13].

We will represent this effect by a term $\chi(\theta)$ that represents an increase in the social benefits per dollar of loan. Thus, the social benefits are proportional to the amount of loans but independent of deposits, and consequently are the same for a regulated or an unregulated bank.

The optimal regulatory mechanism will be exactly the same, since in any case the optimal mechanism imposes no holding of riskless assets.

5.4 CHARTERING BANKS

Since the asymmetric information between the banker and the Deposit Insurance Company implies a social cost, it is reasonable to think that the Deposit Insurance Company will develop procedures to improve its information even if this has an administrative cost. This is precisely the role of a chartering agency. By examining the bank's project, the agency is able to reduce the uncertainty on the bank's quality. By so doing, it changes the distribution

[13] The results obtained by James showing that the borrowers stock price increases with a bank loan gives an empirical support to that assumption.

of θ on $(\underline{\theta}, \overline{\theta})$.

This issue is here particularly important, because it allows to justify that only banks with some minimal capability, $(\theta > \underline{\theta})$, that are the ones that would operate in a free banking setting, will enter the regulated industry. With the type of incentives that are generated, this hypothesis becomes crucial, because any type of agent will have an incentive to declare that it has sufficient capability to develop a banking activity and obtain the corresponding subsidy. If agents are able to do so, this will affect negatively the whole distribution $F(\theta)$, increasing the cost of banking. Consequently our result confirms the intuition of Buser, Chen and Kane (1981) that the same institution has to perform both the functions of insuring deposits and entry regulation.

6 CONCLUDING REMARKS

We have shown that even in a setting in which the banks'risks are perfectly known, and the moral hazard issue is ignored, determining the optimal banking regulation is not trivial. Our results show that under imperfect information capital requirements are perfectly justified in order to limit the cost of deposit insurance. Our results also suggest that regulation can be improved if the deposit premium is allowed to depend upon the capital, although the optimal regulation is never actuarially fair, but allows for informational rents that are higher for less efficient banks. The intuition for this result is that a more stringent regulation would affect first the high quality banks that will leave the banking industry. Consequently the optimal regulation will provide an actuarially fair deposit insurance to the highest quality bank but will have to concede informational rents to all the other ones.

In order to decrease this rents, it is optimal, since deposit insurance is less valuable for highly efficient banks, to base the self selection on the capital ratio and on the deposit insurance premia. The resulting regulation imposes higher capital ratios to the most efficient banks, contrary to the standard view on capital requirements in a perfect information setting.

To put into perspective our result, it is worthwhile recalling that we have used a partial equilibrium setting, in which deposits, as a possibility of transferring mediums of payment, have a utility. In this way we have dispensed with a complete modeling of the deposit market assuming infinite elasticity of deposits at the equilibrium interest rates and a finite capacity of banks, that limit their assets. Bringing in the equilibrium in the credit market is an important line of future research.

We also have assumed that banks differ in their ability to lend. But a reasonable alternative, that they differ in their volatility has been left unexplored, why we know that this is a crucial assumption. This is also a point we hope will be soon dealt with.

Finally, we have chosen the optimal mechanism in a wide class of discriminatory mechanisms, so as to obtain its characteristics. An alternative is to develop a second best approach, taking as given some institutional characteristics as flat insurance premia or a given rule to determine the required capital. This will yield asset–liabilities regulation rules that are more “realistic” but limits the focus of the analysis.

APPENDIX

To prove Proposition 2, we will first prove the three following lemmas.

Lemma 1 *The solution to the following problem*

$$\begin{cases} \max(\theta-1)(l-1)(1+\lambda)+\nu d+\lambda(l-1)\frac{F(\theta)}{f(\theta)} \\ d+l\leq 1 \\ d\geq 0 \\ l\geq 0 \end{cases}$$

is given by:

$$l=0 \quad \text{and} \quad d=1 \quad \text{if} \quad (\theta-1)(1+\lambda)+\frac{\lambda F(\theta)}{f(\theta)}-\nu<0$$

and

$$l=1 \quad \text{and} \quad d=0 \quad \text{if} \quad (\theta-1)(1+\lambda)+\frac{\lambda F(\theta)}{f(\theta)}-\nu>0$$

The value of the problem is $H^*(\theta)=\left(\nu-(\theta-1)(1+\lambda)-\frac{\lambda F(\theta)}{f(\theta)}\right)f(\theta)$ *in the first case and to zero in the second case.*

Proof: Let γ (resp. S_0,α_0) be the Lagrange multiplier associated to constraint $d+l\leq 1$ (resp. $d\geq 0, l\geq 0$). The first order conditions are

$$(\theta-1)(1+\lambda)+\frac{\lambda F(\theta)}{f(\theta)}-\gamma+\alpha_0=0$$

$$\nu-\gamma+\delta_0=0$$

Therefore we have $\gamma>0$ and $l+d=1$, so that if $\alpha_0>0 \quad \delta_0=0$ and conversely, if $\delta_0>0, \alpha_0=0$.

