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The effects of conventional and unconventional monetary policy: a new approach

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A New Approach to Measuring Economic Policy Shocks, with an Application to Conventional and Unconventional Monetary Policy

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Abstract: We propose a new approach to analyze economic shocks. Our new procedure identifies economic shocks as exogenous shifts in a function; hence, we call them "*functional shocks*". We show how to identify such shocks and how to trace their effects in the economy via VARs using "*VARs with functional shocks*" and "*functional local projections*". Using our new procedure, we address the crucial question of studying the effects of monetary policy by identifying monetary policy shocks as shifts in the whole term structure of government bond yields in a narrow window of time around monetary policy announcements. Our approach sheds new light on the effects of monetary policy shocks, both in conventional and unconventional periods, and shows that traditional identification procedures may miss important effects. Our new procedure has the advantage of identifying monetary policy shocks during both conventional and unconventional monetary policy periods in a unified manner and can be applied more generally to other economic shocks.

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1 Introduction

What is an economic policy shock? And how large and pervasive are the effects of policy shocks? Such questions are of fundamental importance in economics, and have spurred countless and lively debates. In this paper, we propose a novel procedure to analyze economic shocks; then, we use our procedure to shed new light on the important question of identifying monetary policy shocks, questioning the traditional approach and showing that it might have missed important aspects.

Our new procedure identifies economic shocks as exogenous shifts in a function; hence, we refer to these shocks as "functional shocks". There are several important examples where shocks can be identified in this way. An important example, which we focus on throughout the paper, is the identification of monetary policy shocks. Other examples of "functional" shocks include: (i) fiscal policy shocks, measured by shifts in the entire term structure of forecasters' expectations of government spending around major fiscal policy events; (ii) tax policy shocks, involving exogenous changes in the entire tax schedule; (iii) uncertainty shocks, measured by shifts in the whole forecasters' predictive densities during salient episodes; (iv) demand or supply shocks, measured by shifts in the whole demand or supply function;[1] (v) productivity shocks involving exogenous shifts in the production function; (vi) shocks to income or wage distributions, where the entire change in the distribution function is of interest. We describe some of these shocks in detail in Section 2.

More precisely, our new definition of a monetary policy shock is a shift in the *entire* term structure of interest rates in a short window of time around central banks' monetary policy announcement dates. Clearly, the entire term structure contains important information on the duration of the zero lower bound episode and on the expected effects of monetary policy (see Gürkaynak and Wright, 2012, for a survey of the relationship between the term structure and the macroeconomy). Hence, our definition of monetary policy shocks is broader than that used in the existing literature, which typically uses exogenous changes in the short-term interest rate alone, and has the potential to encompass more broadly other changes that monetary policy has on both short- and long-term interest rates, such as announcement effects associated with forward guidance or quantitative easing. While a lot is known about the effects of monetary policy during conventional times – that is, at times in which the monetary authority can freely change the short-term interest rate or money supply – much less is known about the effects of monetary policy during zero lower bound periods, where central banks have to resort to unconventional monetary policy since the short-term interest rate is close to zero and cannot be lowered further. In recent years, a consensus has emerged regarding the effects of unconventional monetary policy on the term structure of interest rates (Wright, 2012; Gürkaynak, Sack and Swanson, 2005a,b, 2007); however, the overall effects on macroeconomic aggregates have been challenging to estimate, delivering sometimes estimates that are different from those expected from theory (Wu and Xia, 2014). Understanding how unconventional monetary policy affects the economy is a crucial task that provides important guidance to policymakers.

[1] In fact, demand and supply shocks may affect a multivariate demand function in different ways, by shifting the demand of a product towards other products or simply by shifting the demand of all products in a similar way.

The effects of our functional shocks on the economy can be estimated with a "VAR with functional shocks" procedure, or, alternatively, with a "functional local projection" (FLP) approach based on Jordà (2005), which we describe in Section 3. The approach can also be implemented using instrumental variables, where the instrument itself can be a function.[2] The structural identification of such models is discussed in Section 4.

As mentioned, we identify economic shocks as exogenous shifts in a function. In our leading example on the identification of a monetary policy shock, where the function of interest is the term structure, we use the Nelson and Siegel (1987) and Diebold and Li (2006) approach to model yields as a function of their maturity. The exogenous movement in the term structure due to the monetary policy action is identified either as the movement in the yield curve on a monetary policy announcement day or using an external instrument approach. The Nelson and Siegel (1987) and Diebold and Li (2006) approach provides a widely-used and parsimonious model of the term structure based on three factors: level, curvature and slope. While the factor model offers a convenient interpretation of the shocks as combinations of underlying factors, one could instead also use raw yield data directly. The factors naturally represent different aspects of monetary policy. In particular, they allow us to distinguish between conventional monetary policy, which typically operates by affecting the short-term interest rate, and monetary policy that affects the medium and long end of the yield curve, summarized by the level and curvature factors; the latter include unconventional monetary policy, such as forward guidance, as well as monetary policy announcements that shift people's expectations about the future path of interest rates or about risk premia without actually changing the short-term interest rate.[3] Our results also provide interesting insights on the curvature factor, which so far has eluded an economic interpretation. Our empirical analysis has both advantages and limitations. One of the advantages is that, as we show, the monetary policy shock that we define is a more comprehensive measure of monetary policy shocks, substantially different from those traditionally defined as an exogenous change in short-term interest rates during conventional monetary policy periods. For example, the monetary policy shock on 1/28/2004 led to no change in the short-term rate and would be ignored by the traditional literature, while in fact it did have large effects on medium- and long-term interest rates, due to its forward-guidance content, as discussed in the newspapers at that time.[4] Thus, our monetary policy shock is a more comprehensive measure of monetary policy than traditional measures. Another appealing feature of our framework is that the shock can be multi-dimensional – that is, could involve several "functions".[5] Among the limitations of our empirical analysis, we note that the monetary policy shocks uncovered by using announcements are only those that move the term structure on days of monetary policy. Furthermore, if there are movements in long-term interest rates on announcements days due to the monetary policy transmission mechanism or

[2]For example, the functional instrument we use in our empirical analysis is the term structure of Fed Funds futures.

[3]Note that, in this paper, we do not disentangle changes in the term structure due to expectations about the future path of interest from those due to risk premia. See Rogers, Scotti and Wright (2015) for an approach to do so.

[4]See Gürkaynak, Sack and Swanson (2005a).

[5]Such an example is a monetary policy shock defined as the shift in both the term structure of interest rates as well as mortgage rates at maturities of either 15 or 30 years.

reasons other than monetary policy actions, the latter are interpreted in our approach as an additional feature of the monetary policy shock. To alleviate these concerns, we complement our analysis by including results based on an instrumental variable approach where we use Fed Funds futures as instruments. The instrumental variable approach allows us to better separate the term structure movements due to exogenous monetary policy actions from those associated with the transmission mechanism. On the other hand, our approach allows us to consider monetary policy shocks due to forward guidance as well as balance sheet announcements, but without separately identifying them and only to the extent that they are unexpected, and, as such, affect agents' expectations and the term structure.

Within our framework, we illustrate how monetary policy considerably changed its behavior over time: on average during the conventional period, monetary policy affected mostly the short end of the yield curve while leaving the long end unaffected; in the unconventional period, short-term interest rates were stuck at the zero lower bound, yet monetary policy successfully shifted the long end of the yield curve. Such changes are mainly explained by changes in the way monetary policy has affected both short- and long-term financial market's expectations of interest rates and risk premia.

Using our framework, we quantitatively estimate the effects of monetary policy shocks during both conventional and unconventional monetary policy periods in a unified manner. In fact, it is important to merge information on both normal and exceptional times to have a large enough sample to estimate the effects of monetary policy: our approach is appropriate in this case, as the change over time in the shape of the term structure (described by e.g. level, slope and curvature) has the potential to summarize both conventional and unconventional monetary policy shocks. The major take-away of our empirical analysis is that monetary policy shocks that unambiguously increase (or decrease) all yields have unambiguous contractionary (expansionary) effects on the economy. However, the macroeconomic responses to shocks that manifest themselves as increases in interest rates at some maturities and decreases at others are more complex: a shock that increases short-term interest rates but decreases long-term ones ends up decreasing output in the short-run while increasing it in the medium-run. An opposite shock, that decreases short-term rates while increasing long-term ones, has the opposite effect on output, increasing output in the short-run while decreasing it in the medium-run. We also show that impulse responses associated to shocks that result in the same change in the yield at any given sub-set of maturities may still be very different from each other. Finally, we show that the traditional approach to the identification of monetary policy shocks may have either missed important shocks or been unable to differentiate between shocks that were very different from one another.

Our work is related to several strands in the literature. On the one hand, one of the contributions of our paper is to propose a new approach to the identification of economic shocks. In this regard, our paper is related to the large literature on shock identification, in particular in VAR settings – see Kilian and Lütkepohl (2017) for a recent review of the literature. While we broadly build on existing approaches to shock identification, our approach is very different, as, unlike the traditional approach, it identifies shocks as shifts in a function rather than being summarized by a scalar. One limitation of the existing approaches is that they yield identical impulse responses up to scale for different policy announcements. In contrast, our approach yields different impulse responses for different policy announcements unless two changes in the yield curve are exact scalar multiples of each other (which is highly unlikely). This allows us to analyze and understand the effects of monetary policy at a deeper level. In particular, Gürkaynak et al. (2005a) have highlighted the importance of considering alternative "dimensions" in which monetary policy affects stock prices. Our framework is inspired by their work and allows researchers to directly evaluate and quantify the importance of these additional "dimensions".

On the other hand, the empirical analysis in our paper is related to the large literature that estimates the effects of monetary policy shocks (see Christiano et al., 1999; Sims and Zha, 2006; Romer and Romer, 2004, among others). More recently, as new and unconventional types of monetary policies have been implemented, such as quantitative easing and forward guidance, the literature has taken advantage of alternative identification schemes, including heteroskedasticity-based and high frequency identification (Wright, 2012; Gürkaynak et al., 2005a; Swanson, 2017). While we use high frequency data to extract the exogenous component of monetary policy, our approach identifies shocks in a different way from the existing literature: namely, as a shift in the entire term structure of interest rates (as opposed to a shift in short-term interest rates, or in interest rates at ad-hoc maturities). Note that our analysis is not confined to high frequency data, and it can be applied more generally to other well-known identification procedures, as we discuss in Section 4. Our work is also related to the literature on the effects of unconventional monetary policy on the macroeconomy. For example, Kulish, Morley and Robinson (2016), Baumeister and Benati (2013) and Wu and Zhang (2017) argue, like we do, that it is important to develop methodologies to estimate monetary policy effects during both periods of conventional monetary policy and at the zero lower bound, and do so by estimating structural DSGE models or time-varying VARs. Alternatively, Wu and Xia (2014) and Krippner (2015) propose "shadow rates" estimated from a finance model of the term structure to measure the stance of monetary policy during unconventional times. As previously discussed, the difference between these approaches and ours is that our shock is a function rather than a scalar, and it can summarize multiple dimensions of monetary policy at the same time.[6]

Our paper is more generally related to the literature that measures the effects of unconventional monetary policy on the yield curve, and, in particular, the literature on the effects of news on the yield curve, such as Kuttner (2001), Wright (2012), Gürkaynak, Sack and Swanson (2005b, 2007) and Altavilla and Giannone (2017). While our work builds on these contributions, it markedly differs from them: unlike these papers, which focus on the effects of monetary policy on the yield curve, we use shifts in the yield curve themselves to identify monetary policy shocks and then study their effects on key macroeconomic variables. Another key aspect that differentiates our work from theirs is that these studies estimate impulse responses to shocks to either level or slope whereas we estimate responses to functional shocks, defined as *joint* changes in the whole shape of the yield curve (level, curvature and slope).[7]

[6]For example, Baumeister and Benati (2013) identify monetary policy shocks as exogenous movements in the spread between the 10 years and the 3 month rates. In our case, it is the whole profile of yields as a function of maturity.

[7]Other papers have identified the effects of unconventional monetary policy using external instruments. For example, Gertler and Karadi (2014) identify unconventional monetary policy shocks using high frequency

Finally, the model we use to fit the term structure is the dynamic Nelson and Siegel (1987) and Diebold and Li (2006) model augmented with macroeconomic data. In principle this is not necessary and a non-parametric model for the term structure could be fit to the data. However, as previously mentioned, the Nelson and Siegel (1987) model has the advantage of interpreting the monetary policy shocks as a combination of movements in level, curvature and slope. We are not the first to attempt to link movements in level, curvature and slope of the term structure to the macroeconomy. In particular, Moench (2012) uses a three factor model to predict the yield curve and the macroeconomy, and finds evidence that an increase in the curvature factor predicts a flattening of the yield curve and future declines in output. The main difference between our approach and Moench (2012) is that the latter does not identify monetary policy shocks and separately identifies shocks to level, curvature and slope, while we identify the exogenous, joint movement in level, curvature and slope on days of monetary policy announcements as the monetary policy shock. To address the potential existence of non-linearities, we estimate a time-varying parameter model with state dependence. Alternatively, one could rely on models with time-varying GARCH volatility (Koopman et al., 2010); one could also use more general parametric models that allow for measurement error in the extracted yield curve factors, or, even more generally, macro-yield models with no-arbitrage restrictions (see Diebold et al., 2005; Diebold and Rudebusch, 2012; Moench, 2012; Altavilla et al., 2017; Ang and Piazzesi, 2003, among others).

Section 2 sketches a series of motivating examples. Section 3 presents our novel approach and Section 4 discusses various identification schemes that can be used within our framework. Section 5 presents the monetary policy shock analysis and highlights the differences between our approach and those existing in the literature. Section 6 explains why it is important to consider functional aspects of monetary policy shocks. Section 7 discusses the empirical results on the effects of monetary policy shocks on the macroeconomy in both conventional and unconventional times. Section 8 concludes.

2 Motivation

This paper proposes to study "functional shocks". But what are functional shocks? This section sketches a few examples that help understand and visualize such shocks and clarifies how they differ from existing (scalar) shocks.

As a first example we consider functional monetary policy shocks, which we will discuss in detail in our empirical analysis later on. Monetary policy actions shift the yield curve by affecting agents' expectations about current and future yields. Thus, monetary policy shocks can be identified by changes in the yield curve on days in which there are monetary policy announcements. Let us consider one such date, January 28, 2004, when the Federal Open Market Committee decided to keep its target for the Federal Funds rate fixed at 1 percent but, instead of indicating that interest rates will stay low for a "considerable period of time", as it usually did when planning to keep it fixed in the future, it said instead it

changes in interest rates around the date of the announcements as external instruments, and study the effects of the policies on key macroeconomic aggregates. Our work differs from theirs since we identify the unconventional monetary policy shock as the shift in the whole term structure.

could be "patient in removing its policy accommodation". This monetary policy "action" was interpreted by the markets as the intention of the monetary authority to increase rates in the medium-run. Panel A in Figure 1 depicts the term structure before and after the announcement by a dotted line without and with asterisks, respectively. Clearly, the long-end of the yield curve increased substantially while the short-end did not change. The functional monetary policy shock, depicted on the right by a dotted line, summarizes all these different aspects of monetary policy. A shock based only on short-term interest rates would be negligible, and would not be able to convey the actual monetary policy action on that day.

As a second example, we consider a functional fiscal policy shock. Ramey (2011) constructs a government spending shock based on the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia. The shock is defined as the difference between actual government spending growth and its forecast made one quarter earlier by the survey participants. The survey's forecast of nominal spending is converted to a forecast of real spending using the forecasters' predictions about the GDP deflator. The shock is the difference between actual real defense spending growth and the forecasted growth of defense spending. Ramey (2011) focuses on one-quarter ahead forecasts; however, the survey's dataset includes predictions at multiple horizons. The survey's forecast error, viewed as a function of the forecast horizon, is a functional shock. We depict two such shocks in Figure 1, Panel B: the Carter-Reagan buildup following the Soviet invasion of Afghanistan in 1980Q1 and the 9/11 terrorist attack (2001Q3).[8] The panel on the left shows how the forecast and the realized values differ, across horizons, in the two episodes. In the Carter-Reagan buildup case, shown in the top panel by a dashed line, the difference between the realization and the forecast is almost the same across horizons. Thus, it would be indifferent to focus on the one month or other horizons. However, in the 9/11 case, depicted in the bottom left panel by a continuous line, the difference is negligible at the one-quarter ahead horizon and becomes progressively bigger as the horizon increases. The Carter-Reagan and 9/11 shocks themselves are depicted in the picture on the right by the dashed and continuous lines, respectively. In the 9/11 case, focusing only the value of the shock at the one-quarter ahead forecast horizon makes the shock look negligible, while the shock indeed was comparable to that in the Carter-Reagan buildup at longer forecast horizons.

The above examples show that traditional shocks may miss important policy aspects, and motivate our interest in proposing functional shocks. The next section discusses how to measure their effects on the macroeconomy. The Not-for-Publication Appendix sketches two additional examples: functional tax and uncertainty shocks.

INSERT FIGURE 1 HERE

[8] The shock is smoothed across horizons using a quadratic trend.

3 The "VAR with Functional Shocks" and "Functional LP" Approaches

We propose to construct impulse responses to a shock which is defined as a function (not simply as a scalar); this requires a new and more general methodological approach. Appendix A provides some general definitions. In this section, we define the VAR approach that we utilize and show that it has a functional VAR interpretation. Hence, we will refer to our proposed methodology as the "VAR with functional shocks". We also describe an alternative "functional local projection" (FLP) approach.

For a given $\lambda > 0$, consider a class of possibly time-varying functions of the form:

$$f_t(\tau;\lambda) = \sum_{j=1}^{q} \beta_{j,t} g_j(\tau;\lambda), \tag{1}$$

where the function is a linear combination of q time-varying factors ($\beta_{j,t}$, where t denotes time) with coefficients that are functions of a scalar τ and depend on tuning parameters λ. The special type of function we consider is inspired by the Nelson and Siegel (1987)/Diebold and Li (2006) model, which we will describe in detail in the next Section.[9] For notational simplicity, in what follows we ignore the dependence of the function on λ.

For a given weight function $w(\cdot) : \mathcal{T} \rightarrow \Re$, we let

$$G_j \equiv \int_{\mathcal{T}} w(\tau) g_j(\tau) d\tau,\ j = 1, ..., q, \tag{2}$$

and assume that they exist and are finite.

The Functional VAR Model. Consider the following stationary p–th order linear (reduced-form) functional VAR model that consists of an $(n \times 1)$ vector of random variables X_t and a random function $f_t(\cdot)$:

$$\Phi_{11}(L)X_t + \Phi_{12}(L) \int_{\mathcal{T}} w(\tau)\, f_t(\tau)\, d\tau = \mu_x + u_{x,t}, \tag{3}$$

$$\Phi_{21}(L,\cdot)X_t + \Phi_{22}(L) f_t(\cdot) = \mu_f(\cdot) + u_{f,t}(\cdot), \tag{4}$$

where $\Phi_{11}(L) = \Phi_{11,0} - \Phi_{11,1}L - ... - \Phi_{11,p}L^p$, $\Phi_{12}(L) = -\Phi_{12,1}L - ... - \Phi_{12,p}L^p$, $\Phi_{21}(L;\cdot) = -\Phi_{21,1}(\cdot)L - ... - \Phi_{21,p}(\cdot)L^p$, $\Phi_{22}(L) = \Phi_{22,0} - \Phi_{22,1}L - ... - \Phi_{22,p}L^p$, $\Phi_{22,0}$ and $\Phi_{11,0}$ are identity matrices.

The Functional MA Representation. As shown in Appendix A.2, eqs.(3)-(4) have a reduced-form functional moving average (MA) representation; focusing on the MA representation of X_t, we have:

$$X_t = \Psi_{11}(L)\, u_{x,t} + \Psi_{12}(L,\cdot) \int_{\mathcal{T}} w(\tau) u_{f,t}(\tau) d\tau, \tag{5}$$

[9] In the Nelson and Siegel (1987) model, $q = 3$, τ is the maturity, $g_1(\tau;\lambda) = 1$, $g_2(\tau;\lambda) = [1 - \exp(\tau/\lambda)]/(\tau/\lambda)$ and $g_3(\tau;\lambda) = [1 - \exp(\tau/\lambda)]/(\tau/\lambda) - \exp(-\tau/\lambda)$.

where the matrices $\Psi_{11}(L)$, $\Psi_{12}(L,\cdot)$ are defined in the Appendix.

The Finite-Order MA Representation. Let $\mu_f(\cdot)$, $u_{f,t}(\cdot)$ and $\Phi_{21,s}(\cdot)$, $s=1,..,p$, belong to the class of functions in eq.(1) and be linear:

$$\begin{aligned}\mu_f(\tau) &= \sum_{j=1}^{q} g_j(\tau)\widetilde{\mu}_j, &(6)\\ \Phi_{21,s}(\tau) &= \sum_{j=1}^{q} g_j(\tau)\widetilde{\Phi}_{21,s,j},\ s=1,..,p &(7)\\ u_{f,t}(\tau) &= \sum_{j=1}^{q} g_j(\tau)\tilde{u}_{j,t}. &(8)\end{aligned}$$

Also, for notational convenience, let $\beta_{1:q,t}\equiv\left[\beta_{1,t},...,\beta_{q,t}\right]'$, $\widetilde{u}_{1:q,t}\equiv[\widetilde{u}_{1,t},...,\widetilde{u}_{q,t}]'$ and $G\equiv[G_1\ G_2\ \cdots\ G_q]'$. Then, Appendix A.2 shows that eq.(5) can equivalently be rewritten as the following finite order MA:

$$\begin{aligned}X_t &= \left[I_n+\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'\widetilde{\Phi}_{21,1:q}(L)\right]^{-1}\Phi_{11}^{-1}(L)u_{x,t} \qquad (9)\\ &\quad-\left[I_n+\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'\widetilde{\Phi}_{21,1:q}(L)\right]^{-1}\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'\tilde{u}_{1:q,t}\\ &= C_{11}(L)u_{x,t}+C_{12}(L)\tilde{u}_{1:q,t},\end{aligned}$$

where $G'\tilde{u}_{1:q,t}=\sum_{j=1}^{q}\left(\int_{\mathcal{T}}w(\tau)g_j(\tau)d\tau\right)\tilde{u}_{j,t}=\int_{\mathcal{T}}w(\tau)u_{f,t}(\tau)d\tau$, $C_{12}(L)=\sum_{j=0}^{\infty}C_{12,j}L^j\equiv-\left[I_n+\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'\widetilde{\Phi}_{21,1:q}(L)\right]^{-1}\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'$ and $\widetilde{\Phi}_{21,1:q}(L)$, $C_{11}(L)$ are lag polynomials defined in Appendix A.2.

