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# Efficiency and Equilibrium with Locally Increasing Aggregate Returns Due to Demand Complementarities

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# Abstract

Do relatively minor dynamic non–convexities at the disaggregate level smooth out in the aggregate? We address this question in a model where the production of intermediate inputs is subject to dynamic non–convexities due to start–up costs. Our answer is simple and intuitive: If intermediate inputs are demand complements, then the non–convexities at the disaggregate level translate into non–convexities at the aggregate level. A partial reverse also holds: If intermediate inputs are strong demand substitutes, then the non–convexities at the disaggregate level smooth out in the aggregate. We use these insights to characterize efficient and equilibrium allocations in dynamic models with aggregate increasing returns.

# 1 Introduction

After many years of intensive research our understanding of efficient and equilibrium allocations in dynamic, convex economic models has become fairly complete. The recent literature has turned its focus to increasing returns. At least two approaches can be distinguished. The first characterizes efficient investment in a framework where aggregate production is a convex-concave function of the aggregate stock of capital, see Skiba (1978), Majumdar and Mitra (1982), Dechert and Nishimura (1983) and Brock and Malliaris (1989) among others. The main drawbacks of this approach—referred to as the aggregative framework by Majumdar and Mitra (1982)—are that nothing can be said about decentralized market equilibrium allocations, and that convex-concave production functions—quite natural at the disaggregate level because of fixed costs or indivisibilities for example—are quite difficult to defend in the aggregate. The second approach—which starts from dynamic non-convexities at the disaggregate level, see Judd (1985), Romer (1987), and Grossman and Helpman (1992) among others—can be used to illustrate this last point: Despite the non-convexities at the disaggregate level, returns to aggregate investment are decreasing for the efficient as well as the decentralized market equilibrium. Dynamic non-convexities at the disaggregate level smooth out in the aggregate, and dynamic market equilibria with disaggregate non-convexities are qualitatively similar to the equilibria in convex models. (Of course, equilibrium allocations are no longer Pareto-efficient.)

We re-examine the aggregate implications of relatively minor non-convexities at the disaggregate level for efficient as well as equilibrium allocations. Consider the following simple investment problem: There is a variety of intermediate inputs into final goods production, and the production of each input is subject to relatively minor dynamic non-convexities due to start-up costs. Do the dynamic non-convexities at the disaggregate level translate into dynamic non-

convexities at the aggregate level? We show that the answer is surprisingly simple and intuitive: If intermediate inputs are demand complements, then the non-convexities at the disaggregate level translate into non-convexities at the aggregate level. Furthermore, a partial reverse also holds: If intermediate inputs are strong demand substitutes, then the non-convexities at the disaggregate level smooth out in the aggregate.

Input demand complementarities can therefore provide micro-economic foundations for the convex-concave aggregate production function in the aggregative framework. This micro-economic approach to aggregate non-convexities enables us to define and characterize decentralized market equilibrium allocation. Input demand complementarities imply that relatively minor non-convexities at the disaggregate level will lead to patterns of growth which differ drastically from those predicted by convex models. Furthermore—as we show by comparing efficient with equilibrium allocations—they also imply that minor non-convexities at the disaggregate level can result in "global inefficiencies" which—in contrast to the "local inefficiencies" identified in Judd (1985), Romer (1987), and Grossman and Helpman (1992) for example—cannot be dealt with by marginal, Pigouvian tax policies but must be addressed with nonlinear policy instruments.

The rest of the paper is organized as follows. After describing the model we work with, we characterize the dynamically efficient allocations in section 3; subsections show the relationship between input demand complementarities and locally increasing aggregate returns and how to determine the globally efficient path in the presence of aggregate non-convexities. Section 4 defines and describes the dynamic market equilibria and compares efficient and equilibrium growth.

# 2 The Model

Time is continuous and extends from zero to infinity. The economy consists of identical households who consume a single homogenous consumption good over an infinite horizon. Their preferences over current and future consumption are ordered by

$$V = \int_0^\infty e^{-\rho\tau} U(C_\tau) d\tau$$

where $U(C)$ is strictly increasing and concave. Total labor supplied in this economy is normalized to unity. Three types of goods are produced: Investment goods, consumption goods, and an endogenous variety of differentiated intermediate inputs.

Intermediate inputs are indexed by $i \geq 0$. Although the space of intermediate inputs is unbounded, only a finite range $0 \leq i \leq n$ is produced at any moment in time. Over time, this range can be increased by allocating $I$ units of the investment good to start-up operations; the investment technology is $\dot{n} = I$ —all variables depend on time but time subscripts will generally be suppressed; dots denote time derivatives. All intermediate inputs are produced with constant returns to scale at the margin, $x_i$ units of labor produce $m_i = x_i$ units of intermediate inputs.

The consumption goods technology is given most generally by $C = F(m_i : i \in [0,\infty], L)$ where $L$ denotes the quantity of labor and $m_i$ the quantity of the $i$-th intermediate input employed. We assume weak separability in intermediate inputs and labor $C = F(M(\circledast), L)$; the intermediate inputs aggregator function $M(\circledast)$ is

$$M = \left( \int_0^\infty m_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = \left( \int_0^n m_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where we refer to $M$ as intermediate input composites. Because we assume $\sigma > 1$, no single intermediate input is essential for producing intermediate input composites. The second equality follows because only intermediate inputs in the range $0 \leq i \leq n$ are produced at any moment in time. All intermediate inputs enter symmetrically into production of the intermediate input

composite, and the elasticity of substitution between any pair of inputs is constant and equal to $\sigma$. Production of investment goods employs the same production factors, intermediate input composites $\overline{M}$ and labor $L$, as production of consumption goods but a potentially different technology $I = G(M, L)$. This model will turn out to be *quite general* because no assumptions other than convexity and constant returns are placed on the consumption goods technology $F(\circ)$ and the investment goods technology $G(\circ)$.

The form of product differentiation specified in the intermediate goods composite in (1) has an important property for the analysis of economic growth: Total factor productivity increases with the variety of intermediate inputs available. To see this, let $X$ be the total amount of labor used in the production of intermediate inputs. Because of symmetry and convexity, it is efficient to produce the same quantity of each existing variety $m_i = x_i = X / n$ for all $i \le n$. The quantity of intermediate input composites which can be produced with $X$ units of labor is $M = n^{\sigma/(\sigma-1)} m = n^{1/(\sigma-1)} X$. Since $\sigma > 1$, the average productivity of labor in producing intermediate input composites increases with the existing variety of intermediate inputs $n$. Wilfred Ethier (1982) describes this property as increasing returns due to specialization, and Paul Romer (1987) observes that this captures Allyn Young's (1928) notion of increasing returns due to the progressive division and specialization of industries.

