Universitat Pompeu Fabra Barcelona
Department of Economics and Business

Economics Working Paper Series

Working Paper No. 1647

Monotone contracts

Daniel Bird and Alexander Frug

April 2019

Monotone Contracts*

Daniel Bird Alexander Frug

April 3, 2019

Abstract

A common feature of dynamic interactions is that the environment in which they occur typically changes, perhaps stochastically, over time. We consider a general fluctuating contracting environment with symmetric information, and identify a systematic effect of the fluctuations in the environment on optimal contracts. We develop a notion of a *separable activity* that corresponds to a large class of contractual components, and provide a tight condition under which these components manifest a form of seniority: any change that occurs in these components over time, under an optimal contract, favors the agent. We illustrate how our results can be applied in various economic settings.

Keywords: Dynamic contracting, Stochastic opportunities.
JEL Classification: D86.

1 Introduction

In a long-term principal-agent relationship, it is rarely the case that the parties involved face exactly the same situation period after period. Seasonality and random shocks affect periodic demands; workers accumulate general and firm-specific skills; business opportunities and threats arrive and disappear randomly; and technological innovations make existing practices obsolete. With several notable exceptions (which we address in detail below), the literature on (dynamic) contracts has traditionally focused on the effects of asymmetric information and

*Bird: Eitan Berglas School of Economics, Tel Aviv University (e-mail: dbird@post.tau.ac.il); Frug: Department of Economics and Business, Universitat Pompeu Fabra and Barcelona GSE (e-mail: alexander.frug@upf.edu). We are grateful to Larbi Alaoui, Jose Apesteguia, Eddie Dekel, Jan Eeckhout, Juan Jose Ganuza, Benny Moldovanu, Antonio Penta, Sandro Shelegia, and Joel Sobel for valuable suggestions and comments.

overlooked the impact of the inherent fluctuations in the environment in which the interaction occurs.[1] In this paper, we identify a general feature of optimal contracts that arises directly from the fluctuations in the contracting environment. Our results offer new insights into the general phenomenon of seniority and can be applied in various contexts. In addition, the results afford a unified and more general perspective on several seemingly unrelated classic as well as more recent papers that study dynamic contracting with symmetric information in a broad range of economic settings.

To illustrate the effect of fluctuations in the contracting environment in the most transparent way, we first consider a stylized model of random opportunities where different types of "tasks" arrive stochastically over time in a manner that is i.i.d. across periods. The agent can exert effort on available tasks, and the principal incentivizes him to do so by offering a periodic wage. We make the usual assumptions that the agent's marginal productivity of effort and marginal utility from wage are decreasing, and that his utility from wage and the cost of effort are additively separable. To isolate the effect we want to study, we abstract away frictions that arise from informational asymmetries and assume that the agent's effort and the arrival of tasks are perfectly observed by both players. Although the arrival of tasks is governed by a stationary distribution, the unique optimal contract generates a "promotion-based" dynamics where the periodic wage increases and the required effort (on every type of task) decreases over time.

The bare-bones model of random opportunities offers new insights into wage ladders and seniority in the context of long-term labor contracts by drawing a clear connection between exogenous intertemporal variability in business conditions and the dynamics of effort and compensation. The derivation of the optimal contract, however, crucially relies on a number of simplifying and restrictive assumptions. In many cases, the distribution under which business opportunities arrive is neither identical nor independent across periods. Moreover, real-world long-term interactions usually take place in richer and more complex environments; in practice, contracting environments include many components, of which some vary in a predictable manner, some are constant, and some are determined endogenously by actions taken jointly by the players.

[1]For discussions on adverse selection and moral hazard see Bolton and Dewatripont (2005) and Edmans and Gabaix (2016).

Our objective is to identify, independently of setting-specific details, a robust feature of optimal contracts that arises in a large class of contracting environments. Therefore, while the optimal contract in the aforementioned model of random opportunities exhibits several interesting features, we emphasize the monotonic way in which the terms of the contract are updated over time: *when the required effort or wage is updated, the levels always shift in the direction that favors the agent*. This type of monotonicity turns out to be a general feature of optimal contracts in dynamic interactions with symmetric information and one-sided commitment. Establishing this robust monotonicity property for abstract contracting environments is the main theme of the paper.

In brief, our main result (Theorem 1 and Theorem 2) shows that, as time goes by, certain "components" of optimal contracts change only in the direction that favors the agent. We provide a tight characterization of these components and discuss the economic interpretation. We develop our results for a large class of contracting environments: the only restriction we impose is that information is symmetric. Examples of environments that fall into this class include Harris and Holmström (1982), Holmström (1983), Marcet and Marimon (1992), Thomas and Worrall (1994), Postal-Vinay and Robin (2002a,b), Albuquerque and Hopenhayn (2004), Krueger and Uhlig (2006), and Forand and Zápal (2019). Since we aim to identify a general feature of contracts that holds in different economic settings, we deliberately leave many parts of the contracting environment unspecified. Due to the flexibility and generality of our modeling approach, our results can be easily be applied in various contexts. Moreover, our detail-free approach can be used to draw partial, though economically meaningful, implications even in complex settings in which there is little hope of obtaining a complete characterization of optimal contracts.

The main notion we develop is that of "activity." Broadly speaking, an activity represents a reoccurring component of the interaction that can be measured in (unidimensional) units on which the players have monotone and opposite preferences. Examples include level of periodic wage, level of effort exerted on either routine or randomly arriving tasks, volume of production, level of periodic financing to an entrepreneur, level of authority of a bureaucrat or a unit in an organization, quality of a supplied product, degree of risk-sharing, etc. As these examples imply, in many cases the agent has some of control over the activity. For instance, most production processes require a combination of the principal's resources and the agent's labor. In such cases, it is crucial to consider the agent's incentives to select the suggested level of the activity and, in particular, how much his incentives depend on the principal's activity-related actions. As we show below, these incentives are integral to analyzing the dynamics of an activity.

At the *contractual* level, all elements of the interaction are interrelated through incentives. Clearly, one cannot determine the optimal wage without considering the agent's duties. On the other hand, at the *environmental* level, there is a sense in which certain activities can be "separated" from other elements of the interaction. For example, in the model of random opportunities the distribution of task availability is i.i.d. across periods (an assumption that creates a separation between current actions and future task availability), and all payoffs are additively separable.

The ability to separate a specific activity from the rest of the interaction at the environmental level is key in our modeling approach in which the main focus is on the activity rather than the whole contracting environment. To achieve this property, we impose requirements that are weaker than those satisfied in the model of random opportunities. Our main requirement is that changes in the current activity level should not affect the environment in the future (or, in the language of the model of random opportunities, changes in the agent's wage or effort should not affect the distribution of task availability in future periods). We also require that payoffs in the activity-related and other parts of the interaction are additively separable. Unlike in the model of random opportunities, the availability of separable activities can depend on past realizations as well as players' past actions. In other words, situations where the availability of a separable activity is endogenous and/or path-dependent fall under the framework of our general model.

Consider an arbitrary activity that is separable with respect to a given contracting environment. Since the players have opposing interests regarding the activity levels, each level corresponds to an (activity-related) payoff profile that is located on the "activity-related payoff frontier." As the players' payoffs from the activity are additively separable from their payoffs from other parts of the interaction, the activity-related payoff frontier is well defined and can be fully constructed in isolation from other elements of the contracting environment. Separable activities for which the corresponding payoff frontier is convex are the main object of interest of this paper. We refer to such activities as "convex separable activities." The agent's periodic wage and task-specific effort in the above model of random opportunities are examples of convex separable activities.

We are now ready to describe the main result of the paper. We present an intuitive condition on convex separable activities, referred to as "Condition $\mathcal{D}$," that guarantees that, as time goes by, the levels of the activity shift in the direction that favors the agent in *any* optimal contract in *any* contracting environment. Furthermore, the condition is tight in that, for any activity that violates Condition $\mathcal{D}$, there exist contracting environments where, under the optimal contract, the activity levels change in the opposite direction over time.

To understand Condition $\mathcal{D}$, fix a convex separable activity and let x and y be two activity levels. Let $u(x)$ and $u(y)$ be the agent's activity-related payoffs when the activity is set to levels x and y, respectively. Next, let $D(x)$ and $D(y)$ denote the agent's *maximal* activity-related payoffs when the activity is set to x and y, respectively. For example, if the activity under consideration is the level of the agent's (costly) effort, a reasonable formulation is that for any "intended" level of the activity (i.e.,for any required effort), the agent's maximal activity-related payoff is zero and it is obtained by exerting no effort.[2] Intuitively, $D(x)$ and $D(y)$ capture the agent's activity-specific and level-specific incentives to deviate when the activity is set to the corresponding levels.

Now, let $A = u(x) - u(y)$ and $B = D(x) - D(y)$. Condition $\mathcal{D}$ is satisfied if, for all x and y, the corresponding magnitudes A and B do not have opposing signs, and the absolute value of A is (weakly) greater than that of B. Roughly speaking, this means that the agent's (activity-specific) profit from deviating is not too sensitive to changes in the activity levels. In many cases, Condition $\mathcal{D}$ is easy to verify. For instance, consider again the activity of the agent's (task-specific) effort in the model of random opportunities. Since $D(\cdot) \equiv 0$, Condition $\mathcal{D}$ is satisfied. According to our general result, as time goes by, effort does not increase in an optimal contract. Note that our result implies that this conclusion remains true even if we change the contracting environment, as long as the effort on that specific task is separable with respect to the environment. On the other hand, if changes in the intended levels of an activity create new potential deviations for the agent that disproportionately change the incentives

[2]This is the formulation we assumed in the model of random opportunities. In Section 4.2, we illustrate an alternative specification.

to deviate (e.g., a production change that requires a significant increase in the agent's access to organizational resources), Condition $\mathcal{D}$ will be violated.

The rest of the paper is organized as follows. Section 2 introduces the general contracting environment. Section 3 analyzes the optimal contract in a stylized model of random opportunities. In Section 4 we define and discuss the notion of an activity and in Section 5 we present and discuss our main results. In Section 6 we address the robustness of our result, and in Section 7 we present some of its applications. Section 8 offers a review of the related literature and Section 9 concludes. All proofs are relegated to the appendix.

2 Contracting Environment

We consider dynamic contracting settings that can be represented as follows. In each period $t \in \{1, 2, ..., T\}$, where $T \leq \infty$, the players observe a randomly drawn (strategic-form) game, take actions in the game, observe its outcome, and receive payoffs. The game of period t is drawn from a commonly known distribution $f(h_t)$, where the (public) history h_t contains the full description of the realized (periodic) games and the players' actions in periods $1, 2, ..., t-1$.

As the calendar time, previous realizations of periodic games, and the players' past moves may affect the periodic games the players will play in the future, this specification is fairly general. In addition to the contracting environments with symmetric information mentioned in the Introduction, this class of environments can accommodate a wide variety of scenarios, including, but not limited to, settings where the agent's cost of effort depends on past events, there is seasonality in demand, there is uncertainty about the principal's ability to provide compensation in the future, there are long-term (or storable) investment opportunities, or there are R&D-type investment opportunities that may change future production methods and costs.

A *contract* in this setting specifies, for every finite history, a suggested strategy profile for any game that may be played after that history. A contract is *incentive compatible* if, for every finite history and game that may be played after that history, the agent's suggested continuation strategy is a best response to the principal's continuation strategy specified by the contract. The players maximize (discounted) expected utility and use the same discount factor δ.

In order to study a particular setting, one needs to specify the stochastic process $f(\cdot)$. If the specification is sufficiently narrow, or "stylized," it may be possible to fully characterize the optimal contract. In Section 3, we offer an example of an analysis along these lines and study a model of random opportunities. The role of this analysis is twofold. First, it provides a transparent and easy illustration of the effect of exogenous fluctuations in task arrival on the dynamics of effort and wage. As such, the model is arguably interesting in its own right since it offers new insights into seniority in labor contracts. Second, and perhaps more importantly, it lays the groundwork for the more abstract exposition of the main concepts and results of the paper.

3 A Model of Random Opportunities

Consider an interaction where the set of possible (strategic-form) games is $\Gamma = \{G^1, ..., G^I\}$, and the probability of G^i being played after any history is $q_i > 0$, $\sum_{i=1}^{I} q_i = 1$. The set of the principal's and the agent's possible actions in every game in Γ is $[0, \infty)$, with generic elements denoted by u and e, respectively. The payoffs from the strategy profile $< u, e >$ in G^i are $u - e$ for the agent and $\pi_i(e) - c(u)$ for the principal, where $c(\cdot)$ is an increasing, strictly convex, and differentiable function for which $c(0) = 0$, and $\pi_i(\cdot)$, for all $i \leq I$, is an increasing, strictly concave, and differentiable function for which $\pi_i(0) = 0$. In addition, we assume that for all $i < I$ and $e \in [0, \infty)$, $\pi'_i(e) < \pi'_{i+1}(e)$, and that for all $i \leq I$, $\pi'_i(0) > c'(0)$ and $\lim_{e_i \to \infty} \pi'_i(e_i) < \lim_{u \to \infty} c'(u)$.