It follows from eqs. (8) and (9) that the impulse response of X_t to $u_{f,t}(\cdot)$ is:[10]

Definition 1 (Reduced-form Functional IRFs) *The h-step ahead impulse response of X_t to $u_{f,t}(\cdot)$ is the differential of X_{t+h} in the direction*

$$u_{f,t}^{*}(\tau)=\sum_{j=1}^{q} g_j(\tau)\tilde{u}_{j,t}^{*} \qquad (10)$$

and it is:

$$C_{12,h}\tilde{u}_{1:q,t}^{*}. \qquad (11)$$

The question is how to estimate $C_{12,h}$ conveniently. In what follows, we will show how to estimate $C_{12,h}$ from the finite-dimensional VAR.

The Finite-Order VAR Representation. As shown in Appendix A.2, the model in eqs. (3)-(4) can be written as an $(n+q)$-variable finite-order reduced-form VAR model:

$$A(L)\begin{bmatrix}X_t\\ \beta_{1:q,t}\end{bmatrix}=\begin{bmatrix}u_{x,t}\\ \tilde{u}_{1:q,t}\end{bmatrix}, \qquad (12)$$

$$\text{where } A(L)\equiv\begin{bmatrix}\Phi_{11}(L) & \Phi_{12}(L)G'\\ \widetilde{\Phi}_{21,1:q}(L) & I_q\Phi_{22}(L)\end{bmatrix} \qquad (13)$$

[10] As we discuss in Appendix A.1, the differential we define here is a Gateaux differential.

where $A(L) \equiv I - A_1 L - ... - A_p L^p$.

As discussed in Appendix A.2, under stationarity, eq.(12) has a reduced-form vector moving average representation:

$$\begin{bmatrix} X_t \\ \beta_{1:q,t} \end{bmatrix} = C(L) \begin{bmatrix} u_{x,t} \\ \tilde{u}_{1:q,t} \end{bmatrix}, \tag{14}$$

where $C(L) = A(L)^{-1} = I + C_1 L + ... + C_h L^h + ...$ and C_h is the h-th moving average coefficient matrix, partitioned as $\begin{bmatrix} C_{11,h} & C_{12,h} \\ C_{21,h} & C_{22,h} \end{bmatrix}$.

The moving average representation in eq.(9) is identical to the moving average in eq. (14) associated to the $(q+n)$-variable VAR model (12), as the integration is a linear operator and the space of functions is finite-dimensional. This allows us to focus on the conventional VAR model to calculate the moving average coefficients to obtain the impulse responses without having to estimate eq. (9) directly. The differential of X_{t+h} is the inner product of the moving average coefficient of X_{t+h} on $\tilde{u}_{1:q,t}$ in eq.(14), which can be directly estimated from the finite order VAR in eq.(12), and $\tilde{u}^*_{1:q,t}$.

The Functional Local Projection (FLP) Representation. Note that, if the data follow the VAR(p) model in eq. (12), the latter provides a basis for local projections (Jordà, 2005). Omitting the intercept terms, and expressing the local projection in terms of the reduced-form shocks, it follows from eq. (12) that

$$\begin{aligned} X_{t+1} &= C_{12,1}\beta_{1:q,t} + \varphi_1' W_t + e_{1,t+1}, \\ X_{t+2} &= C_{12,2}\beta_{1:q,t} + \varphi_2' W_t + e_{2,t+2}, \\ \vdots & \quad \vdots \end{aligned} \tag{15}$$

where $e_{h,t+h}$ is an error term, $h = 1, 2, ...$, and W_t is a vector of control variables such as lags of both X_t and $\beta_{1:q,t}$. In the conventional local projection model, a scalar variable is regressed on another scalar variable as well as control variables, and the coefficient on the scalar variable is the parameter of interest. In our FLP model, however, a scalar variable needs to be regressed jointly on vector $\beta_{1:q,t}$, not on each $\beta_{j,t}$ individually, to estimate our functional impulse responses.

4 Identification

Eqs. (12) and (15) are reduced-form functional VARs and LP representations. The structural interpretation could be achieved by external instruments, recursive, sign-restrictions, high-frequency or heteroskedasticity approaches (see Kilian and Lütkepohl, 2017, for a review). However, note that such approaches are to be applied to the whole function, and that is where our identification differs from the literature. In fact, while we broadly build on existing approaches to shock identification, our approach is very different, as it identifies shocks as shifts in a function, rather than being summarized by a scalar.

To see the differences more clearly, consider the VAR in eq. (12) and consider identifying the shocks using a Cholesky (recursive) approach. The standard Cholesky approach would impose a triangularity assumption on the vector $[X_t', \beta_{1,t} ... \beta_{q,t}]'$, thus separately identifying the shocks to each of the $\beta's$. In our approach, the shock is instead identified by a contemporaneous change in all the $\beta's$ without separately identifying each of them. Our approach is really about identifying shifts in a function which is summarized by *a specific combination of the* $\beta's$. Thus, it is very different from identifying the VAR in eq. (12) simply using a recursive identification on the $\beta's$.

We now discuss detailed examples of identification restrictions within our general framework. Let the structural functional shock of interest be denoted by

$$\varepsilon_{f,t}(\tau) \equiv \sum_{j=1}^{q} g_j(\tau)\widetilde{\varepsilon}_{j,t},$$

where $\widetilde{\varepsilon}_{1:q,t} \equiv [\widetilde{\varepsilon}_{1,t}, \widetilde{\varepsilon}_{2,t}, ..., \widetilde{\varepsilon}_{q,t}]'$ denotes the $(q \times 1)$ vector of components of the functional shock; let $\varepsilon_{x,t}$ denote the remaining $(n \times 1)$ structural shocks and $\varepsilon_t \equiv [\varepsilon_{x,t}, \widetilde{\varepsilon}'_{1:q,t}]'$.

Identification in Structural Functional VARs

We have shown that the functional VAR(p) model in eqs. (3)-(4) has the finite-dimensional VAR(p) representation described in eq.(12). Let the relationship between the reduced-form and structural shocks be such that:

$$\begin{bmatrix} u_{x,t} \\ \widetilde{u}_{1:q,t} \end{bmatrix} = \Theta_0 \begin{bmatrix} \varepsilon_{x,t} \\ \widetilde{\varepsilon}_{1:q,t} \end{bmatrix}, \tag{16}$$

$$\text{where } \Theta_0 \equiv \begin{bmatrix} \Theta_{11,0} & \Theta_{12,0} \\ \Theta_{21,0} & \Theta_{22,0} \end{bmatrix} \tag{17}$$

Then the structural functional MA representation can be written as:

$$\text{Structural } MA \quad : \quad \left(X'_{t+h}, \beta'_{1:q,t+h}\right)' = \Theta_0\varepsilon_{t+h} + \Theta_1\varepsilon_{t+h-1} + ... + \Theta_h\varepsilon_t + ..., \tag{18}$$

$$\text{where } \Theta(L) \equiv C(L)\Theta_0 = \Theta_0 + \Theta_1 L + \Theta_2 L^2 + \cdots,$$

and $\Theta_h \equiv C_h\Theta_0$ is partitioned accordingly as $\begin{bmatrix} \Theta_{11,h} & \Theta_{12,h} \\ \Theta_{21,h} & \Theta_{22,h} \end{bmatrix}$, for $h = 0, 1, 2, ...$

The structural functional impulse responses are defined as follows.

Definition 2 (Structural Functional IRFs) *The* h*-step ahead impulse response of* X_t *to* $\widetilde{\varepsilon}_{1:q,t}$ *in the direction of* $\varepsilon^*_{f,t}(\tau) = \sum_{j=1}^{q} g_j(\tau)\widetilde{\varepsilon}^*_{j,t}$ *is given by*

$$\Theta_{12,h}\widetilde{\varepsilon}^*_{1:q,t}. \tag{19}$$

We are interested in the upper-right $(n \times q)$ sub-matrix of Θ_h, $\Theta_{12,h}$. Although C_h is identified from the VAR model, it is well-known in the literature that Θ_0 is not identified without an additional condition.

As in other structural VAR models, Θ_0 can be identified using short-run restrictions, long-run restrictions and heteroskedasticity-based restrictions, although with the appropriate differences explained below. As an example of short-run restrictions, one can argue that a macroeconomic aggregate does not contemporaneously respond to the monetary policy shock, but not vice-versa. In order to achieve this identification, one imposes $\Theta_{12,0} = 0$ in eq.(17). Notice that this does not require imposing a lower-triangular Θ_0. Thus, in this case, our approach has the advantage that one can still identify the structural impulse responses of the macroeconomic variables to the monetary policy shock even if it may be difficult to justify a recursive ordering among the functional structural shocks themselves. Similarly, an oil price shock can be identified by including a block recursive structure in the term structure of oil price futures. To impose long-run identification restrictions, analogous block-diagonal conditions can be imposed on the long-run impact matrix. As for heteroskedasticity-based identification, in the case of monetary policy, for example, one could impose that the volatility of interest rates is higher on a day of a monetary policy shock (e.g., Wright, 2012); in the case of oil prices, one could impose that the variance of oil prices is larger than that of other financial variables on a day of an oil price shock.

Sign restrictions can also be used and imposed directly on the matrix Θ_0. For example, a typical restriction in the context of monetary policy is that an unexpected monetary policy contraction is associated with an increase in the short-term interest rate, a decrease in non-borrowed reserves and a decrease in prices for a few months after the shock (see Uhlig, 2005). Similarly, an oil price shock can be identified by imposing the relevant sign restrictions on the matrix Θ_0 in a VAR that includes the term structure of oil futures.

Identification in Functional Local Projections

A convenient way to identify structural shocks in Functional Local Projections is via External Instruments (FLP-IV). Let X_{t+h} be an $(n \times 1)$ vector of macroeconomic time series (e.g. output and inflation) at time $t + h$. From eq. (18),

$$X_{t+h} = \Theta_{11,h}\varepsilon_{x,t} + \Theta_{12,h}\widetilde{\varepsilon}_{1:q,t} + \varepsilon_{t+h,t}, \tag{20}$$

where $\varepsilon_{t+h,t}$ is a function of $\varepsilon_{t+h}, ..., \varepsilon_{t+1}, \varepsilon_{t-1}, ...$ Note how eq. (20) differs from eq. (15), as the former is written in terms of components of the functional structural shock while the latter depends on the $\beta_t' s$. Furthermore, from eq. (18),

$$\beta_{1:q,t} = \Theta_{22,0}\widetilde{\varepsilon}_{1:q,t} + e_{\beta,t}, \tag{21}$$

where $e_{\beta,t}$ is a function of $\varepsilon_{x,t}, \varepsilon_{t-1}, ...$ We will later impose as an identification condition that $\Theta_{22,0} = I_q$, the $(q \times q)$ identity matrix. The identification condition that the off-diagonal elements of the matrix $\Theta_{22,0}$ are zero is necessary to identify the parameters, while the fact that the components on the main diagonal are one is a normalization condition. The latter implies that the units of the shocks $\widetilde{\varepsilon}_{1:q,t}$ are the same as those of the $\beta_t' s$, and hence helps with the interpretation of the unit of measure of the shocks. Thus,

$$\widetilde{\varepsilon}_{1:q,t} = \Theta_{22,0}^{-1}\beta_{1:q,t} - \Theta_{22,0}^{-1}e_{\beta,t}, \tag{22}$$

under the assumption that the number of components of the true functional shock is the same as those of β_t. By substituting eq. (22) in eq. (20), we have:

$$X_{t+h} = \Theta_{12,h}\Theta_{22,0}^{-1}\beta_{1:q,t} - \Theta_{12,h}\Theta_{22,0}^{-1}e_{\beta,t} + \Theta_{11,h}\varepsilon_{x,t} + \varepsilon_{t+h,t} \tag{23}$$

Assumption FLP-IV. We assume there exists an $(m \times 1)$ *vector of instruments* Z_t *such that: (i)* $E\left(\tilde{\varepsilon}_{1:q,t} Z_t'\right) = \alpha$*,* α *being a* $(q \times m)$ *matrix with rank* q*; (ii)* $E\left(\varepsilon_{x,t} Z_t'\right) = 0$*; (iii)* $E\left(\varepsilon_{t+j} Z_t'\right) = 0$ *for every* $j \neq 0$*; (iv)* $\Theta_{22,0} = I_q$*.*

Under Assumption FLP-IV, we have:

$$\begin{aligned} \alpha &= E\left(\beta_{1:q,t} Z_t'\right), & (24) \\ \Theta_{12,h}' &= \left[E\left(\beta_{1:q,t} Z_t'\right) \Omega E\left(Z_t \beta_{1:q,t}'\right)\right]^{-1} \left[E\left(\beta_{1:q,t} Z_t'\right) \Omega E\left(Z_t X_{t+h}'\right)\right], & (25) \end{aligned}$$

where Ω is a symmetric and positive definite matrix of dimension $(m \times m)$. The proof is in Appendix A.3. The parameters can be estimated using the sample moments counterparts to the expectations. In practice, one can let Ω be the long-run covariance of the orthogonality condition and estimate it using the Newey and West (1987) HAC estimator.

When the structural shocks, $\tilde{\varepsilon}_{1:q,t}$, are observables (e.g., by high frequency identification), the structural local projection provides directly the impulse responses:

$$\begin{aligned} X_{t+1} &= \Theta_{12,1} \tilde{\varepsilon}_{1:q,t} + \varphi_1' W_t + e_{1,t+1}, & (26) \\ X_{t+2} &= \Theta_{12,2} \tilde{\varepsilon}_{1:q,t} + \varphi_2' W_t + e_{2,t+2}, \\ &\vdots \end{aligned}$$

Alternatively, if Θ_0 is identified using an identification restriction, $\Theta_{12,h}$ is identified from (15) and the local projection of X_t on $\beta_{1:q,t}$.

Impulse response function estimates from local projections may be somewhat volatile. To circumvent excessive variations in the local projection estimates in the responses, one could use polynomials, splines or other smoothing devices. In the empirical analysis of this paper, we use fourth-order polynomials. Barnichon and Brownlees (2019) propose smooth local projections based on B-spline smoothing and generalized ridge estimation. One can generalize their estimator to accommodate multi-dimensional shocks by applying the B-splines basis approximation to the coefficients on the multi-dimensional shock in the local projection regression. Because their generalized ridge estimator shrink the estimates toward polynomials, the estimator that we use in this paper can be thought of as a limiting case of their estimator.

The discussion above has shown that the theory applies to any impulse response, no matter whether it is estimated by local projections or VAR procedures. While our approach is general, in this paper it turns out to be convenient to use a high-frequency identification approach and to estimate impulse responses via local projections.

5 A New Approach to the Identification of Monetary Policy Shocks

We illustrate our approach in the leading case of the identification of monetary policy shocks. It is well-known that monetary policy operates (directly or indirectly) by affecting interest rates, which we plot in Figure 2. Panel A depicts daily US zero-coupon bond yields over time between January 1995 and June 2016.[11] The data are from Gürkaynak, Sack and Wright (2007). At every point in time, we have data on yields at different maturities, from 3 months to 10 years.[12] The panel shows clearly the zero lower bound period, which starts in 2008:11 with the beginning of the first large-scale asset purchase program (LSAP-I). The yield curve as a function of maturity is depicted in Panel B of Figure 2. As the figure shows, the term structure of yields changed considerably over time in terms of its intercept, slope and curvature; we are interested, in particular, in exploring episodes of such shifts to identify monetary policy shocks in a more comprehensive manner.

INSERT FIGURE 2 HERE

We define a monetary policy shock as the shift in the entire term structure due to an exogenous monetary policy action. To illustrate how our functional shock can summarize monetary policy events within a theoretical macroeconomic model, we rely on a simple rule monetary policy rule a' la Taylor augmented with forward guidance shocks (Campbell et al., 2012; Del Negro et al., 2015). Let the interest rate at time t, i_t, obey the following monetary policy rule (up to a constant, which we ignore):

$$i_t = \mu + \rho i_{t-1} + (1-\rho)\left[\phi_\pi \pi_t + \phi_u u_t^{gap}\right] + \sum_{j=0}^{\tau_{\max}} \varepsilon_{t-j,j}, \tag{27}$$

where π_t and u_t^{gap} are the inflation rate and the unemployment gap,[13] the parameter ρ describes the degree of interest rate smoothing and the parameters ϕ_π, ϕ_u describe the inflation and unemployment gap aversion of the central bank, respectively. The monetary policy shock, $\sum_{j=0}^{\tau_{\max}} \varepsilon_{t-j,j}$, is a convolution of shocks at different maturities in the future ($\tau = 0, 1, ..., \tau_{\max} - 1$): $\varepsilon_{t,0}$, $\varepsilon_{t-1,1}$, ..., $\varepsilon_{t-\tau_{\max},\tau_{\max}}$. We refer to $\varepsilon_{t,0}$ as the conventional monetary policy shock, that is, the monetary policy shock that appears in the conventional monetary policy rules.[14] The remaining shocks are forward guidance shocks, revealed to the public earlier than the time in which they are implemented in practice. For example, $\varepsilon_{t-1,1}$ is the monetary policy shock announced at time $(t-1)$ to be implemented in practice by the central bank one period hence, that is at time t. Each of these announcements affects the expected path of interest rates at the time the announcement is made.

Let the expectation of the interest rate τ-period ahead given information at the start of period t be denoted by $i_{t+\tau}^{\tau}$. Note that, from eq. (27):

[11] We start the sample in 1995 as the Fed did not release statements of monetary policy decision after its FOMC meetings before 1994. Also, importantly, Gürkaynak et al. (2005a) show that, after 1995, daily data provide an accurate identification of monetary policy shocks, which provides another rationale for using daily yields from 1995 onward in our analysis. Appendix B describes the data in detail.

[12] The analysis of longer maturities requires a more general model and markets may also be illiquid at such maturities.

[13] The unemployment gap is the difference between the unemployment rate and the natural rate of unemployment.

[14] We will refer to $\varepsilon_{t,0}$ later as ε_t^{trad}.

$$i^{\tau}_{t+\tau} = \mu + \rho i^{\tau-1}_{t+\tau-1} + (1-\rho)\left[\phi_{\pi}\pi^{\tau}_{t+\tau} + \phi_u u^{gap,\tau}_{t+\tau}\right] + \sum_{j=\tau+1}^{\tau_{\max}} \varepsilon_{t+\tau-j,j}.$$

Hence, monetary policy shocks announced at time t for $\tau = 0, 1, .., \tau_{\max} - 1$ periods into the future will affect the whole term-structure at those maturities. The sequence of shocks $(\varepsilon_{t,0}, \varepsilon_{t,1}, ..., \varepsilon_{t,\tau_{\max}})'$ is the shock that we are identifying with our functional approach.

We use a high frequency identification inspired by Kuttner (2001) and Gürkaynak et al. (2005a,b, 2007) in our benchmark analysis, where the shock is identified as the shift in the term structure in a short window of time around monetary policy announcements; in a later section, we use an instrumental variable approach to identification. The novelty in our paper relative to the latter contributions is that we identify the whole change in the term structure at a given point in time as the monetary policy shock. There is nothing special about using a high frequency identification within our approach: we could have alternatively used other identification approaches – for example, those discussed in the previous section. The dates of unconventional monetary policy announcements are from Wright (2012),[15] which we extend ourselves to a longer sample up to 2016:6, while those of conventional monetary policy are from Nakamura and Steinsson (2017). Note that, in principle, it is possible to control for concurrent news, such as macroeconomic releases, although for simplicity we do not.

Panel A in Figure 3 shows how the monetary policy shock is identified in a few representative episodes of conventional monetary policy in US history. Each sub-panel in the figure depicts the shift in the term structure at the time of a monetary policy announcement, reported on top of the panel. Each circle represents the value of a yield at a given maturity (in months) before an exogenous monetary policy action, while the asterisk denotes its value afterwards. We define the monetary policy shock as the *joint* shift in yields *at all maturities* caused by the exogenous monetary policy action.

INSERT FIGURE 3 HERE

The monetary policy shock that we define is substantially different from the monetary policy shock traditionally defined as the change in short-term interest rates during conventional monetary policy periods. Such difference can be appreciated by looking closely at Figure 3. In the figure, the traditional monetary policy shock can be loosely approximated in our context by the shift in the interest rate at the shortest maturity, that is the difference between the circle and the asterisk closest to the origin. Clearly, shifts of the same magnitude in short-term interest rates are interpreted in the traditional monetary policy literature as carrying the same information about monetary policy. For example, both monetary policy shocks in 5/16/2000 and 11/6/2001 decreased the short-term rate by a similar magnitude, and would be considered similar monetary policy shocks in the traditional literature. In our approach, instead, it is clear that the shocks are very different: the former decreased proportionally all the yields, while the latter decreased short-term yields, increased medium-term yields and left unchanged long-term ones. Similarly, the shock on 1/28/2004 led to

[15] The unconventional monetary policy dates are reported in the Not-for-Publication Appendix.

no change in the short-term rate and would be ignored by the traditional literature, while in fact it did have large effects on medium- and long-term interest rates. The difference between the monetary policy shock that we identify and that traditionally identified in the literature, thus, is that the latter is typically measured by a scalar (e.g. exogenous changes in the short-term interest rate) while our shock is a function: it is the whole shift in the term structure. Thus, each monetary policy shock can be different not only because it changes the short-term interest rate, but also because, at the same time, it changes the medium- and the long-term ones, and each of them in a potentially different way. In addition, it also matters how the whole term structure shifts, as opposed to how the short- or the long-term rates separately shift, as it is the joint combination of changes in the intercept, slope or curvature of the term structure that matters, as opposed to shifts in a specific maturity of the term structure.

We identify the economic shock as a shift in a function using two approaches. The first approach is parametric while the second uses raw yield data directly. In the parametric approach, we use the Nelson and Siegel (1987)/Diebold and Li (2005) approach to model yields as a function of their maturity. The approach provides a widely-used and parsimonious term structure model. Alternatively, one could use raw yield data directly, which does not require any model. Notice, however, that even if one uses raw yield data, our approach is very different from that in the existing literature as the shock is a simultaneous change in all the yields.

