Economics Working Paper 151

Consistency and the Walrasian Allocations Correspondence*

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January 1996†

Abstract

We characterize the Walrasian allocations correspondence, in classes of exchange economies with smooth and convex preferences, by means of consistency requirements and other axioms. We present three characterization results; all of which require consistency, converse consistency and standard axioms. Two characterizations hold also on domains with a finite number of potential agents, one of them requires envy freeness (with respect to trades) and the other–core selection; a third characterization, that requires core selection, applies only to a variable number of agents domain, but is valid also when the domain includes only a small variety of preferences.

1. INTRODUCTION

The most important solution concept in economics is price equilibrium. However most of the literature which characterizes the price mechanism as a consequence of more basic assumptions concentrates on large economies (e.g. Aumann, 1964). To a certain extent characterizing the Walrasian outcomes as derived by assumptions different than price taking behavior in small economies seems as a more important issue since in these economies the price taking assumption per se is much less natural than in large economies. In other words, an assumption that the economy is large provides by itself a convincing argument for the price taking hypothesis.

This paper provides axiomatic characterizations of the Walrasian allocations correspondence in small economies. Moreover, two of these characterizations hold when the number of agents in each of the economies in the domain of the solution is arbitrarily restricted. On the other hand, these characterizations resemble well known characterizations in large economies, namely, those based on the "core property" and those based on "envy free" (or "anonymous") allocations. Clearly, in order to establish such results one has to require that the solution would satisfy other properties that replace the assumption of a large number of agents.

This paper concentrates on solution concepts that satisfy certain consistency properties. Similar concepts were applied sucssesfully in cooperative game theory and bargaining. Almost all the major solution concepts in cooperative game theory and bargaining satisfy a certain consistency property (the Davis-Maschler ,1965, reduced game property) and can be exactly characterized by this property and some more additional requirements. These solutions include the core of TU and NTU games (Peleg 1986,1985), the Nash bargaining solution (Lensberg 1988), the prenucleolus (Sobolev 1975), and the

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prekernel (Peleg 1986).[1]

Due to the fact that there is no self-evident way to define reduced economies in models were the agents have initial endowments, most of the research on consistency in models other than cooperative games or bargaining problems is confined to situations where the agents have a collective endowment, and thus the major descriptive solutions in economics cannot be studied in those frameworks.

Consistency properties in exchange economies with private endowments were first considered by Thomson (1992). Assume a certain allocation is a solution of a given economy. The question is how to define a reduced economy whose members are a subset of the whole set of agents and their possibilities are derived from the assumption that they should keep their agreement with the agents outside the reduced economy. Thomson (1992) proposed several definitions. The one we adopt is the following. First, in the definition of an economy, in addition to the preferences and endowments of the agents, a trade that defines the market clearing condition is given. In a reduced economy with respect to a coalition and an allocation the agents have the same preferences and endowments as in the original economy and the trade is adjusted according to the underlying allocation. Preliminary characterization results with this model are due to Dagan (1994) and Van den Nouweland, Peleg, and Tijs (1994).[2]

After providing a characterization result that employs envy freeness with respect to trades, we turn to address a problem ignored by previous authors. Previous authors assumed that every agent's preferences are maximized on the budget set induced by the equilibrium prices and the agent's endowment; in economies with a nonzero trade vector this implies that

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[1]An exception is the Shapley value; however it was characterized by a different reduced game property by Hart and Mas-Colell (1989).

[2]The main result of Dagan (1994) is a variant of Theorem 3. The main result of Van den Nouweland, Peleg, and Tijs (1994) is closely related to Lemmas 2 and 3.

the value of the trade of the economy is zero. Moreover, it implies that an economy that has

a trade positive in all coordinates, and whose agents' preferences are monotonic has no

equilibrium. We propose an alternative generalization of Walrasian equilibrium to economies

with a nonzero trade vector. According to this alternative definition the trade that the

economy has with the rest of the world may have a positive value which is divided among

the agents. We provide two axiomatic characterizations of the Walrasian allocations

correspondence by requiring core selection in addition to consistency requirements. The

first, as noted above, does not require a variable number of agents, however it assumes that

there are suficiently many different potential characteristics of agents. The second one shows

that with a variable number of agents, and an arbitrary set of agents' characteristics a great

deal of the result can be maintained. Intuitively, the two results show that "the richness of

the characteristic space" can be restricted in the price of making the size of economies in the

domain "rich enough." The result with a variable number of agents employs a variant of

Debreu and Scarf's (1963) replica theorem, for economies with a nonzero trade vector.

The paper is organized as follows. The model, solution concepts, including our

generalization of Walrasian equilibrium to economies with nonzero trade vector, and some

of their well known properties are presented in Section 2; in Section 3 we discuss the concept

of consistency; axiomatic characterizations are given in Sections 4 and 5; Section 6 presents

the generalization of Debreu and Scarf's theorem; and the proofs of the results of Sections

4 and 5 are given in Section 7.

2. THE MODEL

2.0 Notation

We denote the k-dimensional Euclidian space by $\mathbf{R}^k$; the non negative orthant by $\mathbf{R}^k_+$;

and the interior of $\mathbf{R}^k_+$ by $\mathbf{R}^k_{++}$. The set of natural numbers is denoted by N. The interior of

a set A is denoted by int(A). The cardinality of a set S is denoted by #S.