A possible interpretation (to which we adhere throughout the section) is that the principal offers a periodic wage to incentivize the agent to exert costly effort on randomly arriving tasks (or short-lived "opportunities"). The game G^i is played in periods in which a task of type i is available. In every period, u and e represent, respectively, the agent's utility from compensation and (the cost of) effort.[3] The marginal cost of compensating the agent is increasing. This may reflect, for example, the agent's decreasing marginal utility from a nominal wage. For reasons that will become clear later on, we measure compensation directly in terms of the agent's "utils" and assume that the principal's cost of compensation $c(u)$ is convex. The marginal productivity of effort on all tasks is taken to be decreasing. Moreover, we assume that tasks can be ordered with respect to the marginal productivity of effort. Since $\pi_i(0) = 0$ for all i, this

[3]The assumption that $u \geq 0$ reflects a type of limited liability condition for the agent.

ordering assumption means that tasks are also ordered with respect to the total productivity of effort. Accordingly, we say that task i is *better* than task j if $i > j$.

3.1 The Phase Mechanism

We construct a particular contract, referred to as the "phase mechanism" (henceforth PM), that we will later prove to be uniquely optimal.[4] In brief, PM consists of multiple hierarchical phases. Within each phase, the agent enjoys a fixed periodic compensation and, whenever a task of a given type arrives, he is instructed to exert the same (type-specific) level of effort. The contract transitions to a new phase upon the arrival of a task that is better than all previously available tasks. Thus, the contract moves monotonically (perhaps with jumps) through the phases until the final absorbing phase is reached.

The key qualitative property of PM is that when the contract changes phases, the agent's periodic compensation increases and the task-specific effort requirements decrease. Consequently, as time goes by, both the periodic compensation and the effort exerted on every type of task shift monotonically in the direction that favors the agent. This qualitative result offers new insights into the phenomenon of seniority that is common in many workplaces. In particular, it shows that when a worker's tasks arrive stochastically over time, his inability to commit leads to a dynamics that generates seniority in both of these key features (wage and effort) of an employment contract.[5]

Auxiliary Problems. — The crux of the argument in the characterization of the optimal contract is to show that whenever a best-to-date task becomes available, there should be no debt carried over from the past (i.e., the agent's continuation utility should be zero). Given this insight (which we establish below), the whole interaction can be split into parts that can be analyzed separately. We now define I auxiliary problems that constitute the building blocks

[4] Up to a redundant multiplicity that exists off the path of play.

[5] This is in contrast to other explanations in which seniority arises as a means to solve a specific problem, e.g., fluctuations in outside offers: Harris and Holmström (1982) and Holmström (1983); asymmetric information: Lazear (1981), Carmichael (1983), Milgrom and Roberts (1988), and Prendergast and Topel (1996); relationship specific investment: Becker (1962), Parsons (1972), and Topel (1991); and collective bargaining constraints: Blair and Crawford (1984) and Abraham and Farber (1988).

of the optimal contract.

For $i \in \{1, 2, \ldots, I\}$, let $P^{(i)}$ (referred to as "auxiliary problem i") denote the principal's optimization problem in an auxiliary setting where:

(1) In the first period, G^i is played with certainty.

(2) The interaction ends upon the arrival of a task that is better than i. The period when that task arrives is referred to as the "last period" of the interaction.

(3) The principal is restricted to selecting a stationary effort vector $(e_j)_{j \le i}$ and a fixed compensation u such that e_j is the required effort in game G^j, and the selected periodic compensation u is granted from the *second* to the last period of the interaction, inclusive.

For a contract to be incentive compatible in this auxiliary setting, the agent's cost of effort on the currently available task must not exceed the expected discounted difference between future compensation and the cost of future effort. Since the arrival of tasks is i.i.d. and the principal is restricted to contracts as specified in point (3), the incentive compatibility constraint for game G^j in $P^{(i)}$ is

$$e_j \le \frac{\delta}{1 - \delta \lambda_i} (u - \sum_{k \le i} q_k e_k), \qquad (IC_j^{(i)})$$

where $\lambda_i = 1 - \sum_{k > i} q_k$ is the probability that a task better than i does not arrive in a given period.

Formally, $P^{(i)}$ can be defined as

$$\max_{u, (e_j)_{j \le i}} \pi_i(e_i) + \frac{\delta}{1 - \delta \lambda_i} (\sum_{j \le i} q_j \pi_j(e_j) - c(u)),$$

such that $IC_j^{(i)}$ holds for all $j \le i$.

Notice that $P^{(i)}$ is a convex optimization problem; thus it has a unique solution, which we denote by $< u^{(i)}, (e_j^{(i)})_{j \le i} >$. We now derive several important properties of the solutions of the auxiliary problems; we will later use these properties to establish the optimality of PM.

Lemma 1. *The only binding constraint in the solution of $P^{(i)}$ is $IC_i^{(i)}$.*

The intuition behind this lemma is as follows. In optimum, at least one constraint must be binding. Suppose that there is $j < i$ for which $IC_j^{(i)}$ is binding. As all the incentive constraints of $P^{(i)}$ are the same on the RHS, we get $e_j \geq e_i$. This implies that the marginal return on tasks of type j is strictly lower than the marginal return on tasks of type i. As task i is both better than j and available in the initial period, it is profitable to marginally increase e_i and decrease e_j.

By Lemma 1, the solution of $P^{(i)}$ can be obtained by maximizing the following Lagrangian function:

$$\max_{u,(e_j)_{j\leq i}} \pi_i(e_i) + \frac{\delta}{1-\delta\lambda_i}\left(\sum_{j\leq i} q_j\pi_j(e_j) - c(u)\right) - \mu_i\left(e_i - \frac{\delta}{1-\delta\lambda_i}(u - \sum_{j\leq i} q_j e_j)\right)$$

Lemma 2. *For all $j \in \{1, ..., i\}$, $\pi_j'(e_j^{(i)}) \leq c'(u^{(i)})$, with equality if $e_j^{(i)} > 0$.*

This lemma, which follows directly from the FOCs of the Lagrangian, stipulates that the marginal cost of periodic compensation is equal to the marginal productivity of effort from every implemented task. This condition may seem more obvious than it actually is. It would not hold if, for example, the auxiliary problem did not assume availability of its best admissible task in the initial period, or if compensation were offered from the first rather than from the second period onwards.

The next lemma, which underpins the main result of this section, ranks the agent's periodic compensation in the different auxiliary problems.

Lemma 3. *Let $i > i'$; then $u^{(i)} > u^{(i')}$.*

To see why this is so, suppose that $u^{(i+1)} \leq u^{(i)}$ and note that the combination of Lemma 2, the concavity of $\pi_j(\cdot)$, and the convexity of $c(\cdot)$ would then imply that $e_j^{(i+1)} \geq e_j^{(i)}$, for all $j \leq i$. Now, consider the continuation of the interaction in auxiliary problem $P^{(i+1)}$ that begins with the arrival of a task of type i. Compared to the solution of $P^{(i)}$, from that period to the first arrival of a task of type $k \geq i+1$, the agent provides weakly more effort on all tasks, and receives a weakly lower periodic compensation. By Lemma 1, none of the constraints $IC_j^{(i+1)}$, $j \leq i$, is binding in the solution of $P^{(i+1)}$. Therefore, a periodic compensation strictly lower than $u^{(i)}$ would suffice to incentivize (weakly) more effort than $\{e_j^{(i)}\}_{j=1}^{i}$ until the first arrival of a task of type $k \geq i+1$. This contradicts the optimality of the solution of $P^{(i)}$.

The combination of Lemmas 2 and 3 shows that higher effort is exerted on better tasks within each auxiliary problem, and that, for every type of task that is incentivized in two different auxiliary problems, the effort on tasks of that type in the higher-indexed problem (one that starts with a better task) is lower than that in the lower-indexed problem.

Corollary 1. *Let* $j \leq i$*,*
1. For $j > 1$*,* $e_j^{(i)} \geq e_{j-1}^{(i)}$*, with a strict inequality whenever* $e_j^{(i)} > 0$*,*
2. For $i < I$*,* $e_j^{(i)} \geq e_j^{(i+1)}$*, with a strict inequality whenever* $e_j^{(i)} > 0$*.*

Definition of the Phase Mechanism. — Denote by $\mathscr{I}(h_t)$ the index of the best task that has been available at least once h_t (for the null history set $\mathscr{I}(\emptyset) = 0$). For every h_t and G^i, let $r(h_t, i) = e_i^{(max\{i, \mathscr{I}(h_t)\})}$ denote the required effort if G^i is played after h_t. The principal commits to the following strategy: after $h_t \neq \emptyset$ that is consistent with $r(\cdot, \cdot)$, the compensation is $u = u^{(\mathscr{I}(h_t))}$; after all other histories, the compensation is zero.[6]

Proposition 1. *PM is the unique optimal contract.*

Even though task arrival is i.i.d. across periods, the terms of the contract offered to the agent exhibit a seniority-based dynamics. The relationship between the principal and the agent can be described using the metaphor of a ratchet that allows advancement only in the direction that favors the agent. That is, for any realization of task arrival, the periodic wage and effort exerted on every type of task are given by monotonic step functions. When the periodic wage or effort requirements are updated, they jump to a new level where they stay until the next stochastic event causes another jump.

It is worth noting that if the concavity and convexity assumptions are replaced with their weak versions, PM remains an optimal contract, albeit not necessarily the unique optimal contract. In the extreme case of a linear cost of compensation, in addition to PM, a trivial optimal contract exists where, upon observing a desired effort, the principal fully compensates the agent in the following period. While this contract seems natural in the case of a linear cost of compensation, it cannot be approached as a limit of optimal contracts

[6] A history h_t is consistent with $r(\cdot, \cdot)$ if, along that history, the agent's effort equals the one specified by $r(\cdot, \cdot)$ in every period.

where the cost of compensation is strictly convex. It is the strict convexity of the cost of compensation that creates the intertemporal link in this environment.

Sketch of Proof. — It is easy to see that PM is an incentive-compatible contract under which, whenever a best-to-date task arrives, the agent's continuation utility is zero. We show that any local modification of PM is detrimental. This is sufficient for determining the global optimum since the general optimization problem is convex. We start by showing that modifications "within a phase" reduce value for the principal. To do so, we show that the solutions characterized by Lemma 2 remain the only optimal ones even when the restrictions imposed on the principal's strategy in the auxiliary problems are lifted.

To show that modifications "across phases" are detrimental we first argue that PM is the unique optimal contract among all contracts where the agent's continuation utility is restricted to be zero whenever a best-to-date task arrives. We then show that dispensing with this restriction does not introduce new contracts that outperform PM. To develop some intuition, consider a modification that becomes possible when the restriction is removed: the principal can now incentivize more effort in an early phase in return for a higher compensation in a later phase. This, however, would reduce the value for the principal as, by Lemma 2, within each phase, the marginal productivity of effort equals the marginal cost of compensation and, by Lemma 3, the marginal cost of compensation increases across phases. This example is, of course, only one out of many modifications that have become possible after dispensing with the restriction; the principal can shift compensation between phases while leaving the required efforts unchanged, shift effort without changing the compensation, or any combination thereof. However, since the agent's continuation utility can never drop below zero (due to IC constraints), *all* such modifications are essentially similar to the one illustrated above. It thus follows that PM is the unique optimal contract.

4 Separable Activities

We now return to our general environment. In this section, we develop and discuss extensively the notion of separable activities, which are components of an interaction over which the players have opposed preferences that satisfy certain separability conditions and can be measured in unidimensional units. In the next section we identify a tight condition on the primitives of a separable activity that guarantees that in any optimal contract the level of the activity over time changes only in the direction that favors the agent.

Before we present our reduced-form definition, it is useful to motivate our modeling approach by an informal discussion and illustration. A possible (non-reduced-form) way to model a particular activity A is to specify a simultaneous-move game G_A whose outcome determines the players' activity-related payoffs. The game G_A can be thought of as a ready-to-use "add-on" that contains all the relevant information about the activity A that, in principle, could be added to any contracting environment.

The activity A is *separable* with respect to a given (dynamic) interaction if there are histories after which the players may face a periodic game that can be decomposed into G_A and some other game that corresponds to the rest of that period's interaction[7] such that (1) the periodic payoff of each player is the sum of his payoffs in the two games, and (2) the outcome of G_A does not affect the distribution of periodic games to be played in the future.

We illustrate this decomposition using the model of random opportunities from the previous section. Consider the game G^i (which is played when a task of type i is available). This game can be artificially decomposed into two "degenerate" games, G_e^i and G_u^i, that represent different activities: in the former, the agent chooses an effort level and the principal is inactive, while in the latter, the principal chooses a compensation level and the agent is inactive.

