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Economics Working Paper 152

Recontracting and Competition*

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January 1996†

Keywords: Abstract equilibrium, recontracting, Walrasian equilibrium, core.

Journal of Economic Literature classification: D51, D41.

* Previous versions were entitled "Abstract Equilibria: A Unified Approach to Economic Competition" and "Recontracting and Economic Competition".

This research is based on Chapter 2 of the authors Ph.D. thesis at the Hebrew University of Jerusalem. The author acknowledges helpful comments from Sergiu Hart, Andreu Mas-Colell, Roberto Serrano, Oscar Volij, Eyal Winter, Myrna Wooders, Joseph Zeira, and participants of seminars at the Hebrew University, Universitat Pompeu Fabra, Universidad de Alicante, Universidad Carlos III de Madrid, Ben Gurion University, and Université Catholique de Louvain. The author gratefully acknowledges Joram Mayshar's suggestion that the author should write papers on economics; this suggestion was the major motivation for this research.

† First draft: August 1994.

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Abstract

We characterize the set of Walrasian allocations of an economy as the set of allocations which can be supported by abstract equilibria that satisfy a recontracting condition which reflects the idea that agents can freely trade with each other. An alternative (and weaker) recontracting condition characterizes the core. The results are extended to production economies by extending the definition of the recontracting condition to include the possibility of agents to recontract with firms. However, no optimization requirement is imposed on firms. In pure exchange economies, an abstract equilibrium is a feasible allocation and a list of choice sets, one for each agent, that satisfy the following conditions: an agent's choice set is a subset of the commodity space that includes his endowment; and each agent's equilibrium bundle is a maximal element in his choice set, with respect to his preferences. The recontracting condition requires that any agent can buy bundles from any other agent's choice set by offering the other agent a bundle he prefers to his equilibrium bundle.

1. INTRODUCTION

The notion of competition is central in economics at least since the publication of Walras (1874). In the Walrasian framework there exists a price system and agents are assumed to treat this price system as given.

The motivation of the present research is to give an alternative reasoning for Walrasian competition. We study a notion of abstract equilibrium that reflects the postulates that markets clear and agents optimize. No assumption regarding prices is assumed. Instead, we impose a recontracting condition that resembles the freedom of agents to contract and recontract. The main result of this paper (Theorem 1) is a characterization of the Walrasian allocations by means of three postulates: Markets clear, agents optimize, and agents are free to contract and recontract.

The condition that is interpreted as "agents are free to contract and recontract" can be modified in several ways. One modification characterizes the core; another characterizes the set of strongly fair net trades studied by Schmeidler and Vind (1972); a third characterizes the arbitrage equilibrium of Makowski and Ostroy (1995). All these results as a system provide a unified theory, in which various notions of competition fit in to.

A similar concept of abstract equilibrium was studied by Schmeidler and Vind (1972) and McLennan and Sonnenschein (1991). The main difference is that in their formulation the choice sets are generated from one additive set of net trades. In contrast, our definition of choice sets does not require that the choice sets are derived from one set of net trades, not to mention an additive one. However, it turns out that the recontracting condition that characterizes the Walrasian allocations implies that the choice sets of the agents can be presented as generated from the same additive set of net trades. Thus in our theory the postulate that all the agents face the same market opportunities is derived rather than assumed. Moreover the set of abstract equilibria satisfying our conditions is equivalent to that in McLennan and Sonnenschein (1991, Theorem A). In particular we generalize McLennan and Sonnenschein (1991, Theorem A) to economies in which agents preferences satisfy much weaker assumptions, and to production economies.

We exactly characterize the set of allocations supported by abstract equilibria with practically no restrictions on preferences, endowments, and consumption sets. In general the recontracting condition characterizes the shrunk core (or Edgeworth equilibria)--the set of all allocations that remain in the core when the economy is replicated infinitely many times. Sufficient conditions for the equivalence between this set and the Walrasian allocations are well known. It is worthwhile to note that convexity or differentiability of preferences are not needed.

We also study production economies. We consider economies in which the firms are distinct economic agents from the consumers (as opposed to coalition production economies). We do not make a direct assumption concerning the behavior of firms. It is assumed, by the recontracting condition, that each consumer can buy from each firm any feasible production plan in exchange of the one the firm chose. In addition, each consumer's endowment is modified by adding to it his share in the production plans chosen by the firms. These assumptions lead to a characterization of Walrasian allocations similar to one in exchange economies.

The approach taken in this paper may be considered as axiomatic. A possible criticism to this approach is that it is not evident what properties of the actual trading procedure imply that agents' choice sets are characterized by the axioms. We believe that the results presented here are closely related to non-cooperative models of decentralized trade, and discuss the possible applicability of our results to these models in the concluding section.

The paper is organized as follows. Basic definitions are presented in Section 2; the main result is given in Section 3; Section 4 discuses equilibria generated by additive sets of net trades; Section 5 provides a characterization of the core; Section 6 compares some of the results of Makowski and Ostroy (1995) to ours; Section 7 discuses prodution economies; the proofs are given in Section 8; and Section 9 concludes.

2. DEFINITIONS

2.1 Notation

We denote the k-dimensional Euclidian space by $\mathbb{R}^k$; the non negative orthant by $\mathbb{R}^k_+$; and the interior of $\mathbb{R}^k_+$ by $\mathbb{R}^k_{++}$. Let A and B be two subsets of $\mathbb{R}^k$; define $A + B = \{x: \exists a \in A \text{ and } b \in B, \text{ such that } a+b=x\}$. We shall write $a + B$ instead of $\{a\} + B$.

2.2 Pure Exchange Economies

Let $\mathbb{R}^k$ be the commodity space. An *(exchange) economy* is a list $(X_i,R_i,\omega_i)_{i\in N}$, where N is a finite nonempty set of agents; for all agents $i \in N$ $X_i \subset \mathbb{R}^k$ is the consumption set, R_i is a reflexive preference relation on X_i, and $\omega_i \in \mathbb{R}^k$ is an endowment.

For each agent $i \in N$ define the strict preference relation P_i as follows: for all $x,x' \in X_i$ xP_ix' if and only if xR_ix' and not $x'R_ix$. Note that by construction P_i is irreflexive. For all $x \in X_i$ let $R_i(x) = \{x' \in X_i: x'R_ix\}$ and $P_i(x) = \{x' \in X_i: x'P_ix\}$.

An *allocation* is a list $(x_i)_{i\in N}$ where for all $i \in N$ $x_i \in X_i$, and $\Sigma_{i\in N}\omega_i = \Sigma_{i\in N}x_i$.

Another possible definition of an allocation is by requiring $\Sigma_{i\in N}\omega_i \geq \Sigma_{i\in N}x_i$. Our definition does not exclude the possibility of free disposal, since one may assume that the preferences satisfy a free disposal property. On the other hand we do not impose free disposal.

3. RECONTRACTING AND THE MAIN RESULT

3.1 Abstract Equilibria

Let $e = (X_i,R_i,\omega_i)_{i\in N}$ be an economy. An *abstract equilibrium* is a list $[x_i,C_i]_{i\in N}$ where $(x_i)_{i\in N}$ is an allocation; for all $i\in N$ $C_i\subset\mathbb{R}^k$ is i's choice set that satisfies:

(1) $\omega_i\in C_i$ (possibility not to trade)

(2) $x_i\in C_i$ (feasibility of actual trade) and

(3) $C_i\cap P_i(x_i) = \varnothing$ (optimality).