2.1 A Model

Let $\mathbb{R}^k$ be the commodity space. A *type* on $\mathbb{R}^k$ is a triple (X,R,ω) where $X \subset \mathbb{R}^k$ is the consumption set, R is a complete and transitive preference relation on X, and $\omega \in X$ is an endowment. For a type (X,R,ω) define the strict preference relation P as follows: For all $x,y \in X$ xPy if and only if xRy and not yRx. For all $x \in X$ let $R(x) = \{y \in X : yRx\}$ and $P(x) = \{y \in X : yPx\}$.

For the characterization results we consider types that satisfy additional assumptions.

A1 *Upperhemi continuity*: For all $x \in X$ $P(x)$ is open relative to X.

A2 *Local non satiation*: For all $x \in X$ $P(x) \cap O(x) \neq \varnothing$, for all open balls $O(x)$ around x.

A3 *Smoothness*: $X = \mathbb{R}^k_{++}$ and for all $x \in X$ there exists at most one hyperplane through x with normal p such that for all $y \in P(x)$ $px \leq py$.

A4 *Convexity*: For all $x \in X$ $R(x)$ is convex.

A5 *Weak monotonicity*: If $x \in X$ and $y \gg x$ then $y \in X$ and yPx.

We denote by T^* the set of all types satisfying A1-A4, and by T^{**} the set of all types satisfying A1-A5.

An *economy* is a pair $[(X_i,R_i,\omega_i)_{i \in N}, t]$, where N is a finite nonempty set of names of agents; for all agents $i \in N$ (X_i,R_i,ω_i) is a type on $\mathbb{R}^k$; and $t \in \mathbb{R}^k$ is a trade vector that the economy is comitted to have with the rest of the world.

A *closed economy* is an economy whose trade vector t is equal zero, and an *open economy* is an economy whose trade vector t is different from zero.

An *allocation* is a list $(x_i)_{i \in N}$ where for all $i \in N$ $x_i \in X_i$ and $t + \Sigma_{i \in N} \omega_i = \Sigma_{i \in N} x_i$.

Let Q be a set of names and T a set of types. We denote by $\Omega(Q,T)$ the set of all economies whose agents' names belong to Q and agents' types belong to T.

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2.2 Walrasian Equilibria

Let $e = [(X_i, R_i, \omega_i)_{i \in N}, t]$ be an economy. A *Walrasian equilibrium* is a pair $[(x_i)_{i \in N}, p]$ where $(x_i)_{i \in N}$ is an allocation; $p \in \mathbb{R}^k \backslash \{0\}$; for all $i \in N$ $x_i \in \{y \in \mathbb{R}^k : py \leq p\omega_i\}$ and $P_i(x_i) \cap \{y \in \mathbb{R}^k : py \leq p\omega_i\} = \varnothing$. The allocation $(x_i)_{i \in N}$ associated with a Walrasian equilibrium is called a *Walrasian allocation*. We denote by $W(e)$ the set of all Walrasian allocations of e.

The concept of walrasian equilibrium as defined above seems to be quite restrictive, as it assumes that the value (evaluated by the equlibrium prices) of the trade t is zero. Consider, for example an economy whose net trade t is strictly positive in all coordinates, and all agents have monotonic preferences. According to the above definition there does not exists an equilibrium in this economy; however it is natural to think that in such a situation the value of the trade t may be divided among the agents in a way that everyone gets a positive share and that an equilibrium exists. The following definition of equilibrium formalizes this intuition.

A *Walrasian equilibrium with a nonnegative profit distribution* is a triple $[(x_i)_{i \in N}, p, (s_i)_{i \in N}]$ where $(x_i)_{i \in N}$ is an allocation; $p \in \mathbb{R}^k \backslash \{0\}$; for all $i \in N$ $s_i \geq 0$, $x_i \in \{y \in \mathbb{R}^k : py \leq p\omega_i + s_i\}$ and $P_i(x_i) \cap \{y \in \mathbb{R}^k : py \leq p\omega_i + s_i\} = \varnothing$. The allocation $(x_i)_{i \in N}$ associated with a Walrasian equilibrium with a nonnegative profit distribution is called a *Walrasian allocation with a nonnegative profit distribution*. We denote by $\tilde{W}(e)$ the set of all Walrasian allocations with nonnegative profit distributions of e.

The definition requires that each agent's "profit share" is nonnegative; this means that the agent's responsability of the society's trade cannot be imposed upon him. Note that if one allows for negative shares the notion of individual endowments looses its contents, and if preferences are convex, all Pareto optimal allocations can be supported by some equilibria (this is simply the second welfare theorem). The definition of a Walrasian equilibrium with

nonnegative profit distribution is actually the generalization of Walrasian equilibrium with slack to open economies. Mas-Colell (1988) proposed this equilibrium concept to economies with satiated preferences. There the slack is possible in closed economies as agents with satiated preferences do not exhaust their income, and the unexhausted income is divided among the other agents; in our model the "slacks" are a consequence of the fact that the economy has a nonzero trade with the rest of the world. The term "nonegative profit distribution" is inspired by a similar term used in coalition production economies.