An outcome of G_e^i specifies an effort e and the corresponding payoffs to the agent and principal, namely, $-e$ and $\pi_i(e)$, from this particular component of G^i, namely, the *activity* of the agent's effort on a task of type i. This activity is part of the periodic interaction only in periods when a task of type i is available. Similarly, the outcome of G_u^i determines a level of compensation u and the players' payoffs, u and $-c(u)$, generated by the activity of compensating the agent. This activity is identical for all games G^i because, by assumption, all aspects of compensation provision do not depend on the identity of the available task. We can therefore drop the index i from G_u^i and identify the activity of compensating the agent by G_u.

[7]This second game need not be identical for all occurrences of G_A.

In addition to the players' payoffs from each possible level of periodic wage and effort, the games G_u and G_e^i specify what payoffs the players can obtain from deviations. In G_u, only the principal's action set contains more than one action. Since he is committed to a long-term strategy, the details of G_u are important only as far as the principal's ex-ante considerations are concerned.

The situation is different in the game G_e^i, where the action set of the agent (the player who lacks commitment power) contains more than one element. When the periodic game G_e^i is considered in isolation, the action that gives the agent his *maximal* payoff is to exert zero effort. In an incentive-compatible contract, the agent finds it optimal to select the level of effort suggested by the contract, and the incentives for him to do so this come from other parts of the interaction. The agent's maximal activity-related payoff as a function of the intended level of the activity will play a key role in our characterization.

The agent's periodic wage and task-specific effort are among the simplest possible examples of separable activities. First, since G_u^i and G_e^i are single-player decision problems, the levels of these activities are unilaterally determined by one of the players. Second, every outcome of each game can be described as a level of the activity. However, many activities do not satisfy these two properties. Consider, for example, a variant of the model of random opportunities where, occasionally, there is an opportunity for joint production from "labor" (or effort) provided by the agent and "capital" provided by the principal. Assume that the production process works as follows. First, the principal provides the agent with the capital needed to meet the intended production level, and then, the agent decides whether to supply the appropriate labor or to reallocate the capital to alternative uses for his private benefit.[8]

The game underlying this activity is a nondegenerate two-player game in which the agent's maximal activity-related payoff depends on the principal's action. Furthermore, a deviation by the agent creates a discrepancy between the capital provided by the principal and the capital used for production by the agent. Thus, a deviation induces an outcome that is very different in nature from outcomes that can be simply described as a possible production *level* when production is performed correctly. This demonstrates that a formal definition of an activity as a fully specified game must separate the outcomes of the proposed game into outcomes that correspond to *activity levels* (e.g., the good was

[8]This is exactly the production process described in Albuquerque and Hopenhayn (2004).

produced), and outcomes where the activity is not properly used (e.g., the agent stole the capital).[9]

4.1 Reduced-Form Approach

Decomposing the complex game played by the players after each history into a fully specified game that represents the activity and a component that corresponds to the rest of the interaction is cumbersome. The additional need to further define the activity within the isolated game makes such a direct approach even less appealing. Thus, as our sole objective is to emphasize the dynamic monotonicity in the use of an activity, we can follow a "reduced-form" approach.[10] Our definition will specify an ordered set of activity levels (e.g., possible effort levels), the players' payoffs at every intended level, and the agent's maximal activity-related payoff at every intended level (e.g., exert no effort or enjoy the stolen capital). We will then impose certain separability conditions between the activity and the rest of the contracting environment.

Formal Definition. — An *activity* (L, u, κ, D) consists of an interval $L \subset \mathbb{R}$, two continuously increasing functions $u, \kappa : L \to \mathbb{R}$, where $u(l)$ and $\kappa(l)$ are, respectively, the agent's utility and the principal's *cost* associated with the activity level $l \in L$, and a function $D : L \to \mathbb{R}$ that specifies the supremum of the agent's activity-related payoff at every intended level $l \in L$.

In applications, it is often natural to use ad-hoc activity-specific units to measure an activity (e.g., quantity of good produced, duration of work, amount of compensation paid, etc...). To abstract away the activity-specific details, it is useful to reduce all of the activities to a common denominator.

A *standardized* activity (L, κ, D) is an activity that is measured in terms of the agent's utility from the activity, that is, an activity for which $u(l) = l$. Note that every activity can be expressed in standardized form.

For example, in the model of random opportunities, the agent's periodic wage is measured in agent's utils and hence it is an example of a standardized activity: the set of possible levels is $[0, \infty)$, $\kappa(\cdot)$ coincides with $c(\cdot)$, and $D(\cdot)$ is the identity function since there are no possible deviations for the agent within

[9] In Appendix B we provide a non-reduced-form definition of an activity.

[10] This approach is akin to the modeling strategies used in Ray (2002) and Albuquerque and Hopenhayn (2004).

the activity. By contrast, the agent's task specific effort is measured in the *cost* of effort, hence to standardize this activity one needs to look at the *negative* of effort. Thus, for task i, in the standardized form: the set of possible levels is $(-\infty, 0]$, $\kappa_i(l) = -\pi_i(-l)$,[11] and $D(\cdot) \equiv 0$ as the most profitable deviation for the agent within the activity is to exert no effort.

The advantages of reducing all possible activities to the same common denominator are analogous to the advantages of transforming a general normal distribution into a standard normal distribution: seemingly unrelated activities can be considered and compared using the units of measurement. In what follows, all general results will be stated in terms of standardized activities; when there is no risk of confusion, the qualifier "standardized" will be dropped.

The next definition specifies when an activity is separable with respect to a particular contracting environment. It is useful to note that the conditions in the definition are unit-free, and so they apply equally to standardized and non-standardized activities. Thus, one does not need to express an activity in its standardized form in order to check its separability.

Separability Conditions. — Fix a contracting environment $f(\cdot)$. A given activity is *separable* with respect to $f(\cdot)$ if:

(1) For every two histories h_t and $\hat{h}_t$ that differ *only* in the selected levels of the activity, $f(h_t) = f(\hat{h}_t)$.

(2) In periods in which the activity is available, the action set of each player can be written as a cross product of the actions related to the activity and the actions that are unrelated to the activity.

(3) The payoff from the activity-related component is additively separable from all other components of the interaction.

The first two conditions state that altering the level of the activity in a given period has no impact on the environmental terms of the interaction: the first condition states that changing the level of the activity does not affect the stochastic process of future periodic games, and the second condition states that a changing the level of the activity does not force a player to change his actions related to other components of the interaction. These conditions, together with the condition that payoffs are additively separable (condition 3), enable us to analyze the optimal use of a separable activity while imposing minimal struc-

[11] Recall that $\kappa_i(\cdot)$ is the principal's cost, while $\pi_i(\cdot)$ is his profit.

ture on $f(\cdot)$. Such an approach allows us to derive qualitative results over the activity regardless of the exact details of the contractual environment that contains the activity.

In the model of random opportunities the agent's periodic wage and effort on every type of task are separable activities. However, in that model the availability of activities is i.i.d., and hence the degree of separability between the activities and the environment is considerably stronger than that which satisfies our separability conditions. While the possibility of affecting the future environment through the selection of specific levels of the separable activity is ruled out, the separability conditions are compatible with contracting environments in which the *availability* of the separable activity is determined endogenously by past actions or affected by previous realizations of the periodic games.

As the players are assumed to have opposing preferences regarding the activity levels, a separable activity can be regarded as a method of transferring utility between the players. The main object of interest in the present paper is the class of separable activities for which the payoff frontier is convex. For standardized activities, this means that $\kappa(l)$ is a strictly convex function. Such activities are referred to as *convex separable activities.*

4.2 Additional Examples of Separable Activities

In this section, we offer additional examples of separable activities to emphasize the richness and the potential applicability of the notion. An important part of the discussion will be devoted to how the activities may differ in their functions $D(\cdot)$. Our objective here is not to advocate a specific form of $D(\cdot)$, but rather to show the variety of such functions that can arise naturally in applications.

Recall that in our model of random opportunities, $D(\cdot)$ is constant at zero for effort, and it is the identity function for periodic wage. These two examples of $D(\cdot)$ will turn out to be important bounds in our main result. The following examples will demonstrate that there exist activities for which the function $D(\cdot)$ falls within these bounds and activities for which this is not the case.

Consider the activity of the agent's daily working hours. Every day, an agent chooses how many hours to spend in the office and, while at work, he chooses between *working* and *wasting time*. Moreover, consider the typical case in which spending time at the office is costly for the agent, yet wasting time is less costly than working. In general, the amount of time the agent spends at the office is accurately and objectively measured; thus, if the agent stays at home (or leaves work early) it is a verifiable breach of contract that allows the principal to punish the agent. By contrast, even if the principal can observe whether the agent is wasting time or working, it is unlikely that he can prove this in court. If the punishment for breaching the contract is severe, the action that gives the agent his maximal activity-specific payoff is to arrive at work and waste time. Since wasting time only partially saves the cost of working, in the standardized form of the activity we obtain an increasing function $D(\cdot)$ with $D(l) \in (l, 0)$, where the upper-bound utility of 0 corresponds to the agent's utility from working zero hours.

Activities in which complex functions $D(\cdot)$ arise even more naturally are those that are jointly controlled by both players. Formally, these are activities that correspond to nondegenerate games where the set of possible actions for each player contains at least two elements. We refer to these activities as "joint-production" activities. For such activities, the agent's payoffs depend not only on his but also on the principal's actions. We now illustrate how the agent's payoff from optimal deviation may depend on the intended production levels, using traditional complement/substitute production technologies.

Consider again the example of production from labor and capital. First, the principal provides y units of capital to the agent, who then decides whether to combine the capital with x units of labor (to produce the good as intended), or to reallocate the capital to alternative uses for his private benefit. Assume that both the marginal benefit from misusing the capital and the marginal cost of labor equal one. The latter assumption implies that the standardized activity levels are measured by the negative of the agent's labor: $l(x) = -x$.

To illustrate the case of substitute inputs, suppose that the production function is $x+\alpha y$, and that there is a production target of one unit. Thus, the amount of capital provided by the principal as a function of labor is $y(x) = \frac{(1-x)}{\alpha}$, and it follows that $D(l) = \frac{1+l}{\alpha}$. In particular, the function $D(l)$ is *increasing*: an increase in l is equivalent to a decrease in x, which leads to an increase in y and a larger potential benefit from deviating. Whether the slope is above or below 1 depends on α. Alternatively, for the case of complementary inputs, let the production function be $min\{x, \alpha y\}$. In this case, $y(x) = \frac{x}{\alpha}$, and $D(l) = -\frac{l}{\alpha}$ is a decreasing function.

5 Main Result

We define the following condition on a *standardized* activity (L, κ, D).

Condition $\mathcal{D}$.

$$For\ all\ l' < l'' \in L, \quad 0 \leq \frac{D(l'') - D(l')}{l'' - l'} \leq 1.$$

The condition, which we state in terms of the slope of the function $D(\cdot)$, has an alternative intuitive representation: Condition $\mathcal{D}$ is satisfied if and only if the agent's maximal attainable payoff, $D(l)$, is a (weakly) increasing function and his *incentive to deviate*, $[D(l) - l]$, is a (weakly) decreasing function. The discussion in Section 4.2 offered examples of activities that satisfy Condition $\mathcal{D}$ and of those that do not satisfy it (either the right or left inequality of). Note that the activities in the model of random opportunities satisfy Condition $\mathcal{D}$ on the boundaries: for wage, the slope $D'(l) \equiv 1$ (this property is common to all activities that are unilaterally controlled by the principal), and for effort, $D'(l) \equiv 0$.

In many cases, it is not necessary to express an activity in its standardized form in order to verify Condition $\mathcal{D}$. A convenient way to think about the condition for a nonstandardized activity is as follows. Consider two possible levels l^1 and l^2 of a given activity. Two magnitudes need to be compared: (i) the difference between the agent's payoffs from activity levels l^1 and l^2, and (ii) the difference between the agent's maximal attainable payoffs when the activity is set at levels l^1 and l^2. The condition is satisfied if, *for any two levels*, the values of (i) and (ii) do not have opposing signs, and the *absolute* value of (i) is larger than that of (ii). This alternate expression shows directly that the agent's (nonstandardized) effort on every type of task in the model of random opportunities satisfies Condition $\mathcal{D}$.

Specifications that satisfy Condition $\mathcal{D}$ have been used in the past to derive monotonicity results. The activity of wage/transfer that is the focus of the wage dynamics literature (e.g., Holmström 1983) and risk sharing literature (e.g., Marcet and Marimon 1992) is unilaterally controlled by the principal and thus, as we mentioned before, it satisfies the condition.[12] Hence, the assumption the Condition $\mathcal{D}$ holds remains implicit. Two more recent papers make this assumption explicit. Albuquerque and Hopenhayn (2004) study entrepreneur financing and impose a bound on the change in the liquidation value of the firm (to the entrepreneur) due to an increase in the firm's capital. In a more abstract setting, where a deterministic contracting problem is faced repeatedly and the principal can only commit to the current period's actions, Ray (2002) requires that increasing the transfer the agent receives has a greater impact on his utility if he stays on the job than if he quits.

