

MONOTONE STOCHASTIC CHOICE MODELS: THE CASE OF RISK AND TIME PREFERENCES*

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ABSTRACT. Suppose that, when evaluating two alternatives x and y by means of a parametric utility function, low values of the parameter indicate a preference for x and high values indicate a preference for y . We say that a stochastic choice model is monotone whenever the probability of choosing x is decreasing in the preference parameter. We show that the standard use of random utility models in the context of risk and time preferences may sharply violate this monotonicity property, and argue that their use in preference estimation may be problematic. In particular, they may pose identification problems and yield biased estimations. We then establish that the alternative random parameter models, in contrast, are always monotone. We show in an empirical application that standard risk-aversion assessments may be severely biased.

Keywords: Stochastic Choice; Preference Parameters; Random Utility Models; Random Parameter Models; Risk Aversion; Delay Aversion.

JEL classification numbers: C25; D81.

1. INTRODUCTION

Consider gamble x , which gives \$1 with probability .9 and \$60 with probability .1, and gamble y which gives \$5 for sure. Let these gambles be evaluated by constant relative risk aversion (CRRA) expected utilities, that is, $U_{\omega}^{crra}(x) = .1\frac{1^{1-\omega}}{1-\omega} + .9\frac{60^{1-\omega}}{1-\omega}$ and $U_{\omega}^{crra}(y) = \frac{5^{1-\omega}}{1-\omega}$. Notice that gamble x is riskier than y , since it is only for

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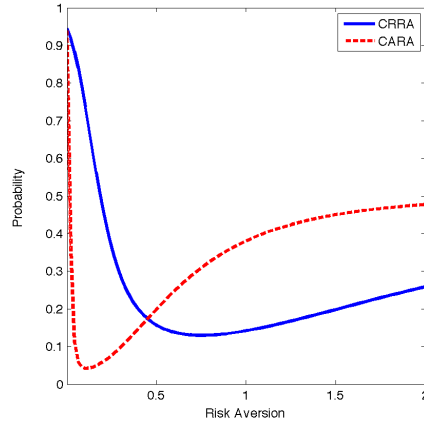


FIGURE 1.—RUM probabilities of choosing gamble x over y , for CRRA and CARA expected utilities.

low levels of risk aversion ω that option x is preferred to y .¹ The most standard approach to stochastic choice modeling is to adopt the logit random utility model (logit RUM), which, in this case, implies that the probability of choosing x over y equals $\frac{e^{U_{\omega}^{crra}(x)}}{e^{U_{\omega}^{crra}(x)} + e^{U_{\omega}^{crra}(y)}}$. Clearly, the logit RUM should be consistent with the underlying utility representation, and hence fulfil the following monotonicity condition: lower levels of risk aversion ω must be associated with higher probabilities of choosing gamble x . Figure 1 shows that this is unfortunately not the case.² There is a large range of risk-aversion parameters for which the probability of choosing the riskier gamble x is increasing with the level of risk aversion. The existence of this anomaly makes this logit RUM theoretically flawed and, presumably, problematic for use in the estimation of risk preferences.

In this paper, we address a number of issues surrounding the problem described in the previous paragraph. First, we precisely characterize the type of gambles and classes of RUMs that are problematic. Second, given a violation of the monotonicity property, we portray the exact structure of the problem. That is, we describe the range of parameters for which the RUM is non-monotone. Third, again given a violation of the monotonicity property, we ascertain whether the range of parameters for which the problem arises has any economic significance. Fourth, we show that the non-monotonicity problem is not restricted to risk preferences, but also affects other important preference dimensions, such as time preferences. Fifth, we show that an alternative random choice model,

¹Specifically, for every risk-aversion level below .19, gamble x is preferred to y , and vice-versa.

²The figure also reports the RUM probability of choosing x for constant absolute risk aversion (CARA) expected utilities. All the formal definitions are given in Section 3.

the random parameter model (RPM), is free from these problems, and hence is always monotone. Finally, we illustrate the practical importance of these results by using actual choice data to evaluate differences in estimates from standard RUMs and RPMs.

We now elaborate on our results in more detail, starting with the type of RUMs and gambles that are problematic. RUMs are typically constructed by introducing additive i.i.d. random shocks on the utility evaluation of the alternatives, which is usually assumed to be given by a CRRA or CARA expected utility functions. We say that one gamble x is riskier than another gamble y if low values of the risk-aversion parameter indicate a preference for x and high values indicate a preference for y . Notice that this definition encompasses the standard definitions in the literature for one gamble being more risk averse than another. We show in Corollary 1 that *for every* pair of gambles where one is riskier than the other, *every* i.i.d. RUM using either CRRA or CARA utilities, and not the logit RUM exclusively, violates monotonicity, thus showing that the problem is ubiquitous.

We then turn to analyze the structure of the anomaly. Proposition 2 shows that, for every such RUM and every such pair of gambles, there is always a level of risk aversion beyond which the probability of choosing the riskier gamble increases. The practical implications of this internal inconsistency in the empirical estimation of risk aversion are apparent. First, there is an identification problem arising from the fact that the same choice probabilities may be associated to two different levels of risk aversion. Second, there is an upper limit to the level of risk aversion that can be estimated when using maximum likelihood techniques, even for extremely risk-averse individuals.

The question arises whether the range of risk-aversion levels for which the RUM is non-monotone has any economic relevance, or only involves risk-aversion parameters that are too high to be of practical importance. The example in Figure 1 already suggests that the problematic range of parameters may be of first order relevance when low payoffs are involved, since the probability of choosing the riskier gamble starts increasing at reasonable risk-aversion levels of .75 in the CRRA case, and .11 in the CARA case. Notably, Proposition 3 shows that the problem is worsened by increasing the payoffs involved in the gambles. As the payoffs increase, the level of risk aversion at which the RUM becomes non-monotone eventually converges to that at which the two original gambles become indifferent.³ This implies that any individual

³As described above, for the gambles in the example this value is as low as .19 for the CRRA case, and .02 for the CARA case.

who is more likely to choose the safer gamble over the riskier one cannot be assigned an estimated level of risk aversion higher than the one at which the two gambles become indifferent. Finally, using standard experimental pairs of gambles, we also illustrate that the problematic range of parameters may be very large.

For the extension of the non-monotonicity results to other key preferences, first consider the following two streams of payoffs. Stream x , the longer-delay stream, provides \$15,000 yearly, except in period 10, where it provides \$21,000. Stream y , the shorter-delay stream, provides the same yearly \$15,000, except in period 5, where it provides \$20,000. Assume, for the moment, risk neutrality and power discounted utility, leading to $U_{\omega}^{pow}(x) = \sum_{t \neq 10} \frac{1}{(1+\omega)^t} 15,000 + \frac{1}{(1+\omega)^{10}} 21,000$ and $U_{\omega}^{pow}(y) = \sum_{t \neq 5} \frac{1}{(1+\omega)^t} 15,000 + \frac{1}{(1+\omega)^5} 20,000$. Clearly, the two options can be ordered in terms of delay aversion, since the higher the value of the delay-aversion parameter ω , the less attractive stream x becomes.⁴ Figure 2 represents the logit RUM probability of selecting x over y , dependent upon the delay-aversion parameter.⁵ It is apparent that exactly the same anomaly emerges. The probability of selecting the longer-delay stream x should decrease with the level of delay aversion, but fails to do so for levels above .19. Corollary 2 and Proposition 4 basically reproduce the situation described in Corollary 1 and Proposition 2 for the case of risk preferences. Namely, under the standard discount functions, that is, the power, the $\beta - \delta$, or the hyperbolic discount functions, for every i.i.d. RUM, with any increasing and continuous utility function over monetary payoffs, and for basically every pair of streams that can be ordered in terms of delay aversion, the probabilities are non-monotone. Moreover, the probability of choosing the longer-delay stream starts to increase beyond a certain level of delay aversion, and hence, in principle, the above two estimation problems apply here also. There is an important difference with respect to the practical relevance of the results. Although there are pairs of streams for which the problematic range of delay-aversion parameters has economic relevance, as shown in Figure 2, we argue in Section 4.2 that the practical relevance is more limited. In particular, there is a need to consider markedly distant payoffs, which are not typical in the streams considered in the experimental literature, for example. In fact, using a set of such streams we illustrate how the critical delay-aversion levels at which the problematic range starts can be absurdly high.

⁴Every delay aversion below .04 prefers stream x over y , and vice-versa.

⁵Figure 2 also reports the probabilities of choosing x when using the $\beta - \delta$ or the hyperbolic discount functions.

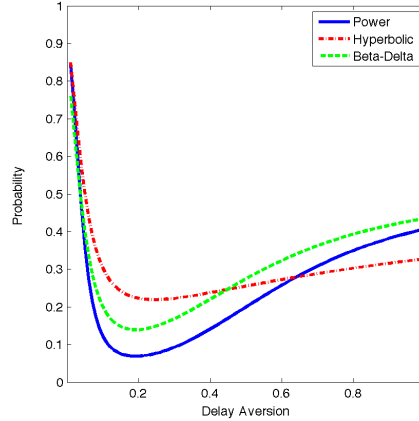


FIGURE 2.—RUM probabilities of choosing stream x over stream y , for the power, hyperbolic and $\beta - \delta$ discounted utilities.

Continuing with the extension of the results to other settings, we provide general results for any i.i.d. RUM contingent on any preference parameter, not just those representing risk or time preferences. Proposition 1 provides a necessary and sufficient condition for monotonicity, and another easy-to-check necessary condition based on the limiting behavior of the utility functionals. These conditions offer the analyst simple tools for elucidating the appropriateness of a particular RUM in a particular context.

We then show that an alternative stochastic choice model, the random parameter model (RPM), is free of the problems just described. In an RPM, the choice probabilities are obtained by introducing random shocks on the preference parameter, rather than on the utility evaluations of the alternatives. As a consequence, the choice probabilities are given by the mass of shocks for which the utility function ranks one option higher than the other. Crucially for our present purposes, when a pair of alternatives can be ordered in terms of the preference parameter under consideration, be it risk, time or any other, we establish that this mass of shocks is monotone in the preference parameter and hence, the RPM is monotone. Thus, RPMs offer a safe method for the treatment of stochastic choice contingent on a preference parameter. This can be appreciated in Figure 3, where we have plotted the RPM choice probabilities for the gambles (Figure 3a) and streams (Figure 3b) considered in Figures 1 and 2, using exactly the same utility families. Figure 3a uses the logistic distribution while, since we have assumed that the discount factor takes values in the positive reals, Figure 3b uses the log-logistic distribution.

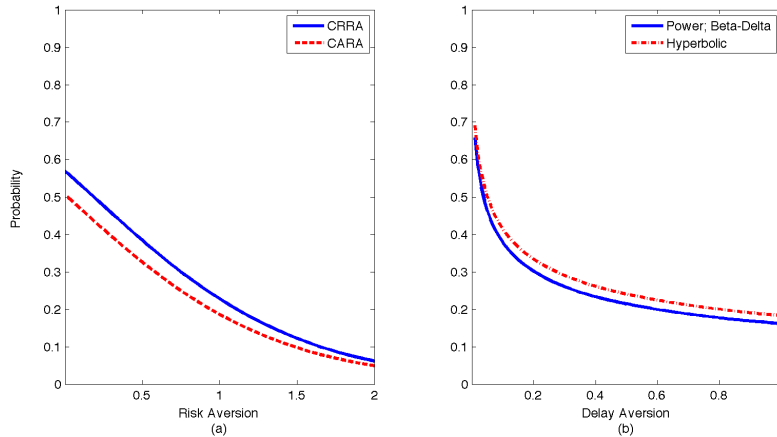


FIGURE 3.—RPM probabilities of choosing (a) gamble x over gamble y and (b) stream x over stream y .

We then turn to an empirical illustration of the established theoretical results in which the two stochastic choice models under scrutiny are used to obtain separate risk and time preference estimates based on the experimental data of Andersen et al. (2008). In our empirical analysis of risk preferences we show that, consistent with our theoretical results, the standard RUM significantly underestimates the population risk-aversion level. When considering the full sample of 253 subjects, the RUM gives a CRRA risk-aversion level of .66, while that given by the RPM is .75, which is about 15% higher. Our theoretical results further indicate that the severity of the estimation bias associated with the RUM increases with more risk-averse individuals, which is fully consistent with our results. Taking a sample of the most risk averse subjects, the RUM risk-aversion estimate is 1.46, while the RPM estimate is 1.87, which is about 30% higher. We consider these results a clear indication of the importance of making the right choice of random model for the estimation of risk preferences. The results of the estimation of time preferences are, again, fully in line with our theoretical results. Both, the RUM and the RPM delay aversion estimates are practically identical. For risk neutrality and power discounted utility these are 0.27 and 0.26, respectively.