Replacing γ we have:

$$(\theta-1)(1+\lambda)+\frac{\lambda F(\theta)}{f(\theta)}-\nu=\delta_0-\alpha_0$$

If the LHS is negative, $\delta_0=0$ and $\alpha_0>0$, implying $d=1$ and $l=0$, if it is positive, then $\delta_0>0$ and $\alpha_0=0$ yielding the result.

Lemma 2 *If $d + l - 1 \geq 0$, the optimal contract involves $l = 1$.*

Proof Notice, first, that we can dispense with the positivity constraints for the variables d and l, since they are implied by the three constraints we impose on d and l when we solve:

$$\begin{cases} \max_{d,l} \left\{ (\theta - 1)(l - 1)(1 + \lambda) + \nu d + \lambda [lN(x) - 1] \frac{F(\theta)}{f(\theta)} \right\} f(\theta) \\ d + l - 1 \geq 0 \\ \quad 1 - l \geq 0 \\ \quad 1 - d \geq 0 \end{cases}$$

A sufficient condition to have $\frac{\partial H}{\partial l} > 0$ is that

$$E = N(x) + lN'(x)\frac{dx}{dl} \geq 0$$

In order to show that E is positive, notice first that

$$E = N(x) - N'(x)\frac{1 - d}{l + d - 1}$$

and

$$\frac{\partial E}{\partial x} = N'(x) + xN'(x)\frac{1 - d}{l + d - 1} > 0$$

Since x is an increasing function of θ it is sufficient to show that the result holds for $\underline{\theta} = 1$.

Define $X = \frac{1-d}{l}$ where $X \geq 0$ and $1 - X = \frac{d+l-1}{l} \geq 0$ implies $X \in [0, 1]$

Then,

$$x = -\frac{1}{\sqrt{s}}\log(1 - X) + \frac{\sqrt{s}}{2}$$

and

$$E[X] = N[x(X)] - N'[x(X)]\frac{1}{\sqrt{s}}\frac{X}{1 - X}$$

We will show that

$$\begin{cases} \min E(X) > 0 \\ X \in [0, 1] \end{cases}$$

In order to do that, we first study the sign of

$$\frac{\partial E[X]}{\partial X}$$

We obtain:

$$\frac{\partial E[X]}{\partial X} = \frac{N'(x)}{\sqrt{s}(1-X)}\left\{1+\left[-\frac{1}{\sqrt{s}}\log(1-X)+\frac{\sqrt{s}}{2}\right]\frac{1}{\sqrt{s}}\frac{X}{1-X}-\frac{1}{1-X}\right\} =$$

$$= \frac{N'(x)}{\sqrt{s}(1-X)}\left\{-\frac{1}{s}\frac{X}{1-X}\log(1-X)+\frac{1}{2}\frac{X}{1-X}-\frac{X}{1-X}\right\} =$$

$$= \frac{N'(x)X}{\sqrt{s}(1-X)^2}\left[-\frac{1}{s}\log(1-X)-\frac{1}{2}\right]$$

So that $\frac{\partial E[X]}{\partial X} = 0$ for $\log(1-X) = -\frac{s}{2}$ that is $X = 1-e^{-s/2}$, with $\frac{\partial E}{\partial x} < 0$ (resp $\frac{\partial E}{\partial x} > 0$) for $X < 1-e^{-s/2}$ (resp. $X > 1-e^{-s/2}$) so that the minimum value of E is attained for $x = 1-e^{-s/2}$. The value of this minimum is

$$E\min(\sqrt{s}) = E\left[1-e^{-s/2}\right] = N[\sqrt{s}] - N'[\sqrt{s}]\frac{1}{\sqrt{s}}\frac{1-e^{-s/2}}{e^{-s/2}}$$

To examine the sign of this expression, let $\sqrt{s} = v$.

$$\frac{\partial E}{\partial v} = N'(v)\left[1-\frac{1-e^{-v^2/2}}{e^{-v^2/2}}-\frac{d}{dv}\left(\frac{1}{v}\frac{1-e^{-v^2/2}}{e^{-v^2/2}}\right)\right]$$

That is, after simplification:

$$\frac{\partial E}{\partial v} = N'(v)\frac{1-e^{-v^2/2}}{v} > 0$$

Consequently to prove that $E(v)$ is positive, it suffices to prove it for $v = 0$.