## 3 Dynamically Efficient Allocations

The efficient growth path will be determined in two stages: Static efficiency is imposed first, and the dynamically efficient allocation is determined second.

PROPOSITION 1. The static production possibility frontier is

$$1 = \frac{C}{\hat{F}(n)} + \frac{I}{\hat{G}(n)} \tag{2}$$

.where $\hat{F}(n)$ and $\hat{G}(n)$ denote the average labor productivity in the production of consumption and investment goods respectively.

**PROOF.** Let $N$ be the total amount of labor used for the production of consumption goods and $X$ the amount of labor employed to produce intermediate inputs; $X/N$ denotes the fraction of total labor employed to produce intermediate inputs. Because of symmetry and convexity, it is efficient to produce the same quantity of all existing intermediate inputs $m = x = X/n$; hence, the production of intermediate input composites is equal to $M = n^{1/(\sigma-1)}X$. Making use of constant returns to scale, average labor productivity in the consumption goods sector therefore is

$$\hat{F}(n) = \underset{\{X/N\}}{Max} \quad F(n^{1/(\sigma-1)}(X/N), 1-(X/N)): \tag{3}$$

Average labor productivity depends only on the variety of intermediate inputs produced. Similarly, average labor productivity in the investment sector is

$$\hat{G}(n) = \underset{\{X/N\}}{Max} \quad G(n^{1/(\sigma-1)}(X/N), 1-(X/N)). \tag{4}$$

Using these definitions, the minimum amount of labor required to produce a quantity of consumption goods $C$ and investment goods $I$ are $C/\hat{F}$ and $I/\hat{G}$ respectively. Full employment requires $C/\hat{F} + I/\hat{G} = 1$ which proves the proposition.　　**Q.E.D.**

Dynamic efficiency requires that the intertemporal consumption profile maximizes the representative household's welfare subject to static efficiency. Given an initial variety of intermediate inputs $n_0$, the dynamically efficient allocation solves

$$\underset{\{C_\tau\}_0^\infty}{\text{Maximize}} \quad \int_0^\infty e^{-\rho\tau}U(C_\tau)d\tau \tag{5}$$

subject to

$$0 \leq \dot{n} = I = \hat{G}(n) - \frac{\hat{G}(n)}{\hat{F}(n)}C. \tag{6}$$

**PROPOSITION 2.** The necessary conditions for dynamic efficiency can be summarized by (6),

$$\frac{\dot{C}}{C} = \begin{cases} \gamma(C)\left(\dfrac{\hat{F}'(n)\hat{G}(n)}{\hat{F}(n)} - \rho\right) & \text{if} \quad C < \hat{F}(n) \\[4mm] 0 & \text{if} \quad C = \hat{F}(n) \quad \text{and} \quad \rho \geq \dfrac{\hat{F}(n)\hat{G}(n)}{\hat{F}(n)} \end{cases} \tag{7}$$

—where $\gamma(C) \equiv -U'(C)/U''(C)C$ denotes the intertemporal elasticity of substitution—and the transversality condition $\lim_{\tau\to\infty} e^{-\rho\tau}U'(C_\tau)n_\tau\hat{F}(n_\tau)/\hat{G}(n_\tau) = 0$.

**PROOF.** From the Lagrangian of the problem

$$L(n,C,\lambda,\theta) = U(C) + (\lambda + \theta)\hat{G}(n)\left(1 - C / \hat{F}(n)\right) \qquad (8)$$

we derive the necessary conditions for dynamic efficiency

$$\rho\lambda - \dot{\lambda} \qquad = (\lambda + \theta)\hat{G}(n)\left(\frac{\hat{G}'(n)}{\hat{G}(n)}\left(1 - \frac{C}{\hat{F}(n)}\right) + \frac{\hat{F}'(n)}{\hat{F}(n)}\frac{C}{\hat{F}(n)}\right) \qquad (a)$$

$$U'(C) \qquad = (\lambda + \theta)\frac{\hat{G}(n)}{\hat{F}(n)} \qquad (b) \qquad\qquad (9)$$

$$\theta\left(\hat{G}(n) - \frac{\hat{G}(n)C}{\hat{F}(n)}\right) \quad = 0 \qquad (c)$$

$$\lim_{\tau\to\infty} n_\tau \lambda_\tau e^{-\rho\tau} \quad = 0 \qquad (d)$$

where $\lambda$, and $\theta \geq 0$ denote the continuously differentiable adjoint variables and the Kuhn-Tucker multiplier associated with the non-negativity constraint on investment. To reduce the necessary conditions for optimality (9a)-(9d) to a dynamical system in the $(n,C)$-phase plane we distinguish two cases:

(i) Suppose that $C < \hat{F}(n)$ and hence $\dot{n} > 0$ and $\theta = 0$. Differentiating (9b) with respect to time and substituting (9a) and (9c) yields

$$\frac{\dot{C}}{C} = -\gamma(C)\left(\frac{\dot{\lambda}}{\lambda} + \left(\frac{\hat{G}'(n)}{\hat{G}(n)} - \frac{\hat{F}'(n)}{\hat{F}(n)}\right)\dot{n}\right) = \gamma(C)\left(\frac{\hat{F}'(n)\hat{G}(n)}{\hat{F}(n)} - \rho\right).$$

(ii) To characterize the dynamical system on the boundary note from (i) that the boundary is absorbing for all $n$ such that $\rho < \hat{F}'(n)\hat{G}(n) / \hat{F}(n)$. This implies that, if $C_t = \hat{F}(n_t)$ and $\rho < \hat{F}'(n_t)\hat{G}(n_t) / \hat{F}(n_t)$ for some $t$, then $n_\tau = n_t$ and $C_\tau = \hat{F}(n_t)$ for all $\tau > t$. Integrating (9a), and using (9b) and (9d) we therefore obtain that $\theta_\tau$ is equal to $(U'(C_t)\hat{F}(n_t) / \hat{G}(n_t)\rho)(\rho - \hat{F}'(n_t)\hat{G}(n_t) / \hat{F}(n_t))$ for all $\tau > t$, which is inconsistent with the non-negativity of $\theta$. If, on the other hand, $C_t = \hat{F}(n_t)$ and $\rho \geq \hat{F}'(n_t)\hat{G}(n_t) / \hat{F}(n_t)$ for some $t$, it is straightforward to check that $n_\tau = n_t$, $C_\tau = \hat{F}(n_t)$, $\lambda_\tau = U'(C_t)\hat{F}'(n_t) / \rho$ and that $\theta_\tau$ is equal to $(U'(C_t)\hat{F}(n_t) / \hat{G}(n_t)\rho)(\rho - \hat{F}'(n_t)\hat{G}(n_t) / \hat{F}(n_t)) \geq 0$ which satisfy all necessary conditions for efficiency (9a)-(9d).  Q.E.D.