It follows directly from the definitions that for all economies e $W(e) \subset \tilde{W}(e)$. Moreover if the economy is closed and preferences satisfy local non satiation then $W(e) = \tilde{W}(e)$.

2.3 Envy Free Allocations

Let $e = [(X_i, R_i, \omega_i)_{i \in N}, t]$ be an economy. An allocation $(x_i)_{i \in N}$ is *envy free* if there are no $i,j \in N$ with $(\omega_i + x_j - \omega_j) P_i x_i$.

Although the term envy free has an normative connotation, this property has a positive interpretation. Consider a situation where the market mechanism gives all agents the access to the same set of possible trades. If all agents choose their trades optimally, the resulting allocation is envy free. The seemingly abstract notion of a "market mechanism" may be supported, for example, by McLennan and Sonnenschein (1991) who study a sequantial bargaining game in a large market. In their game each subgame perfect equilibrium induces a set of trades available to all agents.

It is well known that Walrasian allocations are envy free. Note that Walrasian allocations with a nonnegative profit distribution need not be envy free as agents may receive different profit shares.

2.4 Pareto Optimal and Core Allocations

An allocation $(x_i)_{i \in N}$ is *Pareto optimal* if there does not exist an allocation y such that

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for all $i \in N$ $y_iR_ix_i$, and for some $i \in N$ $y_iP_ix_i$.

When preferences are locally non satiated Walrasian allocations with nonnegative profit distributions are Pareto optimal.

An allocation $(x_i)_{i \in N}$ is a *Core allocation* if it is Pareto optimal and there does not exist a coalition $S \subset N$, and and a list $(y_i)_{i \in S}$ such that $\Sigma_{i \in S} y_i = \Sigma_{i \in S} \omega_i$, for all $i \in S$ $y_iR_ix_i$, and for some $i \in S$ $y_iP_ix_i$. We denote the set of all core allocations of e by *Core(e)*.

It is implicit in our definition of the core that agents can form coalitions without having any commitment with respect to the trade t that the economy has with the rest of the world. This is consistent with the idea that the commitment of the economy cannot be imposed on the agents, like in the definition of Walrasian equilibrium with a nonnegative profit distribution. An alternative approach would be to divide t among the individuals by some rule; we study this alternative approach elsewhere. (Dagan 1995).

There are some well known relations between Walrasian allocations and core allocations in closed economies. A variant of Debreu and Scarf (1963) relating the core and Walrasian allocations with nonnegative profit distribution is established in Section 6.

3. CONSISTENCY

Let Ω be a non-empty set of economies. A *solution* on Ω is a correspondence that assigns each economy in Ω a set (possibly empty) of allocations.

We continue with the definition of a reduced economy:

Let $e = [(X_i,R_i,\omega_i)_{i \in N},t]$ be an economy, and $x=(x_i)_{i \in N}$ be an allocation, and $S \subset N$. The *reduced economy* of e with respect to S and x is the economy:

$$e^{S,x} = [(X_i,R_i,\omega_i)_{i \in N},t]^{S,x} = [(X_i,R_i,\omega_i)_{i \in S},t+\Sigma_{i \in N \setminus S}\omega_i-x_i].$$

The reduced economy of e with respect to S and x is the economy whose members are the members of S, and in which the imports-exports vector is determined by the ongoing

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agreement $x|N\backslash S$ with the rest of the agents in the economy.

Now we are ready to define consistency.

A solution f on Ω is *consistent* if for all economies $e = [(X_i, R_i, \omega_i)_{i \in N}, t]$ in Ω, and for all $S \subset N$, for all $x \in f(e)$, if $e^{S,x} \in \Omega$, then $x|S \in f(e^{S,x})$.

If a consistent solution assigns an allocation to a given economy, then it assigns the reduced allocation to the reduced economy, provided the reduced economy is in the domain of the solution.

It turns out that the Walrasian allocations correspondence and the Walrasian allocations with nonnegative profit distribution correspondence are consistent for all domains of economies. Formally we have:

Proposition 1: Let Ω be a non empty set of economies. The solution that assigns all economies in Ω their Walrasian allocations with nonnegative profit distributions is consistent.

Proof: Let $e = [(X_i, R_i, \omega_i)_{i \in N}, t]$ be an economy in Ω, and let $x = (x_i)_{i \in N}$ be a Walrasian allocation with a nonnegative profit distribution. Thus there exists a price vector p and a list $s = (s_i)_{i \in N}$ such that $[x,p,s]$ is a Walrasian equilibrium with a nonnegative profit distribution. Now let $S \subset N$. Clearly $[x|S,p,s|S]$ is a Walrasian equilibrium with a nonnegative profit distribution of the economy $e^{S,x}$. ☺

Proposition 2: Let Ω be a non empty set of economies. The solution that assigns all economies in Ω their Walrasian allocations is consistent.

The proof is analogous to that of Proposition 1, and is therefore left to the reader.

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Now we present a converse consistency property.