In the Nelson and Siegel (1987) framework, the yield curve at any point in time is summarized by a time-varying three dimensional parameter vector (β_{1t}, β_{2t} and β_{3t}) capturing latent level, slope and curvature factors. The model for the yield curve is the following:

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{28}$$

where $y_t(\tau)$ is the yield to maturity, τ is the maturity.[16]

The continuous lines in Figure 3 help visualize the monetary policy shock identified parametrically as a shift in $y_t(\tau)$ in eq. (28).[17] The solid line depicts the term structure before the exogenous monetary policy move, while the dashed line depicts it afterwards. Clearly, monetary policy shocks (i.e., the difference between the solid and the dashed lines) come in many diverse shapes. Note how monetary policy shocks differ between the conventional and unconventional monetary policy periods: in the unconventional period, the shocks mainly affect medium and long-term maturities while leaving short-term maturities unaffected. For example, consider the shock on November 25, 2008 (depicted in Figure 3, Panel B), when the Fed announced the purchase of mortgage backed securities and agency bonds and the start of the LSAP-I program, and compare it with the shock on November 6, 2001, after the terrorist attacks of 9/11, depicted in Figure 3, Panel A. The figure illustrates how different the shocks are: even if they are both expansionary, the first shock tilts the function (as the

[16] Recall that the parametric term structure model depends on the tuning parameter λ as well, which is omitted for notational convenience.

[17] The R^2 of the estimates for the yield curves are very high, and equal to 0.9981, 0.9995, 0.9977, 0.9993, 0.9999, 0.9991, 0.9986, 0.9989, 0.9996, 1.0000, 0.9992, 0.9971 for the maturities that we consider, that is 3, 6, 12, 24, 36, 48, 60, 72, 84, 96 and 120 months.

short-term rates were fixed at the zero lower bound) while the second is a parallel shift in the function. Thus, each monetary policy shock can be different due to a variety of factors (how it affects short-term expectations *and* how it affects long-term expectations or risk premia) as well as their combination (how it affects short-term expectations *versus* how it affects long-term expectations or risk premia).

The functional monetary policy shocks themselves are depicted in Figures 4 and 5. They are defined as:

$$\varepsilon_{f,t}(\tau) \equiv \Delta y_t(\tau) \cdot d_t, \tag{29}$$

where d_t is a dummy variable equal to one if there is a monetary policy shock at time t and Δ denotes time differences: $\Delta y_t(\tau) \equiv y_t(\tau) - y_{t-1}(\tau)$. Not only do the shocks have different shapes in the conventional and unconventional periods, which is clear from comparing Figures 4 and 5, but they also differ from each other even in the conventional monetary policy period, as Figure 4 shows. For example, notice again how the change in the short-end of the yield curve is similar for both the 11/6/2001 and the 5/16/2000 shocks, while their shape is very different. The shocks of 1/28/2004 and 2/3/1999 are instead an example of similar effects on long-term yields but very different effects on short- and medium-term ones: no effects on short-term yields and large effects on medium-term yields for the 1/28/2004 shock and negative effects on short-term yields but positive effects on medium-term ones on 2/3/1999.

INSERT FIGURE 4 AND 5 HERE

The Nelson and Siegel (1987) model that we use to describe our monetary policy shock has several advantages. In particular, the model is quite flexible and the factors in eq. (28) have an economically interesting interpretation. Since $\beta_{1,t}$ does not vanish as τ approaches infinity, it can be interpreted as the long-term factor (or level factor, since it equally increases all yields independently of their maturity τ); $\beta_{2,t}$ is the factor with a coefficient $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)$ that equals unity at $\tau = 0$ but then decays to zero as τ increases; thus, it reflects a factor that is important in the short-term (this factor can also be interpreted as the slope, as it equals $y_t(\infty) - y_t(0)$); finally, $\beta_{3,t}$ is the factor with a coefficient $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$ that equals zero at $\tau = 0$, increases and subsequently decreases as a function of τ, thus reflects neither short-term nor long-run factors but a factor that is important in the medium-term, where the medium-term definition depends on the value chosen for λ (this factor is also known as the curvature). The estimation follows Diebold and Li (2006) by calibrating λ to 0.0609, which is the value that maximizes the loading on the medium-term factor at 30 months.

On the other hand, it is important to note that a limitation of our approach is that, if there are movements in long-term interest rates on the same day as the monetary authority's announcement due the transmission mechanism or to reasons other than monetary policy actions, the latter are interpreted in our approach as an additional feature of the monetary policy shock. To alleviate the concerns that some movements in the term structure may not be truly a monetary policy shock, we complement our analysis by including robustness results based on an instrumental variable approach where we use Fed Funds futures as instruments in Section 7.2. The instrumental variable approach allows us to better separate term structure movements due to exogenous monetary policy actions from other shocks or from the transmission mechanism.[18]

Importantly, note that $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ embody different aspects of monetary policy. In particular, $\beta_{2,t}$ describes conventional monetary policy, which typically operates by affecting short-term interest rates. $\beta_{3,t}$, instead, embodies monetary policy shocks that affect the medium-term; these include unconventional monetary policy shocks, such as forward guidance, where the short-term is at the zero lower bound, as well as monetary policy announcements that shift people's expectations of future interest rates or risk premia without actually changing the short-term interest rate (such as, for example, the FOMC announcement of January 28, 2004, depicted in Figure 2).[19] Finally, $\beta_{1,t}$ embodies any effects of monetary policies that simultaneously shift all interest rates, and derives from the central bank's ability to shift proportionally both short- and long-term expectations at the same time.

Certain linear combinations of the factors may also carry valuable information. For example, the instantaneous yield equals $\left(\beta_{1,t}+\beta_{2,t}\right)$,[20] while $\left(\beta_{3,t}-\beta_{1,t}\right)$ represents changes in long-run expectations or risk premia that do not result in parallel shifts in the term structure. That is, the latter summarizes additional information that monetary policy shocks contain exclusively about the future path of monetary policy not already contained in shifts in the short-term policy instrument, i.e. additional and potentially important "dimensions" of monetary policy. For example, Panel A in Figure 3 shows several interesting patterns arising from these linear combinations, whose values are reported in Table 1. The top left panel (labeled "11/6/2001") depicts a parallel downward shift in the term structure, which corresponds to a decrease in $\left(\beta_{1,t}+\beta_{2,t}\right)$ due, in large part, to a decrease in $\beta_{1,t}$. The figures labeled "5/16/2000" and "2/3/1999" depict a change in short-term interest rates associated with an increase in medium-term rates, and with an increase in the long-term rates in the latter but unchanged long-term rates in the former. These changes correspond to a large and negative change in $\left(\beta_{1,t}+\beta_{2,t}\right)$ in the first and a small change in $\left(\beta_{1,t}+\beta_{2,t}\right)$ in the second, combined with relatively large increases in both $\beta_{1,t}$ and $\beta_{3,t}$ for the latter, and almost no change in $\beta_{1,t}$ for the former. The panel labeled "1/28/2004" depicts a situation in which the instantaneous interest rate is unchanged ($\beta_{1,t}+\beta_{2,t}\simeq 0$) yet monetary policy affects medium- and long-term interest rates by increasing $\left(\beta_{3,t}-\beta_{1,t}\right)$.

INSERT TABLE 1 HERE

One might be concerned that the shocks identified in the Nelson and Siegel's (1987) model might be reduced-form; however, we can still identify the shocks in the VAR with high frequency identification under some assumptions. We define the h-step ahead impulse response of macroeconomic variable X_t to the true functional structural shock summarized

[18] Our definition of functional shocks based on high frequency identification is broad and includes any change in the yield curve on the day of an announcement, such as a shift in the monetary policy rule and a change in the volatility on the announcement date. Teasing out these effects in our framework may require structural modeling at high frequency and is left for future research.

[19] See Gurkaynak et al. (2005, p. 56) for a discussion of the FOMC announcement of January 28, 2004.

[20] Note that $y_t(0)=\beta_{1,t}+\beta_{2,t}$.

by the $(q \times 1)$ vector $\tilde{\varepsilon}^{\dagger}_{1:q,t}$ as $E[X_{t+h}|\tilde{\varepsilon}^{\dagger}_{1:q,t} + \tilde{\varepsilon}^{\dagger *}_{1:q,t}, \mathcal{I}_t] - E[X_{t+h}|\tilde{\varepsilon}^{\dagger}_{1:q,t}, \mathcal{I}_t]$,where $\mathcal{I}_t$ is the information set at time t excluding $\tilde{\varepsilon}^{\dagger}_{1:q,t}$. When X_{t+h} is linear in $\tilde{\varepsilon}^{\dagger}_{1:q,t}$, the impulse response is given by $\Theta^{\dagger}_{12,h}\tilde{\varepsilon}^{\dagger *}_{1:q,t}$, where $\Theta^{\dagger}_{12,h}$ is the h-th moving average coefficient matrix of X_{t+h} on $\tilde{\varepsilon}^{\dagger}_{1:q,t}$. Now let $\tilde{\varepsilon}_{1:q,t}$ denote a vector of high frequency changes in the Nelson-Siegel factors, and let them be observed, as we assume in our benchmark model in the empirical section. Suppose that $\tilde{\varepsilon}_{1:q,t} = \Lambda \tilde{\varepsilon}^{\dagger}_{1:q,t}$,where Λ is a $(q \times q)$ non-singular matrix that may be unknown to the econometrician. Our impulse responses of X_{t+h} to $\tilde{\varepsilon}_{1:q,t}$ in the direction of $\tilde{\varepsilon}^{*}_{1:q,t}$ at time t can be written as

$$\Theta_{12,0}\tilde{\varepsilon}^{*}_{1:q,t}, \Theta_{12,1}\tilde{\varepsilon}^{*}_{1:q,t}, \Theta_{12,2}\tilde{\varepsilon}^{*}_{1:q,t}, ..., \tag{30}$$

whereas the structural impulse responses in the direction of $\tilde{\varepsilon}^{\dagger *}_{1:q,t}$ can be written as

$$\Theta^{\dagger}_{12,0}\tilde{\varepsilon}^{\dagger *}_{1:q,t}, \Theta^{\dagger}_{12,1}\tilde{\varepsilon}^{\dagger *}_{1:q,t}, \Theta^{\dagger}_{12,2}\tilde{\varepsilon}^{\dagger *}_{1:q,t}, ..., \tag{31}$$

where $\Theta_{12,h}$ is the h-th moving average coefficient matrix of X_{t+h} to the $\tilde{\varepsilon}_{1:q,t}$, and $\Theta^{\dagger}_{12,h}$ is the h-th moving average coefficient matrix to the structural shock $\tilde{\varepsilon}^{\dagger}_{1:q,t}$. Note that the moving average coefficient matrix of X_{t+h} to $\tilde{\varepsilon}_{1:q,t}$ can be written as $\Theta_{12,h} = \Theta^{\dagger}_{12,h}\Lambda^{-1}$. It follows from $\tilde{\varepsilon}_{1:q,t} = \Lambda\tilde{\varepsilon}^{\dagger}_{1:q,t}$ and $\Theta_{12,h} = \Theta^{\dagger}_{12,h}\Lambda^{-1}$ that $\Theta_{12,h}\tilde{\varepsilon}_{1:q,t} = \Theta^{*}_{12,h}\Lambda^{-1}\Lambda\tilde{\varepsilon}^{*}_{1:q,t} = \Theta^{*}_{12,h}\tilde{\varepsilon}^{*}_{1:q,t}$. In other words, even though we cannot jointly identify $\Theta^{*}_{12,h}$ and $\tilde{\varepsilon}^{*}_{1:q,t}$, we can identify our structural impulse responses (31) because they are identical to the impulse responses to $\tilde{\varepsilon}_{1:q,t}$, (30), that are identified from the data.

Our analysis is related, although distinct, from that of recent works, such as Gürkaynak et al. (2005a) and Rogers, Scotti and Wright (2014). In their work, Gürkaynak et al. (2005a) extract factors from changes in bond yields and stock prices around monetary policy announcements and find that two factors are important. To give factors an economic interpretation, they rotate the second factor so that it is independent of changes in the Federal Funds rate (FFR) in the current month. Thus, the first factor is labeled the "current FFR factor", which corresponds to a surprise change in the current FFR target, and the second factor is labeled the "future path of policy factor", which corresponds to changes in future one-year ahead rates independent of changes in the first factor. They find that both monetary policy actions and statements affect asset prices, and the latter have more effects on long-term yields. They show that monetary policy announcements affect asset prices primarily via changing financial markets' expectations of future monetary policy (rather than changing their expectations on the current FFR). Swanson (2017) extends Gürkaynak et al.'s (2005) methodology to include the zero lower bound period, and aims at separately identifying changes in the FFR, forward guidance and LSAP by extracting three factors from a dataset of asset prices that includes the FFR, exchange rates, Treasury bond yields and the stock market. While these works have inspired ours, the differences between our approach and theirs are several. First, and most importantly, differently from Gürkaynak et al. (2005) and Swanson (2017), we do not separately identify factor shocks, as the entire change in the yield curve is the shock itself. We define a monetary policy shock as a specific and time-varying combination of changes in the various factors that we identify: therefore, in our work, each monetary policy shock is potentially different from another. In contrast, the previously cited works are interested mainly in determining how many factors provide a good description of the movements in asset prices at the time of a monetary policy shock and how they evolve over time.[21] Second, our factors are derived directly from the Nelson and Siegel (1987)/ Diebold and Li (2006) model. While the first two principal components in the yield curve are typically level and slope, and thus may correspond to our first two factors, in our work we find that a third factor, the curvature, is potentially important in selected monetary policy episodes.[22] Relative to theirs, our measure has the advantage that it is real-time and does not need to rely on in-sample factor estimates. A third, substantial difference is that, unlike them, we study the effects of monetary policy on macroeconomic variables rather than asset prices. Rogers, Scotti and Wright (2014), like Gürkaynak et al. (2005a), extract two principal components; they notice that the first principal component is correlated with an increase in all the yields, and interpret it as an LSAP shock, while the second seems to rotate the yield curve (pushing short rates down and long rates up), and interpret this as a forward guidance shock. By arguing that forward guidance cannot be credible at long horizons, they can also distinguish between forward guidance and risk premia: they interpret changes in yields that are concentrated in forward rates five years and beyond as caused by shifts in risk premia. Our approach, instead, allows us to directly and jointly estimate the various dimensions of monetary policy shocks. The next section provides a more formal analysis of the empirical importance of alternative dimensions of monetary policy.

6 A More Comprehensive Measure of Monetary Policy Shocks

More formally, how do traditional monetary policy shocks identified in the existing literature compare with the monetary policy shock that we identify as the change in the whole yield curve over time? If their correlation is high, then they are likely measuring the same unobserved shock and researchers can use either one of them; however, if their correlation is low, the existing literature may have missed important information.

First, we discuss anecdotal evidence that using information from the whole term structure is useful in identifying monetary policy shocks. As anticipated in Section 2, the bottom left panel in Figure 3A depicts our monetary policy shock in January 28, 2004. Newspapers at the time noted that the announcement was interpreted by financial markets as implying that policy would have been tightened sooner than expected (Gürkaynak et al., 2004). The traditional shock is the change in the short-term interest rate; the latter was zero on January 28, 2004, as the Fed Fund rate did not change. In this case, our shock and the traditional shock are very different, and the discrepancy is due to the fact that the traditional shock is unable to detect the forward guidance component of monetary policy.

Second, we investigate the relationship between our shock and traditional ones. Let

[21] The fact that Swanson (2017) finds three factors is not inconsistent with our findings, as his dataset includes not only yields but other asset prices as well.

[22] Another minor difference is that Gürkaynak et al. (2005a) and Swanson (2007) extract factors from a joint panel of Treasury yields and stock prices, while we mainly use the former to identify a monetary policy shock.

ε_t^{trad} denote a traditional measure of monetary policy shocks, e.g. a narrative measure. We consider the following regression:

$$\varepsilon_{f,t}(\tau) = \kappa(\tau) + \gamma(\tau)\,\varepsilon_t^{trad} + \eta_t, \tag{32}$$

which we separately estimate in the conventional and unconventional monetary policy periods. Panel A in Figure 6 plots the estimates of $\gamma(\tau)$ as a function of the maturity τ using the traditional Romer and Romer (2004) monetary policy shock as a proxy for the traditional monetary policy shock, ε_t^{trad}.[23] Interestingly, the coefficient $\gamma(\tau)$ during the conventional monetary policy period, depicted on the left, is the highest for short-term maturities, while it becomes the highest for the longest-term maturities in the unconventional monetary policy portion of the sample, depicted on the right. This means that monetary policy considerably changed its behavior: *on average*, during the conventional monetary policy period, monetary policy affected mostly the short end of the yield curve while leaving the long end unaffected; in the unconventional period, short-term interest rates were stuck at the zero lower bound, yet monetary policy successfully shifted the long end of the yield curve, although short-term rates were unaffected. Indeed, the data show strong evidence of a structural change: we filtered the daily yields by a VAR(1) model and then tested the equality of the means between the two sub-samples. The p-values of the Wald tests are all zero. Thus, the mean of the yields has indeed changed over time. Panel B in Figure 6 repeats the analysis using a monetary policy shock based on Wu and Xia's (2014) shadow rate as the proxy for the traditional monetary policy shock, ε_t^{trad}.[24] The latter is estimated in a VAR with inflation, output and the shadow rate, and identified using a Cholesky identification with the variables in the same order. The figure shows that the results are qualitatively similar.[25]

INSERT FIGURE 6 HERE

In order to understand the difference between our identified monetary policy shock and the traditional shocks, we investigate the relationship between the components of our shock and conventional monetary policy shocks. Note that we can decompose our functional shock in eq. (29) as:

$$\begin{aligned} \varepsilon_{f,t}(\tau) &= \Delta\beta_{1,t}^d + \Delta\beta_{2,t}^d\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \Delta\beta_{3,t}^d\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \\ &\equiv \Delta y_t^{(1)}(\tau) + \Delta y_t^{(2)}(\tau) + \Delta y_t^{(3)}(\tau). \end{aligned} \tag{33}$$

where $\Delta\beta_{j,t}^d \equiv d_t \cdot \Delta\beta_{j,t}$. Consider the following regressions:

[23] We use the traditional Romer and Romer (2004) shock up to 12/2007 and we proxy the traditional monetary policy shock after that by the change in the 3-month Treasury yield in a one-day window around monetary policy announcement dates.

[24] The data are available at: https://www.frbatlanta.org/cqer/research/shadow_rate.aspx?panel=1.

[25] The Not-for-Publication Appendix repeats the analysis using Nakamura and Steinsson's (2017) shock and shows that the results are qualitatively similar.

$$\Delta y_t^{(1)}(\tau) = \kappa_1(\tau) + \gamma_1(\tau)\varepsilon_t^{trad} + \eta_{1,t} \tag{34}$$

$$\Delta y_t^{(2)}(\tau) = \kappa_2(\tau) + \gamma_2(\tau)\varepsilon_t^{trad} + \eta_{2,t} \tag{35}$$

$$\Delta y_t^{(3)}(\tau) = \kappa_3(\tau) + \gamma_3(\tau)\varepsilon_t^{trad} + \eta_{3,t}, \tag{36}$$

which we separately estimate in the conventional and unconventional monetary policy subsamples, respectively. To evaluate the instantaneous effects, which are summarized by $\Delta y_t^{(1)}(0) + \Delta y_t^{(2)}(0)$, we also estimate the regression:

$$\Delta y_t^{(1)}(\tau) + \Delta y_t^{(2)}(\tau) = \kappa_4(\tau) + \gamma_4(\tau)\varepsilon_t^{trad} + \eta_{4,t}. \tag{37}$$

Figures 7 and 8 report estimates of $\gamma_i(\tau)$ for the two traditional monetary policy shocks that we consider: Romer and Romer (2004) and Wu and Xia (2014), respectively. In each figure, the top panel (A) shows the values of $\gamma_i(\tau)$ for the conventional monetary policy period (2/1995-10/2008) while the bottom panel (B) shows those for the unconventional monetary policy period (11/2008-4/2014).

INSERT FIGURES 7 AND 8 HERE

Clearly, the figures show drastic changes in the regression coefficients. While in the conventional period $\gamma_4(\tau)$ is the highest at short maturities, it is the highest at the long maturities in the unconventional period. This suggests that the conventional shock is measuring only the short-term effects of monetary policy and does not contain much information regarding its medium to long-term effects, which instead our shock can summarize. Furthermore, the relationship between $\Delta y_t^{(1)}(\tau)$ and the monetary policy shock, which is constant by construction across maturities, changes from very small and negative in the conventional monetary policy period to positive and large in the unconventional period. In addition, with our identification procedure, we find that the regression coefficient between the curvature $(\Delta y_t^{(3)}(\tau))$ and the monetary policy shock changes from negligible to negative values between the two periods, and features a hump-shape in the unconventional period, peaking around 30 months. Thus, our analysis can identify important channels describing how monetary policy has changed over time when moving to the unconventional period. Figure 8 shows that the results are similar for Wu and Xia's (2014) shock. The Not-for-Publication Appendix shows that our empirical results are qualitatively the same if we use Krippner's (2015) shadow rate shock.

Figure 9 and 10 plot the components of the estimated functional monetary policy shocks over time. Note how the nature of the monetary policy shock changes over time. The behavior of $\Delta\beta_{1,t}$ is somewhat constant over time. The behavior of $\Delta\beta_{2,t}$ and $\Delta\beta_{3,t}$ also changed, becoming larger in magnitude in 2008-2009, suggesting important changes in the short-run and medium-run components of the monetary policy as well. The fact that the nature of the shocks has changed over time is confirmed by an outlier detection (Tukey's range) test.[26]

INSERT FIGURES 9 AND 10 HERE

7 The Effects of Monetary Policy Shocks

We estimate the effects of monetary policy by using our functional shocks approach. Ideally, Vector Autoregressions (VARs) allow comparisons between our empirical results and those of existing methods during the conventional period, where the VAR is a frequently used approach. This would require including monetary policy shocks as variables in the VAR; however, since the monetary policy shocks can be zero at times when there is no monetary policy shock, this is not possible. Therefore, we estimate the responses using local projections (Jordà, 2005), which are also more robust than VARs to lack of invertibility (Stock and Watson, 2018).

We consider two approaches. The baseline approach is the functional local projection, which relies on estimating eq. (38) by OLS; the second is the FLP-IV, which relies on estimating eq. (38) by IV. The FLP-IV-based framework is robust to the possibility that changes in the yield curve may be due to shocks other than monetary policy, as it uses the term structure of Fed Fund futures as the instrument.