As can be seen from (7), the dynamically efficient consumption profile depends crucially on $\hat{F}'\hat{G} / \hat{F}$. This schedule can be interpreted in two equivalent ways. For both interpretations it is useful to rewrite (2) as

$$\hat{F}(n) = C + \frac{\hat{F}(n)}{\hat{G}(n)}I; \qquad (10)$$

here $\hat{F}$ is best interpreted as total potential output in terms of consumption goods and $\hat{F} / \hat{G}$ as the cost of investment in terms of the consumption good. With this interpretation, $\hat{F}'\hat{G} / \hat{F}$ is

equal to the intertemporal rate of transformation of current into future potential consumption. For an alternative interpretation, notice that $\hat{G}$ equals the amount of investment goods which can be produced with one unit of labor. Because $N = \Upsilon / \hat{F}$ units of labor are required to produce a level of output $\Upsilon$, $\hat{F}' / \hat{F}$ is equal to the amount of labor saved in the production of future output per unit of investment, $\partial N / \partial I]_{N=1} = \hat{F}' / \hat{F}$; thus, $\hat{F}'\hat{G} / \hat{F}$ can alternatively be interpreted as future labor saved per unit of current labor invested. Whatever the interpretation, we refer to $\hat{F}'\hat{G} / \hat{F}$ as the aggregate return schedule and denote it by $\hat{r}(n)$,

$$\hat{r}(n) \equiv \frac{\hat{F}'(n)\hat{G}(n)}{\hat{F}(n)}. \tag{11}$$

An assumption maintained throughout is that the returns to additional intermediate inputs peter out eventually: $\hat{r}(n) \to 0$ as $n \to \infty$ (restrictions on technology for this to be the case will be discussed in the next sections). This accords with our focus on locally increasing returns as opposed to asymptotically increasing returns and ever accelerating growth.

## A  Locally Increasing Returns Due to Input Demand Complementarities

Locally increasing aggregate returns are defined as instances where the return to investment in new intermediate input varieties $\hat{r}(n)$ increases with the aggregate level of investment in intermediate inputs $\partial \hat{r} / \partial I > 0$, or equivalently $\partial \hat{r} / \partial n > 0$. We now state and prove the main result of this section.

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**PROPOSITION 3.** There will locally increasing aggregate returns if intermediate inputs are complements in the sense of Hicks-Allen.

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**PROOF.** We prove the result in two steps. First, we show that the amount of labor saved per unit of investment is equal to the average productivity of intermediate input varieties in the production of intermediate inputs. Then, we establish the necessary relationship between labor saved and Hicks-Allen complementarities.

To establish the first result, it will be convenient to pursue the interpretation of the production possibility frontier suggested in (10): All resources are used to produce a level of output $\hat{F}(n)$, measured in units of the consumption good, with the rate of transformation of consumption goods into investment goods equal to $\hat{G}(n)/\hat{F}(n)$. Let $\hat{X}^c(n)$ denote the efficient amount of labor employed in the production of intermediate inputs (and therefore $1 - \hat{X}^c(n)$ the efficient amount of labor used directly in the production of consumption goods). Then, by definition

$$\hat{F}(n) \equiv F\left(n^{1/(\sigma-1)}\hat{X}^c(n), 1 - \hat{X}^c(n)\right).$$

Because $\hat{X}^c(n)$ is also equal to the value of intermediate inputs in units of labor, $\hat{x}^c = \hat{X}^c / n$ is equal to the average productivity of intermediate input varieties in the production of intermediate inputs. To link the average productivity of intermediate input varieties in the production of intermediate inputs to the aggregate returns schedule, notice that the envelope theorem implies that

$$\hat{F}'/\hat{F} = \left(\hat{F}_M\hat{M}^c / \hat{X}^c\hat{F}\right)\hat{X}^c / (\sigma-1).$$

Because of constant returns to scale, the increase in output from one additional unit of labor used in the production of intermediate input composites $\hat{F}_M\hat{M}^c / \hat{X}^c$ is equal to the average productivity of labor $\hat{F}$,

$$\hat{F}_M\hat{M}^c / \hat{X}^c = \hat{F}. \tag{12}$$

Therefore,

$$\hat{r}(n) = \hat{x}^C(n)\hat{G}(n)/(\sigma-1). \tag{13}$$

**Remark 1.** Whether the returns to intermediate inputs investment increases with the aggregate level of investment depends on two factors: Whether the average productivity of labor in the production of investment goods $\hat{G}(n)$ increases with the variety of intermediate inputs (locally increasing aggregate returns because of "falling costs" of investment), and whether the average productivity of intermediate input varieties in the production of intermediate inputs $\hat{x}^c(n)$ increases with the variety of intermediate inputs (locally increasing aggregate returns because of "increasing revenues" from investment).

Because the average productivity of labor in the production of investment goods $\hat{G}(n)$ cannot decrease with the variety of intermediate inputs, this implies that the aggregate return to investment $\hat{r}(n)$ slopes up if the average productivity of intermediate input varieties in the production of intermediate inputs increases with new intermediate inputs.