We close this introduction by reviewing the closest study to our own, Wilcox (2008, 2011). Wilcox first shows that, in our language, RUMs may be non-monotone. His setting is very particular in that it involves only risk preferences, focuses exclusively on three-outcome gambles related by mean-preserving spreads, and uses the logit RUM. Analysts may therefore be unconvinced of the scope and relevance of the result, and continue using RUMs for the sake of convenience. In contrast to Wilcox, this paper establishes general theoretical findings that show the problem to be pervasive. Starting

with the context of risk preferences, we show that, not only the logit, but basically *every* i.i.d RUM is non-monotone for *every* pair of gambles ordered by risk aversion, not only three-outcome gambles related by mean-preserving spreads, in a range of parameters which we characterize and show to be of practical importance. This implies that there is no way around this inconsistency. Importantly, we also show that this problem extends to other key preference parameters, such as time preferences. We also establish the conditions guaranteeing the monotonicity of RUMs based on any preference parameter. These conditions are easy to check and hence practical when contemplating the implementation of a random utility model. Wilcox further proposes the use of a novel model, contextual utility, which is monotone for his particular choice of gambles. We discuss contextual utility in Appendix A.1, showing that it does not, alas, solve the problem for the case of gambles involving more than three possible outcomes. Notably, we provide a general and easily-implementable solution to this problem: the use of RPMs. We show that these models are *always* monotone and therefore safe for use in applications.

The rest of the paper is organized as follows. Section 2 reviews the remaining relevant literature. Section 3 lays down the basic definitions. Section 4 is devoted to the study of RUMs, and Section 5 to that of RPMs. Section 6 contains the empirical application, and Section 7 presents the conclusions. Several extensions of the theoretical and empirical parts are reported in the Appendix.

2. RELATED LITERATURE

Thurstone (1927) and Luce (1959) are two of the first key contributions from mathematical psychology to stochastic choice theory. Recent stochastic choice models that have appeared in the theoretical literature are Gul, Natenzon and Pesendorfer (2014), Manzini and Mariotti (2014) and Caplin and Dean (2015). Discrete choice models in general settings are surveyed in McFadden (2001). See also Train (2009) for a detailed textbook introduction.

For theoretical papers recommending the use of random utility models in risk settings, see Becker, DeGroot and Marschak (1963) and Busemeyer and Townsend (1993). Wilcox (2008, 2011), as reviewed in the Introduction, criticizes the use of these models in risky settings. In addition, Blavatsky (2011) shows that there is always one comparison of gambles, where the safe gamble is degenerate, for which random utility models based on expected utility differences are non-monotone. The literature using random

utility models in the estimation of risk aversion is immense, and certainly too large to be exhaustively cited here. We therefore cite only a few of the most influential pieces of work, such as Friedman (1974), Cicchetti and Dubin (1994), Hey and Orme (1994), Holt and Laury (2002), Harrison, List and Towe (2007), Andersen et al. (2008), Post et al. (2008), von Gaudecker, van Soest and Wengstrom (2011), Toubia et al. (2013), and Noussair, Trautmann and van de Kuilen (2014). Our results for risk preferences immediately extend to situations where strategic uncertainty causes the individual to replace objective probabilities with beliefs. A prominent example of this approach in game theory is the quantal response equilibrium of McKelvey and Palfrey (1995), which assumes a random utility model using (subjective) expected utility. Hence, for given beliefs, our results show that there is a level of risk aversion beyond which more risk averse individuals may have a higher probability of choosing the riskier action. The random utility model is also the most commonly used approach in the estimation of time preferences. See, e.g., Andersen et al. (2008), Chabris et al. (2008), Ida and Goto (2009), Tanaka, Camerer and Nguyen (2010), Toubia et al. (2013), and Meier and Sprenger (2015).

Starting with the seminal papers by Wolpin (1984) and Rust (1987), dynamic discrete choice models have been used to tackle issues such as fertility (Ahn, 1995), health (Gilleskie, 1998; Crawford and Shum, 2005), labor (Berkovec and Stern, 1991; Rust and Phelan, 1997), or political economy (Diermeier, Keane and Merlo, 2005). Our results may be relevant for this literature, for two reasons. The first is that some of these settings involve risk and are modeled by means of random utility models with errors over expected utility. The second is the dynamic nature of the setting, which makes our results with respect to time preferences also relevant.

Finally, the use of random parameter models in settings involving gambles has been theoretically discussed in Eliashberg and Hauser (1985), Loomes and Sugden (1995), and Gul and Pesendorfer (2006). For papers using such models to estimate risk aversion, see Barsky et al. (1997), Fullenkamp, Tenorio and Battalio (2003), Cohen and Einav (2007), and Kimball, Sahm and Shapiro (2008, 2009). Coller and Williams (1999) and Warner and Pleeter (2001) are two examples of the use of this approach in the context of time preferences. Our results guarantee that one can be confident that the use of random parameter models for the estimation of preference parameters is free from the kind of internal inconsistencies studied in this paper.

3. PRELIMINARIES

Let X be a set of alternatives and consider a collection of utility functions $\{U_\omega\}_{\omega \in \Omega}$ defined on X . Ω represents the space of possible values of a given preference parameter, which consists of the set of all real numbers, unless otherwise explicitly stated. The preference parameter represents the aversion to choose some alternatives over others. For instance, higher values of ω may represent greater risk aversion or delay aversion causing the individual to be less inclined towards riskier gambles or monetary streams involving more distant payoffs.

Some pairs of alternatives (x, y) are clearly ordered with respect to the preference parameter. Formally, we say that x and y are Ω -ordered whenever $U_{\omega^L}(y) > U_{\omega^L}(x)$ implies that $U_{\omega^H}(y) > U_{\omega^H}(x)$, for every $\omega^L, \omega^H \in \Omega$ such that $\omega^L < \omega^H$. That is, x and y are Ω -ordered if, when the low-type ω^L prefers alternative y over x , so does the high-type ω^H . In other words, option x generates more aversion than y and hence can only be chosen by individuals with low aversion, that is, with low values of the preference parameter ω .⁶ The notion of Ω -ordered pairs of alternatives is natural and applicable to the key comparisons in the different settings, as the following two examples show. In a risk context, a gamble x involving risk is riskier than the degenerate gamble y which gives the expected payoff of x with certainty. It is immediate that, if $\{U_\omega\}_{\omega \in \Omega}$ is a collection of expected utility functions ordered by the Arrow-Pratt coefficient, x and y are Ω -ordered. Similarly, in a context of temporal decision-making, a payoff stream x with a later bonus payout is more delayed than a stream y with an earlier bonus payout. Clearly, if $\{U_\omega\}_{\omega \in \Omega}$ is a collection of standard discounted utility functions, x and y are again Ω -ordered. Sections 4.1 and 4.2 further illustrate the generality of the definition of Ω -ordered pairs.

We now introduce the two main stochastic choice models used in the literature. In the most standard *random utility model (RUM)*, a random error is introduced in the cardinal evaluation of alternatives. Namely, the individual chooses the alternative that provides maximal utility, which is assumed to be additively composed by two terms: (i) the representative utility, $U_\omega(x)$, based on the characteristics of the alternative x and the relevant preference attribute ω and, (ii) a random i.i.d. term, $\epsilon(x)$, that

⁶Notice that the definition of Ω -ordered pairs of alternatives is related to the influential single-crossing condition of Milgrom and Shannon (1994). Basically, a pair of alternatives (x, y) is Ω -ordered if the collection of utilities $\{U_\omega\}_{\omega \in \Omega}$ satisfies the single-crossing condition with respect to (x, y) . For a discussion on the consideration of more than two alternatives see Section A.3.

follows a continuous cumulative distribution Ψ on \mathbb{R} . Given a pair of alternatives (x, y) , the probability assigned to choosing x , $\rho_\omega^{rum}(x, y)$, is given by the probability that $U_\omega(x) + \epsilon(x)$ is greater than $U_\omega(y) + \epsilon(y)$. By far the most widely-used error distributions are the type I extreme value and the normal, which lead to the logit model and the probit model, respectively. The former, also known as the Luce model, has closed-form probability of choosing x over y equal to $\frac{e^{\lambda U_\omega(x)}}{e^{\lambda U_\omega(x)} + e^{\lambda U_\omega(y)}}$, where λ is a precision parameter.⁷

In a *random parameter model (RPM)*, the random error distorts the agent's preference parameter.⁸ Hence, the agent opts for the alternative that maximizes $U_{\omega+\epsilon}$, where the random error ϵ on the preference parameter follows a continuous cumulative distribution Ψ on Ω . Then, given a pair of alternatives (x, y) , the RPM probability of choosing x over y , $\rho_\omega^{rpm}(x, y)$, is simply the probability mass of realizations ϵ from Ψ such that $U_{\omega+\epsilon}(x)$ is greater than $U_{\omega+\epsilon}(y)$. Notice that in the case of Ω -ordered pairs, this probability mass is 1 whenever x is preferred to y for every value of ω , and 0 whenever y is preferred to x for every value of ω . Otherwise, denoting by $\omega^{(x,y)}$ the value such that $U_{\omega^{(x,y)}}(x) = U_{\omega^{(x,y)}}(y)$, this probability is $\Psi(\omega^{(x,y)} - \omega)$.⁹ To illustrate, let Ψ be the logistic distribution, and suppose that (x, y) is Ω -ordered. Then, the closed-form probability of selecting x over y is $\frac{e^{\lambda \omega^{(x,y)}}}{e^{\lambda \omega^{(x,y)}} + e^{\lambda \omega}}$.

Notice that the main difference between the two stochastic choice models lies in where the disturbance occurs, which has the following implications. First, in the case of the RUM, the error distorts the utility evaluation of each alternative independently, while, in the case of the RPM, the error distorts the preference parameter, thereby implying that the evaluation of the alternatives is not distorted independently. Second, in the case of the RUM, the distortion of the utility function leads to it not necessarily belonging to the family $\{U_\omega\}_{\omega \in \Omega}$, while in the case of the RPM, by construction, the utility is transformed into another utility within the same family.

We are now in a position to introduce the main notion in this paper. We say that the stochastic model ρ^{rum} (respectively, ρ^{rpm}) is *monotone for the Ω -ordered pair (x, y)* , whenever $\rho_\omega^{rum}(x, y)$ (respectively, $\rho_\omega^{rpm}(x, y)$) is decreasing in ω . That is, the larger the value of the parameter ω , the lower the probability of choosing alternative x from the

⁷Parameter λ is inversely related to the variance of the initial distribution Ψ and is typically interpreted as a rationality parameter. The larger λ , the more rational the individual.

⁸There is no consensus in the literature as for the denomination of these models. Some authors refer to them as random preference models, random utility functions, or random utility models.

⁹For convenience, we assume that there is at most one such value $\omega^{(x,y)}$.

Ω -ordered pair (x, y) , which captures the aforementioned aversion. This is a minimal condition for the internal consistency of the stochastic model and for accurate empirical estimation of the preference parameter.

4. RANDOM UTILITY MODELS

The following proposition specifies simple conditions to check whether a RUM is monotone for the Ω -ordered pair (x, y) . It first establishes a sufficient and necessary condition, which is based on the fact that RUM probabilities depend on the utility differences between the alternatives. It then provides an even simpler necessary condition based on the limiting behavior of utilities.

Proposition 1. *Let (x, y) be an Ω -ordered pair. Then:*

- (1) ρ^{rum} is monotone for (x, y) if and only if the function $U_\omega(x) - U_\omega(y)$ is decreasing in ω .
- (2) If $\lim_{\omega \rightarrow \infty} [U_\omega(x) - U_\omega(y)] = 0$ and there exists $\omega^* \in \Omega$ such that $U_{\omega^*}(y) > U_{\omega^*}(x)$, then ρ^{rum} is non-monotone for (x, y) .

Proof of Proposition 1. Consider a RUM and an Ω -ordered pair (x, y) . Notice that the probability of $U_\omega(x) + \epsilon(x)$ being greater than $U_\omega(y) + \epsilon(y)$ can be expressed as the probability of $\epsilon(y) - \epsilon(x)$ being smaller than $U_\omega(x) - U_\omega(y)$. Hence, denoting by Ψ^* the cumulative distribution of the difference between two i.i.d. error terms with distribution Ψ , it is the case that $\rho_\omega^{rum}(x, y) = \Psi^*(U_\omega(x) - U_\omega(y))$. Since Ψ^* is a continuous cumulative distribution, $\rho_\omega^{rum}(x, y)$ is decreasing in ω if and only if $U_\omega(x) - U_\omega(y)$ is decreasing in ω , which proves the first part.