7.1 The Functional Local Projection Approach

We estimate the responses directly from the following regression, based on eq. (26):

$$X_{t+h} = \mu_h + \Theta_{12,h}^{(1)}(L)\,\Delta\beta_{1,t}^d + \Theta_{12,h}^{(2)}(L)\,\Delta\beta_{2,t}^d + \Theta_{12,h}^{(3)}(L)\,\Delta\beta_{3,t}^d + \varphi'(L)\,X_{t-1} + e_{h,t+h}, \quad (38)$$

where X_t contains inflation and industrial production; $h = 1, 2, ..., H$ is the horizon of the response and the lag length is 2. The coefficients $\Theta_{12,h}^{(j)}$ are the responses at time $(t+h)$ to a structural shock in $\beta_{j,t}$ at time t, $j = 1, 2, 3$.

Since we are working with data estimated at different frequencies (the term structure is daily, while inflation and industrial production are monthly), we need to attribute the shock (i.e., the daily change in the term structure at the time of a monetary policy announcement) to a given month. We attribute the shock to the month in which it took place. Also, since $\Delta\beta_{1,t}^d$ and $\Delta\beta_{2,t}^d$ appear to be collinear, two factors may be sufficient to describe changes in the term structure during the conventional period. Thus, in practice, we include only $\Delta\beta_{2,t}^d$ and $\Delta\beta_{3,t}^d$ in eq. (38).

We assume that, on monetary policy announcements dates, unexpected changes in monetary policy shift the entire yield curve by simultaneously changing the $\beta_t's$. We then use

[26] The outliers are in the last months of 2008. The Not-for-Publication Appendix visually shows, using scatterplots, that the dimension of the monetary policy shock is not one (i.e. $q \neq 1$).

the chain rule to identify the response of macroeconomic variables to the unconventional monetary policy shock as follows:

$$\frac{\partial X_{t+h}}{\partial \varepsilon_{f,t}(.)} = \frac{\partial X_{t+h}}{\partial \Delta \beta_t^{d\prime}} \frac{\partial \Delta \beta_t^d}{\partial \varepsilon_{f,t}(.)} = \sum_{j=1}^{q} \Theta_{12,h}^{(j)} \Delta \beta_t^d, \tag{39}$$

where the first component on the right hand side, $\frac{\partial X_{t+h}}{\partial \Delta \beta_t^{d\prime}}$, is estimated in the eq. (38), and the second component, $\Delta \beta_t^d = \Delta \beta_t \cdot d_t$, is the change in the term structure (proxied by $\Delta \beta_t$) times a dummy variable (d_t) equal to unity if there is a monetary policy announcement at time t.[27]

We use a high frequency identification that relies on the following set of identification conditions:

Assumption I.

(a) *Shock identification condition*: Inflation and output are not contemporaneously affected by yield curve shocks.

(b) *Relevance condition*: A change in the yield curve on an announcement date is only due to the monetary policy shock.

(c) *Exogeneity condition*: The change in the yield curve after an announcement date in the sampling period is not due to the monetary policy shock.

Under Assumption I, the method described in the paper correctly identifies the effects of monetary policy shocks.

The particular type of identification that we choose (the high frequency identification in Assumption I) follows Gürkaynak et al. (2005a). However, note that our "functional shock" approach does not necessarily rely on a high frequency identification: recursive, sign-restrictions or other typical restrictions can be used as well, as highlighted in Section 4. Assumption I(a) implies that output and inflation do not respond to a monetary policy shock within a few weeks, due to the fact that it takes time to change prices and to adjust production. Importantly, note that we do not need to separately identify shocks to each of the different components in the yield curve (i.e. each of the $\beta_t^{\prime} s$): the monetary policy shock is a simultaneous change in the whole yield curve. Note that Assumption I(a) could be removed, as one might leave the coefficient unrestricted under the assumption that the shock is strictly exogenous contemporaneously; we prefer to be robust and impose this assumption in our estimation.

Assumption I(b) is not as restrictive as it may seem. The assumption is still empirically valid if, on announcement days, the magnitude of the monetary policy shock is significantly bigger than that of any other shock. In principle, it is possible to improve the likelihood that this assumption holds by shortening the window of time in which the shock is identified. In the empirical application in this paper, we assume a one-day window, consistently with the finding in Gürkaynak et al. (2005a) that a window of one day is sufficient to describe monetary policy behavior. Assumption I(c) requires that, for example, there is only one

[27] As discussed in Section 4, we report a smoothed estimate, obtained by fitting a fourth-order polynomial to $\Theta_{12,h}^{(j)}$ across horizons.

monetary policy shock in any given month in a monthly dataset. In practice, there are a handful of months with more than one shock, in which case we take the average of the shocks. Finally, one should interpret the empirical results as if the monetary policy shock realizes at the end of the month. Note that this is the implicit assumption underlying VARs estimated at the monthly frequency for the conventional period.[28]

Equation (39) shows that each monetary policy announcement has a different impulse response, which is realistic and enhances our understanding of monetary policy. In contrast, the conventional analysis imposes that impulse responses are identical up to scale across different announcements.

To allow for changes in the transmission mechanism coefficients $\Theta_{12,h}^{(j)}$ in different monetary policy periods, we estimate eq. (38) separately in two sub-samples: the conventional monetary policy period (1995:1-2008:10) and the unconventional period (2008:11-2016:6). Note that the second sub-sample starts in November 2008, given that November 25 2008 marked the start of the first large scale asset purchasing program, LSAP-I.

7.1.1 Empirical Results on the Effects of Conventional Monetary Policy

Traditional VAR approaches typically identify monetary policy shocks during conventional times as changes in the short-term interest rate that are not caused by an endogenous reaction to the current state of the economy. In those approaches, the effects of monetary policy are estimated as the reaction to, say, an exogenous unitary increase in the short-term interest rate.[29] Thus, there is one impulse-response, and the effects of monetary policy proportionally depend on the magnitude of the increase (or decrease) in the short-term interest rate. The responses to a monetary policy shock in a traditional VAR in the literature typically show that output and inflation decrease after an unexpected monetary policy tightening (e.g. see Stock and Watson, 2001, p. 107). In our framework, instead, the responses of the macroeconomic variables depend on the combination of the shocks, and can, in principle, differ depending on the way the term structure changes beyond just the short-run effect. We depict responses for selected episodes in Figure 11, Panels A and B. For each episode, the figures depict the change in the term structure (panel on the right) and the corresponding response of the macroeconomic variable (panel on the left). Notice how a similar decrease in the short-run interest rate may result in different output responses by comparing the 11/6/2001 and the 9/29/1998 announcements (depicted in the top two panels in Figure 11, Panel A). Both announcements resulted in a decrease in short-term interest rates of similar magnitude ($\Delta\beta_{1,t}^{d} + \Delta\beta_{2,t}^{d}$ around -0.2 from Table 1); yet, the former resulted in a short-run decrease in output while output increased in the latter. The reason is the very different behavior of $\Delta\beta_{2,t}^{d}$ and $\Delta\beta_{3,t}^{d}$: in the former, one decreased and the other increased, while

[28]Alternatively, one could design alternative weighting schemes to take into account the day of the month in which the shock realized, and adjust for the length of time in which output could have responded to the shock. The Not-for-Publication Appendix shows that our results are robust to rescaling the shocks by the number of days in the month after the shock took place, as well as to summing the shocks over the month (rather than averaging them).

[29]Alternatively, the response can be measured as the reaction to a one standard deviation increase in the short-term interest rate. The logic of the argument that follows is unaffected by choice of the unit or measure.

in the latter both increased. Their opposite behavior resulted in a proportionally larger decrease in long-term interest rates in the latter episode. A similar result holds for the response of inflation in these episodes: inflation decreases in the former and increases in the latter.

INSERT FIGURE 11 HERE

7.1.2 Empirical Results on the Effects of Unconventional Monetary Policy

Our results in Section 5 show that, with some exceptions (e.g. in 1/28/2009), typically after a quantitative easing the term structure rotates towards the origin, implying a decrease in both the short- and the medium-term interest rates (cfr. Figure 3, Panel B, and Figure 5). In most cases, the decrease in the level of the term structure is associated with an increase in the slope and an increase in the curvature, whose combined action results in stronger effects of monetary policy at the long end of the term structure.

INSERT FIGURE 12 HERE

Figure 12 (Panels A and B) plot the responses of macroeconomic aggregates to selected unconventional monetary policy shocks. Panel A in Figure 12 shows that quantitative easing typically increases output after a few months (about six), as one would expect from theory; the response is hump-shaped, with the largest effects after about one to one and a half year after the shock, and starting to disappear after two years. The magnitude of the effect varies depending on the episode: the maximum effect is typically between one and two percent. Some of the largest output responses (peaking around one percent) are on 11/25/2008 and 12/16/2008: the first is associated with the announcement that started LSAP-I, and the second with the reduction of the FFR to its effective zero lower bound. Hence, indeed, we find that the announcement of the large scale asset purchases did change the yield curve substantially. There are two occasions where the monetary policy easing decreased subsequent industrial production, and are these are two dates where the term structure moved in the opposite direction, that is 1/28/2009 and 9/13/2012. The first is in line with well-known fact that the Federal Open Market Committee (FOMC) statement of 1/28/2009 was considered disappointing by financial markets, as it did not contain concrete language regarding the purchase and timing of long-term Treasuries in the secondary markets (Gilchrist et al., 2013); the second episode is the announcement of LSAP-III. In both cases, however, the level increased while both the slope and the curvature decreased and long-term interest rates actually decreased (see Table 1).

The effects on inflation are also similar to what would be expected by theory – see Figure 12, Panel B. In particular, one would expect inflation to increase after a monetary policy easing; this is what we find in most cases, again except 1/28/2009 and 9/13/2012. In general, we find that the response of inflation is hump-shaped and peaks about 6 to 10 months after the shock, similarly to industrial production. However, the effects on inflation die away more slowly than those on output, and are still different from zero even after 20 months.

Note that the confidence bands are large. This is potentially due to the local projection approach: on the one hand, the approach is useful to guard against potential non-invertibilities; on the other hand, it leads to less precise estimates of the responses since it does not impose the constraints associated with a parametric VAR structure.

Overall, our main conclusion is that unconventional monetary policy shocks lead to an expansion in output and an increase in inflation when the financial markets interpret the monetary policy easing as a decrease in interest rates in the medium to long run. However, their overall effects in terms of magnitude differ across episodes.

7.2 A Time-Varying FLP-IV Model

In the second approach, we estimate the effects of monetary policy using the functional local projections with external instruments (FLP-IV) method.

To take into account potential non-linearities in the effects of policy changes when monetary policy switches to quantitative easing in the zero lower bound period, we consider the following state dependent FLP model:

$$X_{t+h} = \mu_{h,t} + \Theta^{(1)}_{12,h,t}(L)\,\Delta\beta^d_{1,t} + \Theta^{(2)}_{12,h,t}(L)\,\Delta\beta^d_{2,t} + \Theta^{(3)}_{12,h,t}(L)\,\Delta\beta^d_{3,t} + \varphi'_t(L)\,X_{t-1} + e_{h,t+h}, \quad (40)$$

where the time-varying parameters are state dependent: $\Theta_{12,h,t}(L) = \Theta^{\dagger}_{12,h,t}(L)$ and $\varphi_t(L) = \varphi^{\dagger}(L)$ if $d_{t-1} = 1$, and $\Theta_{12,h,t}(L) = \Theta^{\ddagger}_{12,h,t}(L)$ and $\varphi_t(L) = \varphi^{\ddagger}(L)$ if $d_{t-1} = 0$, for $j = 0, 1, 2, 3$; d_{t-1} is a dummy variable that indicates the state of monetary policy when the shock hits and $h = 1, 2, ..., H$. In practice, we use the short-term interest rate as the indicator of the state of monetary policy, such that $d_{t-1} = 1$ if the short-term interest rate is above 0.75.[30]

The instruments set is the change in high frequency futures data from the Chicago Board of Trade (CME group), and includes 2-, 5- and 10-year T-note futures, as well as the 30-year T-bond futures.[31] The change is calculated in a narrow window of time around the announcement, starting 10 minutes before and ending 20 minutes after the announcement.

The approach relies on the following set of identification conditions:

Assumption I-FLP-IV.

(a) *Shock identification condition*: Inflation and output are not contemporaneously affected by yield curve shocks.

(b) *Relevance condition*: A change in the futures on an announcement date is only due to the monetary policy shock.

(c) *Exogeneity condition*: The change in the futures after an announcement date in the sampling period is not due to the monetary policy shock.

Under Assumption I-FLP-IV, the method described in the paper correctly identifies the effects of monetary policy shocks.

[30] As before, X_t contains inflation and industrial production and the lag length is two.

[31] The data are available at: https://www.tickdata.com/historical-market-data-products/futures-data/available-futures-data/

Figure 13 reports the results. Again, unambiguous decreases in the yield curve typically increase output and inflation, while unambiguous increases typically decrease both output and inflation. On the other hand, movements in the term structure that affect different maturities in opposite ways may result in either increases or decreases in output and inflation; in state 1, in particular, the responses are often negative in the short-run if the short end of the term structure increases, and positive in the medium-run if the long end of the term structure decreases. State 1 is typically associated to episodes of conventional monetary policy.

INSERT FIGURE 13 HERE

7.3 Which Features of Monetary Policy Shocks Matter The Most To Explain Macroeconomic Fluctuations?

How much of the responses of output and inflation to monetary policy shocks are associated with changes in specific features of the shape of the term structure of interest rates? Or, in other words, what are the effects of the various dimensions of monetary policy on output and inflation over time? Figures 14, Panels A and B, report such a decomposition for episodes in state 1, while Figure 15 does the same for state 0.

By comparing Panels A,B in Figure 14 with Figure 13, it is clear that, in state 1, the responses of output and inflation are mainly explained by changes in level ($\Delta\beta_{1,t}^{d}$); the contribution of $\Delta\beta_{2,t}^{d}$ is mostly negligible, hinting at the fact that most of the responses of output and inflation are related to changes in the short-term movements in the term structure proxied by $\Delta\beta_{1,t}^{d}+\Delta\beta_{2,t}^{d}$. There are exceptions, however; for example, the shocks in 5/2000 and 1/2004 are mainly attributed to the curvature ($\Delta\beta_{3,t}^{d}$) – that is, how monetary policy affects medium-term expectations. In both episodes, in fact, the curvature shows up prominently in the shock, changing the slope from positive to negative around medium-term maturities. While the level factor is typically related to expected inflation and the slope is typically related to expected real activity, the curvature factor has so far eluded an economic interpretation in the literature. Our results suggest an interesting interpretation of the elusive curvature factor in some monetary policy episodes: the curvature is correlated with the unanticipated effects of monetary policy and with how inflation and output respond to unexpected changes in monetary policy in the conventional period.

INSERT FIGURES 14 AND 15 HERE

Turning to the unconventional period, a comparison of Panels C,D in Figure 13 with Panels A,B in Figure 15 similarly reveals that the way monetary policy affects future output is mainly explained by the level effect. However, notice now that the contribution of $\Delta\beta_{2,t}^{d}$ is not irrelevant anymore; in fact, it typically has the opposite effect; thus, the level matters, namely $\Delta\beta_{1,t}^{d}$, and not just the short-run anymore.

7.4 Take-Away Points

Depending on the shape of the shock, the output responses can have rich dynamics. But how do different shocks translate into different output responses? Answering this question sheds light on what we can learn from our new approach. Figure 16 shows the major take-away point. Each panel in the figure analyzes how different shocks affect the response of output; we focus on state 0. Each of Panels (i)-(iv) has two pictures: the picture on the right depicts the shock to the term structure while the picture to the left depicts the response of output corresponding to that shock. For example, Panel (i) in the figure investigates how changing the shock to the level factor ($\Delta\beta_{1,t}$) affects the response of output. A shock to the level factor equally affects interest rates at all maturities. Panel (i) shows that, as the level factor shifts the term structure from positive to negative (with monetary policy shifting from contractionary to expansionary), the response of output changes from being negative to being positive. Panels (ii) and (iii) show that shocks to the slope and curvature factors have similar effects. Most interesting is the case of a shock that increases short-term interest rates but decreases long-term ones; such a shock is depicted in Panel (iv) by a long-dashed line. Output reacts by decreasing in the short-run and increasing in the medium-term (see the long-dashed line in the left graph). An opposite shock, that decreases short-term rates while increasing long-term ones, has the opposite effect on output, increasing output in the short-run while decreasing it in the medium-run. Thus, the major take-away point is that monetary policy shocks that unambiguously increase (or decrease) all yields result in an unambiguous negative (positive) output response; however, the response of output to shocks that increase interest rates at some maturities and decrease them at other maturities is more complex: typically, an increase (decrease) at the short-end of the yield curve is associated with a negative (positive) response of output in the short-run, while a decrease (increase) at the long-end is associated with an increase (decrease) in output in the medium-run.

Panel (iv) in the figure also shows another interesting point. Compare the shocks depicted by asterisk and triangle markers on the right hand side of the panel: they share the same movement in the instantaneous yield and at the 50 month maturity. However, the associated output responses on the left-hand-side of the panel show that the responses are the opposite. Thus, impulse responses associated to shocks that result in the same change in the yield at two maturities (or, more generally, a given sub-set of maturities) may still be very different from each other.

INSERT FIGURE 16 HERE

8 Conclusions

This paper proposes a novel approach to the analysis of economic shocks. We view shocks as exogenous shifts in a function, as opposed to changes in a scalar variable, and we propose to estimate their effects on the economy via functional VARs and functional local projection approaches.

In our empirical analysis, in particular, we define monetary policy shocks as shifts in the whole term structure in a short window of time around monetary policy announcements

– as opposed to exogenous changes in just short-term interest rates. This allows us to summarize more broadly the effects that monetary policy has, including the information that it transmits to financial markets regarding the medium and long run path of interest rates. In addition, by being more comprehensive, our identification procedure allows us to estimate unconventional monetary policy shocks in a way similar to that in the conventional monetary policy period.

We find that monetary policy events that manifest themselves as unambiguous decreases in the whole term structure of interest rates have expansionary effects in both conventional and unconventional times, and vice-versa. However, the shape of the monetary policy shock significantly affects the shape of the macroeconomic responses: shocks that manifest themselves as decreases at the short end of the term structure and increases at the long end have expansionary effects on output and inflation in the short-run, but contractionary effects in the long-run.

More generally, our "functional shocks" approach can be applied to many other settings where the shock is a shift in a function, such as demand and supply, uncertainty, fiscal and tax policy or productivity shocks, some of which we are currently investigating.

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Appendix A

A.1 Technical Definitions

Yield curves can be viewed as functions that map $\Re_+$ to $\Re$, which we will denote by $f_t(\cdot)$. Define a space of such yield curves by $\mathcal{B}$ with norm $\|\cdot\|$. Also, let

$$\psi_t(f_t(\cdot)) \equiv E(x_{t+h}|f_t(\cdot), \mathcal{I}_t)$$

where x_t is a variable of interest, such as inflation and output. To simplify the notation, we drop the subscript t from this point on.

The h-step ahead impulse response of a variable is the "derivative" of its expected value with respect to a yield curve. Let $f(\cdot) \in \mathcal{B}$ and $f^*(\cdot) \in \mathcal{B}$. If

$$\partial\psi(f(\cdot); f^*(\cdot)) = \lim_{\alpha \to 0} \frac{\psi(f(\cdot) + \alpha f^*(\cdot)) - \psi(f(\cdot))}{\alpha} \tag{41}$$

exists, it is called the *Gateaux* differential of ψ at $f(\cdot)$ with direction (or increment) $f^*(\cdot)$. If the limit exists for each $f^*(\cdot) \in \mathcal{B}$, it is said to be *Gateaux* differentiable.

A.2 Finite-dimensional representation

Suppose that $g_1(\cdot)$, ..., $g_q(\cdot)$ are known functions that map the set of maturities, $\mathcal{T}$, to $\Re$, where q is a known positive integer. Define a class of functions of the form:[32]

$$\{f : f(\tau) = \sum_{j=1}^{q} c_j g_j(\tau), \ for\ some\ c_1, c_2, ..., c_q\}. \tag{42}$$

For example, $q = 3$, $g_1(\tau) = 1$, $g_2(\tau) = (1 - e^{-\lambda\tau})/(\lambda\tau)$ and $g_3(\tau) = (1 - e^{-\lambda\tau})/(\lambda\tau) - e^{-\lambda\tau}$ in the Nelson and Siegel (1987) model, where, for simplicity, we ignore the dependence of the function $g(.)$ on nuisance parameters. It should be noted that the linear specification is not necessary for local projections, however.

