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Implicit Collusion on Wide Spreads

Bruno Biais[†]
Thierry Foucault[‡]
and
François Salanié^{**}

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[†] Université de Toulouse.

[‡] Universitat Pompeu Fabra.

^{**} INRA, Toulouse.

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Abstract

Recent empirical findings suggest that spreads quoted in dealership markets might be uncompetitive. This paper analyzes theoretically if price competition between risk-averse market-makers leaves room for implicit collusive behavior. We compare the spread and risk-sharing efficiency arising in several market structures differing in terms of i) the priority rule followed in case of ties, and ii) the type of schedules market makers may use, namely: general schedules, linear schedules, or limit orders. In general, competitive pricing does not arise in equilibrium, and there is a conflict between risk sharing efficiency and the tightness of the spread. This conflict can be mitigated by an appropriate market structure design. The limit order market is the only market structure in which the competitive equilibrium is the unique equilibrium.

IMPLICIT COLLUSION ON WIDE SPREADS¹

1 Introduction.

It is commonly assumed in market microstructure models that market makers quote prices competitively. If they are risk neutral, this leads to prices being equal to conditional expectations, as in Kyle (1985), Glosten and Milgrom (1985), Rock (1995) or Glosten (1994). If market makers are risk averse, and the asset is indivisible, competitive pricing corresponds to quotes being reservation prices, as in Ho and Stoll (1983). In these different models, competitive pricing is alleged to stem from price competition between market makers –in line with the original Bertrand analysis.

Empirical evidence, however, suggests that in practice market makers may not behave competitively. Christie and Schultz (1994 a and 1994 b) offer evidence consistent with collusive behaviour on NASDAQ. For example Christie and Schultz (1994 a) page 1813 write:

“In a market where the inside spread is determined by the actions of multiple dealers, competitive spreads might be considered a natural outcome. However, our results suggest otherwise.”

This paper attempts to resolve this contradiction between empirical findings and theoretical assumptions by analyzing theoretically if price competition between market makers leads to competitive pricing, or if (as Christie and Schultz (1994) write) it leaves room for dealers “to implicitly collude to maintain large spreads.”² Furthermore, we study the consequences of the structure of the market on the extent to which

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²Our analysis is cast in terms of non cooperative game theory. Hence by “collusive” equilibria we will not mean equilibria resulting from cooperation between market makers. Rather we mean Nash equilibria with outcomes favourable to the market makers. For example, when the liquidity trader wants to buy, in a “collusive” equilibrium market makers will quote high prices.

For tractability, we consider a very stylized and simple one period, symmetric information model, whereby i) one liquidity trader submits an order to buy,³ ii) N market makers quote offers, and iii) the liquidity trader selects from these offers, and splits his trade across the market makers, in order to purchase his desired amount at the lowest possible cost. Crucial ingredients of the model are that market makers are i) strategic, which leaves the door open to “collusive” pricing and large spreads, and ii) risk averse, which gives rise to inventory effects à la Ho and Stoll (1983). Our model is therefore in line with the inventory paradigm (see Ho and Stoll, 1983).⁴ Focusing on inventory effects and risk aversion enables us to analyze the risk sharing efficiency of the equilibria and market structures we analyze. Thus we can investigate the interplay between risk sharing efficiency, pricing and market structure.⁵

In this framework we consider a variety of possible market structures, differing with respect to the offers that the market makers are allowed to make, and with respect to the priority rule followed in the market place, i.e., how the trade is allocated in case of ties. As regards the offers the market makers can post, we consider i) the case where they can post fully general and unrestricted schedules (which is quite a large strategy space), ii) the case where they can use only limit orders, and iii) the case where they can use only linear schedules (i.e., a constant unit price, whatever the quantity sold). Note that the third case corresponds to the original formulation of Bertrand competition. In each of these cases, we offer a simple characterization of all the equilibria of the game.

We find that in general competitive pricing does not obtain ! In many of the equilibria we characterize, market makers in equilibrium post ask quotes that are way above their competitive prices. We also find that in general the equilibrium outcome is not efficient from the point of view of risk sharing.

The first point (“collusive pricing”) reflects that market makers can use strategically their schedules to threaten not to participate to the trade if they do not obtain very good prices. Facing this threat, the public prefers to trade with them, in spite of the large price they demand, rather than not trading with them. The second point (“inefficient allocation of risks”) reflects that coordination issues between market makers raise the possibility of coordination on inefficient equilibria.

³Symmetric arguments apply in the case of a sell order.

⁴Hansch, Naik and Vishwanathan (1994) offer empirical evidence consistent with the presence of inventory effects in the London SEAQ.

⁵We realize that our simple one period, perfect information game fails to capture a lot of the richness of the dynamic, asymmetric information games actually played in financial markets. Our stylization enables us to concentrate on one single form of market imperfection, namely imperfect competition. We believe that some of the economic intuition of our results should be robust to more realistic and complex settings.

In addition, we evidence a trade-off between risk sharing efficiency and the tightness of the spread. Suppose the liquidity trader wants to buy, and consider a candidate equilibrium. If, in this candidate equilibrium, risk sharing is very efficient, then if one market maker deviated from his equilibrium trade and undercut the others this deviation would reduce risk sharing efficiency, and thus increase risk bearing for the undercutting dealer. This makes undercutting unattractive and implies that large transfers will be somewhat immune from undercutting if they lead to efficient risk sharing. Consequently more (less) efficient equilibria are associated with larger (smaller) transfers to the market makers.

The results discussed above obtain not only when market makers can post fully general schedules, but also when market makers trade at constant unit prices, i.e., in the standard Bertrand case. When one imposes the additional restriction that the entire trade be allocated to (one of) the market maker(s) quoting the best price, this is the market structure considered in Ho and Stoll (1983). We also analyze the case where the trade can be split between market makers. We show that the ability to split the trade has significant consequences, and in particular raises the possibility of non competitive prices and of the above mentioned trade-off between risk sharing efficiency and tight spreads.

In striking contrast with these results, we find that when market makers are restricted to quote limit orders, the unique Nash equilibrium entails competitive pricing and efficient risk sharing.⁶ The intuition is that when market makers post limit orders, then each market maker can always undercut his competitors until he trades at a price equal to his marginal valuation of the asset. This implies competitive pricing and efficient risk sharing. Hence in a pure limit order market there is no tension between fair prices and efficient risk sharing.

From all the market structures studied in the present paper, limit order competition is the only one for which the competitive equilibrium is the unique equilibrium. Thus, within our simple theoretical framework, we find that the limit order market is the optimal market structure. One contribution of our paper is therefore to offer some elements of a normative theory of market microstructure.⁷ From an applied point of view, our analysis suggests that competitive pricing and efficient allocation of trades should not be taken for granted. Maybe our message to market organizers or regulators is that they should be careful not to leave too much flexibility to market makers to

⁶The efficiency of such a market structure had already been studied in general equilibrium theory, see e.g: Dubey (1982).

⁷Glosten (1989) also offers insights on the optimal design of a market. Our analysis is simpler than his, since we do not consider asymmetric information. But our focus on, and analysis of, imperfect competition between market makers differs from his paper.

construct complex schedules (using minimum quantity requirements, fill-or-kill orders, or large price variations from one quantity to another) because such schedules can be used to support “collusion” on large spreads.

Three recent papers discuss issues related to those analyzed in the present paper.

- Bernhardt and Hughson (1994) show in a Kyle (1985) environment that splitting of trades between market makers can lead to quite uncompetitive prices. Our focus on inventory effects and our study of different market structures differentiates our analysis from Bernhardt and Hughson (1994).
- Simultaneously and independently from our work, Simmons and Gerber (1994) have analyzed, in the context of the inventory paradigm, the case where market makers must post linear schedules. Our results on this point (presented in section 6 of the paper) are similar to theirs, although we offer a more general treatment of the effect of priority rules, i.e., the way trades are allocated in the case of ties. But unlike in the present paper, Simmons and Gerber (1994) do not analyze general schedules or competition in limit orders.
- Finally, Dutta and Madhavan (1995) analyze the possibility of collusion when dealers compete in prices. However, in their model, dealers are risk neutral and collusion arises because of repeated interactions. In contrast, we consider a one period model, where implicit collusion can emerge because of the risk aversion of the market-makers and the market structure.