For the second part, suppose that there exists ω^* such that $U_{\omega^*}(y) > U_{\omega^*}(x)$. If the RUM is monotone for (x, y) , we know from the first part of the proposition that $U_\omega(x) - U_\omega(y)$ must be decreasing in ω . Clearly, therefore, either $\lim_{\omega \rightarrow \infty} [U_\omega(x) - U_\omega(y)]$ does not exist, or it must be the case that $\lim_{\omega \rightarrow \infty} [U_\omega(x) - U_\omega(y)] < 0$. In both cases we reach a contradiction, thus proving the result. ■

In the following sections, we show the relevance of these results in the context of risk and time preferences. In particular, the second part of Proposition 1 enables us to show immediately that most of the RUMs used in these contexts are non-monotone for basically every Ω -ordered pair of alternatives. Meanwhile, the first part of Proposition 1 allows us to exploit the functional structure of these models to obtain results strong enough to characterize the extent of the problem for every Ω -ordered pair.

4.1. Risk Preferences. A gamble $x = [x_1, \dots, x_N; p(x_1), \dots, p(x_N)]$ consists of a finite collection of monetary outcomes with $x_i \in \mathbb{R}_+$, and associated probabilities such that $p(x_i) > 0$ and $\sum_i p(x_i) = 1$. In most standard analysis, utility functions over gambles take the form of expected utility $U_\omega^{eu}(x) = \sum_i p(x_i) u_\omega(x_i)$, where u_ω is a monetary utility function that is strictly increasing and continuous in outcomes. The CARA and the CRRA families of monetary utility functions are by far the most widely-used specifications in real applications. The following are standard definitions. CARA utility functions are such that the utility of monetary outcome m is $u_\omega^{cara}(m) = \frac{1-e^{-\omega m}}{\omega}$ for $\omega \neq 0$, and $u_0^{cara}(m) = m$, while CRRA utility functions are defined by $u_\omega^{crra}(m) = \frac{m^{1-\omega}}{1-\omega}$ for $\omega \neq 1$, and $u_1^{crra}(m) = \log m$.¹⁰ We write U_ω^{cara} and U_ω^{crra} for the corresponding expected utilities, and $\rho^{rum(cara)}$ and $\rho^{rum(crra)}$ for the corresponding RUM choice probabilities.

We focus on the interesting case of Ω -ordered pairs of gambles that are not stochastic-dominance related. This implies that some types prefer x to y , while others prefer y to x . We now mention three examples of classes of Ω -ordered pairs of gambles often used in applications, which serve to illustrate the largeness of the class of Ω -ordered pairs of gambles. The standard textbook treatment of risk aversion uses pairs of gambles (x, y) where y involves no risk at all, i.e. $y = [y_1; 1]$. The monetary value y_1 is sometimes taken to be the expected value of x , but any y_1 in the interval $(\min\{x_1, \dots, x_N\}, \max\{x_1, \dots, x_N\})$ forms an Ω -ordered pair with x . Another widely-used comparison involves pairs (x, y) where x is a mean-preserving spread of y .¹¹ Finally, experimental studies often use simple nested pairs of gambles where $x = [x_1, x_2; p, 1 - p]$ and $y = [y_1, y_2; p, 1 - p]$, with $x_1 < y_1 < y_2 < x_2$ and $p \in (0, 1)$.¹² As in the first case, although the mean of x is sometimes assumed to be larger than that of y , no further assumptions are required for them to constitute an Ω -ordered pair of gambles.

We start the analysis by noting that Proposition 1 has immediate bite in this setting. Corollary 1 shows that for every Ω -ordered pair of gambles, both $\rho^{rum(cara)}$ and $\rho^{rum(crra)}$, are non-monotone.

¹⁰For ease of exposition, when dealing with CARA and CRRA we assume that $m \geq 1$.

¹¹Gamble x is a mean-preserving spread of gamble y through outcome y_{j^*} and gamble z , if x can be expressed as a compound gamble that replaces outcome y_{j^*} in gamble y with gamble z , which has y_{j^*} as its expected value. Then, x is a mean-preserving spread of y if there is a sequence of such spreads from y to x .

¹²See, e.g., the gambles used in the influential elicitation procedure of Holt and Laury (2002).

Corollary 1. *Let (x, y) be an Ω -ordered pair of gambles. Then, $\rho^{rum(cara)}$ and $\rho^{rum(crra)}$ are non-monotone for (x, y) .*

Proof of Corollary 1. The proof is an immediate implication of the second part of Proposition 1. Notice that $\lim_{\omega \rightarrow \infty} U_{\omega}^{cara}(x) = \lim_{\omega \rightarrow \infty} U_{\omega}^{cara}(y) = \lim_{\omega \rightarrow \infty} U_{\omega}^{crra}(x) = \lim_{\omega \rightarrow \infty} U_{\omega}^{crra}(y) = 0$. Furthermore, since gamble x does not stochastically dominate y , there are levels of risk aversion for CARA and CRRA utilities for which gamble y is preferred. Hence, the second part of Proposition 1 leads to the result. ■

Having established that $\rho^{rum(cara)}$ and $\rho^{rum(crra)}$ are problematic for every possible Ω -ordered pair of gambles, we now exploit the functional structure of U_{ω}^{cara} and U_{ω}^{crra} to characterize the nature of the problems. In the next result we show that, for every Ω -ordered pair of gambles (x, y) , there always exists a level of risk aversion $\bar{\omega}_{(x,y)}$ above which the probability of choosing the riskier gamble x is strictly increasing. For further emphasis, note the implication that higher levels of risk aversion are associated with greater probabilities of choosing the riskier gamble. This is an obvious lack of internal consistency with immediate practical implications for the estimation of the model. Firstly, there is an identification problem arising from the fact that different levels of risk aversion are compatible with the same probability of choice. Secondly, when using the standard maximum likelihood technique, there is an upper bound in the level of risk aversion that can be estimated, $\bar{\omega}_{(x,y)}$, which can, potentially, affect estimates of intensely risk averse individuals.

Proposition 2. *Let (x, y) be an Ω -ordered pair of gambles. Then, there exists $\bar{\omega}_{(x,y)}$ such that $\rho_{\omega}^{rum(cara)}(x, y)$ and $\rho_{\omega}^{rum(crra)}(x, y)$ are strictly increasing in ω whenever $\omega \geq \bar{\omega}_{(x,y)}$.*

Proof of Proposition 2. Consider an Ω -ordered pair of gambles (x, y) , with $x = [x_1, \dots, x_N; p(x_1), \dots, p(x_N)]$ and $y = [y_1, \dots, y_M; q(y_1), \dots, q(y_M)]$. With reasoning analogous to that used in Proposition 1, we need to show that there exists a risk-aversion level $\bar{\omega}_{(x,y)}$ such that the difference between the utility values of x and y is strictly increasing in values of ω above $\bar{\omega}_{(x,y)}$. Since, by assumption, x does not stochastically dominate y , the two gambles are different. Let \underline{m} be the minimum monetary payoff to which gambles x and y assign different probabilities. Since (x, y) is an Ω -ordered pair of gambles and x does not stochastically dominate y , it must be that $q(\underline{m}) < p(\underline{m})$. This is so because for sufficiently large values of ω , the utility evaluations of the gambles is determined by the first payoff where they differ, \underline{m} . We now prove

that we can consider, w.l.o.g., that $y_j > \underline{m} = \min\{x_i\}$ for all payoffs in gamble y . To see this, suppose that $q^* = \sum_{j:y_j \leq \underline{m}} q(y_j) > 0$. Since $q(\underline{m}) < p(\underline{m})$, the definition of \underline{m} guarantees that it is also the case that $q^* < 1$ and hence, we can express gamble y as a compound gamble that assigns probability q^* to gamble y' and probability $1 - q^*$ to gamble \hat{y} . Gamble y' contains payoffs in y that are below \underline{m} , with associated probability $q'(y_j) = \frac{q(y_j)}{q^*}$. Gamble \hat{y} contains payoffs in y that are strictly above \underline{m} , with associated probability $\hat{q}(y_j) = \frac{q(y_j)}{1 - q^*}$. We can also express x as a compound gamble that assigns probability q^* to gamble y' and probability $1 - q^*$ to gamble \hat{x} . Gamble \hat{x} contains all payoffs in x above \underline{m} , with associated probabilities $\hat{p}(\underline{m}) = \frac{p(\underline{m}) - q(\underline{m})}{1 - q^*}$ and $\hat{p}(x_i) = \frac{p(x_i)}{1 - q^*}$ whenever $x_i > \underline{m}$. By the additive nature of expected utility, we know that the utility difference between gambles x and y is proportional to the utility difference between gambles \hat{x} and \hat{y} , which proves the claim.

We start by considering the case of CARA and focus on $\omega \neq 0$, where the family is differentiable.¹³ In this domain, $\rho_\omega^{rum(cara)}(x, y)$ is strictly increasing in ω if and only if $\frac{\partial[U_\omega^{cara}(x) - U_\omega^{cara}(y)]}{\partial\omega} > 0$ which, by expected utility, is equivalent to $\sum p(x_i) \frac{\partial u_\omega^{cara}(x_i)}{\partial\omega} - \sum q(y_j) \frac{\partial u_\omega^{cara}(y_j)}{\partial\omega} > 0$. Since $-\frac{\partial u_\omega^{cara}(m)}{\partial\omega} = -\frac{e^{-\omega m}(1 + \omega m) - 1}{\omega^2}$ is a strictly increasing and continuous utility function over monetary outcomes, $\rho_\omega^{rum(cara)}(x, y)$ is strictly increasing in ω if and only if $V_\omega^{cara}(y) > V_\omega^{cara}(x)$, where V_ω^{cara} is the expected utility using $-\frac{\partial u_\omega^{cara}(m)}{\partial\omega}$. Denoting by $CE(x, V_\omega^{cara})$ and $CE(y, V_\omega^{cara})$ the certainty equivalents of V_ω^{cara} for gambles x and y , it follows that $\rho_\omega^{rum(cara)}(x, y)$ is strictly increasing in ω if and only if $CE(y, V_\omega^{cara}) > CE(x, V_\omega^{cara})$.¹⁴ Now, notice that the Arrow-Pratt coefficient of risk aversion for $-\frac{\partial u_\omega^{cara}(m)}{\partial\omega}$ is simply $\omega - \frac{1}{m}$.¹⁵ When ω grows, the Arrow-Pratt coefficient goes to infinity, thereby guaranteeing that $\lim_{\omega \rightarrow \infty} CE(x, V_\omega^{cara}) = \underline{m} < \min\{y_1, \dots, y_M\} = \lim_{\omega \rightarrow \infty} CE(y, V_\omega^{cara})$. Hence, we can find a value, which we denote by $\bar{\omega}_{(x,y)}$, such that for every $\omega \geq \bar{\omega}_{(x,y)}$, $CE(y, V_\omega^{cara}) > CE(x, V_\omega^{cara})$, which proves the result.

The proof of the CRRA case can be obtained analogously by considering that, for any $\omega \neq 1$, CRRA utility functions are differentiable, $-\frac{\partial u_\omega^{crra}(m)}{\partial\omega} = -\frac{m^{1-\omega}(1-(1-\omega)\log m)}{(1-\omega)^2}$ is a continuous and strictly monotone utility function over monetary outcomes, and

¹³Note that the discontinuity of the CARA family at this point is not relevant for the result.

¹⁴In general, the certainty equivalent of a gamble x for some utility function U , is the amount of money $CE(x, U)$ such that $U(x) = U([CE(x, U); 1])$.