A necessary and sufficient condition for this to be the case is that intermediate inputs are complements in the sense of Hicks-Allen. To see this, consider the problem of producing one unit of the consumption good with a minimum amount of labor, assuming that $a_i$ units of labor are required to produce one unit of the $i$-th intermediate input. The Lagrangian for this problem is

$$\int_0^n a_i m_i^c \, di + L^c + a(1 - F(M^c, L^c)) + a_M(M^c - M(\circ)),$$

where $a$, $a_M$ denote the Lagrange-multipliers and $M(\circ)$ is defined in (1). Efficient intermediate input demand as a function of the shadow prices of intermediate inputs $a_i$ and the intermediate input composite $a_M$ is

$$\hat{m}_i^c = \left(\frac{a_i}{a_M}\right)^{-\sigma} \hat{M}^c(a_M) \tag{14}$$

for all $i \leq n$, where $\hat{M}^c(a_M)$ denotes the efficient demand for intermediate input composites as a function of their shadow price $a_M$,

$$a_M = \left(\int_0^\infty a_i^{1-\sigma} di\right)^{1/(1-\sigma)}. \tag{15}$$

To establish the result we need the following definitions:

**Definition 1.** Two <u>intermediate inputs</u> $i, j$ <u>are substitutes (complements) in the sense of Hicks-Allen</u> if cost minimization implies that the demand of input $i$ increases (decreases) with the price of input $j$.

**Definition 2.** The <u>Hicks-Allen partial elasticity of substitution</u> between any intermediate input and the intermediate input composite is defined as the percent change in demand for the intermediate input in response to a one percent increase of the price of the intermediate input composite—holding the price of the intermediate input constant. This elasticity will be denoted by $\xi(a_M)$,

$$\xi(a_M) \equiv \partial \log \hat{m}^C / \partial \log a_M.$$

From (14) and (15) it is seen that, because of weak separability, any pair of intermediate inputs will be Hicks-Allen substitutes if and only if $\xi(a_M) > 0$; furthermore, (15) and symmetry imply $a_M = n^{1/(1-\sigma)}$, and hence $\partial \log \hat{x}^c / \partial \log n = -\xi(\bullet)/(\sigma-1)$. Thus, there will be locally increasing returns to investment if intermediate inputs are Hicks-Allen complements. $\qquad$ **Q.E.D.**

**Remark 2.** Notice that whether intermediate inputs are Hicks-Allen complements or substitutes depends on the variety of intermediate inputs already available. It is shown below, for example, that intermediate inputs may be Hicks-Allen complements if few of them are available and become substitutes as their availability increases. Loosely speaking, locally increasing aggregate returns arise when the "next group" of intermediate inputs to be supplied are Hicks-Allen complements.

Intermediate input complementarities are sufficient for locally increasing aggregate returns. A partial converse also holds: If substitutability between intermediate inputs is relatively high, there will be locally decreasing aggregate returns. To state and prove the result it is useful to define:

**Definition 3.** Intermediate <u>inputs are strong substitutes in the sense of Hicks-Allen</u> if $\xi(a_M) \geq 1$, that is demand for any intermediate input increases by more than one percent in response to a one percent increase in the price of the intermediate input composite.

> **PROPOSITION 4.** There will be locally decreasing aggregate returns if intermediate inputs are strong substitutes in the sense of Hicks-Allen.

PROOF. Differentiating (13) making use of $\hat{G}'(n)/\hat{G}(n) = \hat{X}^I(n)/(\sigma-1)n$, where $\hat{X}^I$ is defined (analogously to $\hat{X}^C$) by $\hat{G}(n) = G(n^{1/(\sigma-1)}\hat{X}^I(n), 1-\hat{X}^I(n))$, we get that there will be locally increasing returns if and only if

$$\xi(a_M) < \hat{X}^I(a_M^{1-\sigma}). \tag{16}$$

The result follows because $\hat{X}^I(n) < 1$.                                                     Q.E.D.

We conclude this section with a necessary and sufficient condition for locally increasing aggregate return in terms of the elasticity of substitution between intermediate input composites and labor in the production of the consumption good. This result will prove useful later.

PROPOSITION 5. There will be locally increasing returns to aggregate investment if and only if

$$(\sigma-1)-(\varepsilon(a_M)-1)(1-\hat{X}^C(a_M^{1-\sigma})) < \hat{X}^I(a_M^{1-\sigma}), \tag{17}$$

where $\varepsilon(a_M) \equiv -\partial \log(\hat{M}/\hat{L})/\partial \log a_M$ is the elasticity of substitution between intermediate input composites and labor in the production of the consumption good.

PROOF. From (1) and (15) it follows that (14) can be written as

$$\hat{m}(a_M) = a_M^{-\sigma}\left(\hat{X}^C(a_M^{1-\sigma})/a_M\right) \tag{18}$$

From the definition of the elasticity of substitution between intermediate input composites and labor we obtain that $\hat{M}/\hat{L} = \beta \exp(\int_{a_M}^1 \varepsilon(s)/s\, ds)$. Combined with (12) and constant returns to scale $\hat{X}^C = \hat{F}_M\hat{M}/(\hat{F}_M\hat{M}+\hat{F}_L L) = 1/(1+a_M\hat{L}/\hat{M})$ it follows that

$$\hat{X}^C(a_M^{1-\sigma}) = \left(1+\beta \exp\left(\int_0^{a_M^{1-\sigma}}(\varepsilon(s^{1/(1-\sigma)})-1)/((1-\sigma)s)ds\right)\right)^{-1}. \tag{19}$$

Differentiating (18) with respect to $a_M$ and making use of (19) we get that

$$\xi(a_M) = (\sigma-1)-\left(\varepsilon(a_M)-1\right)\left(1-\hat{X}^C(a_M^{1-\sigma})\right). \tag{20}$$

Combined with (16) this proves the result.                                                     Q.E.D.

Remark 3. *Proposition 5* and (20) imply that Hicks-Allen complementarities are necessary and sufficient for locally increasing aggregate returns if the production of the investment good does not require any intermediate inputs, $\hat{X}^I = 0$. If intermediate inputs are used in the production of investment goods, then there may locally increasing aggregate returns even if intermediate inputs are Hicks-Allen substitutes.

Remark 4. Another immediate consequence of *Proposition* 5 is that there will be globally decreasing aggregate returns if the consumption goods technology satisfies $\varepsilon(a_M) \le 1 \le \sigma - 1$. This will be the case if—for example—the consumption goods technology is Cobb-Douglas and the elasticity of substitution between any pair of inputs is sufficiently large.

## B  Efficient Allocations with Locally Increasing Aggregate Returns

The main complication introduced by locally increasing aggregate returns is that multiple paths may satisfy the necessary conditions for dynamic efficiency (6) and (9). The following two propositions allow us to select the dynamically efficient consumption profile.