We now turn to our main result. Fix a contracting environment and an incentive-compatible contract therein. We say that an activity A is *nondecreasing* under the contract if, from the ex-ante perspective, there is a zero probability of observing two periods $t_1 < t_2$ in which A is available and the level of A in t_1 is strictly higher than that in t_2.

Theorem 1. *Let A be a standardized convex activity that is separable with regard to $f(\cdot)$. If A satisfies Condition $\mathcal{D}$, then A is nondecreasing under any optimal contract.*

The powerful implications of Theorem 1 can be illustrated already in the environment of our simple model of random opportunities. The theorem significantly generalizes the key qualitative results that we obtain under arguably restrictive assumptions by solving directly for the optimal contract. In particular, even if arrival of tasks were not i.i.d. and the tasks could not be ranked according to the marginal productivity of effort, the task-specific effort would still decrease, and wage would still increase, over time. Furthermore, this holds even if the players were able to affect the arrival of tasks endogenously. This result would be all but impossible to derive directly by fully characterizing the optimal contract for such complex environments.

More generally, Theorem 1 unifies and generalizes a large class of results that show that an employee's wage rises over time when there are fluctuations in the value of his outside option. See, for example, Harris and Holmström (1982), Holmström (1983), Postal-Vinay and Robin (2002a,2002b), and Burdett and Coles (2003) for exogenous fluctuations, or Shi (2009) for endogenous

[12]For all such activities, $D'(\cdot) \equiv 1$.

ones.[13] To derive this result, these papers specify a full-blown model of the labor market, that embeds fluctuations in the worker's outside option, and then use the specific structure to obtain the downward rigidity of wage directly. This standard approach is inherently limited as it only derives the downward rigidity of wage for the particular, and oftentimes narrow, specification under consideration. By contrast, our result establishes the downward rigidity of wage for any environment where the separability conditions hold. Therefore, fluctuations in the agent's outside option (or business conditions in general) generically lead to increasing wages over time.

Finally, the generality of the framework developed in this paper enables us to draw connections between seemingly unrelated monotonicity results. For example, Marcet and Marimon (1992), Krueger and Uhlig (2006), and Grochulski and Zhang (2011) study dynamic risk sharing, Forand and Zápal (2019) study dynamic project selection, and the papers mentioned in the previous paragraph study wage dynamics. The object of interest in each of these papers is a convex separable activity[14] that satisfies Condition $\mathcal{D}$, and thus Theorem 1 delivers the qualitative monotonicity results derived directly in all of these papers.

Intuition behind Theorem 1. — The next discussion uses a simple example to illustrate the intuition behind Theorem 1 in an informal way. Consider a four-period contracting problem where the agent "works" in periods 1 and 3, and the principal compensates him in periods 2 and 4. The agent can quit the contract at any time, his outside option is zero, and the players do not discount the future.

Let x_1 and x_3 be the (exogenously given) effort levels the agent must provide in periods 1 and 3, respectively. Assume first that compensation is provided via the convex activity of paying a "periodic wage" (as in Section 3). The cheapest

[13] Although many of these papers study general equilibrium models, their settings can be easily adapted to our framework. Competition over the worker can be incorporated into the model by assuming that occasionally the agent has a "quit" action that: 1) provides him with an immediate payoff equal to the value of his outside offer and 2) after this action is used both players have a payoff of zero in all future games.

[14] In Forand and Zápal (2019) projects can be thought of as *weakly* convex separable activities (that is, activities where κ is only weakly convex) that satisfy Condition $\mathcal{D}$. Therefore, a small adaptation of Theorem 1 to weakly convex cost functions would imply that there exists an optimal contract in which each project is used monotonically over time and all shifts are in the direction that favors the agent. The strict convexity of $\kappa(\cdot)$ is only required in order to establish that this occurs in every optimal contract.

way to compensate the agent for his total effort $(x_1 + x_3)$ is to pay him half of that amount in periods 2 and 4. If $x_1 \geq x_3$, this compensation plan satisfies all incentive constraints and is thus uniquely optimal. However, if $x_1 < x_3$, this form of compensation is not incentive compatible in period 3. To restore incentive compatibility (without increasing the total compensation), the principal must shift some compensation from period 2 to 4. A decreasing compensation plan (where the wage in period 4 is strictly lower than that in period 2) is suboptimal for all x_1 and x_3. An alternative way to frame the argument is as follows. Suppose that a decreasing compensation plan is proposed. Reducing the compensation in period 2 by a small amount and increasing it in period 4 so that the total remains the same decreases the overall cost of compensation (this is a basic smoothing argument). If the original compensation plan was part of an incentive-compatible contract, then, a fortiori, so is the modified compensation plan: back-loading compensation can only *relax* some incentive constraints of the forward-looking agent. This simple argument relies on the implicit assumption that changing the level of compensation in a given period does not create new deviation opportunities for the agent. Formally, this is reflected in the property $D'(l) = 1$, since this form of compensation is unilaterally controlled by the principal.

Assume now that compensation in our example is provided via a more complex convex activity under which the agent's deviation options vary with the level of compensation. Again, start with a decreasing and incentive-compatible compensation plan, and consider the same smoothing modification as before. What is now unclear is whether the modification leads to an incentive-compatible contract. Condition $\mathcal{D}$ guarantees that this is indeed the case. To see this, we now analyze the agent's considerations in periods 2 and 4:

Period 2: The compensation in this period is lowered. By Condition $\mathcal{D}$, the function $D(\cdot)$ is nondecreasing, and so the agent's maximal payoff in case of deviation does not increase. The compensation schedule is modified in a way that the agent's continuation payoff from following the contract does not change. Thus, it remains optimal for the agent to adhere to the terms of the (modified) contract in period 2.

Period 4: The compensation in this period is increased, and so the agent's payoff from following the contract is now higher. The other part of Condition $\mathcal{D}$ states that the total increase in $D(\cdot)$ is bounded by the increase in the agent's payoff from compensation in period 4. Hence, the modified contract is incentive compatible in period 4 as well.

Tightness of Condition $\mathcal{D}$

The characterization in Theorem 1 is tight in the sense that, whenever Condition $\mathcal{D}$ is violated, there exist contracting environments in which the level of the activity shifts in the principal's favor over time. Condition $\mathcal{D}$ stipulates that a small change in the level of the activity does not have a large impact on his deviation payoff (since the slope of $D(\cdot)$ is bounded). A natural class of activities that violate Condition $\mathcal{D}$ are those for which $D(\cdot)$ is not continuous. For example, consider the joint production example described above, and assume that the principal can only assign capital in discrete increments. Due to the usual potential nonexistence problems, a general treatment of activities with discontinuous $D(\cdot)$ involves technicalities and notation that do not add substance. Therefore, in the main text we circumvent this problem and consider the case where $D(\cdot)$ is continuous. In Appendix C we generalize this result and show that if an activity does not satisfy Condition $\mathcal{D}$, there exist contracting problems in which the activity is decreasing in all approximately optimal contracts.[15]

To state the converse to Theorem 1, we need the following definition. Fix a contracting environment and an incentive-compatible contract therein. We say that an activity A is *decreasing* under the contract if, from an ex-ante perspective, there is a probability of one of observing two periods $t_1 < t_2$ in which A is available and the level of A in t_1 is strictly higher than that in t_2.

Theorem 2. *Let A be a standardized convex activity with a continuous $D(\cdot)$ that violates Condition $\mathcal{D}$. There exist contracting environments with respect to which A is separable and in which A is decreasing in any optimal contract.*

To better understand this result we revisit the joint-production activity discussed in Section 4.2. For such activity the agent's incentive to deviate depends on the amount of capital he is entrusted with. However, different production functions lead to different relations between the the capital under the agent's control and the standardized level of the activity, and hence to different slopes of $D(\cdot)$. We consider two different production functions—one for each type of violation of Condition $\mathcal{D}$—and, for each function, construct a counterexample by choosing appropriate compensation opportunities. In both examples we assume that, in addition to the periodic incentive compatibility constraints, the

[15]We use the standard notion of ϵ-optimality. A formal definition is given in the appendix.

agent can reject the contract at time zero, and that the agent and principal do not discount the future.

Pay at the End. — For the case where $D(\cdot)$ is decreasing, suppose that the production function is $\min\{x, \frac{y}{2}\}$, where, as before, x stands for the agent's effort and y is the capital provided by the principal, and assume that there are production opportunities in periods 1 and 2. To further simplify the example, suppose that the principal has very limited discretion on how to provide compensation to the agent: the principal can only decide whether to pay the agent a compensation of 1 util at the end of the interaction (period 3). We can express the capital provided by the principal as a function of the agent's effort by $y(x) = 2x$. Assuming that the marginal cost of effort and the marginal utility from reallocating capital is 1, we obtain a standardized form where $L = (-\infty, 0]$ and $D(l) = -2l$.

The principal's objective is to find the optimal effort schedule (x_1, x_2). As the main focus of the present illustration is on the agent's incentives, we prefer not to add details on how to directly turn the activity into a convex one. Instead, we simply assume that the principal seeks to maximize the agent's *aggregate* effort and, among the profiles of the agent's effort (x_1, x_2) that add up to the same total, the principal prefers the one with the minimal difference between x_1 and x_2. Given this, the agent's effort in the optimal contract can be identified by solving the following linear programming problem:

$$
\begin{aligned}
&max \quad \{x_1 + x_2\} \\
&\text{subject to} \\
&(IC_0) \quad 1 - x_1 - x_2 \geq 0; \\
&(IC_1) \quad 1 - x_1 - x_2 \geq 2 \cdot x_1; \\
&(IC_2) \quad 1 - x_2 \geq 2 \cdot x_2,
\end{aligned}
$$

where IC_t is written such that the LHS corresponds to the agent's payoff from following the (continuation of) the contract from period t onward, and the RHS corresponds to the agent's payoff from (optimally) deviating in period t. The unique solution to this LP problem is $(x_1 = \frac{2}{9}, x_2 = \frac{1}{3})$. Therefore, the optimal contract has this *increasing* effort schedule.

To understand the intuition behind the example, observe that the agent's continuation utility at the end of the second period is 1, while his continuation utility at the end of the first period is $1 - x_2$. Therefore, the *loss* of continuation utility from deviating is greater in the second period than in the first period, which, in turn, implies that the activity-related incentive to deviate $(D(l) - l)$ can be greater in the second period than in the first period. Under the assumed production technology, this observation leads to an increasing effort schedule. It is worth pointing out that the same logic generalizes to more than two periods. In fact, the example can be extended to any number of periods during which the effort will gradually (and monotonically) increase until the compensation is paid upon full accomplishment of all tasks.

Carrot and Stick. — To see why a (standardized) separable activity for which the slope of $D(\cdot)$ is greater than one may be decreasing, consider the production function $x + \frac{y}{2}$ and suppose that there is an "output target" of 1 that must be fulfilled in each of the periods 1 and 3. As in the previous example, the principal can provide a compensation of 1 at the end of the interaction (period 4), but, in addition, in period 2 he can either offer a compensation of 1/2 (carrot) or impose a fine of 1 (stick). Under the assumed production technology, the principal's capital as a function of the agent's effort is $y(x) = 2(1 - x)$. Maintaining the same assumption on the marginal cost of effort and reallocated capital, we obtain a standardized form where $L = [-1, 0]$ and $D(l) = 2 + 2l$.

In this environment the *only* incentive-compatible contract has $x_1 = \frac{1}{2}$ and $x_3 = 1$, and hence the agent's effort *increases* over time in the optimal contract. To see this, note that the agent's period-3 continuation payoff from following the contract is $1 - x_3$, and his payoff from (optimally) deviating in period 3 is $D(l(x_3)) = D(-x_3) = 2(1 - x_3)$. Hence, a contract is incentive-compatible only if $x_3 = 1$. The agent's incentive compatibility constraint in period 1 and his participation constraint are given, respectively, by $1 + \frac{1}{2} - x_1 - x_3 \geq 2(1 - x_1) - 1$ and $\frac{1}{2} + 1 \geq x_1 + x_3$. These constraints and $x_3 = 1$, jointly imply that $x_1 = \frac{1}{2}$.

6 Robustness

When an activity does not satisfy Condition $\mathcal{D}$, then, by Theorem 2, there exist contracting environments where, over time, the activity shifts in the direction that favors the principal. A natural question that arises is whether activities that "almost" satisfy Condition $\mathcal{D}$ can admit arbitrary fluctuations, or whether only small changes in the principal's favor can be observed. It turns out that violations of the upper bound and lower bound of Condition $\mathcal{D}$ have an asymmetric impact on the possible dynamics of a separable activity. In particular, if the slope of $D(\cdot)$, of a standardized separable activity, is negative but close to zero, only small decreases in the activity can be observed under an optimal contract. By contrast, if the slope of $D(\cdot)$ exceeds one, there exist interactions in which large decreases in the level of the activity are observed.

For the robustness result, we need the following definition. Fix a contracting environment and an incentive-compatible contract therein. We say that an activity A is ϵ-*nondecreasing* under the contract if, from the ex-ante perspective, there is a zero probability of observing two periods $t_1 < t_2$ in which A is available and the level of A is higher in t_1 than in t_2 by more than ϵ.