¹⁵The coefficient has a strictly positive derivative with respect to ω and thus, from the classic result of Pratt (1964), it follows that the certainty equivalent of a non-degenerate gamble is strictly decreasing in ω .

the corresponding Arrow-Pratt coefficient is $\frac{\omega \log m - 1}{m \log m}$. ■

Using the differentiability of CARA and CRRA, the proof of Proposition 2 shows that the model is monotone if and only if gamble x has more expected utility than gamble y under the monetary utility function $-\frac{\partial u_\omega}{\partial \omega}$. It establishes that there is always a level of risk aversion, $\bar{\omega}_{(x,y)}$, beyond which this no longer holds, and hence the models are non-monotone for every Ω -ordered pair of gambles. The proof also helps to explain how the critical values, $\bar{\omega}_{(x,y)}$, vary with the pair of gambles involved. Consider, for instance, two Ω -ordered pairs of gambles, (x, y) and (x, y') , where y' first-order stochastically dominates y . Then, it is evident from the proof that $\bar{\omega}_{(x,y)} < \bar{\omega}_{(x,y')}$. Hence, the better the safer option, the wider the range of problems. In order to formally establish the influence of the magnitude of payoffs on the critical value $\bar{\omega}_{(x,y)}$, consider an Ω -ordered pair of gambles (x, y) , and let (x_{+t}, y_{+t}) , $(x_{\times t}, y_{\times t})$ and $(x_{\wedge t}, y_{\wedge t})$ denote the Ω -ordered pairs of gambles where all the payoffs in gambles x and y are increased by, multiplied by, and raised to the power of $t > 0$, respectively.

Proposition 3. *Let (x, y) be an Ω -ordered pair of gambles.*

- *CARA: (i) $\lim_{t \rightarrow \infty} \bar{\omega}_{(x_{+t}, y_{+t})} = \omega^{(x,y)}$, and (ii) for every $t > 0$, $\bar{\omega}_{(x_{\times t}, y_{\times t})} = \frac{\bar{\omega}_{(x,y)}}{t}$.*
- *CRRA: (i) $\lim_{t \rightarrow \infty} \bar{\omega}_{(x_{\times t}, y_{\times t})} = \omega^{(x,y)}$, and (ii) for every $t > 0$, $1 - \bar{\omega}_{(x_{\wedge t}, y_{\wedge t})} = \frac{1 - \bar{\omega}_{(x,y)}}{t}$.*

Proof of Proposition 3. Since the Arrow-Pratt coefficient of risk aversion for $-\frac{\partial u_\omega^{cara(m)}}{\partial \omega}$ is $\omega - \frac{1}{m}$, when the monetary outcomes grow, the Arrow-Pratt coefficient becomes as close to ω as desired. Then, $V_\omega^{cara}(x_{+t}) \geq V_\omega^{cara}(y_{+t})$ becomes essentially $U_\omega^{cara}(x_{+t}) \geq U_\omega^{cara}(y_{+t})$. Given that (x_{+t}, y_{+t}) is an Ω -ordered pair for U_ω^{cara} , the latter inequality holds if and only if $\omega \leq \omega^{(x_{+t}, y_{+t})}$. By noting that CARA utilities imply that $\omega^{(x_{+t}, y_{+t})} = \omega^{(x,y)}$, we show the first claim with respect to CARA. Now, given that $-\frac{\partial u_\omega^{cara(m)}}{\partial \omega} = -\frac{e^{-\omega m(1+\omega m)} - 1}{\omega^2}$, it is immediate that $V_\omega^{cara}(x) = V_\omega^{cara}(y)$ if and only if $V_{\frac{\omega}{t}}^{cara}(x_{\times t}) = V_{\frac{\omega}{t}}^{cara}(y_{\times t})$, and then, by Proposition 2, $\bar{\omega}_{(x_{\times t}, y_{\times t})} = \frac{\bar{\omega}_{(x,y)}}{t}$.

In the CRRA case, note that the relative Arrow-Pratt coefficient for $-\frac{\partial u_\omega^{crra(m)}}{\partial \omega}$ is $\omega - \frac{1}{\log m}$, and, using the same argument as that used in the case of CARA, we obtain that, as t grows, $\bar{\omega}_{(x_{\times t}, y_{\times t})}$ converges towards $\omega^{(x,y)}$. Finally, since $-\frac{\partial u_\omega^{crra(m)}}{\partial \omega} = -\frac{m^{1-\omega}(1-(1-\omega)\log m)}{(1-\omega)^2} = -\frac{e^{(1-\omega)\log m}(1-e^{(1-\omega)\log m})}{(1-\omega)^2}$, the same argument as that used in the CARA case leads to $1 - \bar{\omega}_{(x_{\wedge t}, y_{\wedge t})} = \frac{1 - \bar{\omega}_{(x,y)}}{t}$. ■

Proposition 3 is particularly disquieting, since it implies that, in an estimation exercise involving large payoffs, the more risk averse the individual, the greater the bias in the estimation of her risk-aversion level. Specifically, the first part of the result establishes that, as the monetary payoffs increase (in the case of CARA through the addition of a positive constant, and in the CRRA case through multiplication by a positive constant), the critical value $\bar{\omega}_{(x,y)}$ converges towards the risk-aversion level $\omega^{(x,y)}$, that makes the two original gambles indifferent. This means that every individual who shows a larger probability of choosing the safer gamble y , over the riskier gamble x , cannot be assigned an estimated risk-aversion level higher than $\omega^{(x,y)}$. In the second part of the results, it is shown that, under CARA, if the monetary payoffs are multiplied by a positive scalar, the critical value of risk aversion $\bar{\omega}_{(x,y)}$ diminishes, eventually converging to 0, implying that there is an upper limit of 0 in the risk-aversion level that can be estimated, no matter how risk averse the individual is. Under CRRA, if the monetary payoffs are raised to the power of a positive scalar, the critical risk-aversion value eventually converges to 1. This implies that when $\bar{\omega}_{(x,y)}$ is above (below) 1, the increasing (decreasing) of payoffs becomes more problematic, since the critical value diminishes.

It is worth stressing, however, that one does not need to use implausible payoffs to obtain small critical values $\bar{\omega}_{(x,y)}$. This can be immediately appreciated in Table 1, where we report the CRRA critical values for the 36 Ω -ordered pairs of gambles used in the risk experimental part of Andersen et al. (2008). All the corresponding CARA critical values are very close to 0, and hence omitted herein.

TABLE 1. Critical Risk Aversion Levels for the Pairs of Gambles of Andersen et al. (2008)

	$p = .1$	$p = .2$	$p = .3$	$p = .4$	$p = .5$	$p = .6$	$p = .7$	$p = .8$	$p = .9$	$p = 1$
Task 1	-1.578	-0.810	-0.348	-0.002	0.289	0.556	0.823	1.119	1.520	-
Task 2	-1.320	-0.586	-0.112	0.266	0.607	0.941	1.297	1.722	2.344	-
Task 3	-1.706	-0.877	-0.379	-0.005	0.310	0.600	0.891	1.216	1.658	-
Task 4	-0.611	-0.176	0.095	0.306	0.491	0.668	0.851	1.063	1.361	-

Note: The classes of pairs of gambles used in the four different tasks are, respectively, $([3850, 100; p, 1 - p], [2000, 1600; p, 1 - p])$, $([4000, 500; p, 1 - p], [2250, 1500; p, 1 - p])$, $([4000, 150; p, 1 - p], [2000, 1750; p, 1 - p])$, and $([4500, 50; p, 1 - p], [2500, 1000; p, 1 - p])$. The last pair of gambles in each of the four tasks is not Ω -ordered, since in all four cases one gamble dominates the other.

We close this section by noting that the negative results characterized in Proposition 2 can be extended in a number of dimensions, as reported in Appendix A.1.

4.2. Time Preferences. A monetary stream $x = (x_0, x_1, \dots, x_T)$ describes the amount of money $x_t \in \mathbb{R}_+$ realized at every time period t .¹⁶ The standard approach uses discounted utility $U_\omega^{du}(x) = \sum_t D_\omega(t)u(x_t)$, with discount functions for which $D_\omega(0) = 1$ and $\lim_{t \rightarrow \infty} D_\omega(t) = 0$ and with a parameter space $\Omega = \mathbb{R}_+$.¹⁷ The utility function over monetary outcomes u is strictly increasing and continuous. The most commonly-used discount function is the power function where $D_\omega^{pow}(t) = \frac{1}{(1+\omega)^t}$. The behavioral literature offers two alternative discount functions that appear better able to capture certain behavioral patterns.¹⁸ These are the hyperbolic discount function $D_\omega^{hyp}(t) = \frac{1}{1+\omega t}$, and the $\beta - \delta$ discount function where $D_\omega^{beta}(0) = 1$ and $D_\omega^{beta}(t) = \beta D_\omega^{pow}(t)$ whenever $t > 0$, with $\beta \in (0, 1]$.¹⁹ We write U_ω^{pow} , U_ω^{hyp} and U_ω^{beta} for the corresponding discounted utilities, and $\rho^{rum(pow)}$, $\rho^{rum(hyp)}$ and $\rho^{rum(beta)}$ for the corresponding RUM probabilities.

As in the case of the treatment of risk preferences, we consider here Ω -ordered pairs of streams such that neither dominates the other, in the sense that some types prefer x to y and others prefer y to x . For an illustration of such pairs, consider streams where there is a period of time \bar{t} such that $y_t > x_t$ for every $t \leq \bar{t}$ and $y_t < x_t$ for every $t > \bar{t}$. A simple version of these comparisons used in common practice is one where there is a unique conflict between waiting a shorter period for a larger monetary payoff, or waiting a longer period for some other monetary payoff. That is, $x_t = y_t$ except for two periods $t_1 < t_2$, with $y_{t_1} > x_{t_1}$ and $y_{t_2} < x_{t_2}$. These have been shown to be key streams in the treatment of time preferences, since they are instrumental in the characterization of the notion of more delay aversion (see Benoît and Ok, 2007; see also Horowitz, 1992).

Proposition 1 directly applies here, thereby implying that $\rho^{rum(pow)}$, $\rho^{rum(hyp)}$ and $\rho^{rum(beta)}$ are non-monotone for every Ω -ordered pair of streams such that $x_0 = y_0$.

Corollary 2. *Let (x, y) be an Ω -ordered pair of streams such that $x_0 = y_0$. Then, $\rho^{rum(pow)}$, $\rho^{rum(hyp)}$ and $\rho^{rum(beta)}$ are non-monotone for (x, y) .*

¹⁶Whether streams are finite or infinite is immaterial to this analysis.

¹⁷Notice that Proposition 1 follows immediately with $\Omega = \mathbb{R}_+$.

¹⁸See Loewenstein and Prelec (1992), Laibson (1997), and O'Donoghue and Rabin (1999).

¹⁹ That the exponential function $D_\omega^{exp}(t) = \exp^{-\hat{\omega}t}$ is equivalent to the power function becomes clear by considering $\hat{\omega} = \log(1 + \omega)$. For the $\beta - \delta$ case, the alternative representation based on the exponential function is sometimes called quasi-hyperbolic. That is, $D_\omega^{qh}(0) = 1$ and $D_\omega^{qh}(t) = \beta D_\omega^{exp}(t)$ whenever $t > 0$, with $\beta \in (0, 1]$. Given the equivalence, these functional forms are omitted from the analysis.

Proof of Corollary 2. Consider an Ω -ordered pair of streams (x, y) such that $x_0 = y_0$. Notice that $\lim_{\omega \rightarrow \infty} [U_{\omega}^{pow}(x) - U_{\omega}^{pow}(y)] = u(x_0) - u(y_0) + \lim_{\omega \rightarrow \infty} [\sum_{t=1}^T \frac{1}{(1+\omega)^t} (u(x_t) - u(y_t))] = 0$. Now, since there are, by assumption, some types of individuals for whom y is preferable to x , Proposition 1 applies directly, thereby implying that $\rho^{rum(pow)}$ is non-monotone. The cases of $\rho^{rum(hyp)}$ and $\rho^{rum(beta)}$ are analogous and hence omitted. ■

Note that whenever the Ω -ordered streams x and y differ in terms of present payoffs, that is $x_0 \neq y_0$, the choice probabilities $\rho^{rum(pow)}$, $\rho^{rum(hyp)}$ and $\rho^{rum(beta)}$ may be monotone. Consider, for example, a pair of streams (x, y) differing in only two periods, the present and a period t . Clearly, for (x, y) to be Ω -ordered, it must be the case that $x_0 < y_0$ and $x_t > y_t$. Now, since $u(x_0) - u(y_0)$ is independent of ω and $D_{\omega}(t)[u(x_t) - u(y_t)]$ is decreasing in the delay-aversion coefficient ω , the first part of Proposition 1 guarantees that choice probabilities are monotone for such pairs (x, y) . Importantly, this is the class of streams used in Chabris et al. (2008) and Tanaka, Camerer and Nguyen (2010).

Analogously to Proposition 2, the next result exploits the functional structure of U_{ω}^{pow} , U_{ω}^{hyp} and U_{ω}^{beta} to reach stronger results.