Now

$$\lim_{v\to 0} E\min = N(0) - \lim_{v\to 0} N'(v)\frac{1-e^{-v^2/2}}{ve^{-v^2/2}} = N(0) = \frac{1}{2} > 0$$

Lemma 3 *If,* $\log\theta - \sqrt{2s\log\left(\lambda F(\theta)/\theta f(\theta)\nu\sqrt{2\Pi s}\right)}$ *is decreasing in* θ *(assumption H) the solution to the following problem*

$$\begin{cases} \max\left\{(\theta-1)(l-1)(1+\lambda)+\nu d+\lambda[lN(x(l,d))-1]\frac{F(\theta)}{f(\theta)}\right\}f(\theta) \\ d+l-1\geq 0 \\ 1-d\geq 0 \\ 1-l\geq 0 \end{cases} \tag{12}$$

has the following structure:
There exits $\theta^*, \theta^* \neq \underline{\theta}$, such that

$$l^*(\theta)=1, d^*(\theta)=1 \quad \text{for} \quad \theta\in(\underline{\theta},\theta^*)$$

$$l^*(\theta)=1, d^*(\theta)=\theta\exp\left[-s/2-\sqrt{2s\log k(\theta)}\right] \quad \text{for} \quad \theta\in(\theta^*,\overline{\theta})$$

where

$$k(\theta)=\lambda F(\theta)/(f(\theta)\theta\nu\sqrt{2\Pi s})$$

The value of the problem is:

$$H^*(\theta)=\left\{\nu d^*(\theta)+\lambda\left[N[x(1,d^*(\theta))]-1\right]\frac{F(\theta)}{f(\theta)}\right\}f(\theta)$$

Proof Let γ (resp. δ_1) be the Lagrange multiplier associated with the constraint $d+l-1\geq 0$ (resp. $1-d\geq 0$). Lemma 2 establishes the optimality of $l=1$. The first order condition for d is given by:

$$\nu-\frac{\lambda F(\theta)}{f(\theta)\sqrt{2\Pi s}}\exp[-x^2(1,d)/2]d^{-1}+\gamma-\delta_1=0 \tag{1}$$

Since we have:

$$\begin{aligned}\exp[-x^2(1,d)/2]\left(\frac{\theta}{d}\right)\frac{1}{\theta} &= \exp\left[-\frac{1}{2}\left[\frac{s}{4}+\frac{\log^2(\theta/d)}{s}+\log(\theta/d)-2\log(\theta/d)\right]\right]\theta^{-1} = \\ &= \exp[-y^2(\theta,d)/2]\theta^{-1}\end{aligned}$$

where

$$y(\theta, d) = \frac{\sqrt{s}}{2} - \frac{\log(\theta/d)}{\sqrt{s}} \tag{2}$$

we are able to rewrite (1) as:

$$\nu - \nu k(\theta) \exp[-y^2(\theta, d)/2] + \gamma - \delta_1 = 0 \tag{3}$$

The second derivative is:

$$\nu k(\theta) y \exp[-y^2/2] \frac{\partial y}{\partial d} \leq 0$$

$\frac{\partial y}{\partial d} = \frac{1}{d\sqrt{s}} > 0$, so that the objective function is concave if:

$$y \leq 0 \tag{4}$$

and convex otherwise. (4) is equivalent to $d \leq \theta \exp[-s/2]$. If $\theta \exp[-s/2] \geq 1$, the objective function is always concave. Otherwise, we have two possible local maxima, one given by (3) with $\delta_1 = 0$ and the other by $d = 1$ $(\delta_1 > 0)$.

We first study the solutions to (3).

To obtain the solutions to (3), notice, first, that $\gamma > 0$ will never hold, since for $d + l = 1$, $d \to 0$ and $\exp[-y^2/2] \to 0$ would imply $\delta_1 > 0$, a contradiction.

Under assumption H, $k(\theta)$ is increasing in θ. We have also $k(\underline{\theta}) = 0$ since $F(\underline{\theta}) = 0$.
We set $\theta_0 = \bar{\theta}$ if $k(\bar{\theta}) < 1$ and we take θ_0 such that $k(\theta_0) = 1$ otherwise. Now we will proceed to characterize the solution on $(\underline{\theta}, \theta_0)$ (Step 1) and then on $(\theta_0, \bar{\theta})$ (Step 2) depending on the sign of a function $\tilde{\delta}(\theta_0)$ (Steps 2A and 2B) we will introduce hereafter.