The next section presents and motivates our model and assumptions. The third section presents a few examples illustrating our main results, and motivating the general analysis of the next sections. Section 4 analyzes the case where market makers can use unrestricted general schedules. Section 5 analyzes competition in limit orders. Section 6 analyzes competition in linear schedules. Section 7 offers concluding comments. Proofs not in the text are in the appendix.

2 The model.

2.1 Agents and preferences.

Consider the market for a risky asset, with end of period liquidation value $v = \mu + \epsilon$, where μ is a constant and $E(\epsilon) = 0$. For simplicity, we assume that ϵ is normal with standard deviation σ , but this does not affect qualitatively the results.

There is one liquidity trader, who must buy D shares of the asset.⁸ The utility function of the liquidity trader is lexicographic. He must buy D , but under this constraint the liquidity trader seeks to minimize the price paid for these shares.

There are N suppliers of liquidity competing to accommodate the liquidity demand. Abusing notations a little bit, we will also denote N the set of market makers.

Since the liquidity trader wants to buy, these suppliers of liquidity are hereafter referred to as “the market makers.” They are strategic, rational and risk averse. For simplicity and again without consequences for the qualitative nature of the results, we assume they have CARA utilities, i.e.,

$$\forall i = 1, 2, \dots, N, \exists \gamma_i \geq 0, \mathcal{U}_i(x) = -\exp(-\gamma_i x)$$

Market makers start the game with endowments in the risky asset I_i , as well as cash C_i . There is no asymmetric information about the value of the asset, the positions of the market makers or the liquidity demand.

The expected utility of market maker i if he does not trade is:

$$EU_i(I_i v + C_i) = \mathcal{U}_i(C_i + I_i \mu - \frac{\gamma_i \sigma^2}{2} I_i^2)$$

which we will denote $\mathcal{U}_{i,0}$.

The expected utility of market maker i if he sells q_i in exchange for cash transfer t_i is :

$$EU_i((I_i - q_i)v + C_i + t_i) = \mathcal{U}_i(C_i + t_i + (I_i - q_i)\mu - \frac{\gamma_i \sigma^2}{2} (I_i - q_i)^2) = \mathcal{U}_{i,0} \exp^{-\gamma_i [t_i - V_i(q_i)]}$$

where

$$V_i(q_i) = q_i [\mu - \frac{\gamma_i \sigma^2}{2} (2I_i - q_i)]$$

Note that $V_i(q_i)$ is the reservation transfer of market maker i for quantity q_i , i.e., this is the transfer such that he is indifferent between selling q_i or not trading. To put it differently, V_i measures the value that i attaches to the asset. Note that V_i is convex, i.e., the marginal valuation of market maker i for the asset increases as the amount sold by i increases. This is because as i sells larger and larger amounts, he takes more and more unbalanced positions (maybe short) in the asset, which makes i more and more reluctant to sell more.

⁸For simplicity and brevity, we focus only on the buy side of the market. Symmetric results obtain when the liquidity trader wants to sell.

For now let's denote the utility of market maker i if he trades q_i as:⁹

$$U_i(t_i - V_i(q_i)) = \mathcal{U}_{i,0} \exp^{-\gamma_i(t_i - V_i(q_i))}$$

This rewriting of the objective function of market maker i emphasizes the trade-off faced by i between obtaining transfer t_i and selling quantity q_i , to which i attaches value $V_i(q_i)$.

2.2 A benchmark : the competitive equilibrium.

We now describe briefly the competitive equilibrium arising in this context. This will serve as a benchmark, to evaluate how far from the competitive outcome are the prices and allocations generated by the different market structures we consider. Since the competitive equilibrium is Pareto optimal, if one market structure gives rise to the competitive outcome, it is optimal.

Denote p^* the competitive equilibrium price and q_i^* the competitive supply of market maker i . In the competitive case, market maker i chooses the optimal quantity to offer at the equilibrium price p^* , which he takes as exogenous :

$$\text{Max}_{q_i} U_i(p^* q_i - V_i(q_i))$$

The first order condition is : $p^* = V_i'(q_i^*)$ Hence the competitive outcome is $\{p^*, q_1^*, \dots, q_N^*\}$ such that $\sum_i q_i^* = D$ and $\forall i, V_i'(q_i^*) = p^*$ Because valuation functions V_i are convex, this equilibrium exists and is unique. It corresponds to the optimal allocation of risks.

2.3 Market structure.

There are two stages in the trading game : first the market makers post offers, second the buyer selects the offers at which he can buy D while paying the lowest possible price. The types of offers that the market makers can post, as well as the allocation of trades and price formation resulting from these offers and the reaction of the buyer are presented in this subsection. These features of the trading game define the trading market structure. They are set by the regulators and the market organizers. In this paper we will consider different market structures and compare the equilibrium outcomes to which they give rise.

⁹A similar rewriting of the utilities of market makers is in Biais (1993).

2.3.1 The offers made by the market makers.

The first crucial feature of the market structure is the type of offers market makers are allowed to make, i.e., the type of schedule they are allowed to quote.¹⁰ The schedules of offers posted by the N market makers are denoted : $\{t_i(\cdot)\}_{i=1,\dots,N}$, where $t_i(q)$ is the price at which market maker i offers to sell q .

In the next section, we consider the case where there is no restriction on the type of schedules the market makers can post. We will show that the ability for market makers to link the transfer to the amount traded in a quite general way gives them significant ability to influence prices. One special type of schedule which will prove important, as a tool to solve for equilibria, is fill-or-kill-orders. Such orders are available for example on the NYSE or the Paris Bourse. They specify a price and a quantity, and require that the order be filled at this price and *for the entire quantity* or not at all: partial execution is not allowed.

In the Paris Bourse, the New York Stock Exchange (NYSE) or the Tokyo Stock Exchange liquidity suppliers quote limit orders, specifying the price and the maximum quantity they are willing to trade at this price. We analyze this case in Section 4.

In open-outcry markets (e.g. in futures markets such as the CBOT in Chicago), brokers announce the desired quantity of their customer (in our notations : D), and floor traders (our N market makers), respond with prices at which they are willing to accomodate this quantity. When traders simply respond a price to the announced quantity, they implicitly accept to trade all the announced quantity at this price. This case is in fact competition in linear prices, as in the original Bertrand case. We study it in Section 5.

Those different market structures are analyzed separately because the computations used to solve for the equilibria vary somewhat when the strategies available to the traders vary. Throughout the paper, we relate the results obtained in the different market structures and we compare the economic forces at work in the different cases.

2.3.2 Trades

After the broker has expressed the demand of the buyer (D), and the market makers have posted offers, the buyer selects from these offers to minimize the total price paid

¹⁰We will not discuss in this paper one interesting dimension of the market makers possible strategies, i.e., the discreteness of the pricing grid. On this topic see Bernhardt and Hughson (1993), Cordella and Foucault (1995) and Harris (1994).

for D :

$$\begin{cases} \text{Min}_{(q_i)} \sum_i t_i(q_i) \\ \sum_i q_i \geq D \end{cases} \quad (1)$$

In general, there may not exist solutions to this program. To avoid this we impose some weak constraints to the schedules. First, for obvious economic reasons, we restrict our attention to schedules that do not generate any payment if the market maker does not trade ($t_i(0) = 0$). Second, the schedules should not involve negative monetary transfers to the market makers : $t_i(q) \geq 0, \forall q, \forall i$. Third we impose that an allowed schedule t be defined on a closed subset A (containing zero) of the positive real line, and be lower hemi-continuous on A . Under these conditions, either it is impossible to buy D or more and we impose that no transactions take place,¹¹ or solutions to program (1) exist.