Proposition 4. *Let (x, y) be an Ω -ordered pair of streams with $x_0 = y_0$. Then, there exists $\bar{\omega}_{(x,y)}$ such that $\rho_{\omega}^{rum(pow)}(x, y)$, $\rho_{\omega}^{rum(beta)}(x, y)$ and $\rho_{\omega}^{rum(hyp)}(x, y)$ are strictly increasing in ω whenever $\omega \geq \bar{\omega}_{(x,y)}$.*

Proof of Proposition 4. Let (x, y) be an Ω -ordered pair of streams with $x_0 = y_0$, and denote by $t^* > 0$ the first period for which streams x and y differ. Consider first the case of the power discount function. We first claim that $u(x_{t^*}) - u(y_{t^*}) < 0$. To see this, notice that since (x, y) is an Ω -ordered pair, when ω is sufficiently large, stream y must be preferred to stream x , or equivalently, the sign of $\sum_t D_{\omega}^{pow}(t)[u(x_t) - u(y_t)] = D_{\omega}^{pow}(t^*) \sum_{t:t \geq t^*} (1+\omega)^{t^*-t} [u(x_t) - u(y_t)]$ must be negative. By standard arguments, for ω sufficiently large, the latter sign is equivalent to the sign of $u(x_{t^*}) - u(y_{t^*})$, proving the claim.

Now, $\rho^{rum(pow)}$ is strictly increasing in ω if and only if $\sum_t D_{\omega}^{pow}(t)[u(x_t) - u(y_t)]$ is strictly increasing in ω . Given the differentiability of D_{ω}^{pow} , the latter condition is equivalent to $\sum_t \frac{\partial D_{\omega}^{pow}(t)}{\partial \omega} [u(x_t) - u(y_t)] = \sum_{t:t \geq t^*} -t(1+\omega)^{-t-1} [u(x_t) - u(y_t)] = \sum_{t:t \geq t^*} -t D_{\omega}^{pow}(t+1) [u(x_t) - u(y_t)]$. When ω grows, the sign of the previous expression coincides with the sign of $-[u(x_{t^*}) - u(y_{t^*})]$, which we have shown to be positive. Hence,

there exists $\bar{\omega}_{(x,y)}$ such that $\rho^{rum(pow)}$ is strictly increasing in ω for every $\omega \geq \bar{\omega}_{(x,y)}$, as desired.

The additivity of discounted utility, the fact that $x_0 = y_0$, and that $D^{beta}(t) = \beta D^{pow}(t)$ whenever $t > 0$, makes the proof of the $\beta - \delta$ case analogous. For the hyperbolic case, we start by claiming that $\sum_{t:t \geq t^*} \frac{1}{t}[u(x_t) - u(y_t)]$ is negative. To see this, notice that since (x, y) is an Ω -ordered pair, when ω is sufficiently large, stream y must be preferred to stream x , or equivalently, the sign of $\sum_t D_\omega^{hyp}(t)[u(x_t) - u(y_t)] = D_\omega^{hyp}(t^*) \sum_{t:t \geq t^*} \frac{1+\omega t^*}{1+\omega t}[u(x_t) - u(y_t)]$ must be negative. As ω increases, the limit of $\frac{1+\omega t^*}{1+\omega t}$ is $\frac{t^*}{t}$, and hence $\sum_{t:t \geq t^*} \frac{1}{t}[u(x_t) - u(y_t)]$ must be negative. Now, notice that $\sum_t \frac{\partial D_\omega^{hyp}(t)}{\partial \omega}[u(x_t) - u(y_t)] = \sum_{t:t \geq t^*} -t(1+\omega t)^{-2}[u(x_t) - u(y_t)]$ which in turn is equal to $[D_\omega^{hyp}(t^*)]^2 \sum_{t:t \geq t^*} -t \left(\frac{1+\omega t^*}{1+\omega t}\right)^2 [u(x_t) - u(y_t)]$. Clearly, the sign when ω grows is equal to the sign of $\sum_{t:t \geq t^*} \frac{-1}{t}[u(x_t) - u(y_t)]$, that we know to be positive, concluding the proof. ■

Using the differentiability of the standard discount functions, the proof of Proposition 4 establishes that for every Ω -ordered pair (x, y) such that $x_0 = y_0$, there is always a level of delay aversion, $\bar{\omega}_{(x,y)}$, beyond which the models give an increasing probability for the choice of x over y . The proof also helps to explain how the critical values $\bar{\omega}_{(x,y)}$ vary with the pair of streams involved. To illustrate, we focus on the power case and use streams in which there is a unique conflict, that is, $x_t = y_t$, except for two periods, $0 < t_1 < t_2$, with $y_{t_1} > x_{t_1}$ and $y_{t_2} < x_{t_2}$. In this case, Proposition 4 shows that $\bar{\omega}_{(x,y)}$ is characterized by $\sum_{t:t \geq t^*} -t D_\omega^{pow}(t+1)[u(x_t) - u(y_t)] = 0$, which leads to $\bar{\omega}_{(x,y)} = \left(\frac{t_2[u(x_{t_2}) - u(y_{t_2})]}{t_1[u(y_{t_1}) - u(x_{t_1})]}\right)^{\frac{1}{t_2 - t_1}} - 1$. As in the case of risk, one can immediately appreciate that the better stream y is in relation to x , the lower the critical value $\bar{\omega}_{(x,y)}$. Focusing on time, it is also easy to see that, with the temporal gap $t_2 - t_1$ fixed, as t_1 increases, $\bar{\omega}_{(x,y)}$ decreases. That is, as the first difference between x and y becomes more distant in time, the range of problems widens. This is consistent with the fact discussed above that, for this type of streams, whenever $x_0 \neq y_0$, the RUM choice probabilities are monotone. Finally, with t_1 fixed, there is an interior value of t_2 that minimizes $\bar{\omega}_{(x,y)}$.²⁰

Having shown that for every Ω -ordered pair (x, y) with $x_0 = y_0$ there is a delay-aversion level, $\bar{\omega}_{(x,y)}$, at which the RUM probabilities of selecting the longer-delay stream x increase, we now argue that the practical relevance of the problem is, in fact, limited, because the critical values $\bar{\omega}_{(x,y)}$ obtained with the standard streams used in

²⁰This interior value is the solution to $t_1 + t_2(-1 + \log \frac{t_2[u(x_{t_2}) - u(y_{t_2})]}{t_1[u(y_{t_1}) - u(x_{t_1})]}) = 0$.

the experimental literature are clearly too large to be economically relevant. This can be appreciated with the pairs of streams used in Andersen et al. (2008). The lowest critical value is a yearly discount rate of 4.25 that corresponds to the pair of streams giving 3,313 Danish kroner in 25 months versus 3,000 in 1 month, which is clearly absurdly high in empirical terms. Given the above discussion on the determination of $\bar{\omega}_{(x,y)}$, it is clear that to obtain critical values of practical relevance with streams à la Andersen et al. (2008), we would need to vary the payoff times. In order to see this point consider linear utility functions over monetary payoffs, and note that the standard population discount rate estimated for this case in the literature is about .25. Hence, critical values around this discount rate would be economically relevant, since they would be binding for a large fraction of the population. Then, with the monetary payoffs and the temporal gap of 2 years fixed, when we increase t_1 from one month to 5 years the critical value becomes $\bar{\omega}_{(x,y)} = .24$.

4.3. Other Cases. Beyond risk and time, another preference parameter of interest is the one governing the degree of complementarity between two different inputs. These inputs may be the monetary payoffs to oneself and to another subject, as in a distributive problem with social preferences (see, e.g., Andreoni and Miller, 2002). Another case of interest in this respect is when the inputs refer to present consumption and future consumption, as in the influential Epstein and Zin (1989) preferences.²¹ Yet, another example is when the inputs refer to different consumption goods in general, as in a standard CES utility function. Our results advise caution when the complementarity parameter enters non-linearly into the utility function in a RUM estimation framework.

5. RANDOM PARAMETER MODELS

The following result establishes that RPMs are monotone for every Ω -ordered pair of alternatives.

Proposition 5. *ρ^{rpm} is monotone for every Ω -ordered pair of alternatives.*

Proof of Proposition 5. Let $\omega^L, \omega^H \in \Omega$, with $\omega^L < \omega^H$. Consider a realization ϵ of Ψ such that $U_{\omega^L+\epsilon}(y) > U_{\omega^L+\epsilon}(x)$. Since (x, y) is an Ω -ordered pair of alternatives, it must be the case that $U_{\omega^H+\epsilon}(y) > U_{\omega^H+\epsilon}(x)$. Consequently, the set of realizations

²¹These preferences also introduce risk attitudes and time preferences, so Sections 4.1 and 4.2 are of interest here too.

for which x is preferred to y shrinks with the preference parameter, implying that $\rho_{\omega}^{rpm}(x, y)$ is decreasing in ω , as desired. ■

Proposition 5 implies that RPMs can be safely used for the estimation of preference parameters. In the contexts of risk and time preferences, in particular, RPMs are immune to the problems characterized in Propositions 2 and 4. Furthermore, they are easily implementable. That is, given an Ω -ordered pair of alternatives (x, y) , all that is required to obtain $\rho_{\omega}^{rpm}(x, y)$ is to compute the value $\omega^{(x,y)}$ and the corresponding probability $\Psi(\omega^{(x,y)} - \omega)$.

Note that a distinguishing feature of RPMs is that when, for a given pair of options (x, y) , every utility function regards one option as better than the other, then the probability of choosing the former is one. This is sometimes seen as a limitation of the model, as in the case of stochastic-dominance related gambles, for instance, where the observed probability of choosing the dominated gamble is typically above zero. One way to deal with this in the context of RPMs is to add a trembling stage, in the spirit of the trembling hand approach used in game theory. This would work as follows. After a particular utility has been realized, with a large probability $1 - \kappa$ the choice is made according to the realized utility, and with probability κ there is a tremble and the reverse choice is made. It is easy to see that such a model is also monotone for every Ω -ordered pair of alternatives.

6. AN EMPIRICAL APPLICATION

In an influential paper, Andersen et al. (2008) implement a field experiment to jointly estimate risk and time preferences, using a representative sample of the adult Danish population comprised of 253 subjects. In this section we use the data of Andersen et al. (2008) to obtain separate risk and time preference estimates, using both random utility models and random parameter models. The purpose of this exercise is not to attempt to replicate the original results of Andersen et al. (2008), but rather to illustrate the difference in the estimations obtained by using the two random models under scrutiny. Hence, we depart in a number of ways from the identification strategy used by Andersen et al. (2008).

6.1. Estimation of Risk Preferences. There were four different risk-aversion choice tasks in the style of the multiple-price lists of Holt and Laury (2002). Each task comprised of ten pairs of nested gambles, as described in Table 1. All 253 subjects

were confronted with the four tasks, but 116 of them were presented with all 40 pairs of gambles, 67 with pairs 3, 5, 7, 8, 9, and 10 in every task, and the remaining 70 subjects were presented with pairs 1, 2, 3, 5, 7, and 10, again in every task. Subjects had to make a choice from each pair of gambles presented, which made a total of 7,928 choices. In every pair, subjects could either choose one of the gambles, or express indifference between the two. In the latter case, they were told that the experimenter would settle indifferences by tossing a fair coin.²²

We use CRRA utilities and include a tremble parameter, as defined in Section 5.²³ Every expression of indifference is transformed by assigning a half-choice to each of the two gambles. In the RUM estimations we assume the error distribution to be type I extreme value, while in the RPM estimations it is logistic. This gives the closed-form probabilities of selecting the riskier gamble x over the safer one y described in Section 3. We then use standard maximum likelihood procedures to estimate the population risk-aversion level ω , the tremble parameter κ , and the precision parameter λ . The RUM log-likelihood function contingent on these three parameters is

$$\sum_{i=1}^{i=40} \left[\left(X^i + \frac{I^i}{2} \right) \log \left[\frac{(1-2\kappa)e^{\lambda U_{\omega}^{crra}(x^i)}}{e^{\lambda U_{\omega}^{crra}(x^i)} + e^{\lambda U_{\omega}^{crra}(y^i)}} + \kappa \right] + \left(Y^i + \frac{I^i}{2} \right) \log \left[\frac{(1-2\kappa)e^{\lambda U_{\omega}^{crra}(y^i)}}{e^{\lambda U_{\omega}^{crra}(x^i)} + e^{\lambda U_{\omega}^{crra}(y^i)}} + \kappa \right] \right]$$

where $i = 1, \dots, 40$ denotes the i -th pair of gambles, and X^i , Y^i , and I^i the number of subjects expressing a preference for the riskier gamble, for the safer gamble, or indifference between the two gambles in pair i , respectively. Analogously, the RPM log-likelihood function is

$$\sum_{i=1}^{i=36} \left[\left(X^i + \frac{I_i}{2} \right) \log \left[\frac{(1-2\kappa)e^{\lambda\omega(x^i, y^i)}}{e^{\lambda\omega(x^i, y^i)} + e^{\lambda\omega}} + \kappa \right] + \left(Y^i + \frac{I^i}{2} \right) \log \left[\frac{(1-2\kappa)e^{\lambda\omega}}{e^{\lambda\omega(x^i, y^i)} + e^{\lambda\omega}} + \kappa \right] \right] + \sum_{i=37}^{i=40} \left(Y^i + \frac{I^i}{2} \right) \log[\kappa]$$

where $i = 1, \dots, 36$ denotes the 36 Ω -ordered pairs of gambles and $i = 37, \dots, 40$ the 4 pairs where x^i dominates y^i .