The Functional VAR Model. Consider the following p-th order functional VAR model in eqs.(3)-(4):

$$\Phi_{11}(L)X_t + \Phi_{12}(L)\int_{\mathcal{T}} w(\tau)\, f_t(\tau)\, d\tau = \mu_x + u_{x,t}, \tag{43}$$

$$\Phi_{21}(L,\cdot)X_t + \Phi_{22}(L)f_t(\cdot) = \mu_f(\cdot) + u_{f,t}(\cdot). \tag{44}$$

The Functional MA Representation. Solving eq.(44) for $f_t(\cdot)$, and omitting the intercept terms μ_x and $\mu_f(.)$ for notational simplicity, we obtain

$$f_t(\cdot) = -\Phi_{22}^{-1}(L)\Phi_{21}(L,\cdot)X_t + \Phi_{22}^{-1}(L)u_{f,t}(\cdot), \tag{45}$$

[32] The class of functions define functions to be a linear combination of q basis functions. A function at time t is an element of this set and so is a function at time t.

and substituting it into eq.(43),

$$
\begin{aligned}
X_t &= \Phi_{11}^{-1}(L)\Phi_{12}(L)\int_{\mathcal{T}} w(\tau)\left[\Phi_{22}^{-1}(L)\Phi_{21}(L,\tau)X_t - \Phi_{22}^{-1}(L)u_{f,t}(\tau)\right]d\tau + \Phi_{11}^{-1}(L)u_{x,t} \\
&= \Phi_{11}^{-1}(L)\Phi_{12}(L)\int_{\mathcal{T}} w(\tau)\Phi_{22}^{-1}(L)\Phi_{21}(L,\tau)d\tau X_t + \Phi_{11}^{-1}(L)u_{x,t} \\
&\quad -\Phi_{11}^{-1}(L)\Phi_{12}(L)\int_{\mathcal{T}} w(\tau)\Phi_{22}^{-1}(L)u_{f,t}(\tau)d\tau.
\end{aligned}
$$

Solving the above equation for X_t, we have

$$
\begin{aligned}
X_t &= \left[I_n - \Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)\int_{\mathcal{T}} w(\tau)\Phi_{21}(L,\tau)d\tau\right]^{-1}\Phi_{11}^{-1}(L)u_{x,t} \\
&\quad -\left[I_n - \Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)\int_{\mathcal{T}} w(\tau)\Phi_{21}(L,\tau)d\tau\right]^{-1}\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)\int_{\mathcal{T}} w(\tau)u_{f,t}(\tau)d\tau.
\end{aligned} \tag{46}
$$

The first term in the above equation is the effect of lags of $u_{x,t}$ on X_t, $\Phi_{22}^{-1}(L)$ is a scalar and the second term is the effect of $u_{f,t}(\cdot)$ and its lags on X_t. Substituting eq.(46) in eq.(45), we have:

$$
\begin{aligned}
f_t(\cdot) &= \Phi_{22}^{-1}(L)\Phi_{21}(L,\cdot)\left[I_n - \Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)\int_{\mathcal{T}} w(\tau)\Phi_{21}(L,\tau)d\tau\right]^{-1}\times \\
&\quad \times\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)\int_{\mathcal{T}} w(\tau)u_{f,t}(\tau)d\tau + \Phi_{22}^{-1}(L)u_{f,t}(\cdot) \\
&\quad -\Phi_{22}^{-1}(L)\Phi_{21}(L,\cdot)\left[I_n - \Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)\int_{\mathcal{T}} w(\tau)\Phi_{21}(L,\tau)d\tau\right]^{-1}\Phi_{11}^{-1}(L)u_{x,t}.
\end{aligned} \tag{47}
$$

Eqs. (46) and (47) provide the functional MA representation.

The Impulse Responses Based on the Functional VAR. Let $\mu_f(\cdot)$, $f_t(\cdot)$, $\Phi_{21,1}(\cdot)$,...,$\Phi_{21,p}(\cdot)$ and $u_{f,t}(\cdot)$ belong to the class of functions described in eq. (42) and let $\beta_{1,t}, ..., \beta_{q,t}$; $\widetilde{\Phi}_{21,s,1}, ..., \widetilde{\Phi}_{21,s,q}$; $\widetilde{\mu}_1, ..., \widetilde{\mu}_q$; and $\widetilde{u}_{1,t}, ..., \widetilde{u}_{q,t}$ denote the constants $c_1, ..., c_q$ of $f_t(\cdot)$, $\Phi_{21,s}(\cdot)$, $\mu_f(\cdot)$ and $u_{f,t}(\cdot)$, respectively, $s = 1, ..., p$. For notational convenience, let $\beta_{1:q,t} \equiv \left[\beta_{1,t}, ..., \beta_{q,t}\right]'$, $g(\tau) \equiv [g_1(\tau), ..., g_q(\tau)]'$ and $\widetilde{u}_{1:q,t} \equiv [\widetilde{u}_{1,t}, ..., \widetilde{u}_{q,t}]'$. Furthermore, let G_j be defined as in eq.(2), $G \equiv [G_1, G_2, \cdots, G_q]'$, $\widetilde{\Phi}_{21,j}(L) \equiv \sum_{s=1}^{p}\widetilde{\Phi}_{21,s,j}L^s$ and $\widetilde{\Phi}_{21,1:q}(L) \equiv -[\widetilde{\Phi}'_{21,1}(L), \widetilde{\Phi}'_{21,2}(L), \cdots, \widetilde{\Phi}'_{21,q}(L)]'$. Note that, using eq.(7) and the definitions above, we have: $\Phi_{21}(L;\cdot) = -\sum_{j=1}^{q}\widetilde{\Phi}_{21,j}(L)\,g_j(\cdot)$. Thus, using eqs. (8) and (2), we have: $\int_{\mathcal{T}} w(\tau)\Phi_{21}(L,\tau)d\tau = -\sum_{s=1}^{p}\sum_{j=1}^{q}\widetilde{\Phi}_{21,s,j}L^sG_j$ $= -\sum_{j=1}^{q}\widetilde{\Phi}_{21,j}(L)\,G_j = G'\widetilde{\Phi}_{21,1:q}(L)$ and $\int_{\mathcal{T}} w(\tau)u_{f,t}(\cdot)d\tau = G'\tilde{u}_{1:q,t}$. Thus, eq.(46) can be rewritten as:

$$
\begin{aligned}
X_t &= -\left[I_n - \Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'\widetilde{\Phi}_{21,1:q}(L)\right]^{-1}\Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'\tilde{u}_{1:q,t} \\
&\quad +\left[I_n - \Phi_{11}^{-1}(L)\Phi_{12}(L)\Phi_{22}^{-1}(L)G'\widetilde{\Phi}_{21,1:q}(L)\right]^{-1}\Phi_{11}^{-1}(L)u_{x,t} \\
&\equiv C_{11}(L)u_{x,t} + C_{12}(L)\tilde{u}_{1:q,t},
\end{aligned} \tag{48}
$$

where $C_{11}(L) = \sum_{j=0}^{\infty} C_{11,j} L^j$ and $C_{12}(L) = \sum_{j=0}^{\infty} C_{12,j} L^j$.

Thus the h-step ahead impulse response of X_t to $u_{f,t}(\cdot)$ in the direction of $\tilde{u}^*_{1:q,t}$ is given by $C_{12,h}\tilde{u}^*_{1:q,t}$. In what follows, we will show how to conveniently estimate $C_{12,h}$ from the finite-dimensional VAR.

The Finite-dimensional VAR Model. Note that, again omitting the intercept terms μ_x and $\mu_f(.)$ for notational simplicity, eqs. (43)-(44) can be written as:

$$\begin{aligned}
\Phi_{11}(L)X_t + \Phi_{12}(L)\sum_{j=1}^{q}\beta_{j,t}G_j &= u_{x,t}, \\
-\sum_{s=1}^{p}\sum_{j=1}^{q}\widetilde{\Phi}_{21,s,j}g_j(.)\,X_{t-s} + \sum_{j=1}^{q}\beta_{j,t}g_j(.) - \sum_{s=1}^{p}\Phi_{22,s}\sum_{j=1}^{q}\beta_{j,t-s}g_j(.) &= \sum_{j=1}^{q}g_j(.)\,\widetilde{u}_{j,t}.
\end{aligned}$$

Because the last equation must hold at each $\tau \in \mathcal{T}$, it can be written as a finite-dimensional VAR model:

$$\begin{bmatrix} X_t \\ \beta_{1,t} \\ \vdots \\ \beta_{q,t} \end{bmatrix} = \begin{bmatrix} \Phi_{11,1} & \Phi_{12,1}G_1 & \Phi_{12,1}G_2 & \cdots & \Phi_{12,1}G_q \\ \widetilde{\Phi}_{21,1,1} & \Phi_{22,1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \widetilde{\Phi}_{21,1,q} & 0 & 0 & \cdots & \Phi_{22,1} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \beta_{1,t-1} \\ \vdots \\ \beta_{q,t-1} \end{bmatrix} + \cdots \tag{49}$$

$$+ \begin{bmatrix} \Phi_{11,p} & \Phi_{12,p}G_1 & \Phi_{12,p}G_2 & \cdots & \Phi_{12,p}G_q \\ \widetilde{\Phi}_{21,p,1} & \Phi_{22,p} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \widetilde{\Phi}_{21,p,q} & 0 & 0 & \cdots & \Phi_{22,p} \end{bmatrix} \begin{bmatrix} X_{t-p} \\ \beta_{1,t-p} \\ \vdots \\ \beta_{q,t-p} \end{bmatrix} + \begin{bmatrix} u_{x,t} \\ \widetilde{u}_{1,t} \\ \vdots \\ \widetilde{u}_{q,t} \end{bmatrix} \tag{50}$$

$$\text{i.e. } \underbrace{\begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L)G' \\ \widetilde{\Phi}_{21,1:q}(L) & \Phi_{22}(L) \end{bmatrix}}_{\equiv A(L)} \begin{bmatrix} X_t \\ \beta_{1:q,t} \end{bmatrix} = \begin{bmatrix} u_{x,t} \\ \widetilde{u}_{1:q,t} \end{bmatrix}, \tag{51}$$

where again, the intercept terms are omitted for notational simplicity; $A(L) \equiv I - A_1 L - ... - A_p L^p$, and the matrices $A_1, ..., A_p$ are defined according to eqs. (49)-(50).

The Finite Dimensional MA Representation. To derive the exact expressions in the vector moving average representation, rewrite the finite-dimensional reduced-form VAR in eq.(49) as follows. Provided X_t and $\beta_{1:q,t}$ are stationary, the reduced-form finite-dimensional VAR can be inverted to obtain the reduced-form vector finite-dimensional MA representation:

$$\begin{aligned}
\begin{bmatrix} X_t \\ \beta_{1:q,t} \end{bmatrix} &= C(L)\begin{bmatrix} u_{x,t} \\ \tilde{u}_{1:q,t} \end{bmatrix} \\
&= \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} u_{x,t} \\ \tilde{u}_{1:q,t} \end{bmatrix},
\end{aligned} \tag{52}$$

where $C(L) = A(L)^{-1}$, and, in particular, from eq.(51), using the partitioned inverse matrix formula and recalling that $\Phi_{22}(L)$ is a scalar lag polynomial, we have:

$$C_{12}(L) = -\left[\Phi_{11}(L) - \Phi_{12}(L)\Phi_{22}(L)^{-1}G'\widetilde{\Phi}_{21,1:q}(L)\right]^{-1}\Phi_{12}(L)\Phi_{22}(L)^{-1}G', \tag{53}$$

which is equivalent to $C_{12}(L)$ defined in eq.(48). In other words, one can estimate the finite-dimensional VAR model and calculate $C_{12}(L)$ in the standard fashion rather than estimate the functional VAR model. The Not-for-Publication Appendix derives the moving average coefficient matrices when $n = p = 1$.

Appendix A.3. Proof of eqs.(24)-(25)

Following Stock and Watson (2018), from Assumption FLP-IV (ii-iii), we have:

$$\begin{aligned} & E\left(Z_t X_{t+h}'\right) \\ = \;& E\left(Z_t \left[\Theta_{12,h}\Theta_{22,0}^{-1}\beta_{1:q,t} - \Theta_{12,h}\Theta_{22,0}^{-1}e_{\beta,t} + \Theta_{11,h}\varepsilon_{x,t} + \varepsilon_{t+h,t}\right]'\right) \\ = \;& E\left(Z_t\beta_{1:q,t}'\right)\Theta_{22,0}^{-1'}\Theta_{12,h}' + 0 \ . \end{aligned}$$

Note also that, from eq. (21), $E\left(Z_t\beta_{1:q,t}'\right) = E\left(Z_t\widetilde{\varepsilon}_{1:q,t}'\Theta_{22,0}'\right) = \alpha'\Theta_{22,0}' = \alpha'$ under Assumptions FLP-IV(i,iv). Thus, for any symmetric and positive definite matrix Ω of dimension $(m \times m)$:

$$\left[E\left(\beta_{1:q,t}Z_t'\right)\Omega E\left(Z_t\beta_{1:q,t}'\right)\right]^{-1}\left[E\left(\beta_{1:q,t}Z_t'\right)\Omega E\left(Z_t X_{t+h}'\right)\right] = \Theta_{12,h}'. \tag{54}$$

Under Assumptions FLP-IV, we can identify α' from $E\left(Z_t\beta_{1:q,t}'\right)$ and $\Theta_{12,h}'$ from eq. (54).

Appendix B

Data Description

We collect data from January 1995 to June 2016 on the term structure of yields, industrial production and inflation. We start the sample in 1995 as the Fed did not release statements of monetary policy decision after its FOMC meetings before 1994. Also, importantly, Gürkaynak et al. (2005a) show that, after 1995, daily data provide an accurate identification of monetary policy shocks, which provides another rationale for using daily yields from 1995 onward in our analysis. We end the sample at the end of the zero lower bound period.

Term structure

The term structure data used in Sections 3-5 are daily zero-coupon yields (mnemonics "SVENY") from Gürkaynak, Sack and Wright (2007) and include yields at 1 to 30 years maturities. The daily frequency is dictated by the availability of data: the highest frequency at which the term structure of yields is available is daily. While one might be interested in investigating the identification at a higher frequency, Gürkaynak, Sack and Swanson (2007a) show that daily data are sufficient for extracting monetary policy shocks using a high-frequency identification if the sample is limited to post-1995 data, which is our case. The 3- and 6-month zero-coupon yields are from the Federal Reserve Board H-15 release.

Inflation

Data on inflation is from the Federal Reserve Bank of St. Louis' FRED. Inflation is measured as the annual percentage change in the Consumer Price Index for All Urban Consumers – All Items; it is a monthly, seasonally adjusted time series. The mnemonics for the price definition we use is CPIAUCSL.

Output

Output is measured by the industrial production index and is transformed in an annual percent change. The data is from the Federal Reserve Bank of St. Louis' FRED. This series is monthly and seasonally adjusted as well, and the mnemonics of industrial production is INDPRO.

Tables and Figures

Table 1, Panel A. Monetary Policy Shocks in Selected Conventional Episodes

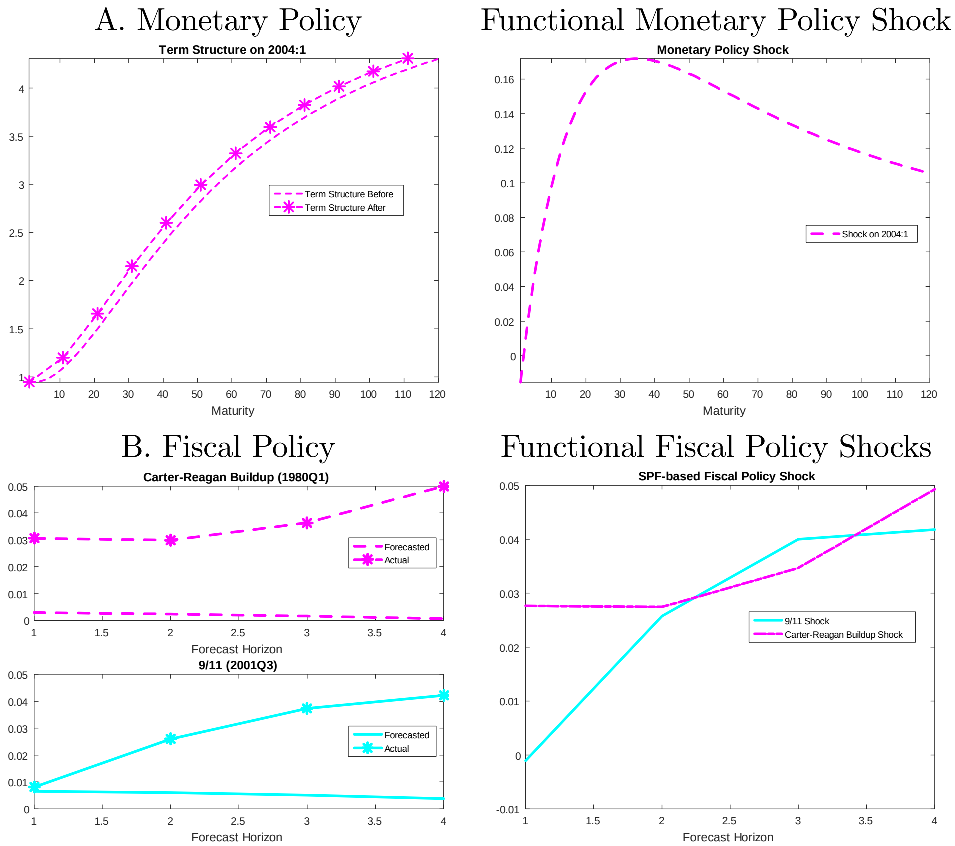
Date			Summary Statistics				
Month	Day	Year	$\Delta\beta_{1t}$	$\Delta\beta_{2t}$	$\Delta\beta_{3t}$	$\Delta\left(\beta_{1t}+\beta_{2t}\right)$	$\Delta\left(\beta_{3t}-\beta_{1t}\right)$
11	6	2001	-0.141	-0.092	0.153	-0.234	0.295
9	29	1998	-0.196	0.036	0.125	-0.159	0.322
2	3	1999	0.116	-0.195	0.222	-0.079	0.106
5	16	2000	-0.060	-0.157	0.574	-0.217	0.634
1	31	2007	-0.051	0.047	0.025	-0.004	0.076
1	28	2004	0.041	-0.074	0.547	-0.033	0.506

Table 1, Panel B. Monetary Policy Shocks in Selected Unconventional Episodes

Date			Summary Statistics				
Month	Day	Year	$\Delta\beta_{1t}$	$\Delta\beta_{2t}$	$\Delta\beta_{3t}$	$\Delta\left(\beta_{1t}+\beta_{2t}\right)$	$\Delta\left(\beta_{3t}-\beta_{1t}\right)$
11	25	2008	-0.392	0.360	0.063	-0.032	0.455
12	1	2008	-0.308	0.391	-0.133	0.083	0.174
12	16	2008	-0.609	0.504	1.182	-0.104	1.792
1	28	2009	0.347	-0.315	-0.131	0.032	-0.478
3	18	2009	-0.673	0.662	0.399	-0.010	1.072
8	10	2010	-0.219	0.218	0.246	-0.001	0.466
9	21	2010	-0.249	0.233	0.309	-0.015	0.558
11	3	2010	-0.217	0.235	0.168	0.017	0.386
8	9	2011	-0.334	0.318	0.314	-0.015	0.648
9	21	2011	-0.447	0.350	1.109	-0.096	1.556
1	25	2012	-0.196	0.237	0.030	0.040	0.226
6	20	2012	-0.023	0.014	0.143	-0.009	0.167
9	13	2012	0.162	-0.134	-0.321	0.027	-0.483

Note to the table. The table reports the estimated value of the shocks to the factors (or linear combinations thereof) at dates of selected monetary policy announcements.

Figure 1



Notes. The figure plots examples of functional shocks.

Figure 2, Panel A. US Yields Over Time

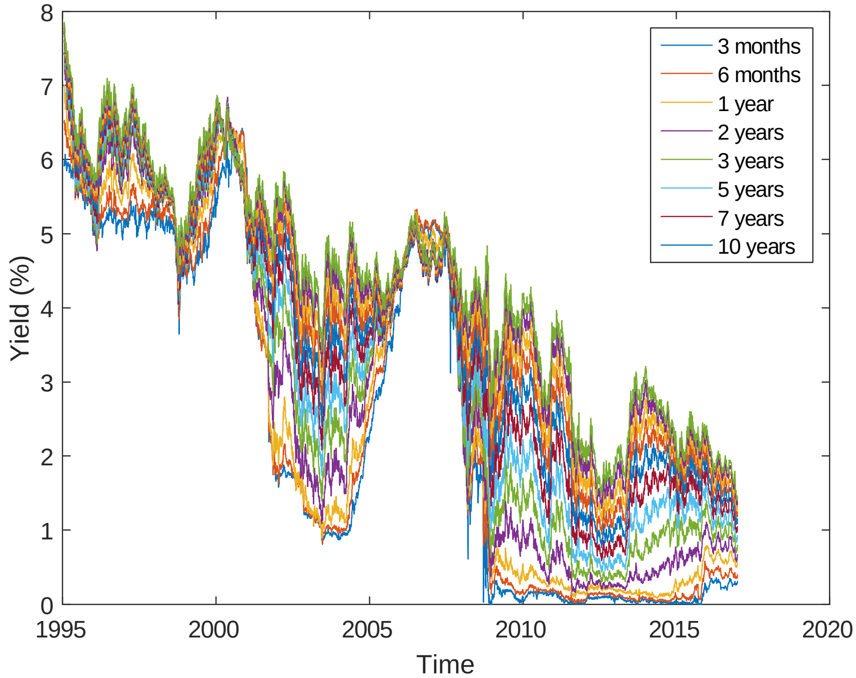
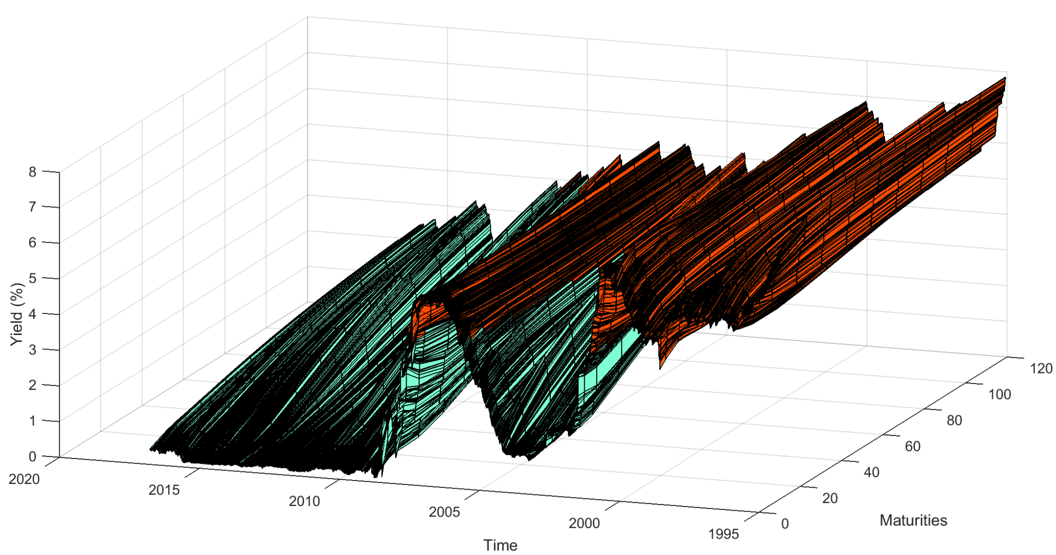


Figure 2, Panel B. The US Term Structure



Notes to the Figure. Panel A plots daily US Treasury yields over time; panel B plots the term structure of daily Treasury yields as a function of time and maturity.