Note that the notion of abstract equilibria is not at all restrictive, since any allocation $(x_i)_{i\in N}$, that satisfies $\omega_i \notin P_i(x_i)$ for all $i\in N$, can be supported by the trivial equilibrium in which $C_i = \{\omega_i,x_i\}$ for all $i\in N$.

The concept of abstract equilibrium is very similar to the notion of generalized games[1], in which agents' strategy sets are not given as a part of the description of the economy, but are determined in equilibrium. In the next subsection we impose an internal consistency property on the choice sets.

Walrasian equilibria can be viewed as a private case of an abstract equilibria. Formally, a *Walrasian equilibrium* is an abstract equilibrium $[x_i,C_i]_{i\in N}$ in which there exists a price vector $p\in\mathbb{R}^k\setminus\{0\}$ such that for all $i\in N$:

(4) $C_i = \{x\in\mathbb{R}^k: px\leq p\omega_i\}$.

3.2 Recontracting

Let $e = (X_i,R_i,\omega_i)_{i\in N}$ be an economy. An abstract equilibrium $[x_i,C_i]_{i\in N}$ satisfies *the recontracting condition* if for all $i,j\in N$, $i\neq j$

[1]This concept was introduced by Debreu (1952) in proving the existence of a Walrasian equilibrium.

$$C_i \supset C_i + C_j - R_j(x_j). \tag{3.1}$$

The interpretation of the recontracting condition is as follows. Assume the agents expect to get their part in an allocation $(x_i)_{i\in N}$. Fix an agent i; if all agents $j\neq i$ were to expect to get x_j, than they would accept to to take any bundle in $R_j(x_j)$. The recontracting condition states that agent i may offer any other agent j these bundles in $R_j(x_j)$ in exchange for any bundle in j's choice set C_j. Now, this condition should hold for all agents, and also in equilibrium. Thus, in equilibrium agents have the possibility to recontract but they do not want to, as they are maximized on their choice sets.

This condition makes sense, in the case where agents "take their choice sets as given" as implicitly assumed in the definition of an abstract equilibrium.

Note that when preferences satisfy local-non-satiation all Walrasian *equilibria* (the budget sets) satisfy the recontracting condition. This is since any feasible bundle for an agent is worth no more than his income, and any improving bundle is worth no less than his income.

Consider economies that satisfy the following assumptions:

(A1) For all $i\in N$, X_i is convex and closed.

(A2) For all $i\in N$, for all $x\in X_i$, $P_i(x)$ is open relative to X_i. (upper hemicontinuity)

(A3) For all $i\in N$, for all $x\in X_i$, for all $\delta>0$, $\{x'\in X_i: \|x'-x\|<\delta\} \cap P_i(x) \neq \varnothing$ (local non satiation).

And one of the following:

(A4) For all $i\in N$, $\omega_i\in \mathrm{int}(X_i)$.

(A5) For all $i\in N$, $X_i = \mathbf{R}^k_+$; for all x and x' in X_i $x'\geq x$ implies $x'\in P_i(x)$, and $\Sigma_{i\in N}\omega_i \gg 0$.

The main result of this paper is as follows:

Theorem 1: Let $e = (X_i, R_i, \omega_i)_{i \in N}$ be an economy that satisfies assumptions A1-A4 (or A1,A2 and A5) and has at least two agents. An allocation $(x_i)_{i \in N}$ of e is a Walrasian allocation if and only if the allocation $(x_i)_{i \in N}$ may be supported by an equilibrium that satisfies the recontracting condition.

Theorem 1 gives a new interpretation of Walrasian equilibrium. The key assumption leading to Walrasian equilibrium is not necessarily that "prices are quoted" and "agents take prices as given" but "agents take choice sets as given" combined with a recontracting condition that is natural when "agents take choice sets as given." Another way of putting this interpretation is by providing the following "auctioneer story". Edgeworth (1881, p18) assumed that "individuals [are] collected at a point or connected by telephones". We assume that agents are collected at a point *and* connected by telephones; as they are collected at a point they can contract and recontract; in addition all agents are connected by telephones to the auctioneer who offers each of them a choice set. Now, the auctioneer is assumed to be "complete" in following sense: He must assure that any deal the agents can strike by trading goods they buy from the auctioneer, the agents can do directly with him. In this case, the auctioneer is bound to offer the agents choice sets that correspond to an equilibrium that satisfies the recontracting condition. Thus, Theorem 1 can be interpreted as a characterization of the Walrasian Auctioneer: The Walrasian Auctioneer is essentially the only auctioneer that can outsmart the agents of an economy who always try to outsmart the auctioneer. The word "essentially" is used since Theorem 1 does not characterize the equilibria (the choice sets) but only the allocations associated with it.

4. Recontracting and Equal Market Opportunities

Schmeidler and Vind (1972) and McLennan and Sonnenschein (1991) studied abstract equilibria in which the choice sets are generated from additive sets of net trades.

For an economy $e = (X_i, R_i, \omega_i)_{i \in N}$, and for a choice set C_i of agent i we say that a set $Z_i \subseteq \mathbb{R}^k$ *generates the choice set* C_i *if* $C_i = \omega_i + Z_i$.

The questions considered in this section are whether the recontracting condition studied in the previous section restricts choice sets to to be generated by additive sets of net trades, and are the sets of net trades of all agents identical. The answer is that any abstract equilibrium that satisfies the recontracting condition can be represented by assigning all agents the same additive set of net trades, and this set satisfies the conditions studied by McLennan and Sonnenschein (1991).

Consider an economy $e = (X_i, R_i, \omega_i)_{i \in N}$, and an allocation $(x_i)_{i \in N}$. McLennan and Sonnenschein (1991) studied the case where there exists a set $Z \subset \mathbb{R}^k$ of net trades that satisfies the following:[2]

i) $0 \in Z = Z + Z$ (possibility not to trade and additivity).

For all $i \in N$:

ii) $x_i - \omega_i \in Z$ (feasibility of actual trade)

iii) $(\omega_i + Z) \cap P_i(x_i) = \varnothing$ (optimality)

iv) $x_i - R_i(x_i) \subset Z$ (expost recontracting).

Theorem 2 below establishes the equivalence between McLennan and Sonnenschein equilibria and equilibria that satisfy the recontracting condition.

[2]As they considered economies in which agents have C^2 utility functions their assumptions were slightly different. The differences are discussed after the statement of Theorem 2.

Theorem 2 Let $e = (X_i, R_i, \omega_i)_{i\in N}$ be an economy.

2.1 Let $(x_i)_{i\in N}$ be an allocation and $Z \subset \mathbb{R}^k$ be a set of net trades that satisfies conditions i)-iv) above. For all $i \in N$ let $C_i = \omega_i + Z$. Then $[x_i, C_i]_{i\in N}$ is an equilibrium that satisfies the recontracting condition.

2.2 Let $[x_i, C_i]_{i\in N}$ be an equilibrium that satisfies the recontracting condition. If there are at least two agents in the economy, then there exists a set Z, such that for all $i \in N$ $C_i = \omega_i + Z$, and Z satisfies conditions i)-iv).