Table 2 presents the estimates. When considering the entire population of 253 subjects, we see that the RPM risk-aversion estimate is about 14% higher than that of the RUM. We see this as a considerable bias in the RUM estimation. We test the significance of this difference using the bootstrap method, which we consider particularly appropriate in the light of our theoretical results on the non-monotonicity of

²²25% of all choices were expressions of indifference. With indifferences omitted, the estimates are practically identical.

²³Given the large values of the payoffs involved in the gambles, we avoid the use of CARA utilities.

TABLE 2. RUM and RPM estimations of risk-aversion

	RUM			RPM			RPM - RUM			
	ω	λ	κ	ω	λ	κ	$\Delta\omega$	CI($\Delta\omega$)	$\%\omega$	
All Subjects	0.661 (0.032)	0.275 (0.067)	0.034 (0.012)	0.752 (0.043)	2.495 (0.137)	0.051 (0.008)	0.091 (0.018)	0.065	0.134	0.137
<i>z</i> % more risk-averse individuals										
<i>z</i> = 50	1.031 (0.036)	4.596 (1.28)	0.016 (0.006)	1.231 (0.056)	2.793 (0.192)	0.029 (0.006)	0.2 (0.029)	0.149	0.262	0.194
<i>z</i> = 45	1.076 (0.038)	6.5 (1.89)	0.01 (0.005)	1.283 (0.06)	2.891 (0.204)	0.025 (0.005)	0.207 (0.033)	0.15	0.28	0.192
<i>z</i> = 40	1.127 (0.041)	9.564 (3.086)	0.01 (0.005)	1.358 (0.067)	2.871 (0.22)	0.024 (0.006)	0.231 (0.039)	0.165	0.317	0.205
<i>z</i> = 35	1.198 (0.049)	16.29 (6.134)	0.011 (0.007)	1.468 (0.074)	2.818 (0.222)	0.025 (0.006)	0.27 (0.044)	0.19	0.367	0.225
<i>z</i> = 30	1.249 (0.056)	23.24 (9.555)	0.004 (0.005)	1.577 (0.083)	2.853 (0.239)	0.023 (0.006)	0.328 (0.055)	0.226	0.442	0.263
<i>z</i> = 25	1.366 (0.068)	62.1 (43.64)	0.011 (0.006)	1.699 (0.1)	2.857 (0.267)	0.022 (0.007)	0.333 (0.066)	0.223	0.482	0.244
<i>z</i> = 20	1.465 (0.102)	128.8 (117.6)	0.013 (0.006)	1.869 (0.147)	2.676 (0.287)	0.026 (0.008)	0.404 (0.094)	0.234	0.606	0.276

Note: Block bootstrap standard errors clustered at the individual level, shown in parentheses, calculated using 10,000 resamples. ω , λ and κ are the population risk-averse level, precision parameter and tremble parameter, respectively. $\Delta\omega$ reports the difference between the RPM and RUM estimated risk-aversion levels. CI($\Delta\omega$) is the 95% bootstrap confidence interval for $\Delta\omega$. $\%\omega$ reports the percentage increase in the estimated risk-aversion level when using RPM as opposed to RUM.

RUMs. We perform 10,000 resamples at the individual level, where we estimate the corresponding RUM and RPM parameters, and compute the difference between them, which is our statistic. Figure 4 reports the density functions of the RUM and RPM risk-aversion estimates (subfigure a), and the density of the difference in the bootstrap estimations (subfigure b). It is immediate to see that RPM risk-aversion estimates are systematically greater than those obtained by the RUM, as predicted by our theoretical results. When we compute the bootstrap confidence interval of our statistic at standard confidence levels, we note that the confidence interval never includes zero.

Our theoretical results show, moreover, that the greater the risk aversion of subjects, the greater the potential bias in the RUM estimates. In order to empirically test this prediction, we rank subjects in terms of their revealed risk aversion, using a simple method that relies neither on RUMs, nor on RPMs.²⁴ The method focuses

²⁴In Appendix B, we explore another possible simple method, with similar results.

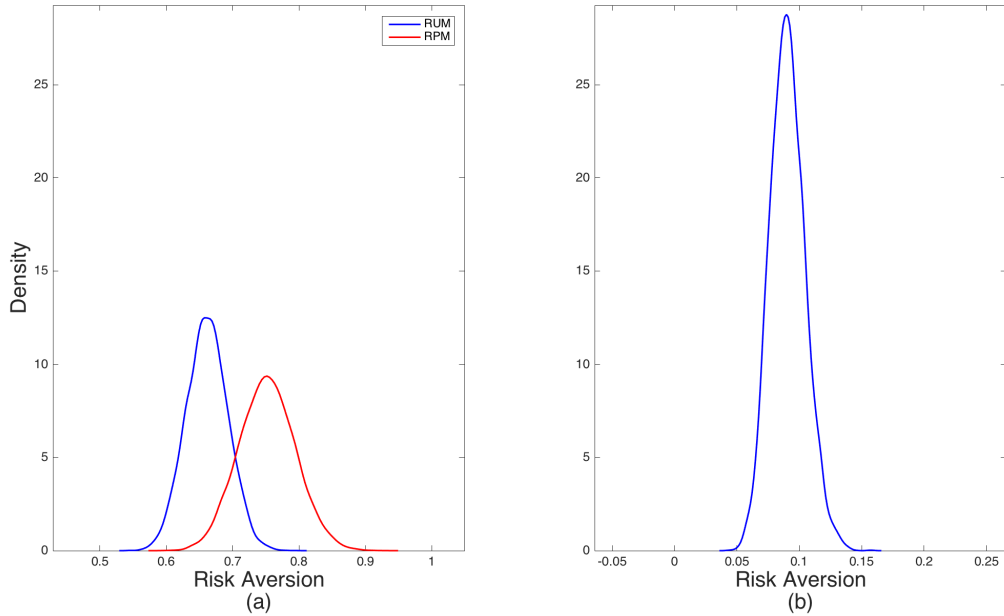


FIGURE 4.—(a) Density functions of the RUM and RPM bootstrap risk-aversion estimates. (b) Density function of the difference between the RUM and RPM bootstrap risk-aversion estimates.

on the 36 Ω -ordered pairs of gambles and computes, for each individual, the proportion of the pairs in which they opted for the riskier gamble x^i , recording a half-choice to each gamble whenever indifference was expressed. In order to break possible ties between individuals, we consider all values $\omega^{(x^i, y^i)}$ that make the gambles x^i and y^i indifferent, and focus on the first value where the individual chooses the riskier gamble over the safer one. Now, given the ranking of the individuals provided by this method, Table 2 reports the estimates for the $z\%$ more risk averse individuals, where $z \in \{50, 45, 40, 35, 30, 25, 20\}$.²⁵ First, we see that the risk averse estimates for both RUMs and RPMs are increasing, suggesting the appropriateness of the selected ranking method. Secondly, as predicted by our theoretical results, we see that the gap between the two methods is increasing, up to a sizable 28%, for the 20% most risk averse individuals. Following the same bootstrap method explained above, we obtain that all these differences are statistically significant: it is always the case that the differences in the estimated RPM and RUM coefficients are systematically positive. Finally, it is

²⁵We stop at the 20% mark to ensure some choice variability.

worth noting that, while the RPM estimates of the precision parameter λ are very robust, the RUM λ estimates increase substantially as the estimation progresses towards more risk averse individuals.²⁶

6.2. Estimation of Time Preferences. There were six delay-aversion choice tasks of the multiple-price list type, each comprised of ten pairs of streams differing in only two periods, as described in Table 3. All 253 subjects were confronted with all the tasks, which made a total of 15,180 choices. Indifferences were again allowed and dealt with in the same way as in the case of risk preferences. We use power and hyperbolic discounted utility functions, including a tremble parameter.²⁷ In the RUM estimations, we assume the error distribution to be type I extreme value, leading to a similar log-likelihood function as in the risk-aversion treatment, with the appropriate utility representation. In the RPM estimations, notice that all pairs of streams are now Ω -ordered, and, since the discount parameter takes only positive values, we use a log-logistic instead of a logistic distribution. Again, using the appropriate utility representation, the log-likelihood function is similar to that employed in the risk-aversion analysis. We then use standard maximum likelihood procedures to estimate the population delay-aversion level ω , the tremble parameter κ , and the precision parameter λ .

TABLE 3. Streams of Payoffs used in Andersen et al. (2008)

	x_{t_2}									
$t_2 = 2$	3012	3025	3037	3049	3061	3073	3085	3097	3109	3120
$t_2 = 5$	3050	3100	3151	3202	3253	3304	3355	3407	3458	3510
$t_2 = 7$	3075	3152	3229	3308	3387	3467	3548	3630	3713	3797
$t_2 = 13$	3153	3311	3476	3647	3823	4006	4196	4392	4595	4805
$t_2 = 19$	3232	3479	3742	4020	4316	4630	4962	5315	5687	6082
$t_2 = 25$	3313	3655	4027	4432	4873	5350	5869	6431	7039	7697

Note: Every pair of streams involves a comparison of x_{t_2} , as detailed in the table, with $y_1 = 3000$.

In consonance with our theoretical results, the RUM and RPM estimations of the delay aversion parameter are very close, as can be appreciated in Table 4.

²⁶Taking the estimated precision parameters for the full sample of 253 individuals (0.275 in the RUM case and 2.495 in the RPM case), and estimating RUM and RPM risk aversion coefficients for the 50% more risk averse individuals, we obtain 0.687 and 1.268, respectively, which represents a bias

TABLE 4. RUM and RPM estimations of delay-aversion

	RUM			RPM			RPM - RUM			
	ω	λ	κ	ω	λ	κ	$\Delta\omega$	CI($\Delta\omega$)	$\%\omega$	
Power	0.274 (0.022)	0.103 (0.087)	0.228 (0.01)	0.265 (0.011)	9.335 (0.72)	0.064 (0.018)	-0.009 (0.018)	-0.04	0.022	-0.033
Hyperbolic	0.245 (0.014)	0.04 (0.009)	0.221 (0.011)	0.262 (0.011)	11.36 (0.798)	0.107 (0.015)	0.017 (0.009)	-0.026	0.007	0.069

Note: Block bootstrap standard errors clustered at the individual level, shown in parentheses, calculated using 10,000 resamples. ω , λ and κ are the population delay-averse level, precision parameter and tremble parameter, respectively. $\Delta\omega$ reports the difference between the RPM and RUM estimated delay-aversion levels. CI($\Delta\omega$) is the 95% bootstrap confidence interval for $\Delta\omega$. $\%\omega$ reports the percentage increase in the estimated delay-aversion level when using RPM as opposed to RUM.

7. CONCLUSIONS

We have introduced here the notion of monotonicity of a stochastic choice model with respect to a preference parameter. Namely, consider a pair of alternatives (x, y) and a utility evaluation of them where x is preferred to y for low values of the parameter and y is preferred to x for larger values. That is, the preference parameter represents an aversion to x with respect to y . Monotonicity implies then that the probability of selecting x should decrease as the aversion to choosing x increases. We argue that this is a minimal property for a stochastic model.

We have focused on two popular stochastic models, random utility models and random parameter models. After establishing the conditions for these models to be monotone, we have focused on the particular cases of risk and delay aversion. We have shown that the standard application of random utility models to risk or time settings is subject to serious theoretical inconsistencies. In the main results we have shown that there is a level of risk aversion (respectively, of delay aversion) beyond which the probability of choosing the riskier gamble (respectively, the more delayed stream) increases with the level of risk aversion (respectively, of delay aversion). We have then established that random parameter models are free from all these inconsistencies. These findings should constitute an alert to exercise caution when directly applying sound stochastic choice models to settings other than those originally contemplated.

of 84%. Reproducing this exercise for the 20% more risk averse individuals, we obtain 0.712 and 1.915, respectively, which is a difference of 169%.