The price setting mechanism we assume has a discriminatory or first price auction flavour, since each market maker trades at her own offer, so that different traders may trade at different prices. This corresponds to the actual structure of continuous markets, such as the Paris Bourse or the NYSE, whereby market orders can be executed at different prices against different orders in the Book. Rock (1995), Glosten (1995) in the context of stock markets, and Back and Zender (1993) in the context of Treasury auctions consider this type of discriminatory price formation. This differs from uniform price auctions such as in Kyle (1989), or Klemperer and Meyer (1989).

2.3.3 The allocation rule.

In general the minimization problem (1) faced by the buyer may have multiple solutions. Actual market structures as well as the analysis presented in this paper show that this is not simply a technical problem but has important economic consequences.

In financial markets, it is often the case that there are more than one way to carry a trade, while leaving the price paid unchanged. For example suppose the buyer needs to buy 100 shares and faces an order book where at the best ask price there are two orders to sell, one for 100 shares and the other for 50. In general, the liquidity trader could choose any combination of the two orders that would add up to 100 shares. In fact, in most financial markets, there are formal rules used to solve this indetermination, i.e., priority rules. Hasbrouck, Sofianos and Sosebee (1993) provide a detailed description of priority rules used in the NYSE.

One commonly used priority rule is time priority, under which orders entered first are

¹¹Such an outcome will never happen at equilibrium, since any market maker could then propose to produce D at a sufficiently high price.

executed first. In our simple one-period model this rule cannot play a role, since we assume that all sell orders are placed simultaneously. There also exists other priority rules based on quantities, such as prorata allocation for example.

We will show that allocation rules used in the case of ties can play an important role. However, such rules may be difficult to implement unless the organization of the market and its trading rules are very explicit. This is an argument in favor of organized, centralized and computerized markets, such as e.g. the Paris Bourse.

An allocation rule α , defined on the cross-product of the strategy sets of the market makers, associates to the market makers' offers a probability distribution over the set of total payment-minimizing allocations. If this set is not a singleton, and if the distribution is not degenerate, we call the allocation rule stochastic. Also, we require the allocation rule to be anonymous, i.e., the allocation rule depends only on the number of market makers involved in a tie and on the offers but not on the index of the market makers.

We use the following notations: S is the n -tuple of offers of the market makers; α is the allocation rule; $\alpha(S)$ is the probability distribution used when the market makers' offers are S ; q_i is the trade of market maker i ; E is the expectation operator taken with respect to $\alpha(S)$. Given offers S , we will say that market maker i is "active" if $Eq_i > 0$.

2.4 Equilibrium.

For simplicity, we restrict this study to pure-strategy equilibria, so that the strategy set of a market maker is identified to the set of offers he is allowed to make.

An equilibrium of the game is a n -tuple S of allowed offers such that no market maker wants to deviate, given that the allocation rule α will be applied¹². An outcome of the game is a demand allocation (q_1, \dots, q_n) , together with associated transfers (t_1, \dots, t_n) which might be selected (with positive probability) by the allocation rule at an equilibrium of the game.

¹²Even if the buyer could change the rule once the market makers' offers are known, he would have no incentive to do so, since α always selects an allocation within the set of allocations which minimize total payment.

3 A few simple examples.

In this section we motivate the analysis carried in the following sections by presenting equilibria arising in four different trading market structures. For simplicity assume in these examples that there are 2 market makers, with identical valuation functions:

$$V_1(q) = V_2(q) = q^2$$

In this simple case, efficient risk-sharing requires to split the trade equally between the two market makers.

We consider the following trading market structures:

1. **Market Structure 1 : linear schedules/no split.** The market makers compete in linear schedules. In case of ties, the liquidity demand is allocated with probability 0.5 to one of the two market makers. This Bertrand competition is assumed in Ho and Stoll (1983).
2. **Market Structure 2 : linear schedules/equal splitting.** The market makers compete in linear schedules but in case of ties, the liquidity demand is split equally between the two market makers.
3. **Market Structure 3 : limit orders/pro-rata allocation.** Each market maker can post one limit order. In case of ties, a pro-rata allocation rule is used.
4. **Market Structure 4 : general schedules/splitting.** Market makers can announce any schedule, differentiable and continuous, except perhaps in 0. In case of ties, the allocation rule favors allocations in which both market makers participate.

Equilibrium outcomes arising in these 4 cases are as follows:¹³

- In the case of linear schedules, and when trades are not split between market makers, as shown in Ho and Stoll (1983), there exists a unique equilibrium, where both market makers quote a price equal to their average reservation transfer ($\frac{V(D)}{D}$) for the quantity D and are allocated this quantity with probability 0.5. This leads to inefficient risk-sharing.

¹³The proofs of these results stem directly from the propositions presented in the following sections.

- With linear schedules and when trades are equally split between the market makers posting the best price, many equilibria can be obtained, with lower or higher prices than in the first market structure. The following quotes are one of those possible equilibria :

$$p_1 = p_2 = \frac{1}{4} \frac{V(D)}{D} = \frac{D}{4}$$

- In the case of limit orders and pro-rata allocation, the competitive outcome is the unique equilibrium, both market makers place the same limit order : they accept to sell at price $p = V'(\frac{D}{2}) = D$ up to a maximum quantity $\bar{q} = D$.
- In the case of general schedules there are multiple equilibria. For example, the following schedules form an equilibrium in which each market maker trades $\frac{D}{2}$ and receives a payment equal to $\frac{3D^2}{4}$:

$$t_i(q) = q^2 + \frac{D^2}{2} \quad \text{for } i \in \{1, 2\}, q > 0$$

$$t_i(0) = 0 \quad \text{for } i \in \{1, 2\}$$

Table 1 (in the Appendix) summarizes the outcomes of the trading game in each market structure when the market makers use these equilibrium strategies. The diversity in possible outcomes shows that the trading rules influence strongly the efficiency of the allocations, the gains from trade obtained by the market makers, the transaction prices and the total payment of the liquidity trader.

In the first market structure the market makers obtain zero gains from trade. In contrast, they strictly benefit from trading in the second market structure. Still, the price paid by the liquidity trader is lower in the second market structure. This is made possible by the greater risk sharing efficiency stemming from the splitting of trades. Yet, the ability to split trades, and the risk sharing efficiency of the allocation of the trade do not guarantee low payments for the liquidity trader : in the fourth market structure, the allocation is efficient *and* the price paid by the liquidity trader is the highest. Finally, when market makers must use limit orders, the competitive outcome is obtained. The prices and the payments of the liquidity trader are the same as in market structure 1 but the market makers extract gains from trade because the allocation of risk is efficient.

In the next sections, we explain why the outcomes of the trading game are so sensitive to the type of orders the market makers can use and to the allocation rule. We also show that market structures which lead to efficient allocations can give rise to high transfers for the market makers. We identify restrictions on order types and allocation rules which help to mitigate this problem.

4 Competition in general schedules.

In this section we characterize the equilibrium outcomes arising when the type of schedules the market makers can use is unrestricted. Although this space of strategies is very large, we are able to characterize *all* the equilibrium outcomes, in a simple and economically meaningful way. We find that the ability for the market makers to link the quantity they sell to the price at which they sell it, and in particular to set a minimum to the quantity they are willing to sell, plays a crucial role. Minima on the quantity for sale are a feature of fill-or-kill orders (hereafter FK). We show in this section that considering these FK orders is useful in solving for and constructing the equilibria.

4.1 Equilibrium conditions when market makers can use general schedules.

Given the offers of her competitors, market maker i selects the schedule which maximizes her expected utility. In fact, the schedules of her competitors and the liquidity demand D define for market maker i the equivalent of a residual inverse demand function. That is, for any quantity q_i that market maker i would like to trade, there exists a maximum transfer that she can ask for, while making sure that she will indeed sell q_i .