Note that the set Z in Theorem 2.2 can be computed by setting $Z = C_i - \omega_i$, for some arbitrary $i \in N$.

McLennan and Sonnenschein (1991) did not require condition ii). However they have noted in their footnote 2 that this condition may replace the interiority and smoothness assumptions they made on the allocation and preferences respectively.

The implication of Theorem 2.2 is that "the completeness of the auctioneer" implies that all agents have identical market opportunities. These opportunities in addition are "simple" as they are represented by an additive set Z.

McLennan and Sonneschein (1991) interpreted the additivity assumption on Z as a consequence of an assumption that "agents are not prevented from trading with each other repeatedly." Clearly, this interpretation applies to an economy with an infinite number of agents. However, even in such a situation this interpretation is not clear, when the economy does not have a finite type structure. In a finite agent setting, our definition of the recontracting condition is more natural than additivity.

McLennan and Sonnenschein (1991) have noted that the implicit anonymity assumption (identical sets of net trades to all agents) of their Theorem A is not needed for the characterization of Walrasian allocations. It turns out that this observation is true only for the smooth case they considered. The following example shows that the anonymity assumption is indeed needed.

Example 1 Let $e = (X_i, R_i, \omega_i)_{i\in N}$ be an economy, and $(x_i)_{i\in N}$ an allocation. Consider a list of sets of net trades $(Z_i)_{i\in N}$ that satisfies the following: for all $i\in N$ $Z_i \subset \mathbb{R}^k$ and:

1) $0 \in Z_i = Z_i + Z_i$ (possibility not to trade and additivity).

2) $x_i - \omega_i \in Z_i$ (feasibility of actual trade)

3) $(\omega_i + Z_i) \cap P_i(x_i) = \varnothing$ (optimality)

4) $x_j - R_j(x_j) \subset Z_i$ for all $j \neq i$ (expost recontracting).

We now show that allocations that are not Walrasian can be supported by sets of net trades satisfying the above conditions. Let $\mathbb{R}^2$ be the commodity space, and let $e = (X_i, R_i, \omega_i)_{i\in N}$. Where $N = \{1,2,3\}$; $X_1 = X_2 = X_3 = \mathbb{R}^2_+$; the preferences R_i are represented by the utility functions $\mathrm{Min}\{ x_i^1, x_i^2 \}$ (superscripts denote the different goods); $\omega_1 = (5,0)$, $\omega_2 = (0,3)$, $\omega_3 = (0,2)$. The allocation $[x_1,x_2,x_3] = [(3,3),(1,1),(1,1)]$ is supported by the following "budget sets": $Z_i = \{ z\in \mathbb{R}^2 \text{: } p_i z \le 0 \}$, where $p_1 = (3,2)$, $p_2 = (2,1)$, $p_3 = (1,1)$. Preference may be modified to satisfy strict convexity and strict monotonicity. It is also possible to construct examples (with more than two goods) where the allocation is not Pareto optimal. One may suggest that the correct formulation of 1) should be: for all i and j $Z_i = Z_i + Z_j$. However this condition implies that all the Z_i's are identical.

Schmeidler and Vind (1972) considered a notion of "strongly fair net trades." A strongly fair net trade can be defined as a net trade that admits a set Z that satisfy the conditions in Theorem 2 excluding the condition $x_i - R_i(x_i) \subset Z$. Schmeidler and Vind (1972) showed that for all strongly fair net trades $(z_i)_{i\in N}$ one can associate a price vector p such that for all i $pz_i = 0$. In addition they provided sufficient conditions for a strongly fair net trade to be associated with a Walrasian allocation. It can be shown that the following condition on abstract equilibria characterizes the set of strongly fair net trades:

Let $e = (X_i,R_i,\omega_i)_{i\in N}$ be an economy. An abstract equilibrium $[x_i,C_i]_{i\in N}$ satisfies the *Schmeidler and Vind recontracting condition* if for all $i,j\in N$, $i\neq j$

$$C_i \supset C_i + C_j - x_j .$$

According to the above condition agent i can buy from agent j any bundle in C_j in exchange to x_j (as opposed to any bundle in $R_j(x_j)$).

5. A Characterization of the Core

In this section we present a characterization of the core by a variant of the recontracting condition.

Let $e = (X_i,R_i,\omega_i)_{i\in N}$ be an economy. A nonempty subset S of N is called a *coalition*. Let S be a coalition, an S-*allocation* is a list $(x_i)_{i\in S}$ where for all $i\in S$ $x_i\in X_i$ and $\Sigma_{i\in S}\omega_i = \Sigma_{i\in S}x_i$. An N-allocation is called an *allocation*. An allocation $(x_i)_{i\in N}$ is a *core allocation* if there does not exist a coalition S and an S-allocation $(x'_i)_{i\in S}$ such that for all $i\in S$ $x'_i\in R_i(x_i)$ and for some $i\in S$ $x'_i\in P_i(x_i)$.

An abstract equilibrium $[x_i,C_i]_{i\in N}$ satisfies the *weak recontracting condition* if for all $i\in N$ for all coalitions S with $i\notin S$

$$C_i \supset \omega_i + \Sigma_{j\in S}\omega_j - \Sigma_{j\in S}R_j(x_j). \tag{5.1}$$

There are two differences between the weak recontracting condition and the recontracting condition. One is that the recontracting condition concentrates on bilateral trades and the weak recontracting condition on multilateral trades. However it is easy to see by finite application of the recontracting condition 3.1 that it implies the following multilateral condition. For all $i \in N$ for all coalitions S with $i \notin S$

$$C_i \supset C_i + \Sigma_{j\in S}C_j - \Sigma_{j\in S}R_j(x_j). \tag{5.2}$$

The other difference is that in the weak one we have ω_i and ω_j in the right hand side of the equation instead of C_i and C_j. The weak recontracting condition for agent i states that if all the agents expect to get an allocation x, then agent i can sell other agents bundles they prefere to x in exchange of their endowments. Following the auctioneer story of Section 3, the weak recontracting condition requires that the auctioneer makes sure that the choice sets he assigns to the agents include all the deals the agents can strike before they have access to their choice sets. This is opposed to the stronger condition that assumes that the choice sets also include all the deals the agents can strike after meeting the auctioneer.

The following theorem characterizes all the allocations associated with the weak recontracting condition.

Theorem 3 Let $e = (X_i,R_i,\omega_i)_{i\in N}$ be an economy. An allocation $(x_i)_{i\in N}$ may be supported by an equilibrium that satisfies the weak recontracting condition if and only if $(x_i)_{i\in N}$ is a core allocation.

Proof: Let $e = (X_i,R_i,\omega_i)_{i\in N}$ be an economy, and $(x_i)_{i\in N}$ an allocation. Define for all $i \in N$ the following choice sets :

$$A_i = \omega_i + Z_i \quad .$$

Where $$Z_i = [\cup_{S\subset N\setminus i}[\Sigma_{j\in S}\omega_j - \Sigma_{j\in S}R_j(x_j)]] \cup \{0\}.$$

The following facts and lemmas are easy to verify and together they imply Theorem 3.

Fact 1: A system $[x_i,C_i]_{i\in N}$ satisfies the weak recontracting condition if and only if for all $i\in N$ $C_i \supset A_i$.