Step 1

Since on $[\underline{\theta}, \theta_0)$ $k\ (\theta) \leq 1$ this implies:
$\forall \theta, \theta \leq \theta_0, \forall d \in [0,1], k(\theta) \exp\left[-\frac{y^2(\theta, d)}{2}\right] \leq 1.$

so that the optimal solution $d^*(\cdot)$ is caracterised by:

$$d^*(\theta) = 1, \quad \forall \theta < \theta_0.$$

irrespectively of the convexity of the objective function.

Step 2

We now proceed to analyze the solution on $(\theta_0, \bar{\theta}]$
$\forall \theta, \theta > \theta_0, k(\theta) \geq 1$ and there exists a unique function $\tilde{d}(\theta)$ that satisfies the first and second order conditions:

a) $k(\theta) \exp\left[\frac{-y^2(\theta, \tilde{d}(\theta))}{2}\right] = 1$

and

b) $y(\theta, \tilde{d}(\theta)) \leq 0$

The function $\tilde{d}(\theta)$ is the solution to equation (3) on $\left(\theta_0, \bar{\theta}\right]$ when we drop the constraint $1 - d \geq 0$. Using a) b) and (2), we obtain:

$$\log \tilde{d}(\theta) = \log \theta - \frac{s}{2} - \sqrt{2s \log k(\theta)}$$

or equivalently:

$$\tilde{d}(\theta) = \theta \exp\left[-\frac{s}{2} - \sqrt{2s \log k(\theta)}\right]$$

It is easy to verify that, under the hypothese H, $\tilde{d}(\theta)$ is decreasing on its domain $[\theta_0, \bar{\theta}]$.

Two cases are possible: $d(\tilde{\theta}_0) \geq 1$ (Step 2A) and $\tilde{d}(\tilde{\theta}_0) < 1$ (Step 2B)

Step 2A

$\tilde{d}(\theta_0) \geq 1$ implying $\theta_0 \exp -\frac{s}{2} \geq 1)$. Then $\forall \theta, \theta \in \left[\theta_0, \bar{\theta}\right]$, the programme is concave in d on all the interval $[0, 1]$ and the optimal solution has the following structure:

$$\forall \theta \in [\theta_0, \theta_1], \quad d^*(\theta) = 1$$

$$\forall \theta \in [\theta_1, \bar{\theta}], \quad d^*(\theta) = \tilde{d}(\theta) = \theta exp\left[-\frac{s}{2} - \sqrt{2s \log k(\theta)}\right]$$

where θ_1 is such that $\tilde{d}(\theta_1) = 1$ if $\tilde{d}(\bar{\theta}) \leq 1$ and $\theta_1 = \bar{\theta}$ otherwise.

Step 2B

For any θ, $\theta > \theta_0$, the optimality necessary conditions are satisfied both at point $\tilde{d}(\theta) < 1$ and at $d = 1$.

Since $f(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$, we use the objective function H to define

$$\begin{aligned}
\Delta(\theta) &= \{H(\theta, 1, 1) - H(\theta, 1, \tilde{d}(\theta)\}/f(\theta) \\
&= \left\{\gamma(1 - \tilde{d}(\theta)) + \lambda[N(x(1, 1) - N(x, (1, \tilde{d}(\theta))] \frac{F(\theta)}{f(\theta)}\right\}
\end{aligned}$$

We now compute

$$\begin{aligned}
\dot{\Delta}(\theta) &= -\gamma \dot{\tilde{d}} - \lambda N'(x(1, \tilde{d}) \left(\frac{1}{\theta} - \frac{\dot{\tilde{d}}}{d}\right) \quad \frac{1}{6\sqrt{t}} \quad \frac{F(\theta)}{f(\theta)} \\
&+ \lambda \left[N(x(1, 1)) - N(x(1, \tilde{d}))\right] \left(\frac{\dot{F(\theta)}}{f(\theta)}\right)
\end{aligned}$$

and using the envelope theorem, we obtain:

$$\dot{\Delta}(\theta) = -\lambda N'(x(1, \tilde{d})) \quad \frac{1}{\theta} \quad \frac{1}{6\sqrt{t}} \quad \frac{F(\theta)}{f(\theta)} + \lambda[N(x, (1, 1)) - N(x(1, \tilde{d}))] \quad \frac{\dot{F(\theta)}}{f(\theta)}$$

or $N(x(1, 1)) - N(x(1, \tilde{d}) < 0$

Now since $\tilde{d} < 1$ we have

$$x(1,\tilde{d}) = \frac{\ln\theta - \ln\tilde{d}}{6\sqrt{t}} + \frac{6\sqrt{t}}{2} > \frac{\ln\theta}{6\sqrt{t}} + \frac{6\sqrt{t}}{2}$$

implying $N(x(1,1)) - N(x(1,\tilde{d})) < 0$.