²⁷Notice that $\beta - \delta$ discounted utility is indistinguishable from power discounted utility, since the relevant payoffs take place in the future.

APPENDIX A. THEORETICAL EXTENSIONS

A.1. Random Utility Models and Risk Preferences. We now comment on several extensions to Proposition 2.

Logarithmic Transformation of the Representative Utility. This approach starts by assuming that the representative utility of every option is strictly positive. Thus, the probability of selecting option x over option y is $P(\log(U_\omega(x)) + \epsilon(x) \geq \log(U_\omega(y)) + \epsilon(y)) = \Psi^*(\log(U_\omega(x)) - \log(U_\omega(y)))$, where Ψ^* is the distribution function of the difference of the i.i.d. errors. Paralleling the first part of Proposition 1, it is now immediate that a RUM based on the logs of the utilities, LRUM, is monotone for the Ω -ordered pair (x, y) if and only if the ratio $\frac{U_\omega(x)}{U_\omega(y)}$ is decreasing in ω . The second part of Proposition 1 can also be directly reproduced considering $\lim_{\omega \rightarrow \infty} [\log(U_\omega(x)) - \log(U_\omega(y))] = 0$. Denote by $\rho^{lrum(cara)}$ the CARA probabilities.²⁸

Proposition 6. *Let (x, y) be an Ω -ordered pair of gambles. Then, there exists $\bar{\omega}_{(x,y)}$ such that $\rho_\omega^{lrum(cara)}(x, y) > \rho_{\omega_{(x,y)}}^{lrum(cara)}(x, y)$ whenever $\omega > \bar{\omega}_{(x,y)}$.*

Proof of Proposition 6. Consider any Ω -ordered pair of gambles (x, y) . Given the structure of gambles and that of the CARA family, there always exists $\hat{\omega} > 0$ such that $U_{\hat{\omega}}^{cara}(x) < U_{\hat{\omega}}^{cara}(y)$, or $\log(U_{\hat{\omega}}^{cara}(x)) < \log(U_{\hat{\omega}}^{cara}(y))$. It also can be immediately seen that the limits of $\log(\omega U_\omega^{cara}(x))$ and $\log(\omega U_\omega^{cara}(y))$ as ω increases are both 0; hence, $\lim_{\omega \rightarrow \infty} [\log(U_\omega(x)) - \log(U_\omega(y))] = \lim_{\omega \rightarrow \infty} [\log(\omega U_\omega(x)) - \log(\omega U_\omega(y))] = 0$. Therefore, there exists $\tilde{\omega} > \hat{\omega}$ such that $\log(U_\omega^{cara}(x)) - \log(U_\omega^{cara}(y)) > \log(U_{\tilde{\omega}}^{cara}(x)) - \log(U_{\tilde{\omega}}^{cara}(y))$ for every $\omega \geq \tilde{\omega}$. This, together with the logarithmic counterpart of Proposition 1, implies that $\rho_\omega^{lrum(cara)}(x, y) > \rho_{\tilde{\omega}}^{lrum(cara)}(x, y)$ for every $\omega \geq \tilde{\omega}$. Now, the function $\rho_\omega^{lrum(cara)}(x, y)$ is continuous on $[\hat{\omega}, \infty)$ and, hence, achieves a minimum ω^* in the closed interval $[\hat{\omega}, \tilde{\omega}]$, and, by the above reasoning, we know that ω^* is also a minimum in $[\hat{\omega}, \infty)$. Given continuity, we only need to consider $\bar{\omega}_{(x,y)}$ to be the largest value of ω for which $\rho_{\omega_{(x,y)}}^{lrum(cara)}(x, y) = \rho_{\omega^*}^{lrum(cara)}(x, y)$ and the result follows. ■

Generalized Expected Utility. Proposition 2 works under the assumption of expected utility. Clearly, generalizations of expected utility, such as cumulative prospect theory, rank-dependent expected utility, disappointment aversion, etc, are susceptible to

²⁸Notice that CRRA functions are not entirely appropriate in this context, because for values of ω above 1 the utilities become negative, which is incompatible with the use of log-transformations. The function $x^{1-\omega}$, without the normalization $\frac{1}{1-\omega}$, is positive for values of $\omega > 1$, but in this case is not monotone in outcomes, and thus is also problematic.

the problems identified above, since they include expected utility as a special case. More importantly, however, the additive nature of these models makes them vulnerable to similar anomalies, even when considering only non-expected utilities. Formally, consider a function π that associates every gamble $x = [x_1, \dots, x_N; p(x_1), \dots, p(x_N)]$ with another gamble $\pi(x) = [x_1, \dots, x_N; q(x_1), \dots, q(x_N)]$ over the same set of outcomes. We assume that, for any given vector of outcomes, the distortion of probabilities is a one-to-one, continuous and monotone function over each argument. Then, the generalized CARA expected utility is $U_\omega^{gcara}(x) = U_\omega^{cara}(\pi(x))$, while the generalized CRRA expected utility is $U_\omega^{gcrra}(x) = U_\omega^{crra}(\pi(x))$, and the corresponding RUM choice probabilities are denoted by $\rho^{rum(gcara)}$ and $\rho^{rum(gcrra)}$. It can be immediately seen that, whenever $(\pi(x), \pi(y))$ is an Ω -ordered pair, the logic behind Proposition 2 can be applied directly. Without assuming that the transformed gambles are Ω -ordered, Proposition 7 nevertheless establishes analogous results.²⁹

Proposition 7. *Let (x, y) be an Ω -ordered pair of gambles such that $\min\{x_1, \dots, x_N\} \neq \min\{y_1, \dots, y_M\}$. Then, there exists $\bar{\omega}_{(x,y)}$ such that $\rho_\omega^{rum(gcara)}(x, y)$ and $\rho_\omega^{rum(gcrra)}(x, y)$ are strictly increasing in ω whenever $\omega \geq \bar{\omega}_{(x,y)}$.*

Proof of Proposition 7. Consider an Ω -ordered pair of gambles (x, y) such that $\min\{x_1, \dots, x_N\} \neq \min\{y_1, \dots, y_M\}$. Since x and y are Ω -ordered and not stochastic-dominance related, it must be the case that $\min\{x_1, \dots, x_N\} < \min\{y_1, \dots, y_M\}$. Following the same logic as in the proof of Proposition 2, it follows that $\rho_\omega^{rum(gcara)}(x, y)$ is strictly increasing in ω if and only if $CE(\pi(y), V_\omega^{cara}) > CE(\pi(x), V_\omega^{cara})$. These certainty equivalents converge, with increasing ω , towards the corresponding minimum outcomes in $\pi(x)$ and $\pi(y)$, which are, by construction, the corresponding minimum outcomes in x and y . Now, the rest of the proof proceeds as in the proof of Proposition 2. The case of CRRA utilities is completely analogous and hence omitted. ■

As already discussed, CARA and CRRA utilities are by far the most used utility specifications used in the literature. In our previous results, the use of CARA and CRRA utilities allows to characterize the structure of the choice probabilities involved in the problem, and thereby to find the global minimum $\bar{\omega}_{(x,y)}$. Notice, however,

²⁹For the result, we assume that gambles x and y in the original Ω -ordered pair do not share the same minimum outcome. Notice, however, that when the gambles do share the same minimum monetary outcome, the second part of Proposition 1 becomes immediately applicable, thereby showing the model to be non-monotone for such gambles.

that one can show that every RUM based on generalized expected utilities using any monetary utility function that is strictly increasing and continuous in outcomes is non-monotone for some Ω -ordered pairs of gambles.³⁰

Certainty Equivalents. The certainty equivalent is sometimes used to replace the expected utility as the representative utility. The main intuition behind this approach is that the certainty equivalent is a monetary representation of preferences, where the use of a common scale facilitates interpersonal comparisons. Thus, it is not beyond reason that, by creating a common scale, this method could provide a solution to the problem under discussion, as is indeed the case in instances, such as whenever the Ω -ordered pair (x, y) involves a degenerate gamble. This can be appreciated by noticing that the certainty equivalent of the non-degenerate gamble x decreases with the level of risk aversion, while the certainty equivalent of the degenerate gamble y is constant across risk-aversion levels. Thus, the difference between the certainty equivalents of the two gambles decreases with the level of risk aversion and, by Proposition 1, the probability of choosing the risky gamble decreases, as desired. However, caution is required when using certainty equivalents, because problems may arise with other comparisons. We illustrate this point by considering Ω -ordered pairs (x, y) such that $\min\{x_1, \dots, x_N\} = \min\{y_1, \dots, y_M\}$. We denote by $\rho^{rum(cecara)}$ and $\rho^{rum(cecrra)}$ the choice probabilities associated with this model, when using the certainty equivalent representation of CARA and CRRA expected utilities, respectively.

Corollary 3. $\rho^{rum(cecara)}$ and $\rho^{rum(cecrra)}$ are non-monotone for every Ω -ordered pair of gambles (x, y) such that $\min\{x_1, \dots, x_N\} = \min\{y_1, \dots, y_M\}$.

Proof of Corollary 3. Consider an Ω -ordered pair of gambles (x, y) such that $\min\{x_1, \dots, x_N\} = \min\{y_1, \dots, y_M\}$. Since the Arrow-Pratt coefficients of u_ω^{cara} and u_ω^{crra} are ω and ωm , respectively, it follows that $\lim_{\omega \rightarrow \infty} [CE(x, U_\omega^{cara}) - CE(y, U_\omega^{cara})] = \lim_{\omega \rightarrow \infty} [CE(x, U_\omega^{crra}) - CE(y, U_\omega^{crra})] = \min\{x_1, \dots, x_N\} - \min\{y_1, \dots, y_M\} = 0$. Now, since by assumption x and y are not stochastic-dominance related, there is a level of risk aversion for which y is preferred to x . Hence, Proposition 1 is immediately applicable, and the claim is proved. ■

Mean-Variance Utilities. Let us now consider mean-variance utilities, which are much used in portfolio theory and macroeconomics. Markowitz (1952) was the first to propose

³⁰We can provide details upon request.

a mean-variance evaluation of risky asset allocations. Roberts and Urban (1988) and Barseghyan et al. (2013) provide examples of the use of mean-variance utilities in a RUM, for the estimation of risk preferences. Formally, given a gamble x , let us denote the expected value and variance of x by $\mu(x) = \sum_i p_i x_i$ and $\sigma^2(x) = \sum_i p_i (x_i - \mu(x))^2$, respectively. Mean-variance utilities are then described by $U_\omega^{mv}(x) = \mu(x) - \omega \sigma^2(x)$. We now argue that the corresponding RUM choice probabilities $\rho^{rum(mv)}$ are always monotone. This follows from the linear dependence of the utility function with respect to the parameter.

Proposition 8. $\rho^{rum(mv)}$ is monotone for every Ω -ordered pair of gambles (x, y) .

Proof of Proposition 8. Consider an Ω -ordered pair of gambles (x, y) . Notice that $U_\omega(x) - U_\omega(y) = \mu(x) - \mu(y) - \omega(\sigma^2(x) - \sigma^2(y))$. Since (x, y) are Ω -ordered, it cannot be that $\sigma^2(x) < \sigma^2(y)$. If the case were otherwise, individuals with an ω that goes to $-\infty$ would prefer gamble y to x , while those with an ω that goes to ∞ would prefer gamble x to y , thereby contradicting that the pair (x, y) is Ω -ordered. Hence, it must be that $\sigma^2(x) \geq \sigma^2(y)$. In this case, $U_\omega(x) - U_\omega(y)$ is decreasing in ω , and Proposition 1 is directly applicable. ■

Contextual Utility. To conclude, Wilcox (2011) suggests normalizing the utility difference between the gambles by the difference between the utilities of the best and worst of all the outcomes involved in the two gambles under consideration. This variation of a RUM goes under the name of contextual utility. The author shows that the suggested normalization solves the problem for cases in which both gambles are defined over the same three outcomes (thus covering the important Marschak-Machina triangles) and related through the notion of mean-preserving spreads. However, this normalization does not solve the problem beyond the case mentioned. We illustrate this point by contemplating the Ω -ordered pair of gambles (x, y) , with $x = [0, 10, 50, 90, 100; .1, .4, 0, .4, .1]$ and $y = [0, 10, 50, 90, 100; .05, 0, .9, 0, .05]$, where x is a mean-preserving spread of y . It can be seen that the RUM probability of choosing x using expected utility with CRRA is lower for the risk-aversion coefficient $\omega_1 = .7$ than for $\omega_2 = .9$.