The following IIA property is a direct consequence of the definition of an abstract equilibrium:

Fact 2 (IIA): Let $e = (X_i,R_i,\omega_i)_{i\in N}$ be an economy, and $[x_i,C_i]_{i\in N}$ an abstract equilibrium. Let $(C_i')_{i\in N}$ satisfy for all $i\in N$ $\omega_i \in C_i' \subset C_i$ and $x_i \in C_i'$. Then $[x_i,C_i']_{i\in N}$ is an abstract equilibrium.

Lemma 3: (restricted IIA) If $[x_i,C_i]_{i\in N}$ is an equilibrium that satisfies the weak recontracting condition and for all i $C_i \supset C_i' \supset A_i$, then $[x_i,C_i']_{i\in N}$ is also an equilibrium that satisfies the weak recontracting condition.

Proof: follows directly from Facts 1 and 2.

Lemma 4: The system $[x_i,A_i]_{i\in N}$ is an equilibrium if and only if $(x_i)_{i\in N}$ is a core allocation.

Proof: Follows from the definition of the core.

Now, if $(x_i)_{i\in N}$ is a Core allocation then by Lemma 4 $[x_i,A_i]_{i\in N}$ is an equilibrium, and by Fact 1 it satisfies the weak recontracting condition.

If $[(x_i,C_i]_{i\in N}$ is an equilibrium that satisfies the weak recontracting condition, then by Fact 1 $C_i \supset A_i$ and by Lemma 3 $[x_i,A_i]_{i\in N}$ is also an equilibrium that satisfies the weak recontracting condition. Lemma 4 implies that $(x_i)_{i\in N}$ is a core allocation. Q.E.D.

The characterization of the core (Theorem 3) can be extended to continuum economies. Following the approach of Hammond, Kaneno, and Wooders (1989), one can impose the following condition: there exists a set of full measure in which for all members of this set, the weak recontracting condition holds (with respect to all finite coalitions in this set). It is easy to verify that this requirement characterizes the core with respect to finite coalitions.

As the core with respect to finite coalitions coincides with the set of Walrasian allocations (Hammond, Kaneko, and Wooders, 1989) there is no need to require the stronger conditions of Theorems 1 or 2 in order to characterize the Walrasian allocations of a continuum economy. McLennan and Sonnenschein (1991, Theorem 1) provide such a characterization with an additive set of net trades as in Theorem 2.

6. On Makowski and Ostroy's Arbitrage Equilibria

In this section we discuss some of the results of Makowski and Ostroy (1995). Consider an economy $e = (X_i, R_i, \omega_i)_{i \in N}$. An abstract equilibrium $[x_i, C_i]_{i \in N}$ is an *arbitrage equilibrium* if for all $i \in N$ for all coalitions S with $i \notin S$:

$$C_i \supset \omega_i + \Sigma_{j \in S} x_j - \Sigma_{j \in S} R_j(x_j) \qquad (6.1)$$

This definition is similar to the one in Makowski and Ostroy (1995). The difference is that instead of the sets $R_j(x_j)$ they have the subsets of $R_j(x_j)$ visible from x_j. (Thus 6.1 characterizes a smaller set of allocations).

The weak recontracting condition that characterizes the core says that if agents expect to get x, they may recontract before the trade was done. Condition 6.1 is exante with respect to i, and expost with respect to $j \in S$. McLennan and Sonnenschein (1991) apply a similar idea: they characterize the set of trades available to an agent in a subgame perfect equilibrium of a bargaining game by the trades he may do with agents that are about to leave the market. We believe that without references to a specific proof of a theorem concerning a specific bargaining game, it is difficult to support the asymmetry between "the arbitrager" i and the other agents $j \in S$. The recontracting condition (in Theorem 1) says that agents may recontract at any point of the trade procedure, and is symmetric between the abitrager and the other agents.

Condition 6.1 is implied by the recontracting condition.[3] Theorem 1 implies then that the set of allocations associated with Makowski-Ostroy arbitrage equilibria is a superset of the Walrasian allocations. Makowski and Ostroy (1995) note that their allocations include all core allocations, in a continuum economy. This observation does not extend to finite agent economies; here there is no general inclusion relation between the two sets.

Also in continuum economies arbitrage equilibria may be outside the core. With an additional flatness condition on the choice set Makowski and Ostroy (1995) characterize a subset of the Walrasian allocations in continuum economies. They claim that in this subset price taking behavior is more appealing in and out of equilibrium. Our approach, on the other hand, attempts to characterize Walrasian outcomes as derived from alternative patterns of behavior, rather than characterize price taking per se.

Makowski and Ostroy (1995) also show that the set of net trades the characterizes their notion of equilibrium is additive (in a continuum economy). This shows, from a different point of view of that of Section 5, that McLennan and Sonnenschein's (1991) assumptions in characterizing Walrasian outcomes are too strong than necessary.

[3]This follows from our previous observation that the recontracting condition 3.1 implies the condition 5.2.

7. Production Economies

In this section the results of Sections 3, 4 and 5 are extended to production economies.

7.1 Definitions

Let J be a nonempty finite set (the firms). A *production sector* (with respect to J and $\mathbb{R}^k$) is a list $(Y_j)_{j\in J}$ where for all $j\in J$ $Y_j \subset \mathbb{R}^k$ is a production set.

A *share distribution* (with respect to J and N) is a matrix $(\theta_{ij})_{ij\in NxJ}$ where for all $ij\in NxJ$ $\theta_{ij} \geq 0$, and for all $j\in J$ $\Sigma_{i\in N}\theta_{ij} = 1$.

A *production economy* is a triple $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$, where $(X_i,P_i,\omega_i)_{i\in N}$ is an exchange economy; $J\cap N=\varnothing$; $(Y_j)_{j\in J}$ is a production sector; and $(\theta_{ij})_{ij\in NxJ}$ is a share distribution.

An *allocation* is a pair $[(x_i)_{i\in N},(y_j)_{j\in J}]$ where for all $i\in N$ $x_i\in X_i$, and for all $j\in J$ $y_j\in Y_j$ and $\Sigma_{j\in J}y_j + \Sigma_{i\in N}\omega_i = \Sigma_{i\in N}x_i$.

7.2 Abstract Equilibria

Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$ be a production economy. An *abstract equilibrium* is a triple $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ where $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is an allocation; for all $i\in N$ $C_i\subset\mathbb{R}^k$ is i's choice set that satisfies:

(1) $\omega_i + \Sigma_{j\in J}\theta_{ij}y_j \in C_i$;

(2) $x_i\in C_i$; and

(3) $C_i\cap P_i(x_i) = \varnothing$.

In the above definition the endowment is modified by adding the agent's share in the production plans chosen by the firms. Note that a consumer does not have access to a proportion of the production sets of the firms he owns, but only receives the rights (and obligations) induced by the actual choice of the firms.

A *Walrasian equilibrium* is an abstract equilibrium $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ in which there exists a price vector $p\in\mathbb{R}^k\backslash\{0\}$ such that for all $i\in N$ $C_i = \{x\in\mathbb{R}^k: px \leq p(\omega_i + \Sigma_{j\in J}\theta_{ij}y_j)\}$ and for all $j\in J$, for all $y_j'\in Y_j$, $py_j \geq py_j'$.