Under assumption H we know that $F(\theta)/f(\theta)$ is increasing so that $\dot{\Delta}(\theta) < 0$.

We also know that $\Delta(\theta) \geq 0$ on $[\underline{\theta}, \theta_0]$ since in Step 1 we have proved $d^* = 1$ to be the optimum. Since $\Delta(\theta)$ is continuous, either $\Delta(\bar{\theta}) \geq 0$ and $d^* = 1$ is optimal on $[\underline{\theta}, \bar{\theta}]$ or else there exists $\theta_2, \theta_2 \in [\theta_0, \bar{\theta}]$, such that $\Delta(\theta_2) = 0$, in which case the optimal solution will be

$$d^*(\theta) = 1 \quad \text{for} \quad \theta \geq \theta_2$$

$$d^*(\theta) = \tilde{d}(\theta) \quad \text{for} \quad \theta > \theta_2$$

(notice that here $d^*(\theta)$ may be discontinuous).

To conclude, there exists θ^* such that

$$\forall\theta \in [\underline{\theta}, \theta^*[,\ d^*(\theta) = 1$$

$$\forall\theta \in [\theta^*, \bar{\theta}],\ d^*(\theta) = \theta\exp[-\frac{s}{2} - \sqrt{2s\log k(\theta)}]$$

(where θ^* is equal to θ_1 or θ_2).

Proof of proposition 2

The proof involves three steps. In step 1 we pose an underconstrained problem, in step 2 we find its solution and in step 3 we show that the solution satisfies the incentive compatibility constrain (7).

Step 1

The underconstrained problem will be set using conditions (10) (11) instead of condition (7).

To write the problem with a function that depends on the sign of the expression $l + d - 1$, we introduce an auxiliary variable q. In addition, we will use the variable $v(\theta) = U(\theta) - \theta + 1$. Since condition (11) implies the continuity of $U(\theta)$, $v(\theta)$ is also continuous.

The underconstrained problem is then, equivalent to:

$$(P1)\begin{cases} \max_{(d,l,q)} \int_{\underline{\theta}}^{\overline{\theta}} [(l(\theta)-1)(\theta-1)(1+\lambda) + \nu d - \lambda v(\theta)] f(\theta) d\theta + E(\theta) - 1 \\ \dot{v}(\theta) = q(\theta)[l(\theta)N(x(l(\theta), d(\theta))) - 1] + [1 - q(\theta)][l(\theta) - 1] \\ v(\theta) \geq 0 \\ 0 \leq d(\theta) \leq 1 \\ 0 \leq l(\theta) \leq 1 \\ (q\left((\theta) - \frac{1}{2}\right)(l(\theta) + d(\theta) - 1) \geq 0 \\ q(\theta) \in \{0, 1\} \end{cases}$$

with $E(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta$.

Since $l(\theta) \leq 1$ and $N(x) \leq 1$ we have $\dot{v}(\theta) < 0$, so that if the individually rationality constraint holds it does so only at the point $\overline{\theta}$. Since an increase in v decreases the value of the objective function, we will have $v(\overline{\theta}) = 0$.

On the other hand, integrating the differential constraint by parts we have

$$\int_{\underline{\theta}}^{\overline{\theta}} v(\theta) f(\theta) d\theta = [v(\theta)F(\theta)]_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} \left\{ q(\theta)[l(\theta)N(x(l(\theta), d(\theta))) - 1] + [1 - q(\theta)][l(\theta) - 1] \right\} F(\theta) d(\theta)$$

so that it is possible to replace the variable $v(\theta)$ in the objective function.

The problem is therefore reduced to a pointwise maximization problem:

$$(P2)\begin{cases} \max_{l,d,q} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ (l-1)(\theta-1)(1+\lambda) + \nu d + \lambda[q(lN(x) - 1) + (1-q)(l-1)] \frac{F(\theta)}{f(\theta)} \right\} f(\theta) d\theta \\ 0 \leq l \leq 1 \\ 0 \leq d \leq 1 \\ \left(q - \frac{1}{2}\right)(l + d - 1) \geq 0 \\ q \in \{0, 1\} \end{cases}$$

Step 2

We will solve the problem for $q = 0$ and $q = 1$, then compare the values of the objective function.

For

$$q = 0, \quad d + l \leq 1, \quad d \geq 0 \quad \text{and} \quad l \geq 0$$

implies $d \leq 1, l \leq 1$. Therefore the solution of $(P2)$ is given by Lemma 1. In the same way, the solution to $(P2)$ for $q = 1$ is given by Lemma 3.