To establish this more explicitly denote $A_{-i}(q)$ the minimum payment for which the liquidity trader can buy quantity q from all N market makers except i . Now suppose i would like to sell q_i . To make sure she will conduct this trade, i can make a take it or leave it offer (by placing a single FK) to sell q_i against payment t_i . i is certain that the buyer will accept this offer, as long as

$$A_{-i}(D - q_i) + t_i \leq A_{-i}(D) \quad (2)$$

Since the market maker is interested in maximizing transfers she will saturate inequality (2), and set ¹⁴

$$t_i = A_{-i}(D) - A_{-i}(D - q_i) \quad (3)$$

¹⁴Strictly speaking, making the constraint bind leads to a tie, since the buyer could also exclude market maker i and bear the same total payment. To avoid such complications, market maker i may ask $A_{-i}(D) - A_{-i}(D - q_i) - \epsilon$ ($\epsilon > 0$ and arbitrarily small), so that the allocation rule does not matter anymore. In the next sections we show that in other market structures the allocation rule plays a much more important role.

Given the “inverse demand curve” defined in equation (3), market maker i can select an optimal trade by placing a single FK, for quantity q_i solving :

$$\text{Max}_{q_i} U_i(A_{-i}(D) - A_{-i}(D - q_i) - V_i(q_i)) \quad (4)$$

Hence we can state our first result :

Lemma 1 : *The equilibrium trades of market maker i must solve :*

$$\text{Min}_{q_i} A_{-i}(D - q_i) + V_i(q_i)$$

Lemma 1 provides a characterization of the equilibrium quantities traded. We now discuss further the equilibrium transfers. Consider a candidate equilibrium characterized by schedules $\{t_i(\cdot)\}_{i=1,\dots,N}$ and by actual trades and transfers $\{t_i, q_i\}_{i=1,\dots,N}$ such that $\sum_{i=1,\dots,N} q_i = D$. Suppose that in this candidate equilibrium, the total price paid by the buyer $F = \sum_{i=1,\dots,N} t_i$ is strictly lower than the price he would pay if he did not trade with i : $A_{-i}(D)$. In this case, market maker i is indispensable, in the sense that if the buyer did not trade with him he would be worse off (since he would pay a larger price). But in this case, market maker i would be better deviating from his candidate equilibrium offer, and post a single FK, for quantity q_i against transfer $t_i + \epsilon$, where ϵ is a small positive number, lower than $A_{-i}(D) - F$. Hence it cannot be an equilibrium that $A_{-i}(D) > F$. On the other hand, if $F > A_{-i}(D)$, then F cannot be the equilibrium payment, since the buyer would be better off buying at $A_{-i}(D)$. Consequently, as stated in the next lemma, in equilibrium $A_{-i}(D) = F$.

Lemma 2 : *In equilibrium, no market maker can be indispensable, in the sense that :*

$$A_{-i}(D) = F \quad \forall i \quad (5)$$

where F is the total price paid by the buyer in equilibrium.

Building on lemmas 1 and 2, we can state our first proposition :¹⁵

Proposition 1 : *Schedules $\{t_1(\cdot), \dots, t_n(\cdot)\}$ form an equilibrium if and only if they generate outcomes such that, for any market maker i :*

a) $F = A_{-i}(D)$ where F is the total payment made by the buyer.

¹⁵That conditions a) and b) in Proposition 1 are necessary stems from lemmas 1 and 2. The sufficiency of a) and b) is straightforward and omitted for brevity.

b) Possible trades for market maker i are solution to

$$\text{Min}_q A_{-i}(D - q) + V_i(q)$$

To establish Proposition 1 we have used the fact that, for given schedules of her competitors, market maker i has always a best response which consists in posting a single fill-or-kill order (t_i, q_i) solution of (3) and (4). However, the result of Proposition 1 is not dependent on the possibility for the market makers to use fill-or-kill orders. Actually there exists other (and smoother) schedules which are best responses for the market makers, which do not require the use of fill-or-kill orders and which lead to trades solution of (3) and (4).

Consider the following strategy for market maker i :

$$t_i(q) = V_i(q) + (t_i - V_i(q_i)), \quad \forall q > 0, \quad t_i(0) = 0 \quad (6)$$

where (t_i, q_i) are as defined in Proposition 1. Facing this schedule the liquidity buyer will select the quantity q traded with i , to minimize :

$$A_i(D - q) + V_i(q)$$

and hence will choose to trade with i , q_i satisfying condition b) in Proposition 1.¹⁶ Substituting this in (6), the transfer to i will be t_i as defined in Proposition 1. Hence we can state the following Corollary:

Corollary 1 : *Given the schedules of his competitors, seller i can make sure he will obtain his optimal trade (t_i, q_i) , solution of (3) and (4) by posting the schedule defined in equation (6).*

The schedule considered in Corollary 1 is discontinuous in zero. Now, remember we have only considered the sell side of the problem, i.e., the formation of ask quotes. The results obtained in the case of bid quotes would be symmetric. Combining the analyses of these two sides of the market, discontinuity in zero can be interpreted as a bid ask spread. This is stated in the next corollary.

Corollary 2 *For the equilibrium schedules described in Corollary 1 there is a discontinuity in zero, i.e., there is a bid-ask spread.*

¹⁶Strictly speaking the buyer would be indifferent between this and excluding i from the trade. To avoid this either assume that i in fact quotes a transfer arbitrarily close to t_i but smaller than t_i , or assume that the allocation rule is such that no market maker associated to a tie is excluded from the trade.

Equilibrium schedules, giving rise to a discontinuity in zero, and interpreted as a bid-ask spread arise in Glosten (1989), but for different reasons than in the present paper.¹⁷ In Glosten (1989) the discontinuity is a feature of the optimal schedule posted by a monopolist, facing privately informed investors. In the present paper, the discontinuity does not arise because of asymmetric information but because of imperfect competition.¹⁸

4.2 An explicit necessary condition for equilibrium.

In order to better characterize the possible equilibria we now provide *explicit* necessary conditions on equilibrium outcomes, that is conditions which do not involve the implicit residual inverse demand function $A_{-i}(\cdot)$.

Consider an outcome $(t_i, q_i)_{i \in N}$. In equilibrium, market maker i obtains expected utility :

$$U_i(t_i - V_i(q_i)) \quad (7)$$

In equilibrium market maker i is better off selling q_i than not trading, i.e. :

$$\forall i \in N, t_i \geq V_i(q_i) \quad (8)$$

Now, consider a set J of active market makers to which i belongs : $i \in J \subset N$. If i wanted to undercut this set of market makers, and sell instead of them, he would have to offer to sell $\sum_J q_j$ for a transfer slightly lower than $\sum_J t_j$. In this case i would obtain expected utility :

$$U_i(\sum_J t_j - V_i(\sum_J q_j)) \quad (9)$$

In equilibrium, no market maker has incentive to undercut her competitors, hence the utility in equation (7) must be larger than the utility in equation (9). This implies that :

$$\forall i \in J \subset N, \sum_{J-\{i\}} t_j \leq V_i(\sum_J q_j) - V_i(q_i) \quad (10)$$

The LHS of equation (10) is the increase in revenues due to undercutting, the RHS is the increase in risk bearing. market maker i does not undercut if the latter is larger than the former.

The next proposition follows from the above discussion (in particular from Equations (8) and (10)).

¹⁷Note that this type of bid-ask spread differs from that analyzed by Ho and Stoll (1983) or Kyle (1985), whereby the schedules posted by market makers do not exhibit discontinuities in 0.

¹⁸More generally, schedules with discontinuities in 0 arise often in the context of optimal pricing models, such as two-part tariffs models for instance. Note that, as in Glosten (1989), in those models the discontinuity reflects information extraction, in contrast with our symmetric information model.

Proposition 2 : *Equilibrium outcomes* $\{(t_i, q_i)\}_{i=1, \dots, N}$ *must satisfy* :

$$\forall J, \forall i \in J, 0 < \sum_{j \in J, j \neq i} [t_j - V_j(q_j)] \leq V_i(\sum_{j \in J} q_j) - \sum_{j \in J} V_j(q_j) \quad (11)$$

Condition (11) implies that :

$$\sum_J V_j(q_j) \leq V_i(\sum_J q_j)$$

which expresses that as regards the allocation of risks, it is more efficient that market makers in J trade their equilibrium quantities, than if one of them undercut all the others. That is, in equilibrium some risk sharing takes place. But the extent of this risk sharing is limited. Optimal risk sharing (as in the competitive equilibrium) would require equality of marginal valuations,

$$\forall (i, j) \in N^2, V'_i(q_i) = V'_j(q_j)$$

which is obviously not granted by condition (11)¹⁹.