A *quasi Walrasian equilibrium* differs from an equilibrium in that the optimization condition of the consumers $C_i\cap P_i(x_i) = \varnothing$ is replaced with the following weaker condition: (3') $C_i\cap P_i(x_i) = \varnothing$ or $C_i\cap X_i$ has an empty interior.

7.3 Recontracting

Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in N\times J}]$ be an economy. An abstract equilibrium $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ satisfies the *recontracting condition with firms* if for all $i\in N$ and for all $j\in J$

$$C_i \supset C_i + Y_j - y_j \ . \tag{7.1}$$

The recontracting condition with firms allows each consumer to recontract with each firm by offering it the plan it is about to produce in exchange to another feasible plan. Note that nowhere in the definitions it is assumed that firms make optimal choices in any sense.

If we were to define profits somehow, and assume that the firms maximize profits then a consumer could recontract with the firms by offering them at least as profitable plans than they choose, an thus have a bigger choice set than in our definition. The definitions adopted here, both for the endowments and the recontracting condition, make the choice sets as small as possible and therefore the sets of allocations asociated with them are the largest possible.

We consider economies that satisfy the following assumption instead of A4 or A5:
(A6) For all $i\in N$, $X_i = \mathbb{R}^k_+$ and $\omega_i\in X_i$; for all $j\in J$ $0\in Y_j$ and there exists an production

plan $(y_j)_{j\in J}$ that satisfies $\Sigma_{j\in J}y_j + \Sigma_{i\in N}\omega_i \gg 0$.

The main result of this section is:

Theorem 4: Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in N\times J}]$ be a production economy that satisfies A1-A3 and A6, and has at least two agents. If an allocation $[(x_i)_{i\in N},(y_j)_{j\in J}]$ of e may be supported by an equilibrium that satisfies the recontracting condition and the recontracting condition with firms, then $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is a quasi Walrasian allocation.

7.4 Recontracting and Equal Market Opportunities

In Section 4 we provided a characterization of Walrasian allocations in terms of a set of net trades. In this subsection we extend this characterization to production economies.

Consider an economy $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in N\times J}]$, and an allocation $[(x_i)_{i\in N},(y_j)_{j\in J}]$. First we modify the conditions studied in section 4 to the concept of endowment presented in the previous subsection. Consider a set $Z \subset \mathbb{R}^k$ of net trades that satisfies the following:

i') $0\in Z = Z + Z$ (possibility not to trade and additivity).

For all $i\in N$:

ii') $x_i - (\omega_i + \Sigma_{j\in J}\theta_{ij}y_j) \in Z$ (feasibility of actual trade)

iii') $(\omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z) \cap P_i(x_i) = \varnothing$ (optimality)

iv') $x_i - R_i(x_i) \subset Z$ (expost recontracting in consumption).

Now we introduce a new condition related to production.

v') $Y_j - y_j \subset Z$, for all $j\in J$ (expost recontracting in production).

The condition v') may be interpreted as a profit maximizing condition in the following sense. Any production plan can be bought in exchange of the actual production plan through the market presented by Z. Thus, the actual production plan is as valuable at least as the other plans. Another interpretation is related to the motivation presented by McLennan and Sonnenschein (1991) for the condition iv). They interpreted Z as a set of trades available to the agents in a subgame perfect equilibrium of a bargaining game. The idea behind condition iv) is that if an agent i is about to leave the market with the bundle x_i, then any other agent can buy x_i from i in exchange to a bundle in $R_i(x_i)$. Now, assume that the bargaining model is adjusted for production economies, by assuming that each firm produces after it leaves the market. Now, assume an firm j is about to leave the market and produce the plan y_j . Although we do not know the "preferences" of firms, it makes sense to assume that they are least reflexive and thus any consumer may sell the plan y_j to j by buying another plan from Y_j .

Theorem 5 below establishes the equivalence between the equilibria asociated with conditions i')-v') and equilibria that satisfy the recontracting condition.

Theorem 5 Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$ be an economy.

5.1 Let $[(x_i)_{i\in N},(y_j)_{j\in J}]$ be an allocation and $Z \subset \mathbb{R}^k$ be a set of net trades that satisfies conditions i')-v') above. For all $i\in N$ let $C_i = \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z$. Then $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ is an equilibrium that satisfies the recontracting condition.

5.2 Let $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ be an equilibrium that satisfies the recontracting condition. If there are at least two agents in the economy, then there exists a set Z, such that for all $i\in N$ $C_i = \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z$, and Z satisfies conditions i')-v').

As noted above condition v') may be interpreted as a profit maximization condition. Following this interpretation we may define a relation on production plans: The plan y is as least as valuable as the plan y' with respect to Z if $y'-y \in Z$. Condition v') requires that the actual production plan of each firm is as least as valuable as all the other feasible plans. Note that the relation "as least as valuable" is not necessarily a complete relation. Therefore one may be tempted to weaken condition v') to:

v'') For all $y_j' \in Y_j$ if $y_j - y_j' \in Z$ then $y_j' - y_j \in Z$.

Condition v'') requires that the chosen production plan is maximal in the production set. In words it says that for any feasible plan that is at least as valuable as the chosen plan, also the converese holds, i.e. the chosen plan is at least as valuable as this other plan. Unfortunately, this weakening is insufficient to characterize an intersting set of allocations as shown in the following example.

Example 2 Let $e = [(X_i,R_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$. Where $\mathbf{R}^2$ is the commodity space; N $= \{1,2\}$; $X_1 = X_2 = \mathbf{R}^2_+$; the preferences R_i are represented by the utility functions Min{ x_i^1, x_i^2 } (again, superscripts denote the different goods); $\omega_1 = (5,2)$; $\omega_2 = (2,5)$; $J=\{3,4\}$; $Y_3 = \{ y\in\mathbf{R}^2: y^1\leq 0\ y^2 \leq -y^1/2 \}$; $Y_4 = \{ y\in\mathbf{R}^2: y^2\leq 0\ y^1\leq -y^2/2 \}$; $\theta_{13} = \theta_{24} = 1$; $\theta_{14} = \theta_{23} = 0$. The allocation $x_1 = (3,3)$, $x_2 = (3,3)$, $y_1 = (-2,1)$, $y_2 = (1,-2)$ is supported by the set of net trades $Z = -\mathbf{R}^2_+$. This allocation is Pareto inferior to all Walrasian allocations of the economy. Preferences may be modified in a way that they will satisfy strict monotonicity and strict convexity.

7.5 The Core

Following Theorem 3 we can require a similar condition to the weak recontracting condition in production economies. This condition provides a new definition for the core of a production economy. Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$ be an economy. An allocation $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is a core allocation if there exists an abstract equilibrium $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ that satisfies the following condition: for all $i\in N$, for all coalitions

S with $i \notin S$

$$C_i \supset (\omega_i + \Sigma_{j\in J}\theta_{ij}y_j) + \Sigma_{k\in S}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j - y_j).$$

8. Proofs

It is clear that Theorem 1 is essentially a private case of Theorem 4, and that Theorem 2 is a private case of Theorem 5. Thus we do not prove Theorems 1 and 2, but only 4 and 5. We begin with the proof of Theorem 5.