**PROPOSITION 6.** Let $\{\hat{n}_\tau, \hat{C}_\tau, \hat{\lambda}_\tau, \hat{\theta}_\tau : 0 \le \tau\}$ satisfy the necessary conditions for optimality (6) and (9). If $\hat{r}(n_\tau) \to 0$ as $\tau \to \infty$, then welfare along the intertemporal consumption path $\hat{V}_0 = \int_0^\infty e^{-\rho\tau} U(\hat{C}_\tau) d\tau$ is equal to $L(\hat{n}_0, \hat{C}_0, \hat{\lambda}_0, \hat{\theta}_0)/\rho$ where $L(\bullet)$ is defined in (8).

The proof of this proposition is a variation on a result in Skiba (1978) and therefore omitted.

**PROPOSITION 7.** The dynamically efficient allocation maximizes initial investment.

**PROOF.** Let $\hat{C}_0^i, \hat{C}_0^\ddot{u}, \hat{\lambda}_0^i, \hat{\lambda}_0^\ddot{u}$ and $\hat{L}_0^i, \hat{L}_0^\ddot{u}$ denote consumption, shadow prices, and the value of Lagrangian at $\tau = 0$ along path $i$ and $\ddot{u}$ respectively. Using *Proposition 6*, concavity of $U$, and (9b) and (9c)

$$
\begin{aligned}
(\hat{V}_0^\ddot{u} - \hat{V}_0^i)\rho &= \hat{L}_0^\ddot{u} - \hat{L}_0^i \\
&= U(\hat{C}_0^\ddot{u}) - U(\hat{C}_0^i) + \hat{\lambda}_0^\ddot{u} \hat{G}_0(\hat{F}_0 - \hat{C}_0^\ddot{u})/\hat{F}_0 - \hat{\lambda}_0^i \hat{G}_0(\hat{F}_0 - \hat{C}_0^i)/\hat{F}_0 \\
&> U'(\hat{C}_0^\ddot{u})(\hat{C}_0^\ddot{u} - \hat{C}_0^i) + U'(\hat{C}_0^\ddot{u})(\hat{F}_0 - \hat{C}_0^\ddot{u}) - U'(\hat{C}_0^i)(\hat{F}_0 - \hat{C}_0^i) \\
&= (U'(\hat{C}_0^\ddot{u}) - U'(\hat{C}_0^\ddot{u}))\hat{F}_0
\end{aligned}
$$

but this last expression is larger than zero if $\hat{C}_0^\ddot{u} \le \hat{C}_0^i$ or equivalently $\hat{I}_0^\ddot{u} \ge \hat{I}_0^i$. Q.E.D.

We can now apply the model (and propositions) to fully characterize the dynamically efficient allocation in a one and a two sector growth model.

In the one sector model of growth consumption and investment goods are produced with identical technologies, $F(M,L) = G(M,L)$. In this case, the dynamic system in (6) and (7) simplifies to

$$\dot{C}/C = \gamma(C)\left(\hat{F}'(n) - \rho\right)$$
$$\dot{n} = \hat{F}(n) - C$$

The dynamic system has the same reduced form as the standard one sector growth model of capital accumulation (*e.g.* Cass (1965)), with $\hat{F}(n)$ corresponding to the aggregate production function and the aggregate return schedule $\hat{F}'(n)$ to the marginal productivity schedule of capital.

We first determine sufficient conditions for "decreasing returns to capital" and "increasing returns to capital." Making use of *Proposition 5* we find that $\sigma - 2 > Max\{\varepsilon(0) - 2, 0\}$ in the one sector model implies that $\hat{F}''(n) < 0$: There are decreasing returns to investment everywhere. In this case, the efficient intertemporal allocation is qualitatively identical to the efficient allocation in the neoclassical model of growth. The same proposition also establishes that, if $\sigma - 2 \leq 0$ then returns to investment will be increasing no matter how many intermediate inputs are already produced. In this case, the efficient growth paths will be characterized by ever accelerating growth rates.

But our main interest is to characterize explicitly the dynamically efficient growth path in the presence of locally but not globally increasing aggregate returns. To do so we consider the following example.

*EXAMPLE 1:* Let both consumption and investment goods be produced according to

$$F(M,L) = G(M,L) = \left(M^{(\varepsilon-1)/\varepsilon} + \beta^{1/\varepsilon}L^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}$$

where $\varepsilon$ is the elasticity of substitution between labor and the intermediate input composites. The necessary and sufficient condition for locally increasing returns in (17) then simplifies to

$$\beta(\varepsilon - \sigma)n^{-(\varepsilon-1)/(\sigma-1)} > \sigma - 2.$$

Thus, the implied aggregate production function $\hat{F}(n)$ can be globally concave, if $\varepsilon < \sigma$ and $\sigma > 2$; globally convex, if $\varepsilon > \sigma$ and $\sigma < 2$; and concave-convex, if $2 > \sigma > \varepsilon$. The case of most interest to us is when the implied aggregate production function $\hat{F}(n)$ is convex-concave $\varepsilon > \sigma > 2$; it is in this case that our disaggregate framework provides a micro-economic foundation for the convex-concave aggregate production functions in Skiba (1978), Majumdar and Mitra (1982), Dechert and Nishimura (1983) and Brock and Malliaris (1989). We illustrate the efficient intertemporal allocation in *Figure* 1, making use of *Proposition* 7. In contrast to convex models of growth, investment rates and growth rates will be first increasing and then decreasing along the efficient growth path. Furthermore, for very low initial intermediate input variety, any investment will be inefficient because of the low current and future returns.

*A Two Sector Model*

Now consider the following two sector model of economic growth: Start-up operations for intermediate inputs require labor: $G(M,L) = L$. In this case we get:

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**PROPOSITION 8.** Input demand complementarities are necessary and sufficient for locally increasing aggregate returns.

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The proof follows immediately from (16).

When intermediate inputs are substitutes, then the efficient intertemporal allocation is qualitatively identical to the one in convex models of growth. To characterize the dynamically efficient allocation in the presence of intermediate input demand complementarities we generalize the growth models of Judd (1985) and Grossman and Helpman (1991, Chapter 3).