Condition (11) also implies an upper bound on the transfers that the market makers can receive in equilibrium. If one market maker demanded too high a transfer, he would be undercut by one of his competitors, as soon as the revenue generated by undercutting would exceed the increase in risk bearing that it would entail.

Interestingly, there is a link between the two above remarks. The more efficient the allocation of risks, the less attractive it is for one market maker to undercut another and thus increase his risk-bearing, and consequently the larger the transfers the other market maker can ask for, without fear of being undercut. *Hence, there is a trade-off between the efficiency of the allocation of risk and the price paid by the buyer.* We will examine this trade-off further in the next subsections.

4.3 Equilibrium Strategies.

Condition (11) delineates the set of possible equilibrium outcomes but does not characterize the equilibrium strategies which can lead to those outcomes. We now compute

¹⁹However it is possible to show that the competitive outcome always satisfy condition (11) because the valuation functions $V_i(\cdot)$ are convex. Therefore the competitive equilibrium is always an equilibrium candidate. The proof of this property is skipped for brevity.

explicitly such strategies. First we show that schedules based on two fill-or-kill orders are sufficient to generate in equilibrium all the outcomes which satisfy (10). This implies that condition (10) is not only necessary but also sufficient. This is the next proposition.

Proposition 3 : *Consider an equilibrium outcome $\{(t_i, q_i)\}_{i=1, \dots, N}$ which satisfies condition (11). If the allocation rule is such that in case of a tie no market maker in the tie is excluded from the trade, then the following schedules based on two fill-or-kill orders :*

$$t_i(q_i) = t_i \quad \text{and} \quad t_i(D) = F \quad \forall i \in N$$

form an equilibrium.

The equilibrium strategies described in Proposition 3 involve :

- one “serious offer”, (t_i, q_i) which will be the equilibrium trade of the market maker.
- one out of the equilibrium path threat $(\sum t_j, D)$ which ensures that the others will not raise their prices.

This decomposition of the equilibrium strategy is common in the literature on competition in price schedules and can be found for instance in Wilson (1979), Klemperer-Meyer (1989) and Back-Zender (1993).

Proposition 3 complements Proposition 2 by showing that the rather natural necessary condition stated in that proposition is also sufficient. Further, by showing that all outcomes satisfying condition (11) can be equilibrium outcomes it shows that there is a large multiplicity of equilibria. In particular it is easy to see that the competitive allocation is an equilibrium trade but that there is a multiplicity of possible equilibrium transfers corresponding to this trade. (We will analyze the multiplicity of equilibria further in the next subsection.)

Proposition 3 also complements Proposition 1 and Proposition 2 by describing explicitly equilibrium strategies that give rise to the outcomes analyzed in these propositions.

4.4 Properties of the equilibria.

We now discuss the properties of the equilibria. For simplicity we focus on the case of two market makers. In this case, from Proposition 2 and Proposition 3, equilibrium outcomes are characterized by

$$V_1(q_1) \leq t_1 \leq V_2(D) - V_2(q_2)$$

$$V_2(q_2) \leq t_2 \leq V_1(D) - V_1(q_1) \quad (12)$$

The following remarks can be made :

- **Equilibrium multiplicity:** There is a large multiplicity of equilibria. This stems from coordination issues, reflecting strategic complementarity : the less attractive the schedules offered by his competitors, the larger the residual demand market maker i faces, the larger the transfer he can demand for q_i . This generates equilibrium multiplicity.
- **Inefficient allocations:** As implied by equation (12), *all* allocations $(q_i, q_j = D - q_i), q_i \in [0, D]$ such that

$$V_i(q_i) + V_j(q_j) \leq \min(V_1(D), V_2(D)) \quad (13)$$

are possible equilibrium outcomes. If, for example, $V_1(D) > V_2(D)$ this precludes allocations whereby q_1 would be larger than a given q^* (defined by $V_i(q^*) + V_j(D - q^*) = V_2(D)$), i.e., whereby market maker 1 would serve nearly all the demand. Still the set of possible allocations is quite large, and contains many inefficient allocations, in addition to the unique efficient allocation. Again this reflects the presence of coordination issues : the market makers can coordinate on bad equilibria, i.e., inefficient allocations. This reinforces our earlier statement (in the discussion of Proposition 2) that the efficiency of risk sharing in this market structure is limited.

- **Non competitive pricing:** Define unit prices as $p_i = t_i/q_i$. The unit prices at which the market makers sell the shares are such that:

$$\begin{aligned} V_1(q_1)/q_1 \leq p_1 &\leq \frac{V_2(D) - V_2(q_2)}{D - q_2} \\ V_2(q_2)/q_2 \leq p_2 &\leq \frac{V_1(D) - V_1(q_1)}{D - q_1} \end{aligned} \quad (14)$$

Hence, for each possible allocation of trades, there are many possible prices, ranging from low reservation prices ($p_i = \frac{V_i(q_i)}{q_i}$), to large prices ($p_i = \frac{V_j(D) - V_j(q_j)}{D - q_j}$) at which the market makers earn large rents, and which are well above competitive prices.

- **Equilibrium price dispersion:** As can also be seen from equation (14), in equilibrium the two market makers may well trade at different unit prices. Hence there can be equilibrium price dispersion although there is no asymmetric information, or differences in preferences among the buyers, or search costs. It is only due to the market structure and the strategic interactions between market makers it generates.

4.5 Selecting an equilibrium.

In this section we investigate which of the many equilibria are more likely to be selected by the market makers.

4.5.1 Pareto dominant equilibrium.

First we search for an equilibrium that would be Pareto dominant from the point of view of the market makers. In the case of two market makers (i and $-i$), for given equilibrium trades q_i, q_{-i} there exists a Pareto Dominant equilibrium for the two market makers, the equilibrium which maximizes the transfers.²⁰ It yields transfers equal to :

$$\begin{aligned}t_i &= V_{-i}(D) - V_{-i}(q_{-i}) \\t_{-i} &= V_i(D) - V_i(q_i)\end{aligned}$$

In this equilibrium expected utilities are :

$$\begin{aligned}U_i(V_{-i}(D) - [V_{-i}(q_{-i}) + V_i(q_i)]) \\U_{-i}(V_i(D) - [V_{-i}(q_{-i}) + V_i(q_i)])\end{aligned}$$

For these transfers, the equilibrium trade (q_i, q_{-i}) which maximizes expected utility for *both* market makers is the efficient allocation, which maximizes risk sharing and leads to:

$$V'_i(q_i) = V'_{-i}(q_{-i})$$

Note however that it is also the trade which maximizes transfers ! We summarize these results in the next proposition.

Proposition 4 *Suppose there are two competing market makers, among all the Nash equilibria of the game, the equilibrium, which from the point of view of both market makers is Pareto dominant, maximizes the efficiency of trades and the price paid by the buyer.*

Hence the most efficient allocation is also the allocation which maximizes the payment for the buyer. This generates a conflict of interests between the market makers and the buyer, and a trade-off between risk sharing efficiency and fairness of prices.

²⁰Note however that with more than two market makers there does not exist a Pareto dominant equilibrium, because of conflicts of interests between potential coalitions of market makers.

The intuition is the following. By definition, in an efficient allocation, each market maker shares risk efficiently. Consequently, any deviation by one of the market makers, whereby he would undercut the others, is likely to lead to less efficient risk sharing, and therefore is not attractive for the market maker. Hence even if the market makers ask for large transfers they will not be undercut. Although we analyze non cooperative Nash equilibria, this association between risk sharing and high prices has a flavour of collusion.

4.5.2 Elimination of dominated strategies.

Another way to focus on equilibria which are particularly likely to emerge is to select equilibria by iterated elimination of dominated strategies (IEDS). Using this criterion leads us to considering offers that are not executed in equilibrium. Considering these offers is important in the present context, since they play an important role (as threats) in the construction of the equilibria.