A solution f on Ω is *conversely-consistent* if x is a Pareto optimal allocation of $e = [(X_i,R_i,\omega_i)_{i\in N},t]\in\Omega$ and for all strict subsets $S\subset N$ $(S\neq N)$ $e^{S,x}\in\Omega$ and $x|S\in f(e^{S,x})$, then $x\in f(e)$.

Converse consistency requires that if a Pareto optimal allocation of an economy solves for all reduced economies, then this allocation solves for the entire economy. This variant of converse consistency is due to Van den Nouweland, Peleg, and Tijs (1994).

Proposition 3: Let $T\subset T^*$, and $Q\neq\varnothing$. The the Walrasian allocations with nonnegative profit distribution correspondence on $\Omega(Q,T)$ is conversely-consistent.

Proof: Let $e = [(X_i,R_i,\omega_i)_{i\in N},t]\in\Omega$, and x be a Pareto optimal allocation that satisfies $x|S\in\tilde{W}(e^{S,x})$ for all strict subsets $S\subset N$. By smoothness and since x is a Walrasian allocation with a nonnegative profit distribution in all reduced economies it follows that there exists a price p and a list $s=(s_i)_{i\in N}$ such that for all $i\in N$ $s_i\geq 0$, $x_i\in\{y\in\mathbb{R}^k:py\leq p\omega_i+s_i\}$ and $P_i(x_i)\cap\{y\in\mathbb{R}^k:py\leq p\omega_i+s_i\} = \varnothing$. It follows then that $[x,p,s]$ is a Walrasian equilibrium with a nonnegative profit distribution, and $x\in\tilde{W}(e)$. ☻

Proposition 4: Let $T\subset T^*$, and $Q\neq\varnothing$. The the Walrasian allocations correspondence on $\Omega(Q,T)$ is conversely-consistent.

The proof is analogous to that of Proposition 3, and is therefore left to the reader.

4. A CHARACTERIZATION BASED ON ENVY-FREENESS

4.1 Theorem

Now we present some standard and well-known properties.

A solution f is *nonempty* if for all closed economies with $W(e) \neq \varnothing$, $f(e) \neq \varnothing$.

A solution f is *neutral* if for all economies e, if $x \in f(e)$ and y is an allocation that satisfies $x_i R_i y_i$ and $y_i R_i x_i$ for all $i \in N$, then $y \in f(e)$.

Theorem 1: Let Q be a set with at least two elements. A solution f on $\Omega(Q, T^{**})$ satisfies nonemptiness, neutrality, consistency, converse consistency and assigns Pareto optimal and envy free allocations if and only if for all economies e $f(e) = W(e)$.

4.2 Discussion

It is important to note that the nonemptiness assumption requires the solution to be nonempty only on *closed* economies that have Walrasian allocations. Van den Nouweland, Peleg, and Tijs (1994) require nonemptiness of the solution for all one person economies in the domain of the solution, which is defined as *all* the economies in $\Omega(Q, T^{**})$ that have Walrasian allocations. In addition, their definition of consistency requires that if $e \in \Omega$ and $x \in f(e)$ then $e^{S,x} \in \Omega$. Their requirements are essentially equivalent to requiring that the solution coincides with the Walrasian correspondence on one person economies.

Champsaur and Laroque (1981) and others (see Mas-Colell, 1985, chapter 7) characterized Walrasian allocations of continuum economies by Pareto optimality and envy freeness. These results require that agents characteristics are dispersed.[3] The proof of Theorem 1 exploits the fact that consistency and converse consistency enable to reach

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[3]These results hold also on some cases where the characteristics are not dispersed, such as the case where all agents are identical.

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conclusions on a small economy by embedding it in economies with many different agents. However there is no need to construct large economies as it is possible to "match" many different individuals in many two agent economies.

Zhou (1992) studied a stronger property than envy free, called strictly envy free, and provided a characterization similar to Champsaur and Laroque (1981) which is valid without any requirement on the distribution of characteristics. Zhou's characterization was applied by Thomson and Zhou (1993) and Dagan (1995) in deriving characterizations of the Walrasian correspondence that use a different consistency property than the one employed here. Those two latter results hold, like Zhou's, without requiring the domain to contain a large variety of preferences.

Now we provide examples that show that the axioms in Theorem 1 are independent. Examples 1, 3, 4, and 6 are valid for domains of the kind $\Omega(Q,T)$ as long as $\#Q \geq 2$ and $T \subset T^*$ (and in particular for $T = T^{**}$). Examples 2 and 5 are valid only when there are only two goods. We do not know whether neutrality and Pareto optimality are both needed in the case when there are more than two goods. However at least one of them is needed as shown by example 7.