Figure 3, Panel A. The Monetary Policy Shock in Conventional Times

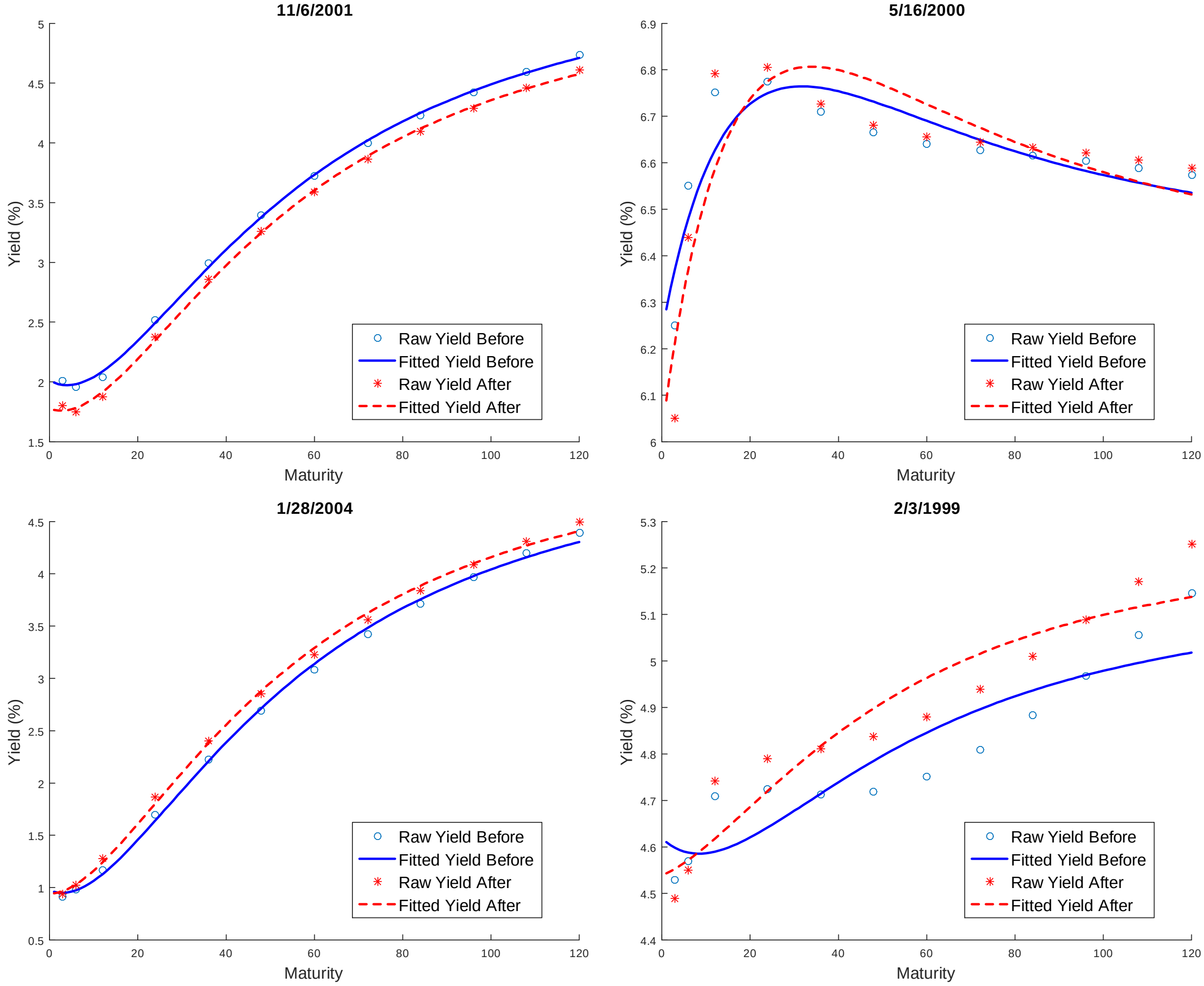
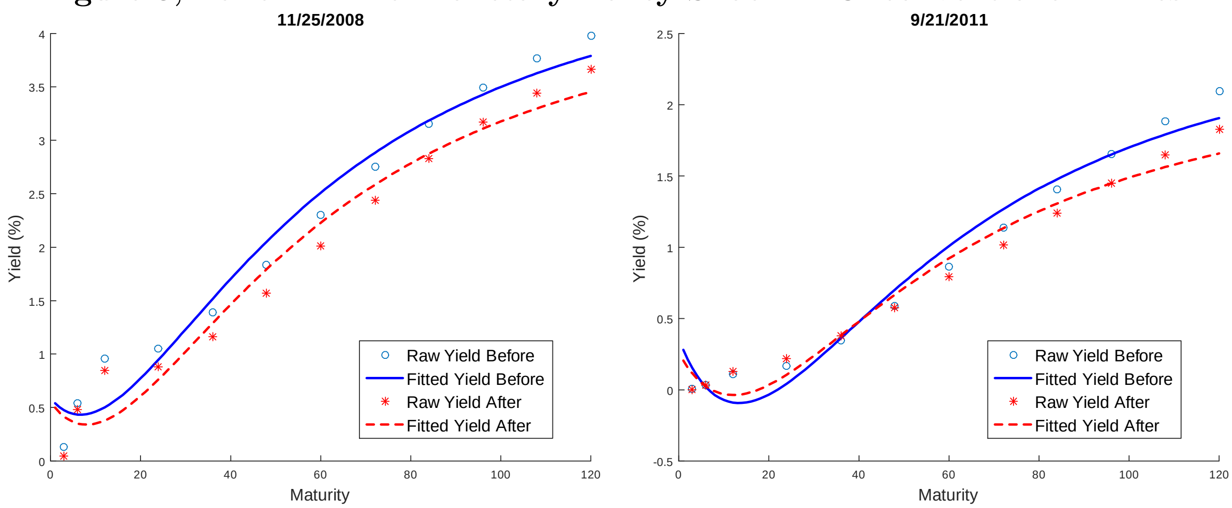


Figure 3, Panel B. The Monetary Policy Shock in Unconventional Times



Notes. The figure depicts a few representative examples of our newly defined monetary policy shock. The date is reported in the title.

Figure 4. The Monetary Policy Shock in Conventional Times

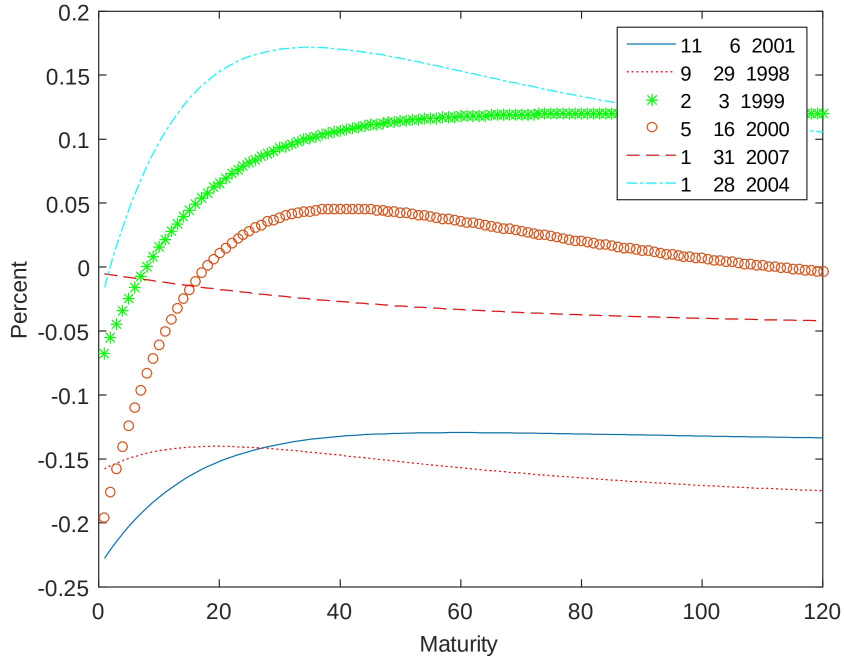
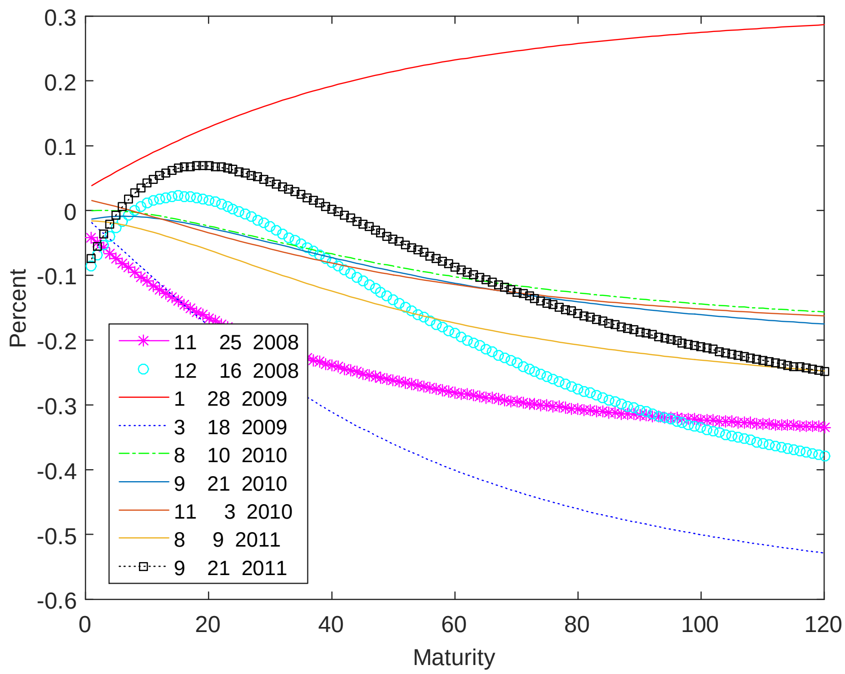


Figure 5. The Monetary Policy Shock in Unconventional Times

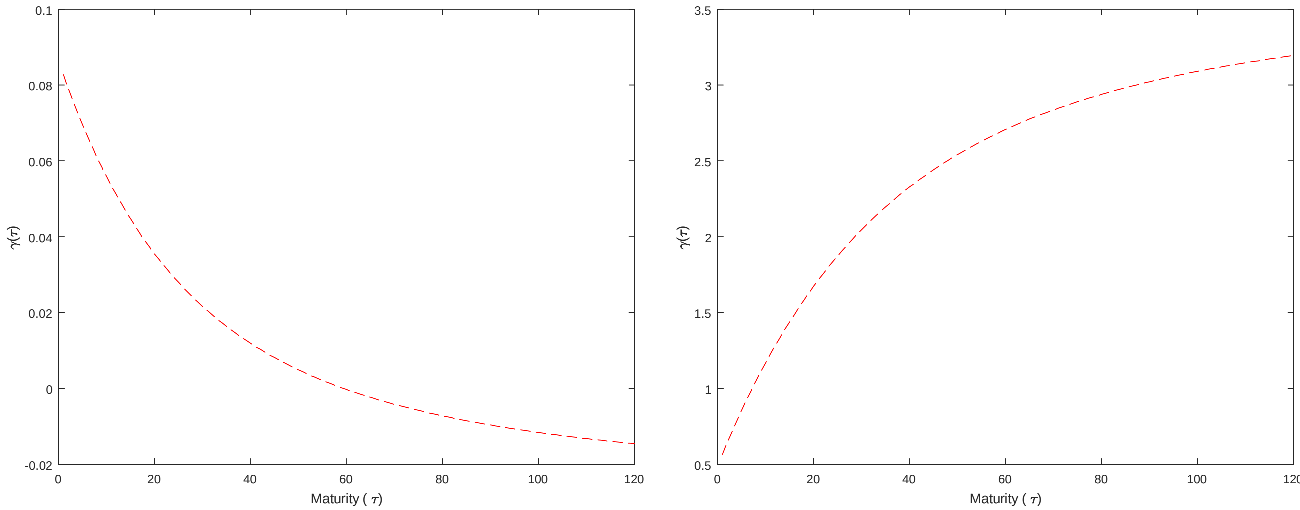


Notes. The figure depicts representative examples of our newly defined monetary policy shock during the conventional and unconventional monetary policy periods. The shock date is reported in the legend.

Figure 6: Relationship Between Our Monetary Policy Shock and Traditional Monetary Policy Shocks

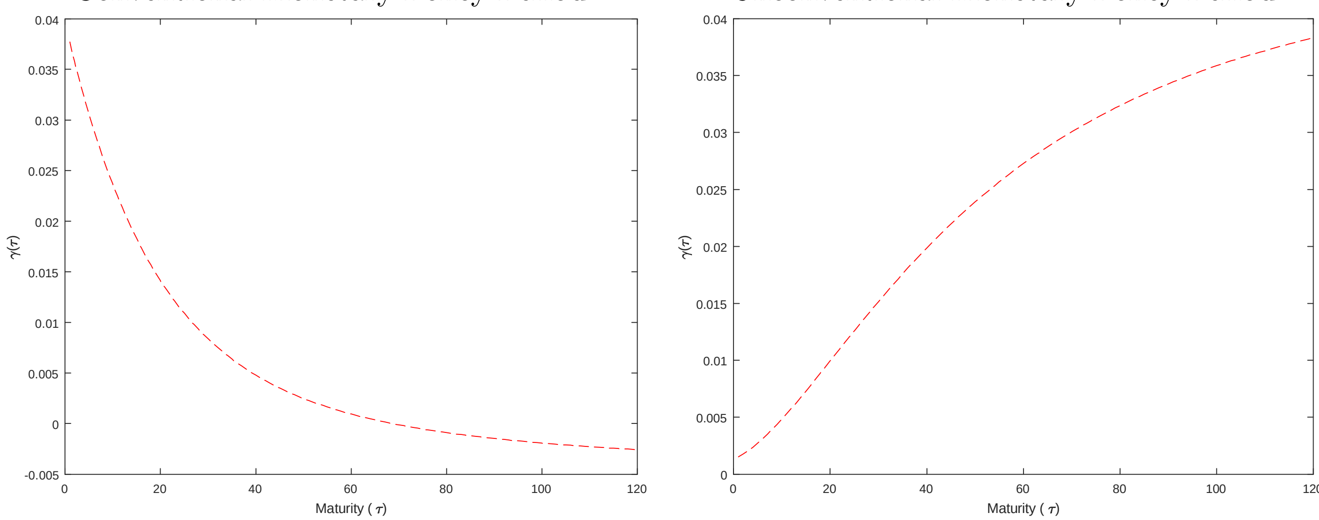
Panel A. Romer and Romer's Shock

Conventional Monetary Policy Period | Unconventional Monetary Policy Period



Panel B. Wu and Xia's Shock

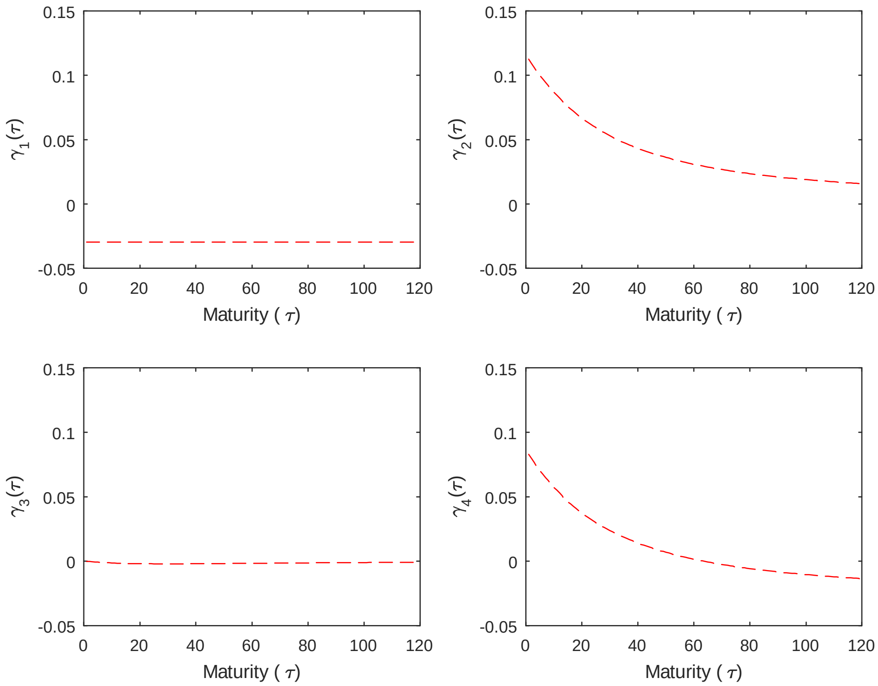
Conventional Monetary Policy Period | Unconventional Monetary Policy Period



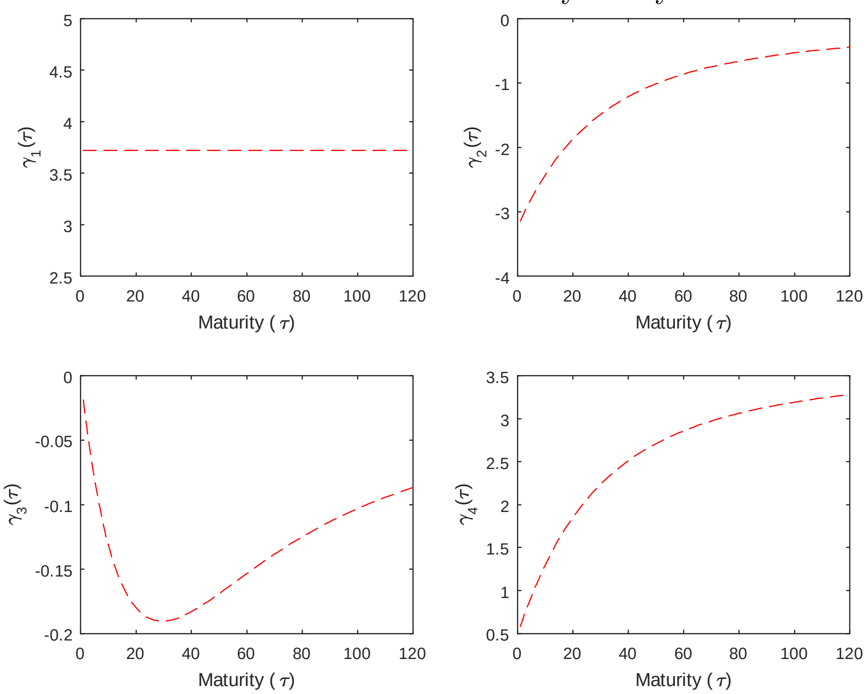
Notes. The figure depicts the coefficient $\gamma(\tau)$ in the regression of our functional monetary policy shock, $\varepsilon_{f,t}(\tau)$, on a traditional (narrative) monetary policy shock: Romer and Romer (2004) in the top panels and Wu and Xia (2014) in the bottom panels.

Figure 7. Our Shock vs. Romer and Romer (2004)

Panel A. Conventional Monetary Policy Period



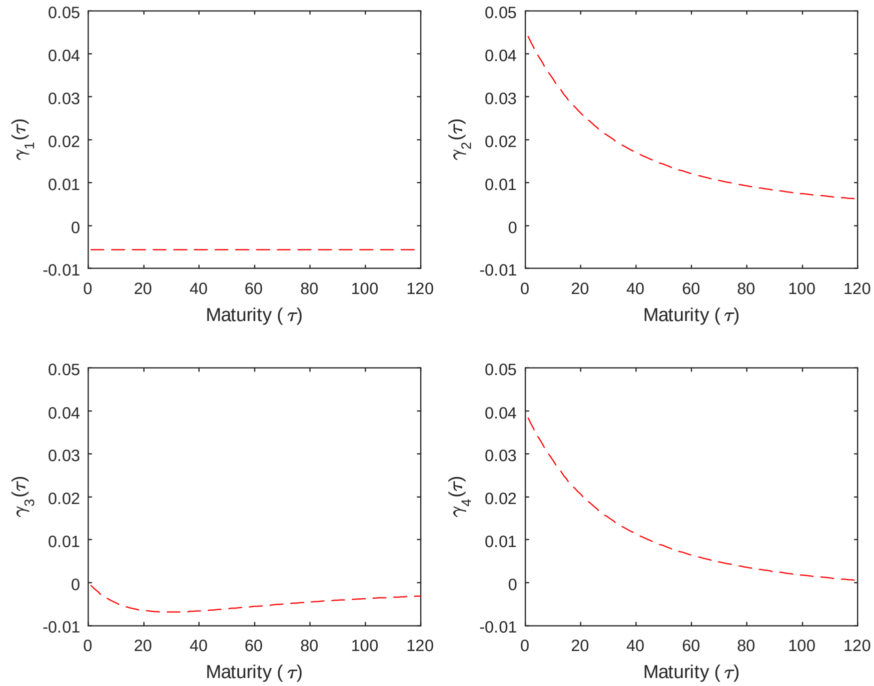
Panel B. Unconventional Monetary Policy Period



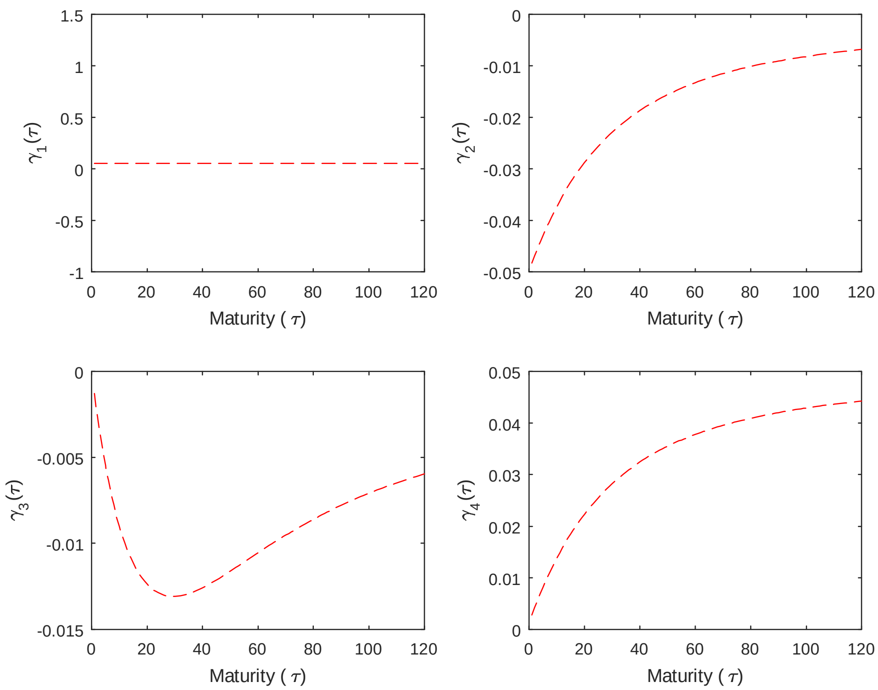
Notes. The figure depicts the coefficient $\gamma_j(\tau)$ in the regression of the components of our functional monetary policy shock, $\varepsilon_{f,t}(\tau)$, and Romer and Romer's (2004) traditional (narrative) monetary policy shock.