First we note that from a similar argument mentioned in section 5, the recontracting condition 3.1 and the recontracting condition with firms together are equivalent to: for all $i \in N$ for all coalitions $S \subset N$ with $i \notin S$

$$C_i \supset C_i + \Sigma_{k\in S}C_k - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j - y_j). \tag{8.1}$$

In the sequel, whenever we say the recontracting condition we mean condition 8.1.

Proof of Theorem 5: 5.1 Let $[(x_i)_{i\in N},(y_j)_{j\in J}]$ be an allocation and $Z \subset \mathbb{R}^k$ be a set of net trades that satisfies conditions i')-v') (see Section 7.4). For all $i \in N$ let $C_i = \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z$. The condition $0 \in Z$ together with conditions ii'), iii) and v') implies that $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ is an equilibrium. We have to show that for all $i \in N$ for all coalitions $S \subset N$ with $i \notin S$

$$C_i \supset C_i + \Sigma_{k\in S}C_k - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j - y_j).$$

If $\#N = 1$, the recontracting condition is trivially satisfied.

If $\#N > 1$ we have :

$$C_i + \Sigma_{k\in S}C_k - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j - y_j) =$$

$$= \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z + \Sigma_{k\in S}[\omega_k + \Sigma_{j\in J}\theta_{kj}y_j + Z] - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j - y_j) \tag{8.2}$$

By ii') and additivity of Z we have:

$$Z + \Sigma_{k\in S}[x_k - \omega_k - \Sigma_{j\in J}\theta_{kj}y_j] \subset Z + Z = Z. \tag{8.3}$$

Since condition ii) holds for all i, Z is additive, and $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is an allocation we have that:

$$\Sigma_{i\in N\setminus S}(x_i - \omega_i - \Sigma_{j\in J}\theta_{ij}y_j) = -\Sigma_{k\in S}(x_k - \omega_k - \Sigma_{j\in J}\theta_{kj}y_j) \in Z \tag{8.4}$$

By 8.3, 8.4 and additivity of Z we get:

$$Z = Z + \Sigma_{k\in S}(x_k - \omega_k - \Sigma_{j\in J}\theta_{kj}y_j) \tag{8.5}$$

Now, from 8.2 and 8.5 we get:

$$C_i + \Sigma_{k\in S}C_k - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j-y_j) =$$

$$= \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z + \Sigma_{k\in S}(x_k - \omega_k - \Sigma_{j\in J}\theta_{kj}y_j) + \Sigma_{k\in S}[\omega_k + \Sigma_{j\in J}\theta_{kj}y_j + Z] - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j-y_j) \tag{8.6}$$

or:

$$C_i + \Sigma_{k\in S}C_k - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j-y_j) =$$

$$= \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z + \Sigma_{k\in S}Z + \Sigma_{k\in S}[x_k - \Sigma_{k\in S}R_k(x_k)] + \Sigma_{j\in J}(Y_j-y_j) \tag{8.7}$$

Applying conditions iv'), v') and the additivity of Z to 8.7 completes the proof.

5.2 Let $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ be an equilibrium that satisfies the recontracting condition. For all $i\in N$ let $Z_i = C_i - \omega_i - \Sigma_{j\in J}\theta_{ij}y_j$. As for all $i\in N$ $\omega_i + \Sigma_{j\in J}\theta_{ij}y_j \in C_i$, we get $0\in Z_i$ for all $i\in N$. Fix two agents i and m. By the recontracting condition:

$$C_m \supset C_m + C_i - R_i(x_i) + \Sigma_{j\in J}(Y_j-y_j) \supset C_m + C_i - R_i(x_i) \tag{8.8}$$

and similarly

$$C_i \supset C_i + \Sigma_{k\in N\setminus i}[C_k - R_k(x_k)] \tag{8.9}$$

Substituting 8.8 in 8.9 we get:

$$C_i \supset C_i + \Sigma_{k\in N}[C_k - R_k(x_k)] \tag{8.10}$$

Now substitute for all $k\in N$ $C_k = \omega_k + \Sigma_{j\in J}\theta_{kj}y_j + Z_k$ in 8.10:

$$\omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z_i \supset \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z_i + \Sigma_{k\in N}[\omega_k + \Sigma_{j\in J}\theta_{kj}y_j + Z_k - R_k(x_k)] \tag{8.11}$$

Deduct $\omega_i + \Sigma_{j\in J}\theta_{ij}y_j$ from both sides of 8.11:

$$Z_i \supset Z_i + \Sigma_{k\in N}[\omega_k + Z_k - R_k(x_k)] \tag{8.12}$$

As $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is an allocation and for all $k\in N$ $x_k \in R_k(x_k)$ we get:

$$Z_i \supset Z_i + \Sigma_{k\in N}Z_k \tag{8.13}$$

As for all $k\in N\backslash m$ $0\in Z_k$ we receive:

$$Z_i \supset Z_i + Z_m \tag{8.14}$$

As $0\in Z_m$:

$$Z_i \supset Z_i + Z_m \supset Z_i \tag{8.15}$$

From 8.15 we get that for all $i,m\in N$ $Z_i = Z_i + Z_m = Z_m := Z$, and that Z is additive.

Now by the construction of the Z_i's, for all $i\in N$:

ii') $x_i - (\omega_i + \Sigma_{j\in J}\theta_{ij}y_j) \in Z_i = Z$ (feasibility of actual trade).

Substituting $C_k = Z_k + \omega_k + \Sigma_{j\in J}\theta_{kj}y_j$ for k =i,m in 4.7:

$$Z_m + \omega_m + \Sigma_{j\in J}\theta_{mj}y_j \supset Z_m + \omega_m + \Sigma_{j\in J}\theta_{mj}y_j + Z_i + \omega_i + \Sigma_{j\in J}\theta_{ij}y_j - R_i(x_i) \tag{8.16}$$

Noting that $x_i - \omega_i - \Sigma_{j\in J}\theta_{ij}y_j \in Z_i$, $0\in Z_m$ and deducting $\omega_m + \Sigma_{j\in J}\theta_{mj}y_j$ from both sides of 8.16:

$$Z_m \supset x_i - R_i(x_i) \tag{8.17}$$

By 8.17:

iv') $x_i - R_i(x_i) \subset Z_m = Z$ (expost recontracting).

As for all $i\in N$ $x_i \in C_i \cap R_i(x_i)$, the recontracting condition implies:

v') $Y_j - y_j \subset Z$, for all $j\in J$ (expost recontracting in production).

As $[(x_i)_{i\in N},(y_j)_{j\in J},(C_i)_{i\in N}]$ is an equilibrium we get for all $i\in N$:

iii') $(\omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z) \cap P_i(x_i) = \varnothing$ (optimality). Q.E.D.

Given an economy $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$, the *m-fold replica of e* is the economy *e(m)* $= [(X_{ir},R_{ir},\omega_{ir})_{ir\in NxM},(Y_{jr})_{jr\in JxM},(\theta_{ijr})_{ijr\in NxJxM}]$ where $M = \{1,2,\ldots,m\}$ and for all $ij\in NxM$ $(X_{ij},R_{ij},\omega_{ij}) = (X_i,R_i,\omega_i)$; for all $jr\in JxR$ $Y_{jr} = Y_j$; and for all $ijr\in NxJxM$ $\theta_{ijr} = \theta_{ij}$. The term θ_{ijr} is the share of agent ir in the firm jr; it is assumed that for all $q\neq r$ the share of ir in the firm jq is zero.