*EXAMPLE 2:* The production technology for the consumption good is taken to be a CES-function,

$$F(M,L) = \left( M^{(\varepsilon-1)/\varepsilon} + \beta^{1/\varepsilon} L^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)},$$

# Figure 1: Dynamically Efficient Allocations
# with Input Demand Complementarities
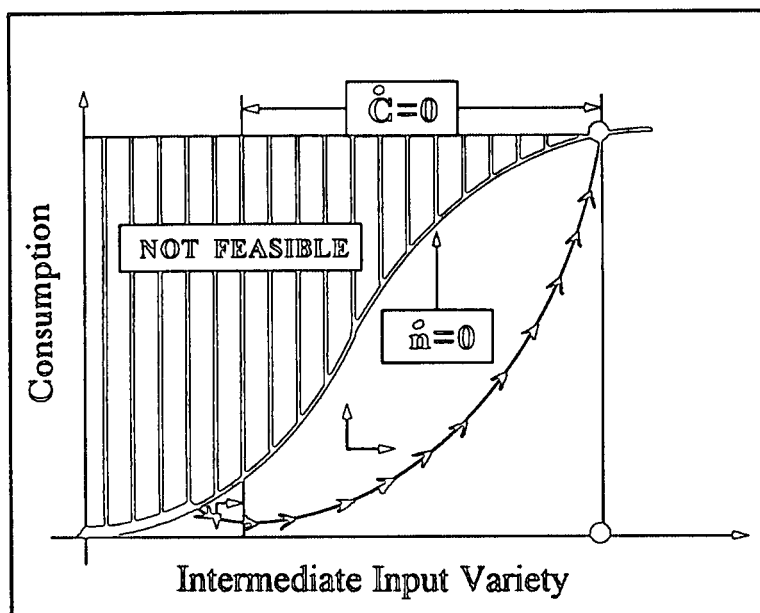


Notes: The thick line corresponds to the region where it is efficient to not invest in intermediate inputs because of low current and future returns.

where $\varepsilon$ is the elasticity of substitution between labor and the intermediate input composites. From (17) we obtain that there will be increasing returns if and only if

$$\beta(\varepsilon - \sigma)n^{-(\varepsilon-1)/(\sigma-1)} > \sigma - 1.$$

Thus, there are only two possibilities. Globally decreasing aggregate returns, $\varepsilon \leq \sigma$; and that the aggregate returns schedule is hump-shaped, $\varepsilon > \sigma$. The efficient intertemporal allocation in the latter case is qualitatively identical to growth models with convex-concave aggregate production functions.

# 4  Equilibrium Growth

One advantage of our approach compared with Skiba (1978), Majumdar and Mitra (1982), Dechert and Nishimura (1983), and Brock and Malliaris (1989) is that we can define and characterize decentralized market equilibria and assess their efficiency.

First, we define the market structure. Both consumption goods and investment goods are produced by competitive firms. Making use of weak separability of both the consumption and the investment goods technologies—and denoting the wage rate, the price of investment goods, and the price of the $i$-th intermediate input (all in terms of the consumption good) with $w$, $q$ and $p_i$—we can define profit maximizing input demand schedules by the usual conditions: $F_L(M^c, L^c) = w$, $F_M(M^c, L^c) = p_M$, and $m_i^c / M^c = (p_i / p_M)^{-\sigma}$ in the consumption goods sector and $qG_L(M^I, L^I) = w$, $qG_M(M^I, L^I) = p_M$, and $m_i^I / M^I = (p_i / p_M)^{-\sigma}$ in the investment goods sector; all variables are defined analogously to the efficient case; and

$$p_M = \left(\int_0^\infty p_i^{1-\sigma} di\right)^{1/(1-\sigma)}$$

denotes the minimum cost of purchasing intermediate inputs to produce one unit of the intermediate good composite. Intermediate inputs are produced by monopolistically competitive firms. Each firm produces one input and maximizes profits taking all other prices as given.

Because they face downward sloping demand curves with equal and constant price elasticity $\sigma$, profit maximization implies constant and identical markups: $p = \mu w$ for all $i \leq n$ where $\mu = \sigma / (\sigma - 1) > 1$, and the price of the intermediate input composite relative to labor $p_M / w$ depends on the available intermediate input variety only $p_M / w = \mu n^{1/(1-\sigma)}$. This allows us to prove an analogue to *Proposition 1*.

PROPOSITION 9. Labor market clearing implies that

$$1 = \frac{C}{F(n)} + \frac{I}{G(n)}, \tag{21}$$

where $F(n)$ and $G(n)$ denote the average labor productivity of the market economy in producing consumption and investment goods.

PROOF. Define $A(n)$ as the factor share of intermediate inputs in the production of the consumption goods,

$$A(n) \equiv \left\{ \frac{F_M(M,L)M}{F(M,L)} : \frac{F_M(M,L)}{F_L(M,L)} = \mu n^{1/(1-\sigma)} \right\}. \tag{22}$$

Using this notation, equilibrium in the market for inputs in to the production of consumption goods implies $A(n)C = npm^c = \mu w X^c$ for intermediate inputs and $(1 - A(n))C = wL^c$ for labor. This allows us to calculate the total labor used in the production of the consumption goods

$$X^c + L^c = (1 - A(n)/\sigma)(C/w), \tag{23}$$

and the level produced $C = F(n^{1/(\sigma-1)}X^c, L^c) = (C/w)F((\mu^{1-\sigma}n)^{1/(\sigma-1)}A(n), 1 - A(n))$. Simplifying the last equation yields

$$w = (1 - A(n)/\sigma)F(n), \tag{24}$$

where we have defined $F(n) = F((\mu^{1-\sigma}n)^{1/(\sigma-1)}A(n), 1 - A(n))/(1 - A(n)/\sigma)$. This last expression is equal to the average labor productivity in the consumption goods sector. To see this, substitute (23) and (24) in the expression for consumption to get $C = F(n)(X^c + L^c)$.

Similarly, let $B(n)$ denote the factor share of intermediate inputs in the investment goods sector. Then,

$$w/q = (1 - B(n)/\sigma)G(n) \tag{25}$$

and $I = G(n)(X^I + L^I)$, where $G(n) = G((\mu^{1-\sigma}n)^{1/(\sigma-1)}B(n), 1 - B(n))/(1 - B(n)/\sigma)$. Collecting results, labor market clearing $X^I + L^I + X^c + L^c = 1$ implies the result. Q.E.D.