Proposition 2. *Fix a domain of a standardized activity $\hat{L}$ and an increasing convex function $\hat{\kappa} : \hat{L} \to \mathbb{R}$. There exists a function $\epsilon : \mathbb{R}_+ \to \mathbb{R}_+$ for which $\lim_{c\to 0} \epsilon(c) = 0$ such that: if $(\hat{L}, \hat{\kappa}, D)$ is separable with respect to $f(\cdot)$, and for all $l' < l'' \in \hat{L}$, $\frac{D(l'')-D(l')}{l''-l'} \in [-c, 1]$, then the activity $(\hat{L}, \hat{\kappa}, D)$ is $\epsilon(c)$-nondecreasing under any optimal contract.*

To illustrate the intuition behind this result, fix $\hat{L} = [0, 1]$ and $\hat{\kappa}(l) = l^2$, and consider a parametrized family of functions $D_c(l) = 1 + c - cl$, for $c > 0$. This specification is convenient as $\frac{D_c(l'')-D_c(l')}{l''-l'} = -c$ for all $l'' > l' \in \hat{L}$. Furthermore, assume that the players do not discount the future and that the activity is available in periods 1 and 2.

Assume that, in an optimal contract, $l_1 > l_2$ where l_t is the level of the standardized activity $(\hat{L}, \hat{\kappa}, D_c)$ in period t. By a standard smoothing argument, decreasing the level of the activity in period 1 by a small ε and increasing it in period 2 by the same ε is profitable for the principal. Therefore, our assumption that $l_1 > l_2$ is part of an optimal contract implies that the aforementioned modification is not incentive compatible. Since the slope of $D_c(\cdot)$ is below 1, increasing the level of the activity in period 2 cannot create any incentive problems in that period. Therefore, the above modification must violate the incentive constraint in period 1.

Suppose that the incentive constraint in period 1 is binding. Reducing the activity by ε in period 1 has two effects. First, it reduces the agent's periodic utility (on the path of play) by ε; second, it *increases* the agent's payoff from deviating in period 1 by εc (due to the assumed form of $D_c(\cdot)$). Thus, increasing the level of the activity in period 2 by $(1 + c)\varepsilon$ restores incentive compatibility. We refer to the modification where l_1 is decreased by ε and l_2 is increased by $(1 + c)\varepsilon$ as an *ε-modification*. The principal's marginal profit from an ε-modification is $\hat{\kappa}'(l_1) - \hat{\kappa}'(l_2)(1+c)$. Plugging in the assumed function $\hat{\kappa}(l)$, and using the fact that $\hat{L} = [0, 1]$, leads to the conclusion that the marginal profit from the ε-modification is *positive* whenever $l_1 - l_2 > c$. Therefore, $l_1 > l_2$ can be consistent with optimality only if $l_1 - l_2 \leq c$.

While the above example has a very specific structure, the main part of the argument—by which smoothing out a decrease in the (standardized) activity can only violate the incentive-constraint in the *earlier* period—is general. This plays a fundamental role in Proposition 2, since it guarantees that the principal can always restore incentive compatibility by sufficiently increasing the level of the activity in the future. This, in turn, enables us to put an upper bound on the size of a decrease in the activity in an optimal contract by (roughly) comparing the marginal gain from smoothing with the marginal cost of increasing the total (discounted) use of the activity.

Proposition 3. *Let (L, κ, D) be a (standardized) convex activity, and assume that $\frac{D(l'')-D(l')}{l''-l'} \geq 1 + c$ for some $c > 0$, for all $l' < l'' \in \tilde{L} \subset L$. Then, there exist contracting environments with respect to which (L, κ, D) is separable and, over time, the level of the activity decreases by $|\tilde{L}|$ in all optimal contracts.*

When the slope of $D(\cdot)$ is greater than one, smoothing out a decrease in the level of the activity between two periods can only violate the incentive-compatibility constraint in the *later* period. Moreover, if the incentive-compatibility constraint in that period is binding, any attempt to smooth the decrease in the activity will violate the constraint. Consequently, if the principal is unable to provide incentives by other means, and later occurrences of the activity may not exist, large reductions in the level of the activity can be consistent with optimality.

7 Applications

Quality Provision over Time. — A long-term contract between a supplier and a client will generally specify the price, quantity, and quality of the goods that the supplier must provide in each period. Price and quality are easy to verifiably measure, and thus clauses related to these dimensions can be enforced by a court. The quality of the goods, however, is often harder to verifiably measure. Consider, for example, the interaction between a farmer and a client (e.g., restaurant, supermarket, ...) to whom he supplies produce. While the client can easily weigh the produce he receives, it is harder for him to prove that the quality of the produce is substandard. Hence, contractual clauses related to quality must often be incentive compatible.

To apply our result, we consider the dynamic interaction between a supplier and a client, and assume that the supplier is also active in a competitive spot market. The quality of the goods that the supplier provides to the client pins down his actions on the spot market, and hence the provision of quality to the client can be modeled as a convex separable activity. First, let us consider the case where the client is large relative to the supplier (e.g., a supermarket chain purchasing produce from a farmer) and thus it is natural to assume (perhaps due to reputational concerns) that the *client has commitment power*. In this case, if the activity of providing quality satisfies Condition $\mathcal{D}$, then, by Theorem 1, quality *decreases* over time.[16] Next, consider the case where the supplier is large (e.g., a farmer selling to a small restaurant) and thus (due to similar reputational concerns) the *supplier has the commitment power*. In this case, the activity is controlled unilaterally by the player with commitment and so Condition $\mathcal{D}$ holds, thus by Theorem 1 quality *increases* over time. Furthermore, if the quality distribution is subject to supply shocks (e.g., in drought years most of the produce is of low quality), the same monotonicity result would hold conditional on each realization of the supply shock.

Pushing our results to the extreme, we hold that this application may also provide insights into complex dynamic allocation problems. The literature on dynamic allocation problems (e.g., Gershkov and Moldovanu 2009, Bergemann and Välimäki 2010, Said 2012, Board and Skrzypacz 2016) generally assumes that it is the seller who has commitment power over all potential buyers, and hence he can design (and commit to) a dynamic allocation mechanism. However, alternative structures of relative commitment power may also be relevant. For example, imagine that it is the same supplier that is active in both of the interactions discussed above (i.e., a farmer deciding how to allocate the quality

[16]Condition $\mathcal{D}$ holds when the client's valuation of quality is lower than the valuation implied by the spot-market prices.

of produce between a restaurant and a supermarket chain) and thus one client has commitment power over the supplier, while the supplier has commitment power over the other client. In this example our results suggest that the supplier will shift quality from the large client to the small one as time goes by.

Power Allocation in Organizations. — The structure of a large firm is often described as a group of divisions that are directed by the central management (M-form model). Moreover, within large organizations it is well known that incentives are often provided via the reallocation of power rather than via monetary transfers (e.g., Cyert and March 1963, Agiohon and Tirole 1997, Dessein 2002, and Li, Matouchek and Poweel 2017). The "excess power," i.e., the power a division manager has beyond what is required for him to perform his job, can sometimes be represented as a separable activity, and, if power is allocated efficiently, as a convex one.

Our result shows that the evolution of a division manager's power is inherently related to his potential benefit from abusing his power. If the potential for abusive use of power is low, then increasing his power should have a small impact on his incentive to deviate and Condition $\mathcal{D}$ is likely to hold. Thus, Theorem 1 implies that the manager's power will increase over time. If, on the other hand, the potential for abuse of power is high, the manager can only be entrusted with a large amount of power when there are sufficient carrots and sticks (in the general interaction) to hold him in line. Hence, his power may increase when such carrots and sticks are available, and decline when they are not.

Foreign Investments. — An additional setting in which our result can provide general insights is that of foreign direct investment or entrepreneur financing à la Thomas and Worrall (1994) and Albuquerque and Hopenhayn (2004). In this setting an entrepreneur must borrow funds from a lender to operate a production facility in an environment with demand shocks. Moreover, the entrepreneur cannot be compelled to repay the loan and if he defaults he may be able to keep some of the working funds (needed to finance the periodic production) he received for that period. Thus, the dynamic financing plan must be incentive compatible.

The level of production/size of loan (conditional on the demand shock) constitutes a convex separable activity if the entrepreneur's profit is concave in the level of capital invested and the periodic profit satisfies the separability conditions. If the entrepreneur's payoff from defaulting satisfies Condition $\mathcal{D}$ our result implies that the level of production will increase over time regardless of the exact specification of the interaction.

The same monotonicity result is derived in Albuquerque and Hopenhayn (2004); however, their result is the consequence of the exact specification of their model.[17] They assume that at time zero the lender finances the project, such that at the start of interaction the entrepreneur owes a large debt to the lender and hence, has a low continuation utility. This, in turn, implies that, initially, periodic production must be low enough to keep the entrepreneur's incentive to deviate small. As time goes by, the initial debt is repaid and the entrepreneur's continuation utility increases, and hence larger and more efficient production levels become incentive compatible. If, however, the lender and entrepreneur have additional dealings that could cause the entrepreneur's continuation utility to vary nonmonotonically (e.g., financing other risky investments that could create a loss for the entrepreneur), Albuquerque and Hopenhayn's (2004) result does not hold.

8 Literature Review

The main motivation for this this paper is to understand the impact of environmental fluctuations in general contracting environments. Thus, this paper is related to papers that have analyzed the impact of such fluctuations in a specific environment. Möbius (2001), Hauser and Hopenhayn (2008), and Samuelson and Stacchetti (2017) study games where each player occasionally has an opportunity to grant a favor to his counterpart (at a cost to himself) and find that efficient equilibria have a cyclical element and hence are nonmonotone. Bird and Frug (2019) and Forand and Zápal (2019) apply a mechanism design approach to (different) variants of the model of random opportunities. Forand and Zápal (2019) allow for arbitrary arrival processes and assume that information is symmetric; they derive directly a particular manifestation of our main result. By contrast, Bird and Frug (2019) assume arrival is i.i.d., but that task availability is *privately* observed by the agent; they find that even though all components of the interaction are (weakly) convex separable activities, the agent's task-specific

[17] Albuquerque and Hopenhayn (2004) impose weaker assumptions than ours. Namely, they do not assume the second part of our separability condition, impose only a weaker version of Condition $\mathcal{D}$, and assume the (standardized) production activity is only quasiconvex.

effort may increase (compensation may decrease) over time under the unique optimal contract. Intuitively, when the agent privately observes task availability, his continuation utility must decrease if no tasks are implemented for a long time in order to incentivize him to reveal available tasks.

This paper derives a general monotonicity result that arises under optimal contracts in a wide range of economic settings. Certain manifestations of this result have been established directly in many specific environments that have been mentioned throughout the paper. Harris and Holmström (1982), Holmström (1983), Postal-Vinay and Robin (2002a, 2002b), Burdett and Coles (2003), and Shi (2009) analyze various types of labor markets and establish that an employee's wage does not decrease over time. Marcet and Marimon (1992), Krueger and Uhlig (2006), and Grochulski and Zhang (2011) study dynamic risk sharing and show that the transfer received by the insured does not decrease over time. Albuquerque and Hopenhayn (2004) show that, conditional on the demand shock, an entrepreneur's access to capital (and the firm's level of production) does not decrease over time. Forand and Zápal (2019) prove that in a model of random project arrival, once the agent is allowed to pursue a project he desires, he is allowed to do so indefinitely. Our work generalizes and (with the exception of Albuquerque and Hopenhayn 2004) unifies the monotonicity results derived in these settings.

Ray (2002) studies a repeated (deterministic) contracting problem where the principal can commit only to his actions in the present period, and derives a monotonicity result that is reminiscent of our own. In particular, he shows that when the agent's utility is quasilinear, the principal's limited commitment power leads to the periodic contract converging to the agent's preferred self-enforcing contract. Moreover, he shows that the agent's constitution utility increases monotonically until this contact is offered. To identify this qualitative impact of limited commitment power, Ray imposes an assumption that is similar to our Condition $\mathcal{D}$. This similarity suggests that conditions of this type may have an even deeper connection to the monotonicity of optimal contracts.

9 Concluding Remarks

Asymmetric Information. — The methodology developed in this paper can be extended beyond symmetric information and used to derive monotonicity results in contracting environments endowed with certain forms of asymmetric information. Consider, for example, a variant of our model of random opportunities where the agent may have multiple types that differ in the cost of effort. Specifically, assume that $\theta \cdot e$ is the cost of effort level e for an agent of type $\theta \in \Theta \subset \mathbb{R}_{+}$. The value θ is privately known to the agent at the beginning of the interaction (and it is constant over time).

This modification introduces a screening element to the principal's problem. Since the arrival of tasks is stochastic and the agent can quit at any time, finding the optimal intertemporal allocation of information rents is a complex problem. Nevertheless, we can establish that in an optimal solution, the (task-specific) effort of every agent type is nonincreasing over time.