For simplicity consider again the case of two market makers $i = 1, 2$. Consider an equilibrium where market maker 1 posts 2 FK :

$$\{(t_1, q_1), (t_1 + t_2, D)\}$$

Is this equilibrium robust to IEDS ? It is not if $t_1 + t_2 < V_1(D)$ – since in this case 1 makes losses if he sells D , while he could have avoided those losses by not placing his order for D at $t_1 + t_2$. Robustness to IEDS implies : $F = t_1 + t_2 \geq \max(V_1(D), V_2(D))$. Hence the price paid by the buyer is larger than that paid in a second price indivisible quantity auction. This is striking, since risk bearing costs are lower in this market structure than in a second price indivisible good auction, and still the payment is higher.

4.6 Conclusion of this section

This section has shown that, when market makers can use general schedules, ask prices can be high, allocations are not necessarily efficient, and there is a multiplicity of equilibria. Also, there is a tension between the tightness of the bid–ask spread and risk sharing efficiency. Finally Pareto dominance select equilibria characterized by high ask prices (which are also selected by IEDS) and efficient risk sharing. In our view these findings suggest that “collusive” pricing is likely to emerge when market makers can use general schedules.

Since market makers can use general schedules to support implicit collusion and thus generate large rents, it may be in the interest of the market maker to restrict the

strategy space of the market makers. Are such restrictions efficient at fighting collusion, and do they come at the cost of lower risk sharing efficiency? We study these issues in the next sections.

5 Competition in Limit Orders.

In this section we consider the case where the market makers can only post limit orders. The equilibrium obtained in this case is rather different from those arising with general schedules.

Proposition 5 : *If the market makers can only use limit orders, then the offers $\{p_i, \bar{q}_i\}_{i=1, \dots, N}$ form an equilibrium where the subset $J \subset N$ of market makers is active iff :*

- All active market makers quote the competitive price, p^* ,

$$\forall i \in J, p_i = p^*$$

- No active market maker is indispensable:²¹ $\forall i \in J, \sum_{\substack{j \neq i \\ j \in J}} \bar{q}_j \geq D,$

- For these offers the allocation rule selects the competitive allocation $\{q_i^*\}_{i \in N}$.

Strikingly, while with general schedules there are many inefficient or high prices equilibria, in the limit order case equilibrium is unique and it involves competitive pricing and efficient risk sharing.²² The intuition is the following. In the general schedules case, the market makers could threaten not to participate to the trade if they did not sell a minimum quantity. This gave them a large bargaining power vis à vis the buyer. In the limit order case the market makers cannot impose such minimum quantity. The buyer can always choose to buy a quantity lower than \bar{q}_i . Under those circumstances, it is much easier for the market makers to undercut each other, by stealing a small fraction of each other's market. In addition, by setting maximum quantities (q_i^*), the

²¹If one is willing to neglect the impact of small differences in price on the amount paid by the buyer, this condition is only sufficient and not necessary, since inactive market makers offering to sell a large quantity at a price slightly above p^* could make the active market makers indispensable.

²²The competitive equilibrium is also a possible equilibrium in the case of general schedules. But as shown in the last sections there exists many other equilibria.

market makers can protect themselves from selling too much (which would be very costly given that their valuation for the asset is convex). Finally, for any given price, as long as one market maker does not sell his competitive supply he has incentives to undercut his competitors to attain his competitive supply. Consequently, the limit order environment is very conducive of undercutting, and this drives prices down to their competitive level.

Proposition 5 is similar to the result obtained by Dubey (1982) in the somewhat different context of general equilibrium implementation.

Note that, in the equilibrium described in Proposition 5, the allocation rule to be used in the case of ties is rather important, since the non-indispensability condition d) implies that in equilibrium there must be a tie and moreover the allocation rule must select the competitive allocation (condition e)). As stated in the next corollary, under the prorata allocation rule, equilibrium does indeed exist (note that Dubey (1982) assume prorata allocation).

Corollary 3 : *If the prorata allocation rule is used in the case of ties then equilibrium exists.*

There are many other allocation rules than prorata under which equilibrium exists. The only requirement on the allocation rule is the following : for each competitive allocation $\{q_i^*\}$ one can find a set of limit quantity $\{\bar{q}_i\}$ such that the allocation rule maps $\{\bar{q}_i\}$ into $\{q_i^*\}$.

Although this requirement leaves a lot of flexibility in the choice of the allocation rule, *one cannot use any rule*. There are some rather natural rules for which equilibrium fails to exist. Two examples are the following:

- o **Quantity priority** : for simplicity consider the case of two market makers quoting the competitive price. Under quantity priority the market maker (say 1) who offers the largest limit quantity ($\bar{q}_1 > \bar{q}_2$) is fully executed ($q_1 = \bar{q}_1$), while the other market maker gets the residual demand ($D - q_1$). Quantity priority, or similar rules, are prevalent in many financial markets. Note however that under this rule, 1 cannot offer a limit quantity larger than his competitive trade, so that if the latter is lower than the demand (D), 2 is indispensable, which is not consistent with the necessary condition for equilibrium c). Hence under this rule equilibrium does not exist.
- o **Equal splitting** : Under this rule (when it is possible) the buyer splits demand equally between market makers quoting the best price. This sounds like a fairly

natural rule, and is the standard rule considered in linear pricing competition models (see next section). Under this rule, equilibrium does not exist in the limit order market, however. This is because (i) equal splitting generically does not allow one to generate competitive allocations and (ii) if market makers were to post limit quantities equal to their competitive supply, to avoid equal split, then each would be indispensable which is impossible in equilibrium.

Note also that for many allocation rules for which equilibrium exists (including prorata) there is an infinity of sets of limit quantities $\{\bar{q}_j\}_{j \in J}$ which yield the competitive allocation $\{q_j\}_{j \in J}$. Hence market makers must coordinate on one. In practice, coordination failures can arise in such cases.

6 Competition in linear schedules.

In this section, we assume that each market maker (after observing the demand D), announces a unit price at which he commits to serve all the demand allocated to him (up to D). Were the demand indivisible, this would be the original Bertrand formulation of price competition, or, in market microstructure, the framework of the analysis of Ho and Stoll (1983). As is well known, in this case, prices are set as in a second price auction, and if there is more than one market maker with minimal valuation for the shares, the price is equal to this valuation.

In this section we extend this analysis to the case where the demand can be split between the market makers. As will be shown below, this apparently minor change in assumptions generates a dramatic change in equilibrium outcomes.

In addition to analyzing the consequences of trade splitting, we also analyze the impact of the allocation (or priority) rule used in case of ties, i.e., when there is more than one way for the buyer to purchase D at the minimum cost.²³ In the case of linear schedules, the allocation rule, to be used in the case of ties, plays a more important role than for general schedules or limit orders, as will be shown below.

A simple allocation rule is the equal-splitting rule, which shares equally the demand

²³As mentioned above in the introduction, the results obtained in this section are similar to the results derived independently by Simmons and Gerber (1994), but our analysis is a little more general since we allow for different allocation rules. Also, our analysis in this section is related to the analysis of Bertrand competition in an IO context in Dastidar (1994). Our analysis in this section differs from his because we consider risk averse market makers, while he considers risk neutral producers, and because we consider different allocation rules.

between the best quoters.²⁴ Note that this rule is deterministic. Stochastic rules may also be considered. Maybe the simplest stochastic allocation rule is the non-splitting rule, which allocates the entire demand with equal probability to any of the market makers quoting the best price. Under the non-splitting rule the market structure we analyze is that of Ho and Stoll (1983). In addition to the non-splitting and the equal splitting rules, there exists a variety of stochastic rules, in which the quantity sold by each of the m best quoters is a random variable, q_m . In the following, E will denote the expectation operator, taken over this variable.