Figure 8. Our Shock vs. Wu and Xia (2014)

Panel A. Conventional Monetary Policy Period



Panel B. Unconventional Monetary Policy Period



Notes. The figure depicts the coefficient $\gamma_j(\tau)$ in the regression of the components of our functional monetary policy shock, $\varepsilon_{f,t}(\tau)$, and Wu and Xia's (2014) monetary policy shock.

Figure 9. The Components of Our Monetary Policy Shock

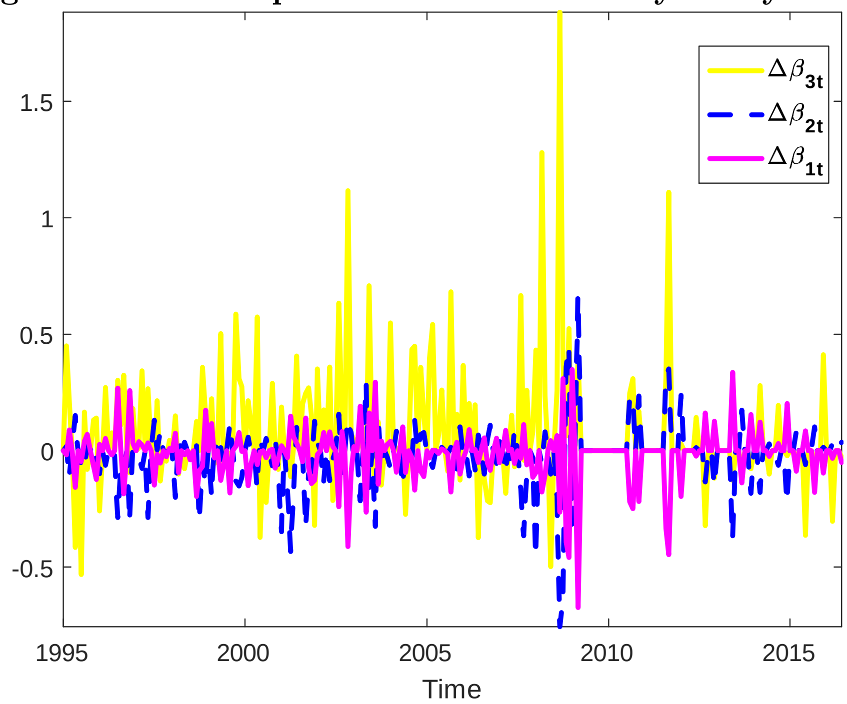
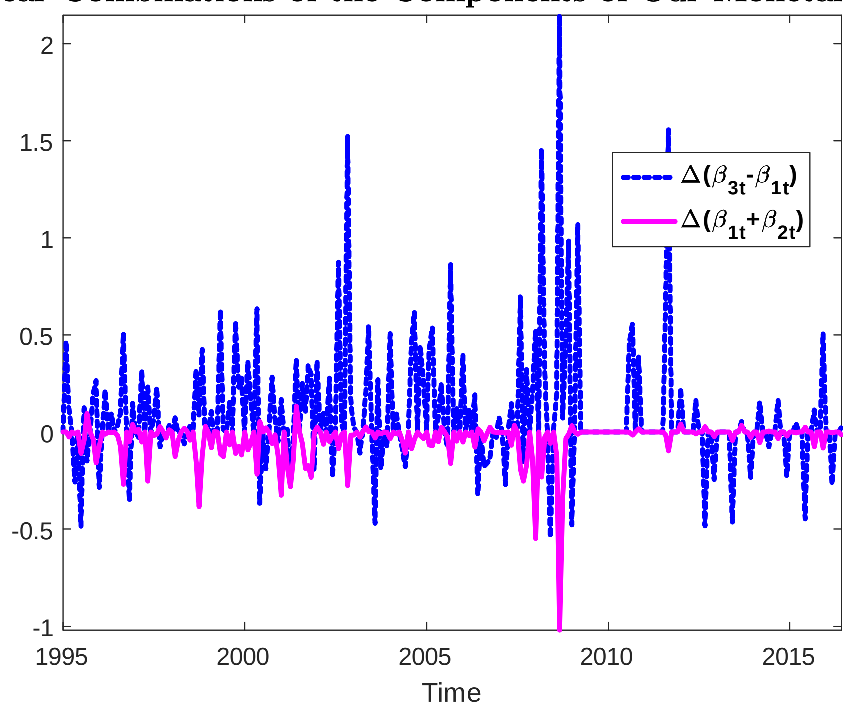


Figure 10. Linear Combinations of the Components of Our Monetary Policy Shock



Notes. Figures 9 and 10 depict the various factors in our functional monetary policy shock.

Figure 11, Panel A. Output Response in Conventional Times

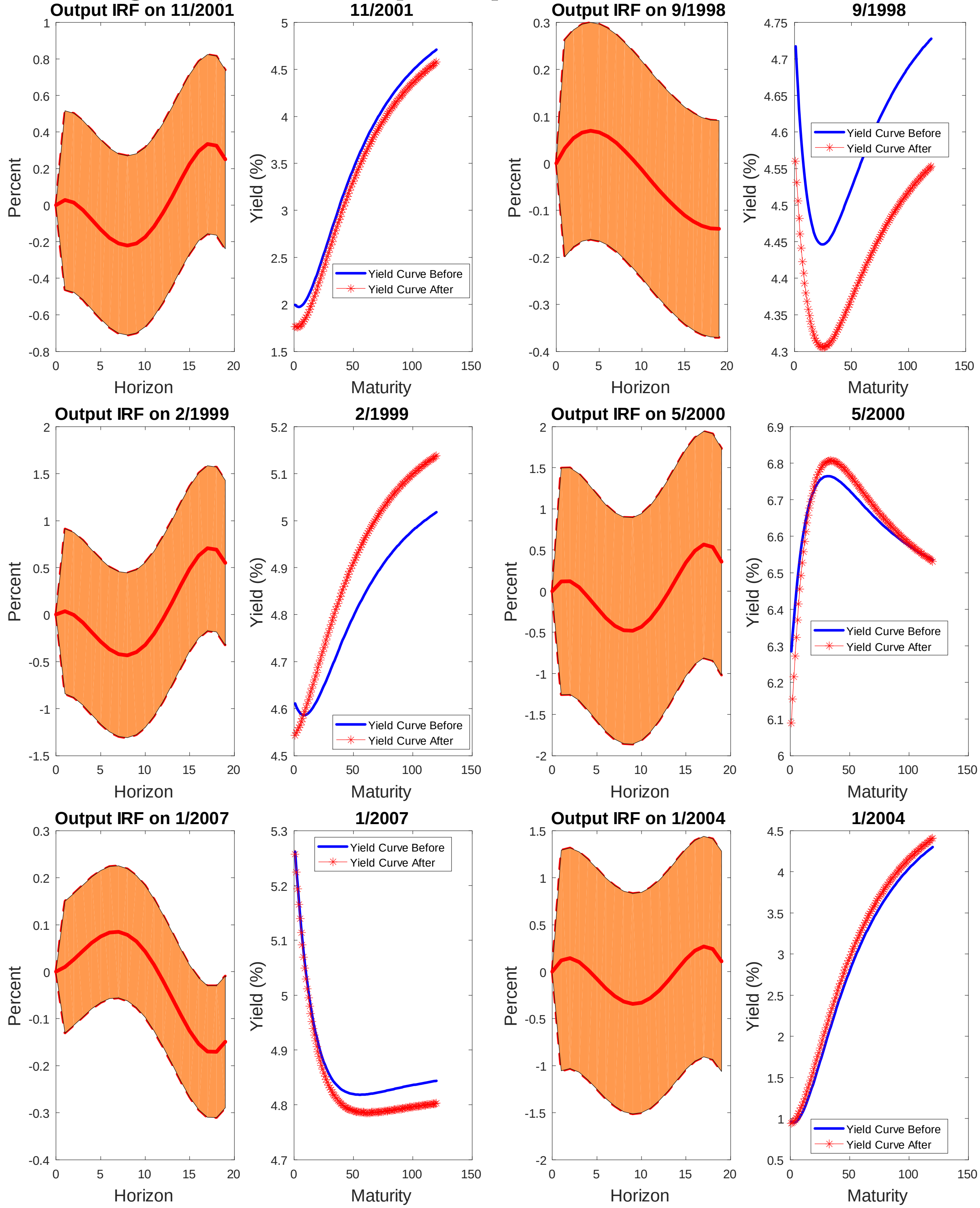


Figure 11, Panel B. Inflation Response in Conventional Times

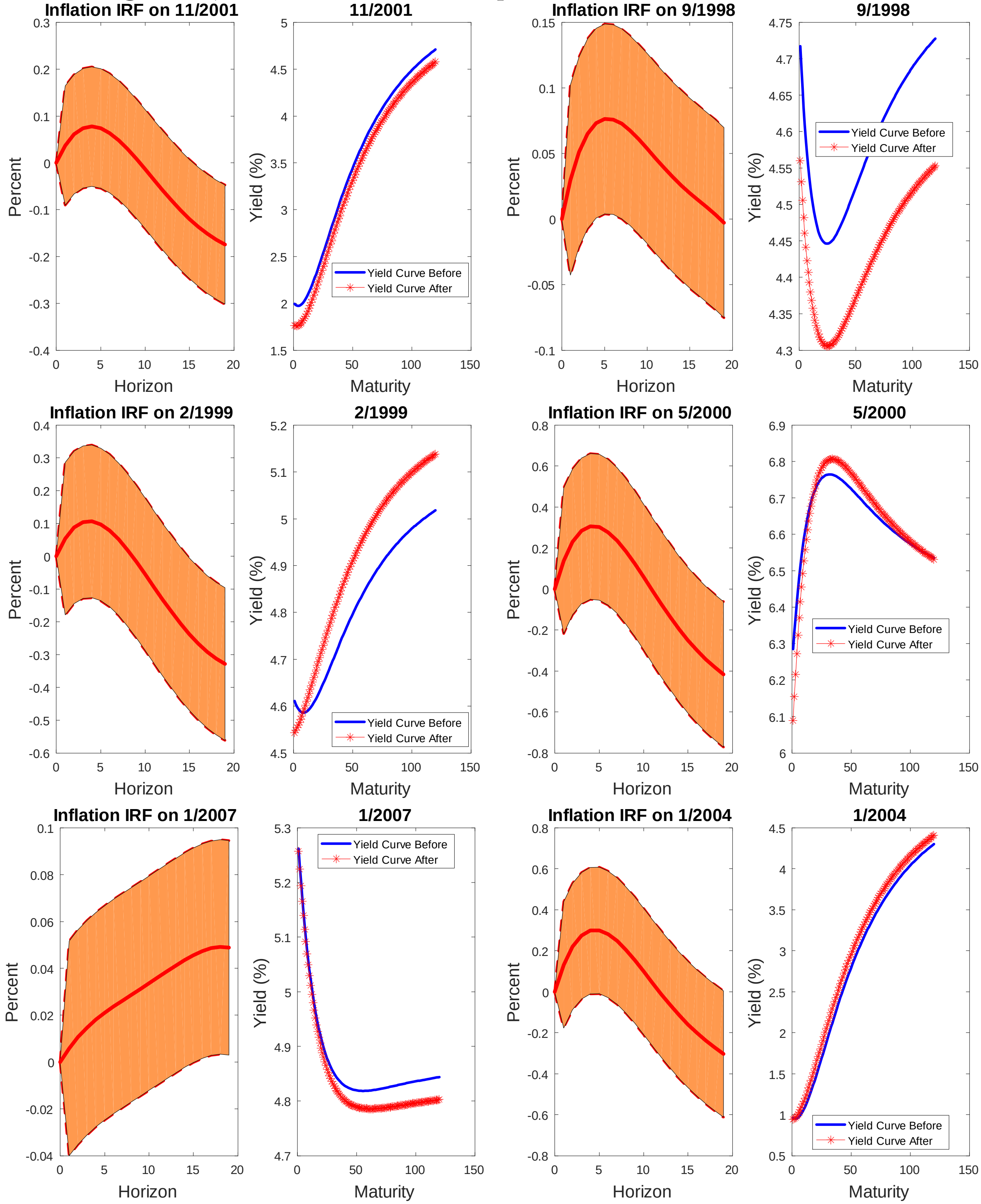
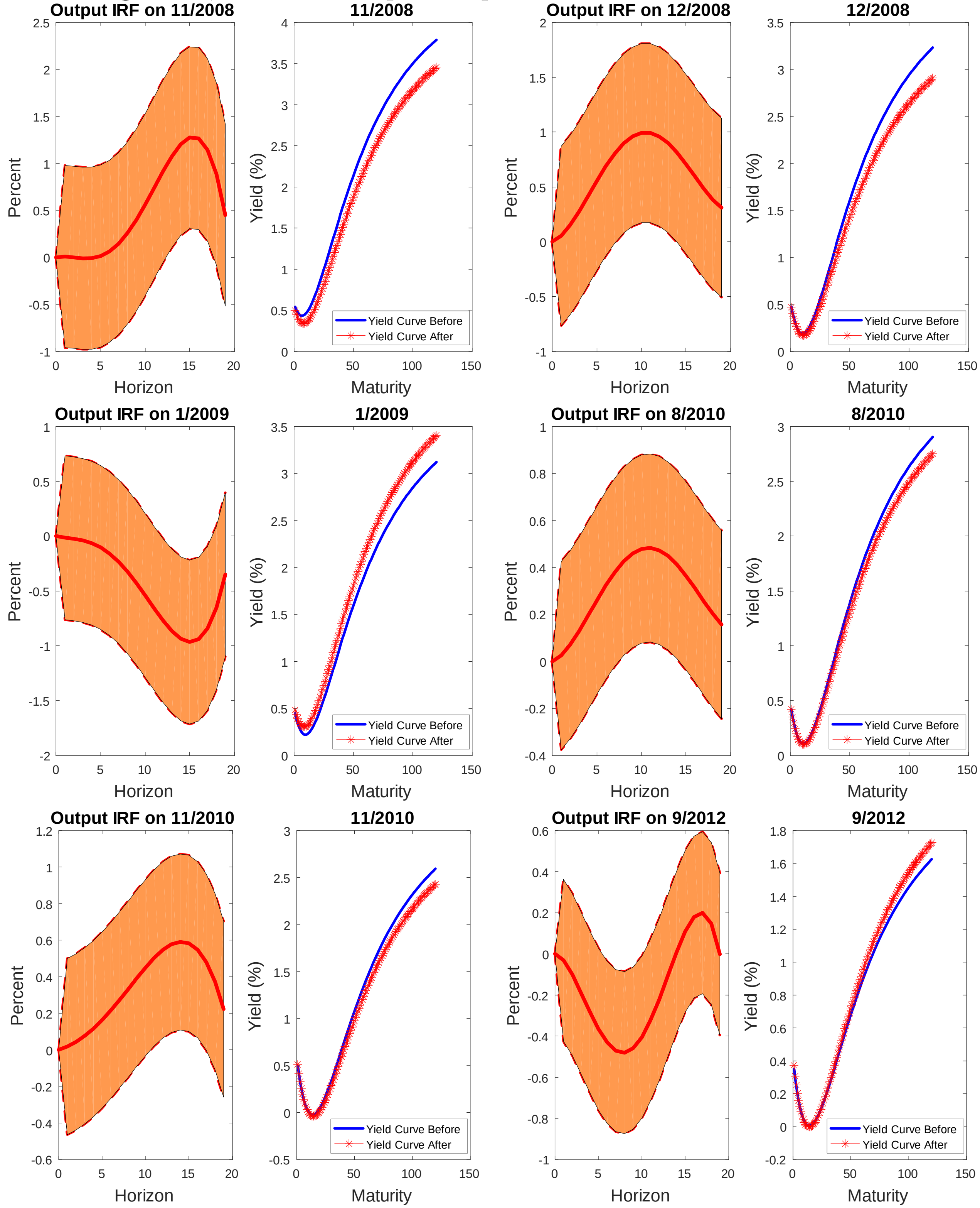
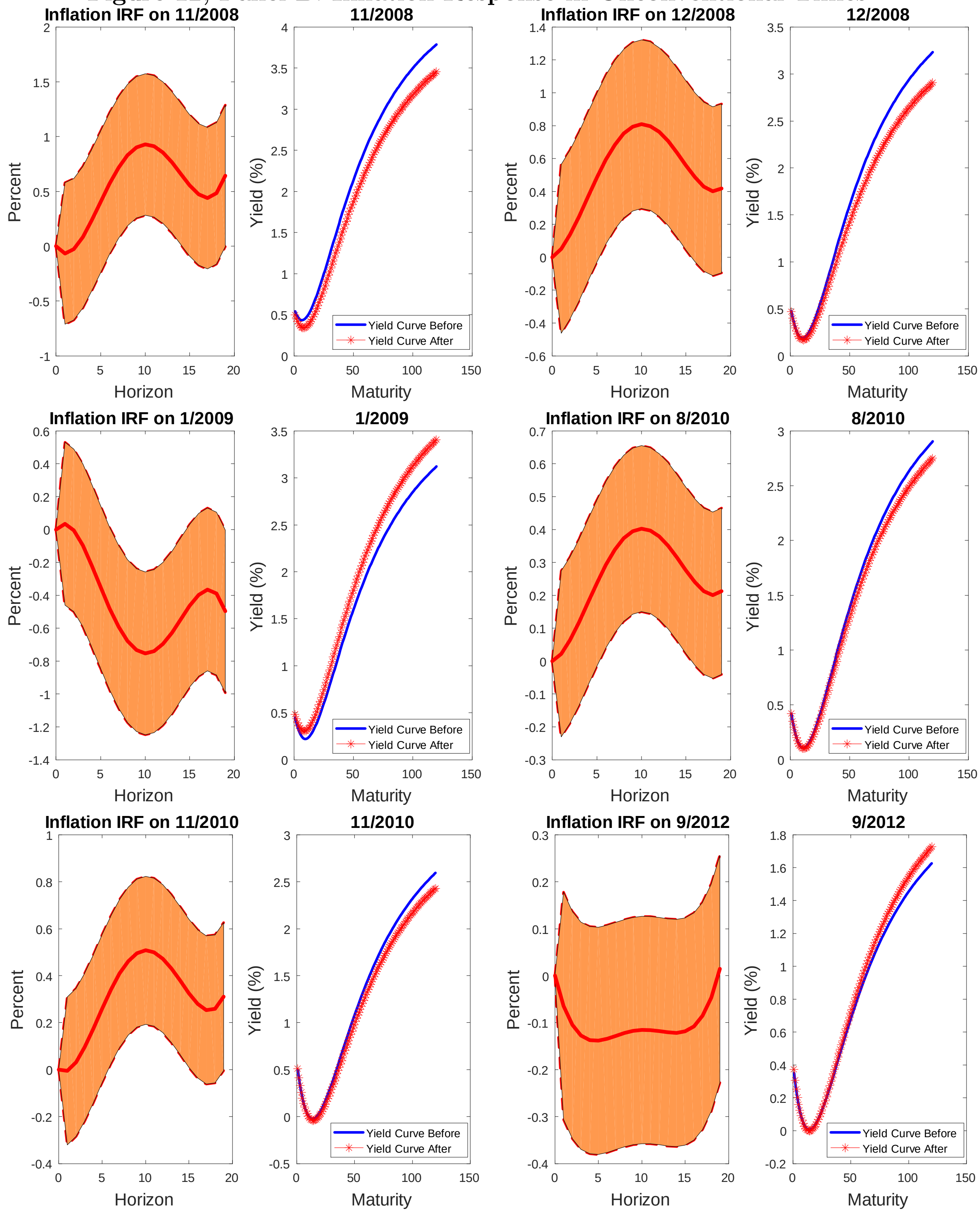


Figure 12, Panel A. Output Response in Unconventional Times



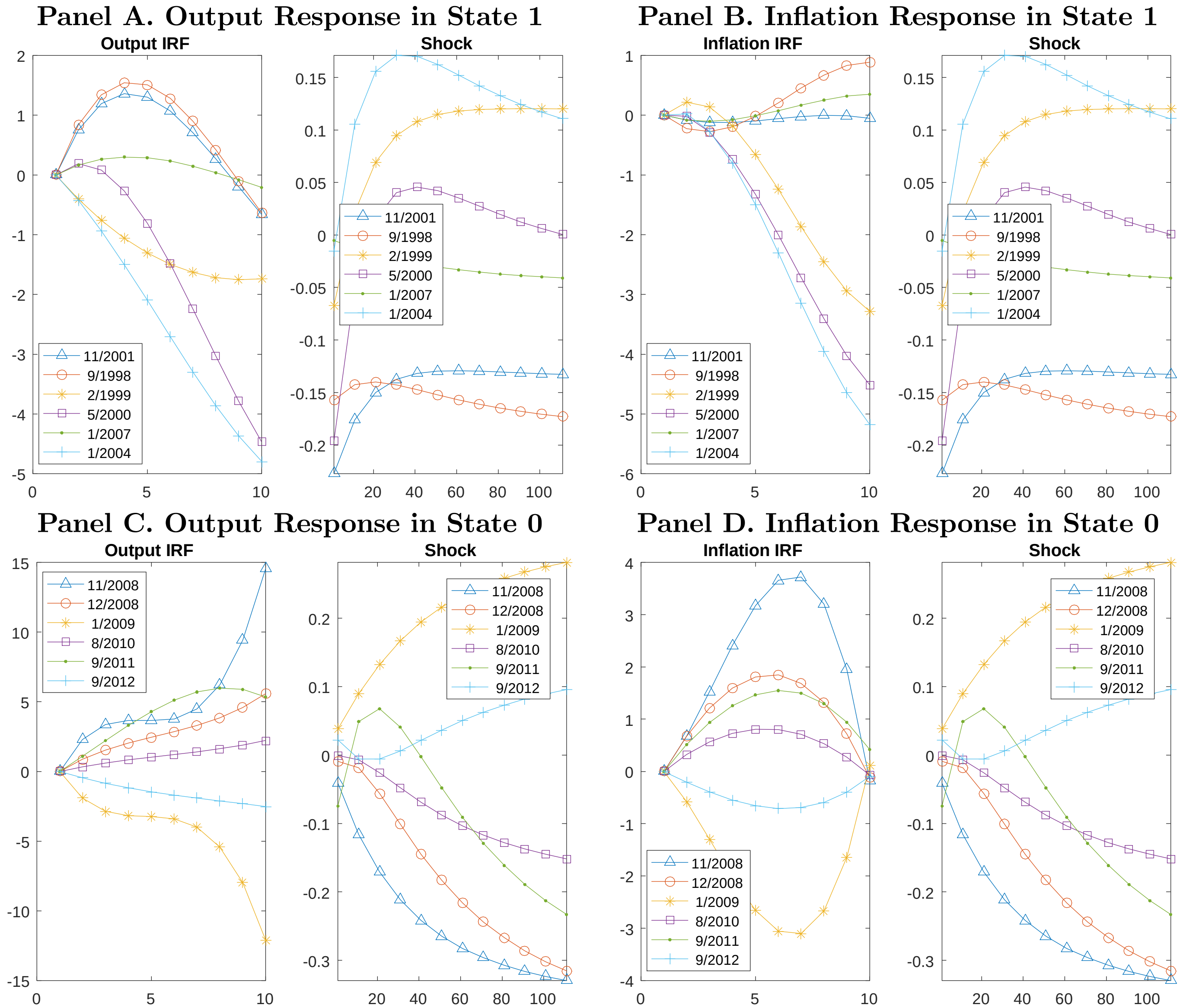
Notes to the figure. The figure plots impulse response functions of industrial production to the monetary policy shock together with 68% confidence bands.