In this screening problem there are two possible reasons for offering a contract with an increasing effort schedule to a certain type of agent. First, it may maximize the principal's profit from the interaction with that agent type. Second, it may reduce the cost of providing information rents to other agent-types. From Theorem 1, we know that an increasing effort schedule cannot be justified by the first reason. Next, we consider the impact of front-loading effort on the cost of providing information rents.

Incentive compatibility in this environment requires that, given a "menu of contracts," every agent-type (weakly) prefers accepting (and never quitting) the contract intended for him, to selecting a contract intended for another type and (perhaps) quitting that contract during the interaction. Notice that front-loading effort (weakly) reduces the agent's payoff from quitting a contract at any point in time. Hence, it does not increase the cost of information rents. Since offering an increasing effort schedule cannot be justified by the second reason either, it follows that, in optimum, the principal will only offer contracts with a nonincreasing effort schedule to all agent types.

Unconnected Support for Activity Levels. — Our definition of an activity requires that the set of possible activity levels is a real-valued interval. This assumption, which at first glance may seem like a mere simplification, is, in fact, necessary for our main result. Consider an infinite interaction with a discount factor of $\delta = \frac{1}{2}$ and assume that in each period the principal may provide compensation worth 2 utils to the agent. In each of the first two periods, the agent can exert an effort of $e \in \{0, 1, 2\}$ on a task and thus generate a profit to the principal that is concave in e. Therefore, effort is a convex separable activity with a discrete support.

Requiring high effort ($e = 2$) in period 1 and low effort ($e = 1$) in period 2 in return for the maximal compensation is not incentive compatible: the agent's discounted utility in period 1 is $\frac{2}{1-\delta} - 2 - \delta = \frac{3}{2}$, while his utility from exerting no effort is 2. Thus, requiring high effort in both periods is also not a viable option. However, requiring low effort in period 1 and high effort in period 2 is incentive compatible: in both periods the agent's discounted continuation payoff is 2, which is exactly his optimal deviation payoff. Therefore, under the optimal contract, the agent's effort increases over time, even though Condition $\mathcal{D}$ holds.

Multiple Activities. — The main theme of this paper is to analyze the dynamic use of a *single* convex separable activity. However, in many contracting problems there exist multiple such activities (e.g., the model of random opportunities). A closer look at the logic behind the proof of Theorem 1 reveals that our methodology and analysis is also informative about the dynamics of multiple convex separable activities that satisfy Condition $\mathcal{D}$. First, it shows that the marginal cost that the principal incurs from an activity today is (weakly) less than the marginal cost he will incur from any activity in the future. Thus, observing the level of a single activity today establishes a lower bound for the level at which *any* activity will be used in the future. Second, it can be shown that, within a given period, the marginal cost of two convex separable activities can be different (as occurs in PM at each phase transition) only if the slope of $D(\cdot)$ for the activity with the lower marginal cost is greater than the slope of $D(\cdot)$ for the activity with the higher marginal cost.

A Proofs

Proof of Lemma 1. We start with two simple observations. First, at least one constraint must be binding, as otherwise slightly reducing u increases the value of the problem without violating any constraint. Second, all tasks are assumed potentially profitable ($\pi_j'(0) > c'(0)$); thus, for at least one task $e_j > 0$.

Assume by way of contradiction that $IC_j^{(i)}$ is binding, for some $j < i$. This implies that $e_j > 0$ and that $e_j \geq e_i$. Consider the following modification for $\epsilon > 0$. Decrease e_j by $\epsilon\frac{1-\delta\lambda_i}{\delta q_j}$ and increase e_i by $\epsilon\frac{1-\delta\lambda_i}{1+\delta(q_i-\lambda_i)}$. It is straightforward to verify that this modification does not violate any of the constraints of the problem $P^{(i)}$.

The first-order effect of this modification on the value of the problem is

$$\epsilon(\pi_i'(e_i)\frac{1-\delta\lambda_i}{1+\delta(q_i-\lambda_i)}(1+\frac{\delta}{1-\delta\lambda_i q_i})-\pi_j'(e_j)\frac{1-\delta\lambda_i}{\delta q_j}\frac{\delta}{1-\delta\lambda_i})$$
$$=\epsilon(\pi_i'(e_i)-\pi_j'(e_j))$$

As $e_j \geq e_i$ the ranking of the tasks implies that this effect is positive; thus, for a small enough ϵ this modification increases the value of the problem. □

Proof of Lemma 2. As the objective is concave, the FOC are necessary and sufficient for optimality. The Lemma is obtained by rearranging the FOC.

□

Proof of Lemma 3. Suppose $u^{(i+1)} \leq u^{(i)}$. Then, $c'(u^{(i+1)}) \leq c'(u^{(i)})$ and, for all $j \leq i$, $e_j^{(i+1)} \geq e_j^{(i)}$ as $\pi_j(\cdot)$ is concave.

The binding constraint of $P^{(i)}$ can be written as

$$e_i^{(i)}(1-\delta\lambda_i)=\delta(u^{(i)}-\sum_{k\leq i}q_k\cdot e_k^{(i)})$$

We now rewrite the binding constraint of $P^{(i+1)}$ in a similar way and use the fact that $\lambda_{i+1}=\lambda_i+q_{i+1}$.

$$e_{i+1}^{(i+1)}(1-\delta\lambda_i)-\delta q_{i+1}\cdot e_{i+1}^{(i+1)}=\delta(u^{(i+1)}-\sum_{k\leq i}q_k\cdot e_k^{(i+1)}-q_{i+1}\cdot e_{i+1}^{(i+1)})$$

Adding $\delta q_{i+1}\cdot e_{i+1}^{(i+1)}$, using the expression for the binding constraint of $P^{(i)}$ and the consequences of our assumption at the beginning of the proof, we get

$$e_{i+1}^{(i+1)}(1-\delta\lambda_i)\leq\delta(u^{(i)}-\sum_{k\leq i}q_k\cdot e_k^{(i)})=e_i^{(i)}(1-\delta\lambda_i)$$

which, as $\delta\lambda_i < 1$, is equivalent to $e_{i+1}^{(i+1)} \leq e_i^{(i)}$. Since $e_i^{(i)} \leq e_i^{(i+1)}$, we get $e_{i+1}^{(i+1)} \leq e_i^{(i+1)}$ which is a contradiction because, $\pi_{i+1}'(e) > \pi_i'(e)$ for all e. The only way we can have $e_{i+1}^{i+1} \leq e_i^{i+1}$ is if both are zero, which cannot be optimal by our assumption that $\pi_i'(0) > c'(0)$. □

Proof of Proposition 1. We start by showing that restricting attention to stationary solutions does not reduce the value of $P^{(i)}$. Consider the general problem for this environment where the principal can choose any incentive-compatible history-dependent effort requirements and compensation schedule. Since this is a convex maximization problem where the objective function is separable in all arguments, if the stationary candidate $u^{(i)}, (e_j^{(i)})_{j \leq i}$ obtained from the solution to $P^{(i)}$ is suboptimal, there exists an improvement such that the required effort on one specific task is modified at one particular history, and only the compensation offered immediately after that history is modified and set at the lowest level under which all IC constraints are satisfied. By Lemma 2, the marginal benefit from every implemented task equals the marginal cost of compensation. Since the cost of compensation is convex and the productivity of effort is concave, every such modification will reduce the total expected value for the principal. Therefore, the stationary solution to $P^{(i)}$ specifies the unique optimal contract in the auxiliary environment.

We now return to the general environment and denote by C_0 the class of all incentive-compatible contracts for which, whenever a task that is better than all previously available tasks arrives, the agent's continuation utility is zero. It is immediate that the PM is the unique optimal contract in the class C_0. To see this, notice that the restriction to contracts in C_0 implies that it is sufficient to show that the PM attains the highest expected value between any two (subsequent) earliest arrivals of tasks that are superior to all previously available ones. But this follows directly from the observation given in the previous paragraph, and the construction of the PM.

Finally, we show that relaxing the restriction that the solution has to be in C_0 is not profitable for the principal. Suppose that the PM is suboptimal in the class of all contracts. Since, as before, the principal solves a convex optimization problem that is separable in all arguments, there must exist a profitable modification of the following form: i) at a given history in phase $k < I$, the PM is marginally altered in the direction that reduces the agent's expected payoff in the phase (i.e., either the required effort is increased or compensation is decreased), and ii) at a later history that is part of phase $k' > k$, the PM is marginally changed such that the resulting contract is incentive compatible. However, since the marginal cost of compensation and the marginal benefit from effort during phase k are below those of $k' > k$ under the PM, any such modification reduces the principal's expected payoff, a contradiction. The optimal contract is unique due to the concavity of the objective function. □

Proof of Theorem 1. Let $l(h_t)$ denote the level of the activity at history h_t and let τ_i denote the time at which the activity is available for the $i-th$ time in a realized infinite history.

Consider an incentive-compatible contract C in which the realized sequence $(l_{\tau_1}, l_{\tau_2}, \ldots)$ decreases with positive probability. Let h_t be a history after which l declines between periods $t = \tau_s$ and $t' = \tau_{s+1}$ with positive probability. There exist $\Delta > 0, p > 0$, such that the set Ω of all histories of length t' that are consistent with h_t and for which $l(h_t) - \Delta \geq l(h_{t'})$ satisfies $Pr(\Omega|h_t) = p$. Fix an $\epsilon > 0$ for which $\epsilon + \frac{\epsilon}{p\delta^{t'-t}} < \Delta$.

Consider the contract $\hat{C}$ that is obtained from C by modifying the level of the activity as follows: $\hat{l}(h_t) = l(h_t) - \epsilon$, and, at every history $h_{t'} \in \Omega$, $\hat{l}(h_{t'}) = l(h_{t'}) + \frac{\epsilon}{\delta^{t'-t}p}$. Notice that the original contract is modified only at histories during which the agent followed the recommendation. Moreover, by assumption, this change does not impact the unmodeled part of the interaction or the future availability of the activity. First, we show that $\hat{C}$ is incentive compatible. Then, we show that it increases the principal's expected value from the interaction.

For all histories h_s such that $s \geq t'$ and $h_s \notin \Omega$, the modified contract is identical to the original one. At $h_{t'} \in \Omega$, the agent's continuation utility from following the contract is increased by $\frac{\epsilon}{\delta^{t'-t}p}$ while his best alternative value increases by $D(l(h_{t'}) + \frac{\epsilon}{\delta^{t'-t}p}) - D(l(h_{t'}))$. By Condition $\mathcal{D}$ this increase is less than $\frac{\epsilon}{\delta^{t'-t}p}$. This, in turn, implies that the agent's incentive to follow the recommendation is weakly greater at all histories with a length of between t and t'. For all histories of length t other than the designated h_t, the contracts C and $\hat{C}$ are identical. Consider h_t where $\hat{l}(h_t) < l(h_t)$. By construction, if the agent follows the modified contract at period t, the expected increase in l at t' balances the decrease in l at h_t. Moreover, by Condition $\mathcal{D}$, decreasing l at h_t weakly reduces the agent's value for violating the contract at h_t. Thus, the modified contract is IC at h_t. Finally, for all histories h_s such that $s < t$ (regardless of whether these histories are consistent with h_t), the modified contract is IC as the agent's continuation utility from any action is unchanged.

To show that this modification is profitable for the principal it is sufficient to show that his expected cost from the activity conditional on reaching h_t under $\hat{C}$ is lower than that under C. Let μ denote the distribution of $l_{t'}$ induced by the distribution of histories in Ω conditional on h_t (a well defined distribution as a truncation of the distribution of $l_{t'}$, conditional on h_t). It is sufficient to show that

$$\kappa(l(h_t)-\epsilon)) + \delta^{t'-t}p\int \kappa(l(h_t') + \frac{\epsilon}{\delta^{t'-t}p})d\mu < \kappa(l(h_t)) + \delta^{t'-t}p\int \kappa(l(h_t'))d\mu$$

Since $\kappa(\cdot)$ is convex, it has a right-hand derivative. With a slight abuse of notation, we denote this derivative by $\kappa'(\cdot)$:

$$\begin{aligned}
&\kappa(l(h_t)-\epsilon) + \delta^{t'-t}p\int \kappa(l(h_t') + \frac{\epsilon}{\delta^{t'-t}p})d\mu < \\
&\kappa(l(h_t)) - \epsilon\kappa'(l(h_t)-\epsilon) + \delta^{t'-t}p\Bigg(\int \kappa(l(h_{t'}))d\mu + \frac{\epsilon}{\delta^{t'-t}p}\kappa'(l(h_t)-\Delta+\frac{\epsilon}{\delta^{t'-t}p})\Bigg) = \\
&\kappa(l(h_t)) + \delta^{t'-t}p\int \kappa(l(h_{t'}))d\mu - \epsilon\Bigg(\kappa'(l(h_t)-\epsilon) - \kappa'(l(h_t)-\Delta+\frac{\epsilon}{\delta^{t'-t}p})\Bigg) < \\
&\kappa(l(h_t)) + \delta^{t'-t}p\int \kappa(l(h_{t'}))d\mu,
\end{aligned}$$

where the last inequality follows from the convexity of $\kappa(\cdot)$ and the choice of ϵ.