1. A solution f that satisfies neutrality, consistency, converse consistency and assigns Pareto optimal and envy free allocations, but does not satisfy nonemptiness is the solution $f(e) = \varnothing$ for all e.⊗

2. In the case when there are two goods, a solution f on $\Omega(Q,T^*)$ that satisfies nonemptiness, consistency, converse consistency and assigns Pareto optimal and envy free allocations, but does not satisfy neutrality can be constructed as follows. The solution f will be a selection from W. One can find a type for which there is an open set of price ratios in which the demand set is not a singleton. Now take a price p^* in the interior of these set of price ratios and a point x^* in the relative interior of the demand set for this price, and that x^* is not the

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endowment of this type. Let f assign all the Walrasian allocations, but those in which this certain type is assigned the bundle x^*. By construction, any closed economy in which assigning x^* to this type is part of an Walrasian equilibrium, there exist other equilibria, thus nonemptiness is not violeted. The other axioms can be easily verified. ⊗

3. A solution f that satisfies nonemptiness, neutrality, converse consistency and assigns Pareto optimal and envy free allocations, but is not consistent is: $f(e) = W(e)$ if e has more than one agent or is closed, and $f(e) = \varnothing$ if e has one agent and is open. ⊗

4. A solution f that satisfies nonemptiness, neutrality, consistency, and assigns Pareto optimal and envy free allocations but is not conversely consistent is the subcorrespondence of $f(e) = \tilde{W}(e)$ in which the allocations are supported by an equilibrium in which all agents have equal profit shares. ⊗

5. A solution f that satisfies nonemptiness, neutrality, consistency, converse consistency and assigns envy free but not Pareto optimal allocations is the following. For each economy $e = [(X_i, R_i, \omega_i)_{i \in N}, t]$ $f(e) = W(e) \cup \{\omega\}$ if $t = 0$ and there does not exists another allocation $y \neq \omega$ such that $y_i R_i \omega_i$ and $\omega_i R_i y_i$ for all $i \in N$, and $f(e) = W(e)$ otherwise. This solution is different from W when there are only two goods. ⊗

6. A solution f that satisfies nonemptiness, neutrality, consistency, converse consistency and assigns Pareto optimal but not envy free allocations is $f(e) = \tilde{W}(e)$. ⊗

7. A solution f that satisfies nonemptiness, consistency, converse consistency and assigns envy free allocations but does not satisfy neutrality and Pareto optimalilty is the solution that assings the endowments in closed economies and the empty set otherwise. ⊗

5. CHARACTERIZATIONS BASED ON CORE SELECTION

5.1 Theorem

It may be argued that the notion of an anonymous market mechanism is not

appropriate in small economies, and thus one should not consider envy-freeness as a reasonable assumption. In this section we consider an alternative assumption--core selection. This requirement is most appealing in small economies, as the formation of coalitions is more likely when these do not include a large number of participants.

A solution f satisfies *core selection* if for all economies e, $f(e) \subset Core(e)$.

Theorem 2: Let Q be a set with at least two elements. If a solution f on $\Omega(Q,T^*)$ satisfies nonemptiness, neutrality, consistency, converse consistency, and core selection then for all economies $W(e) \subset f(e) \subset \bar{W}(e)$. Moreover W and $\bar{W}$ satisfy the above axioms.

5.2 Discussion

Note that although Theorem 2 does not characterize the set of solutions satisfying the axioms exactly, all the solutions that satisfy the axioms coincide on the class of closed economies (as $W(e) = \bar{W}(e)$ in these economies).

Theorem 2 is also valid on domains that contain a small number of potential agents. The idea of the proof is similar to that of Theorem 1. The requirements of consistency and converse consistency make it possible to "match" many different individuals in many two agent economies and reach strong conclusions without constructing large economies.

It is clear that $W(e)$ and $\bar{W}(e)$ can be characterized exactly by adding the requirements of minimality and maximality respectively. Solutions that are different from both $W(e)$ and $\bar{W}(e)$ and satisfy the axioms can be easilly constructed. For example, consider a selection from $\bar{W}(e)$ that assigns only those allocations in which the supporting equilibria have the property that for all $i \in N$ $s_i/p\omega_i$ is a rational number.

Now we show that the axioms of Theorem 2 are independent. Examples 8 and 9 are valid for domains of the kind $\Omega(Q,T)$ as long as $\#Q \geq 2$ and $T \subset T^*$.

For solutions that satisfy all the axioms but one of the following: nonemptiness, neutrality, or consistency, see examples 1,2, and 3 in Section 4 above.⊗

8. A solution f on $\Omega(Q,T)$ that satisfies nonemptiness, neutrality, consistency, and core selection, but not converse consistency is the core correspondence. (Another one that also satisfies envy-freenes is given in example 4 in Section 4 above.)⊗

9. A solution f on $\Omega(Q,T)$ that satisfies nonemptiness, neutrality, consistency, and converse consistency, but not core selection is the solution that assigns all Pareto optimal allocations.⊗

5.3 Another Theorem

There are circumstances where it is natural to assume that the set of preferences of agents are chosen from an arbitrary (small) set. We now present a result concerning such a situation. However this result is valid only when the size of potential economies is not restricted.

Two economies $[(X_i,R_i,\omega_i)_{i\in N},t]$ and $[(X_i,R_i,\omega_i)_{i\in M},t']$ are *equivalent* if $t=t'$ and there exists a one-to-one and onto function $\sigma:N\to M$ such that $(X_{\sigma(i)},R_{\sigma(i)},\omega_{\sigma(i)})=(X_i,R_i,\omega_i)$ for all $i\in N$.

A solution f is *anonymous* if for all two equivalent economies e and e', $x\in f(e)$ implies $\sigma(x)\in f(e')$.