To determine the function $q(\theta)$ we only have to compare the value of the objective function for $q = 0$ and $q = 1$.

We therefore define

$$\Delta = H^*(0) - H^*(1)$$

where $H^*(q)$ is the value of the problem $(P2)$ for a given q.

Notice, first, that if $(\theta - 1)(1 + \lambda) + \lambda \frac{F(\theta)}{f(\theta)} > \nu$ Lemma 1 implies $H(0) = 0$ and Lemma 3 implies $H(1) > 0$ because $l = 1$, $d = 0$ is a non optimal admissible point for which the objective function reaches a zero value.

Let θ_0 be such that $(\tilde{\theta} - 1)(1 + \lambda) + \lambda \frac{F(\hat{\theta})}{f(\hat{\theta})} = \nu$.

We know that for $\theta \in (\theta_0, \overline{\theta}) \quad q^* = 1$.

For $\theta \in (\underline{\theta}, \theta_0)$,

$$\frac{\Delta(\theta)}{f(\theta)} = \nu(1 - d^*(\theta)) - (\theta - 1)(1 + \lambda) - \lambda \left[N(x(1, d^*(\theta))) \right] \frac{F(\theta)}{f(\theta)}$$

Since $d^*(\theta)$ is the optimal solution when $d + l \geq 1$ we have:

$$H(\theta, 1, 1) - H(\theta, 1, d^*) = \left\{ \nu(1 - d^*(\theta)) + \lambda[N(x(1, 1)) - N(x(1, d^*))] \frac{F(\theta)}{f(\theta)} \right\} f(\theta) < 0$$

This implies that $\dfrac{\Delta(\theta)}{f(\theta)} < 0$.

Step 3

We have to check that the solution to $(P2)$ satisfies the incentive compatibility conditions:

$$\forall(\theta, \hat{\theta}) \in [\underline{\theta}, \bar{\theta}]^2, \quad U(\theta, \theta) \geq U(\theta, \hat{\theta}) \tag{1}$$

A) First, we check that condition (1) is satisfied when $d^*(\theta)$ is continuous and differentiable. In order to do that, we study the sign of $u(\theta, \theta) - u(\theta, \hat{\theta})$.

For that, we introduce the function $\varphi(\theta, d)$ such that:

$$\begin{aligned} u(\theta, \theta) &= t(\theta) + \varphi(\theta, d(\theta)) \\ u(\theta, \hat{\theta}) &= t(\hat{\theta}) + \varphi(\theta, d(\hat{\theta})) \end{aligned}$$

In other words, $\varphi(\theta, d) = \theta - 1 + P(\theta, d)$.

Then, using (8) we have:

$$u(\theta, \theta) - u(\theta, \hat{\theta}) = \int_{\hat{\theta}}^{\theta} [\varphi_d(\theta, d(u)) - \varphi_d(u, d(u))] \dot{d}(u) du$$

or equivalently:

$$u(\theta, \theta) - u(\theta, \hat{\theta}) = \int_{\hat{\theta}}^{\theta} \int_{u}^{\theta} \varphi_{d\theta}(t, d(u)) \dot{d}(u) dt du$$

But condition (9) implies that

$$\varphi_{d\theta}(t, d(u)) \dot{d}(u) \geq 0 \qquad \forall t, u$$

So we obtain:

$$u(\theta, \theta) - u(\theta, \hat{\theta}) \geq 0 \qquad \forall(\theta, \hat{\theta}) \in [\underline{\theta}, \bar{\theta}]^2$$

B) Now, we have to check that condition (1) is satisfied when $d^*(\theta)$ is discontinuous in θ^*, that is to say when $\theta^* = \theta_2$.

The interesting cases which are different from **A)** are the following:

a) $\theta < \theta_2 < \hat{\theta}$

and

b) $\hat{\theta} < \theta_2 < \theta$

a) $\theta < \theta_2 < \hat{\theta}$

$u(\theta,\theta) - u(\theta,\hat{\theta}) = [u(\theta,\theta) - u(\theta,\theta_2^-)] + [u(\theta,\theta_2^-) - u(\theta,\theta_2^+)] + [u(\theta,\theta_2^+) - u(\theta,\hat{\theta})]$

$= A_1 + A_2 + A_3.$

From **A)** we know that:

$$A_1 = u(\theta,\theta) - u(\theta,\theta_2^-) \geq 0$$

$$\begin{aligned} A_2 &= u(\theta,\theta_2^-) - u(\theta,\theta_2^+) \\ &= t(\theta_2^-) + \varphi(\theta, d(\theta_2^-)) - t(\theta_2^+) - \varphi(\theta, d(\theta_2^+)) \end{aligned}$$

But we now that the bank θ_2 is indifferent between the contract $(t(\theta_2^-), d(\theta_2^-))$ and $(t(\theta_2^+), d(\theta_2^+))$.