If such h_t is reached with positive probability, the modified contract is better than the original one. Otherwise, our assumption that the realized level of the activity decreases with positive probability under C implies that there is a t for which there is a positive measure of histories h_t such that 1) the activity is available for the $j-th$ time at h_t and 2) conditional on h_t, there is a positive measure of histories $h_{t'}$ that are consistent with h_t in which the activity is available for the $j+1-th$ time and $l(h_t) > l(h_{t'})$. For each such h_t, perform the modification as specified above and note that these modifications do not interact with one another as they modify distinct histories. It follows that the modified contract outperforms the original one. □

Proof of Theorem 2. Consider the following interaction with no discounting that we use to construct both counterexamples (l', l'' are defined below for each type of violation of condition $\mathcal{D}$).

$t=0$ Agent decides whether to initiate interaction.

$t=1$ Activity is available.

$t=2$ Principal chooses G_1 or B_1, which gives the agent a respective payoff of $-l'$ or $-D(l')$. Both actions give the principal a payoff of zero, but the latter action ends the interaction.

$t=3$ Activity is available.

$t = 4$ Principal chooses G_2 or B_2, which gives the agent a respective payoff of $-l''$ or $-D(l'')$. B_2 gives the principal a payoff of zero, and G_2 gives him a large positive payoff.

In an optimal mechanism the principal must incentivize the agent to participate and then incentivize him to select the correct level of l_t while he chooses actions G_1, G_2. As $l \leq D(l)$ it is w.l.o.g. to assume that after the agent misuses the activity the principal chooses B_t. Thus, the IC constraints are

$$\begin{aligned} &l_1 + l_3 - l' - l'' \geq 0 && IC_0 \\ &l_1 + l_3 - l' - l'' \geq D(l_1) - D(l') && IC_1 \\ &l_3 - l'' \geq D(l_3) - D(l'') && IC_3 \end{aligned}$$

We now show that in the optimal contract the activity is used at level $l_1 = l'$ and then $l_3 = l''$. Note that for this contract all ICs are binding.

First we consider the case where the slope of $D(\cdot)$ between two points in L is negative. Due to the continuity of $D(\cdot)$ there exists an interval $L_1 \subset L$ and $c > 0$ such that for any two points in L_1 the slope of $D()$ is less than $-c$.

Choose $l'' < l' \in L_1$ such that for any $\epsilon \in [0, l' - l'']$,

$$\kappa(l') + \kappa(l'') < \kappa(l' - \epsilon) + \kappa(l'' + \epsilon(1 + c))$$

the existence of such values is established in the following lemma:

Lemma 4. *If the slope of $D(\cdot)$ on an interval X is bounded from above by $-c < 0$, there exist $x_2 < x_1 \in X$ such that for any $\epsilon \in (0, x_1 - x_2)$,*

$$\kappa(x_1) + \kappa(x_2) < \kappa(x_1 - \epsilon) + \kappa(x_2 + \epsilon(1 + c)) \tag{1}$$

Proof. Rearranging inequality (1) and dividing by ϵ gives

$$\frac{\kappa(x_1) - \kappa(x_1 - \epsilon)}{\epsilon} < (1 + c)\frac{\kappa(x_2 + \epsilon(1 + c)) - \kappa(x_2)}{(1 + c)\epsilon} \tag{2}$$

Taking the limit $\epsilon \to 0$, the LHS of (2) converges to the left-handed derivative of κ at x_1 while the RHS converges to $(1 + c)$ times the right-handed derivative of κ at x_2. As κ is convex, it is differentiable a.e., thus there exist $x_2 < x_1$ for which (2) holds at the limit. Moreover, as κ is continuous, (2) will also hold for x_1, x_2 and sufficiently small ϵ.

Next, note that due to the convexity of κ, if (2) holds for (x_1, x_2, ϵ), it will also hold for any (x_1, x_3, ϵ) if $x_3 \in (x_2, x_1)$. □

Since $\kappa(\cdot)$ is convex, if there exists a better contract it must have $l_1, l_3 \in (l'', l')$. Note that in this range IC_3 is non-binding. As $l_1 < l'$ and D is decreasing in this range, IC_1 implies IC_0. Thus, if $l_1 = l' - \epsilon$, a necessary condition for the contract to be IC is

$$l_3 \geq l'' + \epsilon(1+c) \tag{3}$$

Thus, relative to the initial contract, l_3 must be increased by at least $(1+c)$ times the decrease in l_1. However, by the choice of l'', l', any l_3 that satisfies this will increase the principal's total cost from the activity.

Next, we consider the case where the slope of $D(\cdot)$ is greater than one for some $l \in L$. In this case, there exists an interval $L_2 \subset L$ where $\forall x'' < x' \in L_2 \quad D(x') - D(x'') > x' - x''$. With a slight abuse of notation, denote $[l'', l'] \equiv L_2$.

In this interval $D(\cdot)$ is strictly increasing; thus IC_0 implies IC_1. Since $\kappa(\cdot)$ and convex and increasing function, and under the suggested contract IC_0 holds with equality, a contract can be more profitable than the one suggested above only if $l_1, l_3 \in L_2$. However, by construction, $l_3 = l''$ is the only such value that satisfies IC_3. □

Proof of Proposition 2. For any $c > 0$ define $X_c = \{x : x > 0 \& \inf_{l,x+l \in \hat{L}} \frac{\hat{\kappa}'(l+x)}{\hat{\kappa}'(l)} < 1+c\}$ (recall that $\hat{\kappa}'(\cdot)$ is the right-hand derivative of $\hat{\kappa}(\cdot)$), that is, the set of x's for which increasing the level of the activity by x, may for some initial levels of l, increase the marginal cost from the activity by a factor of less than $1+c$. Let $\bar{x}_c = \sup\{X_c\}$. As $\hat{L}$ is compact this supremum is finite, and since $\hat{\kappa}$ is convex $\lim_{c \to 0} \bar{x}_c = 0$.

Assume to the contrary that in an optimal contract with strictly positive probability the use of the activity decreases by more than $\Delta > \bar{x}_c$ between the $i-th$ and $j-th$ use o the activity for $j > i$. Let h_t be a history after which l declines between period $t = \tau_i$ and $t' = \tau_j$ by more than Δ with strictly positive probability. There exist $p > 0$, such that the set Ω of all histories of length t' that are consistent with h_t and for which $l(h_t) > l(h_{t'}) + \Delta$ satisfies $Pr(\Omega | h_t) = p$. For each such history we show that there exists a profitable modification of the contract that does not violate any IC constraint.

Changing the level of the activity at t to $\tilde{l}_t = l_t - \epsilon$ and at t' to $\tilde{l}_{t'} = l_{t'} + \alpha\epsilon$ is profitable for sufficiently small ϵ if $\alpha < \frac{1}{p\delta^{t'-t}} \frac{\hat{\kappa}'(l_t)}{\hat{\kappa}'(l_{t'})}$. Moreover, if $\alpha > \frac{1}{p\delta^{t'-t}}$

such a change slackens the IC constraints at $\{1, 2, \ldots, t-1, t+1, \ldots, t'-1\}$, has no impact on IC constraints after t', and, as the slope of $D(\cdot)$ is less than one, does not violate the IC constraints at t'. Thus, $l_t > l_{t'} + \Delta$ only if any small decrease in l_t, which is offset by a subsequent increase in $l_{t'}$ that maintains IC at t, is not profitable.

The supremum of the marginal increase in the agent's on-path payoff from a profitable modification is $(\frac{\hat{\kappa}'(l_{t'}+\Delta)}{\hat{\kappa}'(l_{t'})} - 1)$. Thus, a sufficiently small modification of the type suggested above is IC at t if $\frac{\hat{\kappa}'(l_{t'}+\Delta)}{\hat{\kappa}'(l_{t'})} - 1 \geq c$. Therefore, a decrease of size Δ in the level of the activity can be part of an optimal contract only if $\inf_{l \in \hat{L}} \frac{\hat{\kappa}'(l+\Delta)}{\hat{\kappa}'(l)} < 1 + c$. However, as $\Delta > \bar{x}_c$, there exists profitable modifications of the contract. $\square$

Proof of Proposition 3. The proof of this result builds on the counterexample used to establish Theorem 2. To construct a decrease of size $|\tilde{L}|$ in the use of the activity consider the counterexample used in the the second part of Theorem 2. By selecting $L_2 = \tilde{L}$ we have an environment in which $l_1 = \max_{l \in \tilde{L}} l, l_3 = \min_{l \in \tilde{L}} l$ under the unique optimal contract. $\square$

B Game Form Definition of an Activity

A two-player simultaneous move game G contains an activity if the following conditions hold. There exists a subset of the possible outcomes of the game, Ω, that are associated with the activity and an interval of possible levels of the activity, L, such that: 1) There exists a surjective ("onto") function $\eta : \Omega \to L$ 2) There exist two continuous and increasing functions, $u, \kappa : L \to \Re$ such that for every $\omega \in \Omega$ the principal's payoff is given by $-\kappa(\eta(\omega))$ and the agent's payoff is given by $u(\eta(\omega))$.

For a given activity in the game form G we can then define the agent's maximal activity specific payoff as follows. Let X and Y denote the, respective, strategy space for the agent and principal in G, and denote by $V(x, y)$ the agent payoff from strategy profile $< x, y >$. For each $l \in L$ define the set of principal strategies that can induce the level l

$$X(l) = \{x \in X : \exists y \in Y, \eta(x, y) = l\}$$

The agent's maximal activity specific payoff is then given by

$$D(l) = \inf_{x \in X(l)} \sup_{y \in Y} V(x, y),$$

that is, the maximal payoff he can obtain when the principal tries to induce the level l in the least tempting way possible.

C Discontinuous $D(\cdot)$

In this appendix we first illustrate via example that if for activity A there exists a discontinuity point in $D(\cdot)$, then there exists contracting environments in which the level of the activity decreases over time. Furthermore, we will use this to example to highlight the technical difficulties that may arise due to such points. Then, we will prove a version of Theorem 2 that allows for discontinuous in $D(\cdot)$.

Consider the joint production activity described above where output is given by $min\{\frac{y}{2}, x\}$, the agent's maximal effort is one, and the principal is restricted to assigning capital in integer units. For this activity,

$$D(x) = \begin{cases} 1 & \text{if } x \in (0, \frac{1}{2}) \\ 2 & \text{if } x \in (\frac{1}{2}, 1) \end{cases}.$$

To construct an environment in which the agent's labor increases over time assume (for now) that $D(\cdot)$ is left continuous, and consider the following environment: In period 1, the agent must incur a cost of $\frac{9}{8}$ to initiate the interaction. If he has done so, a production opportunity occurs in period 2. In period 3, either player can terminate the interaction. If neither player has done so, in period 4 there is another production opportunity and in period 5 the principal can decide whether to pay the agent a wage of $\frac{9}{4}$ or impose a large fine.

The maximal net compensation the agent can receive is $\frac{9}{4} - \frac{9}{8} = \frac{9}{8}$. Therefore, it may be possible to require the agent to provide more than $\frac{1}{2}$ a unit of labor in at least one of the two periods where production occurs. The IC constraint at period 2 is given by

$$-l_2 - l_4 + \frac{9}{4} \geq D(l_2)$$

thus, in order to require the agent to provide more than half a unit of labor in period two,

$$-l_2 - l_4 + \frac{9}{4} \geq 2 \Rightarrow l_2 + l_4 \leq \frac{1}{4}.$$

That is, in order to set $l_2 > \frac{1}{2}$, the agent must provide negative labor in period 4. However, if the fine is sufficiently large it is IC to require an effort that is greater than half in period 4. It is straightforward to verify that (for a sufficiently large fine) $l_2 = \frac{1}{2}, l_4 = \frac{3}{4}$ is IC. Moreover, this contract maximizes both the agent's aggregate effort and his effort in period 2, and thus this is the optimal contract. Hence, the standardized level of the activity decrease over time in the optimal contract.

If $D(\cdot)$ is right continuous, the above contract is no longer IC and, moreover, an optimal contract does not exists. Intuitively, the optimal contract would be given by the limit of the sequence of contracts for which $l_2 = \frac{1}{2} - \epsilon, l_4 = \frac{3}{4} + \epsilon$, however, this limiting contract is not IC. Nevertheless, under any approximately optimal contract of this type (formal definition provided below) the agent's labor still increases from $l_2 \sim \frac{1}{2}$ to $l_4 \sim \frac{3}{4}$. Therefore, for this particular activity, the qualitative result of Theorem 2 remains true. That is, if Condition $\mathcal{D}$ is violated, there exists environments in which the level of the activity changes in the principal's favor over time in all approximately optimal contracts.