We now turn to the intertemporal equilibrium conditions. The range of intermediate inputs increases over time through the process of entry of new specialist firms into the intermediate input sector. Start-up operations require one unit of irreversible investment $I$ which firms finance by issuing shares. In equilibrium, the market value of an intermediate goods producer $v$ never exceeds the start-up cost because of free entry $q \geq v$. Whenever there is entry, market value and start-up costs are equalized

$$\dot{n}(q - v) = 0.$$ (26)

Households supply labor, hold and trade ownership shares, and choose consumption levels to maximize their utility subject to their intertemporal budget constraint

$$\int_0^\infty e^{-R_\tau} C_\tau d\tau \leq n_0 v_0 + \int_0^\infty e^{-R_\tau} w_\tau d\tau,$$ (27)

where $R_\tau \equiv \int_0^\tau r_t dt$ and $r_t$ denotes the instantaneous rate of interest. As usual, the optimal consumption profile satisfies

$$\dot{C} / C = \gamma(C)(r - \rho).$$ (28)

**PROPOSITION 10.** For any initial variety of intermediate inputs, $n_0$, the dynamic market equilibrium is characterized by $C_\tau$ and $n_\tau, \tau \geq 0$, which satisfy

$$\frac{\dot{C}}{C} = \begin{cases} \gamma(C)\left(\dfrac{\dot{q}}{q} + \dfrac{A(n)C + B(n)q\dot{n}}{\sigma n q} - \rho\right) & \text{if} \quad C < F(n) \quad \text{(a)} \\ 0 & \text{otherwise} \end{cases}$$ (29)

$$\dot{n} = \text{Max}\left\{G(n)\left(1 - \frac{C}{F(n)}\right), 0\right\} \quad \text{(b)}$$

and

$$\lim_{\tau \to \infty} e^{-\rho\tau} U'(C_\tau) n_\tau F(n_\tau) / G(n_\tau) = 0$$ (30)

where

$$q = (\sigma - A(n))F(n) / (\sigma - B(n))G(n).$$ (31)

**PROOF.** If at time $t$, $C_t = F(n_t)$, then $C_\tau = F(n_\tau)$ for all $\tau > t$ because no investment takes place. This implies the second part of (29a). If $C < F(n)$, then $\dot{n} = I > 0$, and from (26) $q = v$. Arbitrage between consumption loans and equity then implies $r = \Pi / q + \dot{q} / q$, where $\Pi$ denotes operating profits in the intermediate input sector. Operating profits of intermediate input producers are

$$\Pi = (p - w)(\dot{m}^C + m^I) = \frac{A(n)C + qB(n)I}{\sigma n}, \tag{32}$$

and therefore $r = (AC + qB\dot{n})\sigma qn + \dot{q}/q$. Substituting in (28) yields the first part of (29a). The labor markets clearing condition (21) and irreversibility of the start-up investment implies (29b). The price of the investment good $q$ in (31) follows from (24) and (25). Finally, the national income account identity implies $\partial(n_\tau v_\tau)/\partial \tau = r_\tau n_\tau v_\tau + w_\tau - C_\tau$ and therefore $e^{-R_\tau}(n_t v_t) = n_0 v_0 + \int_0^t e^{-R_\tau}(w_\tau - C_\tau)d\tau$. The necessary condition for optimality of the consumption plan $\eta e^{-R_\tau} = e^{-\rho\tau}U'(C_\tau)$ —where $\eta$ denotes the marginal utility of wealth— and (27) with equality and imply that $e^{-\rho\tau}U'(C_\tau)n_\tau v_\tau \to 0$ as $\tau \to \infty$. Because $v_\tau = q_\tau$ for all $\tau > t$ and $0 \le A(n) \le 1$, $0 \le B(n) \le 1$, (30) follows from (31). $\qquad$ Q.E.D.

We now apply this proposition to characterize the equilibrium dynamics and asses the efficiency of the one and the two sector growth models discussed above.

*The One Sector Model—Continued*

In the one sector model $F(M,L) = G(M,L)$. The market equilibrium conditions (29) become

$$\dot{C}/C = \gamma(C)\left(\frac{A(n)F(n)}{\sigma n} - \rho\right) \tag{33}$$

$$\dot{n} = \text{Max}\left\{F(n) - C, 0\right\}; \tag{34}$$

$A(n)F(n)/\sigma n$ is equal to the rate of return to intermediate input producers measured investment. It is now straightforward to prove the analogue to *Proposition* 3.

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**PROPOSITION 11.** There will be locally increasing aggregate returns if intermediate inputs are complements in the sense of Hicks-Allen.

---

PROOF. From (22) and (12) it follows that $A(n)/n = \hat{X}^C(\mu^{1-\sigma}n)/n$. The proof of *Proposition 4* then implies that if intermediate inputs are Hicks-Allen complements then $A(n)/n$ is increasing. $\qquad$ Q.E.D.

We now fully characterize the dynamic market equilibria and assess their efficiency in the presence of input demand complementarities.

*EXAMPLE 1—cont'd* : A necessary condition for input demand complementarities is that $\varepsilon > \sigma > 2$. This implies that there are locally increasing aggregate returns due to input demand

# Figure 2: Dynamic Market Equilibria with Input Demand Complementarities
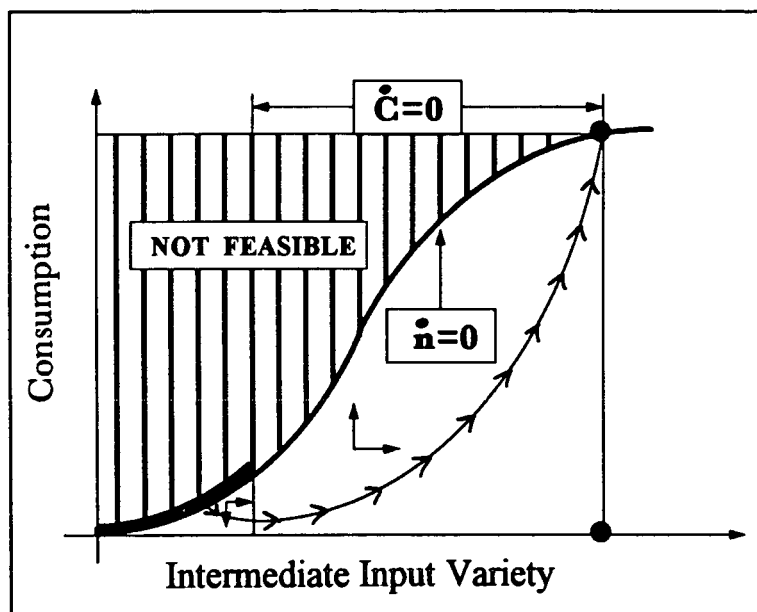


Notes: The thick line corresponds to stationary equilibria. For intermediate input varieties where the thick line and the saddle path overlap, there is dynamic coordination failure.

complementarities when few intermediate inputs are available. As more inputs become available, inputs become demand substitutes. Locally increasing aggregate returns result in multiple intertemporal equilibria as illustrated in *Figure* 2.