So we have

$$t(\theta_2^-) + \varphi(\theta_2, d(\theta_2^-)) = t(\theta_2^+) + \varphi(\theta_2, d(\theta_2^+))$$

This implies

$$\begin{aligned} A_2 &= \varphi(\theta, d(\theta_2^-)) - \varphi(\theta, d(\theta_2^+)) \\ &- [\varphi(\theta_2, d(\theta_2^-)) - \varphi(\theta_2, d(\theta_2^+))] \\ &= \int_{\theta_2}^{\theta} \int_{d(\theta_2^+)}^{d(\theta_2^-)} \varphi_{\theta d}(t,u) du \ dt \\ &\geq 0 \end{aligned}$$

Since $\theta < \theta_2, d(\theta_2^-) > d(\theta_2^+)$ and $\varphi_{\theta d}(t,u) \leq 0$.

$$\begin{aligned} A_3 &= u(\theta,\theta_2^+) - u(\theta,\hat{\theta}) \\ &= \int_{\hat{\theta}}^{\theta_2^+} [\varphi_d(\theta, d(u)) - \varphi_d(u, d(u))]\dot{d}(u)\ du \\ &= \int_{\hat{\theta}}^{\theta_2^+} \int_u^{\theta} \varphi_{d\theta}(t, d(u))\dot{d}(u) dt\ du \\ &\geq 0 \end{aligned}$$

Since $\theta_2^+ < \hat{\theta}, \quad \theta < u$ and $\varphi_{d\theta}(t,d(u))\dot{d}(u) \geq 0$.

Finally we have $u(\theta,\theta) - u(\theta,\hat{\theta}) \geq 0$.

b) $\hat{\theta} < \theta_2 < \theta$

$$\begin{aligned} u(\theta,\theta) - u(\theta,\hat{\theta}) &= [u(\theta,\theta) - u(\theta,\theta_2^+)] + [u(\theta,\theta_2^+) \\ &- u(\theta,\theta_2^-)] + [u(\theta,\theta_2^-) - u(\theta,\hat{\theta})] \\ &= A_1 + A_2 + A_3. \end{aligned}$$

From **A)** we know that

$$A_1 = u(\theta,\theta) - u(\theta,\theta_2^+) \geq 0.$$

As in case **A)** we have :

$$\begin{aligned} A_2 &= u(\theta,\theta_2^+) - u(\theta,\theta_2^-) \\ &= \int_{\theta_2}^{\theta} \int_{d(\theta_2^-)}^{d(\theta_2^+)} \varphi_{d\theta}(t,u) du\ dt \\ &\geq 0 \end{aligned}$$

Since $\theta_2 < \theta, d(\theta_2^-) > d(\theta_2^+)$ and $\varphi_{d\theta}(t,u) < 0$.

$$
\begin{aligned}
A_3 &= u(\theta, \theta_2^-) - u(\theta, \hat{\theta}) \\
&= \int_{\hat{\theta}}^{\theta_2^-} \int_u^{\theta} \varphi_{d\theta}(t, d(u))\dot{d}(u)\ dt\ du \\
&\geq 0
\end{aligned}
$$

Since $\hat{\theta} < \theta_2^-, u < \theta$ and $\varphi_{d\theta}(t, d(u))\dot{d}(u) > 0$.

Finally we have $u(\theta, \theta) - u(\theta, \hat{\theta}) \geq 0$.

Proof of Proposition 4

To prove proposition 4 we have to determine the value of the transfer t.

For θ in $(\underline{\theta}, \theta^*)$ condition (8) implies $\dot{t} = 0$.For θ in $(\theta^*, \overline{\theta})$ condition (8) implies $\dot{t} = -\dot{d}P_2$ and $t(\overline{\theta}) = -P(l(\overline{\theta})\overline{\theta}, l(\overline{\theta}) + d(\overline{\theta}) - 1)$ since $v(\overline{\theta}) = 0$. The total deposit insurance premium $a(\theta) = -t(\theta)$ is therefore positive and decreasing since $P_2 > 0$.

REFERENCES

Acharya, S. and J.F. Dreyfus (1989), "Optimal Bank Reorganization Policies and the Pricing of Federal Deposit Insurance", *Journal of Finance*, **44**, n°5, 1313–33.