As the above example demonstrates, if $D(\cdot)$ is not continuous an optimal contract may not exist. However, this concern is mute if the activity is not used in the vicinity of the discontinuity point. Therefore, if there exists an interval on which $D(\cdot)$ is continuous and Condition $\mathcal{D}$ is violated, then by a counter-example identical to the one used to prove Theorem 2 we can find an environment in which the activity is decreasing under the optimal contract. Thus, we assume the violation of Condition $\mathcal{D}$ is due entirely to the discontinuations of $D(\cdot)$. Moreover, to avoid dealing with pathological example we assume there are a finite number of discontinuous points.[18] Furthermore, in this appendix we consider only jump discontinuities as we find it hard to believe that removable- or singular-discontinuity points exist in any economically relevant activity.

We say that a contract C is ϵ-optimal if for any IC contract C', the difference between the principal's payoff from contract C' and contract C is less than ϵ. Moreover, we say that an activity is decreasing in every approximately optimal contract, if there exists an $\bar{\epsilon} > 0$ such that for any $\epsilon < \bar{\epsilon}$ the activity is decreasing under all ϵ-optimal contracts.

Theorem 3. *If a standardized convex activity A has a jump discontinuity in*

[18] This definition rules our activities for which the set of discontinuity points of $D(\cdot)$ is dense relative to some compact subset of L. However, as we are doubtful if there are economically relevant activities for which this is not the case, we do not consider this definition to be needlessly restrictive.

$D(\cdot)$, then there exists an a contracting environment with respect to which A is separable such that A is decreasing under all approximately optimal contracts.

Proof. In the following proof let L', l^* denote, respectively, an interval in which there is a single discontinuity point l^*. Moreover,let z denote the size of the discontinuity $z = |\lim_{\epsilon\to 0^+} D(l^*+\epsilon) - \lim_{\epsilon\to 0^-} D(l^*+\epsilon)|$. Finally let $r > 0$ be such that $[l^*-r, l^*+r] \subset int(L')$

Case 1: $D(\cdot)$ is increasing at jump point

Consider the following interaction with no discounting. In period 1 the agent chooses whether to exert effort at a cost of E or quit and end the interaction. In periods 2 and 4 the activity is available. In periods 3 and 5, the principal can either fire the agent and terminate the interaction, or reward the agent with a reward worth G_t. Finally, if the agent has not been fired, in period 6 the principal receives a large positive payoff.

First, we show that if $D(\cdot)$ is left-continuous at l^* the activity is decreasing under the unique optimal contract.

The agent's IC constraints are as follows:

IC_1 $l_2 + G_3 + l_4 + G_5 - E \geq 0$

IC_2 $l_2 + G_3 + l_4 + G_5 \geq D(l_2)$

IC_4 $l_4 + G_5 \geq D(l_4)$

We will now show that for a sufficiently small $\varepsilon > 0$ if

$$E > D(l^*+r)$$
$$G_3 = E - D(l^*) - \frac{z}{2} - l^* - \varepsilon$$
$$G_5 = D(l^*) - l^* + \frac{z}{2}$$

then the unique optimal contract is to set $l_2 = l^*+\varepsilon, l_4 = l^*$. Note, that for this candidate IC_1 binds, which implies that this solution minimizes the aggregate level of the activity. Thus, since κ is increasing and convex if there is a contract that is better than the candidate suggested above, under the alternative contract it must be the case that $|l_4' - l_2'| < \varepsilon$ and $l_4' > l^*$.

Since $\lim_{\eta\to 0^+} D(l^*+\eta) = D(l^*) + z$ and $D(\cdot)$ satisfies Condition $\mathcal{D}$ on L', there exists a υ such that IC_4 is violated for all $l_4 \in (l^*, l^*+\upsilon)$. Set $\varepsilon = \frac{\upsilon}{2}$. In this case to have $|l_4' - l_2'| < \varepsilon$ it must be that $l_2' > l^* + \epsilon$, but then $l_4' > l_4 + \varepsilon$, $l_2' > l_2 + \varepsilon$ and this alternative contract is inferior to our candidate. Note, that the difference in value between the optimal contract and any alternative where $l_4 > l^*$ is at least, $2\varepsilon\kappa'(l^*)$.

Next, consider the case where $D(\cdot)$ is right-continuous at l^*. It is straightforward to see that the supremum of the principal's value is given by the (non-IC) contract $l_2 = l^* + \varepsilon, l_4 = l^*$ and that any contract that provide within $2\varepsilon\kappa'(l^*)$ of this payoff must have $l_4 \leq l^*$. Therefore, for $\epsilon < 2\varepsilon\kappa'(l^*)$ the set of ϵ-optimal IC contracts has $l_4 \leq l^*$ and $l_2 \geq l^* + \varepsilon$. Which implies, that the activity is decreasing in all approximately optimal contracts.

Case 2: $D(\cdot)$ is decreasing at jump point

Consider the following interaction. In period 1, the agent decides whether to pay c to initiate the interaction or not. In period 2 the activity is available. In period 3 each player can unilaterally end the interaction. In period 4, the activity is available. Finally, in period 5 the principal can either compensate the agent with W utils or fine him F and fire him, If the agent is working in period 6, the principal gets a large payoff.

Note, that if the agent deviates in period 2, his continuation payoff from period 3 onwards is zero. Moreover, to deter deviations in period 4, the principal will fire and fine in 5. Thus, the agent ICs are

IC_1 $l_2 + l_4 - C + W \geq 0$

IC_2 $l_2 + l_4 + W \geq D(l_2)$

IC_3 $l_4 + W \geq 0$

IC_4 $l_4 + W \geq D(l_4) - F$

If F is large enough, IC_3 implies IC_4, thus we assume F is large and ignore IC_4.

Let $\varepsilon < \frac{z}{2}$ and set $W = D(l^*) - 2l^* + \frac{z}{2}$ and $C = D(l^*) + \frac{z}{2} - \varepsilon$. From IC_1 it follows that $l_1 + l_3 \geq 2l^* - \varepsilon$ in any contract the agent accepts. From IC_2 if follows that $l_2 \in (l^* - \rho, l^*), l_4 \in (l^* - \rho, l^*]$ is not IC for a small enough ρ. To see this note that

$$2l^* + W = D(l^*) + \frac{z}{2} < \lim_{\eta \to 0^+} D(l^* - \eta)$$

which in combination with Condition $\mathcal{D}$ holding on $(l^* - r, l^*)$ implies it is not IC to require a level of l just below l^* in both periods. This implies that the only way to have $l_2, l_4 \leq l^*$ is to set $l_4 < l^* = l_2$. From IC_3 it follows that if $\varepsilon < \rho$ setting $l_4 = l^* - \varepsilon$ is the optimal IC contract of this type. To see this note, that for any other contract both the sum of l_2 and l_4 and the distance between the two points is greater than under the suggested contract. Thus as κ is both increasing and convex, such a contract is sub-optimal. Note, that the difference in value between the optimal contract and any alternative IC contract is at least, $\varepsilon\kappa'(l^* - \varepsilon)$.

Now, consider the case where $D(\cdot)$ is left-continuous at l^*. In this case we show that for sufficiently small ϵ the activity is decreasing in all ϵ-optimal contracts. It is straightforward to see that the supremum of the principal's value is given by the (non-IC) contract $l_2 = l^* + \varepsilon, l_4 = l^*$ and that any contract that provides within $\varepsilon\kappa'(l^* - \varepsilon)$ of this payoff must have $l_2 \geq l^* + \varepsilon$. Therefore, for $\epsilon < \varepsilon\kappa'(l^* - \varepsilon)$ the set of ϵ-optimal IC contracts has $l_4 \leq l^*$ and $l_2 \geq l^* + \varepsilon$. Which implies, that the activity is decreasing in all approximately optimal contracts.

□

References

Abraham, Katharine G., and Henry S. Farber. 1988. "Returns to seniority in union and nonunion jobs: a new look at the evidence." *ILR Review*, 42(1): 3–19.

Aghion, Philippe, and Jean Tirole. 1997. "Formal and real authority in organizations." *Journal of political economy*, 105(1): 1–29.

Albuquerque, Rui, and Hugo A. Hopenhayn. 2004. "Optimal lending contracts and firm dynamics." *The Review of Economic Studies*, 71(2): 285–315.

Becker, Gary S. 1962. "Investment in human capital: A theoretical analysis." *Journal of Political Economy*, 70(5, Part 2): 9–49.

Bergemann, Dirk, and Juuso Välimäki. 2010. "The dynamic pivot mechanism." *Econometrica*, 78(2): 771–789.

Bird, Daniel, and Alexander Frug. 2019. "Dynamic nonmonetary incentives." *American Economic Journal: Microeconomics*, Forthcoming.

Blair, Douglas H., and David L. Crawford. 1984. "Labor union objectives and collective bargaining." *Quarterly Journal of Economics*, 99(3): 547–566.

Board, Simon, and Andrzej Skrzypacz. 2016. "Revenue management with forward-looking buyers." *Journal of Political Economy*, 124(4): 1046–1087.

Bolton, Patrick, and Mathias Dewatripont. 2005. *Contract Theory.* MIT Press.

Burdett, Ken, and Melvyn Coles. 2003. "Equilibrium wage-tenure contracts." *Econometrica*, 71(5): 1377–1404.

Carmichael, Lorne. 1983. "Firm-specific human capital and promotion ladders." *The Bell Journal of Economics*, 251–258.

Cyert, Richard M., and James G. March. 1963. "A behavioral theory of the firm." *Englewood Cliffs, NJ*, 2: 169–187.

Dessein, Wouter. 2002. "Authority and communication in organizations." *Review of Economic Studies*, 69(4): 811–838.

Edmans, Alex, and Xavier Gabaix. 2016. "Executive compensation: A modern primer." *Journal of Economic Literature*, 54(4): 1232–1287.

Forand, Jean Guillaume, and Jan Zápal. 2019. "Production priorities in dynamic relationships." Mimeo.

Gershkov, Alex, and Benny Moldovanu. 2009. "Dynamic revenue maximization with heterogeneous objects: A mechanism design approach." *American economic Journal: microeconomics*, 1(2): 168–98.

Grochulski, Borys, and Yuzhe Zhang. 2011. "Optimal risk sharing and borrowing constraints in a continuous-time model with limited commitment." *Journal of Economic Theory*, 146(6): 2356–2388.

Harris, Milton, and Bengt Holmström. 1982. "A theory of wage dynamics." *Review of Economic Studies*, 49(3): 315–333.

Hauser, Christine, and Hugo Hopenhayn. 2008. "Trading favors: Optimal exchange and forgiveness." *Collegio Carlo Alberto*, Working paper 88.

Holmström, Bengt. 1983. "Equilibrium long-term labor contracts." *Quarterly Journal of Economics*, 98: 23–54.

Krueger, Dirk, and Harald Uhlig. 2006. "Competitive risk sharing contracts with one-sided commitment." *Journal of Monetary Economics*, 53(7): 1661–1691.

Lazear, Edward P. 1981. "Agency, earnings profiles, productivity, and hours restrictions." *The American Economic Review*, 71(4): 606–620.

Li, Jin, Niko Matouschek, and Michael Powell. 2017. "Power dynamics in organizations." *American Economic Journal: Microeconomics*, 9(1): 217–241.

Marcet, Albert, and Ramon Marimon. 1992. "Communication, commitment, and growth." *Journal of Economic Theory*, 58(2): 219–249.

Milgrom, Paul, and John Roberts. 1988. "An economic approach to influence activities in organizations." *American Journal of Sociology*, 94: S154–S179.

Möbius, Markus. 2001. "Trading favors." Harvard University Mimeo.

Parsons, Donald O. 1972. "Specific human capital: An application to quit rates and layoff rates." *Journal of Political Economy*, 80(6): 1120–1143.

Postel–Vinay, Fabien, and Jean-Marc Robin. 2002*a*. "The distribution of earnings in an equilibrium search model with state-dependent offers and counteroffers." *International Economic Review*, 43(4): 989–1016.

Postel–Vinay, Fabien, and Jean-Marc Robin. 2002*b*. "Equilibrium wage dispersion with worker and employer heterogeneity." *Econometrica*, 70(6): 2295–2350.

Prendergast, Canice, and Robert H. Topel. 1996. "Favoritism in organizations." *Journal of Political Economy*, 104(5): 958–978.

Ray, Debraj. 2002. "The time structure of self-enforcing agreements." *Econometrica*, 70(2): 547–582.

Said, Maher. 2012. "Auctions with dynamic populations: Efficiency and revenue maximization." *Journal of Economic Theory*, 147(6): 2419–2438.

Samuelson, Larry, and Ennio Stacchetti. 2017. "Even up: Maintaining relationships." *Journal of Economic Theory*, 169: 170–217.

Shi, Shouyong. 2009. "Directed Search for Equilibrium Wage-Tenure Contracts." *Econometrica*, 77(2): 561–584.

Thomas, Jonathan, and Tim Worrall. 1994. "Foreign direct investment and the risk of expropriation." *The Review of Economic Studies*, 61(1): 81–108.

Topel, Robert. 1991. "Specific capital, mobility, and wages: Wages rise with job seniority." *Journal of Political Economy*, 99(1): 145–176.