A.2. Random Utility Models and Time Preferences. We now consider the LRUM case in the context of time preferences, as introduced in Appendix A.1. In order to impose the condition that the discounted utilities must be strictly positive for Ω -ordered pairs, we assume that $u(0) = 0$. Denote by $\rho^{lrum(pow)}$, $\rho^{lrum(beta)}$ and $\rho^{lrum(hyp)}$ the

LRUM probabilities for the power, $\beta - \delta$ and hyperbolic utilities. Proposition 9 establishes for the LRUM case results analogous to those of Proposition 4. For the hyperbolic case denote by \hat{m}_x and \hat{m}_y the monetary payoffs such that $u(\hat{m}_x) = \sum_{t>t^*} \frac{t^*+1}{t} u(x_t)$ and $u(\hat{m}_y) = \sum_{t>t^*} \frac{t^*+1}{t} u(y_t)$.

Proposition 9.

- (1) Let (x, y) be an Ω -ordered pair of streams with $t^* > 0$ and $y_t > 0$ for some $t < t^*$. Then, there exists $\bar{\omega}_{(x,y)}$ such that $\rho_\omega^{lrum(pow)}(x, y)$ and $\rho_\omega^{lrum(beta)}(x, y)$ are strictly increasing in ω whenever $\omega \geq \bar{\omega}_{(x,y)}$.
- (2) Let (x, y) be an Ω -ordered pair of streams with $\frac{u(y_{t^*})-u(x_{t^*})}{u(\hat{m}_x)-u(\hat{m}_y)} > \frac{t^*}{t^*+1} > 0$ and $y_0 > 0$. Then, there exists $\bar{\omega}_{(x,y)}$ such that $\rho_\omega^{lrum(hyp)}(x, y)$ is strictly increasing in ω whenever $\omega \geq \bar{\omega}_{(x,y)}$.

Proof of Proposition 9. From the logarithmic version of Proposition 1 and the differentiability of U_ω^α , $\alpha \in \{pow, hyp, beta\}$, $\rho^{lrum(\alpha)}$ is strictly increasing in ω if and only if the derivative of $\frac{\sum_t D_\omega^\alpha(t)u(x_t)}{\sum_t D_\omega^\alpha(t)u(y_t)}$ with respect to ω is strictly positive. Clearly, the sign of this derivative is the same as that of $\sum_t \frac{\partial D_\omega^\alpha(t)}{\partial \omega} u(x_t) [\sum_t D_\omega^\alpha(t)u(y_t)] - \sum_t \frac{\partial D_\omega^\alpha(t)}{\partial \omega} u(y_t) [\sum_t D_\omega^\alpha(t)u(x_t)]$.

Now, in the case of $\alpha = pow$, since $\frac{\partial D_\omega^\alpha(0)}{\partial \omega} = 0$, the relevant sign is equivalent to that of $-\sum_t t D_\omega^{pow}(t)u(x_t) [\sum_t D_\omega^{pow}(t)u(y_t)] + \sum_t t D_\omega^{pow}(t)u(y_t) [\sum_t D_\omega^{pow}(t)u(x_t)]$. The latter expression is equivalent to $\sum_r \sum_s (s-r) D_\omega^{pow}(r) D_\omega^{pow}(s) u(x_r) u(y_s)$ or, simply, $\sum_r \sum_s (s-r) D_\omega^{pow}(r+s) u(x_r) u(y_s)$. To analyze the sign of the previous expression when ω is sufficiently large, we only need to consider the smallest integer k for which the term $\sum_r \sum_{s:r+s=k} (s-r) u(x_r) u(y_s)$ is different from zero. Now, let \bar{t} be the smallest integer such that $\bar{t} < t^*$ with $y_{\bar{t}} > 0$, which exists by assumption. Any sum where $k < \bar{t} + t^*$ is equal to zero, while the sum $\sum_r \sum_{s:r+s=\bar{t}+t^*} (s-r) u(x_r) u(y_s)$ is equal to $(t^* - \bar{t}) u(y_{\bar{t}}) (u(y_{t^*}) - u(x_{t^*}))$, which is strictly positive by the assumptions on x and y . This makes the desired derivative strictly positive above a certain value $\bar{\omega}_{(x,y)}$ and hence $\rho_\omega^{lrum(pow)}(x, y)$ is strictly increasing above $\bar{\omega}_{(x,y)}$.

When $\alpha = beta$, the relevant expression becomes $\beta \sum_s s D_\omega^{pow}(s) u(x_0) u(y_s) - \beta \sum_r r D_\omega^{pow}(r) u(x_r) u(y_0) + \beta^2 \sum_{r>0} \sum_{s>0} (s-r) D_\omega^{pow}(r+s) u(x_r) u(y_s)$. Since $\beta > 0$, for sufficiently high values of ω , the sign is equivalent to the sign of $(t^* - \bar{t}) u(y_{\bar{t}}) (u(y_{t^*}) - u(x_{t^*}))$, and the result follows.

Now consider the case of $\alpha = hyp$, where the relevant expression becomes $\sum_r \sum_s (s-r) [D_\omega^{hyp}(r) D_\omega^{hyp}(s)]^2 u(x_r) u(y_s)$. When ω goes to infinity, the expression converges to

zero and the dominant terms are all terms in which either r or s is zero, i.e. those of the forms $s[D_\omega^{hyp}(s)]^2u(x_0)u(y_s)$ and $-r[D_\omega^{hyp}(r)]^2u(x_r)u(y_0)$. To study the sign of their sum, simply notice that, as ω increases, the limit of $\frac{D_\omega^{hyp}(a)}{D_\omega^{hyp}(b)}$ is b/a . Hence, the determining expression is $\sum_s \frac{1}{s}u(x_0)u(y_s) - \sum_r \frac{1}{r}u(x_r)u(y_0)$, which is equal to $\frac{1}{t^*}(u(x_0)u(y_{t^*}) - u(x_{t^*})u(y_0)) + \sum_{t>t^*} \frac{1}{t}(u(x_0)u(y_t) - u(x_t)u(y_0))$, with $x_0 = y_0 > 0$. Note that the summation in the former expression is strictly negative, since (x, y) is an Ω -ordered pair of streams. Then, the extra condition assumed in the $\alpha = hyp$ guarantees that the expression is strictly positive and the result follows. ■

We close the treatment of time preferences by noticing that this problem pervades beyond the usual parametric functions used in the literature. One can show that for every discounted utility RUM there is always an Ω -ordered pair of streams for which the model is not well-defined.³¹

A.3. More than Two Alternatives. We now show that the use of an Ω -ordered pair of alternatives causes no loss of generality.

First, consider the case where stochastic choice is defined over a menu A involving more than two options. Suppose that there is an alternative $x \in A$ such that the pair (x, y) is Ω -ordered for every $y \in A \setminus \{x\}$. In RUMs, our results show that, in the most standard applications, there is a preference parameter $\bar{\omega}_{(x,y)}$ such that $\rho_\omega^{rum}(x, y)$ is strictly increasing whenever $\omega \geq \bar{\omega}_{(x,y)}$, for every $y \in A \setminus \{x\}$. We now contemplate the probability of choosing x from A . It is easy to see that this probability is also strictly increasing whenever $\omega \geq \max_{y \in A \setminus \{x\}} \{\bar{\omega}_{(x,y)}\}$. To see this, notice that the probability of choosing x from A is simply equal to the probability of $\epsilon(y) - \epsilon(x)$ being smaller than $U_\omega(x) - U_\omega(y)$ for every $y \in A \setminus \{x\}$. Our results show that all the utility differences $U_\omega(x) - U_\omega(y)$ are increasing, at least for preference parameters $\omega \geq \max_{y \in A \setminus \{x\}} \{\bar{\omega}_{(x,y)}\}$, and hence follows the result. Nevertheless, it is obvious that RPMs are still monotone when considering the mentioned probability. This follows from observing that the set of realizations from Ψ for which x is maximal in A shrinks with the value of ω , for exactly the same reason as given in Proposition 5.

Now consider the case where there is a collection of Ω -ordered pairs of alternatives $\{(x^i, y^i)\}_{i=1}^K$, and that the exercise revolves around the selection of one alternative from each pair. We comment on two conceptual problems that arise when $K > 1$. These two problems collapse into the one we have studied in this paper when there is just

³¹Again, we avoid the details here, but can provide them upon request.

one pair of alternatives. We first contemplate the conditional probability of choosing vector \mathbf{x} with respect to \mathbf{y} , where \mathbf{x} and \mathbf{y} differ in only one pair, say pair j , where x^j is in \mathbf{x} and y^j is in \mathbf{y} . It is obvious that, in the case of RUMs, this conditional probability is increasing above $\omega_{(x^j, y^j)}$. Now consider the joint probability of selecting each x^i from the corresponding pair (x^i, y^i) , $i = 1, \dots, K$. It is immediate that this joint probability is prey to the same type of problem. Specifically, since the joint probability is simply the product probability of the different choices, it strictly increases at least for values $\omega \geq \max_i \{\bar{\omega}_{(x^i, y^i)}\}$. However, any conditional or joint probability of an RPM is monotone, due to the product nature of the considered probability.

APPENDIX B. EMPIRICAL APPLICATION: FURTHER CONSIDERATIONS

In Section 6.1 we propose a method with which to rank individuals in terms of their revealed risk aversion. We now study another simple method serving this purpose, Method B, and show that we replicate the results reported in Section 6.1. Method B ranks individuals according to the average of the $\omega^{(x_i, y_i)}$ corresponding to the first pair in which the riskier option was taken or indifference expressed and the $\omega^{(x_i, y_i)}$ corresponding to the last pair in which the safer option was taken or indifference expressed. We can now repeat the estimation analysis described in Section 6.1, conditional upon the rankings of individuals given by Method B. Table B.1 reports the results. It is immediately apparent that we reproduce the main conclusions reached in Section 6.1. Notably, RPM risk-aversion estimates are always significantly greater than those provided by RUM, and the differences show an increasing trend, with the magnitude of bias reaching as high as 33%.

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TABLE B.1. RUM and RPM estimations for the $z\%$ more risk averse individuals classified by Method B

	RUM			RPM			RPM - RUM			
	ω	λ	κ	ω	λ	κ	$\Delta\omega$	CI($\Delta\omega$)	$\% \omega$	
$z = 50$	1.019 (0.031)	4.572 (1.09)	0.008 (0.004)	1.179 (0.044)	3.254 (0.208)	0.023 (0.005)	0.16 (0.021)	0.122	0.205	0.157
$z = 45$	1.058 (0.032)	6.408 (1.495)	0.009 (0.004)	1.219 (0.046)	3.345 (0.230)	0.023 (0.005)	0.161 (0.022)	0.121	0.209	0.152
$z = 40$	1.109 (0.033)	9.08 (2.211)	0.005 (0.004)	1.287 (0.05)	3.367 (0.256)	0.023 (0.006)	0.178 (0.027)	0.13	0.237	0.161
$z = 35$	1.166 (0.035)	14.22 (3.463)	0.006 (0.004)	1.371 (0.056)	3.364 (0.264)	0.022 (0.006)	0.205 (0.03)	0.152	0.27	0.176
$z = 30$	1.229 (0.037)	23.57 (5.739)	0.005 (0.005)	1.447 (0.065)	3.417 (0.324)	0.021 (0.006)	0.218 (0.039)	0.148	0.302	0.177
$z = 25$	1.306 (0.042)	40.85 (11.91)	0.007 (0.007)	1.578 (0.079)	3.223 (0.338)	0.024 (0.007)	0.272 (0.05)	0.183	0.382	0.208
$z = 20$	1.402 (0.049)	76.89 (25.12)	0.000 (0.001)	1.774 (0.096)	3.13 (0.271)	0.024 (0.007)	0.372 (0.067)	0.251	0.517	0.265

Note: Block bootstrap standard errors clustered at the individual level, shown in parentheses, calculated using 10,000 resamples. ω , λ and κ are the population delay-averse level, precision parameter and tremble parameter, respectively. $\Delta\omega$ reports the difference between the RPM and RUM estimated delay-aversion levels. CI($\Delta\omega$) is the 95% bootstrap confidence interval for $\Delta\omega$. $\% \omega$ reports the percentage increase in the estimated delay-aversion level when using RPM as opposed to RUM.

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