Avery, R.B. and A. N. Berger (1991), "Risk-Based Capital and Deposit Insurance Reform", *Journal of Banking and Finance*, 847–74.

Baron, D. and R. Myerson (1982), "Regulating a Monopolist with Unknown Costs", *Econometrica*, **50**, 911–30.

Bensaid, B., H. Pagès and J. C. Rochet (1995), "Efficient Regulation of Banks Solvency", discussion paper *GREMAQ*, **n°9507370**, Toulouse.

Berlin, M., A. Saunders, and G.F. Udell (1991), "Deposit Insurance Reform: What are the Issues and What Needs to be Fixed?", *Journal of Banking and Finance*, **15**, 735–52.

Boot, A.W.A., and Thakor A.V. (1993), "Self–Interested Bank Regulation", *American Economic Review*, **83**, n° 2, 206–212.

Buser, S. A., A. H. Chen, and E. J. Kane (1981), "Federal Deposit Insurance, Regulatory Policy, and Optimal Bank Capital", *Journal of*

Finance, **36** n° 1, 51–60.

Campbell, T.S., Chan Y., and Marino A.M. , (1992), "An Incentive–Based Theory of Bank Regulation", *Journal of Financial Intermediation*, n° 2, 255-276.

Chan, Y., S.I., Greenbaum, and A. V. Thakor (1992), "Is Fairly Priced Deposit Insurance Possible?", *The Journal of Finance*, **47** n° 1, 227–45.

Dewatripont, M. and J. Tirole (1994), "The Prudential Regulation of Banks", *M.I.T. Press, Cambridge, MA.*

Diamond, D. and Dybvig (1983), "Bank Runs, Deposit Insurance and Liquidity" *Journal of Political Economy*, **91** 401–19.

Eisenbeis, R.A. (1986), "Risk as a Criterion for Expanding Banking Activities", in Kaufman, G. G. and R.C. Kormendi, editors, *Deregulation Financial Service: Public Policy in Flux*, chapter 7, Ballinger.

Fama, E.F. (1980), "Banking in the Theory of Finance", *Journal of Monetary Economics*, **6** n°1, 39–57.

Fama, E.F. (1985), "What's Different about Banks?", *Journal of Monetary Economics*, **15** n° 1, 29–39.

Gianmarino R., T. Lewis and D. Sappington (1993), "An Incentive Approach to Banking Regulation", *The Journal of Finance*, 1523–1542.

Gorton, G. and G. Pennacchi (1990), "Financial Intermediation and Liquidity Creation", *Journal of Finance*, 49–71.

Guesnerie, R. and J.J. Laffont (1984), "A Complete Solution to a Class of Principal Agent Problems with Applications to the Control of the Self-Managed Firm", *Journal of Public Economics*, **25**, 329–69.

James, C. (1987), "Some Evidence on the Uniqueness of Bank Loans", *Journal of Financial Economics*, **19**, 217–35.

Kane, E.I. (1985), "The Gatering Crisis in the Federal Deposit Insurance". MIT Press, Cambridge, Mass.

Kane, E.I. , (1990), "Principal–Agent Problem in S&L Salvage", *Journal of Finance*, **XLV**, n° 3, 755–764.

Kareken, J.H. and N. Wallace (1978), "Deposit Insurace and Bank Regulation: A Partial Equilibrium Exposition", *Journal of Business*, **51**, 413–38.

Laffont, J.J. and J. Tirole (1986), "Using Cost Observation to Regulate Firms", *Journal of Political Economy*, **94** n° 3, 614–41.

Lucas, D. and R.L. McDonald (1987), "Bank Portfolio Choice with Private Information about Loan Quality: Theory and Implications for Regulation", *Journal of Banking and Finance*, **11** n° 3, 473–97.

Lucas, D. and R.L. McDonald (1992), "Bank Financing and Investment Decisions with Asymmetric Information about Loan Quality", *Rand Journal of Economics*, **23** n° 1, 86–105.

Marcus, A. J. and I. Shaked (1984), "The Valuation of FDIC Deposit Insurance Using Option-Pricing Estimates", *Journal of Money, Credit and Banking*, **16** n°4, 446–60, Part 1.

Merton, R. (1977), "An Analytic Derivation of the FDIC Deposit Insurance and Loan Guarantees", *Journal of Banking and Finance*, **1**, 3–11.

Pyle, D. (1984), "Deregulation and Deposit Insurance Reform", *Economic Review*, Federal Reserve Bank of San Francisco.

Tomiyama (1985), "Two-Stage Optimal Control Problems and Optimality Conditions", *Journal of Economics Dynamics and Control*, **9**, n°3, 317–37.