Let T be a set of types. T *is closed under improvements* if $(X,R,\omega)\in T$ and $\omega'R\omega$ imply $(X,R,\omega')\in T$.

Theorem 3: Let T be a nonempty subset of T^* that is closed under improvements. If a solution f on $\Omega(N,T)$ satisfies nonemptiness, consistency, converse consistency, anonymity, and core selection then for all economies $f(e) \subset \tilde{W}(e)$.

6. Cores of Replicas of Open Economies

Debreu and Scarf (1963) showed that if a replica of an allocation is in the core of all replicas of a closed economy then this allocation is a Walrasian allocation. In this section we provide a generalization of that result to open economies. In our theorem we have to require that the allocation cannot be improved upon by any coalition of all replicas, in which some of its members come with their final bundle, and not with their initial bundle. Note that in closed economies, if an allocation can be improved upon by some coalition in some replica, in which some of its members come with their final bundles, one can find an improving coalition in which all members come with their endowments. One has to add agents to the improving coalition as to complete the profiles of those agents who come with their final bundles, and as the sum of the endowments of each profile is the sum of final bundles, an improvement can be realized by assigning all members of the original improving coalition their improving allocation, and to the rest their final bundles.

Let $e = [(X_i, R_i, \omega_i)_{i \in N}, t]$ be an economy, and let x be an allocation. *The adjoint economy of e with respect to* x, is the economy $e_x = [(X_{ij}, R_{ij}, \omega_{ij})_{i \in Nx\{1,2\}}, t]$ where $(X_{ij}, R_{ij}) = (X_i, R_i)$ for all $ij \in Nx\{1,2\}$; $\omega_{i1} = \omega_i$ and $\omega_{i2} = x_i$ for all $i \in N$. *The adjoint allocation of* x is an allocation of e_x such that for all $ij \in Nx\{1,2\}$ $x_{ij} = x_i$.

Let $e = [(X_i, R_i, \omega_i)_{i \in N}, t]$ be an economy. *The m-fold replica of e* is the economy $\epsilon(m) = [(X_{ij}, R_{ij}, \omega_{ij})_{ij \in NxM}, mt]$ where $M = \{1, 2, ..., m\}$ and for all $ij \in NxM$ $(X_{ij}, R_{ij}, \omega_{ij}) = (X_i, R_i, \omega_i)$. The *m-fold replica of an allocation* $(x_i)_{i \in N}$ *of e* is an allocation $(x_{ij})_{ij \in NxM}$ of $e(m)$ in which for all $ij \in NxM$ $x_{ij} = x_i$.

We consider types that satisfy the following:

A1 *Upperhemi continuity*: For all $x \in X$ $P(x)$ is open relative to X.

A2 *Local non satiation*: For all $x \in X$ $P(x) \cap O(x) \neq \varnothing$, for all open balls $O(x)$ around x.

A6 *Interior endowments*: X is convex and $\omega \in \text{int}(X)$.

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Lemma 4: Let $Q \neq \emptyset$, and $T \subset T^*$. If a solution f on $\Omega(Q,T)$ satisfies consistency, converse consistency and for all one agent economies $f(e) \subset \tilde{W}(e)$, and assigns Pareto optimal allocations then for all economies $e \in \Omega(Q,T^*)$ $f(e) \subset \tilde{W}(e)$.

Proof: Practically identical to Lemma 3, and is therefore left to the reader. ☺

Proof of Theorem 1: We will prove only the uniqueness part.

Step 1: Let Q be a set with at least two elements. If a solution f on $\Omega(Q,T^*)$ satisfies nonemptiness, neutrality, consistency, converse consistency and assigns envy free allocations then for all one agent economies e with linear preferences $f(e) = W(e)$.

Proof: Let $e = [(R_i,\omega_i),t]$ be an economy with one agent, and R_i is a linear preference with indifference surfaces with slope p. If the economy has no allocations the statement is trivially satisfied. So assume that the economy has an allocation. First, consider the case where $tp=0$. Now consider an economy $e' = [(R_j,\omega_j)_{j \in \{i,k\}},0]$ in which in addition to our friend i, there is an agent k with the same preferences and a large enough endowment ω_k such that $[\omega_i+t,\omega_k-t]$ is an allocation. Since e' is a closed economy and $W(e') \neq \emptyset$, nonemptiness implies that $f(e') \neq \emptyset$. Now since $[\omega_i+t,\omega_k-t]$ is the unique, up to neutrality, envy free allocation of this economy, it follows that $[\omega_i+t,\omega_k-t] \in f(e')$. As f is consistent $\omega_i+t \in f(e)$. Now consider the case where $tp \neq 0$. We have to show that $\omega_i+t \notin f(e)$. Assume that $\omega_i+t \in f(e)$. Recall our friend k from the previous argument, and assume without loss of generality that $\omega_k+t \in \mathbb{R}_{++}^k$. Consider the economy $e'' = [(R_j,\omega_j)_{j \in \{i,k\}},t]$. It follows from the above argument that $\omega_k \in f([(R_k,\omega_k),0])$. By assumption $\omega_i+t \in f(e)$. As $[\omega_i+t,\omega_k]$ is Pareto optimal in e'', and f is conversely consistent it follows that $[\omega_i+t,\omega_k] \in f(e'')$. Note, however, that this allocation is not envy free. A contradiction. ☺

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Step 2: Let Q be a set with at least two elements. If a solution f on $\Omega(Q,T^*)$ satisfies nonemptiness, neutrality, consistency, converse consistency and assigns Pareto optimal and envy free allocations then for all one agent economies e, $f(e) = W(e)$.