Figure 12, Panel B. Inflation Response in Unconventional Times



Notes. The figures plot impulse response functions of output (Panel A) and inflation (Panel B) to the monetary policy shock together with 68% confidence bands.

Figure 13. Results for the Time-Varying FLP-IV Framework



Notes. For each panel, the figures on the left plot impulse response functions of output (Panels A,C) and inflation (Panels B,D) to the monetary policy shock; the figure on the right depicts the monetary policy shock.

Figure 14(A). Decomposition of Output Responses in State 1

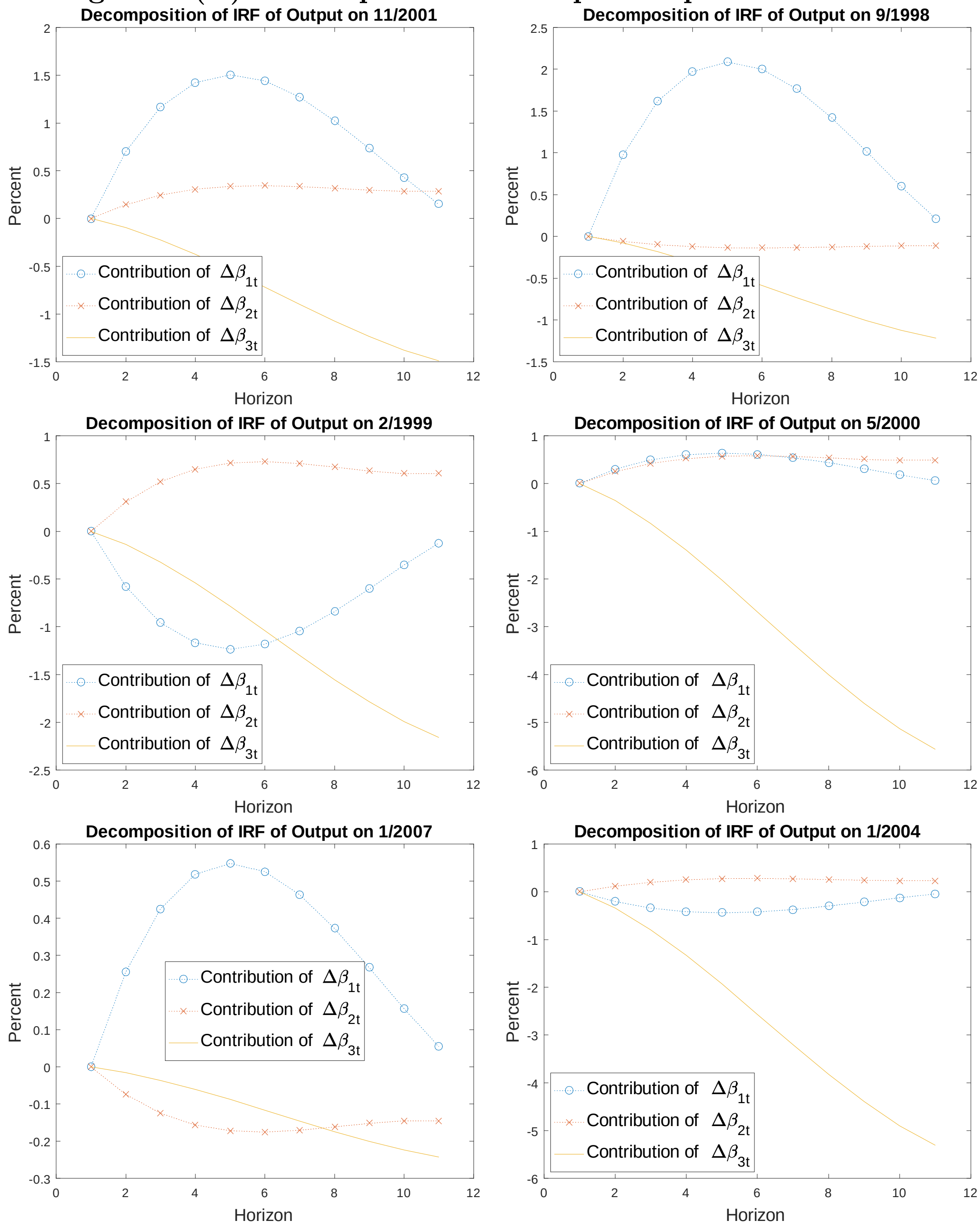


Figure 14(B). Decomposition of Inflation Responses in State 1

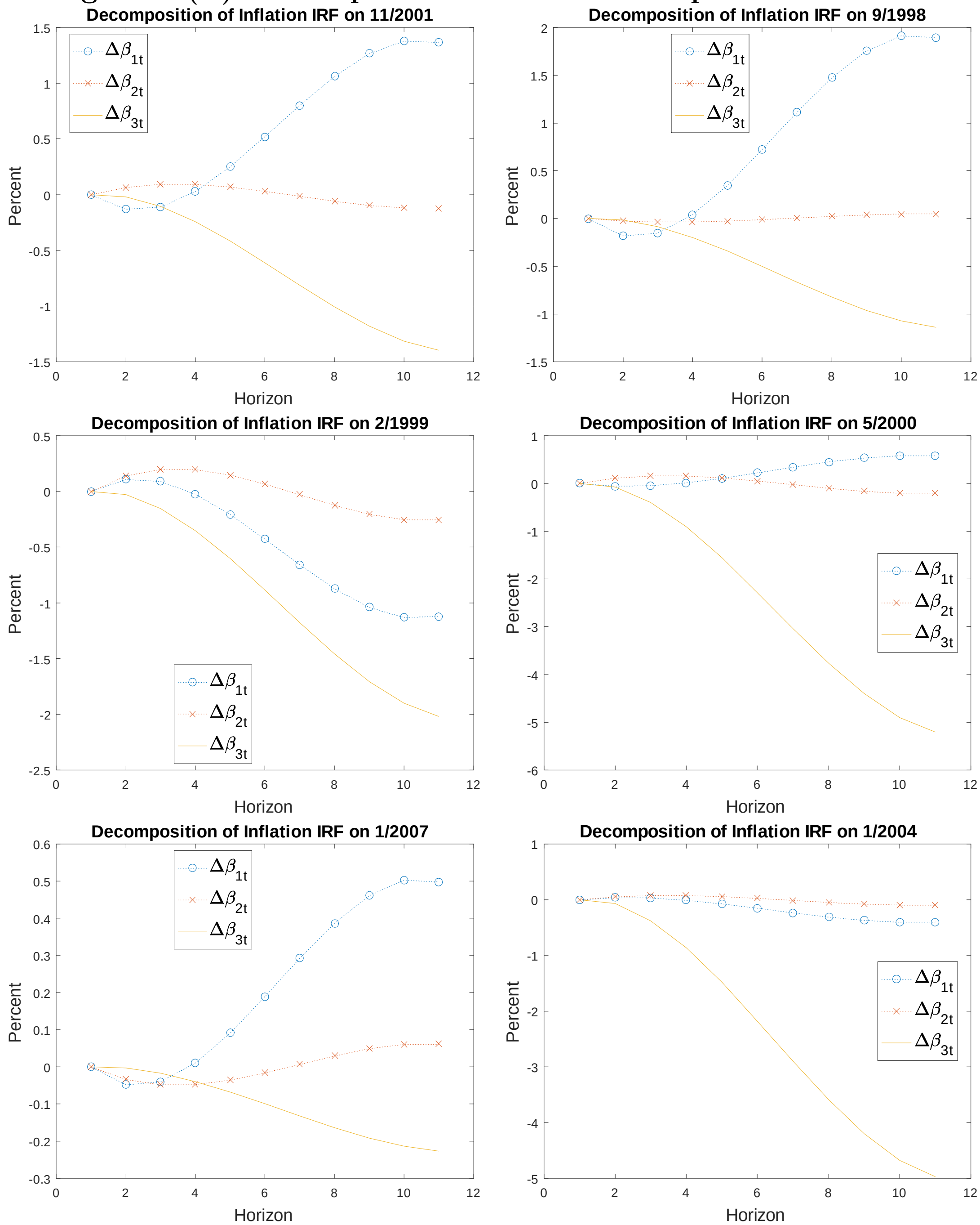


Figure 15(A). Decomposition of Output Responses in State 0

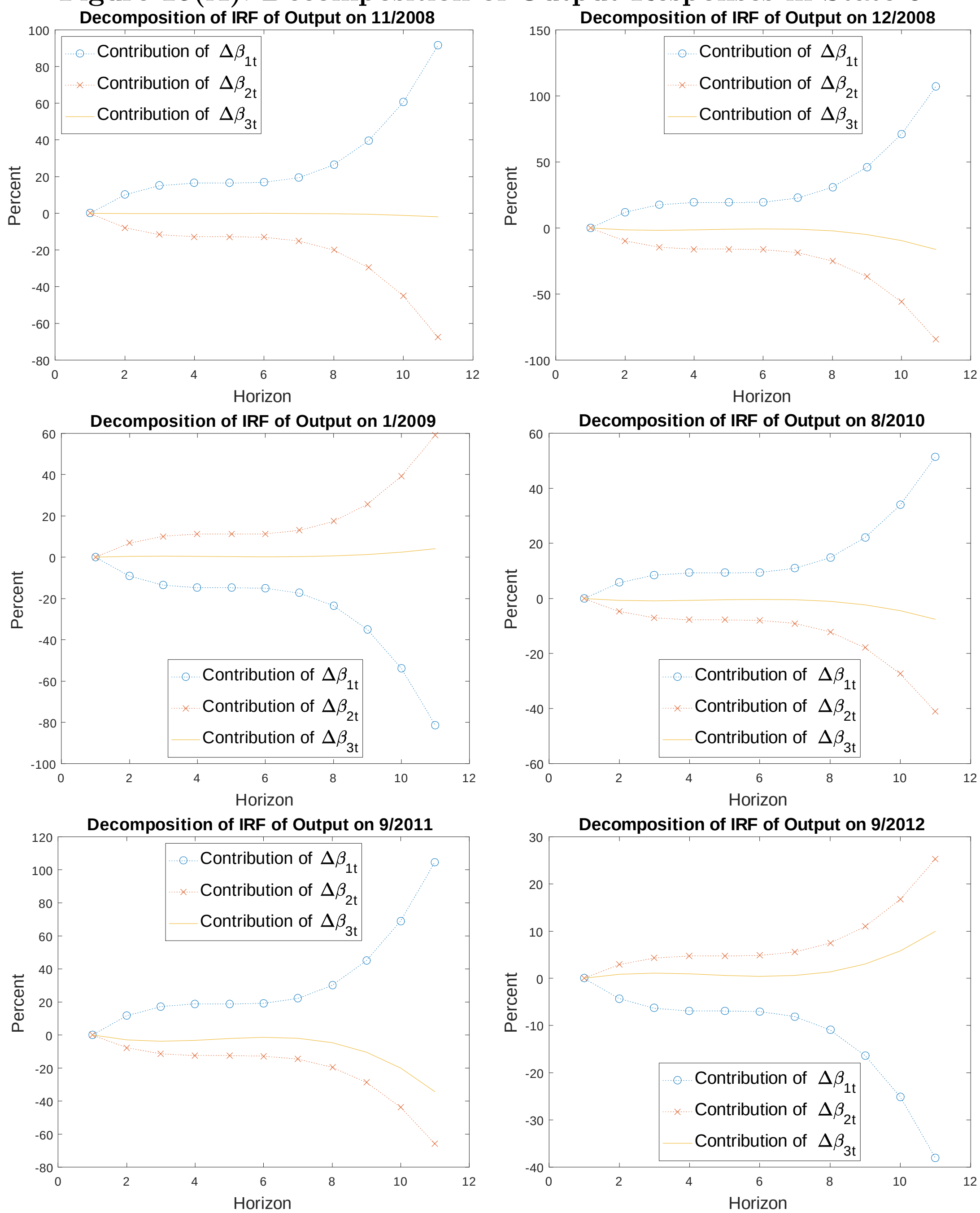
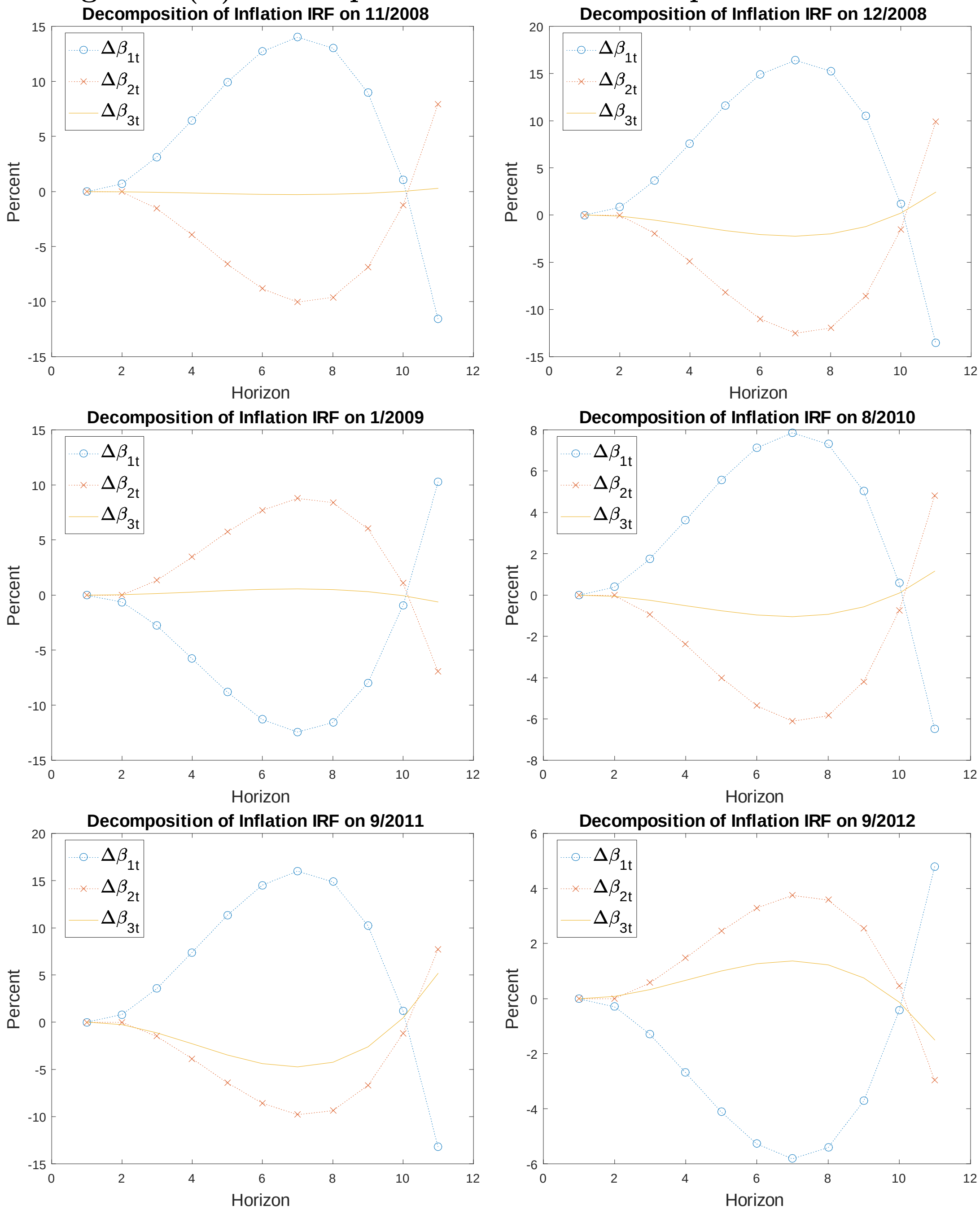
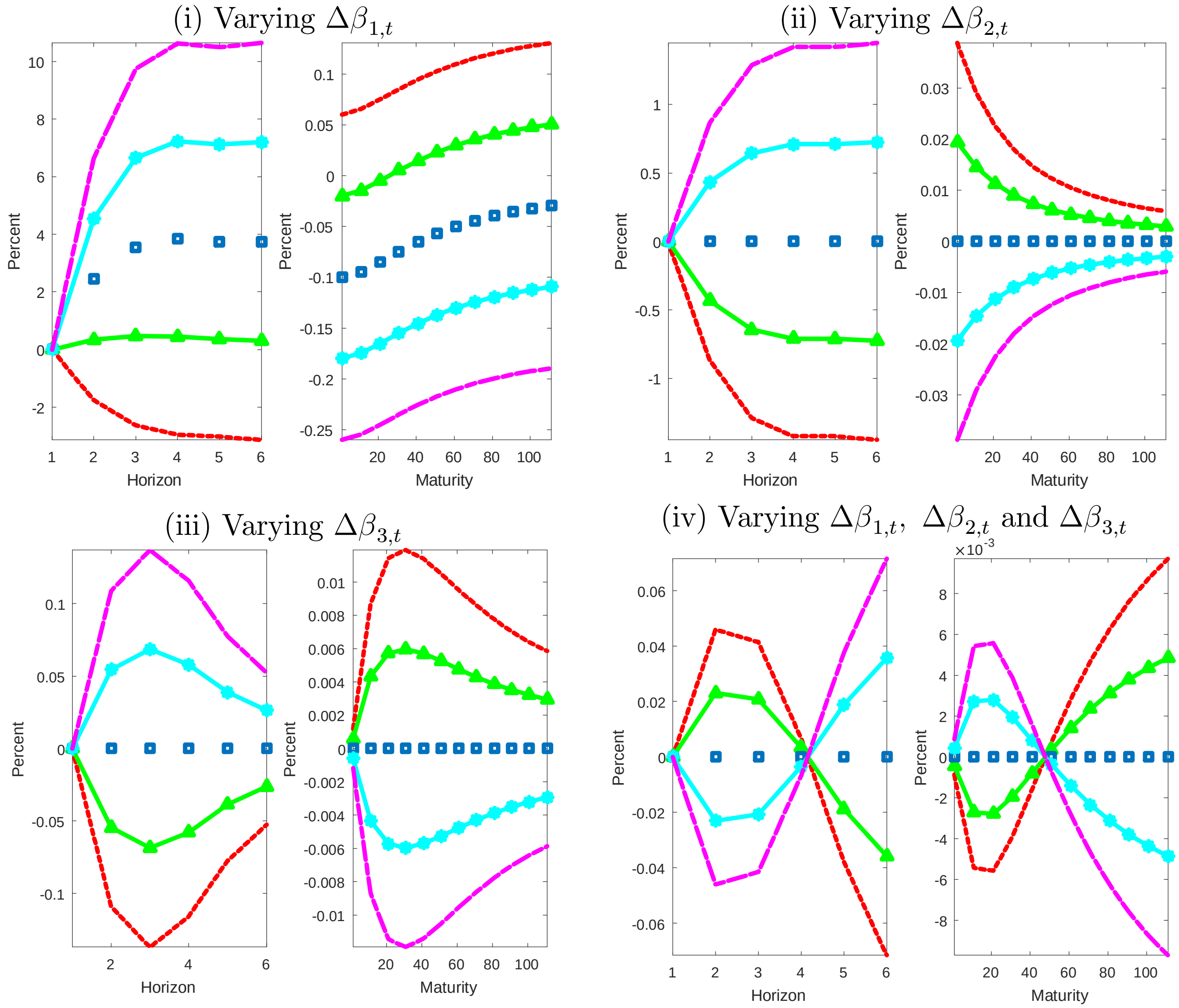


Figure 15(B). Decomposition of Inflation Responses in State 0



Notes. Figures 14-15 plot the decomposition of the responses of output and inflation in the parts related to shocks associated with level, curvature and slope of the term structure, respectively.

FIGURE 16. Understanding How Shocks Affect the Response of Output



Notes. (A) $\Delta\beta_{1,t}$ varies between -0.16 and 0.16; $\Delta\beta_{2,t} = -0.1$ and $\Delta\beta_{3,t} = -0.1$ are constant. (B) $\Delta\beta_{2,t}$ varies between -0.04 and 0.04; $\Delta\beta_{1,t} = 0$ and $\Delta\beta_{3,t} = 0$ are constant. (C) $\Delta\beta_{3,t}$ varies between -0.04 and 0.04; $\Delta\beta_{1,t} = 0$ and $\Delta\beta_{2,t} = 0$ are constant. (D) $\Delta\beta_{1,t}$ varies between 0.02 and -0.02, $\Delta\beta_{2,t}$ varies between -0.02 and 0.02 and $\Delta\beta_{3,t}$ varies between -0.05 and 0.05.