The *m-fold replica of an allocation* $[(x_i)_{i\in N},(y_j)_{j\in J}]$ *of e* is an allocation $[(x_{ir})_{ir\in NxM},(y_{jr})_{jr\in JxM}]$ of *e(m)* in which for all $ir\in NxM$ $x_{ir} = x_i$ and for all $jr\in JxM$ $y_{jr} = y_j$.

A nonempty subset S of N is called a *coalition*. Let $[(x_i)_{i\in N},(y_j)_{j\in J}]$ be an allocation. A coalition S can improve upon $[(x_i)_{i\in N},(y_j)_{j\in J}]$ if there exists a list $(x_i)_{i\in S}$ where for all $i\in S$ $x_i\in X_i$ and $\Sigma_{i\in S}x_i \in \Sigma_{i\in S}\omega_i + \Sigma_{j\in J}\theta_{ij}y_j + \Sigma_{j\in J}(Y_j-y_j)$, for all $i\in S$ $x_i' \in R_i(x_i)$ and for some $i\in S$ $x_i' \in P_i(x_i)$. An allocation $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is a *core allocation* if there does not exist a coalition S that can improve upon $[(x_i)_{i\in N},(y_j)_{j\in J}]$.

An allocation $[(x_i)_{i\in N},(y_j)_{j\in J}]$ of *e* is a *shrunk-core allocation* if for all m the m-fold replica of $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is a core allocation in the m-fold replica of *e*.

Theorem 0 Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$ be an economy in which there are at least two agents. An allocation $[(x_i)_{i\in N},(y_j)_{j\in J}]$ may be supported by an equilibrium that satisfies the recontracting condition if and only if $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is a shrunk-core allocation.

Proof: Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$ be an economy and let $[(x_i)_{i\in N},(y_j)_{j\in J}]$ an allocation. Let $M=\{1,\ldots,m\}$ and:

$$Z^m = [\cup_{S\subset NxM}[\Sigma_{k\in S}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S}R_k(x_k) + \Sigma_{jr\in JxM}(Y_{jr}-y_{jr})]]$$

$$Z^* = \cup_{m=1}^{\infty} Z^m .$$

Lemma 5: Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$ be an economy, and $[(x_i)_{i\in N},(y_j)_{j\in J}]$ an allocation. Let Z^* defined as above. Then Z^* satisfies:

i') $0 \in Z^* = Z^* + Z^*$ (possibility not to trade and additivity).

For all $i \in N$:

ii') $x_i - (\omega_i + \Sigma_{j\in J}\theta_{ij}y_j) \in Z^*$ (feasibility of actual trade)

iv') $x_i - R_i(x_i) \subset Z^*$ (expost recontracting in consumption).

v') $Y_j - y_j \subset Z^*$, for all $j \in J$ (expost recontracting in production).

Proof: For convinience we will identify agent i1 with agent i, and firm j1 with j.

i) First note that since for all $i \in N$ $x_i \in R_i(x_i)$ and for all $j \in J$ $y_j \in Y_j$ we have

$0 = \Sigma_{k\in N}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in N}(x_k) + \Sigma_{j\in J}(y_j-y_j) \in \Sigma_{k\in N}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j-y_j)$.

Let $z,z' \in Z^*$. Thus there exist coalitions $S \subset NxM$ and $T \subset NxM'$ such that

$z \in \Sigma_{k\in S}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S}R_k(x_k) + \Sigma_{jr\in JxM}(Y_{jr}-y_{jr})$, and

$z' \in \Sigma_{k\in T}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in T}R_k(x_k) + \Sigma_{jr\in JxM'}(Y_{jr}-y_{jr})$.

As the preferrred sets are determined by the type of the agent, S and T can be assumed to be disjoint and subsets of $NxM''=Nx\{1,...,m+m'\}$. Now it is clear that

$z+z' \in \Sigma_{k\in S\cup T}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S\cup T}R_k(x_k) + \Sigma_{jr\in JxM''}(Y_{jr}-y_{jr}) \subset Z^*$.

ii') Let $S=N\backslash\{i\}$. As $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is an allocation, $x_i \in R(x_i)$ for all i, and $0 \in \Sigma_{j\in J}(Y_j-y_j)$ we get that

$x_i - (\omega_i + \Sigma_{j\in J}\theta_{ij}y_j) = \Sigma_{k\in S}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S}x_k + 0 \in \Sigma_{k\in S}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S}R_k(x_k) + \Sigma_{j\in J}(Y_j-y_j)$.

iv') Let $z \in x_i - R_i(x_i)$. Now

$z = \Sigma_{k\in N}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in N\backslash i}x_k - (x_i-z) + 0 \in \Sigma_{k\in N}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in N}R_k(x_k) + \Sigma_{j\in J}(Y_j-y_j)$.

v') Let $z \in Y_j - y_j$. Now

$z = \Sigma_{k\in N}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in N}x_k + \Sigma_{i\in J\setminus j}(Y_i - y_i) + z \in \Sigma_{k\in N}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in N}R_k(x_k)$
$+ \Sigma_{j\in J}(Y_j - y_j)$.

Lemma 6: Let $e = [(X_i, P_i, \omega_i)_{i\in N}, (Y_j)_{j\in J}, (\theta_{ij})_{ij\in NxJ}]$ be an economy, and $[(x_i)_{i\in N}, (y_j)_{j\in J}]$ an allocation. Let Z be a set that satisfies:

i') $0 \in Z = Z + Z$ (possibility not to trade and additivity).

For all $i \in N$:

ii') $x_i - (\omega_i + \Sigma_{j\in J}\theta_{ij}y_j) \in Z$ (feasibility of actual trade)

iv') $x_i - R_i(x_i) \subset Z$ (expost recontracting in consumption).

v') $Y_j - y_j \subset Z$, for all $j \in J$ (expost recontracting in production).

Then $Z^* \subset Z$.

Proof: Let $z \in Z^*$. Thus there exists a coalition $S \subset NxM$ for some M, bundles $x'_k \in R_k(x_k)$ for all $k \in S$, and production plans $y'_{jr} \in Y_{jr}$ for all $jr \in JxM$, such that

$z = \Sigma_{k\in S}(\omega_k + \Sigma_{j\in J}\theta_{kj}y_j) - \Sigma_{k\in S}x'_k + \Sigma_{jr\in JxM}(y'_{jr} - y_{jr})$.

Now, for all $k \in S$, by 8.4 (in the proof of Theorem 5.1)

$$\omega_k + \Sigma_{j\in J}\theta_{kj}y_j\omega_j - x_k \in Z$$

and by iv')

$$x_k - x'_k \in Z$$

By additivity of Z and the above we get that for all $k \in S$

$$\omega_k + \Sigma_{j\in J}\theta_{kj}y_j\omega_j - x_k + x_k - x'_k = \omega_k + \Sigma_{j\in J}\theta_{kj}y_j\omega_j - x'_k \in Z. \qquad (8.18)$$

and by v')

$$y'_{jr} - y_{jr} \in Z \qquad (8.19)$$

By additivity, and summing 8.18 over all $k \in S$ and 8.19 over all $jr \in JxM$, we get the required result.