Proof: First we show that $f(e) \subset W(e)$. Let $e = [(R_i,\omega_i),t]$ be an economy with one agent, and assume $\omega_i+t \in f(e)$. Let p be the unique supporting price at ω_i+t. Consider an agent k with linear preferences and indifference curves with slope p, and an endowment $\omega_k \geq \omega_i$. Now by Step 1 for all trades $z \in \{z \mid zp=0 \ \omega_i+z \in \mathbf{R}^k_{++}\}$, $\omega_k+z \in f([(R_k,\omega_k),z]$. As $[\omega_i+t,\omega_k+z]$ is Pareto optimal in $e'' = [(R_j,\omega_j)_{j\in\{i,k\}},t+z]$ and f is conversely consistent we have $[\omega_i+t,\omega_k+z] \in f(e'')$. By envy freeness, we get $P_i(\omega_i+t) \cap \{\omega_i+z \mid zp=0 \ \omega_i+z \in \mathbf{R}^k_{++}\}$ $= \varnothing$, it follows by weak monotonicity of preferences that $P_i(\omega_i+t) \cap \{\omega_i+z \mid zp\leq 0$ $\omega_i+z \in \mathbf{R}^k_{++}\} = \varnothing$ as well, and therefore, by local non satiation, $tp\geq 0$. By envy freeness applied to k we get $tp\leq 0$, thus $tp=0$, and $\omega_i+t \in W(e)$. Now let $e = [(R_i,\omega_i),t]$ be an economy with one agent, and assume $\omega_i+t \in W(e)$. Recall our above friend k and assume without loss of generality that $\omega_k-t \in \mathbf{R}^k_{++}$. Now consider the economy $e''=[(R_j,\omega_j)_{j\in\{i,k\}},0]$. Note that $[\omega_i+t,\omega_k+z]$ is a Walrasian allocation of e''. Thus by nonemptiness $f(e'')\neq\varnothing$. By the previous argument and Lemma 2 $f(e) \subset W(e)$ for all e, and as $[\omega_i+t,\omega_k+z]$ is the unique, up to neutrality, Walrasian allocation of e'', $[\omega_i+t,\omega_k+z] \in f(e'')$. By consistency applied to the set $\{i\}$ the proof of this step is completed. ☺

It follows from Step 2 and Lemmas 2 and 3 that if f satisfies the axioms then for all economies e $f(e) = W(e)$. ☺

Proof of Theorem 2: We will not show that W and $\bar{W}$ satisfy the axioms.

Step 1: Let Q be a set with at least two elements. If a solution f on $\Omega(Q,T^*)$ satisfies

nonemptiness, neutrality, consistency, converse consistency, and core selection then for all one agent economies e with linear preferences $W(e) \subset f(e)$.

Proof: Let $e = [(R_i,\omega_i)_{i\in N}, t]$ be an economy with one agent and R_i is a linear preference with indifference surfaces with slope p. If the economy has no allocations the statment is trivially satisfied. So assume that the economy has an allocation. If $tp \neq 0$ $W(e) = \varnothing$, and the statement is trivially satisfied. Now, consider the case where $tp=0$. Consider an economy $e' = [(R_j,\omega_j)_{j\in\{i,k\}}, 0]$ in which in addition to our friend i, there is an agent k with the same preferences and a large enough endowment ω_k such that $[\omega_i+t, \omega_k-t]$ is an allocation. Since e' is a closed economy and $W(e') \neq \varnothing$, nonemptiness implies that $f(e') \neq \varnothing$. Now since $[\omega_i+t, \omega_k-t]$ is the unique, up to neutrality, core allocation of this economy, it follows that $[\omega_i+t, \omega_k-t] \in f(e')$. As f is consistent $\omega_i+t \in f(e)$. ☺

Step 2: Let Q be a set with at least two elements. If a solution f on $\Omega(Q,T^*)$ satisfies nonemptiness, neutrality, consistency, converse consistency, and core selection then for all one agent economies e $f(e) \subset \tilde{W}(e)$.