Consider a candidate equilibrium, where m market makers in a subset J of N quote the best price p , while the other market makers quote strictly greater prices. Suppose that $m \geq 2$. The expected utility of market maker j in the set J of best-quoters is :

$$EU_j(pq_m - V_j(q_m))$$

For the market maker to be willing to quote p rather than a larger price at which he would not trade, we need :

$$EU_j(pq_m - V_j(q_m)) \geq \mathcal{U}_{j,0} \quad (15)$$

or:

$$Ee^{-\gamma(pq_m - V_j(q_m))} \leq 1 \quad (16)$$

The left-hand-side of equation (16) is continuous and decreasing in p . For $p = 0$ it is larger than 1, and as p goes to infinity it goes to 0. Consequently, we can state the following lemma.

Lemma 3 : *There exists a positive price \underline{p}_j^m which solves:*

$$Ee^{-\gamma(\underline{p}_j^m q_m - V_j(q_m))} = 1$$

and such that market maker j is willing to share the trade with $m - 1$ other market makers rather than not participating to the trade iff $p > \underline{p}_j^m$

\underline{p}_j^m can be interpreted as the reservation valuation of market maker j , since at this price j is indifferent between sharing the trade with $m - 1$ other market makers or not trading. Note that this reservation price varies as the number of market makers sharing the trade varies, and as the allocation rule to be used varies.

²⁴It is the allocation rule considered in Simmons and Gerber (1994) and Dastidar (1994).

On the other hand, market maker j could also quote a price p' strictly less than p and serve all the demand at this price. Such undercutting would generate utility :

$$U_j(p'D - V_j(D))$$

Taking the limit as p' goes to p , the condition under which j prefers not to undercut is :

$$U_{j,0} E e^{-\gamma(pq_m - V_j(q_m))} \geq U_{j,0} e^{-\gamma(pD - V_j(D))} \quad (17)$$

or:

$$E e^{-\gamma(p(q_m - D) - V_j(q_m) + V_j(D))} \leq 1 \quad (18)$$

The left-hand-side of equation (18) increasing in p . For $p = 0$ it is lower than 1, and as p goes to infinity it also goes to infinity. Consequently, we can state the following lemma.

Lemma 4 : *There exists a positive price \bar{p}_j^m which solves:*

$$E e^{-\gamma(\bar{p}_j^m(q_m - D) - V_j(q_m) + V_j(D))} = 1$$

and such that market maker j prefers to share the trade with $m-1$ other market makers rather than undercutting iff $p < \bar{p}_j^m$

Finally, note that $\underline{p}_j^m < \bar{p}_j^m$ iff inequation (18) holds for \underline{p}_j^m , i.e., iff

$$1 < e^{-\gamma(\underline{p}_j^m D - V_j(D))}$$

that is iff

$$\underline{p}_j^m < \frac{V_j(D)}{D}$$

which holds, since the convexity of the valuation function implies that \underline{p}_j^m , the price at which j is indifferent between sharing the trade or not trading, is lower than $\frac{V_j(D)}{D}$, the price at which the market maker is indifferent between serving all the demand or not trading. The following proposition summarizes the above discussion:

Proposition 6 : *There exists an equilibrium where the m market makers in J (a subset of N , with $m \geq 2$) quote price p while the other market makers quote strictly larger prices iff :*

$$\forall i \in J, \underline{p}_j^m \leq p \leq \bar{p}_j^m \quad (19)$$

and

$$\forall i \notin J, p \leq \underline{p}_j^{m+1}$$

Note that, in contrast with the limit order case, but similarly to the case where the market makers could use general schedules, there is a multiplicity of equilibria. Note also that, except when the price is equal to the lower bound of an interval $[\underline{p}_j^m, \bar{p}_j^m]$, equilibrium prices are above reservation prices. This is in striking contrast with the indivisible demand case, whereby the price is equal to the reservation price of the second best market maker (see Ho and Stoll (1983)). The intuition for this multiplicity of equilibria and for these large equilibrium prices is the following. Consider for simplicity the case of two market makers. Suppose one of them quotes a rather large price. If the other matches this quote both market makers earn large profits. But should the other rather undercut this quote? If he does so then he serves the whole demand D . Now this can be very unattractive since marginal valuations are convex, or, to put it differently, since this would entail large risk bearing. Consequently it may well be that the market maker is better off matching the quote of his competitor, rather than undercutting it, even if this quote is rather large. Note that there is a coordination issue: the two dealers must coordinate on one of the many quotes at which they earn profits and which they are better off matching than undercutting.

In the case where the allocation rule is the equal splitting rule the trade of market maker j if he shares with $m - 1$ other market makers is no longer stochastic. In this case the cut-off prices simplify to:

$$\underline{p}_j^m = \frac{V_j(D/m)}{D/m}$$

and:

$$\bar{p}_j^m = \frac{V_j(D) - V_j(D/m)}{D - D/m}$$

Hence for p to be an equilibrium with m market makers in J active, it must be that for each of them:

$$\underline{p}_j^m = \frac{V_j(D/m)}{D/m} < p < \bar{p}_j^m = \frac{V_j(D) - V_j(D/m)}{D - D/m} \quad (20)$$

and

$$\forall j \notin J \quad p \leq \text{Min}_j \frac{V_j(D/(m+1))}{D/(m+1)} \quad (21)$$

Maybe, Equation (20) evidences more transparently than Proposition 6 that \underline{p}_j^m is a reservation price. Interestingly, Equation (20) is formally similar to Equation (12), obtained in the case of general schedules, and has the same economic interpretation.

The reason for this similarity between the general schedules case and the linear schedules case is the following: In the case of general schedules, the market makers impose minima on the quantities they sell at the equilibrium price, hence if one market maker wanted to undercut the others he would have to serve the whole demand. In the case of linear schedules, the same phenomenon arises, i.e., if one market maker undercuts he must serve the entire demand, but this is not due to the strategies followed by the market makers, but simply to the market structure. Now, as mentioned above, because of the convexity of the valuation functions (V_i), it is quite unattractive to serve the whole demand, because it leads to large risk bearing. \bar{p}_j^m , as well as the right-hand-side of equation (12), reflects the cost of this risk bearing.

While the results obtained for linear schedules are rather similar to those obtained for general schedules, they are in striking contrast with the results obtained for limit order competition. The economic interpretation of this difference is the following: As discussed above, in the linear schedules case, the market makers are unwilling to undercut each other, since it would lead them to serve the whole demand, which is costly in terms of risk bearing. In the case of limit orders, in contrast, the market makers can undercut the quotes of their competitors, while protecting themselves from this large risk bearing, by setting a maximum to the quantity they accept to sell. Hence limit orders are much more conducive of undercutting and aggressive competition than linear schedules.

In the case of the non-splitting allocation rule (which allocates the entire trade to one of the best quoters), the cut-off prices simplify to:

$$\underline{p}_j^m = \frac{V_j(D)}{D} = \bar{p}_j^m$$

and, as discussed above, the equilibrium price is the same as in a second price auction. Clearly, in terms of risk sharing, this allocation rule is dominated by others, such as the equal splitting rule, which share the trade, and thus the risk bearing, between market makers.²⁵ Does this lower risk sharing efficiency translate in less attractive prices for the buyer? To investigate this issue, consider an equilibrium of the linear schedule game under the equal splitting allocation rule, whereby the m market makers in J are active. The maximum possible equilibrium price P_m in this case is:

$$P_m = \text{Min}\left\{\min_{j \in J} \frac{V_j(D) - V_j(D/m)}{D - D/m}, \min_{j \notin J} \frac{V_j\left(\frac{D}{m+1}\right)}{\frac{D}{m+1}}\right\} \quad (22)$$

²⁵Note, however, that with linear schedules generically no allocation rule can implement the optimal allocation of risks in contrast with the limit order market.