Lemma 7: Let $e = [(X_i, P_i, \omega_i)_{i\in N}, (Y_j)_{j\in J}, (\theta_{ij})_{ij\in NxJ}]$ be an economy, and $[(x_i)_{i\in N}, (y_j)_{j\in J}]$ an allocation.

The set Z^* defined above satisfies:

iii') $(\omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z^*) \cap P_i(x_i) = \varnothing$ for all $i \in N$ (optimality) if and only if the allocation $[(x_i)_{i\in N}, (y_j)_{j\in J}]$ is a shrunk-core allocation.

Proof: Follows from the definition of the shrunk-core. (Analogous to Lemma 4, in Section 5).

Now let $[(x_i)_{i\in N}, (y_j)_{j\in J}]$ be a shrunk-core allocation. Let $C_i = \omega_i + \Sigma_{j\in J}\theta_{ij}y_j + Z^*$ for all $i \in N$. By Lemmas 5 and 7 and Theorem 5.1 $[(x_i)_{i\in N}, (y_j)_{j\in J}, (C_i)_{i\in N}]$ is an equilibrium that satisfies the recontracting condition.

Let $[(x_i)_{i\in N}, (y_j)_{j\in J}, (C_i)_{i\in N}]$ be an equilibrium that satisfies the recontracting condition. By Theorem 5.2 there exists a set Z that satisfies i)-v) and by Lemma 6 $Z^* \subset Z$. It follows from $(\omega_i + Z) \cap P_i(x_i) = \varnothing$ that $(\omega_i + Z^*) \cap P_i(x_i) = \varnothing$, thus by Lemma 7 we get that $[(x_i)_{i\in N}, (y_j)_{j\in J}]$ is a shrunk-core allocation. This concludes the proof of Theorem 0.

It is well known that in large classes of economies all shrunk core allocations are allocations associated with quasi Walrasian equilibria. The first result concerning this equivalence is due to Edgeworth (1881). More general results are due to Debreu and Scarf (1963) and others. The following result is based on Debreu and Scarf (1963,Theorem 3).

Proposition A: Let $e = [(X_i,P_i,\omega_i)_{i\in N},(Y_j)_{j\in J},(\theta_{ij})_{ij\in NxJ}]$ be an economy that satisfies C1-C3 and CP1. If $[(x_i)_{i\in N},(y_j)_{j\in J}]$ is a shrunk core allocation of e then it is a quasi Walrasian allocation of e.

The proof of Proposition A is similar to that of Debreu and Scarf (1963,Theorem 3). One has to replace the sets $P_i(x_i)-\omega_i$ with $P_i(x_i)-[\omega_i + \Sigma_{j\in J}\theta_{ij}y_j + \Sigma_{m=1}^{\infty}\Sigma_{jr\in JxM}(Y_{jr}-y_j)]$. The rest of the proof follows the arguments of Debreu and Scarf.

9. Conclusion

The definition of strongly fair net trades of Schmeidler and Vind (1972) does not include any condition related to recontracting. On the other hand it concentrates on "geometrical" properties of the set of net trades, such as additivity. As a consequence Schmeidler and Vind (1972) do not characterize the Walrasian allocations, but able only to find sufficient conditions that are far from necessary. McLennan and Sonnenschein (1991) added the expost recontracting condition $x_i - R_i(x_i) \subset Z$, and derived a characterization of the Walrasian allocations. Theorem 1 emphasizes the importance of the recontracting condition, as no "geometrical" requirement is imposed on the choice sets, but is derived as a result (Theorem 2.2). In addition, it is shown that the recontracting condition is closely related to the notion of the core, as it is a natural strengthening of the weak recontracting condition. And thus the Walrasian Theory can be derived from core-like conditions, in a finite agent setting. This insight is not apparent from Schmeidler and Vind (1972) and McLennan and Sonnenschein (1991) who concentrated on the Walrasian theory alone.

McLennan and Sonnenschein (1991) characterized a subgame perfect equilibium of a non-cooperative model of trade by showing that each agent has access to a set of net trades that satisfies certain properties. A natural question would be, given our generalization of the axiomatic result, whether it is possible to derive more general results in the non-cooperative model. The answer to this question is twofold; first, we claim that the derevation of the properties of the net-trades set in the non-cooperative model is possible due to the peculiar assumptions that are made on the preferences of agents, and thus there is little hope to generalize these results; second, the results of the current paper provide us insight concerning the question what properties of the trade procedure are essential in deriving Walrasian outcomes; this may enable us to "correct" the non-cooperative model as to derive more general results, as explained below.

McLennan and Sonnenschein (1991) study a variant of Gale's (1986a) non-cooperative model and characterize its subgame perfect equilibria by applying a variant of Theorem 2 for continuum economies. Although we have shown that differentiability is not needed for Theorem 2, the results of the non-cooperative model are dependent of differentiability in a very strong way. Indeed, Gale (1986a,1986b) and McLennan and sonnenschein (1991) use the fact the "pairwise Pareto efficiency" implies Pareto efficiency. Clearly this is not true if preferences are not differentiable. Moreover, they conclude that every agent may conduct any trade with a negative value (evaluated by the efficiency prices) from very strong requirements on the agents characteristics.[4] In addition, their proofs are based on trades with agents that are about to leave the market, and therefore it seems that the characterization of Makowski and Ostroy (1995) should suffice. Note however that this latter characterization

[4]Gale (1986a) assumed that there is at most a countable number of utility functions, and that for each of them the endowments are dispersed. Gale (1986b) assumed that agents indifference surfaces satisfy a uniform bounded curvature property. McLennan and Sonnenschein (1991) did not relax these assumptions considerably.

is applicable to the non-cooperative model only when preferences are differentiable.[5]

In order to generelize the results of the non-cooperative models to the non-differentiable case one should think of trade mechanisms that induce the weak recontracting condition (that characterizes the core) on the subgame perfect equilibria of the game. We think that by considering a matching process that is not restricted to pairwise meetings but allows meetings in finite groups may prove to be a promising line of research. The players in the bargaining models of Gale (1986a,1986b) and McLennan and Sonnenschein (1991) do not discount the future; therefore they leave the market to consume only when there is no hope to find an improving deal. Moreover, in a continuum economy, the minimal choice sets defined by the weak recontracting condition are unbounded if the allocation is not in the core.[6] This may imply that non-core allocations cannot be supported by a subgame perfect equilbrium as the agents will always have some hope to strike an improving deal, and will never leave the market.

The results concerning production economies imply that the notion of profit maximization can be derived from maximizing behavior of consumers. The concepts developed in this paper also induce a new definition of the core of production economies. This definition does not assign each group of individuals an exante possibilities set, but looks at each group's possibilities in equilibrium. We feel that this concept is more similar in spirit to that of Edgeworth.

It follows from the proof of Theorem 1 that it can be generalized to other situations where the shrunk-core-Walras equivalence hold; for example, various classes of economies

[5]Otherwise the set of allocations characterized may include non-Walrasian allocations.

[6]These choice sets are defined in the proof of Theorem 3 in Section 5, and denoted by A_i .

with an infinite number of goods (e.g. Aliprantis, Brown, and Burkinshaw, 1987).

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