Proof: Assume by contradiction, that for some one agent economy $e = [(R_i,\omega_i),t]$ $f(e) \not\subset \tilde{W}(e)$. Let p be the unique supporting price at ω_i+t, and $s=tp$. By assumption $s<0$. Clearly there exists a bundle y that satisfies: $\omega_i+t < py < p\omega_i$ and $yP_i(\omega_i+t)$. Let $t'=y-\omega_i$. Consider an agent k with linear preferences and indifference surfaces with slope p, and an endowment ω_k such that $\omega_k-t' \in \mathbb{R}^k_{++}$. Note that $p(\omega_k-t') > p\omega_k$, thus, $(\omega_k-t')P_k\omega_k$. By Step 1, $\omega_k \in f([(R_k,\omega_k),0])$, and by assumption $\omega_i+t \in f(e)$. Note that $[\omega_i+t, \omega_k]$ is a Pareto optimal allocation in $[(R_j,\omega_j)_{j\in\{i,k\}}, t]$, thus by converse consistency $[\omega_i+t, \omega_k] \in f([(X_j,R_j,\omega_j)_{j\in\{i,k\}}, t])$. Note that this allocation is not a core allocation, as the coalition of both agents can improve

upon it with the sum of their own endowments, i.e. by the assignment $[y,\omega_k\text{-}t']$. A contradiction. ☺

Step 3: Let Q be a set with at least two elements. If a solution f on $\Omega(Q,T^*)$ satisfies nonemptiness, neutrality, consistency, converse consistency, and core selection then for all one agent economies e $W(e) \subset f(e)$.

Proof: Let $e = [(R_i,\omega_i),t]$ be an economy with one agent, and $\omega_i+t \in W(e)$. Let p be the unique equilibrium price associated to this Walrasian allocation. Consider a closed economy e' with the agent i, and another agent k with linear preferences with indifference surfaces with slope p, such that $\omega_k\text{-}t \in \mathbf{R}^k_{++}$. Clearly $[\omega_i+t,\omega_k\text{-}t] \in W(e')$, thus by nonemptiness $f(e') \neq \varnothing$. By Step 2 and Lemma 3.1, $f(e') \subset \bar{W}(e')$. Since e' is closed $f(e') \subset W(e')$. As $[\omega_i+t,\omega_k\text{-}t]$ is the unique, up to neutrality, Walrasian allocation of e' we have $[\omega_i+t,\omega_k\text{-}t] \in f(e')$ and by consistency, $\omega_i+t \in f(e)$. ☺

It follows from Steps 2 and 3, and Lemmas 2 and 4 that if a solution satisfies the axioms then for all economies e $W(e) \subset f(e) \subset \bar{W}(e)$. ☺

For the next lemma we need another definition.

A solution f on Ω satisfies *stability under juxtaposition* if for all economies $e = [(R_i,\omega_i)_{i \in N},t]$ and $e' = [(R_i,\omega_i)_{i \in M},t']$, if $N \cap M = \varnothing$, $x \in f(e)$, $x' \in f(e')$ and (x,x') is Pareto optimal in $e'' = [(R_i,\omega_i)_{i \in N \cup M}, t+t']$, then $(x,x') \in f(e'')$.

Lemma 5: Let $Q \neq \varnothing$, and $T \subset T^*$. If a solution f on $\Omega(Q,T)$ is consistent, conversely consistent and assigns Pareto optimal allocations then f satisfies stability under juxtaposition.

Proof: Let $e = [(R_i,\omega_i)_{i \in N},t]$ and $e' = [(R_i,\omega_i)_{i \in M},t']$ be economies that satisfy $N \cap M = \emptyset$, $x \in f(e)$, $x' \in f(e')$ and (x,x') is Pareto optimal in $e'' = [(R_i,\omega_i)_{i \in N \cup M},t+t']$. By consistency $x_i \in f(e^{(i),x})$ for all $i \in N$ and $x_i' \in f(e^{(i),x'})$ for all $i \in M$. As (x,x') is Pareto optimal in e'' it follows from converse consistency of f and Lemma 1 that $(x,x') \in f(e'')$. ◎

For the next lemma we need some more definitions.

A solution f on Ω is *replica invariant* if for all economies $e = [(R_i,\omega_i)_{i \in N},t]$ in Ω, and for all $M = \{1,2,\dots,m\}$, for all $(x_i)_{i \in N} \in f(e)$, if e' is equivalent to $e(m)$ and $e' \in \Omega$, then $\sigma(x_{ij})_{ij \in N \times M} \in f(e')$.

Lemma 6: Let $Q \neq \emptyset$, and $T \subset T^*$. If a solution f on $\Omega(Q,T)$ satisfies stability under juxtaposition, anonymity, and assigns Pareto optimal allocations then it satisfies replica invariance.

Proof: Trivial. ◎

The proof of Theorem 3 makes use of definitions and results stated and proved in Section 6.

Proof of Theorem 3: Let T be a nonempty subset of T^* that is closed under improvements. First we show that if a solution f on $\Omega(N,T)$ satisfies nonemptiness, consistency, converse consistency, anonymity, and core selection then for all economies, for all $x \in f(e)$ and for all n, $x \in C(e_x(n))$. Let $e = [(R_i,\omega_i)_{i \in N},t]$ be an economy, and let $x \in f(e)$. As $x \in Core(e)$ we have $e' = [(R_i,x_i)_{i \in N},0] \in \Omega(N,T)$. Since $x \in W(e')$, by nonemptiness $f(e') \neq \emptyset$. As x is the unique, up to neutrality core allocation of e' we have $x \in f(e')$. Let e'' be a juxtaposition of

e and an economy equivalent to *e'*. Note that *e''* is an adjoint of *e* with respect to x. By anonymity and Lemma 5 the adjoint of x is in *f(e'')*; Now by Lemma 6, core selection and Theorem 4 we get the required result. ☺

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