To compare efficient and equilibrium allocation, notice that the $n$-isocline in the equilibrium dynamics lies below the $n$-isocline in the efficient dynamics as $F(n) < \hat{F}(n)$ because of the static price distortion. Furthermore—regarding the relative position of the $C$-isoclines— the market-equilibrium rate of return schedule $A(n)F(n)/\sigma n$ is always lower than the efficient rate of return schedule $\hat{F}'(n) = \hat{X}^c(n)\hat{F}(n)/(\sigma-1)n$; this is because the price of the intermediate input composite relative to labor $p_M / w$ is higher in the decentralized equilibrium than in the efficient allocation. As a result, the steady-state variety of intermediate goods as provided by the market is always too low as illustrated in *Figure* 3. The figure shows "local" as well as "global" inefficiencies. "Local inefficiencies," also identified in Judd (1985), Romer (1987), and Grossman and Helpman (1992) for example, can as usual be undone by subsidizing intermediate input purchases so as to equalize their purchase price with the marginal cost of production. (This makes the equilibrium isoclines coincide with the efficient isoclines.) But in spite of such subsidies, "global inefficiencies" may persist. This is because the return to supplying intermediate inputs increases with the availability of other, complementary inputs when few inputs are available. "Global inefficiencies"—or to put it differently, dynamic coordination failure—must be addressed by nonlinear policy instruments.

*The Two Sector Model—Continued*

In the two sector model only labor is used for start-up operations, $G(M,L) = L$. Assuming $U(C) = \log C$ the dynamical system in (29) simplifies to

$$\dot{V} = \rho V - A(n)/\sigma n$$
$$\dot{n} = \text{Max}\left\{\alpha - (1 - A(n)/\sigma)/V, 0\right\}$$

# Figure 3: Dynamically Efficient and Market Equilibrium Allocations Compared
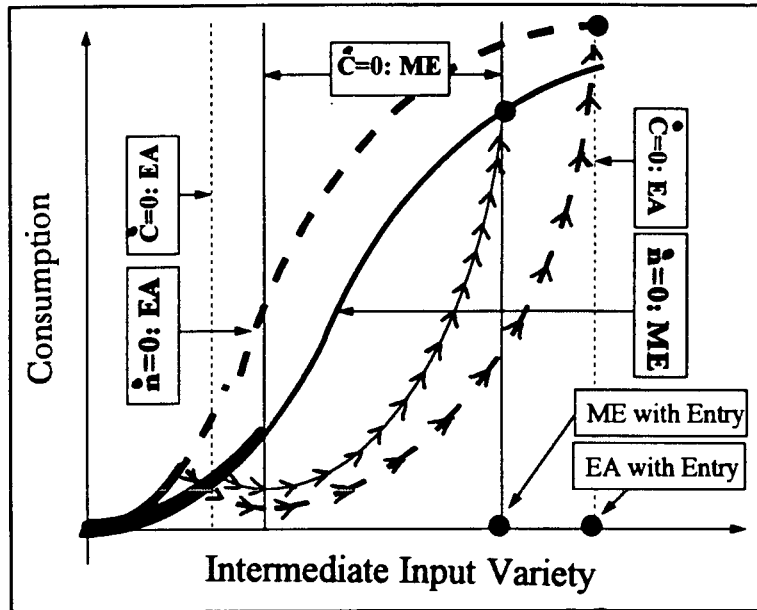


Notes: The thick lines indicate stationary allocations. ME refers to market equilibria and EA to efficient allocations.

where $V = q/C$ denotes the cost of investment measured in utility. Again, the qualitative features of the equilibria depend on the $A(n)/n$ schedule. From the proof of *Proposition* 11, it follows therefore that there will be locally increasing aggregate returns if intermediate inputs are Hicks-Allen complements, and locally decreasing aggregate returns when they are Hicks-Allen substitutes. We turn to a full characterization of dynamic market equilibria, also assessing their efficiency in the presence of input demand complementarities.

EXAMPLE 2—cont'd : A necessary and sufficient condition for the presence of intermediate input demand complementarities is $\varepsilon > \sigma > 1$. This condition implies that intermediate inputs are complements when few of them are available, but become substitutes as the available variety increases. Locally increasing aggregate returns for a low variety of intermediate inputs result in multiple dynamic equilibria, see Ciccone and Matsuyama (1993) for a detailed analysis for this case.

To facilitate the comparison between the efficient and equilibrium allocations, we transform (6) and (7) by defining $W = (1 - 1/\sigma)\hat{F}/\alpha\hat{C}$. This yields

$$\dot{W} = \rho W - \hat{X}^c(n)/\sigma n$$
$$\dot{n} = \text{Max}\left\{\alpha - (1 - 1/\sigma)/W, 0\right\}$$

where use has been made of (12). Again, the $W$-isocline of the efficient allocation lies above the $W$-isocline of the market equilibrium, and the equilibrium $n$-isocline above the respective isocline in the efficient allocation. As a result the market steady-state variety is lower than efficient: There are "local" and global" inefficiencies, which must be addressed with Pigouvian and nonlinear taxes respectively.

# 5 Concluding Remarks

Do relatively minor dynamic increasing returns at the disaggregate level necessarily smooth out in the aggregate? And can we therefore conclude that the distortions from minor dynamic increasing returns at the disaggregate can be readily undone with marginal, Pigouvian taxes and subsidies? These questions are addressed in a model where the production of intermediate inputs is subject to dynamic non-convexities due to start-up costs. The answer comes with two simple key insights: Relatively minor dynamic increasing returns lead to dynamic aggregate increasing returns if inputs are Hicks-Allen complements. Only if inputs are strong substitutes will minor dynamic increasing returns smooth out in the aggregate.

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