Denote by index m the market maker for which it is attained. Now, compare P_m to the equilibrium price prevailing in the case of the non-splitting rule:

$$\frac{V_2(D)}{D} \tag{23}$$

where the index 2 indicates that this price corresponds to the second lowest average value for $D : \frac{V_j(D)}{D}$ across all market makers $j \in N$. Manipulating the expressions in (22) and (23), the equilibrium price under the non splitting rule is lower than the largest possible equilibrium price under the equal splitting rule iff $V_2(D) \leq V_m(D)$ which is quite likely to be satisfied²⁶. Consequently, although the equal splitting rule is more efficient, with respect to risk sharing, than the non splitting rule, and thus imposes lower costs on the market makers, it does not necessarily lead to better prices for the buyer. This result is reminiscent of the result obtained above, in the case of general schedules, that greater risk sharing efficiency came at the cost of higher prices. The intuition is the same. The equal splitting rule leads to better risk sharing than the non splitting rule. Because they share risk better in equilibrium, the market makers are less eager to deviate from this equilibrium and undercut each other, since it would lead to a less efficient allocation of trades. This leads to less competitive pressure and consequently larger prices.

The last issue we investigate is whether the large equilibrium prices obtained in this section would go to competitive prices, if the number of competing market makers went to infinity. To see that it does not, note that the limit of the right-hand-side of equation (20) as m goes to infinity is :

$$\frac{V_j(D) - V_j(0)}{D}.$$

This can be rather large and is definitely above the competitive price. The reason why equilibrium prices may not go to the competitive price as the number of market makers goes to infinity is the following. Suppose a very large number of market makers is present and all quote a very large price. If one individual dealer were to undercut them, he would have to serve the whole demand D . Hence the same reasoning as above applies, unaltered by the large number of competing market makers.

²⁶This is the case for instance in the example presented in section 3. In this case one can check using (22) that the price under equal splitting rule can be as high as $\frac{3D}{2}$ which is strictly greater than the price D obtained under the non-splitting allocation rule.

7 Conclusion.

This paper shows that if market makers can compete in general schedules, or if they compete in linear schedules (à la Bertrand) and can share the trades, collusive pricing is likely to emerge. In fact, as shown in Proposition 4, the Nash equilibrium which is Pareto dominant for the market makers maximizes the bid-ask spread. This theoretical result is consistent with the empirical results of Christie and Schultz (1994a and b), who offer evidence suggesting that NASDAQ market makers implicitly collude on wide spreads.

In contrast with the results obtained for general or linear schedules, in the case of a pure limit order market, the unique Nash equilibrium involves competitive pricing and efficient risk sharing. This theoretical result is consistent with the empirical finding in Christie and Schultz (1994 a) that in limit order markets, such as the NYSE and the AMEX, there is no evidence of collusive pricing.

Although our theoretical model is very stylized, and therefore is bound to miss a number of important features of real world markets, it suggests some policy implications. In unregulated markets, where the market makers could define the rule of the games, collusive pricing is possible. Market organizers or regulators should limit the extent to which market makers can influence the quantity and prices at which they trade, and in particular their ability to impose minima on the quantity they are willing to trade. In view of our stylized theoretical analysis, limit orders appear to constitute an optimal market structure.

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Appendix.

Proofs.

Proof of Proposition 3 :

Consider the strategies whereby the market makers announce two FK,

$$t_i(q_i) = t_i \quad \text{and} \quad t_i(D) = F \quad \forall i \in N$$

We show that these strategies are an equilibrium when the pairs (t_i, q_i) satisfy condition (11). First, note that the allocation (q_1, \dots, q_n) does indeed minimize the total payment for the buyer. The buyer might also choose to purchase all from one market maker only, but this is ruled out by the allocation rule.

To prove that these strategies form an equilibrium we show that they satisfy a) and b) of Proposition 1.

Condition a) : holds because the total amount F paid by the public is equal to $A_{-i}(D)$.

We now prove that b) holds , i.e. that for all i , and all q

$$A_{-i}(D - q_i) + V_i(q_i) \leq A_{-i}(D - q) + V_i(q) \quad (24)$$

Consider a deviation from the conjectured equilibrium in which i wants to sell another quantity than q_i , say q . Two cases can occur. Either there exists a subset J of N (to which i belongs) such that $\sum_{j \in J} q_j = q$. Or there does not exist such a subset J . In the second case, were the buyer to accept i 's deviation, then the buyer would have to buy more than D . But this would generate an inefficiency, and i would be better off lowering q , so that the buyer would only need to buy D if he were to accept the deviation. Hence we consider only deviations for which there exists a subset J of N (to which i belongs) such that $\sum_{j \in J} q_j = q$.

So, to establish that equation (24) holds, we only need to prove that, for all J :

$$A_{-i}(D - q_i) + V_i(q_i) \leq A_{-i}(D - \sum_{j \in J} q_j) + V_i(\sum_{j \in J} q_j)$$

Now, this is equivalent to:

$$\begin{aligned} \forall J \subset N, \forall i \in J, F - t_i(q_i) + V_i(q_i) &\leq A_{-i}(D - \sum_{j \in J} q_j) + V_i(\sum_{j \in J} q_j) \Leftrightarrow \\ \sum_{j \neq i} t_j(q_j) + V_i(q_i) &\leq \sum_{j \notin J} t_j(q_j) + V_i(\sum_{j \in J} q_j) \Leftrightarrow \end{aligned}$$

$$\begin{aligned}
\sum_{j \neq i, j \in J} t_j(q_j) + V_i(q_i) &\leq V_i(\sum_{j \in J} q_j) \Leftrightarrow \\
\sum_{j \neq i, j \in J} [t_j - V_j(q_j)] + \sum_{j \in J} V_j(q_j) &\leq V_i(\sum_{j \in J} q_j) \Leftrightarrow \\
\sum_{j \neq i, j \in J} [t_j - V_j(q_j)] &\leq V_i(\sum_{j \in J} q_j) - \sum_{j \in J} V_j(q_j)
\end{aligned}$$

which is condition (11).

Q.E.D.

Proof of Proposition 5.

First suppose that sellers i and j are active, and $p_i < p_j$. Then i could raise slightly her price, and sell the same quantity \bar{q}_i for a strictly higher profit, a contradiction. Thus at equilibrium all active sellers must quote the same price p . If firm i (active or inactive) were to quote a price slightly under p , she could obtain a profit arbitrarily close to

$$\text{Max}_{\{0 \leq q \leq D\}} pq - V_i(q)$$

by selling her competitive supply. Therefore inactive firms must be such that their competitive supply at price p is zero; while active firms must sell their competitive supply at price p . Since these offers sum to D , p must be the competitive price p^* .

The last point to check is whether an active firm could profitably raise her price. The condition

$$\sum_{j \neq i} \bar{q}_j \geq D$$

ensures that firm i would have no demand in this case (non-indispensability); another possibility is that some inactive firms quote a price higher but close to p , as to deter active firms from raising their prices.

QED

Proof of Corollary 3:

Equilibrium exists for the prorata allocation rule if for each equilibrium outcome : $\{q_i\}_{i \in J}$, satisfying the conditions given in Proposition 5, there exists a set of limit quantities, $\{\bar{q}_i\}_{i \in J}$ such that if these limit quantities are offered then the prorata allocation rule yields the equilibrium trades.

This is the case if :

$$0 = \begin{pmatrix} (\frac{q_1}{D} - 1) & \frac{q_2}{D} & \dots & \frac{q_J}{D} \\ \frac{q_1}{D} & (\frac{q_2}{D}) - 1 & \dots & \frac{q_J}{D} \\ \dots & \dots & \dots & \dots \\ \frac{q_1}{D} & \frac{q_2}{D} & \dots & (\frac{q_J}{D}) - 1 \end{pmatrix} \begin{pmatrix} \bar{q}_1 \\ \bar{q}_2 \\ \dots \\ \bar{q}_J \end{pmatrix}$$

This homogeneous system of linear equations admits an infinity of solutions since the columns of the matrix add up to 0.

Q.E.D.

Table 1: *Market Structures and Examples of Equilibrium Outcomes*

	Is risk sharing efficient?	$t_i - V_i(q_i)$	Price	Total payment
Linear schedule-no splits	No	0	D	D^2
Linear schedules-splits	Yes	$\frac{D^2}{8}$	$\frac{3D}{4}$	$\frac{3D^2}{4}$
Limit orders-prorata	Yes	$\frac{D^2}{4}$	D	D^2
General schedules-splits	Yes	$\frac{D^2}{2}$	$\frac{3D}{2}$	$\frac{3D^2}{2}$

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